EXPERT TUITION

Maths Questions By Topic:

Differentiation
Mark Scheme

A-Level Edexcel

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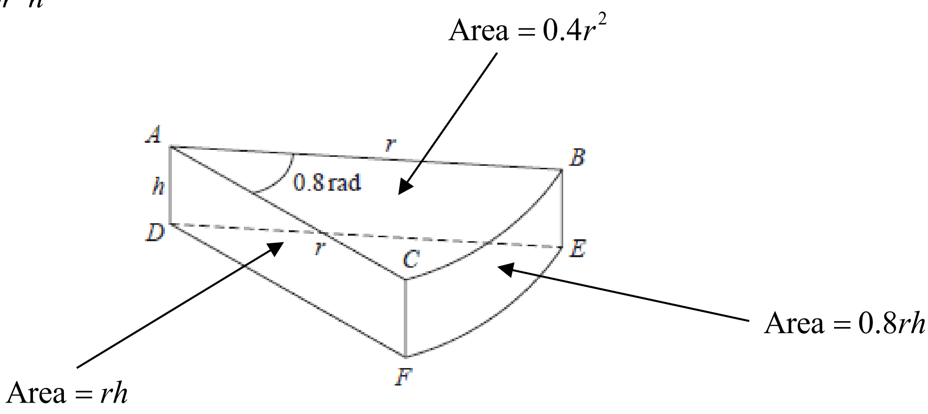
Question	Scheme	Marks	AOs
1 (a)	Sets up an allowable equation using volume $=240$ E.g. $\frac{1}{2}r^2 \times 0.8h = 240 \Rightarrow h = \frac{600}{r^2}$ o.e.	M1 A1	3.4 1.1b
	Attempts to substitute their $h = \frac{600}{r^2}$ into $(S=)\frac{1}{2}r^2 \times 0.8 + \frac{1}{2}r^2 \times 0.8 + 2rh + 0.8rh$	dM1	3.4
	$S = 0.8r^2 + 2.8rh = 0.8r^2 + 2.8 \times \frac{600}{r} = 0.8r^2 + \frac{1680}{r}$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{dS}{dr}\right) = 1.6r - \frac{1680}{r^2}$	M1 A1	3.1a 1.1b
	Sets $\frac{dS}{dr} = 0 \Rightarrow r^3 = 1050$ $r =$ awrt 10.2	dM1 A1	2.1 1.1b
		(4)	
(c)	Attempts to substitute their positive r into $\left(\frac{d^2S}{dr^2}\right) = 1.6 + \frac{3360}{r^3}$ and considers its value or sign	M1	1.1b
	E.g. Correct $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$ with $\left.\frac{d^2S}{dr^2}\right\|_{r=10.2} = 5 > 0$ proving a minimum value of S	A1	1.1b
		(2)	
			(10 marks)
Notes:			

Volume $= 0.4r^2h$



Total surface area $= 2rh + 0.8r^2 + 0.8rh$

1 **(a)**

M1: Attempts to use the fact that the volume of the toy is 240 cm^3

Sight of $\frac{1}{2}r^2 \times 0.8 \times h = 240$ leading to $h = ...$ or $rh = ...$ scores this mark

But condone an equation of the correct form so allow for $kr^2h = 240 \Rightarrow h = ...$ or $rh = ...$

A1: A correct expression for $h = \frac{600}{r^2}$ or $rh = \frac{600}{r}$ which may be left unsimplified.

This may be implied when you see an expression for S or part of S E.g $2rh = 2r \times \frac{600}{r^2}$

dM1: Attempts to substitute their $h = \frac{a}{r^2}$ o.e. such as $hr = \frac{a}{r}$ into a **correct** expression for S

Sight of $\frac{1}{2}r^2 \times 0.8 + \frac{1}{2}r^2 \times 0.8 + rh + rh + 0.8rh$ with an appropriate substitution

Simplified versions such as $0.8r^2 + 2rh + 0.8rh$ used with an appropriate substitution is fine.

A1*: Correct work leading to the given result.

$S =$, $SA =$ or surface area = must be seen at least once in the correct place

The method must be made clear so expect to see evidence. For example

$S = 0.8r^2 + 2rh + 0.8rh \Rightarrow S = 0.8r^2 + 2r \times \frac{600}{r^2} + 0.8r \times \frac{600}{r^2} \Rightarrow S = 0.8r^2 + \frac{1680}{r}$ would be fine.

(b) There is no requirement to see $\frac{dS}{dr}$ in part (b). It may even be called $\frac{dy}{dx}$.

M1: Achieves a derivative of the form $pr \pm \frac{q}{r^2}$ where p and q are non- zero constants

A1: Achieves $\left(\frac{dS}{dr}\right) = 1.6r - \frac{1680}{r^2}$

dM1: Sets or implies that their $\frac{dS}{dr} = 0$ and proceeds to $mr^3 = n$, $m \times n > 0$. It is dependent upon a correct attempt at differentiation. This mark may be implied by a correct answer to their $pr - \frac{q}{r^2} = 0$

A1: $r =$ awrt 10.2 or $\sqrt[3]{1050}$

(c)

M1: Attempts to substitute their positive r (found in (b)) into $\left(\frac{d^2S}{dr^2} =\right) e \pm \frac{f}{r^3}$ where e and f are non zero and finds its value or sign.

Alternatively considers the sign of $\left(\frac{d^2S}{dr^2} =\right) e \pm \frac{f}{r^3}$ (at their positive r found in (b))

Condone the $\frac{d^2S}{dr^2}$ to be $\frac{d^2y}{dx^2}$ or being absent, but only for this mark.

A1: States that $\frac{d^2S}{dr^2}$ or $S'' = 1.6 + \frac{3360}{r^3} =$ awrt $5 > 0$ proving a minimum value of S

This is dependent upon having achieved $r =$ awrt 10 and a correct $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$

It can be argued without finding the value of $\frac{d^2S}{dr^2}$. E.g. $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3} > 0$ as $\underline{r > 0}$, so minimum value of S. For consistency it is also dependent upon having achieved $r =$ awrt 10

Do **NOT** allow $\frac{d^2y}{dx^2}$ for this mark

Question	Scheme	Marks	AOs
2(a)	$V=\pi r^2 h=355\Rightarrow h=\dfrac{355}{\pi r^2}$ $\left(\text{or } rh=\dfrac{355}{\pi r} \text{ or } \pi rh=\dfrac{355}{r}\right)$	B1	1.1b
	$C=0.04\left(\pi r^2+2\pi rh\right)+0.09\left(\pi r^2\right)$	M1	3.4
	$C=0.13\pi r^2+0.08\pi rh=0.13\pi r^2+0.08\pi r\left(\dfrac{355}{\pi r^2}\right)$	dM1	2.1
	$C=0.13\pi r^2+\dfrac{28.4}{r}$ *	A1*	1.1b
		(4)	
(b)	$\dfrac{dC}{dr}=0.26\pi r-\dfrac{28.4}{r^2}$	M1 A1	3.4 1.1b
	$\dfrac{dC}{dr}=0\Rightarrow r^3=\dfrac{28.4}{0.26\pi}\Rightarrow r=...$	M1	1.1b
	$r=\sqrt[3]{\dfrac{1420}{13\pi}}=3.26...$	A1	1.1b
		(4)	
(c)	$\left(\dfrac{d^2C}{dr^2}=\right)\ 0.26\pi+\dfrac{56.8}{r^3}=0.26\pi+\dfrac{56.8}{"3.26"^3}$	M1	1.1b
	$\left(\dfrac{d^2C}{dr^2}=\right)\ (2.45..)>0$ Hence minimum (cost)	A1	2.4
		(2)	
(d)	$C=0.13\pi\left("3.26"\right)^2+\dfrac{28.4}{"3.26"}$	M1	3.4
	$(C=)13$	A1	1.1b
		(2)	
			(12 marks)

Notes

(a)

B1: Correct expression for h or rh or πrh in terms of r. This may be implied by their later substitution.

M1: Scored for the sum of the three terms of the form $0.04...r^2$, $0.09...r^2$ and $0.04\times...rh$

The $0.04\times...rh$ may be implied by eg $0.04\times...r\times\dfrac{355}{\pi r^2}$ if h has already been replaced

dM1: Substitutes h or rh or πrh into their equation for C which must be of an allowable form (see above) to obtain an equation connecting C and r.
It is dependent on a correct expression for h or rh or πrh in terms of r

A1*: Achieves given answer with no errors. Allow Cost instead of C but they cannot just have an expression.
As a minimum you must see

- the separate equation for volume
- the two costs for the top and bottom separate before combining
- a substitution before seeing the $\frac{28.4}{r}$ term

Eg $355 = \pi r^2 h$ and $C = 0.04\pi r^2 + 0.09\pi r^2 + 0.04 \times 2\pi rh = 0.13\pi r^2 + 0.08\pi \times \left(\frac{355}{\pi r}\right)$

(b)

M1: Differentiates to obtain at least $r^{-1} \rightarrow r^{-2}$

A1: Correct derivative.

M1: Sets $\frac{\mathrm{d}C}{\mathrm{d}r} = 0$ and solves for r. There must have been some attempt at differentiation of the equation for C ($...r^2 \to ...r$ or $...r^{-1} \to ...r^{-2}$) Do not be concerned with the mechanics of their rearrangement and do not withhold this mark if their solution for r is negative

A1: Correct value for r. Allow exact value or awrt 3.26

(c)

M1: Finds $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$ at their (positive) r or considers the sign of $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$.

This mark can be scored as long as their second derivative is of the form $A + \frac{B}{r^3}$ where A and B are non zero

A1: Requires

- A correct $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$
- Either
    - deduces $\frac{\mathrm{d}^2C}{\mathrm{d}r^2} > 0$ for $r > 0$ (without evaluating). There must be some minimal explanation as to why it is positive.
    - substitute their positive r into $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$ without evaluating and deduces $\frac{\mathrm{d}^2C}{\mathrm{d}r^2} > 0$ for $r > 0$
    - evaluate $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$ (which must be awrt 2.5) and deduces $\frac{\mathrm{d}^2C}{\mathrm{d}r^2} > 0$ for $r > 0$

(d)

M1: Uses the model and their positive r found in (b) to find the minimum cost. Their r embedded in the expression is sufficient. May be seen in (b) but must be used in (d).

