Maths Questions By Topic:

Coordinate geometry in the (x, y) plane

Mark Scheme

A-Level Edexcel

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Question	Scheme	Marks	AOs
1 (a)	(i) $x^2+y^2-10x+16y=80 \Rightarrow (x-5)^2+(y+8)^2=\ldots$	M1	1.1b
	Centre $(5,-8)$	A1	1.1b
	(ii) Radius 13	A1	1.1b
		(3)	
(b)	Attempts $\sqrt{"5"^2+"8"^2}+"13"$	M1	3.1a
	$13+\sqrt{89}$ but ft on their centre and radius	A1ft	1.1b
		(2)	
			(5 marks)
Notes:			

(a)(i)

M1: Attempts to complete the square on **both** x and y terms.

Accept $(x\pm5)^2+(y\pm8)^2=\ldots$ or imply this mark for a centre of $(\pm5,\pm8)$

Condone $(x\pm5)^2\ldots.(y\pm8)^2=\ldots$ where the first ... could be , or even –

A1: Correct centre $(5,-8)$.

Accept without brackets. May be written $x=5, y=-8$

(a)(ii)

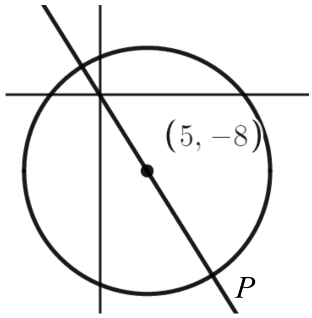
A1: 13. The M mark must have been awarded, so it can be scored following a centre of $(\pm5,\pm8)$.

Do not allow for $\sqrt{169}$ or ±13

(b)

M1: Attempts $\sqrt{"5"^2+"8"^2}+"13"$ for their centre $(5,-8)$ and their radius 13.

Award when this is given as a decimal, e.g. 22.4 for correct centre and radius. Look for $\sqrt{a^2+b^2}+r$ where centre is $(\pm a,\pm b)$ and radius is r



A1ft: $13+\sqrt{89}$ Follow through on their $(5,-8)$ and their 13 leading to an exact answer. ISW for example if they write $13+\sqrt{89}=22.4$

....

There are more complicated attempts which could involve finding P by solving $y="-\frac{8}{5}x"$ and $x^2+y^2-10x+16y=80$ simultaneously and choosing the coordinate with the greatest modulus. The method is only scored when the distance of the largest coordinate from O is attempted. Such methods are unlikely to result in an exact value but can score 1 mark for the method. Condone slips

FYI. Solving $y=-\frac{8}{5}x$ and $x^2+y^2-10x+16y=80 \Rightarrow 89x^2-890x-2000=0 \Rightarrow P=(11.89,-19.02)$

Hence $OP=\sqrt{"11.89"^2+"19.02"^2}\ (=22.43)$ scores M1 A0 but $OP=\sqrt{258+26\sqrt{89}}$ is M1 A1

Question	Scheme	Marks	AOs
2(a)	$(x\pm5)^2+(y\pm4)^2$	M1	1.1b
	(i) Centre is (5, 4)	A1	1.1b
	(ii) Radius is 3	A1	1.1b
		(3)	
(b)	$2y+x+6=0\Rightarrow y=-\frac{1}{2}x+...\Rightarrow-\frac{1}{2}\to2$	B1	2.2a
	$m_N=2\Rightarrow y-4=2(x-5)$ $y-4=2(x-5),\ 2y+x+6=0\Rightarrow x=...,\ y=...$	M1	3.1a
	Intersection is at $\left(\frac{6}{5},\ -\frac{18}{5}\right)$ oe	A1	1.1b
	Distance from centre to intersection is $\sqrt{\left(5-\frac{6}{5}\right)^2+\left(4+\frac{18}{5}\right)^2}$ So distance required is $\sqrt{\left(\text{"}5\text{"}-\text{"}\frac{6}{5}\text{"}\right)^2+\left(\text{"}4\text{"}+\text{"}\frac{18}{5}\text{"}\right)^2}-\text{"}3\text{"}$	dM1	3.1a
	$=\frac{19\sqrt{5}}{5}-3$ (or awrt 5.50)	A1	1.1b
		(5)	
		(8 marks)	

Notes

(a)

M1: Attempts to complete the square for both x and y terms $(x\pm5)^2$ .... $(y\pm4)^2$ which may be implied by a centre of (±5, ±4)

A1: Centre (5, 4)

A1: Radius 3

(b)

B1: Deduces the gradient of the perpendicular to l is 2. May be seen in the equation for the perpendicular line to l

M1: A fully correct strategy for finding the intersection. This requires use of their gradient of the perpendicular which cannot be the gradient of l
Look for $y-\text{"}4\text{"}=\text{"}2\text{"}(x-\text{"}5\text{"})$ where $(5,4)$ is their centre being solved simultaneously with the equation of l
Do not be concerned with the mechanics of their rearrangement when solving simultaneously.
Many are finding the y-intercept of l $(0,-3)$ which is M0

A1: $\left(\frac{6}{5},\ -\frac{18}{5}\right)$ or equivalent eg $(1.2,\ -3.6)$

They do not have to be written as coordinates and may be seen within their working rather than explicitly stated. They may also be written on the diagram.

dM1: Fully correct strategy for finding the required distance e.g. correct use of Pythagoras to find the distance between their centre and their intersection and then completes the problem by subtracting their radius. Condone a sign slip subtracting their $-\frac{18}{5}$.

It is dependent on the previous method mark.

Alternatively, they solve simultaneously their $y = 2x - 6$ with the equation of the circle and then find the distance between this intersection point and the point of intersection between *l* and the normal. They must choose the smaller positive root of the solution to their quadratic.

Eg

$(x-5)^2 + (2x-6-4)^2 = 9 \Rightarrow 5x^2 - 50x + 125 = 9$

$x = \frac{25-3\sqrt{5}}{5},\ y = \frac{20-6\sqrt{5}}{5}$

Distance between two points:

$$\sqrt{\left(\text{"}\frac{25-3\sqrt{5}}{5}\text{"} - \text{"}\frac{6}{5}\text{"}\right)^2 + \left(\text{"}\frac{20-6\sqrt{5}}{5}\text{"} + \text{"}\frac{18}{5}\text{"}\right)^2}$$

A1: Correct value e.g. $\sqrt{\frac{361}{5}} - 3$ or $\frac{19\sqrt{5}-15}{5}$). Also allow awrt 5.50

Isw after a correct answer is seen.

Alt (b) Be aware they may use vector methods:

B1M1: Attempts to find the perpendicular distance between their $(5,4)$ and $x + 2y + 6 = 0$ by substituting the values into the formula to find the distance between a point (x, y) and a line $ax + by + c = 0$

$$\Rightarrow \frac{|ax+by+c|}{\sqrt{a^2+b^2}} = \frac{|\text{"}5\text{"}\times\text{"}1\text{"}+\text{"}4\text{"}\times\text{"}2\text{"}+\text{"}6\text{"}|}{\sqrt{\text{"}1\text{"}^2+\text{"}2\text{"}^2}}$$

A1: $\frac{|5\times1+4\times2+6|}{\sqrt{1^2+2^2}}\ \left(=\frac{19}{\sqrt{5}}\right)$

dM1: Distance $= \text{"}\frac{19\sqrt{5}}{5}\text{"} - 3$

A1: $\frac{19\sqrt{5}-15}{5}$

Question	Scheme	Marks	AOs
3(a)(i)	$(x-5)^2+(y+2)^2=...$	M1	1.1b
	$(5,\,-2)$	A1	1.1b
(ii)	$r=\sqrt{"5"^2+"-2"^2-11}$	M1	1.1b
	$r=3\sqrt{2}$	A1	1.1b
		(4)	
(b)	$y=3x+k\Rightarrow x^2+(3x+k)^2-10x+4(3x+k)+11=0$ $\Rightarrow x^2+9x^2+6kx+k^2-10x+12x+4k+11=0$	M1	2.1
	$\Rightarrow 10x^2+(6k+2)x+k^2+4k+11=0$	A1	1.1b
	$b^2-4ac=0\Rightarrow(6k+2)^2-4\times10\times(k^2+4k+11)=0$	M1	3.1a
	$\Rightarrow 4k^2+136k+436=0\Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a
		(5)	
			(9 marks)
	Notes		

(a)(i)

M1: Attempts to complete the square on by halving both x and y terms.

Award for sight of $(x\pm5)^2,(y\pm2)^2=...$ This mark can be implied by a centre of (±5, ±2).

A1: Correct coordinates. (Allow x = 5, y = −2)

(a)(ii)

M1: Correct strategy for the radius or radius^2 . For example award for $r=\sqrt{"\pm5"^2+"\pm2"^2-11}$

or an attempt such as $(x-a)^2-a^2+(y-b)^2-b^2+11=0\Rightarrow(x-a)^2+(y-b)^2=k\Rightarrow r^2=k$

A1: $r=3\sqrt{2}$. Do not accept for the A1 either $r=\pm3\sqrt{2}$ or $\sqrt{18}$

The A1 can be awarded following sign slips on $(5,\,-2)$ so following $r^2="\pm5"^2+"\pm2"^2-11$

(b) Main method seen

M1: Substitutes y = 3x + k into the given equation (or their factorised version) and makes progress by attempting to expand the brackets. Condone lack of = 0

A1: Correct 3 term quadratic equation.

The terms must be collected but this can be implied by correct a, b and c

M1: Recognises the requirement to use $b^2 - 4ac = 0$ (or equivalent) where both b and c are expressions in k . It is dependent upon having attempted to substitute y = 3x + k into the given equation

M1: Solves 3TQ in k. See General Principles.

The 3TQ in k must have been found as a result of attempt at $b^2 - 4ac$ ... 0

A1: Correct simplified values

<u>Look carefully at the method used. It is possible to attempt this using gradients</u>

(b) Alt 1	$x^2+y^2-10x+4y+11=0\Rightarrow 2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx}=3\Rightarrow x+3y+1=0$ and combines with equation for C $\Rightarrow 5x^2-50x+44=0$ or $5y^2+20y+11=0$ $\Rightarrow x=...$ or $y=...$	M1	3.1a
	$x=\frac{25\pm9\sqrt{5}}{5},\ y=\frac{-10\pm3\sqrt{5}}{5},\ k=y-3x\Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a

M1: Differentiates implicitly condoning slips but must have two $\frac{dy}{dx}$'s coming from correct terms

A1: Correct differentiation.

M1: Sets $\frac{dy}{dx} = 3$, makes y or x the subject, substitutes back into C and attempts to solve the resulting quadratic in x or y.

M1: Uses at least one pair of coordinates and l to find at least one value for k. It is dependent upon having attempted both M's

A1: Correct simplified values

(b) Alt 2	$x^2+y^2-10x+4y+11=0 \Rightarrow 2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx}=3 \Rightarrow x+3y+1=0$ and combines with equation for l $y=3x+k, x+3y=1$ $\Rightarrow x=\ldots$ and $y=\ldots$ in terms of k	M1	3.1a
	$x=\frac{-3k-1}{10},\ y=\frac{k-3}{10},\ x^2+y^2-10x+4y+11=0 \Rightarrow k=\ldots$	M1	1.1b
	$k=-17\pm 6\sqrt{5}$	A1	2.2a

Very similar except it uses equation for l instead of C in mark 3

M1 A1: Correct differentiation (See alt 1)

M1: Sets $\frac{dy}{dx} = 3$, makes y or x the subject, substitutes back into l to obtain x and y in terms of k

M1: Substitutes for x and y into C and solves resulting 3TQ in k

A1: Correct simplified values

(b) Alt 3	$y=3x+k \Rightarrow m=3 \Rightarrow m_r=-\frac{1}{3}$	M1
	$y+2=-\frac{1}{3}(x-5)$	A1
	$(x-5)^2+(y+2)^2=18,\ y+2=-\frac{1}{3}(x-5)$ $\Rightarrow \frac{10}{9}(x-5)^2=18 \Rightarrow x=\ldots$ or $\Rightarrow 10(y+2)^2=18 \Rightarrow y=\ldots$	M1
	$x=\frac{25\pm 9\sqrt{5}}{5},\ y=\frac{-10\pm 3\sqrt{5}}{5},\ k=y-3x \Rightarrow k=\ldots$	M1
	$k=-17\pm 6\sqrt{5}$	A1

M1: Applies negative reciprocal rule to obtain gradient of radius

A1: Correct equation of radial line passing through the centre of C

M1: Solves simultaneously to find x or y

Alternatively solves "$y=-\frac{1}{3}x-\frac{1}{3}$" and $y=3x+k$ to get x in terms of k which they substitute in $x^2+(3x+k)^2-10x+4(3x+k)+11=0$ to form an equation in k.

M1: Applies $k = y - 3x$ with at least one pair of values to find k

A1: Correct simplified values

Question	Scheme	Marks	AOs
4	$(x-3)^2+y^2=\left(\frac{t^2+5}{t^2+1}-3\right)^2+\left(\frac{4t}{t^2+1}\right)^2$	M1	3.1a
	$=\frac{(2-2t^2)^2+16t^2}{(t^2+1)^2}=\frac{4+8t^2+4t^4}{(t^2+1)^2}$	dM1	1.1b
	$\frac{4(t^4+2t^2+1)}{(t^2+1)^2}=\frac{4(t^2+1)^2}{(t^2+1)^2}=4*$	A1*	2.1
		(3)	

M1: Attempts to substitute the given parametric forms into the Cartesian equation or the lhs of the Cartesian equation. There may have been an (incorrect) attempt to multiply out the $(x-3)^2$ term.

dM1: Attempts to combine (at least the lhs) using correct processing into a single fraction, multiplies out and collects terms on the numerator.

A1*: Fully correct proof showing all key steps

Question	Scheme	Marks	AOs
Alt	$x=\frac{t^2+5}{t^2+1}\Rightarrow xt^2+x=t^2+5\Rightarrow t^2=\frac{5-x}{x-1}$ $y=\frac{4t}{t^2+1}\Rightarrow y^2=\frac{16t^2}{(t^2+1)^2}=\frac{16\left(\frac{5-x}{x-1}\right)}{\left(\frac{5-x}{x-1}+1\right)^2}$	M1	3.1a
	$y^2=\frac{16\left(\frac{5-x}{x-1}\right)}{\left(\frac{5-x}{x-1}+1\right)^2}=16\left(\frac{5-x}{x-1}\right)\times\left(\frac{(x-1)}{5-x+x-1}\right)^2\Rightarrow y^2=(5-x)(x-1)$	dM1	1.1b
	$y^2=(5-x)(x-1)\Rightarrow y^2=6x-x^2-5$ $\Rightarrow y^2=4-(x-3)^2$ or other intermediate step $\Rightarrow(x-3)^2+y^2=4*$	A1*	2.1
		(3)	
			(3 marks)
	Notes		

M1: Adopts a correct strategy for eliminating t to obtain an equation in terms of x and y only. See scheme.

Other methods exist which also lead to an appropriate equation. E.g using $t=\frac{y}{x-1}$

dM1: Uses correct processing to eliminate the fractions and start to simplify

A1*: Fully correct proof showing all key steps

Question	Scheme	Marks	AOs
5(a)	Attempts to find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x=2$	M1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=6x \Rightarrow$ gradient of tangent at P is 12	A1	1.1b
		(2)	
(b)	Gradient $PQ = \frac{3(2+h)^2-2-10}{(2+h)-2}$ oe	B1	1.1b
	$=\frac{3(2+h)^2-12}{(2+h)-2}=\frac{12h+3h^2}{h}$	M1	1.1b
	$=12+3h$	A1	2.1
		(3)	
(c)	Explains that as $h \to 0,\ 12+3h \to 12$ and states that the gradient of the chord tends to the gradient of (the tangent to) the curve	B1	2.4
		(1)	
			(6 marks)

Notes

(a)

M1: Attempts to differentiate, allow $3x^2-2 \to ...x$ and substitutes $x=2$ into their answer

A1: cso $\frac{\mathrm{d}y}{\mathrm{d}x}=6x \Rightarrow$ gradient of tangent at P is 12

(b)

B1: Correct expression for the gradient of the chord seen or implied.

M1: Attempts $\frac{\delta y}{\delta x}$, condoning slips, and attempts to simplify the numerator. The denominator must be h

A1: cso $12+3h$

(c)

B1: Explains that as $h \to 0,\ 12+3h \to 12$ and states that the gradient of the chord tends to the gradient of the curve

Question	Scheme	Marks	AOs
6(a)	Deduces the line has gradient "-3" and point $(7,4)$ Eg $y-4=-3(x-7)$	M1	2.2a
	$y=-3x+25$	A1	1.1b
		(2)	
(b)	Solves $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously	M1	3.1a
	$P=\left(\frac{15}{2},\frac{5}{2}\right)$ oe	A1	1.1b
	Length $PN=\sqrt{\left(\frac{15}{2}-7\right)^2+\left(4-\frac{5}{2}\right)^2}=\left(\sqrt{\frac{5}{2}}\right)$	M1	1.1b
	Equation of C is $(x-7)^2+(y-4)^2=\frac{5}{2}$ o.e.	A1	1.1b
		(4)	
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C using vectors Eg: $\begin{pmatrix}7.5\\2.5\end{pmatrix}+2\times\begin{pmatrix}-0.5\\1.5\end{pmatrix}$	M1	3.1a
	Substitutes their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$ to find k	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	
			(9 marks)
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $\frac{10}{9}x^2+\left(\frac{2}{3}k-\frac{50}{3}\right)x+k^2-8k+\frac{125}{2}=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=$...	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	

Notes:
(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PN is -3 with point $(7,4)$ to find the equation of line PN

So sight of $y-4=-3(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=$ ... to score this mark.

A1: Achieves $y=-3x+25$

(b)

M1: Awarded for an attempt at the key step of finding the coordinates of point *P*. ie for an attempt at solving their $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates.

A1: $P=\left(\frac{15}{2},\frac{5}{2}\right)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=\left(\frac{15}{2},\frac{5}{2}\right)$ and $(7,4)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: Full and careful work leading to a correct equation. Eg $(x-7)^2+(y-4)^2=\frac{5}{2}$ or its expanded form. Do not accept $(x-7)^2+(y-4)^2=\left(\sqrt{\frac{5}{2}}\right)^2$

(c)

M1: Attempts to find where $y=\frac{1}{3}x+k$ meets *C* using a vector approach

M1: For a full method leading to k . Scored for substituting their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$

A1: $k=\frac{10}{3}$ only

Alternative I

M1: For solving $y=\frac{1}{3}x+k$ with their $(x-7)^2+(y-4)^2=\frac{5}{2}$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both *b* and *c* are dependent upon *k***. The terms in x^2 and *x* must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $\frac{10}{9}x^2+\left(\frac{2}{3}k-\frac{50}{3}\right)x+k^2-8k+\frac{125}{2}=0$ oe

M1: For using the discriminant condition $b^2-4ac=0$ to find k. It is not dependent upon the previous M and may be awarded from only one term in *k*.
Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=\frac{10}{3}$ only

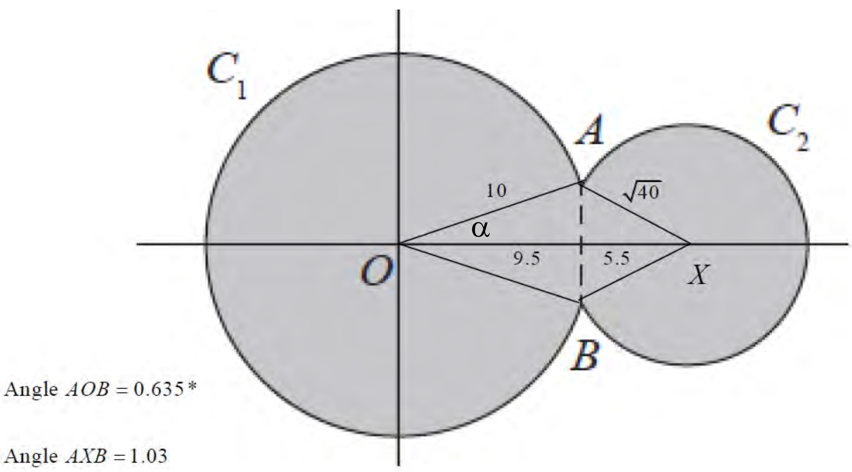
Alternative II

M1: For solving $y=-3x+25$ with their $(x-7)^2+(y-4)^2=\frac{5}{2}$, creating a 3TQ and solving.

M1: For substituting their $\left(\frac{13}{2},\frac{11}{2}\right)$ into $y=\frac{1}{3}x+k$ and finding *k*

A1: $k=\frac{10}{3}$ only

Question	Scheme	Marks	AOs
7 (a)	Solves $x^2+y^2=100$ and $(x-15)^2+y^2=40$ simultaneously to find *x* or *y* E.g. $(x-15)^2+100-x^2=40 \Rightarrow x=...$	M1	3.1a
	Either $\Rightarrow -30x+325=40 \Rightarrow x=9.5$ Or $y=\dfrac{\sqrt{39}}{2}=$ awrt ± 3.12	A1	1.1b
	Attempts to find the angle *AOB* in circle C_1 Eg Attempts $\cos\alpha=\dfrac{"9.5"}{10}$ to find α then $\times 2$	M1	3.1a
	Angle *AOB* = $2\times\arccos\left(\dfrac{9.5}{10}\right)=0.635$ rads (3sf) *	A1*	2.1
		(4)	
(b)	Attempts $10\times(2\pi-0.635)$ = 56.48	M1	1.1b
	Attempts to find angle *AXB* or *AXO* in circle C_2 (see diagram) E.g. $\cos\beta=\dfrac{15-"9.5"}{\sqrt{40}}\Rightarrow\beta=...$ (Note *AXB* =1.03 rads)	M1	3.1a
	Attempts $10\times(2\pi-0.635)+\sqrt{40}\times(2\pi-2\beta)$	dM1	2.1
	= 89.7	A1	1.1b
		(4)	
			(8 marks)
Notes:			



(a)

M1: For the key step in an attempt to find either coordinate for where the two circles meet.
Look for an attempt to set up an equation in a single variable leading to a value for *x* or *y*.

A1: $x=9.5$ (or $y=\dfrac{\sqrt{39}}{2}=$ awrt ± 3.12)

M1: Uses the radius of the circle and correct trigonometry in an attempt to find angle *AOB* in circle C_1

E.g. Attempts $\cos\alpha = \frac{"9.5"}{10}$ to find α then $\times 2$

Alternatives include $\tan\alpha = \frac{\sqrt{100-"9.5"^2}}{"9.5"} = (0.3286...)$ to find α then $\times 2$

And $\cos AOB = \frac{10^2+10^2-\left(\sqrt{39}\right)^2}{2\times 10\times 10} = \frac{161}{200}$

A1*: Correct and careful work in proceeding to the given answer. Condone an answer with greater accuracy. Condone a solution where the intermediate value has been truncated, provided the trig equation is correct.

E.g. $\sin\alpha = \frac{\sqrt{39}}{20} \Rightarrow \alpha = 0.317 \Rightarrow AOB = 2\alpha = 0.635$

Condone a solution written down from awrt $36.4°$ (without the need to shown any calculation.)
E

(b)

M1: Attempts to use the formula $s = r\theta$ with $r = 10$ and $\theta = 2\pi - 0.635$

The formula may be embedded. You may see $\underline{\underline{2\pi 10}}+2\pi\sqrt{40}\underline{\underline{-10\times 0.635}}$... which is fine for this M1

M1: Attempts to use a correct method in order to find angle *AXB* or *AXO* in circle C_2

Amongst many other methods are $\tan\beta = \frac{"3.12"}{15-9.5}$ and $\cos AXB = \frac{40+40-\left(\sqrt{39}\right)^2}{2\times\sqrt{40}\times\sqrt{40}} = \frac{41}{80}$

Note that many candidates believe this to be 0.635. This scores M0 dM0 A0

dM1: A full and complete attempt to find the perimeter of the region.
It is dependent upon having scored both M's.

A1: awrt 89.7

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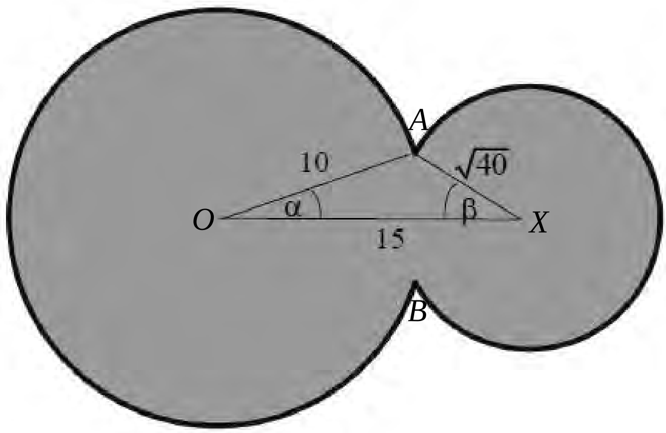

(a)

M1: For the key step in attempting to find all lengths in triangle *OAX*, condoning slips

A1: All three lengths correct

M1: Attempts cosine rule to find α then $\times 2$

A1*: Correct and careful work in proceeding to the given answer

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Question	Scheme	Marks	AOs
8(a)	Attempts $A = mn + c$ with either $(0,190)$ or $(8,169)$ Or attempts gradient eg $m = \pm\frac{190-169}{8} (= -2.625)$	M1	3.3
	Full method to find a linear equation linking A with n E.g. Solves $190 = 0n + c$ and $169 = 8n + c$ simultaneously	dM1	3.1b
	$A = -2.625n + 190$	A1	1.1b
		(3)	
(b)	Attempts $A = -2.625 \times 19 + 190 = ...$	M1	3.4
	$A = 140.125$ g km^{-1}	A1	1.1b
	It is predicting a much higher value and so is not suitable	B1ft	3.5a
		(3)	
			(6 marks)

Notes

(a)

M1: Attempts $A = mn + c$ with either $(0,190)$ or $(8,169)$ considered.
Eg Accept sight of $190 = 0n + c$ or $169 = 8m + c$ or $A - 169 = m(n-8)$ or $A = 190 + mn$ where m could be a value.
Also accept an attempt to find the gradient $\pm\frac{190-169}{8}$ or sight of ± 2.625 or $\pm\frac{21}{8}$ oe

dM1: A full method to find both constants of a linear equation
Method 1: Solves $190 = 0n + c$ and $169 = 8n + c$ simultaneously
Method 2: Uses gradient and a point Eg $m = \pm\frac{190-169}{8} (= -2.625)$ and $c = 190$
Condone different variables for this mark. Eg. y in terms of x.

A1: $A = -2.625n + 190$ or $A = -\frac{21}{8}n + 190$ oe

(b)

M1: Attempts to substitute "n" $= 19$ into their linear model to find A. They may call it $x = 19$
Alternatively substitutes $A = 120$ into their linear model to find n.

A1: $A = 140.125$ from $n = 19$ Allow $A = 140$
or $n = 26 / 27$ following $A = 120$

B1ft: Requires a correct calculation for their model, a correct statement and a conclusion
E.g For correct (a) $A = 140$ is (much) higher than 120 so the model is not suitable/appropriate.
Follow through on a correct statement for their equation. As a guide allow anything within [114,126] to be regarded as suitable. Anything less than 108 or more than 132 should be justified as unsuitable.

Note B0 Recorded value is not the same as/does not equal/does not match the value predicted

Question	Scheme	Marks	AOs
9(i)	$x^2+y^2+18x-2y+30=0 \Rightarrow (x+9)^2+(y-1)^2=...$	M1	1.1b
	Centre $(-9,1)$	A1	1.1b
	Gradient of line from $P(-5,7)$ to "$(-9,1)$" $= \dfrac{7-1}{-5+9}=\left(\dfrac{3}{2}\right)$	M1	1.1b
	Equation of tangent is $y-7=-\dfrac{2}{3}(x+5)$	dM1	3.1a
	$3y-21=-2x-10 \Rightarrow 2x+3y-11=0$	A1	1.1b
		(5)	
(ii)	$x^2+y^2-8x+12y+k=0 \Rightarrow (x-4)^2+(y+6)^2=52-k$	M1	1.1b
	Lies in Quadrant 4 if radius $<4 \Rightarrow$ "$52-k$" $<4^2$	M1	3.1a
	$\Rightarrow k>36$	A1	1.1b
	Deduces $52-k>0 \Rightarrow$ Full solution $36<k<52$	A1	3.2a
		(4)	
			(9 marks)

Notes

(i)

M1: Attempts $(x\pm9)^2....(y\pm1)^2=...$ It is implied by a centre of $(\pm9,\pm1)$

A1: States or uses the centre of C is $(-9,1)$

M1: A correct attempt to find the gradient of the radius using their $(-9,1)$ and P. E.g. $\dfrac{7-\text{"1"}}{-5-\text{"}-9\text{"}}$

dM1: For the complete strategy of using perpendicular gradients and finding the equation of the tangent to the circle. It is dependent upon both previous M's. $y-7=-\dfrac{1}{\text{gradient } CP}(x+5)$

Condone a sign slip on one of the -7 or the 5.

A1: $2x+3y-11=0$ oe such as $k(2x+3y-11)=0, k\in Z$

....

Attempt via implicit differentiation. The first three marks are awarded

M1: Differentiates $x^2+y^2+18x-2y+30=0 \Rightarrow ...x+...y\dfrac{dy}{dx}+18-2\dfrac{dy}{dx}....=0$

A1: Differentiates $x^2+y^2+18x-2y+30=0 \Rightarrow 2x+2y\dfrac{dy}{dx}+18-2\dfrac{dy}{dx}=0$

M1: Substitutes $P(-5,7)$ into their equation involving $\dfrac{dy}{dx}$

....

(ii)

M1: For reaching $(x \pm 4)^2 + (y \pm 6)^2 = P - k$ where P is a positive constant. Seen or implied by centre coordinates $(\mp 4, \mp 6)$ and a radius of $\sqrt{P-k}$

M1: Applying the strategy that it lies entirely within quadrant if "their radius" < 4 and proceeding to obtain an inequality in k only (See scheme). Condone $\ldots \leqslant 4$ for this mark.

A1: Deduces that $k > 36$

A1: A rigorous argument leading to a full solution. In the context of the question the circle exists so that as well as $k > 36$ $52 - k > 0 \Rightarrow 36 < k < 52$ Allow $36 < k \leqslant 52$

Question	Scheme	Marks	AOs
10(a)	$2x+4y-3=0 \Rightarrow y=\mp\frac{2}{4}x+...$ Gradient of perpendicular = $\pm\frac{4}{2}$	M1	1.1b
	Either $m=2$ or $y=2x+7$	A1	1.1b
		(2)	
(b)	Combines 'their' $y=2x+7$ with $2x+4y-3=0 \Rightarrow 2x+4(2x+7)-3=0 \Rightarrow x=...$	M1	1.1b
	$x=-2.5$ oe	A1	1.1b
		(2)	
			(4 marks)

Notes

(a)

M1: Attempts to set given equation in the form $y=ax+b$ with $a=\mp\frac{2}{4}$ oe such as $\mp\frac{1}{2}$ **AND** deduces that $m=-\frac{1}{a}$ Condone errors on the "$+b$"

An alternative method is to find both intercepts to get gradient $l_1=\pm\frac{0.75}{1.5}$ and use the perpendicular gradient rule.

A1: Correct answer. Accept **either** $m=2$ **or** $y=2x+7$

This must be simplified and not left as $m=\frac{4}{2}$ or $m=2x$ unless you see $y=2x+7$.