A1: $(C =)\ 13$ ignore units

Question	Scheme	Marks	AOs
3(a)	$H = ax^2 + bx + c$ and $x=0,\ H=3 \Rightarrow H = ax^2+bx+3$	M1	3.3
	$H = ax^2 + bx + 3$ and $x=120, H=27 \Rightarrow 27 = 14400a + 120b + 3$ **or** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$	M1 A1	3.1b 1.1b
	$H = ax^2 + bx + 3$ and $x=120, H=27 \Rightarrow 27 = 14400a + 120b + 3$ **and** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$ $\Rightarrow a = ..., b = ...$	dM1	3.1b
	$H = -\frac{1}{300}x^2 + \frac{3}{5}x + 3$ o.e.	A1	1.1b
		(5)	
(b)(i)	$x = 90 \Rightarrow H\left(= -\frac{1}{300}(90)^2 + \frac{3}{5}(90) + 3\right) = 30\,\text{m}$	B1	3.4
(b)(ii)	$H = 0 \Rightarrow -\frac{1}{300}x^2 + \frac{3}{5}x + 3 = 0 \Rightarrow x = ...$	M1	3.4
	$x = (-4.868...,)\ 184.868...$ $\Rightarrow x = 185\,(\text{m})$	A1	3.2a
		(3)	
(c)	Examples must focus on why the model may not be appropriate or give values/situations where the model would break down: E.g. • The ground is unlikely to be horizontal • The ball is not a particle so has dimensions/size • The ball is unlikely to travel in a vertical plane (as it will spin) • H is not likely to be a quadratic function in x	B1	3.5b
		(1)	
			(9 marks)

Notes

(a)

M1: Translates the problem into a suitable model and uses $H = 3$ when $x = 0$ to establish $c = 3$
Condone with $a = \pm 1$ so $H = x^2 + bx + 3$ will score M1 but little else

M1: For a correct attempt at **using one of the two other pieces** of information within a quadratic model **Either** uses $H = 27$ when $x = 120$ (with $c = 3$) to produce a linear equation connecting a and b for the model **Or** differentiates and uses $\frac{dH}{dx} = 0$ when $x = 90$. Alternatives exist here, using the symmetrical nature of the curve, so they could use $x = -\frac{b}{2a}$ at vertex or use point (60, 27) or (180,3).

A1: At least one correct equation connecting a and b. Remember "a" could have been set as negative so an equation such as $27 = -14400a + 120b + 3$ would be correct in these circumstances.

dM1: Fully correct strategy that uses $H = ax^2 + bx + 3$ with the two other pieces of information in order to establish the values of **both** a **and** b for the model

A1: Correct equation, not just the correct values of a, b and c. Award if seen in part (b)

(b)(i)

B1: Correct height including the units. CAO

(b)(ii)

M1: Uses $H = 0$ and attempts to solve for x. Usual rules for quadratics.

A1: Discards the negative solution (may not be seen) and identifies awrt 185 m. Condone lack of units

(c)
B1: Candidate should either refer to an issue with one of the four aspects of how the situation has been modelled or give a situation where the model breaks down

- the ball has been modelled as a particle
- there may be trees (or other hazards) in the way that would affect the motion

Condone answers (where the link to the model is not completely made) such as

- the ball will spin
- ground is not flat

Do not accept answers which refer to the situation after it hits the ground (this isn't what was modelled)

- the ball will bounce after hitting the ground
- it gives a negative height for some values for x

Do not accept answers that do not refer to the model in question, or else give single word vague answers

- the height of tee may have been measured incorrectly
- "friction", "spin", "force" etc
- it does not take into account the weight of the ball
- it depends on how good the golfer is
- the shape of the ball will affect the motion
- you cannot hit a ball the same distance each time you hit it

The method using an alternative form of the equation can be scored in a very similar way.
The first M is for the completed square form of the quadratic showing a maximum at $x = 90$
So award M1 for $H = \pm a(x-90)^2 + c$ or $H = \pm a(90-x)^2 + c$. Condone for this mark an equation with $a = 1 \Rightarrow H = (x-90)^2 + c$ or $c = 3 \Rightarrow H = a(x-90)^2 + 3$ but will score little else

Alt (a)	$H = a(x+b)^2 + c$ and $x = 90$ at $H_{max} \Rightarrow H = a(x-90)^2 + c$	M1	3.3
	$H = 3$ when $x = 0 \Rightarrow 3 = 8100a + c$ **or** $H = 27$ when $x = 120 \Rightarrow 27 = 900a + c$	M1 A1	3.1b 1.1b
	$H = 3$ when $x = 0 \Rightarrow 3 = 8100a + c$ **and** $H = 27$ when $x = 120 \Rightarrow 27 = 900a + c$ $\Rightarrow a = \dots, c = \dots$	dM1	3.1b
	$H = -\frac{1}{300}(x-90)^2 + 30$ o.e	A1	1.1b
		(5)	
(b)	$x = 90 \Rightarrow H = 0^2 + 30 = 30\,\text{m}$	B1	3.4
		(1)	
	$H = 0 \Rightarrow 0 = -\frac{1}{300}(x-90)^2 + 30 \Rightarrow x = \dots$	M1	3.4
	$\Rightarrow x = 185\,(\text{m})$	A1	3.2a
		(2)	

Note that $H = -\frac{1}{300}(x-90)^2 + 30$ is equivalent to $H = -\frac{1}{300}(90-x)^2 + 30$

Other versions using symmetry are also correct so please look carefully at all responses

E.g. Using a starting equation of $H = a(x-60)(x-120) + b$ leads to $H = -\frac{1}{300}(x-60)(x-120) + 27$

Question	Scheme	Marks	AOs
4	$y=\dfrac{x-4}{2+\sqrt{x}} \Rightarrow \dfrac{dy}{dx}=\dfrac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$	M1 A1	2.1 1.1b
	$=\dfrac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}=\dfrac{2+\sqrt{x}-\frac{1}{2}\sqrt{x}+2x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}=\dfrac{2\sqrt{x}+\frac{1}{2}x+2}{\sqrt{x}\left(2+\sqrt{x}\right)^2}$	M1	1.1b
	$=\dfrac{x+4\sqrt{x}+4}{2\sqrt{x}\left(2+\sqrt{x}\right)^2}=\dfrac{\left(2+\sqrt{x}\right)^2}{2\sqrt{x}\left(2+\sqrt{x}\right)^2}=\dfrac{1}{2\sqrt{x}}$	A1	2.1
		(4)	
		(4 marks)	

Notes

M1: Attempts to use a correct rule e.g. quotient or product (& chain) rule to achieve the following forms

Quotient : $\dfrac{\alpha\left(2+\sqrt{x}\right)-\beta(x-4)x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$ but be tolerant of attempts where the $\left(2+\sqrt{x}\right)^2$ has been incorrectly expanded   Product: $\alpha\left(2+\sqrt{x}\right)^{-1}+\beta x^{-\frac{1}{2}}(x-4)\left(2+\sqrt{x}\right)^{-2}$

Alternatively with $t=\sqrt{x}$, $y=\dfrac{t^2-4}{2+t}\Rightarrow\dfrac{dy}{dx}=\dfrac{dy}{dt}\times\dfrac{dt}{dx}=\dfrac{2t(2+t)-\left(t^2-4\right)}{(2+t)^2}\times\dfrac{1}{2}x^{-\frac{1}{2}}$ with same rules

A1: Correct derivative in any form. Must be in terms of a single variable (which could be t)

M1: Following a correct attempt at differentiation, it is scored for multiplying both numerator and denominator by $\sqrt{x}$ and collecting terms to form a single fraction. It can also be scored from $\dfrac{uv'-vu'}{v^2}$

For the $t=\sqrt{x}$, look for an attempt to simplify $\dfrac{t^2+4t+4}{(2+t)^2}\times\dfrac{1}{2t}$

A1: Correct expression showing all key steps with no errors or omissions. $\dfrac{dy}{dx}$ must be seen at least once

Question	Scheme	Marks	AOs
4	$y=\dfrac{x-4}{2+\sqrt{x}}\Rightarrow y=\dfrac{\left(\sqrt{x}+2\right)\left(\sqrt{x}-2\right)}{2+\sqrt{x}}=\sqrt{x}-2$	M1 A1	2.1 1.1b
	$\dfrac{dy}{dx}=\dfrac{1}{2\sqrt{x}}$	M1 A1	1.1b 2.1
		(4)	
		(4 marks)	

Notes

M1: Attempts to use difference of two squares. Can also be scored using

$t=\sqrt{x}\Rightarrow y=\dfrac{t^2-4}{t+2}\Rightarrow y=\dfrac{(t+2)(t-2)}{t+2}$

A1: $y=\sqrt{x}-2$ or $y=t-2$

M1: Attempts to differentiate an expression of the form $y=\sqrt{x}+b$

A1: Correct expression showing all key steps with no errors or omissions. $\dfrac{dy}{dx}$ must be seen at least once

Question	Scheme	Marks	AOs
5	Any equation involving an exponential of the correct form. See notes	M1	3.1b
	$n = Ae^{kt}$ (where A and k are positive constants)	A1	1.1b
		(2)	
			(2 marks)

Notes:

M1: Any equation of the correct form, involving n and an exponential in t.

So allow for example $n = e^{\pm t}$, $n = Ae^{\pm t}$, $n = Ae^{\pm kt}$ condoning $n = A + Be^{\pm t}$

Condone an intermediate form where n has not been made the subject.

E.g Allow $\ln n = kt + c$ but also condone $\ln n = kt$

A1: E.g. $n = Ae^{kt}$, $n = e^{kt+c}$, $n = e^{kt}e^{c}$ There is no requirement to state that A and k are positive constants

Note that the two constants need to be different.

Mark the final answer so $n = e^{kt+c}$ followed by $n = e^{kt} + e^{c}$ o.e. $n = e^{kt} + A$ such as is M1 A0

....

You may see solutions that don't include "e".

These are fine so you can include versions of $n = Ak^{t}$ using the same marking criteria as above

FYI $\frac{dn}{dt} = Ak^{t} \times \ln k = \ln k \times n$ so $\frac{dn}{dt} \propto n$

....

Question	Scheme	Marks	AOs
6 (a)	Uses the model to state $\frac{\mathrm{d}V}{\mathrm{d}t}=-c$ (for positive constant c)	B1	3.1b
	Uses $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$ with their $\frac{\mathrm{d}V}{\mathrm{d}t}=-c$ and $\frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$	M1	2.1
	$-c=4\pi r^2\times\frac{\mathrm{d}r}{\mathrm{d}t}\Rightarrow\frac{\mathrm{d}r}{\mathrm{d}t}=-\frac{c}{4\pi r^2}=-\frac{k}{r^2}$ *	A1*	2.2a
		(3)	
(b)	$\frac{\mathrm{d}r}{\mathrm{d}t}=-\frac{k}{r^2}\Rightarrow\int r^2\mathrm{d}r=\int -k\mathrm{d}t$ and integrates with one side "correct"	M1	2.1
	$\frac{r^3}{3}=-kt(+\alpha)$	A1	1.1b
	Uses $t=0, r=40\Rightarrow\alpha=...$ $\quad\alpha=\frac{64000}{3}$	M1	1.1b
	Uses $t=5, r=20$ & $\alpha=...\Rightarrow k=...$	M1	3.4
	$r^3=64000-11200t$ or exact equivalent	A1	3.3
		(5)	
(c)	Uses the equation of their model and proceeds to a limiting value for t E.g. "$64000-11200t$" ... $0\Rightarrow t$...	M1	3.4
	For times up to and including $\frac{40}{7}$ seconds	A1ft	3.5b
		(2)	
			(10 marks)
Notes:			

(a)

B1: Uses the model to state $\frac{dV}{dt} = -c$ (for positive constant c).

Any "letter" is acceptable here including k.

Note that $\frac{dV}{dt} = c$ is B0 unless they state that c is a negative constant.

M1: For an attempt to use $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ with their $\frac{dV}{dt}$ and $\frac{dV}{dr} = 4\pi r^2$

Allow for an attempt to use $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ with their $\frac{dV}{dt}$ and $\frac{dV}{dr} = \lambda r^2$ (Any constant is fine)

There is no requirement to use the correct formula for the volume of a sphere for this mark.