Watch: There may be candidates who look at $2x+4y-3=0$ and incorrectly state that the gradient is 2 and use the perpendicular rule to get $m=-\frac{1}{2}$ They will score M0 A0 in (a) and also no marks in (b) as the lines would be parallel. In a case like this don't allow an equation to be "altered"
Candidates who state $m=2$ or $y=2x+7$ **with no incorrect working** can score both marks

(b)

M1: Substitutes their $y=mx+7$ into $2x+4y-3=0$, condoning slips, in an attempt to form and solve an equation in *x*. Alternatively equates their $y=-\frac{1}{2}x+\frac{3}{4}$ with their $y=mx+7$ in an attempt to form and solve, condoning slips, an equation in *x*. Don't be too concerned by the mechanics of the candidates attempt to solve. (E.g. allow solutions from their calculators).
You may see $2x+4y-3=2x-y+7$ with y being found before the value of *x* appears
It cannot be awarded from "unsolvable" equations (e.g. lines that are parallel).

A1: $x=-2.5$

The answer alone can score both marks as long as both equations are correct and no incorrect working is seen.
Remember to isw after the correct answer and ignore any *y* coordinate

Question	Scheme	Marks	AOs
11(a)	Attempts $H = mt + c$ with both $(3, 2.35)$ and $(6, 3.28)$	M1	3.3
	Method to find both *m* and *c*	dM1	3.1a
	$H = 0.31t + 1.42$ oe	A1	1.1b
		(3)	
(b)	Uses the model and states that the initial height is their '*b*'	B1ft	3.4
	Compares 140 cm with their 1.42 (m) and makes a valid comment. In the case where $H = 0.31t + 1.42$ it should be this fact supports the use of the linear model as the values are close.	B1ft	3.5a
		(2)	
			(5 marks)

Notes

Mark parts (a) and (b) as one

(a)

M1: For creating a linear model with both pieces of information given.
Eg. Accept sight of $2.35 = 3m + c$ and $3.28 = 6m + c$ Condone slips on the 2.35 and 3.28.
Allow for an attempt at the "gradient" $m = \frac{3.28 - 2.35}{6 - 3} (= 0.31)$ or the intercept.
Allow for a pair of simultaneous in any variable even *x* and *y*

dM1: A full method to find both constants. For simultaneous equations award if they arrive at values for *m* and *c*.
If they attempted the gradient it would be for attempting to find "*c*" using $y = mx + c$ with their *m* and one of the points $(3, 2.35)$ or $(6, 3.28)$

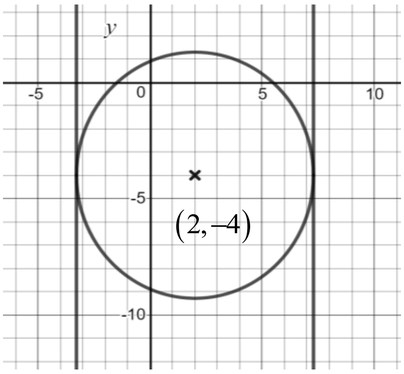
A1: A correct model using allowable/correct variables. $H = 0.31t + 1.42$ Condone $h \leftrightarrow H, t \leftrightarrow T$
Allow equivalents such as $H = \frac{31}{100}t + \frac{142}{100}$, $t = \frac{H - 1.42}{0.31}$ but not $H = \frac{0.93}{3}t + 1.42$
Do not allow $H = 0.31t + 1.42$ m (with the units)

(b) To score any marks in (b) the model must be of the form $H = mt + b$ where $m > 0, b > 0$

B1ft: States or implies that 1.42 (with or without units) or 142 cm (including the units) is the original height or the height when $t = 0$
You should allow statements such as *c* = 1.42 or original height = 1.42 (m)
Follow through on their value of '*c*', so for $H = 0.25t + 1.60$ it is scored for stating the initial height is 1.60 (m) or 160 cm. Do not follow through if $c \leqslant 0$

B1ft: Compares 140 cm with their 1.42 (m) **and** makes a valid comment.
In the case where $H = 0.31t + 1.42$ it should be this fact supports the use of the linear model as the values are close or approximately the same. Allow $1.42\text{m} \approx 1.4\text{m}$ or similar
In the case of $H = 0.25t + 1.60$ it would be for stating that the fact that it does not support the use of the model as the values are too different. If they state $1.60 > 1.40$ this is insufficient. They cannot just state that they are not the same. It must be implied that there is a significant difference.
As a rule of thumb use "good model" for between 135cm and 145 cm.

This requires a correct calculation for their H, a correct statement with an appreciation shown for the units and a correct conclusion.
Notes on Question 11 continue
SC B1 B0 Award SC for incomplete answers which suggest the candidate knows what to do. Eg. In (b) $H = 0.31t + 1.42$ followed by in (c) It supports the model as when $t = 0$ it is approximately 140 cm

Question	Scheme	Marks	AOs
12(a)	$x^2+y^2-4x+8y-8=0$		
	Attempts $(x-2)^2+(y+4)^2-4-16-8=0$	M1	1.1b
	(i) Centre $(2,-4)$	A1	1.1b
	(ii) Radius $\sqrt{28}$ oe Eg $2\sqrt{7}$	A1	1.1b
		(3)	
(b)	 Attempts to add/subtract 'r' from '2'	M1	3.1a
	$k=2\pm\sqrt{28}$	A1ft	1.1b
		(2)	
		(5 marks)	

Notes

(a)

M1: Attempts to complete the square. Look for $(x\pm2)^2+(y\pm4)^2$ ...

If a candidate attempts to use $x^2+y^2+2gx+2fy+c=0$ then it may be awarded for a centre of $(\pm2,\pm4)$ Condone $a=\pm2, b=\pm4$

A1: Centre $(2,-4)$ This may be written separately as $x=2, y=-4$ BUT $a=2, b=-4$ is A0

A1: Radius $\sqrt{28}$ or $2\sqrt{7}$ isw after a correct answer

(b)

M1: Attempts to add **or** subtract their radius from their 2.

Alternatively substitutes $y=-4$ into circle equation and finds x/k by solving the quadratic equation by a suitable method.

A third (and more difficult) method would be to substitute $x=k$ into the equation to form a quadratic eqn in $y \Rightarrow y^2+8y+k^2-4k-8=0$ and use the fact that this would have one root.

E.g. $b^2-4ac=0 \Rightarrow 64-4(k^2-4k-8)=0 \Rightarrow k=..$ Condone slips but the method must be sound.

A1ft: $k=2+\sqrt{28}$ and $k=2-\sqrt{28}$ Follow through on their 2 and their $\sqrt{28}$

If decimals are used the values must be calculated. Eg $k=2\pm5.29 \to k=7.29, k=-3.29$

Accept just $2\pm\sqrt{28}$ or equivalent such as $2\pm2\sqrt{7}$

Condone $x=2+\sqrt{28}$ and $x=2-\sqrt{28}$ but not $y=2+\sqrt{28}$ and $y=2-\sqrt{28}$

Question	Scheme	Marks	AOs
13(a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient "$-\frac{1}{2}$" and point $(7,5)$ $y-5=-\frac{1}{2}(x-7)$	M1	1.1b
	Completes proof $2y+x=17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y+x=17$ and $y=2x+1$ simultaneously	M1	2.1
	$P=(3,7)$	A1	1.1b
	Length $PA=\sqrt{(3-7)^2+(7-5)^2}=\left(\sqrt{20}\right)$	M1	1.1b
	Equation of C is $(x-7)^2+(y-5)^2=20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y=2x+k$ meets C using $\overrightarrow{OA}+\overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11,3)$ in $y=2x+k$ to find k	M1	2.1
	$k=-19$	A1	1.1b
		(3)	
			(10 marks)
(c)	Attempts to find where $y=2x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $5x^2+(4k-34)x+k^2-10k+54=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=$...	M1	2.1
	$k=-19$	A1	1.1b
		(3)	

Notes:
(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PA is $-\frac{1}{2}$. Condone $-\frac{1}{2}x$ if followed by correct work. You may well see the perpendicular line set up as $y=-\frac{1}{2}x+c$ which scored this mark

M1: Award for the method of finding the equation of a line with a changed gradient and the point $(7,5)$

So sight of $y-5=\frac{1}{2}(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=$ ... to score this mark.

A1*: Completes proof with no errors or omissions $2y+x=17$

(b)
M1: Awarded for an attempt at the key step of finding the coordinates of point *P*. ie for an attempt at solving $2y+x=17$ and $y=2x+1$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates. Do not allow where they start $17-x=2x+1$ as they have set $2y=y$ but condone bracketing errors, eg $2\times 2x+1+x=17$

A1: $P=(3,7)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=(3,7)$ and $(7,5)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: $(x-7)^2+(y-5)^2=20$. Do not accept $(x-7)^2+(y-5)^2=\left(\sqrt{20}\right)^2$

(c)
M1: Attempts to find where $y=2x+k$ meets *C*.

Awarded for using $\overrightarrow{OA}+\overrightarrow{PA}$. $(11,3)$ or one correct coordinate of $(11,3)$ is evidence of this award.

M1: For a full method leading to k . Scored for either substituting their $(11,3)$ in $y=2x+k$

or**, in the alternative**, for solving their $(4k-34)^2-4\times5\times\left(k^2-10k+54\right)=0 \Rightarrow k=...$ Allow use of a calculator here to find roots. Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

....

Alternative I

M1: For solving $y=2x+k$ with their $(x-7)^2+(y-5)^2=20$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both *b* and *c* are dependent upon *k***. The terms in x^2 and *x* must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $5x^2+(4k-34)x+k^2-10k+54=0$

M1: For using the discriminant condition $b^2-4ac=0$ to find k. It is not dependent upon the previous M and may be awarded from only one term in *k*.

$(4k-34)^2-4\times5\times\left(k^2-10k+54\right)=0 \Rightarrow k=...$ Allow use of a calculator here to find roots.

Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

....

Alternative II

M1: For solving $2y+x=17$ with their $(x-7)^2+(y-5)^2=20$, creating a 3TQ and solving.

M1: For substituting their $(11,3)$ into $y=2x+k$ and finding *k*

A1: $k=-19$ only

....

Other method are possible using trigonometry.

Question	Scheme	Marks	AOs
14(a)	Either $3y^2 \to Ay\dfrac{dy}{dx}$ or $2xy \to 2x\dfrac{dy}{dx}+2y$	M1	2.1
	$2x-2x\dfrac{dy}{dx}-2y+6y\dfrac{dy}{dx}=0$	A1	1.1b
	$(6y-2x)\dfrac{dy}{dx}=2y-2x$	M1	2.1
	$\dfrac{dy}{dx}=\dfrac{2y-2x}{6y-2x}=\dfrac{y-x}{3y-x}$ *	A1*	1.1b
		(4)	
(b)	$\left(\text{At } P \text{ and } Q\ \dfrac{dy}{dx}\to\infty\Rightarrow\right)$ Deduces that $3y-x=0$	M1	2.2a
	Solves $y=\dfrac{1}{3}x$ and $x^2-2xy+3y^2=50$ simultaneously	M1	3.1a
	$\Rightarrow x=(\pm)5\sqrt{3}$ OR $\Rightarrow y=(\pm)\dfrac{5}{3}\sqrt{3}$	A1	1.1b
	Using $y=\dfrac{1}{3}x \Rightarrow x=..$ AND $y=..$	dM1	1.1b
	$P=\left(-5\sqrt{3},-\dfrac{5}{3}\sqrt{3}\right)$	A1	2.2a
		(5)	
(c)	Explains that you need to solve $y=x$ and $x^2-2xy+3y^2=50$ simultaneously and choose the positive solution	B1ft	2.4
		(1)	
			(10 marks)

Notes:

(a)

M1: For selecting the appropriate method of differentiating either $3y^2 \to Ay\dfrac{dy}{dx}$ or $2xy \to 2x\dfrac{dy}{dx}+2y$

It may be quite difficult awarding it for the product rule but condone $-2xy \to -2x\dfrac{dy}{dx}+2y$ unless you see evidence that they have used the incorrect law vu'-uv'

A1: Fully correct derivative $2x-2x\dfrac{dy}{dx}-2y+6y\dfrac{dy}{dx}=0$

Allow attempts where candidates write $2x\,dx-2x\,dy-2y\,dx+6y\,dy=0$

but watch for students who write $\dfrac{dy}{dx}=2x-2x\dfrac{dy}{dx}-2y+6y\dfrac{dy}{dx}$ This, on its own, is A0 unless you are convinced that this is just their notation. Eg $\dfrac{dy}{dx}=2x-2x\dfrac{dy}{dx}-2y+6y\dfrac{dy}{dx}=0$

M1: For a valid attempt at making $\frac{dy}{dx}$ the subject. with two terms in $\frac{dy}{dx}$ coming from $3y^2$ and $2xy$

Look for $(\ldots\pm\ldots)\frac{dy}{dx}=\ldots\ldots$ It is implied by $\frac{dy}{dx}=\frac{2y-2x}{6y-2x}$

This cannot be scored from attempts such as $\frac{dy}{dx}=2x-2x\frac{dy}{dx}-2y+6y$ which only has one correct term.

A1*: $\frac{dy}{dx}=\frac{y-x}{3y-x}$ with no errors or omissions.

The previous line $\frac{dy}{dx}=\frac{2y-2x}{6y-2x}$ or equivalent must be seen.

(b)

M1: Deduces that $3y-x=0$ oe

M1: Attempts to find either the x or y coordinates of P and Q by solving their $y=\frac{1}{3}x$ with $x^2-2xy+3y^2=50$ simultaneously. Allow for finding a quadratic equation in x or y and solving to find at least one value for x or y.

This may be awarded when candidates make the numerator = 0 ie using $y=x$

A1: $\Rightarrow x=(\pm)5\sqrt{3}$ OR $\Rightarrow y=(\pm)\frac{5}{3}\sqrt{3}$

dM1: Dependent upon the previous M, it is for finding the y coordinate from their x (or vice versa)

This may also be scored following the numerator being set to 0 ie using $y=x$

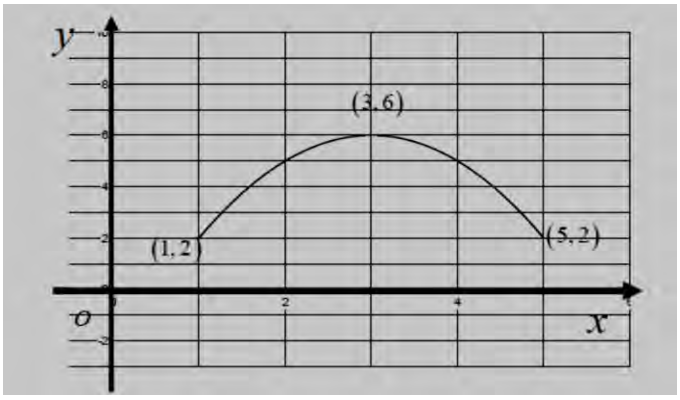
A1: Deduces that $P=\left(-5\sqrt{3},-\frac{5}{3}\sqrt{3}\right)$ OE. Allow to be $x=\ldots$ $y=\ldots$

(c)

B1ft: Explains that this is where $\frac{dy}{dx}=0$ and so you need to solve $y=x$ and $x^2-2xy+3y^2=50$ simultaneously and choose the positive solution (or larger solution).

Allow a follow through for candidates who mix up parts (b) and (c)

Alternatively candidates could complete the square $(x-y)^2+2y^2=50$ and state that y would reach a maximum value when $x=y$ and choose the positive solution from $2y^2=50$

Question	Scheme	Marks	AOs
15(a)	Attempts to use $\cos 2t = 1-2\sin^2 t \Rightarrow \dfrac{y-4}{2} = 1-2\left(\dfrac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y-4 = 2-4\times\dfrac{(x-3)^2}{4} \Rightarrow y = 6-(x-3)^2$ *	A1*	1.1b
		(2)	
(b)	 $\cap$ shaped parabola	M1	1.1b
	Fully correct with 'ends' at (1,2) & (5,2)	A1	1.1b
	Suitable reason : Eg states as $x = 3+2\sin t$, $1 \leqslant x \leqslant 5$	B1	2.4
		(3)	
(c)	Either finds the lower value for $k = 7$ or deduces that $k < \dfrac{37}{4}$	B1	2.2a
	Finds where $x+y=k$ meets $y = 6-(x-3)^2$ $\Rightarrow k-x = 6-(x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $\quad x^2-7x+(k+3)=0$ Or y $\quad y^2+(7-2k)y+(k^2-6k+3)=0$	A1	1.1b
	Uses $b^2-4ac=0 \Rightarrow \quad 49-4\times1\times(k+3)=0 \Rightarrow k=\left(\dfrac{37}{4}\right)$ or $(7-2k)^2-4\times1\times(k^2-6k+3)=0 \Rightarrow k=\left(\dfrac{37}{4}\right)$	M1	2.1
	Range of values for $k = \left\{k : 7 \leqslant k < \dfrac{37}{4}\right\}$	A1	2.5
		(5)	
			(10 marks)

(a)

M1: Uses $\cos 2t = 1-2\sin^2 t$ in an attempt to eliminate t

A1*: Proceeds to $y=6-(x-3)^2$ without any errors

Allow a proof where they start with $y=6-(x-3)^2$ and substitute the parametric coordinates. M1 is scored for a correct $\cos 2t = 1-2\sin^2 t$ but A1 is only scored when both sides are seen to be the same AND a comment is made, hence proven, or similar .

(b)

M1: For sketching a $\cap$ parabola with a maximum in quadrant one. It does not need to be symmetrical

A1: For sketching a $\cap$ parabola with a maximum in quadrant one and with end coordinates of $(1,2)$ and $(5,2)$

B1: Any suitable explanation as to why *C* does not include all points of $y=6-(x-3)^2, \quad x\in\mathbb{R}$

This should include a reference to **the limits on sin or cos** with a **link to a restriction on *x* or *y***.

For example

'As $-1\le \sin t\le 1$ then $1\leqslant x\leqslant 5$' Condone in words '*x* lies between 1 and 5' and strict inequalities

'As $\sin t\le 1$ then $x\leqslant 5$' Condone in words '*x* is less than 5'

'As $-1\le \cos(2t)\le 1$ then $2\leqslant y\leqslant 6$ ' Condone in words '*y* lies between 2 and 6'

Withhold if the statement is incorrect Eg "because the domain is $2\leqslant x\leqslant 5$"

Do not allow a statement on the top limit of *y* as this is the same for both curves

(c)

B1: Deduces either

- the correct that the lower value of $k=7$ This can be found by substituting into $(5,2)$

  $x+y=k \Rightarrow k=7$ or substituting $x=5$ into $x^2-7x+(k+3)=0 \Rightarrow 25-35+k+3=0$
  $\Rightarrow k=7$

- or deduces that $k<\dfrac{37}{4}$ This may be awarded from later work

M1: For an attempt at the upper value for *k*.

Finds where $x+y=k$ meets $y=6-(x-3)^2$ once by using an appropriate method.

Eg. Sets $k-x=6-(x-3)^2$ and proceeds to a 3TQ

A1: Correct 3TQ $x^2-7x+(k+3)=0$ The = 0 may be implied by subsequent work

M1: Uses the "discriminant" condition. Accept use of $b^2=4ac$ oe or $b^2...4ac$ where ... is any inequality leading to a critical value for *k*. Eg. one root $\Rightarrow 49-4\times1\times(k+3)=0 \Rightarrow k=\dfrac{37}{4}$

A1: Range of values for $k=\left\{k:7\leqslant k<\dfrac{37}{4}\right\}$ Accept $k\in\left[7,\dfrac{37}{4}\right)$ or exact equivalent

ALT	As above	B1	2.2a
	Finds where $x+y=k$ meets $y=6-(x-3)^2$ once by using an appropriate method. Eg. Sets gradient of $y=6-(x-3)^2$ equal to -1	M1	3.1a
	$-2x+6=-1 \Rightarrow x=3.5$	A1	1.1b
	Finds point of intersection and uses this to find upper value of *k*. $y=6-(3.5-3)^2=5.75$ Hence using $k=3.5+5.75=9.25$	M1	2.1
	Range of values for $k=\{k:7\leqslant k<9.25\}$	A1	2.5

Question	Scheme	Marks	AOs
16	States gradient of $4y-3x=10$ is $\frac{3}{4}$ oe or rewrites as $y=\frac{3}{4}x+...$	B1	1.1b
	Attempts to find gradient of line joining $(5,-1)$ and $(-1,8)$	M1	1.1b
	$=\frac{-1-8}{5-(-1)}=-\frac{3}{2}$	A1	1.1b
	States neither with suitable reasons	A1	2.4
		(4)	
			(4 marks)

Notes

B1: States that the gradient of line l_1 is $\frac{3}{4}$ or writes l_1 in the form $y=\frac{3}{4}x+...$

M1: Attempts to find the gradient of line l_2 using $\frac{\Delta y}{\Delta x}$ Condone one sign error Eg allow $\frac{9}{6}$

A1: For the gradient of l_2 $=\frac{-1-8}{5-(-1)}=-\frac{3}{2}$ or the equation of l_2 $y=-\frac{3}{2}x+...$

Allow for any equivalent such as $-\frac{9}{6}$ or -1.5

A1: CSO (on gradients)

Explains that they are neither parallel as the gradients not equal nor perpendicular as $\frac{3}{4}\times-\frac{3}{2}\neq-1$ oe

Allow a statement in words "they are not negative reciprocals " for a reason for not perpendicular and "they are not equal" for a reason for not being parallel

Question	Scheme	Marks	AOs
17 (a)	Attempts to complete the square $(x\pm3)^2+(y\pm5)^2=...$	M1	1.1b
	(i) Centre $(3,-5)$	A1	1.1b
	(ii) Radius 5	A1	1.1b
		(3)	
(b)	Uses a sketch or otherwise to deduce $k=0$ is a critical value	B1	2.2a
	Substitute $y=kx$ in $x^2+y^2-6x+10y+9=0$	M1	3.1a
	Collects terms to form correct 3TQ $(1+k^2)x^2+(10k-6)x+9=0$	A1	1.1b
	Attempts $b^2-4ac...0$ for their *a*, *b* and *c* leading to values for *k* $"(10k-6)^2-36(1+k^2)...0"\rightarrow k=...,...$ $\left(0 \text{ and } \frac{15}{8}\right)$	M1	1.1b
	Uses $b^2-4ac>0$ and chooses the outside region (see note) for **their** critical values (Both *a* and *b* must have been expressions in *k*)	dM1	3.1a
	Deduces $k<0, k>\frac{15}{8}$ oe	A1	2.2a
		(6)	
			(9 marks)

Notes

(a)

M1: Attempts $(x\pm3)^2+(y\pm5)^2=..$

This mark may be implied by candidates writing down a centre of $(\pm3,\pm5)$ or $r^2=25$

(i) A1: Centre $(3,-5)$

(ii) A1: Radius 5. Do not accept $\sqrt{25}$

Answers only (no working) scores all three marks

(b)

B1: Uses a sketch or their subsequent quadratic to deduce that $k=0$ is a critical value.
You may award for the correct $k<0$ but award if $k\leqslant 0$ or even with greater than symbols

M1: Substitutes $y=kx$ in $x^2+y^2-6x+10y+9=0$ or their to form an equation in just *x* and *k* . It is possible to substitute $x=\frac{y}{k}$ into their circle equation to form an equation in just *y* and *k*.

A1: Correct 3TQ $(1+k^2)x^2+(10k-6)x+9=0$ with the terms in *x* collected. The "= 0" can be implied by subsequent work. This may be awarded from an equation such as
$x^2+k^2x^2+(10k-6)x+9=0$ so long as the correct values of *a*, *b* and *c* are used in $b^2-4ac...0$.

FYI The equation in *y* and *k* is $(1+k^2)y^2+(10k^2-6k)y+9k^2=0$ oe

M1: Attempts to find two critical values for *k* using $b^2-4ac...0$ or $b^2...4ac$ where ... could be "=" or any inequality.

dM1: Finds the outside region using their critical values. Allow the boundary to be included. It is dependent upon all previous M marks and both *a* and *b* must have been expressions in *k*.
Note that it is possible that the correct region could be the inside region if the coefficient of k^2 in $4ac$ is larger than the coefficient of k^2 in b^2 Eg.

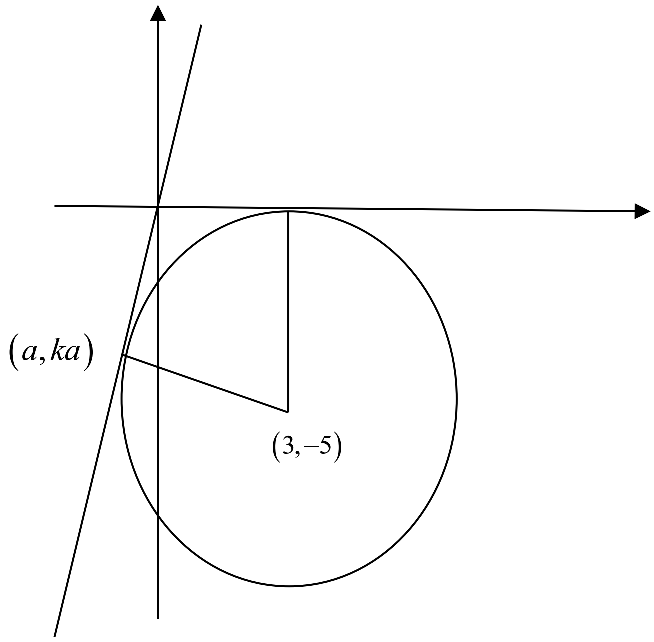
$b^2-4ac=(k-6)^2-4\times(1+k^2)\times9>0\Rightarrow-35k^2-12k>0\Rightarrow k(35k+12)<0$

A1: Deduces $k<0, k>\frac{15}{8}$. This must be in terms of *k*.

Allow exact equivalents such as $k<0 \cup k>1.875$

but not allow $0>k>\frac{15}{8}$ or the above with AND, & or $\cap$ between the two inequalities

Alternative using a geometric approach with a triangle with vertices at $(0,0)$, and $(3,-5)$



Alt (b)	Uses a sketch or otherwise to deduce $k=0$ is a critical value	B1	2.2a
	Distance from (a,ka) to $(0,0)$ is $3 \Rightarrow a^2(1+k^2)=9$	M1	3.1a
	Tangent and radius are perpendicular $\Rightarrow k \times \frac{ka+5}{a-3} = -1 \Rightarrow a(1+k^2)=3-5k$	M1	3.1a
	Solve simultaneously, (dependent upon both M's)	dM1	1.1b
	$k=\frac{15}{8}$	A1	1.1b
	Deduces $k<0, k>\frac{15}{8}$	A1	2.2a
		(6)	

Question	Scheme	Marks	AOs
18(a)	Uses or implies that $V = at + b$	B1	3.3
	Uses both $4 = 24a + b$ and $2.8 = 60a + b$ to get either a or b	M1	3.1b
	Uses both $4 = 24a + b$ and $2.8 = 60a + b$ to get both a and b	M1	1.1b
	$\Rightarrow V = -\frac{1}{30}t + 4.8$	A1	1.1b
		(4)	
(b)	(i) States that the initial volume is 4.8 m^3	B1 ft	3.4
	(ii) Attempts to solve $0 = -\frac{1}{30}t + 4.8$	M1	3.4
	States 144 minutes	A1	1.1b
		(3)	
(c)	States any logical reason • The tank will leak more quickly at the start due to the greater water pressure • The hole will probably get larger and so will start to leak more quickly • Sediment could cause the leak to be plugged and so the tank would not empty.	B1	3.5b
		(1)	
		(8 marks)	

Notes:

(a)
B1: Uses or implies that $V = at + b$
You may award this at their final line but it must be $V = f(t)$
M1: Awarded for translating the problem in context and starting to solve. It is scored when both $4 = 24a + b$ and $2.8 = 60a + b$ are written down and the candidate proceeds to find either a or b. You may just see a line $\pm\frac{4-2.8}{60-24}$
M1: Uses $4 = 24a + b$ and $2.8 = 60a + b$ to find both a and b
A1: $V = -\frac{1}{30}t + 4.8$ or exact equivalent. Eg $30V + t = 144$

(b)(i)
B1ft: Follow through on their 'b'
(b)(ii)
M1: States that $V = 0$ and finds t by attempting to solve their $0 = -\frac{1}{30}t + 4.8$
A1: States 144 minutes

(c)
B1: States any logical reason. There must be a statement and a reason that matches See scheme

Question	Scheme	Marks	AOs
19(a)	Attempts to find the radius $\sqrt{(2--2)^2+(5-3)^2}$ or radius ^{2}	M1	1.1b
	Attempts $(x-2)^2+(y-5)^2='r'^2$	M1	1.1b
	Correct equation $(x-2)^2+(y-5)^2=20$	A1	1.1b
		(3)	
(b)	Gradient of radius *OP* where *O* is the centre of *C* $=\frac{5-3}{2--2}=\left(\frac{1}{2}\right)$	M1	1.1b
	Equation of *l* is $-2=\frac{y-3}{x+2}$	dM1	3.1a
	Any correct form $y=-2x-1$	A1	1.1b
	Method of finding *k* Substitute $x=2$ into their $y=-2x-1$	M1	2.1
	$k=-5$	A1	1.1b
		(5)	
			(8 marks)

Notes:

(a)

M1: As scheme or states form of circle is $(x-2)^2+(y-5)^2='r'^2$

M1: As scheme or substitutes $(-2,3)$ into $(x-2)^2+(y-5)^2='r'^2$

A1: For a correct equation

If students use $x^2+y^2+2fx+2gy+c=0$ **M1:** $f=2, g=5$ **M1:** substitutes $(2,5)$ to find value of c

A1: $x^2+y^2-4x-10y+9=0$

(b)

M1: Attempts to find the gradient of *OP* where *O* is the centre of *C*

dM1: For a complete strategy of finding the equation of *l* using the perpendicular gradient to *OP* and the point $(-2,3)$..

A1: Any correct form of *l* Eg $y=-2x-1$

M1: Scored for the key step of finding *k*. In this method they are required to substitute $(2,k)$ in their $y=-2x-1$ and solve for *k*.

A1: $k=-5$

Alt using Pythagoras' theorem

M1: Attempts Pythagoras to find both *PQ* and *OQ* in terms of *k* (where *O* is centre of *C*)

dM1: For the complete strategy of using Pythagoras theorem on triangle *POQ* to form an equation in *k*

A1: A correct equation in *k* Eg. $20+(k-3)^2+16=(k-5)^2$

M1: Scored for a correct attempt to solve their quadratic to find *k*.

A1: $k=-5$

Question	Scheme	Marks	AOs
20	$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t$		
	$x + y = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t$	M1	3.1a
		M1	1.1b
	$x + y = 2\sqrt{3}\cos t$	A1	1.1b
	$\left(\frac{x+y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$	M1	3.1a
	$\frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$		
	$(x+y)^2 + 3y^2 = 12$	A1	2.1
		(5)	
20 Alt 1	$(x+y)^2 = \left(4\cos\left(t + \frac{\pi}{6}\right) + 2\sin t\right)^2$		
	$= \left(4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t\right)^2$	M1	3.1a
		M1	1.1b
	$= \left(2\sqrt{3}\cos t\right)^2$ or $12\cos^2 t$	A1	1.1b
	So, $(x+y)^2 = 12(1 - \sin^2 t) = 12 - 12\sin^2 t = 12 - 12\left(\frac{y}{2}\right)^2$	M1	3.1a
	$(x+y)^2 + 3y^2 = 12$	A1	2.1
		(5)	
		(5 marks)	

Question **20** Notes:

M1:	Looks ahead to the final result and uses the compound angle formula in a full attempt to write down an expression for $x + y$ which is in terms of *t* only.
M1:	Applies the compound angle formula on their term in *x*. E.g. $\cos\left(t + \frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$
A1:	Uses correct algebra to find $x + y = 2\sqrt{3}\cos t$
M1:	Complete strategy of applying $\cos^2 t + \sin^2 t = 1$ on a rearranged $x + y = "2\sqrt{3}\cos t"$, $y = 2\sin t$ to achieve an equation in *x* and *y* only
A1:	Correctly proves $(x+y)^2 + ay^2 = b$ with both $a = 3, b = 12$, and no errors seen

Question **20** Notes Continued:	
Alt 1	
M1:	Apply in the same way as in the main scheme
M1:	Apply in the same way as in the main scheme
A1:	Uses correct algebra to find $\left(x+y\right)^2 = \left(2\sqrt{3}\cos t\right)^2$ or $\left(x+y\right)^2 = 12\cos^2 t$
M1:	Complete strategy of applying $\cos^2 t+\sin^2 t=1$ on $\left(x+y\right)^2 = \left("2\sqrt{3}\cos t"\right)^2$ to achieve an equation in *x* and *y* only
A1:	Correctly proves $(x+y)^2+ay^2=b$ with both $a=3,\ b=12$, and no errors seen

Question	Scheme	Marks	AOs
21 **Way 1**	Uses $y = mx + c$ with both (3, 1) and (4, −2) and attempt to find m or c	M1	1.1b
	$m = -3$	A1	1.1b
	$c = 10$ so $y = -3x + 10$ o.e.	A1	1.1b
		(3)	
Or **Way 2**	Uses $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$ with both (3, 1) and (4, −2)	M1	1.1b
	Gradient simplified to -3 (may be implied)	A1	1.1b
	$y = -3x + 10$ o.e.	A1	1.1b
		(3)	
Or **Way 3**	Uses $ax + by + k = 0$ and substitutes both $x = 3$ when $y = 1$ and $x = 4$ when $y = -2$ with attempt to solve to find a, b or k in terms of one of them	M1	1.1b
	Obtains $a = 3b$, $k = -10b$ or $3k = -10a$	A1	1.1b
	Obtains $a = 3$, $b = 1$, $k = -10$ Or writes $3x + y - 10 = 0$ o.e.	A1	1.1b
		(3)	
			(3 marks)

Notes:

M1: Need correct use of the given coordinates

A1: Need fractions simplified to -3 (in ways 1 and 2)

A1: Need constants combined accurately

N.B. Answer left in the form $(y - 1) = -3(x - 3)$ or $(y - (-2)) = -3(x - 4)$ is awarded M1A1A0 as answers should be simplified by constants being collected

Note that a correct answer implies all three marks in this question

EXPERT TUITION

Question	Scheme	Marks	AOs
22	Finds $\frac{dy}{dx} = 8x - 6$	M1	3.1a
	Gradient of curve at P is –2	M1	1.1b
	Normal gradient is $-\frac{1}{m} = \frac{1}{2}$	M1	1.1b
	So equation of normal is $(y-2) = \frac{1}{2}\left(x - \frac{1}{2}\right)$ or $4y = 2x+7$	A1	1.1b
	Eliminates y between $y = \frac{1}{2}x + \ln(2x)$ and their normal equation to give an equation in x	M1	3.1a
	Solves their $\ln 2x = \frac{7}{4}$ so $x = \frac{1}{2}e^{\frac{7}{4}}$	M1	1.1b
	Substitutes to give value for y	M1	1.1b
	Point Q is $\left(\frac{1}{2}e^{\frac{7}{4}}, \frac{1}{4}e^{\frac{7}{4}} + \frac{7}{4}\right)$	A1	1.1b
			(8 marks)

Notes:

M1: Differentiates correctly

M1: Substitutes $x = \frac{1}{2}$ to find gradient (may make a slip)

M1: Uses negative reciprocal gradient

A1: Correct equation for normal

M1: Attempts to eliminate y to find an equation in x

M1: Attempts to solve their equation using exp

M1: Uses their x value to find y

A1: Any correct exact form

Question	Scheme		Marks	AOs
23 (a)	**Way 1:** Finds circle equation $(x \pm 2)^2 + (y \mp 6)^2 = (10 \pm (-2))^2 + (11 \mp 6)^2$	**Way 2:** Finds distance between $(-2, 6)$ and $(10, 11)$	M1	3.1a
	Checks whether (10, 1) satisfies their circle equation	Finds distance between $(-2, 6)$ and $(10, 1)$	M1	1.1b
	Obtains $(x+2)^2 + (y-6)^2 = 13^2$ and checks that $(10+2)^2 + (1-6)^2 = 13^2$ so states that (10, 1) lies on C *	Concludes that as distance is the same (10, 1) lies on the circle C *	A1*	2.1
			(3)	
(b)	Finds radius gradient $\dfrac{11-6}{10-(-2)}$ or $\dfrac{1-6}{10-(-2)}$ (m)		M1	3.1a
	Finds gradient perpendicular to their radius using $-\dfrac{1}{m}$		M1	1.1b
	Finds (equation and) y intercept of tangent (see note below)		M1	1.1b
	Obtains a correct value for y intercept of their tangent i.e. 35 or –23		A1	1.1b
	Way 1: Deduces gradient of second tangent	**Way 2:** Deduces midpoint of PQ from symmetry (0, 6)	M1	1.1b
	Finds (equation and) y intercept of second tangent	Uses this to find other intercept	M1	1.1b
	So obtains distance PQ = 35 + 23= 58*		A1*	1.1b
			(7)	
			(10 marks)	

Question 23 continued

Notes:

(a) Way 1 and Way 2:

M1: Starts to use information in question to find equation of circle or radius of circle
M1: Completes method for checking that (10, 1) lies on circle
A1*: Completely correct explanation with no errors concluding with statement that circle passes through (10, 1)

(b)

M1: Calculates $\frac{11-6}{10-(-2)}$ or $\frac{1-6}{10-(-2)}$ (m)

M1: Finds $-\frac{1}{m}$ (correct answer is $-\frac{12}{5}$ **or** $\frac{12}{5}$). This is referred to as m' in the next note

M1: Attempts $y-11=their\left(-\frac{12}{5}\right)(x-10)$ or $y-1=their\left(\frac{12}{5}\right)(x-10)$ and puts $x = 0$, or uses vectors to find intercept e.g. $\frac{y-11}{10}=-m'$

A1: One correct intercept 35 or – 23

Way 1:

M1: Uses the negative of their previous tangent gradient or uses a correct $-\frac{12}{5}$ or $\frac{12}{5}$

M1: Attempts the second tangent equation and puts $x = 0$ or uses vectors to find intercept e.g. $\frac{11-y}{10}=m'$

Way 2:

M1: Finds midpoint of PQ from symmetry. (This is at (0, 6))
M1: Uses this midpoint to find second intercept or to find difference between midpoint and first intercept. e.g. 35 – 6 = 29 then 6 – 29 = –23 so second intercept is at (–23, 0)

Ways 1 and 2:

A1*: Obtain 58 correctly from a valid method

Question	Scheme	Marks	AOs
24(a)	Attempts $(x-2)^2+(y+5)^2=....$	M1	1.1b
	Centre (2, −5)	A1	1.1b
		(2)	
(b)	Sets $k+2^2+5^2>0$	M1	2.2a
	$\Rightarrow k>-29$	A1ft	1.1b
		(2)	
			(4 marks)

Notes:

(a)

M1: Attempts to complete the square so allow $(x-2)^2+(y+5)^2=....$

A1: States the centre is at (2, −5). Also allow written separately $x=2, y=-5$

(2, −5) implies both marks

(b)

M1: Deduces that the right hand side of their $(x\pm...)^2+(y\pm...)^2=...$ is >0 or $\geqslant 0$

A1ft: $k>-29$ Also allow $k\geqslant -29$ Follow through on their rhs of $(x\pm...)^2+(y\pm...)^2=...$

Question	Scheme	Marks	AOs
25	Attempts to substitute $=\dfrac{x+1}{2}$ into $y \Rightarrow y = 4\left(\dfrac{x+1}{2}\right)-7 + \dfrac{6}{(x+1)}$	M1	2.1
	Attempts to write as a single fraction $y = \dfrac{(2x-5)(x+1)+6}{(x+1)}$	M1	2.1
	$y = \dfrac{2x^2-3x+1}{x+1}$ $\quad a=-3, b=1$	A1	1.1b
			(3 marks)

Notes:

M1: Score for an attempt at substituting $t=\dfrac{x+1}{2}$ or equivalent into $y = 4t-7+\dfrac{3}{t}$

M1: Award this for an attempt at a single fraction with a correct common denominator. Their $4\left(\dfrac{x+1}{2}\right)-7$ term may be simplified first

A1: Correct answer only $y = \dfrac{2x^2-3x+1}{x+1}$ $\quad a=-3, b=1$

Question	Scheme	Marks	AOs
26(a)	Attempts $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$	M1	1.1b
	$\dfrac{dy}{dx} = \dfrac{\sqrt{3}\sin 2t}{\sin t}$ $\left(= 2\sqrt{3}\cos t\right)$	A1	1.1b
		(2)	
(b)	Substitutes $t = \dfrac{2\pi}{3}$ in $\dfrac{dy}{dx} = \dfrac{\sqrt{3}\sin 2t}{\sin t} = \left(-\sqrt{3}\right)$	M1	2.1
	Uses gradient of normal = $-\dfrac{1}{dy/dx} = \left(\dfrac{1}{\sqrt{3}}\right)$	M1	2.1
	Coordinates of $P = \left(-1, -\dfrac{\sqrt{3}}{2}\right)$	B1	1.1b
	Correct form of normal $y + \dfrac{\sqrt{3}}{2} = \dfrac{1}{\sqrt{3}}\left(x+1\right)$	M1	2.1
	Completes proof $\Rightarrow 2x - 2\sqrt{3}y - 1 = 0$ *	A1*	1.1b
		(5)	
(c)	Substitutes $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$	M1	3.1a
	Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic in $\cos t$	M1	3.1a
	$\Rightarrow 12\cos^2 t - 4\cos t - 5 = 0$	A1	1.1b
	Finds $\cos t = \dfrac{5}{6}, \cancel{-\dfrac{1}{2}}$	M1	2.4
	Substitutes their $\cos t = \dfrac{5}{6}$ into $x = 2\cos t$, $y = \sqrt{3}\cos 2t$,	M1	1.1b
	$Q = \left(\dfrac{5}{3}, \dfrac{7}{18}\sqrt{3}\right)$	A1	1.1b
		(6)	
			(13 marks)

Question 26 continued
Notes:

(a)

M1: Attempts $\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}$ and achieves a form $k\dfrac{\sin 2t}{\sin t}$ Alternatively candidates may apply the double angle identity for $\cos 2t$ and achieve a form $k\dfrac{\sin t\cos t}{\sin t}$

A1: Scored for a correct answer, either $\dfrac{\sqrt{3}\sin 2t}{\sin t}$ or $2\sqrt{3}\cos t$

(b)

M1: For substituting $t=\dfrac{2\pi}{3}$ in their $\dfrac{dy}{dx}$ which must be in terms of t

M1: Uses the gradient of the normal is the negative reciprocal of the value of $\dfrac{dy}{dx}$. This may be seen in the equation of l.

B1: States or uses (in their tangent or normal) that $P=\left(-1,-\dfrac{\sqrt{3}}{2}\right)$

M1: Uses their numerical value of $-1\Big/\dfrac{dy}{dx}$ with their $\left(-1,-\dfrac{\sqrt{3}}{2}\right)$ to form an equation of the normal at P

A1*: This is a proof and all aspects need to be correct. Correct answer only $2x-2\sqrt{3}y-1=0$

(c)

M1: For substituting $x=2\cos t$ and $y=\sqrt{3}\cos 2t$ into $2x-2\sqrt{3}y-1=0$ to produce an equation in t. Alternatively candidates could use $\cos 2t=2\cos^2 t-1$ to set up an equation of the form $y=Ax^2+B$.

M1: Uses the identity $\cos 2t=2\cos^2 t-1$ to produce a quadratic equation in $\cos t$
In the alternative method it is for combining their $y=Ax^2+B$ with $2x-2\sqrt{3}y-1=0$ to get an equation in just one variable

A1: For the correct quadratic equation $12\cos^2 t-4\cos t-5=0$
Alternatively the equations in x and y are $3x^2-2x-5=0$ $\quad 12\sqrt{3}y^2+4y-7\sqrt{3}=0$

M1: Solves the quadratic equation in $\cos t$ (or x or y) and rejects the value corresponding to P.

M1: Substitutes their $\cos t=\dfrac{5}{6}$ or their $t=\arccos\left(\dfrac{5}{6}\right)$ in $x=2\cos t$ and $y=\sqrt{3}\cos 2t$
If a value of x or y has been found it is for finding the other coordinate.

A1: $Q=\left(\dfrac{5}{3},\dfrac{7}{18}\sqrt{3}\right)$. Allow $x=\dfrac{5}{3},y=\dfrac{7}{18}\sqrt{3}$ but do not allow decimal equivalents.

Question	Scheme	Marks	AOs
27(a)	Attempts $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{4\sec^2 t\tan t}{2\sec^2 t}\left(=2\tan t\right)$	M1 A1	1.1b 1.1b
	At $t=\frac{\pi}{4}$, $\frac{dy}{dx}=2, x=3, y=7$	M1	2.1
	Attempts equation of normal $y-7=-\frac{1}{2}\left(x-3\right)$	M1	1.1b
	$y=-\frac{1}{2}x+\frac{17}{2}$ *	A1*	2.1
		(5)	
(b)	Attempts to use $\sec^2 t=1+\tan^2 t\Rightarrow\frac{y-3}{2}=1+\left(\frac{x-1}{2}\right)^2$	M1	3.1a
	$\Rightarrow y-3=2+\frac{\left(x-1\right)^2}{2}\Rightarrow y=\frac{1}{2}(x-1)^2+5$ *	A1*	2.1
		(2)	
	(b) Alternative 1:		
	$y=\frac{1}{2}(x-1)^2+5=\frac{1}{2}(2\tan t+1-1)^2+5$ $=\frac{1}{2}4\tan^2 t+5=2\left(\sec^2 t-1\right)+5$	M1	3.1a
	$=2\sec^2 t+3=y$*	A1	2.1
	(b) Alternative 2:		
	$x=2\tan t+1\Rightarrow t=\tan^{-1}\left(\frac{x-1}{2}\right)\Rightarrow y=2\sec^2\left(\tan^{-1}\left(\frac{x-1}{2}\right)\right)+3$ $\Rightarrow y=2\left(1+\tan^2\left(\tan^{-1}\left(\frac{x-1}{2}\right)\right)\right)+3$	M1	3.1a
	$\Rightarrow y=2\left(1+\left(\frac{x-1}{2}\right)^2\right)+3=\frac{1}{2}\left(x-1\right)^2+5$*	A1	2.1
	(b) Alternative 3:		
	$\frac{dy}{dx}=2\tan t=x-1\Rightarrow y=\int\left(x-1\right)dx=\frac{x^2}{2}-x+c$ $\left(3,7\right)\to 7=\frac{3^2}{2}-3+c\Rightarrow c=\frac{11}{2}$	M1	3.1a
	$\frac{x^2}{2}-x+\frac{11}{2}=\frac{1}{2}\left(x^2-2x\right)+\frac{11}{2}=\frac{1}{2}\left(x-1\right)^2-\frac{1}{2}+\frac{11}{2}=\frac{1}{2}\left(x-1\right)^2+5$*	A1	2.1

(c)	**Attempts the lower limit for k:** $\frac{1}{2}(x-1)^2+5=-\frac{1}{2}x+k \Rightarrow x^2-x+(11-2k)=0$ $b^2-4ac=1-4(11-2k)=0 \Rightarrow k=\ldots$	M1	2.1
	$(k=)\frac{43}{8}$	A1	1.1b
	Attempts the upper limit for k: $(x,y)_{t=-\frac{\pi}{4}}: t=-\frac{\pi}{4} \Rightarrow x=2\tan\left(-\frac{\pi}{4}\right)+1=-1,\ y=2\sec^2\left(-\frac{\pi}{4}\right)+3=7$ $(-1,\ 7),\ y=-\frac{1}{2}x+k \Rightarrow 7=\frac{1}{2}+k \Rightarrow k=\ldots$	M1	2.1
	$(k=)\frac{13}{2}$	A1	1.1b
	$\frac{43}{8}<k\leqslant\frac{13}{2}$	A1	2.2a
		(5)	
			(12 marks)

Notes:

(a) **Must use parametric differentiation to score any marks in this part and not e.g. Cartesian form**

M1: For the key step of attempting $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$. There must be some attempt to differentiate both parameters however poor and divide the right way round so using $\frac{dy}{dx}=\frac{y}{x}$ scores M0.

This may be implied by e.g. $\frac{dx}{dt}=2\sec^2 t,\ \frac{dy}{dt}=4\sec^2 t\tan t,\ t=\frac{\pi}{4} \Rightarrow \frac{dx}{dt}=4,\ \frac{dy}{dt}=8 \Rightarrow \frac{dy}{dx}=2$

A1: $\frac{dy}{dx}=\frac{4\sec^2 t\tan t}{2\sec^2 t}$. Correct expression in any form. May be implied as above.

Condone the confusion with variables as long as the intention is clear e.g.

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{4\sec^2 x\tan x}{2\sec^2 x}(=2\tan x)$ and allow subsequent marks if this is interpreted correctly

M1: For attempting to find the values of x, y and the gradient at $t=\frac{\pi}{4}$ **AND** getting at least two correct.

Follow through on their $\frac{dy}{dx}$ so allow for any two of $x=3,\ y=7,\ \frac{dy}{dx}=2\left(\text{or their } \frac{dy}{dx} \text{ at } t=\frac{\pi}{4}\right)$

Note that the $x=3,\ y=7$ may be seen as e.g. (3, 7) on the diagram. There must be a non-trivial $\frac{dy}{dx}$ for this mark e.g. they must have a $\frac{dy}{dx}$ to substitute into.

M1: For a correct attempt at the normal equation using their x and y at $t=\frac{\pi}{4}$ with the negative reciprocal of their $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$ having made some attempt at $\frac{dy}{dx}$ and all correctly placed.

For attempts using $y = mx + c$ they must reach as far as a value for c using their x and y at $t=\frac{\pi}{4}$ with the negative reciprocal of their $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$ all correctly placed.

A1*: Proceeds with a clear argument to the given answer with no errors.

(b)

M1: Attempts to use $\sec^2 t = 1+\tan^2 t$ oe to obtain an equation involving y and $(x-1)^2$

E.g. as above or e.g. $y = 2\sec^2 t + 3 = 2\left(1+\tan^2 t\right)+3 = 2\left(1+\left(\frac{x-1}{2}\right)^2\right)+3$ for M1 and then

$y = \frac{1}{2}\left(x-1\right)^2 + 5*$ for A1

A1*: Proceeds with a clear argument to the given answer with no errors

Alternative 1:

M1: Uses the given result, substitutes for x and attempts to use $\sec^2 t = 1+\tan^2 t$ oe

A1: Proceeds with a clear argument to the y parameter and makes a (minimal) conclusion e.g. " $= y$" QED, hence proven etc.

Alternative 2:

M1: Uses the x parameter to obtain t in terms of arctan, substitutes into y and attempts to use $\sec^2 t = 1+\tan^2 t$ oe

A1: Proceeds with a clear argument to the given answer with no errors

Alternative 3:

M1: Uses $\frac{\mathrm{d}y}{\mathrm{d}x}$ from part (a) to express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x, integrates and uses (3, 7) to find "c" to reach a Cartesian equation.

A1: Proceeds with a clear argument to the given answer with no errors

Allow the marks for (b) to score anywhere in their solution e.g. if they find the Cartesian equation in part (a)

(c)

M1: A full attempt to find the **lower** limit for k.

$\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k \Rightarrow x^2 - x + \left(11-2k\right) = 0 \Rightarrow b^2 - 4ac = 1 - 4\left(11-2k\right) = 0 \Rightarrow k = ...$

Score **M1** for setting $\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k$, rearranging to 3TQ form and attempts $b^2 - 4ac ... 0$

e.g. $b^2 - 4ac > 0$ or e.g. $b^2 - 4ac < 0$ correctly to find a value for k.

A1: $k = \frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$

An alternative method using calculus for lower limit:

$$y = \frac{1}{2}(x-1)^2 + 5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = x-1,\ x-1 = -\frac{1}{2} \Rightarrow x = \frac{1}{2}$$

$$x = \frac{1}{2} \Rightarrow y = \frac{1}{2}\left(\frac{1}{2}-1\right)^2 + 5 = \frac{41}{8}$$

$$y = -\frac{1}{2}x + k \Rightarrow \frac{41}{8} = -\frac{1}{4} + k \Rightarrow k = ...$$

Score **M1** for $\frac{\mathrm{d}y}{\mathrm{d}x} =$ "a linear expression in x", sets $= -\frac{1}{2}$, solves a linear equation to find x and then substitutes into the given result in (b) to find y and then uses $y = -\frac{1}{2}x + k$ to find a value for k. **A1**: $k = \frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$

An alternative method using parameters for lower limit:

$$y=-\frac{1}{2}x+k \Rightarrow 2\sec^2 t+3=-\frac{1}{2}(2\tan t+1)+k$$

$$\Rightarrow 2\left(1+\tan^2 t\right)+3=-\frac{1}{2}(2\tan t+1)+k \Rightarrow 2\tan^2 t+\tan t+5.5-k=0$$

$$b^2-4ac=0 \Rightarrow 1-4\times 2(5.5-k)=0 \Rightarrow k=\frac{43}{8}$$

Score **M1** for substituting parametric form of x and y into $y=-\frac{1}{2}x+k$, uses $\sec^2 t=1+\tan^2 t$ rearranges to 3TQ form and attempts $b^2-4ac...0$ or e.g. $b^2-4ac>0$ or $b^2-4ac<0$ correctly to find a value for k.

A1: $k=\frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k>\frac{43}{8}$, $k<\frac{43}{8}$

M1: A full attempt to find the **upper** limit for k. This requires an attempt to find the value of x and the value of y using $t=-\frac{\pi}{4}$, the substitution of these values into $y=-\frac{1}{2}x+k$ and solves for k.

A1: $k=\frac{13}{2}$. Look for this value e.g. may appear in an inequality.

A1: Deduces the correct range for k: $\frac{43}{8}<k\leqslant\frac{13}{2}$

Allow equivalent notation e.g. $\left(k\leqslant\frac{13}{2} \text{ and } k>\frac{43}{8}\right)$, $\left(k\leqslant\frac{13}{2}\cap k>\frac{43}{8}\right)$, $\left(\frac{43}{8}, \frac{13}{2}\right]$

But not e.g. $\left(k\leqslant\frac{13}{2},\ k>\frac{43}{8}\right)$, $\left(k\leqslant\frac{13}{2}\cup k>\frac{43}{8}\right)$, $\left(k\leqslant\frac{13}{2} \text{ or } k>\frac{43}{8}\right)$ and do not allow if in terms of x.

Allow equivalent exact values for $\frac{43}{8}$, $\frac{13}{2}$

There may be other methods for finding the upper limit which are valid. If you are in any doubt if a method deserves credit then use Review.

Question	Scheme	Marks	AOs
28 (a)	C is $(x-r)^2+(y-r)^2=r^2$ or $x^2+y^2-2rx-2ry+r^2=0$	B1	2.2a
	$y=12-2x,\ x^2+y^2-2rx-2ry+r^2=0$ $\Rightarrow x^2+(12-2x)^2-2rx-2r(12-2x)+r^2=0$ or $y=12-2x,\ (x-r)^2+(y-r)^2=r^2$ $\Rightarrow (x-r)^2+(12-2x-r)^2=r^2$	M1	1.1b
	$x^2+144-48x+4x^2-2rx-24r+4rx+r^2=0$ $\Rightarrow 5x^2+(2r-48)x+(r^2-24r+144)=0$ *	A1*	2.1
		(3)	
(b)	$b^2-4ac=0\Rightarrow(2r-48)^2-4\times5\times(r^2-24r+144)=0$	M1	3.1a
	$r^2-18r+36=0$ or any multiple of this equation	A1	1.1b
	$\Rightarrow(r-9)^2-81+36=0\Rightarrow r=...$	dM1	1.1b
	$r=9\pm3\sqrt{5}$	A1	1.1b
		(4)	
			(7 marks)

Notes:

(a)

B1: Deduces the correct equation of the circle

M1: Attempts to form an equation with terms of the form x^2, x, r^2, and xr **only** using $y=12\pm2x$ and their circle equation which must be of an appropriate form. I.e. includes or implies an x^2, y^2, r^2 such as $x^2+y^2=r^2$

If their circle equation starts off as e.g. $(x\pm a)^2+(y\pm b)^2=r^2$ then the B mark and the M mark can be awarded when the "a" and "b" are replaced by r or $-r$ as appropriate for their circle equation.

A1*: Uses correct and accurate algebra leading to the given solution.

(b)

M1: Attempts to use $b^2-4ac...0$ o.e. with $a=5, b=2r-48, c=r^2-24r+144$ and where ... is "=" or any inequality

Allow minor slips when copying the a, b and c provided it does not make the work easier and allow **their** a, b and c if they are similar expressions.

FYI $(2r-48)^2-4\times5\times(r^2-24r+144)=4r^2-192r+2304-20r^2+480r-2880=-16r^2+288r-576$

A1: Correct quadratic **equation** in r (or inequality). Terms need not be all one side but must be collected.

E.g. allow $r^2-18r=-36$ and allow any multiple of this equation (or inequality).

dM1: Correct attempt to solve their 3TQ in r. Dependent upon previous M

A1: Careful and accurate work leading to both answers in the required form (must be simplified surds)

Question	Scheme	Marks	AOs
29	$C_1: \ x = 10\cos t\,, \ y = 4\sqrt{2}\sin t, \quad 0 \le t < 2\pi; \quad C_2: x^2 + y^2 = 66$		
Way 1	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
	$100(1-\sin^2 t) + 32\sin^2 t = 66$ $100\cos^2 t + 32(1-\cos^2 t) = 66$	M1 A1	2.1 1.1b
	$100 - 68\sin^2 t = 66 \ \Rightarrow \sin^2 t = \dfrac{1}{2}$ $\ \Rightarrow\ \sin t = ...$ $68\cos^2 t + 32 = 66 \ \Rightarrow \cos^2 t = \dfrac{1}{2}$ $\ \Rightarrow\ \cos t = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
		(6)	
Way 2	$\{\cos^2 t + \sin^2 t = 1 \Rightarrow\}\ \left(\dfrac{x}{10}\right)^2 + \left(\dfrac{y}{4\sqrt{2}}\right)^2 = 1 \ \{\Rightarrow 32x^2 + 100y^2 = 3200\}$	M1	3.1a
	$\dfrac{x^2}{100} + \dfrac{66-x^2}{32} = 1$ $\dfrac{66-y^2}{100} + \dfrac{y^2}{32} = 1$	M1 A1	2.1 1.1b
	$32x^2 + 6600 - 100x^2 = 3200$ $\ x^2 = 50 \ \Rightarrow\ x = ...$ $2112 - 32y^2 + 100y^2 = 3200$ $\ y^2 = 16 \ \Rightarrow\ y = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the corresponding x-coordinate or y-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
		(6)	
Way 3	$\{C_2: x^2 + y^2 = 66 \Rightarrow\}\quad x = \sqrt{66}\cos\alpha,\ \ y = \sqrt{66}\sin\alpha$ $\{C_1 = C_2 \ \Rightarrow\}\quad 10\cos t = \sqrt{66}\cos\alpha, \quad 4\sqrt{2}\sin t = \sqrt{66}\sin\alpha$ $\{\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow\}\ \left(\dfrac{10\cos t}{\sqrt{66}}\right)^2 + \left(\dfrac{4\sqrt{2}\sin t}{\sqrt{66}}\right)^2 = 1$	M1	3.1a
	then continue with applying the mark scheme for Way 1		
Way 4	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
	$100\left(\dfrac{1+\cos 2t}{2}\right) + 32\left(\dfrac{1-\cos 2t}{2}\right) = 66$	M1 A1	2.1 1.1b
	$50 + 50\cos 2t + 16 - 16\cos 2t = 66 \ \Rightarrow\ 34\cos 2t + 66 = 66$ $\Rightarrow\ \cos 2t = ...$	dM1	1.1b
	Substitutes their solution back into the original equation(s) to get the value of the x-coordinate and value of the y-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
		(6)	
	Note: Give final A0 for writing $x = 5\sqrt{2},\ y = -4$ followed by $S = (-4, 5\sqrt{2})$		
			(6 marks)
	Notes for Question **29**		

	Way 1
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 1: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. *t*) only.
M1:	Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only or $\cos^2 t$ only
A1:	A correct equation in $\sin^2 t$ only or $\cos^2 t$ only
dM1: **Note:**	**dependent on both the previous M marks** Rearranges to make $\sin t = ...$ where $-1 \le \sin t \le 1$ or $\cos t = ...$ where $-1 \le \cos t \le 1$ Condone 3rd M1 for $\sin^2 t = \frac{1}{2} \Rightarrow \sin t = \frac{1}{4}$
M1:	See scheme
A1:	Selects the correct coordinates for *S* Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$
	Way 2
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 2: A complete process of using $\cos^2 t + \sin^2 t \equiv 1$ to convert the parametric equation for C_1 into a Cartesian equation for C_1
M1:	Complete valid attempt to write an equation in terms of *x* only or *y* only not involving trigonometry
A1:	A correct equation in x only or y only not involving trigonometry
dM1: **Note:**	**dependent on both the previous M marks** Rearranges to make $x = ...$ or $y = ...$ their x^2 or their y^2 must be >0 for this mark
M1: **Note:**	See scheme their x^2 and their y^2 must be >0 for this mark
A1:	Selects the correct coordinates for *S* Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$
	Way 3
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 3: A complete process of writing C_2 in parametric form, combining the parametric equations of C_1 and C_2 and applying $\cos^2 \alpha + \sin^2 \alpha \equiv 1$ to give an equation in one variable (i.e. t) only.
	then continue with applying the mark scheme for Way 1
	Way 4
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 4: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. *t*) only.
M1: **Note:**	Uses the identities $\cos 2t \equiv 2\cos^2 t - 1$ and $\cos 2t \equiv 1 - 2\sin^2 t$ to achieve an equation in $\cos 2t$ only At least one of $\cos 2t \equiv 2\cos^2 t - 1$ or $\cos 2t \equiv 1 - 2\sin^2 t$ must be correct for this mark.
A1:	A correct equation in $\cos 2t$ only
dM1:	**dependent on both the previous M marks** Rearranges to make $\cos 2t = ...$ where $-1 \le \cos 2t \le 1$
M1:	See scheme
A1:	Selects the correct coordinates for *S* Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$

29	$C_1: x = 10\cos t,\ y = 4\sqrt{2}\sin t,\quad 0 \le t < 2\pi;\quad C_2: x^2 + y^2 = 66$		
Way 5	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66(\sin^2 t + \cos^2 t)$	M1	2.1
		A1	1.1b
	$100\cos^2 t + 32\sin^2 t = 66\sin^2 t + 66\cos^2 t \Rightarrow 34\cos^2 t = 34\sin^2 t$ $\Rightarrow \tan t = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
		(6)	

	Way 5
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 5: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1:	Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only and $\cos^2 t$ only **with no constant term**
A1:	A correct equation in $\sin^2 t$ and $\cos^2 t$ containing no constant term
dM1:	**dependent on both the previous M marks** Rearranges to make $\tan t = ...$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$