A1*: Proceeds to the given answer with an intermediate line equivalent to $\frac{dr}{dt} = -\frac{c}{4\pi r^2}$

If candidate started with $\frac{dV}{dt} = -k$ they must provide a minimal explanation how

$\frac{dr}{dt} = -\frac{k}{4\pi r^2} \rightarrow \frac{dr}{dt} = -\frac{k}{r^2}$. E.g $\frac{1}{4\pi}$ is a constant so replace $\frac{k}{4\pi}$ with k

It is not necessary to use the full formula for the volume of a sphere, eg allow $V = \kappa r^3$ but if it has been quoted it must be correct. So using $V = 4r^3$ can potentially score 2 of the 3 marks.

(b)

M1: For the key step of separating the variables correctly AND integrating one side with at least one index correct. The integral signs do not need to be seen.

A1: Correct integration E.g. $\frac{r^3}{3} = -kt(+\alpha)$ or equivalent. The $+\alpha$ is not required for this mark.

This may be awarded if k has been given a value.

M1: Uses the initial conditions to find a value for the constant of integration α

If a constant of integration is not present, or k has been given a pre defined value, then only the first two marks can be awarded in part (b)

The mark may be awarded if the equation has been adapted incorrectly. E.g. each term cube rooted.

M1: Uses the second set of conditions with their value of α to find k

This may be awarded if the equation has been adapted incorrectly. E.g. each term cube rooted.

A1: Obtains any correct equation for the model.

E.g. $r^3 = 64000 - 11200t$ or exact equivalent such as $\frac{r^3}{3} = \frac{64000}{3} - \frac{11200}{3}t$.

ISW after sight of a correct answer. Condone recurring decimals e.g. $21333.\dot{3}$ for $\frac{64000}{3}$

Do not award if **only the** rounded/truncated decimal equivalents to say $\frac{64000}{3}$ is used.

(c)

M1: Recognises that the model is only valid when $r \geqslant 0$ **and uses this to find *t***. Condone $r > 0$

Award for an attempt to find the value of t when r =0. See scheme.

It must be from an equation of the form $ar^n = b - ct, \quad a,b,c > 0$ which give + ve values of t.

A1ft: Allow valid for times up to (and including) $\frac{40}{7}$ seconds, 5.71 seconds. Allow $t < \frac{40}{7}$ or $t \leqslant \frac{40}{7}$

There is no requirement for the left hand side of the inequality, 0

States invalid for times greater than $\frac{40}{7}$ seconds, 5.71 seconds.

Follow through on their equation so allow $t <$ their "$\frac{64000}{11200}$" as long as this value is greater than 5 (t = 5 is one of the values in the question)

Question	Scheme	Marks	AOs
7(a)	$x^2 \tan y = 9 \Rightarrow 2x\tan y + x^2\sec^2 y\dfrac{dy}{dx} = 0$	M1 A1	3.1a 1.1b
	Full method to get $\dfrac{dy}{dx}$ in terms of x using $\sec^2 y = 1+\tan^2 y = 1+f(x)$	M1	1.1b
	$\dfrac{dy}{dx} = \dfrac{-2x\times\dfrac{9}{x^2}}{x^2\left(1+\dfrac{81}{x^4}\right)} = \dfrac{-18x}{x^4+81}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{dy}{dx} = \dfrac{-18x}{x^4+81}$ $\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{-18\times(x^4+81)-(-18x)(4x^3)}{(x^4+81)^2} = \dfrac{54(x^4-27)}{(x^4+81)^2}$ o.e.	M1 A1	1.1b 1.1b
	States that when $x < \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} < 0$ when $x = \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} = 0$ **AND** when $x > \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} > 0$ giving a point of inflection when $x = \sqrt[4]{27}$	A1	2.4
		(3)	
			(7 marks)
Notes:			

(a)

M1: Attempts to differentiate $\tan y$ implicitly. Eg. $\tan y \to \sec^2 y\dfrac{dy}{dx}$ or $\cot y \to -\text{cosec}^2 y\dfrac{dy}{dx}$

You may well see an attempt $\tan y = \dfrac{9}{x^2} \Rightarrow \sec^2 y\dfrac{dy}{dx} = ...$

When a candidate writes $x^2\tan y = 9 \Rightarrow x = 3\tan^{-\frac{1}{2}} y$ the mark is scored for $\tan^{-\frac{1}{2}} y \to ...\tan^{-\frac{3}{2}} y\sec^2 y$

A1: Correct differentiation $2x\tan y + x^2\sec^2 y\dfrac{dy}{dx} = 0$

Allow also $\sec^2 y\dfrac{dy}{dx} = -\dfrac{18}{x^3}$ or $2x = -9\text{cosec}^2 y\dfrac{dy}{dx}$ amongst others

M1: Full method to get $\dfrac{dy}{dx}$ in terms of x using $\sec^2 y = 1+\tan^2 y = 1+f(x)$

A1*: Proceeds correctly to the given answer of $\dfrac{dy}{dx} = \dfrac{-18x}{x^4+81}$

(b)

M1: Attempts to differentiate the given expression using the product or quotient rule.

For example look for a correct attempt at $\frac{vu'-uv'}{v^2}$ with $u=-18x, v=x^4+81, u'=\pm18, v'=\ldots x^3$

If no method is seen or implied award for $\frac{\pm18\times\left(x^4+81\right)\pm18x\left(ax^3\right)}{\left(x^4+81\right)^2}$

Using the product rule award for $\pm18\left(x^4+81\right)^{-1}\pm18x\left(x^4+81\right)^{-2}\times cx^3$

A1: Correct **simplified** $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{54\left(x^4-27\right)}{\left(x^4+81\right)^2}$ o.e. such as $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{54x^4-1458}{\left(x^4+81\right)^2}$

Alternatively score for showing that when a correct (unsimplified) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0\Rightarrow x^4=27\Rightarrow x=\sqrt[4]{27}$

Or for substituting $x=\sqrt[4]{27}$ into an unsimplified but correct $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and showing that it is 0

A1: Correct explanation with a minimal conclusion and correct second derivative.
See scheme.

It can be also be argued from $x^4<27$, $x^4=27$ and $x^4>27$ provided the conclusion states that the point of inflection is at $x=\sqrt[4]{27}$

Alternatively substitutes values of x either side of $\sqrt[4]{27}$ and at $\sqrt[4]{27}$, into $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, finds all three values and makes a minimal conclusion.

A different method involves finding $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ and showing that $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\neq0$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$ when $x=\sqrt[4]{27}$

FYI $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{23328x^3}{\left(x^4+81\right)^3}=0.219$ when $x=\sqrt[4]{27}$

Alternative part (a) using arctan

M1: Sets $y=\arctan\frac{9}{x^2}\rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+\left(\frac{9}{x^2}\right)^2}\times\ldots$ where ... could be 1

A2: $y=\arctan\frac{9}{x^2}\rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+\left(\frac{9}{x^2}\right)^2}\times-\frac{18}{x^3}$

A1*: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+\frac{81}{x^4}}\times-\frac{18}{x^3}=\frac{-18x}{x^4+1}$ showing correct intermediate step and no errors.

Question	Scheme	Marks	AOs
8 (a)	Temperature $= 83°\text{C}$	B1	3.4
		(1)	
(b)	$18+65\text{e}^{-\frac{t}{8}}=35 \Rightarrow 65\text{e}^{-\frac{t}{8}}=17$	M1	1.1b
	$t=-8\ln\left(\frac{17}{65}\right)$ $\ln 65-\frac{t}{8}=\ln 17 \Rightarrow t=...$	dM1	1.1b
	$t=10.7$	A1	1.1b
		(3)	
(c)	States a suitable reason • As $t\to\infty, \theta\to 18$ from above. • The minimum temperature is $18°\text{C}$	B1	2.4
		(1)	
(d)	$A+B=94$ or $A+B\text{e}^{-1}=50$	M1	3.4
	$A+B=94$ and $A+B\text{e}^{-1}=50$	A1	1.1b
	Full method to find at least a value for A	dM1	2.1
	Deduces $\mu=\frac{50\text{e}-94}{\text{e}-1}$ or accept $\mu=\text{awrt } 24.4$	A1	2.2a
		(4)	
			(9 marks)

Notes

(a)

B1: Uses the model to state that the temperature $=83°\text{C}$ Requires units as well

(b)

M1: Uses the information and proceeds to $P\text{e}^{\pm\frac{t}{8}}=Q$ condoning slips

dM1: A full method using correct log laws and a knowledge that e^x and $\ln x$ are inverse functions. This cannot be scored from unsolvable equations, e.g $P>0, Q<0$. Condone one error in their solution.

A1: $t=\text{awrt } 10.7$

(c)

B1: States a suitable reason with minimal conclusion

- As $t\to\infty, \theta\to 18$ from above.
- The minimum temperature is $18°\text{C}$ (so it cannot drop to $15°\text{C}$)
- Substitutes $\theta=15$ (or a value between 15 and 18) into $18+65\text{e}^{-\frac{t}{8}}=15$ (may be impied by $15-18=-3$ or similar) and makes a statement that $\text{e}^{-\frac{t}{8}}$ cannot be less than zero or else that $\ln(-ve)$ is undefined and hence $\theta\neq 15$. All calculations must be correct
- (According to the model) the room temperature is $18°\text{C}$ (so cannot fall below this)

(d)

M1: Attempts to use $(0, 94)$ or $(8, 50)$ in order to form at least one equation in A and B
Allow this to be scored with an equation containing e^0

A1: Correct equations $A + B = 94$ and $A + Be^{-1} = 50$ or equivalent. $e^0 = 1$ must have been processed. Condone $A + B = 94$ and $A + 0.37B = 50$ where $e^{-1} =$ awrt 0.37

dM1: Dependent upon having two equations in A and B formed from the information given. It is a full and correct method leading to a value of A. Allow this to be solved from a calculator.
Note $B = 69.6..$ or $\frac{44}{1 - e^{-1}} \Rightarrow A = 94 - "B"$

A1: Deduces that $\mu = \frac{50e - 94}{e - 1}$ or accept $\mu =$ awrt 24.4. Condone $y = \ldots$

Question	Scheme	Marks	AOs
9(a)	Correct method used in attempting to differentiate $y=\dfrac{5x^2+10x}{(x+1)^2}$	M1	3.1a
	$\dfrac{dy}{dx}=\dfrac{(x+1)^2\times(10x+10)-(5x^2+10x)\times 2(x+1)}{(x+1)^4}$ oe	A1	1.1b
	Factorises/Cancels term in $(x+1)$ and attempts to simplify $\dfrac{dy}{dx}=\dfrac{(x+1)\times(10x+10)-(5x^2+10x)\times 2}{(x+1)^3}=\dfrac{A}{(x+1)^3}$	M1	2.1
	$\dfrac{dy}{dx}=\dfrac{10}{(x+1)^3}$	A1	1.1b
		(4)	
(b)	For $x<-1$ Follow through on their $\dfrac{dy}{dx}=\dfrac{A}{(x+1)^n},n=1,3$	B1ft	2.2a
		(1)	
		(5 marks)	

(a)

M1: Attempts to use a correct rule to differentiate Eg: Use of quotient (& chain) rules on $y=\dfrac{5x^2+10x}{(x+1)^2}$

Alternatively uses the product (and chain) rules on $y=(5x^2+10x)(x+1)^{-2}$

Condone slips but expect $\left(\dfrac{dy}{dx}\right)=\dfrac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{(x+1)^4}$ $(A,B,C,D>0)$ or

$\left(\dfrac{dy}{dx}\right)=\dfrac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{\left((x+1)^2\right)^2}$ $(A,B,C,D>0)$ using the quotient rule

or $\left(\dfrac{dy}{dx}\right)=(x+1)^{-2}\times(Ax+B)+(5x^2+10x)\times C(x+1)^{-3}$ $(A,B,C\neq 0)$ using the product rule.