Question	Scheme	Marks	AOs
30	£y is the total cost of making x bars of soap Bars of soap are sold for £2 each		
(a)	$y = kx + c$ {where k and c are constants}	B1	3.3
	Note: Work for (a) cannot be recovered in (b) or (c)	**(1)**	
(b) Way 1	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Applies (800, their 1100) and (300, their 680) to give two equations $1100 = 800k + c$ and $680 = 300k + c \Rightarrow k, c = ...$	dM1	1.1b
	Solves correctly to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	**(3)**	
(b) Way 2	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Complete method for finding both $k = ...$ and $c = ...$ e.g. $k = \frac{1100 - 680}{800 - 300}$ $\{= 0.84\}$ $(800, 1100) \Rightarrow 1100 = 800(0.84) + c \Rightarrow c = ...$	dM1	1.1b
	Solves to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	**(3)**	
(b) Way 3	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	$\{y = 0.84x + 428 \Rightarrow\}$ $x = 800 \Rightarrow y = (0.84)(800) + 428 = 1100$ $x = 300 \Rightarrow y = (0.84)(300) + 428 = 680$	dM1	1.1b
	Hence $y = 0.84x + 428$ *	A1*	2.1
		(3)	
(c)	Allow any of {0.84, in £s} represents • the ***cost*** of {making} each extra bar {of soap} • the direct ***cost*** of {making} a bar {of soap} • the marginal ***cost*** of {making} a bar {of soap} • the ***cost*** of {making} a bar {of soap} **(**Condone this answer) **Note: Do not allow** • {0.84, in £s} is the profit per bar {of soap} • {0.84, in £s} is the (selling) price per bar {of soap}	B1	3.4
		(1)	
(d) Way 1	{Let n be the least number of bars required to make a profit}		
	$2n = 0.84n + 428 \Rightarrow n = ...$ (Condone $2x = 0.84x + 428 \Rightarrow x = ...$)	M1	3.4
	Answer of 369 {bars}	A1	3.2a
		(2)	
(d) Way 2	• Trial 1: $n = 368 \Rightarrow y = (0.84)(368) + 428 \Rightarrow y = 737.12$ {revenue $= 2(368) = 736$ or loss $= 1.12$} • Trial 2: $n = 369 \Rightarrow y = (0.84)(369) + 428 \Rightarrow y = 737.96$ {revenue $= 2(369) = 738$ or profit $= 0.04$} leading to an answer of 369 {bars}	M1 A1	3.4 3.2a
		(2)	
			(7 marks)

EXPERT TUITION

Notes for Question **30**	
(a)	
B1:	Obtains a correct form of the equation. E.g. $y = kx + c$; $k \neq 0, c \neq 0$. **Note:** Must be seen in (a)
Note:	Ignore how the constants are labelled – as long as they appear to be constants. e.g. *k*, *c*, *m* etc.
(b)	**Way 1**
M1:	Translates the problem into the model by finding either • $y = 2(800) - 500$ for $x = 800$ • $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no errors in their working
Note	Allow 1st M1 for any of • $1600 - y = 500$ • $600 - y = -80$
(b)	**Way 2**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no error in their working
(b)	**Way 3**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** Uses the model to test both points $(800, \text{their } 1100)$ and $(300, \text{their } 680)$
A1:	Confirms $y = 0.84x + 428$ is true for both $(800, 1100)$ and $(300, 680)$ and gives a conclusion
Note:	Conclusion could be "$y = 0.84x + 428$" or "QED" or "proved"
Note:	Give 1st M0 for $500 = 800k + c,\ 80 = 300k + c \Rightarrow k = \frac{500-80}{800-300} = 0.84$
(c)	
B1:	see scheme
Note:	Also condone B1 for "rate of change of cost", "cost of {making} a bar", "constant of proportionality for cost per bar of soap" or "rate of increase in cost",
Note:	Do not allow reasons such as "price increase or decrease", "rate of change of the bar of soap" or "decrease in cost"
Note:	Give B0 for incorrect use of units. E.g. Give B0 for "the cost of making each extra bar of soap is £84" Condone the use of £0.84p

Notes for Question **30** Continued	
(d)	**Way 1**
M1:	Using the model and constructing an argument leading to a critical value for the number of bars of soap sold. See scheme.
A1:	369 only. Do not accept decimal answers.
(d)	**Way 2**
M1:	Uses either 368 or 369 to find the cost $y = ...$
A1:	Attempts both trial 1 and trial 2 to find both the cost $y = ...$ and arrives at an answer of 369 only. Do not accept decimal answers.
Note:	You can ignore inequality symbols for the method mark in part (d)
Note:	Give M1 A1 for no working leading to 369 {bars}
Note:	Give final A0 for $x > 369$ or $x > 368$ or $x \geq 369$ without $x = 369$ or 369 stated as their final answer
Note:	Condone final A1 for **in words** "at least 369 bars must be made/sold"
Note:	**<u>Special Case:</u>** Assuming a profit of £1 is required and achieving $x = 370$ scores special case M1A0

Question	Scheme	Marks	AOs
31(a)	E.g. midpoint $PQ = \left(\frac{-9+15}{2}, \frac{8-10}{2}\right)$	M1	1.1b
	$= (3, -1)$, which is the centre point A, so PQ is the diameter of the circle.	A1	2.1
		(2)	
(a) Alt 1	$m_{PQ} = \frac{-10-8}{15--9} = -\frac{3}{4} \Rightarrow PQ: \ y - 8 = -\frac{3}{4}(x--9)$	M1	1.1b
	$PQ: y = -\frac{3}{4}x + \frac{5}{4}$. So $x = 3 \Rightarrow y = -\frac{3}{4}(3) + \frac{5}{4} = -1$ so PQ is the diameter of the circle.	A1	2.1
		(2)	
(a) Alt 2	$PQ = \sqrt{(-9-15)^2 + (8--10)^2}\ \left\{= \sqrt{900} = 30\right\}$ **and either** • $AP = \sqrt{(3--9)^2 + (-1-8)^2}\ \left\{= \sqrt{225} = 15\right\}$ • $AQ = \sqrt{(3-15)^2 + (-1--10)^2}\ \left\{= \sqrt{225} = 15\right\}$	M1	1.1b
	e.g. as $PQ = 2AP$, then PQ is the diameter of the circle.	A1	2.1
		(2)	
(b)	Uses Pythagoras in a correct method to find either the radius or diameter of the circle.	M1	1.1b
	$(x-3)^2 + (y+1)^2 = 225\ \left(\text{or } (15)^2\right)$	M1	1.1b
		A1	1.1b
		(3)	
(c)	Distance $= \sqrt{(\text{"15"})^2 - (10)^2}$ or $= \frac{1}{2}\sqrt{(2(\text{"15"}))^2 - (2(10))^2}$	M1	3.1a
	$\left\{= \sqrt{125}\right\} = 5\sqrt{5}$	A1	1.1b
		(2)	
(d)	$\sin(A\hat{R}Q) = \frac{20}{2(\text{"15"})}$ or $A\hat{R}Q = 90 - \cos^{-1}\left(\frac{10}{\text{"15"}}\right)$	M1	3.1a
	$A\hat{R}Q = 41.8103... = 41.8°$ (to 0.1 of a degree)	A1	1.1b
		(2)	
			(9 marks)

Question **31** Notes:	
(a)	
M1:	Uses a correct method to find the midpoint of the line segment PQ
A1:	Completes proof by obtaining $(3, -1)$ and gives a correct conclusion.
(a)	
Alt 1	
M1:	Full attempt to find the equation of the line PQ
A1:	Completes proof by showing that $(3, -1)$ lies on PQ and gives a correct conclusion.
(a)	
Alt 2	
M1:	Attempts to find distance PQ and either one of distance AP or distance AQ
A1:	Correctly shows either • $PQ = 2AP$, supported by $PQ = 30,\ AP = 15$ and gives a correct conclusion • $PQ = 2AQ$, supported by $PQ = 30,\ AQ = 15$ and gives a correct conclusion
(b)	
M1:	**Either** • uses Pythagoras correctly in order to find the **radius**. Must clearly be identified as the **radius**. E.g. $r^2 = (-9-3)^2 + (8+1)^2$ or $r = \sqrt{(-9-3)^2 + (8+1)^2}$ or $r^2 = (15-3)^2 + (-10+1)^2$ or $r = \sqrt{(15-3)^2 + (-10+1)^2}$ **or** • uses Pythagoras correctly in order to find the **diameter**. Must clearly be identified as the **diameter**. E.g. $d^2 = (15+9)^2 + (-10-8)^2$ or $d = \sqrt{(15+9)^2 + (-10-8)^2}$ **Note:** This mark can be implied by just 30 clearly seen as the **diameter** or 15 clearly seen as the **radius** (may be seen or implied in their circle equation)
M1:	Writes down a circle equation in the form $(x \pm \text{"3"})^2 + (y \pm \text{"}-1\text{"})^2 = (\text{their } r)^2$
A1:	$(x-3)^2 + (y+1)^2 = 225$ or $(x-3)^2 + (y+1)^2 = 15^2$ or $x^2 - 6x + y^2 + 2y - 215 = 0$
(c)	
M1:	Attempts to solve the problem by using the circle property "the perpendicular from the centre to a chord bisects the chord" and so applies Pythagoras to write down an expression of the form $\sqrt{(\text{their "15"})^2 - (10)^2}$.
A1:	$5\sqrt{5}$ by correct solution only
(d)	
M1:	Attempts to solve the problem by e.g. using the circle property "the angle in a semi-circle is a right angle" and writes down either $\sin(A\hat{R}Q) = \dfrac{20}{2(\text{their "15"})}$ or $A\hat{R}Q = 90 - \cos^{-1}\left(\dfrac{10}{\text{their "15"}}\right)$ **Note:** Also allow $\cos(A\hat{R}Q) = \dfrac{15^2 + \left(2\left(5\sqrt{5}\right)\right)^2 - 15^2}{2(15)\left(2\left(5\sqrt{5}\right)\right)}$ $\left\{= \dfrac{\sqrt{5}}{3}\right\}$
A1:	41.8 by correct solution only

Question	Scheme	Marks	AOs
32 (a)	Gradient $AB = -\frac{2}{5}$	B1	2.1
	y coordinate of A is 2	B1	2.1
	Uses perpendicular gradients $y = +\frac{5}{2}x + c$	M1	2.2a
	$\Rightarrow 2y - 5x = 4$ *	A1*	1.1b
		(4)	
(b)	Uses Pythagoras' theorem to find AB or AD Either $\sqrt{5^2+2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2+2^2}$	M1	3.1a
	Uses area $ABCD = AD \times AB = \sqrt{29} \times \sqrt{\frac{116}{25}}$	M1	1.1b
	area $ABCD = 11.6$	A1	1.1b
		(3)	
			(7 marks)

Notes:

(a) It is important that the student communicates each of these steps clearly

B1: States the gradient of AB is $-\frac{2}{5}$

B1: States that y coordinate of $A = 2$

M1: Uses the form $y = mx + c$ with $m =$ their adapted $-\frac{2}{5}$ and $c =$ their 2

Alternatively uses the form $y - y_1 = m(x - x_1)$ with $m =$ their adapted $-\frac{2}{5}$ and $(x_1, y_1) = (0, 2)$

A1*: Proceeds to given answer

(b)

M1: Finds the lengths of AB or AD using Pythagoras' Theorem. Look for $\sqrt{5^2+2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2+2^2}$

Alternatively finds the lengths BD and AO using coordinates. Look for $\left(5+\frac{4}{5}\right)$ and 2

M1: For a full method of finding the area of the rectangle $ABCD$. Allow for $AD \times AB$

Alternatively attempts area $ABCD = 2 \times \frac{1}{2} BD \times AO = 2 \times \frac{1}{2}$ '5.8'×'2'

A1: Area $ABCD = 11.6$ or other exact equivalent such as $\frac{58}{5}$

Question Number	Scheme		Marks
33(a)	$\frac{5}{4}$ oe	$\frac{5}{4}$ or exact equivalents such as 1.25 but **not** $\frac{5}{4}x$.	B1
			(1)
(b)	$y=\frac{5}{4}x+c$	Uses a line with a parallel gradient $\frac{5}{4}$ oe or their gradient from part (a). Evidence is $y=\text{"}\frac{5}{4}\text{"}x+c$ or similar.	M1
	$12,5 \Rightarrow 5='\frac{5}{4}'\times 12+c \Rightarrow c=..$	Method of finding an equation of a line with numerical gradient and passing through $12,5$. Score even for the perpendicular line. Must be seen in part (a).	M1
	$y=\frac{5}{4}x-10$	Correct equation. Allow $-\frac{40}{4}$ for -10	A1
			(3)
(c)	$B=\ 0,-10$	$B=\ 0,-10$ Follow through on their 'c'. Allow also if -10 is marked in the correct place on the diagram. Allow x = 0, y = -10 (the x = 0 may be seen "embedded" but not just y = -10 with no evidence that x = 0)	B1ft
	$C=\ 8,0$	$C=\ 8,0$ Correct coordinates. Allow also if 8 is marked in the correct place on the diagram. Allow y = 0, x = 8 (the y = 0 may be seen "embedded" but not just x = 8 with no evidence that y = 0)	B1
	Do not penalise lack of "0" twice so penalise it at the first occurrence but check the diagram if necessary.		
			(2)

(d) **Way 1**	Area of Parallelogram = $3+'10' \ \times '8'$	Uses area of parallelogram = $bh = \ 3+'10' \ \times ''8''$ Follow through on their 10 and their 8	M1
	= 104	cao	A1
	Correct answer only scores both marks		**(2)**
(d) **Way 2**	Trapezium *AOCD* + Triangle *OCB* $=\frac{1}{2} \ 3+3+'10' \ \times '8' + \frac{1}{2} \times '8' \times '10'$	A correct method using their values for *AOCD* + *OCB*.	M1
	= 104	cao	A1
			(2)
(d) **Way 3**	2 Triangles + Rectangle $=2\times\frac{1}{2} \ '8' \times '10' \ +'8' \times 3$	A correct method using their values for 2x*OBC* + rectangle.	M1
	= 104	cao	A1
			(2)
(d) **Way 4**	Triangle *ACD* + Triangle *ACB* $=2\times\frac{1}{2} \ '10'+3 \ \times '8'$	A correct method using their values for *ACD* + *ABC*.	M1
	= 104	cao	A1
			(2)
			(8 marks)

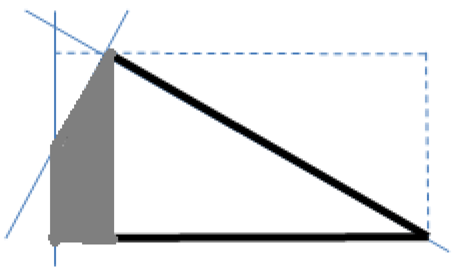
Question Number	Scheme		Marks
34(a)	Gradient of $l_1 = \frac{4}{5}$ oe	States or implies that the gradient of $l_1 = \frac{4}{5}$. E.g. may be implied by a perpendicular gradient of $-\frac{5}{4}$. Do not award this mark for just rearranging to $y = \frac{4}{5}x + ...$ unless they then state e.g. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{5}$	B1
	Point P = (5, 6)	States or implies that P has coordinates (5, 6). y = 6 is sufficient. **May be seen on the diagram.**	B1
	$-\frac{5}{4} = \frac{y - "6"}{x - 5}$ or $y - "6" = -\frac{5}{4}(x - 5)$ or $"6" = -\frac{5}{4}(5) + c \Rightarrow c = ...$	Correct straight line method using P(5, "6") and gradient of $-\frac{1}{\text{grad } l_1}$. Unless $-\frac{5}{4}$ or $-\frac{1}{4}$ is being used as the gradient here, the gradient of l_1 clearly needs to have been identified and its negative reciprocal attempted to score this mark.	M1
	$5x + 4y - 49 = 0$	Accept any integer multiple of this equation including "= 0"	A1
			(4)

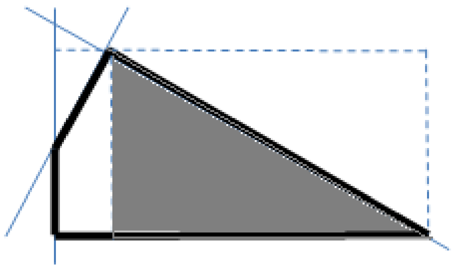
34(b)	$y=0 \Rightarrow 5x+4(0)-49=0 \Rightarrow x=\ldots$ **or** $y=0 \Rightarrow 5(0)=4x+10 \Rightarrow x=\ldots$	Substitutes $y = 0$ into their l_2 to find a value for x **or** substitutes $y = 0$ into l_1 or their rearrangement of l_1 to find a value for x. **This may be implied by a correct value on the diagram.**	M1
	$y=0 \Rightarrow 5x+4(0)-49=0 \Rightarrow x=\ldots$ **and** $y=0 \Rightarrow 5(0)=4x+10 \Rightarrow x=\ldots$	Substitutes $y = 0$ into their l_2 to find a value for x **and** substitutes $y = 0$ into l_1 or their rearrangement of l_1 to find a value for x. **This may be implied by correct values on the diagram.**	M1
	(Note that at T, x = 9.8 and at S, x = -2.5)		
	Fully correct method using their values to find the area of triangle SPT with vertices at points of the form (5, "6"), (p, 0) and (q, 0) where $p \neq q$ **Attempts to use integration should be sent to your team leader** **Method 1:** $\frac{1}{2}ST \times "6"$ $\frac{1}{2}\times('9.8'-'-2.5')\times'6'=\ldots$ **Method 2:** $\frac{1}{2}SP \times PT$ $\frac{1}{2}\times\sqrt{(5-'-2.5')^2+('6')^2}\times\sqrt{('9.8'-5)^2+('6')^2}=\ldots$ $\left(=\frac{1}{2}\times\frac{3\sqrt{41}}{2}\times\frac{6\sqrt{41}}{5}\right)$ **Note that if the method is correct but slips are made when simplifying any of the calculations, the method mark can still be awarded** **Method 3:** 2 Triangles $\frac{1}{2}\times(5+'2.5')\times'6'+\frac{1}{2}\times('9.8'-5)\times'6'=\ldots$ **Method 4:** Shoelace method $\frac{1}{2}\begin{vmatrix}5 & 9.8 & -2.5 & 5\\ 6 & 0 & 0 & 6\end{vmatrix}=\frac{1}{2}\left\|(0+0-15)-(58.8+0+0)\right\|=\frac{1}{2}\left\|-73.8\right\|=\ldots$ **(must see a correct calculation i.e. the middle expression for this determinant method)** **Method 5:** Trapezium + 2 triangles $\frac{1}{2}\times('2.5')\times'2'+\frac{1}{2}("2"+"6")\times5+\frac{1}{2}\times("9.8"-5')\times'6'=\ldots$		**dd**M1
	$=36.9$	36.9 cso oe e.g $\frac{369}{10}$, $36\frac{9}{10}$, $\frac{738}{20}$ but not e.g. $\frac{73.8}{2}$	A1
	Note that the final mark is cso so beware of any errors that have fortuitously resulted in a correct area.		
			(4)
			(8 marks)

Question Number	Scheme	Notes	Marks
35(a)	l_1 : passes through (0, 2) and (3, 7) l_2 : goes through (3, 7) and is perpendicular to l_1		
	Gradient of l_1 is $\frac{7-2}{3-0}\left(=\frac{5}{3}\right)$	$m(l_1)=\frac{7-2}{3-0}$. Allow un-simplified. May be implied.	B1
	$m(l_2) = -1 \div \textit{their}\ \frac{5}{3}$	Correct application of perpendicular gradient rule	M1
	$y-7 = "-\tfrac{3}{5}"(x-3)$ or $y = "-\tfrac{3}{5}"x + c,\ 7 = "-\tfrac{3}{5}"(3) + c \Rightarrow c = \tfrac{44}{5}$	M1: Uses $y-7=m(x-3)$ with their **changed** gradient or uses $y=mx+c$ with $(3, 7)$ and their **changed** gradient to find a value for c A1ft: Correct ft equation for their perpendicular gradient **(this is dependent on both M marks)**	M1A1ft
	$3x+5y-44=0$	Any positive or negative integer multiple. Must be seen in (a) and must include "= 0".	A1
			[5]
(b)	When $y=0$ $x=\frac{44}{3}$	M1: Puts $y = 0$ and finds a value for x from their equation A1: $x=\frac{44}{3}\left(\text{or } 14\frac{2}{3} \text{ or } 14.\dot{6}\right)$ or exact equivalent. ($y = 0$ not needed)	M1 A1
	Condone 3x – 5y – 44 = 0 only leading to the correct answer and condone coordinates written as (0, 44/3) but allow recovery in (c)		
			[2]
(c)	**GENERAL APPROACH:**		
	Correct attempt at finding the area of any one of the triangles or one of the trapezia but not just one rectangle. The correct pair of 'base' and 'height' must be used for a triangle and the correct formula used for a trapezium. If Pythagoras is required, then it must be used correctly with the correct end coordinates. **Note that the first three marks apply to their calculated coordinates e.g. their** $\frac{44}{3}, \frac{44}{5}, -\frac{6}{5}$ **etc. But the given coordinates must be correct e.g. (0, 2) and (3, 7).**		M1
	A correct numerical **expression** for the area of one **triangle** or one **trapezium** for **their coordinates**.		A1ft
	Combines the correct areas together correctly for their chosen "way". Note that if correct numerical expressions for areas have been incorrectly simplified before combining them, then this M1 may still be given. **Dependent on the first method mark.**		dM1
	Correct numerical expression for the area of *ORQP*. **The expressions must be fully correct for this mark i.e. no follow through.**		A1
	Correct exact area e.g. $54\frac{1}{3}$, $\frac{163}{3}$, $\frac{326}{6}$, $54.\dot{3}$ or any exact equivalent		A1

	Shape	Vertices	Numerical Expression	Exact Area	
	Triangle	*TRQ*	$\frac{1}{2}\times 7\times\left(\frac{44}{3}-3\right)$	$\frac{245}{6}$	
	Triangle	*SPO*	$\frac{1}{2}\times\frac{6}{5}\times 2$	$\frac{6}{5}$	
	Triangle	*PWQ*	$\frac{1}{2}\times\left(\frac{44}{5}-2\right)\times 3$	$\frac{51}{5}$	
	Triangle	*PVQ*	$\frac{1}{2}\times(7-2)\times 3$	$\frac{15}{2}$	

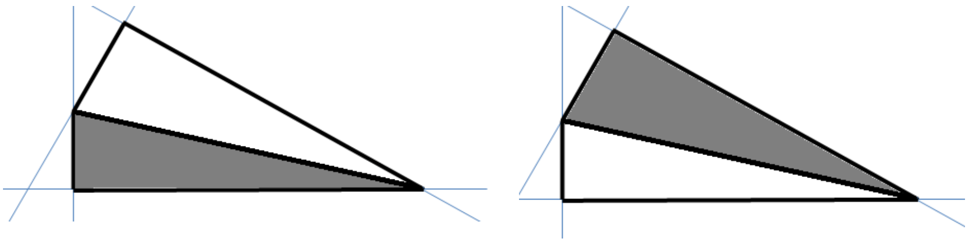
	Triangle	*VWQ*	$\frac{1}{2}\times\left(\frac{44}{5}-7\right)\times3$	$\frac{27}{10}$	
	Triangle	*QUR*	$\frac{1}{2}\times\left(\frac{44}{3}-3\right)\times7$	$\frac{245}{6}$	
	Triangle	*PQR*	$\frac{1}{2}\times\sqrt{34}\times\frac{7}{3}\times\sqrt{34}$	$\frac{119}{3}$	
	Triangle	*PNQ*	$\frac{1}{2}\times\frac{34}{3}\times5$	$\frac{85}{3}$	
	Triangle	*OPQ*	$\frac{1}{2}\times2\times3$	3	
	Triangle	*OQR*	$\frac{1}{2}\times\frac{44}{3}\times7$	$\frac{154}{3}$	
	Triangle	*OWR*	$\frac{1}{2}\times\frac{44}{3}\times\frac{44}{5}$	$\frac{968}{15}$	
	Triangle	*SQR*	$\frac{1}{2}\times\left(\frac{44}{3}+\frac{6}{5}\right)\times7$	$\frac{833}{15}$	
	Triangle	*OPR*	$\frac{1}{2}\times\frac{44}{3}\times2$	$\frac{44}{3}$	
	Trapezium	*OPQT*	$\frac{1}{2}(2+7)\times3$	$\frac{27}{2}$	
	Trapezium	*OPNR*	$\frac{1}{2}\times\left(\frac{34}{3}+\frac{44}{3}\right)\times2$	26	
	Trapezium	*OVQR*	$\frac{1}{2}\times\left(3+\frac{44}{3}\right)\times7$	$\frac{371}{6}$	
	EXAMPLES				
(c)	**WAY 1**				
	$OPQT=\frac{1}{2}(2+7)\times3$ or $TRQ=\frac{1}{2}\times7\times\left(\frac{44}{3}-3\right)$		M1: Correct method for *OPQT* or *TRQ* A1ft: $OPQT=\frac{1}{2}(2+7)\times3$ or $TRQ=\frac{1}{2}\times7\times\left(\frac{44}{3}-3\right)$		M1A1ft
	$\frac{1}{2}(2+7)\times3+\frac{1}{2}\times7\times\left(\frac{44}{3}-3\right)$		dM1: Correct numerical combination of areas that have been calculated correctly A1: **Fully Correct** numerical expression for the area *ORQP*		dM1A1
	$54\frac{1}{3}$		Any exact equivalent e.g. $\frac{163}{3}$, $\frac{326}{6}$, $54.\dot{3}$		A1





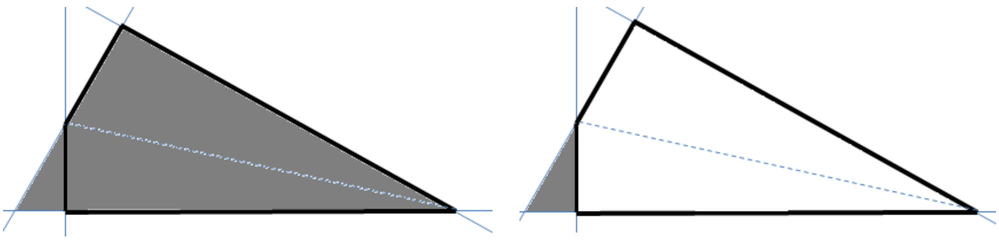
$$\frac{1}{2}\times(7+2)\times3+\frac{1}{2}\times\frac{"35"}{3}\times7$$
$$=\frac{27}{2}+\frac{245}{6}=\frac{326}{6}$$

	WAY 2		
	$PQR = \frac{1}{2}\times\sqrt{34}\times\frac{7}{3}\times\sqrt{34}$ **or** $OPR = \frac{1}{2}\times\frac{44}{3}\times 2$	M1: Correct method for PQR or OPR A1ft: $PQR = \frac{1}{2}\times\sqrt{34}\times\frac{7}{3}\times\sqrt{34}$ or $OPR = \frac{1}{2}\times\frac{44}{3}\times 2$	M1A1ft
	$\frac{1}{2}\times\sqrt{34}\times\frac{7}{3}\times\sqrt{34}+\frac{1}{2}\times\frac{44}{3}\times 2$	dM1: Correct numerical combination of areas that have been calculated correctly A1: **Fully Correct** numerical expression for the area $ORQP$	dM1A1
	$54\frac{1}{3}$	Any exact equivalent e.g. $\frac{163}{3}$, $\frac{326}{6}$, $54.\dot{3}$	A1



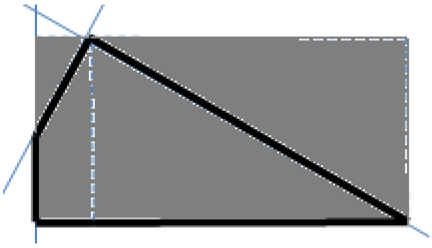
$$\frac{1}{2}\times\frac{"44"}{3}\times 2+\frac{1}{2}\times\sqrt{34}\times"\frac{7}{3}\sqrt{34}"$$
$$=\frac{88}{6}+\frac{238}{6}=\frac{326}{6}$$

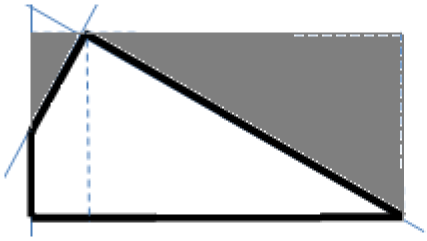
	WAY 3		
	$SQR = \frac{1}{2}\times 7\times\frac{238}{15}$ **or** $SPO = \frac{1}{2}\times\frac{6}{5}\times 2$	M1: Correct method for SQR or SPO A1ft: $SQR = \frac{1}{2}\times 7\times\frac{238}{15}$ or $SPO = \frac{1}{2}\times\frac{6}{5}\times 2$	M1A1ft
	$\frac{1}{2}\times 7\times\frac{238}{15}-\frac{1}{2}\times\frac{6}{5}\times 2$	dM1: Correct numerical combination of areas that have been calculated correctly A1: **Fully Correct** numerical expression for the area $ORQP$	dM1A1
	$54\frac{1}{3}$	Any exact equivalent e.g. $\frac{163}{3}$, $\frac{326}{6}$, $54.\dot{3}$	A1



$$\frac{1}{2}\times\frac{"238"}{15}\times 7-\frac{1}{2}\times\frac{"6"}{5}\times 2$$
$$=\frac{1666}{30}-\frac{6}{5}=\frac{1630}{30}$$

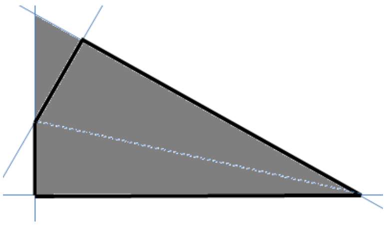
	WAY 4		
	$PVQ = \frac{1}{2}\times 5\times 3$ **or** $QUR = \frac{1}{2}\times 7\times \frac{35}{3}$	M1: Correct method for PVQ or QUR A1ft: $PVQ = \frac{1}{2}\times 5\times 3$ or $QUR = \frac{1}{2}\times 7\times \frac{35}{3}$	M1A1ft
	$OVUR\ 7\times\frac{44}{3} - \frac{1}{2}\times 5\times 3 - \frac{1}{2}\times 7\times\frac{35}{3}$	dM1: Correct numerical combination of areas that have been calculated correctly A1: **Fully Correct** numerical expression for the area $ORQP$	dM1A1
	$54\frac{1}{3}$	Any exact equivalent e.g. $\frac{163}{3}$, $\frac{326}{6}$, $54.\dot{3}$	A1

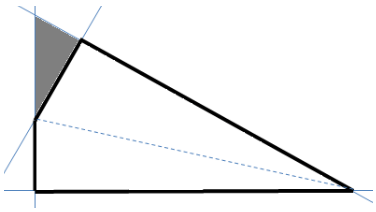




$$7\times\frac{"44"}{3} - \frac{1}{2}\times 5\times 3 - \frac{1}{2}\times\frac{"35"}{3}\times 7$$
$$= \frac{308}{3} - \frac{15}{2} - \frac{245}{6} = \frac{326}{6}$$

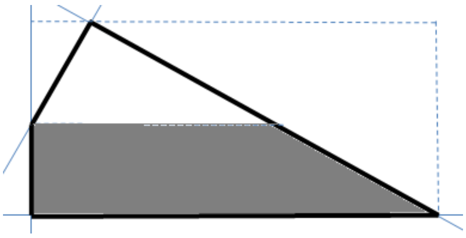
	WAY 5		
	$OWR = \frac{1}{2}\times\frac{44}{3}\times\frac{44}{5}$ **or** $PWQ = \frac{1}{2}\times\left(\frac{44}{5} - 2\right)\times 3$	M1: Correct method for OWR or PWQ A1ft: $OWR = \frac{1}{2}\times\frac{44}{3}\times\frac{44}{5}$ or $PWQ = \frac{1}{2}\times\left(\frac{44}{5} - 2\right)\times 3$	M1A1ft
	$\frac{1}{2}\times\frac{44}{3}\times\frac{44}{5} - \frac{1}{2}\times\left(\frac{44}{5} - 2\right)\times 3$	dM1: Correct numerical combination of areas that have been calculated correctly A1: **Fully Correct** numerical expression for the area $ORQP$	dM1A1
	$54\frac{1}{3}$	Any exact equivalent e.g. $\frac{163}{3}$, $\frac{326}{6}$, $54.\dot{3}$	A1

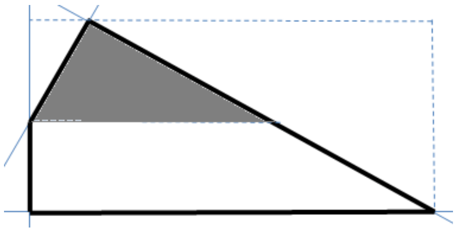




$$\frac{1}{2}\times\frac{"44"}{5}\times\frac{"44"}{3} - \frac{1}{2}\times\left(\frac{44}{5} - 2\right)\times 3$$
$$= \frac{968}{15} - \frac{51}{5} = \frac{163}{3}$$

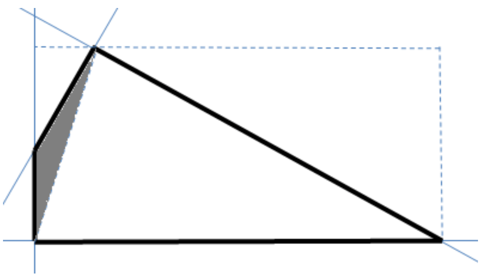
	WAY 6		
	$OPNR = \frac{1}{2}\times\left(\frac{34}{3}+\frac{44}{3}\right)\times 2$ **or** $PNQ = \frac{1}{2}\times\frac{34}{3}\times 5$	M1: Correct method for *OPNR* or *PNQ* A1ft: $OPNR = \frac{1}{2}\times\left(\frac{34}{3}+\frac{44}{3}\right)\times 2$ **or** $PNQ = \frac{1}{2}\times\frac{34}{3}\times 5$	M1A1ft
	$\frac{1}{2}\times\left(\frac{34}{3}+\frac{44}{3}\right)\times 2+\frac{1}{2}\times\frac{34}{3}\times 5$	dM1: Correct numerical combination of areas that have been calculated correctly A1: **Fully Correct** numerical expression for the area *ORQP*	dM1A1
	$54\frac{1}{3}$	Any exact equivalent e.g. $\frac{163}{3}$, $\frac{326}{6}$, $54.\dot{3}$	A1

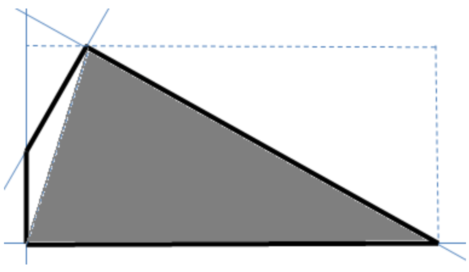




$$\frac{1}{2}\times\left(\frac{"34"}{3}+\frac{"44"}{3}\right)\times 2+\frac{1}{2}\times\frac{"34"}{3}\times 5$$
$$=\frac{156}{6}+\frac{170}{6}=\frac{326}{6}$$

	WAY 7		
	$OPQ = \frac{1}{2}\times 3\times 2$ **or** $OQR = \frac{1}{2}\times\frac{44}{3}\times 7$	M1: Correct method for *OPQ* or *OQR* A1ft: $OPQ = \frac{1}{2}\times 3\times 2$ **or** $OQR = \frac{1}{2}\times\frac{44}{3}\times 7$	M1A1ft
	$\frac{1}{2}\times 3\times 2+\frac{1}{2}\times\frac{44}{3}\times 7$	dM1: Correct numerical combination of areas that have been calculated correctly A1: **Fully Correct** numerical expression for the area *ORQP*	dM1A1
	$54\frac{1}{3}$	Any exact equivalent e.g. $\frac{163}{3}$, $\frac{326}{6}$, $54.\dot{3}$	A1

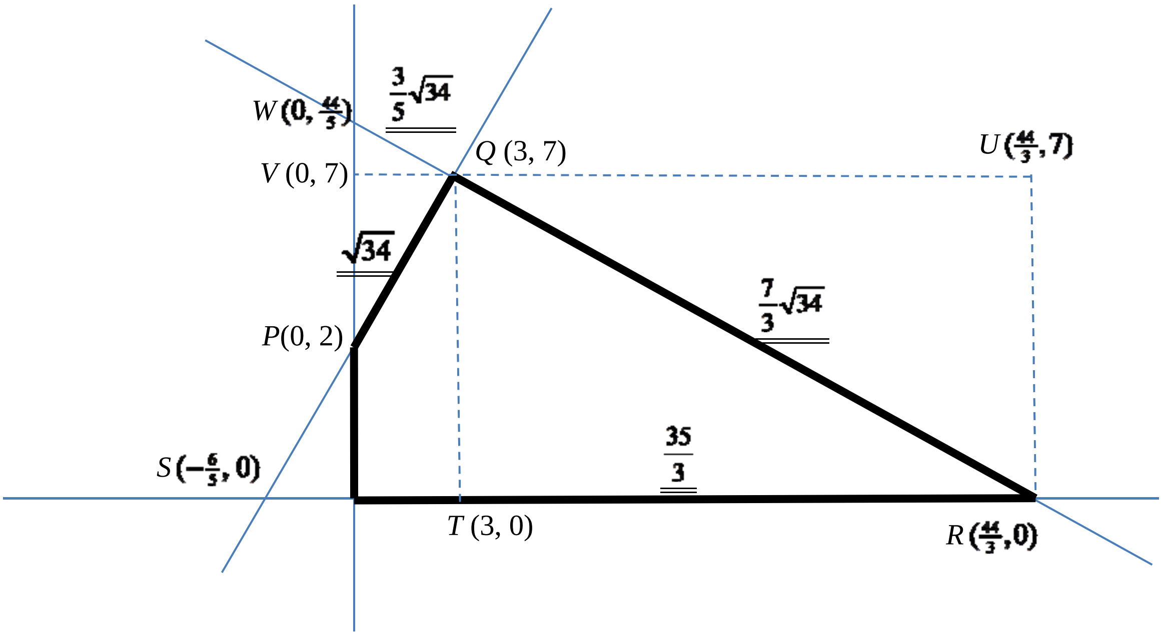


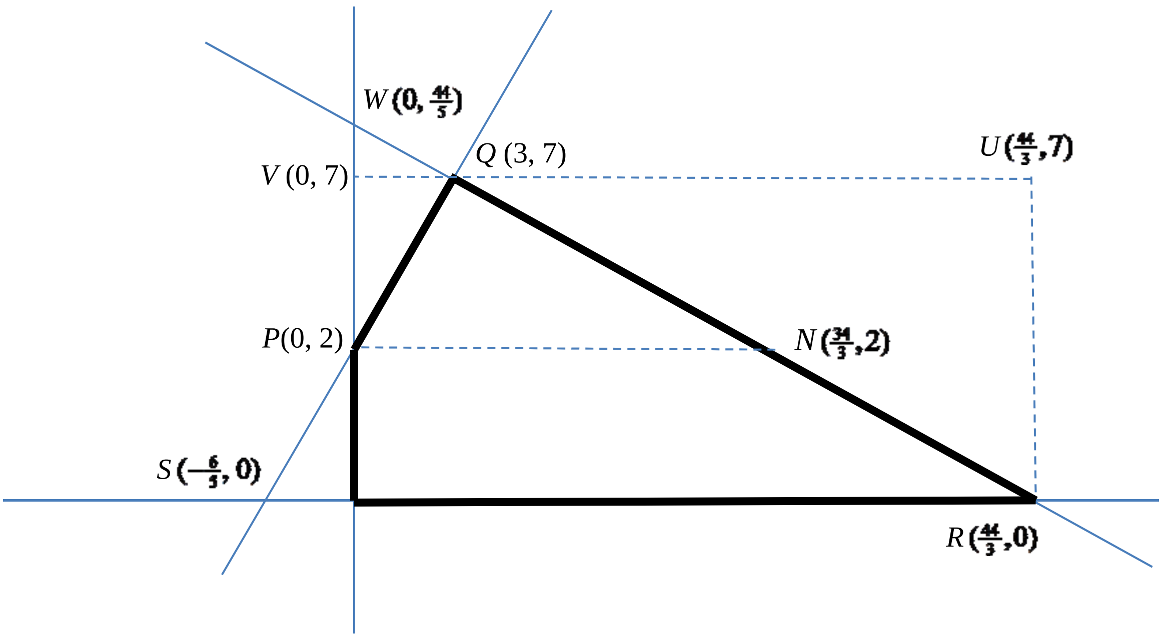


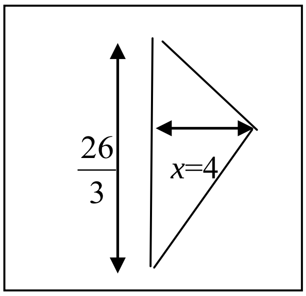
$$\frac{1}{2}\times 3\times 2+\frac{1}{2}\times\frac{"44"}{3}\times 7$$
$$=3+\frac{308}{6}=\frac{326}{6}$$

	WAY 8		
	$\frac{1}{2}\begin{vmatrix} 0 & \frac{44}{3} & 3 & 0 & 0 \\ 0 & 0 & 7 & 2 & 0 \end{vmatrix}$	M1: Uses the vertices of the quadrilateral to form a determinant $\begin{vmatrix} 0 & \frac{44}{3} & 3 & 0 & 0 \\ 0 & 0 & 7 & 2 & 0 \end{vmatrix}$ A1ft: $\frac{1}{2}\begin{vmatrix} 0 & \frac{44}{3} & 3 & 0 & 0 \\ 0 & 0 & 7 & 2 & 0 \end{vmatrix}$	M1A1ft
	$\frac{1}{2}\left(\frac{44}{3}\times 7 + 3\times 2\right)$	dM1: Fully correct determinant method with no errors A1: **Fully Correct** numerical expression for the area *ORQP*	dM1A1
	$54\frac{1}{3}$	Any exact equivalent e.g. $\frac{163}{3}$, $\frac{326}{6}$, $54.\dot{3}$	A1

There will be other ways but the same approach to marking should be applied.





Question Number	Scheme	Marks
36.	(a) $2x+3y=26 \Rightarrow 3y=26\pm 2x$ and attempt to find *m* from *y* = *mx* + *c*	M1
	$\left(\Rightarrow y=\frac{26}{3}-\frac{2}{3}x\right)$ so gradient = $-\frac{2}{3}$	A1
	Gradient of perpendicular $=\frac{-1}{\text{their gradient}}$ $\left(=\frac{3}{2}\right)$	M1
	Line goes through (0,0) so $y=\frac{3}{2}x$	A1
		(4)
	(b) Solves their $y=\frac{3}{2}x$ with their $2x+3y=26$ to form equation in *x* or in *y*	M1
	Solves their equation in *x* or in *y* to obtain *x* = **or** *y* =	dM1
	x=4 or any equivalent e.g. 156/39 or $y=6$ o.a.e	A1
	B= $(0, \frac{26}{3})$ used or stated in (b)	B1
	 **Method 1** (see other methods in notes below)	
	Area $=\frac{1}{2}\times"4"\times\frac{"26"}{3}$	dM1
	$=\frac{52}{3}$ (oe with integer numerator and denominator)	A1
		(6)
		(10 marks)

Notes

(a) M1 Complete method for finding gradient. (This may be implied by later correct answers.)
e.g. Rearranges $2x+3y=26 \Rightarrow y=mx+c$ so $m=$
Or finds coordinates of two points on line and finds gradient e.g. $(13,0)$ *and* $(1,8)$ *so* $m=\frac{8-0}{1-13}$

A1 States or implies that gradient = $-\frac{2}{3}$ - condone $-\frac{2}{3}x$ if they continue correctly. Ignore errors in constant term in straight line equation

M1 Uses $m_1\times m_2=-1$ to find the gradient of l_2. This can be implied by the use of $\frac{-1}{\text{their gradient}}$

A1 $y=\frac{3}{2}x$ or $2y-3x=0$ Allow $y=\frac{3}{2}x+0$ Also accept $2y=3x$, $y=39/26x$ or even $y-0=\frac{3}{2}(x-0)$ and isw

Notes Continued

(b) M1 Eliminates variable between their $y=\frac{3}{2}x$ and their (possibly rearranged) $2x+3y=26$ to form an equation in *x* or *y*. (They may have made errors in their rearrangement)

dM1 (Depends on previous M mark) Attempts to solve their equation to find the value of *x* or *y*

A1 $x = 4$ or equivalent or $y = 6$ or equivalent

B1 *y* coordinate of *B* is $\frac{26}{3}$ (stated or implied) - isw if written as ($\frac{26}{3}$, 0). **Must be used or stated in (b)**

dM1 (Depends on previous M mark) Complete method to find area of triangle *OBC* (using their values of *x* and/or *y* at point *C* and their 26/3)

A1 Cao $\frac{52}{3}$ or $\frac{104}{6}$ or $\frac{1352}{78}$ o.e

Method 1:
Uses the area of a triangle formula ½×*OB* ×(*x* coordinate of *C*)

Alternative methods: Several Methods are shown below. The only mark which differs from Method 1 is the last M mark and its use in each case is described below:

Method 2 in 9(b) using $\frac{1}{2}\times BC\times OC$

dM1 Uses the area of a triangle formula ½×*BC* ×*OC* Also finds OC (=√52) and BC= ($\frac{4}{3}\sqrt{13}$)

Method 3 in 9(b) using $\frac{1}{2}\begin{vmatrix}0 & 4 & 0 & 0\\ 0 & 6 & \frac{26}{3} & 0\end{vmatrix}$

dM1 States the area of a triangle formula $\frac{1}{2}\begin{vmatrix}0 & 4 & 0 & 0\\ 0 & 6 & \frac{26}{3} & 0\end{vmatrix}$ or equivalent with their values

Method 4 in 9(b) using area of triangle *OBX* – area of triangle *OCX* where *X* is point (13, 0)

dM1 Uses the correct subtraction $\frac{1}{2}\times 13\times"\frac{26}{3}"-\frac{1}{2}\times 13\times "6"$

Method 5 in 9(b) using area = ½ (6 × 4) + ½ (4 × 8/3) drawing a line from C parallel to the *x* axis and dividing triangle into two right angled triangles

dM1 for correct method area = ½ (“6” × “ 4”) + ½ (“4” × [“26/3”-“6”])

Method 6 Uses calculus

dM1 $\int_0^4 "\frac{26}{3}"-\frac{2x}{3}-\frac{3x}{2}\,dx = \left[\frac{26}{3}x-\frac{x^2}{3}-\frac{3x^2}{4}\right]_0^4$

Question Number	Scheme	Marks
37.(a)	Method 1: $gradient = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2-(-4)}{-1-7}, = -\frac{3}{4}$ Method 2: $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$, so $\frac{y - y_1}{6} = \frac{x - x_1}{-8}$	M1, A1
	$y - 2 = -\frac{3}{4}(x+1)$ or $y + 4 = -\frac{3}{4}(x-7)$ or $y = their\,'-\frac{3}{4}'x + c$	M1
	$\Rightarrow \pm(4y + 3x - 5) = 0$	A1 (4)
	Method 3: Substitute $x = -1, y = 2$ and $x = 7, y = -4$ into $ax + by + c = 0$	M1
	$-a + 2b + c = 0$ and $7a - 4b + c = 0$	A1
	Solve to obtain $a = 3$, $b = 4$ and $c = -5$ or multiple of these numbers	M1 A1 (4)
(b)	Attempts $gradient\ LM \times gradient\ MN = -1$ so $-\frac{3}{4} \times \frac{p+4}{16-7} = -1$ or $\frac{p+4}{16-7} = \frac{4}{3}$ Or $(y+4) = \frac{4}{3}(x-7)$ equation with $x =$ 16 substituted	M1
	$p + 4 = \frac{9 \times 4}{3} \Rightarrow p = ...$, $p = 8$ So $y =$, $y = 8$	M1, A1
		(3)
Alternative for (b)	Attempt Pythagoras: $(p+4)^2 + 9^2 + (6^2 + 8^2) = (p-2)^2 + 17^2$	M1
	So $p^2 + 8p + 16 + 81 + 36 + 64 = p^2 - 4p + 4 + 289 \Rightarrow p = ...$	M1
	$p = 8$	A1
		(3)
(c)	Either $(y=)\ p + 6$ or $2 + p + 4$ Or use 2 perpendicular line equations through L and N and solve for y	M1
	$(y =)\ 14$	A1
		(2) (9 marks)

(a) M1 Uses the gradient formula with points L and M i.e. quote $gradient = \frac{y_1 - y_2}{x_1 - x_2}$ and attempt to substitute correct numbers. Formula may be implied by the correct $\frac{2-(-4)}{-1-7}$ or equivalent.

A1 Any correct single fraction gradient i.e $\frac{6}{-8}$ or equivalent

M1 Uses their gradient with either (-1, 2) or (7, -4) to form a linear equation

Eg $y - 2 = their\,'-\frac{3}{4}'(x+1)$ or $y + 4 = their\,'-\frac{3}{4}'(x-7)$ or $y = their\,'-\frac{3}{4}'x + c$ then find a value for c by substituting (-1,2) or (7, -4) in the correct way(not interchanging x and y)

A1 Accept $\pm k(4y + 3x - 5) = 0$ with k an integer (This implies previous M1)

(b) M1 Attempts to use $gradient\ LM \times gradient\ MN = -1$. ie. $-\frac{3}{4} \times \frac{p+4}{16-7} = -1$ (allow sign errors)

Or Attempts Pythagoras correct way round (allow sign errors)

M1 An attempt to solve their linear equation in ‘p’. A1 cao $p = 8$

(c) M1 For using their numerical value of p and adding 6 . This may be done by any complete method (vectors, drawing, perpendicular straight line equations through L and N) or by no method. Assuming $x = 7$ is M0

A1 Accept 14 for both marks as long as no incorrect working seen (Ignore left hand side – allow k) . If there is wrong working resulting fortuitously in 14 give M0A0. Allow (8, 14) as the answer.

Question Number	Scheme		Marks
38	(-1, 3) , (11, 12)		
(a)	$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{12-3}{11-(-1)}, = \dfrac{3}{4}$	M1:Correct method for the gradient A1: Any correct fraction or decimal	M1,A1
	$y-3$ = ¾ $(x+1)$ or $y-12$ = ¾ $(x-11)$ or y = ¾ $x + c$ with attempt at substitution to find c	Correct straight line method using either of the given points and a numerical gradient.	M1
	$4y - 3x - 15 = 0$	Or equivalent with integer coefficients (= 0 **is** required)	A1
	This A1 should only be awarded in (a)		
			(4)
(a) Way 2	$\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1} \Rightarrow \dfrac{y-3}{12-3} = \dfrac{x+1}{11+1}$	M1: Use of a correct formula for the straight line A1: Correct equation	M1A1
	$12(y-3) = 9(x+1)$	Eliminates fractions	M1
	$4y - 3x - 15 = 0$	Or equivalent with integer coefficients (= 0 **is** required)	A1
			(4)
(b)	Solves their equation from part (a) and L_2 simultaneously to eliminate one variable	Must reach as far as an equation in x only or in y only. (Allow slips in the algebra)	M1
	$x = 3$ or $y = 6$	One of $x = 3$ or $y = 6$	A1
	Both $x = 3$ **and** $y = 6$	Values can be un-simplified fractions.	A1
	Fully correct answers with no working can score 3/3 in (b)		
			(3)
(b) Way 2	$(-1,3) \to -a + 3b + c = 0$ $(11,12) \to 11a + 12b + c = 0$	Substitutes the coordinates to obtain two equations	M1
	$\therefore a = -\dfrac{3}{4}b, \; b = -\dfrac{4}{15}c$	Obtains sufficient equations to establish values for a, b and c	A1
	e.g. $c = 1 \Rightarrow b = -\dfrac{4}{15}, \; a = \dfrac{3}{15}$	Obtains values for a, b and c	M1
	$\dfrac{3}{15}x - \dfrac{4}{15}y + 1 = 0 \Rightarrow 4y - 3x - 15 = 0$	Correct equation	A1
			(4)
			[7]

Question Number	Scheme	Notes	Marks
39(a)	$4x+2y-3=0 \Rightarrow y=-2x+\frac{3}{2}$	Attempt to write in the form $y =$	M1
	$\Rightarrow$ gradient $=-2$	Accept any un-simplified form and allow even with an incorrect value of "c"	A1
(a) Way 2	Alternative: $4+2\frac{\mathrm{d}y}{\mathrm{d}x}=0$	Attempt to differentiate Allow $p \pm q\frac{\mathrm{d}y}{\mathrm{d}x}=0,\ p,q \neq 0$	M1
	$\Rightarrow$ gradient $=-2$	Accept any un-simplified form	A1
	Answer only scores M1A1		
			[2]
(b)	Using $m_N=-\frac{1}{m_T}$	Attempt to use $m_N = -\frac{1}{gradient\ from\ (a)}$	M1
	$y-5='\frac{1}{2}'(x-2)$ or Uses $y = mx + c$ in an attempt to find c	Correct straight line method using a 'changed' gradient and the point (2, 5)	M1
	$y=\frac{1}{2}x+4$	Cao (Isw)	A1
			(3)
			[5]

Question Number	Scheme	Notes	Marks
40(a)	$y = x + 2 \Rightarrow x^2 + 4(x+2)^2 - 2x = 35$	Substitute $y = \pm x \pm 2$ into $x^2 + 4y^2 - 2x = 35$ to obtain an equation in x only.	M1
	Alternative: $\dfrac{2x - x^2 + 35}{4} = (x+2)^2$ or $\sqrt{\dfrac{2x - x^2 + 35}{4}} = (x+2)$		
	$5x^2 + 14x - 19 = 0$	Multiply out and collects terms producing 3 term quadratic in any form.	M1
	$(5x+19)(x-1) = 0 \Rightarrow x = ..$	Solves their quadratic, usual rules, as far as x = ... **Dependent on the first M1** i.e. a correct method for eliminating y (or x – see below)	dM1
	$x = -\dfrac{19}{5}, x = 1$	Both correct	A1 for both
	$y = -\dfrac{9}{5}, y = 3$	M1: Substitutes back into either given equation to find a value for y	M1
	Coordinates are ($-\dfrac{19}{5}, -\dfrac{9}{5}$) and (1, 3)	Correct matching pairs. Coordinates need not be given explicitly but it must be clear which x goes with which y	A1
			(6)
Alternative to part (a)	$x = y - 2 \Rightarrow (y-2)^2 + 4y^2 - 2(y-2) =$	Substitutes $x = \pm y \pm 2$ into $x^2 + 4y^2 - 2x = 35$	M1
	$5y^2 - 6y - 27 = 0$	Multiply out, collect terms producing 3 term quadratic in any form.	M1
	$(5y+9)(y-3) = 0 \Rightarrow y = ..$	Solves their quadratic, usual rules, as far as y = ... **Dependent on the first M1** i.e. a correct method for eliminating x	dM1
	$y = -\dfrac{9}{5}, y = 3$	Both correct	A1 for both
	$x = -\dfrac{19}{5}, x = 1$	M1: Substitutes back into either given equation to find a value for x	M1
	Coordinates are ($-\dfrac{19}{5}, -\dfrac{9}{5}$) and (1,3)	Correct matching pairs as above.	A1
(b)	$d^2 = (1 - -\dfrac{19}{5})^2 + (3 - -\dfrac{9}{5})^2$ or $d = \sqrt{(1 - -\dfrac{19}{5})^2 + (3 - -\dfrac{9}{5})^2}$	M1: Use of $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$ or $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ where neither $(x_1 - x_2)$ nor $(y_1 - y_2)$ are zero. A1ft: Correct ft expression for d or d^2 (may be un-simplified)	M1A1ft
	$d = \dfrac{24}{5}\sqrt{2}$	Allow $4.8\sqrt{2}$	A1cao
			(3)
			[9]

EXPERT TUITION

Question Number	Scheme	Marks
41.		
(a)	Gradient of l_2 is $\frac{1}{2}$ or 0.5 or $\frac{-1}{-2}$	B1
	Either $y-6="\frac{1}{2}"(x-5)$ or $y="\frac{1}{2}"x+c$ and $6="\frac{1}{2}"(5)+c \Rightarrow c=("\frac{7}{2}")$	M1
	$x-2y+7=0$ or $-x+2y-7=0$ or $k(x-2y+7)=0$ with ***k* an integer**	A1 **[3]**
(b)	Puts $x=0$, or $y=0$ in **their** equation and solves to find appropriate co-ordinate	M1
	x-coordinate of *A* is -7 and *y*-coordinate of *B* is $\frac{7}{2}$.	A1 cao **[2]**
(c)	Area $OAB=\frac{1}{2}(7)\left(\frac{7}{2}\right)=\frac{49}{4}$ (units)^2 Applies $\pm\frac{1}{2}$(base)(height)	M1
	$\frac{49}{4}$	A1**cso** **[2]**
		7 marks
	Notes	
(a)	**B1**: Must have ½ or 0.5 or $\frac{-1}{-2}$ o.e. stated and stops, or used in their line equation **M1:** Full method to obtain an equation of the line through (5,6) with their "*m*". So $y-6=m(x-5)$ with their gradient or uses $y=mx+c$ with $(5,6)$ and their gradient to find *c*. Allow any numerical gradient here including –2 or –1 but **not zero** . (Allow (6,5) as a slip **if** $y-y_1=m(x-x_1)$ is quoted first) **A1:** Accept any multiple of the correct equation, provided that the coefficients are integers and equation = 0 e.g. $-x+2y-7=0$ or $k(x-2y+7)=0$ or even $2y-x-7=0$	
(b)	**M1:** Either one of the *x* or *y* coordinates using their equation **A1:** Needs both correct values. Accept any correct equivalent.. Need not be written as co-ordinates. **Even** just –7 and 3.5 with **no** indication which is which may be awarded the A1.	
(c)	**M1:** Any correct method for area of triangle *AOB*, with their values for co-ordinates of *A* and *B* (may include negatives) *Method usually half base times height but determinants could be used.* **A1**: Any exact equivalent to 49/4, e.g. 12.25. (**negative final** answer is A0 but replacing by **positive** is A1) Do not need units. **c.s.o.** implies if A0 is scored in (b) then A0 is scored in (c) as well**. However** if candidate has correct line equation in (a) of wrong form may score A0 in (a) and A1 in (b) and (c)	
	Note: Special cases: $\frac{1}{2}(-7)\left(+\frac{7}{2}\right)=-\frac{49}{4}$ (units)^2 is M1 A0 but changing sign to area $=+\frac{49}{4}$ gets M1A1 (recovery) **N.B.** Candidates making sign errors in (b) and obtaining +7 and $-\frac{7}{2}$. may also get $\frac{49}{4}$ as their answer following previous errors. They should be awarded A0 as this answer is not ft and is for correct **solution** only **Special Case**: In (a) and (b): Produces parallel line instead of perpendicular line: So uses $m=-2$ This is not treated as a misread as it simplifies the question. The marks will usually be B0 M1 A0, M1 A0, M1 A0 i.e. maximum of 3/7	

Question Number	Scheme	Marks
	L_1: $4y + 3 = 2x \Rightarrow y = \frac{1}{2}x - \frac{3}{4}$; $A(p, 4)$ lies on L_1.	
42. (a)	$\{p =\}\ 9\frac{1}{2}$ or $\frac{19}{2}$ or 9.5	B1
		[1]
(b)	$\{4y + 3 = 2x\} \Rightarrow y = \frac{2x-3}{4} \Rightarrow m(L_1) = \frac{1}{2}$ or $\frac{2}{4}$	M1 A1
	So $m(L_2) = -2$	B1ft
	L_2: $y - 4 = -2(x - 2)$	M1
	L_2: $2x + y - 8 = 0$ or L_2: $2x + 1y - 8 = 0$	A1
		[5]
(c)	$\{L_1 = L_2 \Rightarrow\}\ 4(8-2x) + 3 = 2x$ or $-2x + 8 = \frac{1}{2}x - \frac{3}{4}$	M1
	$x = 3.5,\ y = 1$	A1, A1 cso
		[3]
(d)	$CD^2 = ("3.5" - 2)^2 + ("1" - 4)^2$	"M1"
	$CD = \sqrt{("3.5" - 2)^2 + ("1" - 4)^2}$	A1 ft
	$= \sqrt{1.5^2 + 3^2} = 1.5\sqrt{1^2 + 2^2} = 1.5\sqrt{5}$ or $\frac{3}{2}\sqrt{5}$ **(*)**	A1 **cso**
		[3]
(e)	Area = triangle ABC + triangle ABE	
	$= \frac{1}{2} \times \frac{3}{2}\sqrt{5} \times \sqrt{80} + \frac{1}{2} \times 3\sqrt{5} \times \sqrt{80}$ Finding the area of any triangle.	M1
	$= \frac{3}{4}\sqrt{5} \times 4\sqrt{5} + \frac{3}{2}\sqrt{5} \times 4\sqrt{5}$	
	$= \frac{3}{4}(20) + \frac{3}{2}(20)$	B1
	$= 45$	A1
		[3] **15**

	Notes
42. (a)	**B1:** 9.5 oe.
(b)	**1st M1:** for an attempt to rearrange $4y + 3 = 2x$ into $y = mx + c$. This mark can be implied by the correct gradient of L_1 or L_2. **1st A1:** for gradient of $L_1 = \frac{1}{2}$ or $\frac{2}{4}$. Stating $m(L_1) = \frac{1}{2}$ without working is M1A1. **B1ft:** for applying $m(L_2) = \frac{-1}{\text{their } m(L_1)}$. Need not be simplified. **Note:** Writing down $m(L_2) = -2$ with ***no earlier incorrect working*** gains M1A1B1 **2nd M1:** for applying $y - 4 = \pm\lambda(x - 2)$ where λ is a numerical value, $\lambda \neq 0$. or full method of $y = mx + c$, with $x = 2$, $y = 4$ and (their $\pm\lambda$) to find c. **2nd A1:** $2x + y - 8 = 0$ or $-2x - y + 8 = 0$ or $y + 2x - 8 = 0$ or $4x + 2y - 16 = 0$ or $2x + 1y - 8 = 0$ etc. Must be "$= 0$". So do not allow $2x + y = 8$ etc. **Note:** Condone the error of incorrectly rearranging L_1 to give $y = \frac{1}{2}x - 3 \Rightarrow m(L_1) = \frac{1}{2}$.