Condone missing brackets and slips for the M mark. For instance if they quote $u=5x^2+10$, $v=(x+1)^2$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow where they quote the correct formula, give values of u and v, but only have v rather than v^2 the denominator.

A1: A correct (unsimplified) answer

Eg. $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{(x+1)^2\times(10x+10)-(5x^2+10x)\times 2(x+1)}{(x+1)^4}$ or equivalent via the quotient rule.

OR $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=(x+1)^{-2}\times(10x+10)+(5x^2+10x)\times -2(x+1)^{-3}$ or equivalent via the product rule

M1: A valid attempt to proceed to the given form of the answer.

It is dependent upon having a quotient rule of $\pm\frac{v\mathrm{d}u-u\mathrm{d}v}{v^2}$ and proceeding to $\frac{A}{(x+1)^3}$

It can also be scored on a quotient rule of $\pm\frac{v\mathrm{d}u-u\mathrm{d}v}{v}$ and proceeding to $\frac{A}{(x+1)}$

You may see candidates expanding terms in the numerator. FYI $10x^3+30x^2+30x+10-10x^3-30x^2-20x$ but under this method they must reach the same expression as required by the main method.

Using the product rule expect to see a common denominator being used correctly before the above

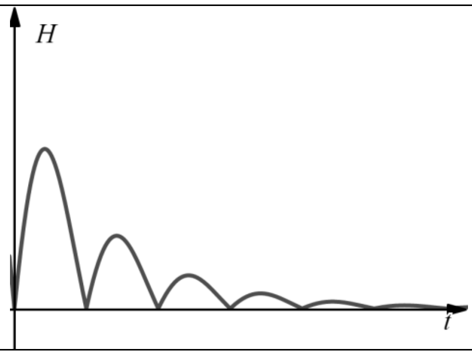
A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{10}{(x+1)^3}$ There is no requirement to see $\frac{\mathrm{d}y}{\mathrm{d}x}=$ and they can recover from missing brackets/slips.

(b)

B1ft: Score for deducing the correct answer of $x<-1$ This can be scored independent of their answer to part (a). Alternatively score for a correct **ft** answer for their $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{A}{(x+1)^n}$ where $A<0$ and $n=1,3$ award for $x>-1$. So for example if $A>0$ and $n=1,3\Rightarrow x<-1$

Question	Scheme	Marks	AOs
Alt via division	Writes $y=\frac{5x^2+10x}{(x+1)^2}$ in form $y=A\pm\frac{B}{(x+1)^2}$ $A,B\neq 0$	M1	3.1a
	Writes $y=\frac{5x^2+10x}{(x+1)^2}$ in the form $y=5-\frac{5}{(x+1)^2}$	A1	1.1b
	Uses the chain rule $\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{C}{(x+1)^3}$ (May be scored from $A=0$)	M1	2.1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{10}{(x+1)^3}$ which cannot be awarded from incorrect value of A	A1	1.1b
		(4)	
(b)	For $x<-1$ or correct follow through	B1ft	2.2a
		(1)	

(5 marks)

Question	Scheme	Marks	AOs
10(a)	$\mathrm{f}(x)=10\mathrm{e}^{-0.25x}\sin x$		
	$\Rightarrow \mathrm{f}'(x)=-2.5\mathrm{e}^{-0\ 25x}\sin x+10\mathrm{e}^{-0\ 25x}\cos x$ oe	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x)=0\Rightarrow -2.5\cancel{\mathrm{e}^{-0\ 25x}}\sin x+10\cancel{\mathrm{e}^{-0\ 25x}}\cos x=0$	M1	2.1
	$\dfrac{\sin x}{\cos x}=\dfrac{10}{2.5}\Rightarrow \tan x=4*$	A1*	1.1b
		(4)	
(b)	 "Correct" shape for 2 loops Fully correct with decreasing heights	M1 A1	1.1b 1.1b
		(2)	
(c)	Solves $\tan x=4$ and substitutes answer into $H(t)$	M1	3.1a
	$H(4.47)=\left\lvert 10\mathrm{e}^{-0\ 25\times 4\ 47}\sin 4.47\right\rvert$	M1	1.1b
	awrt 3.18 (metres)	A1	3.2a
		(3)	
(d)	The **times between** each bounce should not stay the same when the heights of each bounce is getting smaller	B1	3.5b
		(1)	
			(10 marks)

(a)

M1: For attempting to differentiate using the product rule condoning slips, for example the power of e .

So for example score expressions of the form $\pm ...\mathrm{e}^{-0\ 25x}\sin x\pm ...\mathrm{e}^{-0\ 25x}\cos x$ M1

Sight of $vdu-udv$ however is M0

A1: $\mathrm{f}'(x)=-2.5\mathrm{e}^{-0\ 25x}\sin x+10\mathrm{e}^{-0\ 25x}\cos x$ which may be unsimplified

M1: For clear reasoning in setting their $\mathrm{f}'(x)=0$, factorising/ cancelling out the $\mathrm{e}^{-0\ 25x}$ term leading to a trigonometric equation in only $\sin x$ and $\cos x$

Do not allow candidates to substitute $x=\arctan 4$ into $\mathrm{f}'(x)$ to score this mark.

A1*: Shows the steps $\dfrac{\sin x}{\cos x}=\dfrac{10}{2.5}$ or equivalent leading to $\Rightarrow \tan x=4*$. $\dfrac{\sin x}{\cos x}$ must be seen.

(b)

M1: Draws at least two "loops". The height of the second loop should be lower than the first loop.

Condone the sight of rounding where there should be cusps

A1: At least 4 loops with decreasing heights and no rounding at the cusps.

The intention should be that the graph should 'sit' on the x -axis but be tolerant.

It is possible to overwrite Figure 3, but all loops must be clearly seen.

(c)
M1: Understands that to solve the problem they are required to substitute an answer to $\tan t = 4$ into $H(t)$

This can be awarded for an attempt to substitute $t =$ awrt 1.33 or $t =$ awrt 4.47 into $H(t)$

$H(t) = 6.96$ implies the use of $t = 1.33$ Condone for this mark only, an attempt to substitute $t =$ awrt $76°$ or awrt $256°$ into $H(t)$

M1: Substitutes $t =$ awrt 4.47 into $H(t) = \left|10e^{-0.25t}\sin t\right|$. Implied by awrt 3.2

A1: Awrt 3.18 metres. Condone the lack of units. If two values are given the correct one must be seen to have been chosen

It is possible for candidates to sketch this on their graphical calculators and gain this answer. If there is no incorrect working seen and 3.18 is given, then award 111 for such an attempt.

(d)
B1: Makes reference to the fact that the time between each bounce should not stay the same when the heights of each bounce is getting smaller.

Look for " time (or gap) between the bounces will change"

'bounces would not be equal times apart'

'bounces would become more frequent'

But do not accept 'the times between each bounce would be longer or slower'

Do not accept explanations such as there are other factors that would affect this such as "wind resistance", friction etc

Question	Scheme	Marks	AOs
11 (a)	Attempts to differentiate $x=4\sin 2y$ and inverts $\frac{\mathrm{d}x}{\mathrm{d}y}=8\cos 2y \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8\cos 2y}$	M1	1.1b
	At (0,0) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8}$	A1	1.1b
		(2)	
(b)	(i) Uses $\sin 2y \approx 2y$ when y is small to obtain $x \approx 8y$	B1	1.1b
	(ii) The value found in (a) is the gradient of the line found in (b)(i)	B1	2.4
		(2)	
(c)	Uses their $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of y and, using both $\sin^2 2y+\cos^2 2y=1$ and $x=4\sin 2y$ in an attempt to write $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ as a function of x Allow for $\frac{\mathrm{d}y}{\mathrm{d}x}=k\frac{1}{\cos 2y}=..\frac{1}{\sqrt{1-(..x)^2}}$	M1	2.1
	A correct answer $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}=8\sqrt{1-\left(\frac{x}{4}\right)^2}$	A1	1.1b
	and in the correct form $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{16-x^2}}$	A1	1.1b
		(3)	
			(7 marks)

(a)

M1: Attempts to differentiate $x=4\sin 2y$ and inverts.

Allow for $\frac{\mathrm{d}x}{\mathrm{d}y}=k\cos 2y \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{k\cos 2y}$ or $1=k\cos 2y\frac{\mathrm{d}y}{\mathrm{d}x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{k\cos 2y}$

Alternatively, changes the subject and differentiates $x=4\sin 2y \to y=...\arcsin\left(\frac{x}{4}\right) \to \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{...}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

It is possible to approach this from $x=8\sin y\cos y \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y}=\pm 8\sin^2 y \pm 8\cos^2 y$ before inverting

A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8}$ Allow both marks for sight of this answer as long as no incorrect working is seen (See below)

Watch for candidates who reach this answer via $\frac{\mathrm{d}x}{\mathrm{d}y}=8\cos 2x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8\cos 2x}$ This is M0 A0

(b)(i)

B1: Uses $\sin 2y \approx 2y$ when y is small to obtain $x = 8y$ oe such as $x = 4(2y)$.

Do not allow $\sin 2y \approx 2\theta$ to get $x = 8\theta$ but allow recovery in (b)(i) or (b)(ii)

Double angle formula is B0 as it does not satisfy the demands of the question.

(b)(ii)

B1: Explains the relationship between the answers to (a) and (b) (i).

For this to be scored the first three marks, in almost all cases, must have been awarded and the statement must refer to both answers

Allow for example "The gradients are the same $\left(=\frac{1}{8}\right)$" 'both have $m = \frac{1}{8}$'

Do not accept the statement that 8 and $\frac{1}{8}$ are reciprocals of each other unless further correct work explains the relationship in terms of $\frac{dx}{dy}$ and $\frac{dy}{dx}$

(c)

M1: Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4\sin 2y$, attempts to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x. The $\frac{dy}{dx}$ may not be seen and may be implied by their calculation.

A1: A correct (un-simplified) answer for $\frac{dy}{dx}$ or $\frac{dx}{dy}$ Eg. $\frac{dy}{dx} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$

A1: $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$ The $\frac{dy}{dx}$ must be seen at least once in part (c) of this solution

....

Alt to (c) using arcsin

M1: Alternatively, changes the subject and differentiates $x = 4\sin 2y \to y = \ldots\arcsin\left(\frac{x}{4}\right) \to \frac{dy}{dx} = \frac{\ldots}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

Condone a lack of bracketing on the $\frac{x}{4}$ which may appear as $\frac{x^2}{4}$

A1: $\frac{dy}{dx} = \frac{1/8}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$ oe

A1: $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$

Question	Scheme	Marks	AOs
12(a)	$x^n \to x^{n-1}$	M1	1.1b
	$\left(\frac{dy}{dx}\right) = 6x - \frac{24}{x^2}$	A1 A1	1.1b 1.1b
		(3)	
(b)	Attempts $6x - \frac{24}{x^2} > 0 \Rightarrow x >$	M1	1.1b
	$x > \sqrt[3]{4}$ or $x \geqslant \sqrt[3]{4}$	A1	2.5
		(2)	
			(5 marks)

Notes

(a)

M1: $x^n \to x^{n-1}$ for any correct index of x. Score for $x^2 \to x$ or $x^{-1} \to x^{-2}$

Allow for unprocessed indices. $x^2 \to x^{2-1}$ oe

A1: Sight of either $6x$ or $-\frac{24}{x^2}$ which may be un simplified.