(c)	**M1:** for an attempt to solve. Must form a linear equation in one variable. **1st A1:** for $x = 3.5$ (correct solution only). **2nd A1:** for $y = 1$ (correct solution only). **Note:** If $x = 3.5,\ y = 1$ is found from no working, then send to review. **Note:** Use of trial and error to find one of *x* or *y* and then substitution into one of L_1 or L_2 can achieve M1A1A1.
(d)	**M1:** for an attempt at CD^2 - ft their point *D*. Eg: $("3.5" - 2)^2 + ("1" - 4)^2$ or simplified. This mark can be implied by finding *CD*. **1st A1ft:** for finding their *CD* - ft their point *D*. Eg: $\sqrt{("3.5" - 2)^2 + ("1" - 4)^2}$ or correctly simplified. **2nd A1:cso** for no incorrect working seen. **Note:** A candidate initially writing down $\sqrt{1.5^2 + 3^2}$ can be awarded M1A1. ***Alternatives part (d): Final accuracy*** **1.** $\left\{\sqrt{1.5^2 + 3^2} =\right\}\ \sqrt{\frac{9}{4} + 9} = \sqrt{\frac{9}{4} + \frac{36}{4}} = \sqrt{\frac{45}{4}} = \frac{3\sqrt{5}}{2}$ **2.** $\left\{\sqrt{1.5^2 + 3^2} =\right\}\ \sqrt{11.25} = \sqrt{2.25}\sqrt{5} = 1.5\sqrt{5}$
(e)	**M1:** for an attempt at finding the area of either triangle *ABC* or triangle *ABE*. **B1:** Correct method for removing a square root. Eg: $\sqrt{80}\sqrt{5} = \sqrt{400} = 20$ or $\sqrt{5} \times 4\sqrt{5} = 20$ **Note:** This mark can be implied. **A1:** for 45 only. ***Alternative 1 to part (e):*** Area $= \frac{1}{2}\left(\frac{3}{2}\sqrt{5} + 3\sqrt{5}\right)\left(\sqrt{80}\right) = \frac{1}{2}(30 + 60) = 45$ **M1:** $\frac{1}{2}(AB)(CE)$. **B1:** Evidence of correct surd removal. **A1:** for 45. **Note:** Multiplying the diagonals (usually to find 90) is M0, B1 if surds are removed correctly, A0. ***Alternative 2 to part (e):*** Area = triangle DAC + triangle DCB + triangle DEA + triangle DBE $= \left(\frac{1}{2} \times \frac{3}{2}\sqrt{5} \times \sqrt{45}\right) + \left(\frac{1}{2} \times \frac{3}{2}\sqrt{5} \times \left(\sqrt{80} - \sqrt{45}\right)\right) + \left(\frac{1}{2} \times 3\sqrt{5} \times \sqrt{45}\right) + \left(\frac{1}{2} \times 3\sqrt{5} \times \left(\sqrt{80} - \sqrt{45}\right)\right)$ $= \left(\frac{1}{2} \times \frac{3}{2}(15)\right) + \left(\frac{1}{2} \times \frac{3}{2}(5)\right) + \left(\frac{1}{2} \times 3\,(15)\right) + \left(\frac{1}{2} \times 3(5)\right)$ $= \left(\frac{45}{4}\right) + \left(\frac{15}{4}\right) + \left(\frac{45}{2}\right) + \left(\frac{15}{2}\right)$ $= 45$ **M1:** For finding the area of one of the four triangles. **B1:** Evidence of correct surd removal. **A1:** for 45. ***Alternative 3 to part (e):*** $\left\{CE = CD + DE = \frac{3}{2}\sqrt{5} + 3\sqrt{5} = \frac{9}{2}\sqrt{5}\right\}$, $\left\{BD = DA + \underline{AB} = 3\sqrt{5} + \underline{4\sqrt{5}} = 7\sqrt{5}\right\}$ Area = triangle BCE − triangle ACE $= \frac{1}{2}(CE)(BD) - \frac{1}{2}(CE)(BD)$ $= \frac{1}{2} \times \frac{9}{2}\sqrt{5} \times 7\sqrt{5} - \frac{1}{2} \times \frac{9}{2}\sqrt{5} \times 3\sqrt{5}$ **M1:** for an attempt at the area of triangle *BCE* or triangle *ACE*. $= \frac{63(5)}{4} - \frac{27(5)}{4} = \frac{36(5)}{4} = 9(5)$ **B1:** Evidence of correct surd removal. $= 45$ **A1:** for 45 only.

Question	Scheme	Marks
43. (a)	$(m=)\frac{2}{3}$ (or exact equivalent)	B1 (1)
(b)	B: (0, 4) [award when first seen – may be in (c)]	B1
	Gradient: $\frac{-1}{m}=-\frac{3}{2}$	M1
	$y-4=-\frac{3x}{2}$ or equiv. e.g. $\left(y=-\frac{3x}{2}+4, \quad 3x+2y-8=0\right)$	A1 (3)
(c)	A: $(-6,0)$ [award when first seen – may be in (b)]	B1
	C: $\frac{3x}{2}=4 \quad \Rightarrow \quad x=\frac{8}{3}$ [award when first seen – may be in (b)]	B1ft
	Area: Using $\frac{1}{2}(x_C-x_A)y_B$	M1
	$=\frac{1}{2}\left(\frac{8}{3}+6\right)4=\frac{52}{3} \quad \left(=17\frac{1}{3}\right)$	A1 cso (4)
ALT	$BC=\frac{4}{6}\sqrt{52}$ (from similar triangles) (or possibly using C)	2nd B1ft
	Area: Using $\frac{1}{2}(AB\times BC)$ N.B. $AB=\sqrt{6^2+4^2}=\sqrt{52}$	M1
	$=\frac{1}{2}\times\sqrt{52}\times\left(\frac{2}{3}\sqrt{52}\right)=\frac{52}{3} \quad \left(=17\frac{1}{3}\right)$	A1
		8 marks
	Notes	
(a)	B1 for $\frac{2}{3}$ seen. Do not award for $\frac{2}{3}x$ and must be in part (a)	
(b)	B1 for coordinates of B. Accept 4 marked on y-axis (clearly labelled) M1 for use of perpendicular gradient rule. Follow through their value for m A1 for a correct equation (any form, need not be simplified). Answer only 3/3	
(c)	1st B1 for the coordinates of A (clearly labelled). Accept – 6 marked on x-axis 2nd B1ft for the coordinates of C (clearly labelled) or $AC=\frac{26}{3}$. Accept $x=\frac{8}{3}$ marked on x-axis. Follow through from l_2 if >0 M1 for an expression for the area of the triangle (all lengths > 0). Ft their 4, - 6 and $\frac{8}{3}$ A1 cso for $\frac{52}{3}$ or exact equivalent seen but must be a single fraction or $17\frac{1}{3}$ or $17\frac{2}{6}$ etc $17\frac{1}{3}$ on its own can only score full marks if A, B and C are all correct.	
ALT **Use of Det**	2nd B1ft If they use this approach award this mark for C (if seen) or BC 2nd M1 must get as far as: $\frac{1}{2}\left|x_A\times y_B-x_C\times y_B\right|$	

Question Number	Scheme	Marks
44.	Mid-point of *PQ* is (4, 3)	B1
	PQ: $m = \frac{0-6}{9-(-1)}, \left(= -\frac{3}{5}\right)$	B1
	Gradient perpendicular to *PQ* $= -\frac{1}{m} \quad (= \frac{5}{3})$	M1
	$y - 3 = \frac{5}{3}(x-4)$	M1
	$5x - 3y - 11 = 0$ or $3y - 5x + 11 = 0$ or multiples e.g. $10x - 6y - 22 = 0$	A1 (5)
		5
	<u>Notes</u> B1: correct midpoint. B1: correct numerical expression for gradient – need not be simplified 1st M: Negative reciprocal of their numerical value for *m* 2nd M: Equation of a line through **their** (4, 3) with any gradient except 0 or ∞. If the 4 and 3 are the wrong way round the 2nd M mark can still be given if a correct formula (e.g. $y - y_1 = m(x - x_1)$) is seen, otherwise M0. If (4, 3) is substituted into $y = mx + c$ to find *c*, the 2nd M mark is for attempting this. A1: Requires integer form with an = zero (see examples above)	

Question Number	Scheme	Marks
45. (a)	$(8-3-k=0)$ so $\underline{k=5}$	B1 (1)
(b)	$2y=3x+k$ $y=\frac{3}{2}x+...$ and so $m=\frac{3}{2}$ o.e.	M1 A1 (2)
(c)	Perpendicular gradient = $-\frac{2}{3}$ Equation of line is: $y-4=-\frac{2}{3}(x-1)$ $\underline{3y+2x-14=0}$ o.e.	B1ft M1A1ft A1 (4)
(d)	$y=0$, $\Rightarrow$ $B(7,0)$ or $\underline{x=7}$ $x=7$ or $-\frac{c}{a}$	M1A1ft (2)
(e)	$AB^2=(7-1)^2+(4-0)^2$ $AB=\sqrt{52}$ or $2\sqrt{13}$	M1 A1 (2) **11**
	Notes	
(b)	M1 for an attempt to rearrange to $y=\ldots$ A1 for clear statement that gradient is 1.5, can be m =1.5 o.e.	
(c)	B1ft for using the perpendicular gradient rule correctly on their "1.5" M1 for an attempt at finding the equation of the line through A using their gradient. Allow a sign slip 1st A1ft for a correct equation of the line follow through their changed gradient 2nd A1 as printed or equivalent with integer coefficients – allow $\underline{3y+2x=14}$ or $\underline{3y=14-2x}$	
(d)	M1 for use of $y=0$ to find $x=\ldots$in their equation A1ft for $x=7$ or $-\frac{c}{a}$	
(e)	M1 for an attempt to find AB or AB^2 A1 for any correct surd form- need not be simplified	

Question Number	Scheme	Marks
46.		
(a)	$m_{AB} = \dfrac{4-0}{7-2} \quad \left(= \dfrac{4}{5}\right)$	M1
	Equation of AB is: $y - 0 = \dfrac{4}{5}(x-2)$ or $y - 4 = \dfrac{4}{5}(x-7)$ (o.e.)	M1
	$\underline{4x - 5y - 8 = 0}$ (o.e.)	A1 (3)
(b)	$(AB =)\sqrt{(7-2)^2 + (4-0)^2}$	M1
	$= \sqrt{41}$	A1 (2)
(c)	Using isos triangle with $AB = AC$ then $t = 2\times y_A = 2\times 4 = 8$	B1 (1)
(d)	Area of triangle = $\frac{1}{2}t\times(7-2)$	M1
	= $\underline{20}$	A1 (2)
		8

	Notes
	Apply the usual rules for quoting formulae here. For a correctly quoted formula with some correct substitution award M1 If no formula is quoted then a fully correct expression is needed for the M mark
(a)	1^{st} M1 for attempt at gradient of AB. Some correct substitution in correct formula. 2^{nd} M1 for an attempt at equation of AB. Follow through their gradient, not e.g. $-\dfrac{1}{m}$ Using $y = mx + c$ scores this mark when c is found. Use of $\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$ scores 1^{st} M1 for denominator, 2^{nd} M1 for use of a correct point A1 requires integer form but allow $5y + 8 = 4x$ etc. Must have an "=" or A0
(b)	M1 for an expression for AB or AB^2. Ignore what is "left" of the equals sign
(c)	B1 for $t = 8$. May be implied by correct coordinates (2, 8) or the value appearing in (d)
(d)	M1 for an expression for the area of the triangle, follow through their t ($\neq 0$) but must have the $(7-2)$ or 5 and the $\frac{1}{2}$.
DET	e.g. $\begin{matrix} 2 & 7 & 2 & 2 \\ 0 & 4 & t & 0 \end{matrix}$ Area $= \frac{1}{2}\left[8 + 7t + 0 - (0 + 8 + 2t)\right]$ Must have the $\frac{1}{2}$ for M1

Question number	Scheme	Marks
47	(a) Putting the equation in the form $y = mx\ (+c)$ <u>and</u> attempting to extract the m or mx (<u>not</u> the c), or finding 2 points on the line and using the correct gradient formula. Gradient = $-\frac{3}{5}$ (or equivalent)	M1 A1 (2)
	(b) Gradient of perp. line = $\frac{-1}{"\left(-\frac{3}{5}\right)"}$ (Using $-\frac{1}{m}$ with the m from part (a)) $y - 1 = "\left(\frac{5}{3}\right)"(x-3)$ $y = \frac{5}{3}x - 4$ (Must be in this form... allow $y = \frac{5}{3}x - \frac{12}{3}$ but not $y = \frac{5x-12}{3}$) This A mark is dependent upon <u>both</u> M marks.	M1 M1 A1 (3) **[5]**
	(a) Condone sign errors and ignore the c for the M mark, so... both marks can be scored even if c is wrong (e.g. $c = -\frac{2}{5}$) or omitted. <u>Answer only</u>: $-\frac{3}{5}$ scores M1 A1. Any other <u>answer only</u> scores M0 A0. $y = -\frac{3}{5}x + \frac{2}{5}$ with no further progress scores M0 A0 (m or mx not extracted). (b) 2nd M: For the equation, in any form, of a straight line through (3, 1) with <u>any</u> numerical gradient (except 0 or ∞). (Alternative is to use (3, 1) in $y = mx + c$ to <u>find a value</u> for c, in which case $y = \frac{5}{3}x + c$ leading to $c = -4$ is sufficient for the A1). (See general principles for straight line equations at the end of the scheme).	

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>48 (a)

(b)

(c)</td><td>AB: $m = \frac{2-7}{8-6}, \left(= -\frac{5}{2}\right)$

Using $m_1 m_2 = -1$: $m_2 = \frac{2}{5}$

$y - 7 = \frac{2}{5}(x-6)$, $2x - 5y + 23 = 0$ (o.e. with integer coefficients)

Using $x = 0$ in the answer to (a), $y = \frac{23}{5}$ or 4.6

Area of triangle = $\frac{1}{2} \times 8 \times \frac{23}{5} = \frac{92}{5}$ (o.e) e.g. $\left(18\frac{2}{5},\ 18.4,\ \frac{184}{10}\right)$</td><td>B1

M1

M1, A1 (4)

M1, A1ft (2)

M1 A1 (2)

[8]</td></tr>
<tr><td>(a)

(b)

(c)</td><td>B1 for an expression for the gradient of AB. Does not need the = -2.5
1st M1 for use of the perpendicular gradient rule. Follow through their m
2nd M1 for the use of (6, 7) and their changed gradient to form an equation for l.
Can be awarded for $\frac{y-7}{x-6} = \frac{2}{5}$ o.e.
Alternative is to use (6, 7) in $y = mx + c$ to <u>find a value</u> for c. Score when $c = \dots$ is reached.
A1 for a correct equation in the required form and must have "= 0" and integer coefficients
M1 for using $x = 0$ in their answer to part (a) e.g. $-5y + 23 = 0$
A1ft for $y = \frac{23}{5}$ provided that $x = 0$ clearly seen <u>or</u> C (0, 4.6). Follow through their equation in (a)
If $x = 0$, $y = 4.6$ are clearly seen but C is given as (4.6,0) apply ISW and award the mark.
This A mark requires a simplified fraction or an exact decimal
Accept their 4.6 marked on diagram next to C for M1A1ft
M1 for $\frac{1}{2} \times 8 \times y_C$ so can follow through their y coordinate of C.
A1 for 18.4 (o.e.) but their y coordinate of C must be positive

<u>Use of 2 triangles or trapezium and triangle</u>
Award M1 when an expression for area of OCB only is seen

<u>Determinant approach</u>
Award M1 when an expression containing $\frac{1}{2} \times 8 \times y_C$ is seen</td><td></td></tr>
</table>

Question Number	Scheme	Marks
49 (a)	$y-5=-\frac{1}{2}(x-2)$ or equivalent, e.g. $\frac{y-5}{x-2}=-\frac{1}{2}$, $y=-\frac{1}{2}x+6$	M1A1, A1cao (3)
(b)	$x=-2 \Rightarrow$ $y=-\frac{1}{2}(-2)+6$ = 7 (therefore B lies on the line) (or equivalent verification methods)	B1 (1)
(c)	$(AB^2=)$ $(2--2)^2+(7-5)^2$, = 16 + 4 = 20, $AB=\sqrt{20}=2\sqrt{5}$	M1, A1, A1 (3)
(d)	C is $(p,\ -\frac{1}{2}p+6)$, so $AC^2=(p-2)^2+\left(-\frac{1}{2}p+6-5\right)^2$ Therefore $25=p^2-4p+4+\frac{1}{4}p^2-p+1$ $25=1.25p^2-5p+5$ or $100=5p^2-20p+20$ (or better, RHS simplified to 3 terms) Leading to: $0=p^2-4p-16$ (*)	M1 M1 A1 A1cso (4) **[11]**
(a) (b) (c) (d)	M1 A1 The version in the scheme above can be written down directly (for 2 marks), and M1 A0 can be allowed if there is just one slip (sign or number). If the 5 and 2 are the wrong way round the M mark can still be given if a correct formula (e.g. $y-y_1=m(x-x_1)$) is seen, otherwise M0. If (2, 5) is substituted into $y=mx+c$ to find c, the M mark is for attempting this and the 1st A mark is for c = 6. Correct answer without working or from a sketch scores full marks. A conclusion/comment is not required, except when the method used is to establish that the line through $(-2,7)$ with gradient $-\frac{1}{2}$ has the same eqn. as found in part (a), or to establish that the line through $(-2,7)$ and $(2,5)$ has gradient $-\frac{1}{2}$. In these cases a comment 'same equation' or 'same gradient' or 'therefore on same line' is sufficient. M1 for attempting AB^2 or AB. Allow one slip (sign or number) <u>inside</u> a bracket, i.e. do <u>not</u> allow $(2--2)^2-(7-5)^2$. 1st A1 for 20 (condone bracketing slips such as $-2^2=4$) 2nd A1 for $2\sqrt{5}$ or k = 2 (Ignore ± here). 1st M1 for $(p-2)^2+(\text{linear function of } p)^2$. The linear function may be unsimplified but must be equivalent to $ap+b,\ a\neq 0,\ b\neq 0$. 2nd M1 (dependent on 1st M) for forming an equation in p (using 25 or 5) and attempting (perhaps not very well) to multiply out both brackets. 1st A1 for collecting like p terms and having a correct expression. 2nd A1 for correct work leading to printed answer. <u>Alternative, using the result</u>: Solve the quadratic $(p=2\pm 2\sqrt{5})$ and use one or both of the two solutions to find the length of AC^2 or $C_1C_2{}^2$: e.g. $AC^2=(2+2\sqrt{5}-2)^2+(5-\sqrt{5}-5)^2$ scores 1st M1, and 1st A1 if fully correct. Finding the length of AC or AC^2 for both values of p, or finding C_1C_2 with some evidence of halving (or intending to halve) scores the 2nd M1. Getting AC = 5 for both values of p, or showing $\frac{1}{2}C_1C_2=5$ scores the 2nd A1 (cso).	

EXPERT TUITION

Question number	Scheme	Marks
50. (a)	$QR=\sqrt{(7-1)^2+(0-3)^2}$	M1
	$=\sqrt{36+9}$ or $\sqrt{45}$ (condone $\pm$)	A1
	$=3\sqrt{5}$ or $a=3$ ($\pm 3\sqrt{5}$ etc is A0)	A1 (3)
(b)	Gradient of QR (or l_1) = $\dfrac{3-0}{1-7}$ or $\dfrac{3}{-6}, =-\dfrac{1}{2}$	M1, A1
	Gradient of l_2 is $-\dfrac{1}{-\frac{1}{2}}$ or 2	M1
	Equation for l_2 is: $y-3=2(x-1)$ or $\frac{y-3}{x-1}=2$ [or $y = 2x + 1$]	M1 A1ft (5)
(c)	P is (0, 1) (allow "x = 0, y = 1" but it must be clearly identifiable as P)	B1 (1)
(d)	$PQ=\sqrt{(1-x_P)^2+(3-y_P)^2}$	M1
	$PQ=\sqrt{1^2+2^2}=\sqrt{5}$	A1
	Area of triangle is $\frac{1}{2}QR\times PQ=\frac{1}{2}3\sqrt{5}\times\sqrt{5},=\dfrac{15}{2}$ or 7.5	dM1, A1 (4)
	Determinant Method e.g.(0+0+7) - (1+21+0) = - 15 (o.e.) Area = $\frac{1}{2}\lvert -15\rvert$,= 7.5	
		13

Rules for quoting formula: For an M mark, if a correct formula is quoted and <u>some</u> correct substitutions seen then M1 can be awarded, if no values are correct then M0. If no correct formula is seen then M1 can only be scored for a fully correct expression.

(a)
M1 for attempting QR or QR^2. May be implied by 6^2+3^2
1st A1 for as printed or better. Must have square root. Condone $\pm$

(b)
1st M1 for attempting gradient of QR
1st A1 for - 0.5 or $-\frac{1}{2}$, can be implied by gradient of $l_2=2$
2nd M1 for an attempt to use the perpendicular rule on their gradient of QR.
3rd M1 for attempting equation of a line using Q with their changed gradient.
2nd A1ft requires all 3 Ms but can ft their gradient of QR.

$y = 2x + 1$ with no working. Send to review.

(d)
1st M1 for attempting PQ or PQ^2 follow through their coordinates of P
1st A1 for PQ as one of the given forms.
2nd dM1 for correct attempt at area of the triangle. Follow through their value of a and their PQ. This M mark is dependent upon the first M mark
2nd A1 for 7.5 or some exact equivalent. Depends on both Ms. Some working must be seen.

<u>ALT</u>
Use QS where S is (1, 0)
1st M1 for attempting area of $OPQS$ and QSR and OPR. Need all 3.
1st A1 for $OPQS$ = $\frac{1}{2}(1+3)\times 1=2$, QSR = 9, OPR = $\frac{7}{2}$
2nd dM1 for $OPQS$ + QSR − OPR = …Follow through their values.
2nd A1 for 7.5

Determinant Method
M1 for attempt -at least one value in each bracket correct .
A1 if correct ($\pm$ 15)
M1 for correct area formula
A1 for 7.5

<u>MR</u>
Misreading x-axis for y-axis for P. Do NOT use MR rule as this oversimplifies the question. They can only get M marks in (d) if they use PQ and QR.

Question number	Scheme	Marks
51.	(a) $m = \frac{4-(-3)}{-6-8}$ or $\frac{-3-4}{8-(-6)}$, $= \frac{7}{-14}$ or $\frac{-7}{14}$ $\left(= -\frac{1}{2}\right)$	M1, A1
	Equation: $y - 4 = -\frac{1}{2}(x-(-6))$ or $y-(-3) = -\frac{1}{2}(x-8)$	M1
	$x + 2y - 2 = 0$ (or equiv. with <u>integer</u> coefficients… must have '= 0')	A1 (4)
	(e.g. $14y + 7x - 14 = 0$ and $14 - 7x - 14y = 0$ are acceptable)	
	(b) $(-6-8)^2 + (4-(-3))^2$	M1
	$14^2 + 7^2$ or $(-14)^2 + 7^2$ or $14^2 + (-7)^2$ (M1 A1 may be implied by 245)	A1
	$AB = \sqrt{14^2 + 7^2}$ or $\sqrt{7^2(2^2+1^2)}$ or $\sqrt{245}$	
	$7\sqrt{5}$	A1cso (3)
		7
	(a) 1st M: Attempt to use $m = \frac{y_2 - y_1}{x_2 - x_1}$ (may be implicit in an equation of L). 2nd M: Attempting straight line equation in any form, e.g $y - y_1 = m(x - x_1)$, $\frac{y-y_1}{x-x_1} = m$, with any value of m (except 0 or ∞) and either (–6, 4) or (8, –3). N.B. It is also possible to use a different point which lies on the line, such as the midpoint of AB (1, 0.5). Alternatively, the 2nd M may be scored by using $y = mx + c$ with a numerical gradient and substituting (–6, 4) or (8, –3) to find the value of c. Having coords the <u>wrong way round</u>, e.g. $y-(-6) = -\frac{1}{2}(x-4)$, loses the 2nd M mark <u>unless</u> a correct general formula is seen, e.g. $y - y_1 = m(x - x_1)$. (b) M: Attempting to use $(x_2 - x_1)^2 + (y_2 - y_1)^2$. <u>Missing bracket</u>, e.g. $-14^2 + 7^2$ implies M1 if no earlier version is seen. $-14^2 + 7^2$ with no further work would be M1 A0. $-14^2 + 7^2$ followed by 'recovery' can score full marks.	

Question number	Scheme	Marks
52	**You may mark (a) and (b) together** $x^2 + y^2 - 2x + 14y = 0$	
(a)	Obtain LHS as $\underline{(x \pm 1)^2} + \underline{\underline{(y \pm 7)^2}} = \ldots$	M1
	Centre is $(1, -7)$.	A1 **(2)**
(b)	Uses $r^2 = a^2 + b^2$ or $r = \sqrt{a^2 + b^2}$ where their centre was at $(\pm a, \pm b)$ $r = \sqrt{50}$ or $5\sqrt{2}$	M1 A1 **(2)**
(c)	Substitute $x = 0$ in either form of equation of circle and solve resulting quadratic to give $y =$	M1
	$y^2 + 14y = 0$ *so* $y = 0$ *and* -14 **or** $\underline{\underline{(y \pm 7)^2 - 49}} = 0$ *so* $y = 0$ *and* -14	A1 **(2)**
(d)	Gradient of radius joining centre to (2,0) is $\dfrac{"-7"-0}{"1"-2}$ $(= 7)$	M1
	Gradient of tangent is $\dfrac{-1}{m}$ $(= -\dfrac{1}{7})$	M1
	So equation is $y - 0 = -\frac{1}{7}(x - 2)$ and so $x + 7y - 2 = 0$	M1, A1 **(4)** **(10 marks)**
	Alternative Methods which may be seen	
(a)	Method 2: Comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$ to write down centre $(-g, -f)$ directly. Condone sign errors for this M mark**.** Centre is $(1, -7)$.	M1 A1 **(2)**
(b)	Method 2: Using $\sqrt{g^2 + f^2 - c}$. So $r = \sqrt{50}$ or $5\sqrt{2}$	M1 A1 **(2)**
(d)	Method 3: Using Implicit Differentiation	
	$2x + 2y\dfrac{dy}{dx} - 2 + 14\dfrac{dy}{dx} = 0$ **or** $2(x-1) + 2(y+7)\dfrac{dy}{dx} = 0$	M1
	$\dfrac{dy}{dx} = \ldots\left(\dfrac{2-2x}{14+2y} = \dfrac{-2}{14}\right)$	M1
	So equation is $y - 0 = -\frac{1}{7}(x - 2)$ and so $x + 7y - 2 = 0$	M1, A1 (4)
	Method 4: Making y the subject of the formula and differentiating	
	$y = -7 \pm \sqrt{\{50 - (x-1)^2\}}$ so $\dfrac{dy}{dx} = \pm\dfrac{1}{2} \times -2(x-1)\{50 - (x-1)^2\}^{-\frac{1}{2}}$	M1
	At $x = 2$, $\dfrac{dy}{dx} = \mp\dfrac{1}{7}$	M1 (contd next page)

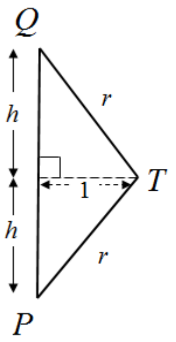
	So equation is $y-0=\mp\frac{1}{7}(x-2)$	M1
	Chooses $\frac{dy}{dx}=-\frac{1}{7}$ and so $x+7y-2=0$	A1

Notes

(a)
M1: as in scheme and can be implied by $(\pm1,\pm7)$ even if this follows some poor working.
A1: (1, −7)
(b)
M1 : Uses $r^2=a^2+b^2$ or $r=\sqrt{a^2+b^2}$ where their centre was at $(\pm a,\pm b)$
A1**:** √50 or 5√2 not 50 only
Special case: if centre is given as $(-1,-7)$ or (1, 7) or (-1, 7) or coordinates given wrong way round- allow M1A1 for $r = 5\sqrt{2}$ worked correctly. $r^2 = "1"+"49"$
If they get $r = 5\sqrt{2}$ after wrong statements such as $r^2 = "-1"+"-49"$ then this is M0A0
$r = 5\sqrt{2}$ with no working earns M1A1 as there is no wrong work.