Condone an additional term e.g. + 2 for this mark

The indices now must have been processed

A1: $\frac{dy}{dx} = 6x - \frac{24}{x^2}$ or exact simplified equivalent. Eg accept $\frac{dy}{dx} = 6x^1 - 24x^{-2}$

You do not need to see the $\frac{dy}{dx}$ and you should isw after a correct simplified answer.

(b)

M1: Sets an allowable $\frac{dy}{dx}$ **...**0 and proceeds to x... via an allowable intermediate equation

$\frac{dy}{dx}$ must be in the form $Ax + Bx^{-2}$ where $A, B \neq 0$

and the intermediate equation must be of the form $x^p ... q$ oe

Do not be concerned by either the processing, an equality or a different inequality.

It may be implied by $x =$ awrt 1.59

A1: $x > \sqrt[3]{4}$ or $x \geqslant \sqrt[3]{4}$ oe such as $x > 4^{\frac{1}{3}}$ or $x \geqslant 2^{\frac{2}{3}}$

Question	Scheme	Marks	AOs
13 (a)	117 tonnes	B1	3.4
		(1)	
(b)	1200 tonnes	B1	2.2a
		(1)	
(c)	Attempts $\left\{1200-3\times(5-20)^2\right\}-\left\{1200-3\times(4-20)^2\right\}$	M1	3.1a
	93 tonnes	A1	1.1b
		(2)	
(d)	States the model is only valid for values of n such that $n \leqslant 20$	B1	3.5b
	States that the total amount mined cannot decrease	B1	2.3
		(2)	
			(6 marks)

Notes

Note: Only withhold the mark for a lack of tonnes, once, the first time that it occurs.

(a)
B1: 117 tonnes or 117 t.
(b)
B1: 1200 tonnes or 1200 t.
(c)
M1: Attempts $T_5 - T_4 = \left\{1200-3\times(5-20)^2\right\}-\left\{1200-3\times(4-20)^2\right\}$ May be implied by $525-432$

Condone for this mark an attempt at $T_4 - T_3 = \left\{1200-3\times(4-20)^2\right\}-\left\{1200-3\times(3-20)^2\right\}$

A1: 93 tonnes or 93 t
(d)
For one mark
Shows an appreciation of the model
- States $n \leqslant 20$ or $n < 20$
- Condone for one mark $n \leqslant 40$ or $n < 40$ **with** "the mass of tin mined cannot be negative" oe
- Condone for one mark $n = 40$ **with** a statement that "the mass of tin mined becomes 0" oe
- after 20 years the (total) amount of tin mined starts to go down (n may not be mentioned and total may be missing)
- after 20 years the (total) mass reaches a maximum value. (Similar to above)
- States T_{max} is reached when $n = 20$

For two marks
States the limitation on n and explains fully. (Total mass, not mass must be used)
- States that $n \leqslant 20$ and explains that the total mass of tin cannot decrease.
- Alternatively states that n cannot be more than 20 and the total mass of tin would be decreasing
- $0 < n \leqslant 20$ as the maximum total amount of tin mined is reached at 20 years

Question	Scheme	Marks	AOs
14(a)	(i) $\frac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2} = 2 + 6x^{-\frac{3}{2}}$	B1ft	1.1b
		(3)	
(b)	Substitutes $x = 4$ into their $\frac{dy}{dx} = 2\times4 - 2 - 12\times4^{-\frac{1}{2}} = ...$	M1	1.1b
	Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" oe	A1	2.1
		(2)	
(c)	Substitutes $x = 4$ into their $\frac{d^2y}{dx^2} = 2 + 6\times4^{-\frac{3}{2}} = (2.75)$	M1	1.1b
	$\frac{d^2y}{dx^2} = 2.75 > 0$ and states "hence minimum"	A1ft	2.2a
		(2)	
			(7 marks)

(a)(i)

M1: Differentiates to $\frac{dy}{dx} = Ax + B + Cx^{-\frac{1}{2}}$ **A1:** $\frac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$ (Coefficients may be unsimplified)

(a)(ii)

B1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx}$ (Their $\frac{dy}{dx}$ must have a negative or fractional index)

(b)

M1: Substitutes $x = 4$ into their $\frac{dy}{dx}$ and attempts to evaluate. There must be evidence $\left.\frac{dy}{dx}\right|_{x=4} = ...$

Alternatively substitutes $x = 4$ into an equation resulting from $\frac{dy}{dx} = 0$ Eg. $\frac{36}{x} = (x-1)^2$ and equates

A1: There must be a reason and a minimal conclusion. Allow ✓ , QED for a minimal conclusion

Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" oe

Alt Shows that $x = 4$ is a root of the resulting equation and states "hence there is a stationary point"
All aspects of the proof must be correct including a conclusion

(c)

M1: Substitutes $x = 4$ into their $\frac{d^2y}{dx^2}$ and calculates its value, or implies its sign by a statement such as when $x = 4 \Rightarrow \frac{d^2y}{dx^2} > 0$. This must be seen in (c) and not labelled (b). Alternatively calculates the gradient of C either side of $x = 4$ or calculates the value of y either side of $x = 4$.

A1ft: For **a correct calculation**, **a valid reason** and **a correct conclusion**. Ignore additional work where candidate finds $\frac{d^2y}{dx^2}$ left and right of $x = 4$. Follow through on an incorrect $\frac{d^2y}{dx^2}$ but it is dependent upon having a negative or fractional index. Ignore any references to the word convex. The nature of the turning point is "minimum".

Using the gradient look for correct calculations, a valid reason…. goes from negative to positive, and a correct conclusion …minimum.

Question	Scheme	Marks	AOs
15	$\frac{dy}{d\theta}=\frac{(2\sin\theta+2\cos\theta)3\cos\theta-3\sin\theta(2\cos\theta-2\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$	M1 A1	1.1b 1.1b
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator or uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{...}{.........C\sin\theta\cos\theta}$	M1	3.1a
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{P}{Q+R\sin 2\theta}$	M1	2.1
	$\Rightarrow\frac{dy}{d\theta}=\frac{3}{2+2\sin 2\theta}=\frac{3/2}{1+\sin 2\theta}$	A1	1.1b
		(5 marks)	

Notes:

M1: For choosing either the quotient, product rule or implicit differentiation and applying it to the given function. Look for the correct form of $\frac{dy}{d\theta}$ (condone it being stated as $\frac{dy}{dx}$) but tolerate slips on the coefficients and also condone $\frac{d(\sin\theta)}{d\theta}=\pm\cos\theta$ and $\frac{d(\cos\theta)}{d\theta}=\pm\sin\theta$

For quotient rule look for $\frac{dy}{d\theta}=\frac{(2\sin\theta+2\cos\theta)\times\pm...\cos\theta-3\sin\theta(\pm...\cos\theta\pm...\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$

For product rule look for

$$\frac{dy}{d\theta}=(2\sin\theta+2\cos\theta)^{-1}\times\pm...\cos\theta\pm3\sin\theta\times(2\sin\theta+2\cos\theta)^{-2}\times(\pm...\cos\theta\pm...\sin\theta)$$

Implicit differentiation look for $(...\cos\theta\pm...\sin\theta)y+(2\sin\theta+2\cos\theta)\frac{dy}{d\theta}=...\cos\theta$

A1: A correct expression involving $\frac{dy}{d\theta}$ condoning it appearing as $\frac{dy}{dx}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator OR uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{...}{.........C\sin\theta\cos\theta}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ in the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in the denominator to reach an expression of the form $\frac{dy}{d\theta}=\frac{P}{Q+R\sin 2\theta}$.

A1: Fully correct proof with $A=\frac{3}{2}$ stated but allow for example $\frac{3/2}{1+\sin 2\theta}$

Allow recovery from missing brackets. Condone notation slips. This is not a given answer

Question	Scheme	Marks	AOs
16 (a)(i)	$C=\frac{1500}{v}+\frac{2v}{11}+60 \Rightarrow \frac{\mathrm{d}C}{\mathrm{d}v}=-\frac{1500}{v^2}+\frac{2}{11}$	M1 A1	3.1b 1.1b
	Sets $\frac{\mathrm{d}C}{\mathrm{d}v}=0 \Rightarrow v^2=8250$	M1	1.1b
	$\Rightarrow v=\sqrt{8250} \Rightarrow v=90.8\ (\mathrm{km\,h^{-1}})$	A1	1.1b
(ii)	For substituting their $v=90.8$ in $C=\frac{1500}{v}+\frac{2v}{11}+60$	M1	3.4
	Minimum cost =awrt (£) 93	A1 ft	1.1b
		(6)	
(b)	Finds $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}=+\frac{3000}{v^3}$ at $v=90.8$	M1	1.1b
	$\frac{\mathrm{d}^2C}{\mathrm{d}v^2}=(+0.004)>0$ hence minimum (cost)	A1 ft	2.4
		(2)	
(c)	It would be impossible to drive at this speed over the whole journey	B1	3.5b
		(1)	
			(9 marks)

Notes

(a)(i)

M1: Attempts to differentiate (deals with the powers of v correctly).

Look for an expression for $\frac{\mathrm{d}C}{\mathrm{d}v}$ in the form $\frac{A}{v^2}+B$

A1: $\left(\frac{\mathrm{d}C}{\mathrm{d}v}\right)=-\frac{1500}{v^2}+\frac{2}{11}$

A number of students are solving part (a) numerically or graphically. Allow these students to pick up the M1 A1 here from part (b) when they attempt the second derivative.

M1: Sets $\frac{\mathrm{d}C}{\mathrm{d}v}=0$ (which may be implied) and proceeds to an equation of the type $v^n=k,\ k>0$

Allow here equations of the type $\frac{1}{v^n}=k,\ k>0$

A1: $v=\sqrt{8250}$ or $5\sqrt{330}$ awrt 90.8 ($\mathrm{km\,h^{-1}}$).

As this is a speed withhold this mark for answers such as $v=\pm\sqrt{8250}$

* Condone $\frac{\mathrm{d}C}{\mathrm{d}v}$ appearing as $\frac{\mathrm{d}y}{\mathrm{d}x}$ or perhaps not appearing at all. Just look for the rhs.

(a)(ii)

M1: For a correct method of finding $C =$ from their solution to $\frac{dC}{dv} = 0$.

Do not accept attempts using negative values of v.
Award if you see $v = ..,\ C = ...$ where the v used is their solution to (a)(i).

A1ft: Minimum cost = awrt (£) 93. Condone the omission of units
Follow through on sensible values of v. $60 < v < 110$

v	C
60	95.9
65	94.9
70	94.2
75	93.6
80	93.3
85	93.1
90	93.0
95	93.1
100	93.2
105	93.4
110	93.6

(b)

M1: Finds $\frac{d^2C}{dv^2}$ (following through on their $\frac{dC}{dv}$ which must be of equivalent difficulty) and attempts to find its value / sign at their v

Allow a substitution of their answer to (a) (i) in their $\frac{d^2C}{dv^2}$

Allow an explanation into the sign of $\frac{d^2C}{dv^2}$ from its terms (as $v > 0$)

A1ft: $\frac{d^2C}{dv^2} = +0.004 > 0$ hence minimum (cost). Alternatively $\frac{d^2C}{dv^2} = +\frac{3000}{v^3} > 0$ as $v > 0$

Requires a correct calculation or expression, a correct statement and a correct conclusion.