(c)
M1: As in the scheme – allow for just one value of y
A1: Accept (0, 0), (0, -14) or y = 0, y = -14 or just 0 and -14

(d) Method 1:
M1: Correct method for gradient – if no method shown answer must be correct to earn this mark
If x and y coordinates are confused and fraction is upside down this is M0 even if the formula is quoted as there is no evidence of understanding.
M1: Correct negative reciprocal of their gradient
M1: Line equation through (2,0) with changed gradient so if they use $y = mx + c$ they need to use (2, 0) to find c
A1: For any multiple of the answer in the scheme. (The answer must be an equation so if "=0" is missing this is A0)
(d) Method 3:
M1: Correct implicit differentiation (no errors)
M1: Rearranges their differentiated expression and substitutes x = 2, y = 0 to obtain gradient – allow slips. (It should be $\frac{dy}{dx}=\frac{2-2x}{14+2y}=\left(\frac{-2}{14}\right)$)
If there is no y term this mark may be earned for substitution of x =2 as y = 0 is not needed
M1: Line equation through (2,0) with their obtained gradient so if they use $y = mx + c$ they need to use (2, 0) to find c
A1: For any multiple of the answer in the scheme (The answer must be an equation so if "=0" is missing this is A0)
Method 4:
M1: Correct rearrangement and differentiation (no errors)
M1: Substitutes x = 2 to obtain gradient – allow minus and plus.
M1: Line equation through (2,0) with their obtained gradient so if they use $y = mx + c$ they need to use (2, 0) to find c
A1: For any multiple of the answer in the scheme (The answer must be an equation so if "= 0" is missing this is awarded A0)

Question number	Scheme	Marks
53	$x^2 + y^2 - 10x + 6y + 30 = 0$	
(a)	Uses any appropriate method to find the coordinates of the centre, e.g achieves $\underline{(x \pm 5)^2} + \underline{\underline{(y \pm 3)^2}} = \ldots$. Accept $(\pm5,\pm3)$ as indication of this.	M1
	Centre is $(5, -3)$.	A1
		(2)
(b) Way 1	Uses $\underline{(x \pm "5")^2 - "5^2"} + \underline{\underline{(y \pm "3")^2 - "3^2"}} + 30 = 0$ to give	M1
	$r = \sqrt{"25" + "9" - 30}$ or $r^2 = "25" + "9" - 30$ (not $30 - 25 - 9$)	
	$r = 2$	A1cao
		(2)
Or Way 2	Using $\sqrt{g^2 + f^2 - c}$ from $x^2 + y^2 + 2gx + 2fy + c = 0$ (Needs formula stated or correct working)	M1
	$r = 2$	A1
		(2)
(c) Way 1	Use $x = 4$ in *an* equation of circle and obtain equation in y only	M1
	e.g $(4-5)^2 + (y+3)^2 = 4$ or $4^2 + y^2 - 10 \times 4 + 6y + 30 = 0$	
	Solve their quadratic in y and obtain **two** solutions for y	dM1
	e.g. $(y+3)^2 = 3$ or $y^2 + 6y + 6 = 0$ so $y = -3 \pm \sqrt{3}$	A1
		(3)
Or Way 2	 Divide triangle PTQ and use Pythagoras with $"r"^2 - ("5" - 4)^2 = h^2$,	M1
	Find h and evaluate $"-3" \pm h$. May recognise $(1, \sqrt{3}, 2)$ triangle.	dM1
	So $y = -3 \pm \sqrt{3}$	A1
		(3) **[7]**

EXPERT TUITION

	Notes
(a)	**Parts (a) and (b) can be marked together** **M1** as in scheme and can be <u>implied</u> by $(\pm5, \pm3)$ May be awarded for writing LHS as or by comparing with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g, -f)$ directly **A1:** (5, −3). **This correct answer implies M1A1**
(b)	**M1** for a **full** correct method leading to *r* = …, or $r^2 =$ with their 5, their −3, their 25 and their 9 and their "−30". Completion of square method errors result in **M0** here. Usually *r* = 4 or *r* = 16 imply M0A0 **A1** 2 cao Do not accept $r = \pm2$ unless it is followed by (*r* =) 2 The correct answer with no wrong work seen implies M1A1 **Special case**: if centre is given as (-5, -3) or (5, 3) or (-5, 3) allow **M1A1** for *r* = 2 worked correctly. i.e. $r^2 = "25" + "9" - 30$
(c)	**M1**: *Way 1*: Use *x* = 4 in a circle equation (may have wrong centre and/or radius) to obtain an equation in *y* only or *Way* 2. Uses geometry to find equation in *h* (ft on their radius and centre) **dM1**: (needs first method mark) Solve their quadratic in *y* or *Way* 2. Uses their *h* and their *y* coordinate correctly **A1**: cao

Question Number	Scheme		Marks
54.	$P(7, 8)$ and $Q(10, 13)$		
(a)	$\{PQ =\}\ \sqrt{(7-10)^2 + (8-13)^2}$ or $\sqrt{(10-7)^2 + (13-8)^2}$	Applies distance formula. Can be implied.	M1
	$\{PQ\} = \sqrt{34}$	$\sqrt{34}$ or $\sqrt{17}.\sqrt{2}$	A1
			[2]
(b) **Way 1**	$(x-7)^2 + (y-8)^2 = 34$ $\left(\text{or } \left(\sqrt{34}\right)^2\right)$	$(x \pm 7)^2 + (y \pm 8)^2 = k$, where k is a positive value.	M1
		$(x-7)^2 + (y-8)^2 = 34$	A1 oe
			[2]
(b) **Way 2**	$x^2 + y^2 - 14x - 16y + 79 = 0$	$x^2 + y^2 \pm 14x \pm 16y + c = 0$, where c is any value < 113.	M1
		$x^2 + y^2 - 14x - 16y + 79 = 0$	A1 oe
			[2]
(c) **Way 1**	$\{\text{Gradient of radius}\} = \frac{13-8}{10-7}$ or $\frac{5}{3}$	This must be seen or implied in part (c).	B1
	Gradient of tangent $= -\frac{1}{m}\left(= -\frac{3}{5}\right)$	Using a perpendicular gradient method on their gradient. So Gradient of tangent $= -\frac{1}{\text{gradient of radius}}$	M1
	$y - 13 = -\frac{3}{5}(x-10)$	$y - 13 = (\text{their changed gradient})(x-10)$	M1
	$3x + 5y - 95 = 0$	$3x + 5y - 95 = 0$ o.e.	A1
			[4]
(c) **Way 2**	$2(x-7) + 2(y-8)\frac{dy}{dx} = 0$	Correct differentiation (or equivalent). Seen or implied	B1
	$2(10-7) + 2(13-8)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{3}{5}$	Substituting **both** $x = 10$ and $y = 13$ into a valid differentiation to find a value for $\frac{dy}{dx}$	M1
	$y - 13 = -\frac{3}{5}(x-10)$	$y - 13 = (\text{their gradient})(x-10)$	M1
	$3x + 5y - 95 = 0$	$3x + 5y - 95 = 0$ o.e.	A1
			[4]
(c) **Way 3**	$10x + 13y - 7(x+10) - 8(y+13) + 79 = 0$	$10x + 13y - 7(x+10) - 8(y+13) + 79 = 0$	B1
		$10x + 13y - 7(x+10) - 8(y+13) + c = 0$ where c is any value < 113	M2
	$3x + 5y - 95 = 0$	$3x + 5y - 95 = 0$ o.e.	A1
			[4]
			8

		Question 54 Notes
(a)	**M1**	Allow for $\{PQ =\}\ \sqrt{(7-10)^2+(8-13)^2}$ or for $\{PQ =\}\ \sqrt{3^2+5^2}$. Can be implied by answer.
	A1	Need to see $\sqrt{34}$. You can ignore subsequent work so $\sqrt{34}$ followed by 5.83 earns M1 A1, but $\{PQ =\}\ \sqrt{3^2+5^2} = 5.83$, with no exact value for the answer given, earns M1A0. Allow $\pm\sqrt{34}$ this time. NB Some use equation of circle to find this distance Achieving $\sqrt{34}$ gets M1A1 Others find half of their $\pm\sqrt{34}$. Do not isw here as it is an error – confusing *d* with diameter. Give M1A0
(b)	**M1**	Either of the correct approaches for equation of circle (as shown on scheme)
	A1	Correct equation (two are shown and any correct equivalent is acceptable)
(c)	**B1**	A correct start to finding the gradient of the tangent (see each scheme)
	1st M1	Complete method for finding the gradient of the tangent (see each scheme) Where implicit differentiation has been used the only slips allowed here should be sign slips.
	2nd M1	Correct attempt at line equation for tangent at correct point (10, 13) with **their tangent** gradient. If the $y = mx + c$ method is used to find the equation, this M1 is earned at the point where the *x*- and *y*-values are substituted to find *c* e.g. 13 = -3/5 ×10 + *c*
	A1	Accept any correct answer of the required format; so integer multiple of $3x+5y-95=0$ or $3x-95+5y=0$ or $-3x-5y+95=0$ (must include "=0") e.g. $6x+10y-190=0$ earns A1 Also allow $5y+3x-95=0$ etc
	Common error	$\frac{dy}{dx} = 2(x-7)+2(y-8) = 6+10 = 16$ so (*y*-13)=16(*x*-10) is marked B0 M0 M1 A0 (Way 2)

Question Number	Scheme	Marks
	Way 1 / Way 2	
55 (a)	Way 1: $(x \mp 2)^2 + (y \pm 1)^2 = k$, $k > 0$ Way 2: $x^2 + y^2 \mp 4x \pm 2y + c = 0$	M1
	Way 1: Attempts to use $r^2 = (4-2)^2 + (-5+1)^2$ Way 2: $4^2 + (-5)^2 - 4\times 4 + 2\times -5 + c = 0$	M1
	Way 1: Obtains $(x-2)^2 + (y+1)^2 = 20$ Way 2: $x^2 + y^2 - 4x + 2y - 15 = 0$	A1
		(3)
	N.B. Special case: $(x-2)^2 - (y+1)^2 = 20$ is not a circle equation but earns M0M1A0	
(b) Way 1	Gradient of radius from centre to (4, -5) = -2 (must be correct)	B1
	Tangent gradient = $-\dfrac{1}{\text{their numerical gradient of radius}}$	M1
	Equation of tangent is $(y+5) = '\tfrac{1}{2}'(x-4)$	M1
	So equation is $x - 2y - 14 = 0$ (or $2y - x + 14 = 0$ or other integer multiples of this answer)	A1
		(4)
b)Way 2	Quotes $xx' + yy' - 2(x+x') + (y+y') - 15 = 0$ and substitutes (4, -5)_	B1
	$4x - 5y - 2(x+4) + (y-5) - 15 = 0$ so $2x - 4y - 28 = 0$ (or alternatives as in Way 1)	M1,M1A1
		(4)
b)Way 3	Use differentiation to find expression for gradient of circle	
	Either $2(x-2) + 2(y+1)\dfrac{dy}{dx} = 0$ **or** states $y = -1 - \sqrt{20-(x-2)^2}$ so $\dfrac{dy}{dx} = \dfrac{(x-2)}{\sqrt{20-(x-2)^2}}$	B1
	Substitute $x = 4$, $y = -5$ after valid differentiation to give gradient =	M1
	Then as Way 1 above $(y+5) = '\tfrac{1}{2}'(x-4)$ so $x - 2y - 14 = 0$	M1 A1
		(4)
		[7]

Notes

(a) **M1:** Uses centre to write down equation of circle in one of these forms. There may be sign slips as shown.

M1: Attempts distance between two points **to establish** r^2 (independent of first M1)- allow one sign slip only using distance formula with -5 or -1, usually $(-5-1)$ in 2nd bracket. Must not identify this distance as diameter.
This mark may alternatively (e.g. way 2)be given for substituting (4, -5) into a **correct circle** equation with one unknown
Can be awarded for $r = \sqrt{20}$ or for $r^2 = 20$ stated or implied but not for $r^2 = \sqrt{20}$ or $r = 20$ or $r = \sqrt{5}$

A1: Either of the answers printed or correct equivalent e.g. $(x-2)^2 + (y+1)^2 = (2\sqrt{5})^2$ is A1 but $2\sqrt{5}^2$ (no bracket) is A0 unless there is recovery

Also $(x-2)^2 + (y-(-1))^2 = (2\sqrt{5})^2$ may be awarded M1M1A1as a correct equivalent.

N.B. $(x-2)^2 + (y+1)^2 = 40$ commonly arises from one sign error evaluating r and earns M1M1A0

(b) **Way 1:**
B1: Must be correct answer -2 if evaluated (otherwise may be implied by the following work)
M1: Uses negative reciprocal of their gradient
M1: Uses $y - y_1 = m(x - x_1)$ with (4,-5) and their **changed** gradient **or** uses $y = mx + c$ and (4, -5) with their changed gradient (not gradient of radius) to find c
A1: answers in scheme or multiples of these answers (must have " = 0"). NB Allow $1x - 2y - 14 = 0$

N.B. $(y+5) = '\tfrac{1}{2}'(x-4)$ following gradient of is ½ after errors leads to $x - 2y - 14 = 0$ but is worth B0M0M0A0

Way 2: Alternative method (b) is rare.
Way 3: Some may use implicit differentiation to differentiate- others may attempt to make y the subject and use chain rule
B1: the differentiation must be accurate and the algebra accurate too. Need to take (-) root not (+)root in the alternative
M1: Substitutes into their gradient function but must follow valid accurate differentiation
M1: Must use "their" tangent gradient and $y + 5 = m(x-4)$ but allow over simplified attempts at differentiation for this mark.
A1: As in Way 1

Question Number	Scheme		Marks
	Mark (a) and (b) together		
56. (a)	$OQ^2 = \left(6\sqrt{5}\right)^2 + 4^2$ or $OQ = \sqrt{\left(6\sqrt{5}\right)^2 + 4^2}$ $\{= 14\}$	Uses the addition form of Pythagoras on $6\sqrt{5}$ and 4. Condone missing brackets on $\left(6\sqrt{5}\right)^2$ **(Working or 14 may be seen on the diagram)**	M1
	$y_Q = \sqrt{14^2 - 11^2}$	$y_Q = \sqrt{(\text{their } OQ)^2 - 11^2}$ **Must include √ and is dependent on the first M1 and requires OQ > 11**	**d**M1
	$= \sqrt{75}$ or $5\sqrt{3}$	$\sqrt{75}$ or $5\sqrt{3}$	A1**cso**
			[3]
(b)	$(x - 11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$	M1: $(x \pm 11)^2 + (y \pm \text{their } k)^2 = 4^2$ **Equation must be of this form and must use *x* and *y* not other letters. *k* could be their last answer to part (a). Allow their *k* ≠ 0 or just the letter *k*.**	M1A1
		A1: $(x - 11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$ or $(x - 11)^2 + \left(y - 5\sqrt{3}\right)^2 = 4^2$ **NB** $5\sqrt{3}$ **must come from correct work in (a) and allow awrt 8.66**	
	Allow in expanded form for the final A1 e.g. $x^2 - 22x + 121 + y^2 - 10\sqrt{3}y + 75 = 16$		
			[2]
			Total 5
	Watch out for:		
	(a) $OQ = \sqrt{\left(6\sqrt{5}\right)^2 + 4^2} = \sqrt{46}$ M1 $y_Q = \sqrt{46 - 11^2}$ M0 (OQ < 11) $y_Q = \sqrt{75}$ A0 (b) $(x - 11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$ M1A0		

Question Number	Scheme		Marks
57(a)	$A\left(\frac{-9+15}{2}, \frac{8-10}{2}\right) = A(3, -1)$	M1: A correct attempt to find the midpoint between *P* and *Q*. Can be implied by one of x or y-coordinates correctly evaluated.	M1A1
		A1: $(3, -1)$	
			[2]
(b)	$(-9-3)^2 + (8+1)^2$ or $\sqrt{(-9-3)^2 + (8+1)^2}$ or $(15-3)^2 + (-10+1)^2$ or $\sqrt{(15-3)^2 + (-10+1)^2}$ Uses Pythagoras correctly in order to find the **radius**. Must clearly be identified as the **radius** and may be implied by their circle equation. Or $(15+9)^2 + (-10-8)^2$ or $\sqrt{(15+9)^2 + (-10-8)^2}$ Uses Pythagoras correctly in order to find the **diameter**. Must clearly be identified as the **diameter** and may be implied by their circle equation. This mark can be implied by just 30 clearly seen as the **diameter** or 15 clearly seen as the **radius** (may be seen or implied in their circle equation) **Allow this mark if there is a correct statement involving the radius or the diameter but <u>must be seen in (b)</u>**		M1
	$(x-3)^2 + (y+1)^2 = 225 \ \left(\text{or } (15)^2\right)$	$(x \pm \alpha)^2 + (y \pm \beta)^2 = k^2$ where $A(\alpha, \beta)$ and k is their radius.	M1
	$(x-3)^2 + (y+1)^2 = 225$	Allow $(x-3)^2 + (y+1)^2 = 15^2$	A1
	Accept correct answer only		
			[3]
	Alternative using $x^2 + 2ax + y^2 + 2by + c = 0$		
	Uses $A(\pm\alpha, \pm\beta)$ and $x^2 + 2ax + y^2 + 2by + c = 0$ e.g. $x^2 + 2(-3)x + y^2 + 2(1)y + c = 0$		M1
	Uses P or Q and $x^2 + 2ax + y^2 + 2by + c = 0$ e.g. $(-9)^2 + 2(-3)(-9) + (8)^2 + 2(1)(8) + c = 0 \Rightarrow c = -215$		M1
	$x^2 - 6x + y^2 + 2y - 215 = 0$		A1
(c)	Distance $= \sqrt{15^2 - 10^2}$	$= \sqrt{(\text{their } r)^2 - 10^2}$ or a correct method for the distance e.g. their $r \times \cos\left[\sin^{-1}\left(\frac{10}{\text{their } r}\right)\right]$	M1
	$\left\{= \sqrt{125}\right\} = 5\sqrt{5}$	$5\sqrt{5}$	A1
			[2]

Question Number	Scheme		Marks
(d)	$\sin\left(A\hat{R}Q\right) = \dfrac{20}{30}$ or $A\hat{R}Q = 90 - \cos^{-1}\left(\dfrac{10}{15}\right)$	$\sin\left(A\hat{R}Q\right) = \dfrac{20}{\left(2 \times \text{their } r\right)}$ or $\dfrac{10}{\text{their } r}$ or $A\hat{R}Q = 90 - \cos^{-1}\left(\dfrac{10}{\text{their } r}\right)$ or $A\hat{R}Q = \cos^{-1}\left(\dfrac{\text{Part}(c)}{\text{their } r}\right)$ or $A\hat{R}Q = 90 - \sin^{-1}\left(\dfrac{\text{Part}(c)}{\text{their } r}\right)$ or $20^2 = 15^2 + 15^2 - 2\times15\times15\cos\left(\mathbf{2}ARQ\right)$ or $15^2 = 15^2 + \left(10\sqrt{5}\right)^2 - 2\times15\times10\sqrt{5}\cos\left(ARQ\right)$ A fully correct method to find $A\hat{R}Q$, where their $r > 10$. Must be a **correct** statement involving angle ARQ	M1
	$A\hat{R}Q = 41.8103...$	awrt 41.8	A1
			[2]
			Total 9

Question Number	Scheme	Marks
58. (a)	Equation of form $(x \pm 5)^2 + (y \pm 9)^2 = k$, $k > 0$	M1
	Equation of form $(x - a)^2 + (y - b)^2 = 5^2$, with values for a and b	M1
	$(x + 5)^2 + (y - 9)^2 = 25 = 5^2$	A1
		(3)
	$P(8, -7)$. Let centre of circle $= X(-5, 9)$	
(b)	$PX^2 = (8 - "-5")^2 + (-7 - "9")^2$ or $PX = \sqrt{(8 - -5)^2 + (-7 - 9)^2}$	M1
	($PX = \sqrt{425}$ or $5\sqrt{17}$) $PT^2 = (PX)^2 - 5^2$ with numerical PX	dM1
	$PT \left\{= \sqrt{400}\right\} = 20$ (allow 20.0)	A1 **cso**
		(3) **[6]**
Alternative 2 for (a)	Equation of the form $x^2 + y^2 \pm 10x \pm 18y + c = 0$	M1
	Uses $a^2 + b^2 - 5^2 = c$ with their a and b or substitutes (0, 9) giving $+9^2 \pm 2b \times 9 + c = 0$	M1
	$x^2 + y^2 + 10x - 18y + 81 = 0$	A1
		(3)
Alternative 2 for (b)	An attempt to find the point T may result in pages of algebra, but solution needs to reach (-8, 5) or $\left(\frac{-8}{17}, 11\frac{2}{17}\right)$ to get first M1 (even if gradient is found first)	M1
	M1: Use either of the correct points with P (8, -7) and distance between two points formula	dM1
	A1: 20	A1cso
		(3)
Alternative 3 for (b)	Substitutes (8, -7) into circle **equation** so $PT^2 = 8^2 + (-7)^2 + 10 \times 8 - 18 \times (-7) + 81$	M1
	Square roots to give $PT \left\{= \sqrt{400}\right\} = 20$	dM1A1
		(3)
	Notes for Question 58	
(a)	**The three marks in (a) each require a circle equation – (see special cases which are not circles)** M1: Uses coordinates of centre to obtain LHS of circle equation (RHS must be r^2 or $k > 0$ or a positive value) M1: Uses $r = 5$ to obtain RHS of circle equation as 25 or 5^2 A1: correct circle equation in any equivalent form **Special cases** $(x \pm 5)^2 + (x \pm 9)^2 = (5^2)$ is **not a circle** equation so M0M0A0 Also $(x \pm 5)^2 + (y - 9) = (5^2)$ **And** $(x \pm 5)^2 - (y \pm 9)^2 = (5^2)$ **are not circles** and gain M0M0A0 **But** $(x - 0)^2 + (y - 9)^2 = 5^2$ gains M0M1A0	
(b)	M1: Attempts to find distance from their **centre of circle** to P (or square of this value). If this is called PT and given as answer this is M0. Solution may use letter other than X, as centre was not labelled in the question. N.B. Distance from (0, 9) to (8, -7) is incorrect method and is M0, followed by M0A0. dM1**:** Applies the **subtraction** form of Pythagoras to find PT or PT^2 (depends on previous method mark for distance from **centre to *P***) or uses appropriate complete method involving trigonometry A1: 20 cso	

59.			
(a)			
(i)	The centre is at (10, 12)	B1: $x = 10$ B1: $y = 12$	B1 B1
(ii)	Uses $(x-10)^2+(y-12)^2=-195+100+144 \Rightarrow r = ...$		M1
	Completes the square for both x and y in an attempt to find r. $(x\pm"10")^2 \pm a$ and $(y\pm"12")^2 \pm b$ and $+195=0,(a,b\neq 0)$ Allow errors in obtaining their r^2 but must find square root		
	$r=\sqrt{10^2+12^2-195}$	A correct numerical expression for r including the square root and can implied by a correct value for r	A1
	$r = 7$	Not $r=\pm 7$ unless -7 is rejected	A1
			(5)
(a) Way 2	Compares the given equation with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g,-f)$ i.e. (10, 12)	B1: $x = 10$ B1: $y = 12$	B1B1
	Uses $r=\sqrt{(\pm"10")^2+(\pm"12")^2-c}$		M1
	$r=\sqrt{10^2+12^2-195}$	A correct numerical expression for r	A1
	$r = 7$		A1
			(5)
(b)	$MN=\sqrt{(25-"10")^2+(32-"12")^2}$	Correct use of Pythagoras	M1
	$MN\left(=\sqrt{625}\right)=25$		A1
			(2)
(c)	$NP=\sqrt{("25"^2-"7"^2)}$	$NP=\sqrt{(MN^2-r^2)}$	M1
	$NP\left(=\sqrt{576}\right)=24$		A1
			(2)
(c) Way 2	$\cos(NMP)=\frac{7}{"25"} \Rightarrow NP="25"\sin(NMP)$	Correct strategy for finding NP	M1
	$NP = 24$		A1
			(2)
			[9]

Question number	Scheme	Marks
60 (a)	Obtain $\underline{(x \pm 10)^2}$ **and** $\underline{\underline{(y \pm 8)^2}}$	M1
	Obtain $\underline{(x - 10)^2}$ **and** $\underline{\underline{(y - 8)^2}}$	A1
	Centre is (10, 8). N.B. This may be indicated on **diagram only** as (10, 8)	A1 (3)
(b)	See $\underline{(x \pm 10)^2} + \underline{\underline{(y \pm 8)^2}} = 25 (= r^2)$ or $(r^2 =) "100" + "64" - 139$	M1
	$r = 5$ * (this is a printed answer so need one of the above two reasons)	**A1** **(2)**
(c)	Use $x = 13$ in either form of equation of circle and solve resulting quadratic to give $y =$	**M1**
	e.g $x = 13 \Rightarrow (13-10)^2 + (y-8)^2 = 25 \Rightarrow (y-8)^2 = 16$ so $y=$ *or* $13^2 + y^2 - 20 \times 13 - 16y + 139 = 0 \Rightarrow y^2 - 16y + 48 = 0$ so $y =$	
	$y = 4$ or 12 (on EPEN mark one correct value as A1A0 and both correct as A1 A1)	**A1, A1** (3)
(d)	Use of $r\theta$ with $r = 5$ and $\theta = 1.855$ (may be implied by 9.275)	**M1**
	Perimeter $PTQ = 2r$ + their **arc** PQ (Finding perimeter of triangle is M0 here)	**M1**
	$= 19.275$ or 19.28 or 19.3	**A1** (3)
		11 marks
Alternatives (a)	*Method 2:* From $x^2 + y^2 + 2gx + 2fy + c = 0$ centre is $(\pm g, \pm f)$ Centre is $(-g, -f)$, **and so centre is** (10, 8).	M1 A1, A1
OR	*Method 3:* Use any value of y to give two points (L and M) on circle. x co-ordinate of mid point of LM is "10" **and** Use any value of x to give two points (P and Q) on circle. y co-ordinate of mid point of PQ is "8" (Centre – chord theorem) . (10,8) is M1A1A1	M1 A1 A1 (3)
(b)	*Method 2*: Using $\sqrt{g^2 + f^2 - c}$ or $(r^2 =) "100" + "64" - 139$ $r = 5$ *	M1 A1
OR	*Method 3:* Use point on circle with centre to find radius. Eg $\sqrt{(13-10)^2 + (12-8)^2}$ $r = 5$ *	**M1** **A1 cao** (2)
(c)	Divide triangle PTQ and use Pythagoras with $r^2 - (13 - "10")^2 = h^2$, then evaluate $"8 \pm h"$ - (N.B. Could use 3,4,5 Triangle and 8 ± 4). Accuracy as before	**M1**
Notes (a) (b)	**Mark (a) and (b) together** **M1** as in scheme and can be <u>implied</u> by $(\pm 10, \pm 8)$ **. Correct centre (10, 8) implies M1A1A1** **M1** for a correct method leading to $r = \ldots$, or $r^2 = "100" + "64" - 139$ (not 139 – "100" –" 64") or for using equation of circle in $\underline{(x \pm 10)^2} + \underline{\underline{(y \pm 8)^2}} = k^2$ form to identify $r=$ **3rd A1** $\boldsymbol{r = 5}$ **(NB This is a given answer so should follow** $k^2 = 25$ or $r^2 = 100 + 64 - 139$ **)** **Special case**: if centre is given as (-10, -8) or (10, -8) or (-10, 8) allow **M1A1** for $r = 5$ worked correctly as $r^2 = 100 + 64 - 139$	
(d)	Full marks available for calculation using major sector so Use of $r\theta$ with $r = 5$ and $\theta = 4.428$ leading to perimeter of 32.14 for major sector	

Question number	Scheme	Marks
61	The equation of the circle is $(x+1)^2+(y-7)^2=(r^2)$ The radius of the circle is $\sqrt{(-1)^2+7^2}=\sqrt{50}$ or $5\sqrt{2}$ or $r^2=50$ So $(x+1)^2+(y-7)^2=50$ or equivalent	M1 A1 M1 A1 (4) **4**
Notes	**M1** is for this expression on left hand side– allow *errors in sign* of 1 and 7. **A1** correct signs (just LHS) **M1** is for Pythagoras or substitution into equation of circle to give r or r^2 Giving this value as diameter is **M0** **A1**, cao for cartesian equation with numerical values but allow $(\sqrt{50})^2$ or $(5\sqrt{2})^2$ or any exact equivalent A correct answer implies a correct method – so answer given with no working earns all four marks for this question.	
Alternative method	Equation of circle is $x^2+y^2\pm 2x\pm 14y+c=0$ Equation of circle is $x^2+y^2+2x-14y+c=0$ Uses (0,0) to give $c = 0$, **or** finds $r=\sqrt{(-1)^2+7^2}=\sqrt{50}$ or $5\sqrt{2}$ or $r^2=50$ So $x^2+y^2+2x-14y=0$ or equivalent	M1 A1 M1 A1

Question Number	Scheme		Marks
62.	$x^2 + y^2 + 4x - 2y - 11 = 0$		
(a)	$\left\{\underline{(x+2)^2 - 4} + \underline{\underline{(y-1)^2 - 1}} - 11 = 0\right\}$	$(\pm 2, \pm 1)$, see notes.	M1
	Centre is $(-2, 1)$.	$(-2, 1)$.	A1 **cao** **[2]**
(b)	$(x+2)^2 + (y-1)^2 = 11 + 1 + 4$	$r = \sqrt{11 \pm "1" \pm "4"}$	M1
	So $r = \sqrt{11+1+4} \Rightarrow r = 4$	4 or $\sqrt{16}$ (Award A0 for ± 4).	A1 **[2]**
(c)	When $x = 0$, $y^2 - 2y - 11 = 0$	Putting $x = 0$ in C or their C.	M1
		$y^2 - 2y - 11 = 0$ or $(y-1)^2 = 12$, etc	A1 aef
	$y = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)}$ $\left\{= \dfrac{2 \pm \sqrt{48}}{2}\right\}$	Attempt to use formula or a method of completing the square in order to find $y = ...$	M1
	So, $y = 1 \pm 2\sqrt{3}$	$1 \pm 2\sqrt{3}$	A1 **cao cso** **[4]**
			8

Note: Please mark parts (a) and (b) together. Answers only in (a) and/or (b) get full marks.
Note in part (a) the marks are now M1A1 and not B1B1 as on ePEN.

(a) M1: for $(\pm 2, \pm 1)$. Otherwise, M1 for an attempt to complete the square eg. $\underline{(x \pm 2)^2 \pm \alpha}$, $\alpha \neq 0$ or $\underline{\underline{(y \pm 1)^2 \pm \beta}}$, $\beta \neq 0$. M1A1: Correct answer of $(-2, 1)$ stated from any working gets M1A1.

(b) M1: to find the radius using 11, "1" and "4", ie. $r = \sqrt{11 \pm "1" \pm "4"}$. By applying this method candidates will usually achieve $\sqrt{16}$, $\sqrt{6}$, $\sqrt{8}$ or $\sqrt{14}$ and not 16, 6, 8 or 14.

Note: $(x+2)^2 + (y-1)^2 = -11 - 5 = -16 \Rightarrow r = \sqrt{16} = 4$ should be awarded M0A0.

Alternative: M1 in part (a): For comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$ to write down centre $(-g, -f)$ directly. Condone sign errors for this M mark. M1 in part (b): For using $r = \sqrt{g^2 + f^2 - c}$. Condone sign errors for this method mark.

$(x+2)^2 + (y-1)^2 = 16 \Rightarrow r = 8$ scores M0A0, but $r = \sqrt{16} = 8$ scores M1A1 isw.

(c) 1st M1: Putting $x = 0$ in either $x^2 + y^2 + 4x - 2y - 11 = 0$ or their circle equation usually given in part (a) or part (b). 1st A1 for a correct equation in *y* **in any form** which can be implied by later working.

2nd M1: See rules for using the formula. Or completing the square on a 3TQ to give $y = a \pm \sqrt{b}$, where $\sqrt{b}$ is a surd, $b \neq$ their 11 and $b > 0$. This mark should not be given for an attempt to factorise.

2nd A1: Need exact pair in simplified surd form of $\{y =\}\ 1 \pm 2\sqrt{3}$. This mark is also cso.

Do not need to see $(0, 1 + 2\sqrt{3})$ and $(0, 1 - 2\sqrt{3})$. Allow 2nd A1 for bod $(1 + 2\sqrt{3}, 0)$ and $(1 - 2\sqrt{3}, 0)$.

Any incorrect working in (c) gets penalised the final accuracy mark. So, **beware:** incorrect $(x-2)^2 + (y-1)^2 = 16$ leading to $y^2 - 2y - 11 = 0$ and then $y = 1 \pm 2\sqrt{3}$ scores M1A1M1A0.

Special Case for setting *y* = 0: Award SC: M0A0M1A0 for an attempt at applying the formula

$$x = \frac{-4 \pm \sqrt{(-4)^2 - 4(1)(-11)}}{2(1)} \quad \left\{= \frac{-4 \pm \sqrt{60}}{2} = -2 \pm \sqrt{15}\right\}$$

Award SC: M0A0M1A0 for completing the square to their equation in *x* which will usually be $x^2 + 4x - 11 = 0$ to give $a \pm \sqrt{b}$, where $\sqrt{b}$ is a surd, $b \neq$ their 11 and $b > 0$.

Special Case: For a candidate not using $\pm$ but achieving one of the correct answers then award SC: M1A1 M1A0 for one of either $y = 1 + 2\sqrt{3}$ or $y = 1 - 2\sqrt{3}$ or $y = 1 + \sqrt{12}$ or $y = 1 - \sqrt{12}$.