Follow through on their v ($v > 0$) and their $\frac{d^2C}{dv^2}$

* Condone $\frac{d^2C}{dv^2}$ appearing as $\frac{d^2y}{dx^2}$ or not appearing at all for the M1 but for the A1 the correct notation must be used (accept notation C'').

(c)

B1: Gives a limitation of the given model, for example

- It would be impossible to drive at this speed over the whole journey
- The traffic would mean that you cannot drive at a constant speed

Any statement that implies that the speed could not be constant is acceptable.

Question	Scheme	Marks	AOs
17	Considers $\dfrac{(x+h)^3-x^3}{h}$	B1	2.1
	Expands $(x+h)^3 = x^3+3x^2h+3xh^2+h^3$	M1	1.1b
	so gradient (of chord) $=\dfrac{3x^2h+3xh^2+h^3}{h}=3x^2+3xh+h^2$	A1	1.1b
	States as $h\to 0$, $3x^2+3xh+h^2\to 3x^2$ so derivative $=3x^2$ *	A1*	2.5
		(4 marks)	

B1: Gives the correct fraction for the gradient of the chord either $\dfrac{(x+h)^3-x^3}{h}$ or $\dfrac{(x+\delta x)^3-x^3}{\delta x}$

It may also be awarded for $\dfrac{(x+h)^3-x^3}{x+h-x}$ oe. It may be seen in an expanded form

It does not have to be linked to the gradient of the chord

M1: Attempts to expand $(x+h)^3$ or $(x+\delta x)^3$ Look for two correct terms, most likely $x^3+...+h^3$

This is independent of the B1

A1: Achieves gradient (of chord) is $3x^2+3xh+h^2$ or exact un simplified equivalent such as $3x^2+2xh+xh+h^2$. Again, there is no requirement to state that this expression is the gradient of the chord

A1*: CSO. Requires correct algebra and making a link between the gradient of the chord and the gradient of the curve. See below how the link can be made. The words "gradient of the chord" do not need to be mentioned but derivative, $f'(x), \dfrac{dy}{dx}, y'$ should be. Condone invisible brackets for the expansion of $(x+h)^3$ as long as it is only seen at the side as intermediate working.

Requires either

- $f'(x) \underset{\lim h\to 0}{=} \dfrac{(x+h)^3-x^3}{h}=3x^2+3xh+h^2=3x^2$
- Gradient of chord $=3x^2+3xh+h^2$ As $h\to 0$ Gradient of chord tends to the gradient of curve so derivative is $3x^2$
- $f'(x) \underset{\lim h\to 0}{=} 3x^2+3xh+h^2=3x^2$
- Gradient of **chord** $=3x^2+3xh+h^2$ when $h\to 0$ gradient of **curve** $=3x^2$
- Do not allow $h=0$ alone without limit being considered somewhere:
  so don't accept $h=0 \Rightarrow f'(x)=3x^2+3x\times 0+0^2=3x^2$

....

Alternative: B1: Considers $\dfrac{(x+h)^3-(x-h)^3}{2h}$ M1: As above A1: $\dfrac{6x^2h^2+2h^3}{2h}=3x^2+h^2$

Question	Scheme	Marks	AOs
18(a)	$y=2x^3-2x^2-2x+8 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=6x^2-4x-2$	M1 A1	1.1b 1.1b
		(2)	
(b)	Attempts $6x^2-4x-2>0 \Rightarrow (6x+2)(x-1)>0$	M1	1.1b
	$x=-\frac{1}{3},1$	A1	1.1b
	Chooses outside region	M1	1.1b
	$\left\{x : x<-\frac{1}{3}\right\} \cup \{x : x>1\}$	A1	2.5
		(4)	
			(6 marks)

Notes:

(a)

M1: Attempts to differentiate. Allow for two correct terms un-simplified

A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=6x^2-4x-2$

(b)

M1: Attempts to find the critical values of their $\frac{\mathrm{d}y}{\mathrm{d}x}>0$ or their $\frac{\mathrm{d}y}{\mathrm{d}x}=0$

A1: Correct critical values $x=-\frac{1}{3},1$

M1: Chooses the outside region

A1: $\left\{x : x<-\frac{1}{3}\right\} \cup \{x : x>1\}$ or $\left\{x : x \in \mathrm{R} \;\; x<-\frac{1}{3} \text{ or } x>1\right\}$

Accept also $\left\{x : x \leqslant -\frac{1}{3}\right\} \cup \{x : x \geqslant 1\}$

Question	Scheme	Marks	AOs
19(a)	0.2 m^2	B1	3.4
		(1)	
(b)	$A=0.2\mathrm{e}^{0.3t}$ Rate of change = gradient $=\dfrac{\mathrm{d}A}{\mathrm{d}t}=0.06\mathrm{e}^{0.3t}$	M1	3.1b
	At $t=5\Rightarrow$ Rate of Growth is $0.06\mathrm{e}^{1.5}=0.269\text{ m}^2\text{/day}$	A1	1.1b
		(2)	
(c)	$100=0.2\mathrm{e}^{0.3t}\Rightarrow\mathrm{e}^{0.3t}=500$	M1 A1	3.1a 1.1b
	$\Rightarrow t=\dfrac{\ln(500)}{0.3}=20.7\text{ days}$ 20 days 17 hours	M1 A1	1.1b 3.2a
		(4)	
	At $t=5\Rightarrow$ Rate of Growth is $0.06\mathrm{e}^{1.5}=0.269\text{ m}^2\text{/day}$	A1	1.1b
		(2)	
(d)	The model given suggests that the pond is fully covered after 20 days 17 hours. Observed data is inconsistent with this as the pond is only 90% covered by the end of one month (28/29/30/31 days). Hence the model is not accurate	B1	3.5a
		(1)	
			(8 marks)

Notes:

(a)

B1: 0.2 m^2 oe

(b)

M1: Links rate of change to gradient and differentiates $0.2\mathrm{e}^{0.3t}\rightarrow k\mathrm{e}^{0.3t}$

A1: Correct answer $0.269\text{ m}^2\text{/day}$

(c)

M1: Substitutes $A=100$ and proceeds to $\mathrm{e}^{0.3t}=k$

A1: $\mathrm{e}^{0.3t}=500$

M1: Correct method when proceeding from $\mathrm{e}^{0.3t}=k\Rightarrow t=..$

A1: 20 days 17 hours

(d)

B1: Valid conclusion following through on their answer to (c).

Question	Scheme	Marks	AOs
20	Gradient of chord $= \dfrac{(2(x+h)^3+5)-(2x^3+5)}{x+h-h}$	B1	1.1b
		M1	2.1
	$(x+h)^3 = x^3+3x^2h+3xh^2+h^3$	B1	1.1b
	Gradient of chord $= \dfrac{(2(x^3+3x^2h+3xh^2+h^3)+5)-(2x^3+5)}{1+h-1}$		
	$= \dfrac{2x^3+6x^2h+6xh^2+2h^3+5-2x^3-5}{1+h-1}$		
	$= \dfrac{6x^2h+6xh^2+2h^3}{h}$		
	$= 6x^2+6xh+2h^2$	A1	1.1b
	$\dfrac{dy}{dx} = \lim\limits_{h \to 0}\left(6x^2+6xh+2h^2\right) = 6x^2$ and so at P, $\dfrac{dy}{dx} = 6(1)^2 = 6$	A1	2.2a
		(5)	
20 Alt 1	Let a point Q have x coordinate $1+h$, so $y_Q = 2(1+h)^3+5$	B1	1.1b
	$\left\{P(1,7), Q(1+h, 2(1+h)^3+3) \Rightarrow \right\}$		
	Gradient $PQ = \dfrac{2(1+h)^3+5-7}{1+h-1}$	M1	2.1
	$(1+h)^3 = 1+3h+3h^2+h^3$	B1	1.1b
	Gradient $PQ = \dfrac{2(1+3h+3h^2+h^3)+5-7}{1+h-1}$		
	$= \dfrac{2+6h+6h^2+2h^3+5-7}{1+h-1}$		
	$= \dfrac{6h+6h^2+2h^3}{h}$		
	$= 6+6h+2h^2$	A1	1.1b
	$\dfrac{dy}{dx} = \lim\limits_{h \to 0}\left(6+6h+2h^2\right) = 6$	A1	2.2a
		(5)	
			(5 marks)

Question 20 Notes:

B1:	$2(x+h)^3+5$, seen or implied
M1:	Begins the proof by attempting to write the gradient of the chord in terms of x and h
B1:	$(x+h)^3 \to x^3+3x^2h+3xh^2+h^3$, by expanding brackets or by using a correct binomial expansion
M1:	Correct process to obtain the gradient of the chord as $\alpha x^2+\beta xh+\gamma h^2$, $\alpha, \beta, \gamma \neq 0$
A1:	Correctly shows that the gradient of the chord is $6x^2+6xh+2h^2$ and applies a limiting argument to deduce when $y=2x^3+5$, $\frac{\mathrm{d}y}{\mathrm{d}x}=6x^2$. E.g. $\lim_{h\to 0}\left(6x^2+6xh+2h^2\right)=6x^2$. Finally, deduces that at the point P, $\frac{\mathrm{d}y}{\mathrm{d}x}=6$. **Note:** δx can be used in place of h
Alt 1	
B1:	Writes down the y coordinate of a point close to P. E.g. For a point Q with $x=1+h$, $\{y_Q\}=2(1+h)^3+5$
M1:	Begins the proof by attempting to write the gradient of the chord PQ in terms of h
B1:	$(1+h)^3 \to 1+3h+3h^2+h^3$, by expanding brackets or by using a correct binomial expansion
M1:	Correct process to obtain the gradient of the chord PQ as $\alpha+\beta h+\gamma h^2$, $\alpha, \beta, \gamma \neq 0$
A1:	Correctly shows that the gradient of PQ is $6+6h+2h^2$ and applies a limiting argument to deduce that at the point P on $y=2x^3+5$, $\frac{\mathrm{d}y}{\mathrm{d}x}=6$. E.g. $\lim_{h\to 0}\left(6+6h+2h^2\right)=6$ **Note:** For Alt 1, δx can be used in place of h

Question	Scheme	Marks	AOs
21 (a)	$f'(x) = k - 4x - 3x^2$		
	$f''(x) = -4 - 6x = 0$	M1	1.1b
	Criteria 1 **Either** $f''(x) = -4 - 6x = 0 \Rightarrow x = \frac{4}{-6} \Rightarrow x = -\frac{2}{3}$ **or** $f''\left(-\frac{2}{3}\right) = -4 - 6\left(-\frac{2}{3}\right) = 0$ **Criteria 2** **Either** • $f''(-0.7) = -4 - 6(-0.7) = 0.2 > 0$ $f''(-0.6) = -4 - 6(-0.6) = -0.4 < 0$ **or** • $f'''\left(-\frac{2}{3}\right) = -6 \neq 0$		
	At least one of Criteria 1 or Criteria 2	B1	2.4
	Both Criteria 1 and Criteria 2 **and** concludes C has a point of inflection at $x = -\frac{2}{3}$	A1	2.1
		(3)	
(b)	$f'(x) = k - 4x - 3x^2,\ AB = 4\sqrt{2}$		
	$f(x) = kx - 2x^2 - x^3\ \{+c\}$	M1 A1	1.1b 1.1b
	$f(0) = 0$ or $(0, 0) \Rightarrow c = 0 \Rightarrow f(x) = kx - 2x^2 - x^3$ $\{f(x) = 0 \Rightarrow\}\ f(x) = x(k - 2x - x^2) = 0 \Rightarrow \{x = 0,\}\ k - 2x - x^2 = 0$	A1	2.2a
	$\{x^2 + 2x - k = 0\} \Rightarrow (x+1)^2 - 1 - k = 0,\ x = \ldots$	M1	2.1
	$\Rightarrow x = -1 \pm \sqrt{k+1}$	A1	1.1b
	$AB = \left(-1 + \sqrt{k+1}\right) - \left(-1 - \sqrt{k+1}\right) = 4\sqrt{2} \Rightarrow k = \ldots$	M1	2.1
	So, $2\sqrt{k+1} = 4\sqrt{2} \Rightarrow k = 7$	A1	1.1b
		(7)	
			(10 marks)

Question 21 Notes:	
(a)	
M1:	E.g. • attempts to find $f''\left(-\frac{2}{3}\right)$ • finds $f''(x)$ and sets the result equal to 0
B1:	See scheme
A1:	See scheme
(b)	
M1:	Integrates $f'(x)$ to give $f(x) = \pm kx \pm ax^2 \pm bx^3$, $a, b \neq 0$ with or without the constant of integration
A1:	$f(x) = kx - 2x^2 - x^3$, with or without the constant of integration
A1:	Finds $f(x) = kx - 2x^2 - x^3 + c$, and makes some reference to $y = f(x)$ passing through the origin to deduce $c = 0$. Proceeds to produce the result $k - 2x - x^2 = 0$ or $x^2 + 2x - k = 0$
M1:	Uses a valid method to solve the quadratic equation to give x in terms of k
A1	Correct roots for x in terms of k. i.e. $x = -1 \pm \sqrt{k+1}$
M1:	Applies $AB = 4\sqrt{2}$ on $x = -1 \pm \sqrt{k+1}$ in a complete method to find $k = ...$
A1:	Finds $k = 7$ from correct solution only

Question	Scheme	Marks	AOs
22	Attempt to differentiate	M1	1.1a
	$\frac{dy}{dx} = 4x - 12$	A1	1.1b
	Substitutes $x = 5 \Rightarrow \frac{dy}{dx} = ...$	M1	1.1b
	$\Rightarrow \frac{dy}{dx} = 8$	A1ft	1.1b
			(4 marks)

Notes:

M1: Differentiation implied by one correct term

A1: Correct differentiation

M1: Attempts to substitute $x = 5$ into their derived function

A1ft: Substitutes $x = 5$ into **their** derived function **correctly** i.e. Correct calculation of their $f'(5)$ so follow through slips in differentiation

Question	Scheme	Marks	AOs
23	Considers $\dfrac{3(x+h)^2-3x^2}{h}$	B1	2.1
	Expands $3(x+h)^2 = 3x^2 + 6xh + 3h^2$	M1	1.1b
	So gradient $=\dfrac{6xh+3h^2}{h}=6x+3h$ or $\dfrac{6x\delta x+3(\delta x)^2}{\delta x}=6x+3\delta x$	A1	1.1b
	States as $h\to 0$, gradient $\to 6x$ so in the limit derivative $=6x$ *	A1*	2.5
			(4 marks)

Notes:

B1: Gives correct fraction as in the scheme above or $\dfrac{3(x+\delta x)^2-3x^2}{\delta x}$

M1: Expands the bracket as above or $3(x+\delta x)^2=3x^2+6x\delta x+3(\delta x)^2$

A1: Substitutes correctly into earlier fraction and simplifies

A1*: Uses Completes the proof, as above (may use $\delta x \to 0$), considers the limit and states a conclusion with no errors

Question	Scheme	Marks	AOs
24(a)	(i) $\dfrac{dy}{dx}=12x^3-24x^2$	M1 A1	1.1b 1.1b
	(ii) $\dfrac{d^2y}{dx^2}=36x^2-48x$	A1ft	1.1b
		(3)	
(b)	Substitutes $x=2$ into their $\dfrac{dy}{dx}=12\times2^3-24\times2^2$	M1	1.1b
	Shows $\dfrac{dy}{dx}=0$ and states "hence there is a stationary point"	A1	2.1
		(2)	
(c)	Substitutes $x=2$ into their $\dfrac{d^2y}{dx^2}=36\times2^2-48\times2$	M1	1.1b
	$\dfrac{d^2y}{dx^2}=48>0$ and states "hence the stationary point is a minimum"	A1ft	2.2a
		(2)	
		(7 marks)	

Notes:

(a)(i)

M1: Differentiates to a cubic form

A1: $\dfrac{dy}{dx}=12x^3-24x^2$

(a)(ii)

A1ft: Achieves a correct $\dfrac{d^2y}{dx^2}$ for their $\dfrac{dy}{dx}=36x^2-48x$

(b)

M1: Substitutes $x=2$ into their $\dfrac{dy}{dx}$

A1: Shows $\dfrac{dy}{dx}=0$ and states "hence there is a stationary point" All aspects of the proof must be correct

(c)

M1: Substitutes $x=2$ into their $\dfrac{d^2y}{dx^2}$

Alternatively calculates the gradient of C either side of $x=2$

A1ft: For a correct calculation, a valid reason and a correct conclusion.

Follow through on an incorrect $\dfrac{d^2y}{dx^2}$

Question	Scheme	Marks	AOs
25	Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$	B1	2.1
	Uses the compound angle identity for $\sin(A+B)$ with $A=\theta,\ B=h$ $\Rightarrow \sin(\theta+h)=\sin\theta\cos h+\cos\theta\sin h$	M1	1.1b
	Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$	A1	1.1b
	$=\dfrac{\sin h}{h}\cos\theta+\left(\dfrac{\cos h-1}{h}\right)\sin\theta$	M1	2.1
	Uses $h\to 0,\ \dfrac{\sin h}{h}\to 1$ and $\dfrac{\cos h-1}{h}\to 0$ Hence the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow \dfrac{dy}{d\theta}=\cos\theta$ *	A1*	2.5
			(5 marks)

Notes:

B1: States or implies that the gradient of the chord is $\dfrac{\sin(\theta+h)-\sin\theta}{h}$ or similar such as $\dfrac{\sin(\theta+\delta\theta)-\sin\theta}{\theta+\delta\theta-\theta}$ for a small h or $\delta\theta$

M1: Uses the compound angle identity for $\sin(A+B)$ with $A=\theta,\ B=h$ or $\delta\theta$

A1: Obtains $\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$ or equivalent

M1: Writes their expression in terms of $\dfrac{\sin h}{h}$ and $\dfrac{\cos h-1}{h}$

A1*: Uses correct language to explain that $\dfrac{dy}{d\theta}=\cos\theta$

For this method they should use all of the given statements $h\to 0,\ \dfrac{\sin h}{h}\to 1,$ $\dfrac{\cos h-1}{h}\to 0$ meaning that the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$

and therefore the gradient of the chord $\to$ gradient of the curve $\Rightarrow \dfrac{dy}{d\theta}=\cos\theta$

Question	Scheme	Marks	AOs
25 alt	Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$	B1	2.1
	Sets $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\dfrac{\sin\left(\theta+\frac{h}{2}+\frac{h}{2}\right)-\sin\left(\theta+\frac{h}{2}-\frac{h}{2}\right)}{h}$ and uses the compound angle identity for $\sin(A+B)$ and $\sin(A-B)$ with $A=\theta+\frac{h}{2},\quad B=\frac{h}{2}$	M1	1.1b
	Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=$ $\dfrac{\left[\sin\left(\theta+\frac{h}{2}\right)\cos\left(\frac{h}{2}\right)+\cos\left(\theta+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]-\left[\sin\left(\theta+\frac{h}{2}\right)\cos\left(\frac{h}{2}\right)-\cos\left(\theta+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]}{h}$	A1	1.1b
	$=\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\times\cos\left(\theta+\frac{h}{2}\right)$	M1	2.1
	Uses $h\to 0,\ \frac{h}{2}\to 0$ hence $\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\to 1$ and $\cos\left(\theta+\frac{h}{2}\right)\to\cos\theta$ Therefore the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$ *	A1*	2.5
		(5 marks)	

Additional notes:

A1*: Uses correct language to explain that $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$. For this method they should use the (adapted) given statement $h\to 0,\ \frac{h}{2}\to 0$ hence $\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\to 1$ with $\cos\left(\theta+\frac{h}{2}\right)\to\cos\theta$ meaning that the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and therefore the gradient of the chord $\to$ gradient of the curve $\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$

Question	Scheme	Marks	AOs
26(a)	Attempts to differentiate using the quotient rule or otherwise	M1	2.1
	$f'(x)=\dfrac{e^{\sqrt{2}x-1}\times 8\cos 2x-4\sin 2x\times\sqrt{2}e^{\sqrt{2}x-1}}{\left(e^{\sqrt{2}x-1}\right)^2}$	A1	1.1b
	Sets $f'(x)=0$ and divides/ factorises out the $e^{\sqrt{2}x-1}$ terms	M1	2.1
	Proceeds via $\dfrac{\sin 2x}{\cos 2x}=\dfrac{8}{4\sqrt{2}}$ to $\Rightarrow \tan 2x=\sqrt{2}$ *	A1*	1.1b
		(4)	
(b)	(i) Solves $\tan 4x=\sqrt{2}$ and attempts to find the 2[nd] solution	M1	3.1a
	$x=1.02$	A1	1.1b
	(ii) Solves $\tan 2x=\sqrt{2}$ and attempts to find the 1[st] solution	M1	3.1a
	$x=0.478$	A1	1.1b
		(4)	
			(8 marks)

Notes:

(a)

M1: Attempts to differentiate by using the quotient rule with $u=4\sin 2x$ and $v=e^{\sqrt{2}x-1}$ or alternatively uses the product rule with $u=4\sin 2x$ and $v=e^{1-\sqrt{2}x}$

A1: For achieving a correct $f'(x)$. For the product rule

$f'(x)=e^{1-\sqrt{2}x}\times 8\cos 2x+4\sin 2x\times -\sqrt{2}e^{1-\sqrt{2}x}$

M1: This is scored for cancelling/ factorising out the exponential term. Look for an equation in just $\cos 2x$ and $\sin 2x$

A1*: Proceeds to $\tan 2x=\sqrt{2}$. This is a given answer.