Question Number	Scheme		Marks
63.			
(a)	$C\left(\dfrac{-2+8}{2}, \dfrac{11+1}{2}\right) = C(3,6)$ **AG**	Correct method (no errors) for finding the mid-point of AB giving $(3, 6)$	B1*
			(1)
(b)	$(8-3)^2 + (1-6)^2$ or $\sqrt{(8-3)^2+(1-6)^2}$ or	Applies distance formula in order to find the radius.	M1
	$(-2-3)^2 + (11-6)^2$ or $\sqrt{(-2-3)^2+(11-6)^2}$	Correct application of formula.	A1
	$(x-3)^2 + (y-6)^2 = 50 \left(\text{or } \left(\sqrt{50}\right)^2 \text{ or } \left(5\sqrt{2}\right)^2\right)$	$(x \pm 3)^2 + (y \pm 6)^2 = k$, k is a positive value.	M1
		$(x-3)^2 + (y-6)^2 = 50$ (Not 7.07^2)	A1
			(4)
(c)	{For $(10, 7)$, } $\underline{(10-3)^2 + (7-6)^2 = 50}$, {so the point lies on C.}		B1
			(1)
(d)	{Gradient of radius} $= \dfrac{7-6}{10-3}$ or $\dfrac{1}{7}$	This must be seen in part (d).	B1
	Gradient of tangent $= \dfrac{-7}{1}$	Using a perpendicular gradient method.	M1
	$y - 7 = -7(x-10)$	$y - 7 = (\text{their gradient})(x-10)$	M1
	$y = -7x + 77$	$y = -7x + 77$ or $y = 77 - 7x$	A1 cao
			(4)
			[10]
	Notes		
(a)	Alternative method: $C\left(-2+\dfrac{8--2}{2}, 11+\dfrac{1-11}{2}\right)$ or $C\left(8+\dfrac{-2-8}{2}, 1+\dfrac{11-1}{2}\right)$		
(b)	You need to be convinced that the candidate is attempting to work out the radius and not the diameter of the circle to award the first M1. Therefore allow 1st M1 generously for $\dfrac{(-2-8)^2+(11-1)^2}{2}$ Award 1st M1A1 for $\dfrac{(-2-8)^2+(11-1)^2}{4}$ or $\dfrac{\sqrt{(-2-8)^2+(11-1)^2}}{2}$. Correct answer in (b) with no working scores full marks.		
(c)	B1 awarded for correct verification of $\underline{(10-3)^2+(7-6)^2=50}$ with no errors. Also to gain this mark candidates need to have the correct equation of the circle either from part (b) or re-attempted in part (c). They cannot verify $(10, 7)$ lies on C without a correct C. Also a candidate could either substitute $x=10$ in C to find $y=7$ or substitute $y=7$ in C to find $x=10$.		

Question Number	Scheme	Marks
(d)	2nd M1 mark also for the complete method of applying $7 = (\text{their gradient})(10) + c$, finding c. **Note**: Award 2nd M0 in (d) if their numerical gradient is either 0 or ∞. **<u>Alternative:</u>** For first two marks (differentiation): $2(x-3) + 2(y-6)\frac{dy}{dx} = 0$ (or equivalent) scores B1. 1st M1 for substituting **both** $x = 10$ and $y = 7$ to find a value for $\frac{dy}{dx}$, which must contain both x and y. (This M mark can be awarded generously, even if the attempted "differentiation" is not "implicit".) **<u>Alternative</u>:** $(10-3)(x-3) + (7-6)(y-6) = 50$ scores B1M1M1 which leads to $y = -7x + 77$.	

Question Number	Scheme	Marks
64	(a) $(10-2)^2+(7-1)^2$ or $\sqrt{(10-2)^2+(7-1)^2}$ $(x\pm 2)^2+(y\pm 1)^2=k$ (k a positive <u>value</u>) $(x-2)^2+(y-1)^2=100$ (Accept 10^2 for 100) (Answer only scores full marks)	M1 A1 M1 A1 (4)
	(b) (Gradient of radius =) $\dfrac{7-1}{10-2}=\dfrac{6}{8}$ (or equiv.) Must be seen in part (b) Gradient of tangent $=\dfrac{-4}{3}$ (Using perpendicular gradient method) $y-7=m(x-10)$ Eqn., in any form, of a line through (10, 7) with any numerical gradient (except 0 or ∞) $y-7=\dfrac{-4}{3}(x-10)$ or equiv (ft gradient of <u>radius</u>, dep. on <u>both</u> M marks) $\{3y=-4x+61\}$ (N.B. The A1 is only available as <u>ft</u> after B0) The unsimplified version scores the A mark (isw if necessary... subsequent mistakes in simplification are not penalised here. The equation must at some stage be <u>exact</u>, not, e.g. $y=-1.3x+20.3$	B1 M1 M1 A1ft (4)
	(c) $\sqrt{r^2-\left(\dfrac{r}{2}\right)^2}$ Condone sign slip if there is evidence of correct use of Pythag. $=\sqrt{10^2-5^2}$ or numerically exact equivalent $PQ\ \left(=2\sqrt{75}\right)=10\sqrt{3}$ Simplest surd form $10\sqrt{3}$ required for final mark	M1 A1 A1 (3) **11**
	(b) 2nd M: Using (10, 7) to find the equation, in any form, of a straight line through (10, 7), with any numerical gradient (except 0 or ∞). <u>Alternative:</u> 2nd M: Using (10, 7) and an m value in $y = mx + c$ to find a value of c. (b) <u>Alternative</u> for first 2 marks (differentiation): $2(x-2)+2(y-1)\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ or equiv. B1 Substitute $x = 10$ and $y = 7$ to find a value for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ M1 (This M mark can be awarded generously, even if the attempted 'differentiation' is not 'implicit'). (c) <u>Alternatives</u>: To score M1, must be a <u>fully</u> correct method to obtain $\dfrac{1}{2}PQ$ or PQ. 1st A1: For alternative methods that find PQ directly, this mark is for an <u>exact numerically correct version</u> of PQ.	

Question Number	Scheme	Marks
65 (a)	$N\,(2, -1)$	B1, B1 (2)
(b)	$r = \sqrt{\dfrac{169}{4}} = \dfrac{13}{2} = 6.5$	B1 (1)
(c)	Complete Method to find x coordinates, $x_2 - x_1 = 12$ and $\dfrac{x_1 + x_2}{2} = 2$ then solve To obtain $x_1 = -4, \quad x_2 = 8$ Complete Method to find y coordinates, using equation of circle or Pythagoras i.e. let d be the distance below N of A *then* $d^2 = 6.5^2 - 6^2 \Rightarrow d = 2.5 \Rightarrow y = ..$ So $y_2 = y_1 = -3.5$	M1 A1ft A1ft M1 A1 (5)
(d)	Let $A\hat{N}B = 2\theta \Rightarrow \sin\theta = \dfrac{6}{\text{"}6.5\text{"}} \Rightarrow \theta = (67.38)...$ So angle ANB is 134.8 *	M1 A1 (2)
(e)	AP is perpendicular to AN so using triangle ANP $\tan\theta = \dfrac{AP}{\text{"}6.5\text{"}}$ Therefore $AP = 15.6$	M1 A1cao (2) **[12]**
(a) (b)	B1 for 2 (α) , B1 for –1 B1 for 6.5 o.e.	
(c)	1st M1 for finding x coordinates – may be awarded if either x co-ord is correct A1ft,A1ft are for $\alpha - 6$ and $\alpha + 6$ if x coordinate of N is α 2nd M1 for a method to find y coordinates – may be given if y co-ordinate is correct A marks is for –3.5 only.	
(d)	M1 for a full method to find θ or angle ANB (eg sine rule or cosine rule directly or finding another angle and using angles of triangle.) **ft their 6.5 from radius or wrong *y*.** (cos ANB = $\dfrac{\text{"}6.5\text{"}^2 + \text{"}6.5\text{"}^2 - 12^2}{2 \times \text{"}6.5\text{"} \times \text{"}6.5\text{"}}$ =-0.704) A1 is a printed answer and must be 134.8 – do not accept 134.76.	
(e)	M1 for a full method to find AP Alternative Methods N.B. May use triangle AXP where X is the mid point of AB. Or may use triangle ABP. From circle theorems may use angle BAP = 67.38 or some variation. Eg $\dfrac{AP}{\sin 67.4} = \dfrac{12}{\sin 45.2}$, $AP = \dfrac{6}{\sin 22.6}$ or $AP = \dfrac{6}{\cos 67.4}$ are each worth M1	

Question Number	Scheme	Marks
66 (a)	$\left(x-3\right)^2-9+\left(y+2\right)^2-4=12$ Centre is (3, –2)	M1 A1, A1
	$\left(x-3\right)^2+\left(y+2\right)^2=12+"9"+"4"$ $r=\sqrt{12+"9"+"4"}=5$ (or $\sqrt{25}$)	M1 A1 (5)
(b)	$PQ=\sqrt{(7--1)^2+(-5-1)^2}$ or $\sqrt{8^2+6^2}$	M1
	= 10 = 2×radius, ∴diam. (N.B. For A1, need a comment or conclusion)	A1 (2)
	[ALT: midpt. of PQ $\left(\frac{7+(-1)}{2},\frac{1+(-5)}{2}\right)$: M1, = (3, –2) = centre: A1]	
	[ALT: eqn. of PQ $3x+4y-1=0$: M1, verify (3, –2) lies on this: A1]	
	[ALT: find two grads, e.g. PQ and P to centre: M1, equal ∴diameter: A1]	
	[ALT: show that point $S(-1,-5)$ or (7, 1) lies on circle: M1 because $\angle PSQ$ = 90°, semicircle∴diameter: A1]	
(c)	R must lie on the circle (angle in a semicircle theorem)… often <u>implied</u> by <u>a diagram with R on the circle</u> or by subsequent working)	B1
	$x=0\Rightarrow$ $y^2+4y-12=0$	M1
	$(y-2)(y+6)=0$ $y=.....$ (M is dependent on previous M)	dM1
	$y=-6$ or 2 (Ignore $y=-6$ if seen, and 'coordinates' are not required))	A1 (4) **[11]**
(a)	1st M1 for attempt to complete square. Allow $(x\pm3)^2\pm k$, or $(y\pm2)^2\pm k$, $k\neq0$. 1st A1 x-coordinate 3, 2nd A1 y-coordinate –2 2nd M1 for a full method leading to r = …, with their 9 and their 4, 3rd A1 5 or $\sqrt{25}$ The 1st M can be <u>implied</u> by $(\pm3,\pm2)$ but a full method must be seen for the 2nd M. Where the 'diameter' in part (b) has <u>clearly</u> been used to answer part (a), no marks in (a), but in this case the M1 (<u>not</u> the A1) for part (b) can be given for work seen in (a). <u>Alternative</u> 1st M1 for comparing with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g,-f)$ directly. Condone sign errors for this M mark. 2nd M1 for using $r=\sqrt{g^2+f^2-c}$. Condone sign errors for this M mark.	
(c)	1st M1 for setting x = 0 and getting a 3TQ in y by using eqn. of circle. 2nd M1 (dep.) for attempt to solve a 3TQ leading to <u>at least one</u> solution for y. <u>Alternative 1</u>: (Requires the B mark as in the main scheme) 1st M for using (3, 4, 5) triangle with vertices $(3,-2),(0,-2),(0,y)$ to get a linear or quadratic equation in y (e.g. $3^2+(y+2)^2=25$). 2nd M (dep.) as in main scheme, but may be scored by simply solving a linear equation. <u>Alternative 2</u>: (Not requiring realisation that R is on the circle) B1 for attempt at $m_{PR}\times m_{QR}=-1$, (<u>NOT</u> m_{PQ}) or for attempt at Pythag. in triangle PQR. 1st M1 for setting x = 0, i.e. (0, y), and proceeding to get a 3TQ in y. Then main scheme. <u>Alternative 2 by 'verification'</u>: B1 for attempt at $m_{PR}\times m_{QR}=-1$, (<u>NOT</u> m_{PQ}) or for attempt at Pythag. in triangle PQR. 1st M1 for trying (0, 2). 2nd M1 (dep.) for performing all required calculations. A1 for fully correct working and conclusion.	

Question Number	Scheme	Marks
67 (a)	PQ: $m_1 = \dfrac{10-2}{9-(-3)}$ $(=\tfrac{2}{3})$ **and** QR: $m_2 = \dfrac{10-4}{9-a}$	M1
(b)	$m_1 m_2 = -1$: $\dfrac{8}{12}\times\dfrac{6}{9-a} = -1$ $a = 13$ (*)	M1 A1 (3)
Alt for (a)	(a) Alternative method (Pythagoras) Finds **all three** of the following	
	$\left(9-(-3)\right)^2 + (10-2)^2$, $(i.e. 208)$, $(9-a)^2 + (10-4)^2$, $\left(a-(-3)\right)^2 + (4-2)^2$	M1
	Using Pythagoras (correct way around) e.g. $a^2 + 6a + 9 = 240 + a^2 - 18a + 81$ to form equation	M1
	Solve (or verify) for a, $a = 13$ (*)	A1 (3)
	(b) Centre is at (5, 3)	B1
	$(r^2 =)\ (10-3)^2 + (9-5)^2$ or equiv., or $(d^2 =)\ \left(13-(-3)\right)^2 + (4-2)^2$	M1 A1
	$(x-5)^2 + (y-3)^2 = 65$ or $x^2 + y^2 - 10x - 6y - 31 = 0$	M1 A1 (5)
Alt for (b)	Uses $(x-a)^2 + (y-b)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$ and substitutes (-3, 2), (9, 10) and (13, 4) then eliminates one unknown	M1
	Eliminates second unknown	M1
	Obtains $g = -5, f = -3, c = -31$ or $a = 5, b = 3,$ $r^2 = 65$	A1, A1, B1cao (5) **[8]**
Notes (a)	**M1**-considers gradients of *PQ* and *QR* -must be *y* difference / *x* difference (or considers three lengths as in alternative method) **M1** Substitutes gradients into product = -1 (or lengths into Pythagoras' Theorem the correct way round) **A1** Obtains a = 13 with no errors by solution or verification. Verification can score 3/3.	
(b)	Geometrical method: **B1** for coordinates of centre – can be implied by use in part (b) **M1** for attempt to find r^2, d^2, r or d (allow one slip in a bracket). **A1** cao. These two marks may be gained implicitly from circle equation **M1** for $(x\pm5)^2 + (y\pm3)^2 = k^2$ or $(x\pm3)^2 + (y\pm5)^2 = k^2$ ft their (5,3) Allow k^2 non numerical. **A1** cao for whole equation and rhs must be 65 or $\left(\sqrt{65}\right)^2$, (similarly B1 must be 65 or $\left(\sqrt{65}\right)^2$, in alternative method for (b))	

Question Number	Scheme	Marks
Further alternatives	(i) A number of methods find gradient of PQ = 2/3 then give perpendicular gradient is –3/2 This is **M1**	M1
	They then proceed using equations of lines through point Q or by using gradient QR to obtain equation such as $\frac{4-10}{a-9}=-\frac{3}{2}$ **M1** (may still have x in this equation rather than a and there may be a small slip)	M1
	They then complete to give (a)= 13 **A1**	A1
	(ii) A long involved method has been seen finding the coordinates of the centre of the circle first. This can be done by a variety of methods Giving centre as (c, 3) and using an equation such as $(c-9)^2+7^2=(c+3)^2+1^2$ (equal radii) or $\frac{3-6}{c-3}=-\frac{3}{2}$ **M1** (perpendicular from centre to chord bisects chord)	M1
	Then using c (= 5) to find a is **M1**	M1
	Finally a = 13 **A1**	A1
	(iii) Vector Method:	M1
	States **PQ. QR** = 0, with vectors stated 12i +8j and (9 – a)**i + 6**j is **M1**	
	Evaluates scalar product so 108 – 12 a + 48 = 0 (**M1**)	M1
	solves to give a = 13 (**A1**)	A1

Question number	Scheme	Marks	
68.	(a) $(8-3)^2+(3-1)^2$ or $\sqrt{(8-3)^2+(3-1)^2}$	M1 A1	
	$(x\pm 3)^2+(y\pm 1)^2=k$ or $(x\pm 1)^2+(y\pm 3)^2=k$ (*k* a positive <u>value</u>)	M1	
	$(x-3)^2+(y-1)^2=29$ (<u>Not</u> $\left(\sqrt{29}\right)^2$ or 5.39^2)	A1	(4)
	(b) Gradient of radius = $\frac{2}{5}$ (or exact equiv.) Must be seen or used in (b)	B1	
	Gradient of tangent = $\frac{-5}{2}$ (Using perpendicular gradient method)	M1	
	$y-3=\frac{-5}{2}(x-8)$ (ft gradient of radius, dependent upon <u>both</u> M marks)	M1 A1ft	
	$5x+2y-46=0$ (Or equiv., equated to zero, e.g. $92-4y-10x=0$) (Must have <u>integer</u> coefficients)	A1	(5) **9**
	(a) For the M mark, condone <u>one</u> slip <u>inside</u> a bracket, e.g. $(8-3)^2+(3+1)^2$, $(8-1)^2+(1-3)^2$ The first two marks may be gained implicitly from the circle equation. (b) 2nd M: Eqn. of line through (8, 3), in any form, with any grad.(except 0 or ∞). If the 8 and 3 are the 'wrong way round', this M mark is only given if a correct general formula, e.g. $y-y_1=m(x-x_1)$, is quoted. <u>Alternative:</u> 2nd M: Using (8, 3) and an *m* value in *y* = *mx* + *c* to find a value of *c*. A1ft: as in main scheme. (Correct substitution of 8 and 3, then a wrong *c* value will still score the A1ft) (b) <u>Alternatives for the first 2 marks</u>: (but in these 2 cases the 1st A mark is <u>not</u> ft) (i) Finding gradient of tangent by <u>implicit</u> differentiation $2(x-3)+2(y-1)\frac{dy}{dx}=0$ (or equivalent) B1 Subs. *x* = 8 <u>and</u> *y* = 3 into a 'derived' expression to find a value for dy/dx M1 (ii) Finding gradient of tangent by differentiation of $y=1+\sqrt{20+6x-x^2}$ $\frac{dy}{dx}=\frac{1}{2}\left(20+6x-x^2\right)^{-\frac{1}{2}}(6-2x)$ (or equivalent) B1 Subs. *x* = 8 into a 'derived' expression to find a value for dy/dx M1 <u>Another alternative</u>: Using $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$ $x^2+y^2-6x-2y-19=0$ B1 $8x+3y,\quad -3(x+8)-(y+3)-19=0$ M1, M1 A1ft (ft from circle eqn.) $5x+2y-46=0$ A1		

EXPERT TUITION

69	(a)	$(x-6)^2 + (y-4)^2 = ;\ 3^2$	B1; B1 (2)
	(b)	Complete method for MP: $= \sqrt{(12-6)^2 + (6-4)^2}$	M1
		$= \sqrt{40}$ (= 6.325)	A1
		[These first two marks can be scored if seen as part of solution for (c)]	
		Complete method for $\cos\theta$, $\sin\theta$ or $\tan\theta$	M1
		e.g. $\cos\theta = \frac{\text{MT}}{\text{MP}} = \frac{3}{candidate's\sqrt{40}}$ (= 0.4743) (θ = 61.6835°)	
		[If TP = 6 is used, then M0]	
		θ = 1.0766 rad **AG**	A1 (4)
	(c)	Complete method for area TMP; e.g. $= \frac{1}{2}\times 3\times\sqrt{40}\sin\theta$	M1
		$= \frac{3}{2}\sqrt{31}$ (= 8.3516..) allow awrt 8.35	A1
		Area (sector) MTQ $= 0.5\times 3^2\times 1.0766$ (= 4.8446…)	M1
		Area TPQ = candidate's (8.3516.. – 4.8446..)	M1
		= 3.507 awrt [Note: 3.51 is A0]	A1 (5) **[11]**
Notes		(a) Allow 9 for 3^2. (b) First M1 can be implied by $\sqrt{40}$ For second M1: May find TP = $\sqrt{(\sqrt{40})^2 - 3^2} = \sqrt{31}$, then either $\sin\theta = \frac{TP}{MP} = \frac{\sqrt{31}}{\sqrt{40}}$ (= 0.8803...) *or* $\tan\theta = \frac{\sqrt{31}}{3}$ (1.8859..) or cos rule **NB. Answer is given, but allow final A1 if all previous work is correct.** (c) First M1: (alternative) $\frac{1}{2}\times 3\times\sqrt{40-9}$	

Question Number	Scheme	Notes	Marks
70.	$x=1+t-5\sin t,\ y=2-4\cos t,\ -\pi \leqslant t \leqslant \pi;\ A(k,\,2),\ k>0$, lies on C		
(a)	$\{$When $y=2,\}\ 2=2-4\cos t \Rightarrow t=-\frac{\pi}{2},\ \frac{\pi}{2}$ $k(\text{or }x)=1+\frac{\pi}{2}-5\sin\left(\frac{\pi}{2}\right)$ **or** $k(\text{or }x)=1-\frac{\pi}{2}-5\sin\left(-\frac{\pi}{2}\right)$	Sets $y=2$ to find t and some evidence of using their t to find $x=...$	M1
	$\left\{\text{When } t=-\frac{\pi}{2},\ k>0,\right\}$ so $k=6-\frac{\pi}{2}$ or $\frac{12-\pi}{2}$	k (or x) $=6-\frac{\pi}{2}$ or $\frac{12-\pi}{2}$	A1
			[2]
(b)	$\frac{dx}{dt}=1-5\cos t,\quad \frac{dy}{dt}=4\sin t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct (Can be implied)	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct (Can be implied)	B1
	$\frac{dy}{dx}=\frac{4\sin t}{1-5\cos t}$ at $t=-\frac{\pi}{2},\ \frac{dy}{dx}=\frac{4\sin\left(-\frac{\pi}{2}\right)}{1-5\cos\left(-\frac{\pi}{2}\right)}\quad\{=-4\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes their t into their $\frac{dy}{dx}$ **Note:** their t can lie outside $-\pi \leqslant t \leqslant \pi$ for this mark	M1
	• $y-2=-4\left(x-\left(6-\frac{\pi}{2}\right)\right)$ • $2=(-4)\left(6-\frac{\pi}{2}\right)+c \Rightarrow y=-4x+2+4\left(6-\frac{\pi}{2}\right)$	Correct straight line method for an equation of a tangent where $m_T\ (\neq m_N)$ is found using calculus **Note:** their k (or x) must be in terms of π and correct bracketing must be used or implied	M1
	$\{y-2=-4x+24-2\pi \Rightarrow\}\quad y=-4x+26-2\pi$	**dependent on all previous marks in part (b)** $y=-4x+26-2\pi$	A1 **cso**
		$(p=-4,\ q=26-2\pi)$	**[5]**
			7

	Question 70 Notes	
5. (a)	**Note**	M1 can be implied by either x or $k=6-\frac{\pi}{2}$ or awrt 4.43 or x or $k=\frac{\pi}{2}-4$ or awrt -2.43
	Note	An answer of 4.429… without reference to a correct **exact** answer is A0
	Note	M1 can be earned in part (a) by working in degrees
	Note	Give M0 for not substituting their t back into x. E.g. $2=2-4\cos t \Rightarrow t=-\frac{\pi}{2} \Rightarrow k=-\frac{\pi}{2}$
	Note	If two values for k are found, they must identify the correct answer for A1
	Note	Condone M1 for $2=2-4\cos t \Rightarrow t=-\frac{\pi}{2},\ \frac{\pi}{2} \Rightarrow x=1-\frac{\pi}{2}-5\sin\left(\frac{\pi}{2}\right)$
(b)	**Note**	The 1st M mark may be implied by their value for $\frac{dy}{dx}$ e.g. $\frac{dy}{dx}=\frac{4\sin t}{1-5\cos t}$, followed by an answer of -4 (from $t=-\frac{\pi}{2}$) or 4 (from $t=\frac{\pi}{2}$)
	Note	Give 1st M0 for **applying** their $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ even if they state $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$
	2nd M1	• **applies** $y-2=(\text{their } m_T)(x-(\text{their } k))$, • **applies** $2=(\text{their } m_T)(\text{their } k)+c$ **leading to** $y=(\text{their } m_T)x+(\text{their } c)$ where k must be in terms of π and $m_T\ (\neq m_N)$ is a numerical value found using calculus
	Note	Correct bracketing must be used for 2nd M1, but this mark can be implied by later working

	Question 70 Notes Continued	
70. (b)	**Note**	The final A mark is dependent on all previous marks in part (b) being scored. This is because the correct answer can follow from an incorrect $\frac{\mathrm{d}y}{\mathrm{d}x}$
	Note	The first 3 marks can be gained by using degrees in part (b)
	Note	Condone mixing a correct t with an incorrect x or an incorrect t with a correct x for the M marks
	Note	Allow final A1 for any answer in the form $y = px + q$ E.g. Allow final A1 for $y = -4x + 26 - 2\pi$, $y = -4x + 2 + 4\left(6 - \frac{\pi}{2}\right)$ or $y = -4x + \left(\frac{52 - 4\pi}{2}\right)$
	Note	Do not apply isw in part (b). So, an incorrect answer following from a correct answer is A0
	Note	Do not allow $y = 2(-2x + 13 - \pi)$ for A1
	Note	$y = -4x + 26 - 2\pi$ followed by $y = 2(-2x + 13 - \pi)$ is condoned for final A1

Question Number	Scheme		Marks
71.	$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t$		
(a)	**Main Scheme**		
	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$	$\cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	So, $\{x + y\} = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t$	Adds their expanded x (which is in terms of t) to $2\sin t$	dM1
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) + 2\sin t$		
	$= 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
(a)	**Alternative Method 1**		
	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$	$\cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) = 2\sqrt{3}\cos t - 2\sin t$		
	So, $x = 2\sqrt{3}\cos t - y$	Forms an equation in x, y and t.	dM1
	$x + y = 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
(b)	**Main Scheme**		
	$\left(\frac{x+y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow \frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$		
	$\Rightarrow (x+y)^2 + 3y^2 = 12$	$(x+y)^2 + 3y^2 = 12$ $\{a = 3, b = 12\}$	A1
			[2]
(b)	**Alternative Method 1**		
	$(x+y)^2 = 12\cos^2 t = 12(1 - \sin^2 t) = 12 - 12\sin^2 t$		
	So, $(x+y)^2 = 12 - 3y^2$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow (x+y)^2 + 3y^2 = 12$	$(x+y)^2 + 3y^2 = 12$	A1
			[2]
(b)	**Alternative Method 2**		
	$(x+y)^2 = 12\cos^2 t$		
	As $12\cos^2 t + 12\sin^2 t = 12$		
	then $(x+y)^2 + 3y^2 = 12$		M1, A1
			[2]
			5

		Question 71 Notes
71. (a)	M1	$\cos\left(t+\frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$ or $\cos\left(t+\frac{\pi}{6}\right) \to \left(\frac{\sqrt{3}}{2}\right)\cos t \pm \left(\frac{1}{2}\right)\sin t$
	Note	**If a candidate states** $\cos(A+B) = \cos A\cos B \pm \sin A\sin B$, but there is an error ***in its application*** then give M1. **<u>Awarding the dM1 mark which is dependent on the first method mark</u>**
Main	dM1 **Note**	Adds their expanded x (which is in terms of t) to $2\sin t$ Writing $x+y=...$ is not needed in the **Main Scheme** method.
Alt 1	dM1	Forms an equation in x, y and t.
	A1*	Evidence of $\cos\left(\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{6}\right)$ evaluated and the proof is correct with no errors.
	Note	$\{x+y\} = 4\cos\left(t+\frac{\pi}{6}\right) + 2\sin t$, by itself is M0M0A0.
(b)	M1 A1	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's. leading $(x+y)^2 + 3y^2 = 12$
	SC	Award **Special Case B1B0** for a candidate who writes down **either** • $(x+y)^2 + 3y^2 = 12$ from no working • $a=3,\ b=12$, but **<u>does not provide a correct proof</u>**.
	Note **Note**	Alternative method 2 is fine for M1 A1 Writing $(x+y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b \Rightarrow a=3,\ b=12$ is SC: B1B0
	Note	Writing $(x+y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b$ • states $a=3,\ b=12$ • and refers to either $\cos^2 t + \sin^2 t = 1$ or $12\cos^2 t + 12\sin^2 t = 12$ • and there is no incorrect working would get M1A1

Question Number	Scheme		Marks
72.	$x = 27\sec^3 t, \quad y = 3\tan t, \quad 0 \leqslant t \leqslant \dfrac{\pi}{3}$		
	$\{1 + \tan^2 t = \sec^2 t\} \Rightarrow 1 + \left(\dfrac{y}{3}\right)^2 = \left(\sqrt[3]{\left(\dfrac{x}{27}\right)}\right)^2 = \left(\dfrac{x}{27}\right)^{\frac{2}{3}}$		M1
	$\Rightarrow 1 + \dfrac{y^2}{9} = \dfrac{x^{\frac{2}{3}}}{9} \Rightarrow 9 + y^2 = x^{\frac{2}{3}} \Rightarrow y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ *		A1 * **cso**
	$a = 27$ and $b = 216$ or $27 \leqslant x \leqslant 216$	$a = 27$ and $b = 216$	B1
			[3]

Notes for Question 72 Continued	
	Note: Please check that their $\dfrac{dx}{dt}$ is differentiated correctly. Eg. Note that $x = 27\sec^3 t = 27(\cos t)^{-3} \Rightarrow \dfrac{dx}{dt} = -81(\cos t)^{-2}(-\sin t)$ is correct. **M1:** Either: • Applying a correct trigonometric identity (usually $1 + \tan^2 t = \sec^2 t$) to give a Cartesian equation in x and y only. • Starting from the RHS and goes on to achieve $\sqrt{9\tan^2 t}$ by using a correct trigonometric identity. • Starts from the LHS and goes on to achieve $\sqrt{9\sec^2 t - 9}$ by using a correct trigonometric identity. **A1*:** For a correct proof of $y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$. **Note** this result is printed on the Question Paper, so no incorrect working is allowed. **B1:** Both $a = 27$ and $b = 216$. **Note** that $27 \leqslant x \leqslant 216$ is also fine for B1.

Notes for Question 72 Continued			
72. **Way 2**	***Alternative responses for M1A1*: STARTING FROM THE RHS**		
	$\{\text{RHS} =\} \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} = \sqrt{\left(27\sec^3 t\right)^{\frac{2}{3}} - 9} = \sqrt{9\sec^2 t - 9} = \sqrt{9\tan^2 t}$	For applying $1 + \tan^2 t = \sec^2 t$ oe to achieve $\sqrt{9\tan^2 t}$	M1
	$= 3\tan t = y \; \{= \text{LHS}\}$ **cso**	Correct proof from $\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ to y.	A1*
	M1: Starts from the RHS and goes on to achieve $\sqrt{9\tan^2 t}$ by using a correct trigonometric identity.		
72. **Way 3**	***Alternative responses for M1A1 in part (b)*: STARTING FROM THE LHS**		
	$\{\text{LHS} =\} \; y = 3\tan t = \sqrt{\left(9\tan^2 t\right)} = \sqrt{9\sec^2 t - 9}$	For applying $1 + \tan^2 t = \sec^2 t$ oe to achieve $\sqrt{9\sec^2 t - 9}$	M1
	$= \sqrt{9\left(\dfrac{x}{27}\right)^{\frac{2}{3}} - 9} = \sqrt{9\left(\dfrac{x^{\frac{2}{3}}}{9}\right) - 9} = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ **cso**	Correct proof from y to $\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$.	A1*
	M1: Starts from the LHS and goes on to achieve $\sqrt{9\sec^2 t - 9}$ by using a correct trigonometric identity.		

Question Number	Scheme	Marks
73.		
(a)	$\dfrac{dx}{dt} = \dfrac{1}{t}$, $\quad \dfrac{dy}{dt} = 2t$	
	$\dfrac{dy}{dx} = 2t^2$	M1 A1
	Using $mm' = -1$, at $t = 3$	
	$m' = -\dfrac{1}{18}$	M1 A1
	$y - 7 = -\dfrac{1}{18}(x - \ln 3)$	M1 A1 (6)
(b)	$x = \ln t \Rightarrow t = e^x$	B1
	$y = e^{2x} - 2$	M1 A1 (3)

Question Number	Scheme		Marks	
74	(a) $y = 0 \Rightarrow t(9 - t^2) = t(3 - t)(3 + t) = 0$			
	$t = 0, 3, -3$	Any one correct value	B1	
	At $t = 0$, $x = 5(0)^2 - 4 = -4$	Method for finding one value of x	M1	
	At $t = 3$, $x = 5(3)^2 - 4 = 41$			
	$\left(\text{At } t = -3,\ x = 5(-3)^2 - 4 = 41\right)$			
	At A, $x = -4$; at B, $x = 41$	Both	A1	(3)