(b) (i)

M1: Solves $\tan 4x=\sqrt{2}$ attempts to find the 2[nd] solution. Look for $x=\dfrac{\pi+\arctan\sqrt{2}}{4}$

Alternatively finds the 2[nd] solution of $\tan 2x=\sqrt{2}$ and attempts to divide by 2

A1: Allow awrt $x=1.02$. The correct answer, with no incorrect working scores both marks

(b)(ii)

M1: Solves $\tan 2x=\sqrt{2}$ attempts to find the 1[st] solution. Look for $x=\dfrac{\arctan\sqrt{2}}{2}$

A1: Allow awrt $x=0.478$. The correct answer, with no incorrect working scores both marks

Question	Scheme	Marks	AOs
27	$\dfrac{2(x+h)^2-2x^2}{h}=\ldots$	M1	2.1
	$\dfrac{2(x+h)^2-2x^2}{h}=\dfrac{4xh+2h^2}{h}$	A1	1.1b
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\lim\limits_{h\to 0}\dfrac{4xh+2h^2}{h}=\lim\limits_{h\to 0}(4x+2h)=4x$ *	A1*	2.5
		(3)	
			(3 marks)
Notes:			

Throughout the question allow the use of δx for h or any other letter e.g. α if used consistently.
If δx is used then you can condone e.g. $\delta^2 x$ for δx^2 as well as condoning e.g. poorly formed δ's

M1: Begins the process by writing down the gradient of the chord and attempts to expand the correct bracket – you can condone "poor" squaring e.g. $(x+h)^2=x^2+h^2$.

Note that $\dfrac{2(x-h)^2-2x^2}{-h}=\ldots$ is also a possible approach.

A1: Reaches a correct fraction oe with the x^2 terms cancelled out.

E.g. $\dfrac{4xh+2h^2}{h}$, $\dfrac{\cancel{2x^2}+4xh+2h^2-\cancel{2x^2}}{h}$, $4x+2h$

A1*: Completes the process by applying a limiting argument and deduces that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=4x$ with no errors seen. The "$\dfrac{\mathrm{d}y}{\mathrm{d}x}=$" doesn't have to appear but there must be something equivalent e.g. "$\mathrm{f}'(x)=$" or "Gradient =" which can appear anywhere in their working. If $\mathrm{f}'(x)$ is used then there is no requirement to see f (x) defined first. Condone e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}\to 4x$ or $\mathrm{f}'(x)\to 4x$.

Condone missing brackets so allow e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\lim\limits_{h\to 0}\dfrac{4xh+2h^2}{h}=\lim\limits_{h\to 0}4x+2h=4x$

Do not allow $h=0$ if there is never a reference to h→0

e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\lim\limits_{h\to 0}\dfrac{4xh+2h^2}{h}=\lim\limits_{h\to 0}4x+2(0)=4x$ is acceptable

but e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{4xh+2h^2}{h}=4x+2h=4x+2(0)=4x$ is not if there is no h→0 seen.

The h→0 does not need to be present throughout the proof e.g. on every line.

They must reach $4x+2h$ at the end and not $\dfrac{4xh+2h^2}{h}$ (without the h's cancelled) to complete the limiting argument.

Question	Scheme	Marks	AOs
28(a)	$(\mathrm{f}'(x)=)4\cos\left(\frac{1}{2}x\right)-3$	M1 A1	1.1b 1.1b
	Sets $\mathrm{f}'(x)=4\cos\left(\frac{1}{2}x\right)-3=0\Rightarrow x=$	dM1	3.1a
	$x=14.0$ Cao	A1	3.2a
		(4)	
(b)	Explains that $\mathrm{f}(4)>0$, $\mathrm{f}(5)<0$ **and** the function is continuous	B1	2.4
		(1)	
(c)	Attempts $x_1=5-\dfrac{8\sin 2.5-15+9}{"4\cos 2.5-3"}$ (NB $\mathrm{f}(5)=-1.212\ldots$ and $\mathrm{f}'(5)=-6.204\ldots$)	M1	1.1b
	$x_1=$ awrt 4.80	A1	1.1b
		(2)	
			(7 marks)
Notes:			

(a)

M1: Differentiates to obtain $k\cos\left(\frac{1}{2}x\right)\pm\alpha$ where α is a constant which may be zero and no other terms. The brackets are not required.

A1: Correct derivative $\mathrm{f}'(x)=4\cos\left(\frac{1}{2}x\right)-3$. Allow unsimplified e.g. $\mathrm{f}'(x)=\frac{1}{2}\times 8\cos\left(\frac{1}{2}x\right)-3x^0$

There is no need for $\mathrm{f}'(x)=\ldots$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\ldots$ just look for the expression and the brackets are not required.

dM1: For the complete strategy of proceeding to a value for x.

Look for

- $\mathrm{f}'(x)=a\cos\left(\frac{1}{2}x\right)+b=0,\quad a,b\neq 0$
- Correct method of finding a valid solution to $a\cos\left(\frac{1}{2}x\right)+b=0$

Allow for $a\cos\left(\frac{1}{2}x\right)+b=0\Rightarrow\cos\left(\frac{1}{2}x\right)=\pm k\Rightarrow x=2\cos^{-1}(\pm k)$ where $|k|<1$

If this working is not shown then you may need to check their value(s).

For example $4\cos\left(\frac{1}{2}x\right)-3=0\Rightarrow x=1.4\ldots$ or $11.1\ldots$ (or $82.8\ldots$ or $637\ldots$. or 803 in degrees) would indicate this method.

A1: Selects the correct turning point $x=14.0$ and not just 14 or unrounded e.g. $14.011\ldots$
Must be this value only and no other values unless they are clearly rejected or 14.0 clearly selected. Ignore any attempts to find the y coordinate.
Correct answer with no working scores no marks.

(b)

B1: See scheme. Must be a full reason, (e.g. change of sign and continuous)
Accept equivalent statements for $\mathrm{f}(4)>0$, $\mathrm{f}(5)<0$ e.g. $\mathrm{f}(4)\times\mathrm{f}(5)<0$,"there is a change of sign", "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because x is continuous" or "because the interval is continuous"

(c)

M1: Attempts $x_1 = 5 - \dfrac{f(5)}{f'(5)}$ to obtain a value following through on their $f'(x)$ as long as it is a "changed" function.

Must be a correct N-R formula used – may need to check their values.

Allow if attempted in degrees. For reference in degrees f (5) = –5.65… and f ′(5) = 0.996… and gives $x_1 = 10.67...$

There must be clear evidence that $5 - \dfrac{f(5)}{f'(5)}$ is being attempted.

so e.g. $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} \Rightarrow x_1 = 4.80$ scores M0 as does e.g. $x_1 = x - \dfrac{8\sin\left(\frac{1}{2}x\right) - 3x + 9}{4\cos\left(\frac{1}{2}x\right) - 3} = 4.80$

BUT evidence may be provided by the accuracy of their answer. Note that the full N-R accuracy is 4.804624337 so e.g. 4.805 or 4.804 (truncated) with no evidence of incorrect work may imply the method.

A1: $x_1 =$ awrt 4.80 not awrt 4.8 but isw if awrt 4.80 is seen. Ignore any subsequent iterations.

Note that work for part (a) cannot be recovered in part (c)

Note also:

$5 - \dfrac{f(5)}{f'(5)} =$ awrt 4.80 following a correct derivative scores M1A1

$5 - \dfrac{f(5)}{f'(5)} \neq$ awrt 4.80 with no evidence that $5 - \dfrac{f(5)}{f'(5)}$ was attempted scores M0

Question	Scheme	Marks	AOs
29(a)	$\mathrm{f}(x)=\dfrac{\mathrm{e}^{3x}}{4x^2+k}\Rightarrow \mathrm{f}'(x)=\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ or $\mathrm{f}(x)=\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}\Rightarrow \mathrm{f}'(x)=3\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}-8x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x)=\dfrac{\left(12x^2-8x+3k\right)\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$	A1	2.1
		(3)	
(b)	If $y=\mathrm{f}(x)$ has at least one stationary point then $12x^2-8x+3k=0$ has at least one root	B1	2.2a
	Applies $b^2-4ac\ (\geqslant)\ 0$ with $a=12, b=-8, c=3k$	M1	2.1
	$0<k\leqslant\dfrac{4}{9}$	A1	1.1b
		(3)	
			(6 marks)
Notes:			

(a)

M1: Attempts the quotient rule to obtain an expression of the form $\dfrac{\alpha\left(4x^2+k\right)\mathrm{e}^{3x}-\beta x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$, $\alpha\beta>0$

condoning bracketing errors/omissions as long as the intention is clear.
If the quotient rule formula is quoted it must be correct.

Condone e.g. $\mathrm{f}'(x)=\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)}$ provided an incorrect formula is not quoted.

May also see product rule applied to $\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}$ to obtain an expression of the form

$\alpha\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}+\beta x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$ $\alpha,\beta 0<0$ condoning bracketing errors/omissions as long as the intention is clear. If the product rule formula is quoted it must be correct.

A1: Correct differentiation in any form with correct bracketing which may be implied by subsequent work.

A1: Obtains $\mathrm{f}'(x)=\left(12x^2-8x+3k\right)\mathrm{g}(x)$ where $\mathrm{g}(x)=\dfrac{\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ **or equivalent**

e.g. $\mathrm{g}(x)=\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$

Allow recovery from "invisible" brackets earlier and apply isw here once a correct answer is seen.
Note that the complete form of the answer is not given so allow candidates to go from e.g.
$\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ or $3\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}-8x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$ to $\dfrac{\left(12x^2-8x+3k\right)\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ for the final mark.

The "$\mathrm{f}'(x) =$" must appear at some point but allow e.g. "$\frac{\mathrm{d}y}{\mathrm{d}x} =$"

(b) Note that B0M1A1 is not possible in (b)

B1: Deduces that if $y = \mathrm{f}(x)$ has at least one stationary point then $12x^2 - 8x + 3k = 0$ has at least one root. There is no requirement to formally state $\frac{\mathrm{e}^{3x}}{\left(4x^2 + k\right)^2} > 0$

This may be implied by an attempt at $b^2 - 4ac \geqslant 0$ or $b^2 - 4ac > 0$ condoning slips.

M1: Attempts $b^2 - 4ac ... 0$ with $a = 12,\ b = -8,\ c = 3k$ where … is e.g. "=", <, >, etc.

Alternatively attempts to complete the square and sets rhs ...0

E.g. $12x^2 - 8x + 3k = 0 \Rightarrow x^2 - \frac{2}{3}x + \frac{1}{4}k = 0 \Rightarrow \left(x - \frac{1}{3}\right)^2 = \frac{1}{9} - \frac{1}{4}k$ leading to $\frac{1}{9} - \frac{1}{4}k \geqslant 0$

A1: $0 < k \leqslant \frac{4}{9}$ but condone $k \leqslant \frac{4}{9}$ and condone $0 \leqslant k \leqslant \frac{4}{9}$

Must be in terms of k not x so do not allow e.g. $0 < x \leqslant \frac{4}{9}$ but condone $\left(0, \frac{4}{9}\right]$ or $\left[0, \frac{4}{9}\right]$

Question	Scheme	Marks	AOs
30(a)(i)	$\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$	M1 A1	1.1b 1.1b
(ii)	$\frac{d^2y}{dx^2} = 60x^2 - 144x + 84$	A1ft	1.1b
		(3)	
(b)(i)	$x = 1 \Rightarrow \frac{dy}{dx} = 20 - 72 + 84 - 32$	M1	1.1b
	$\frac{dy}{dx} = 0$ so there is a stationary point at $x = 1$	A1	2.1
	Alternative for (b)(i)		
	$20x^3 - 72x^2 + 84x - 32 = 4(x-1)^2(5x-8) = 0 \Rightarrow x = \ldots$	M1	1.1b
	When $x = 1$, $\frac{dy}{dx} = 0$ so there is a stationary point	A1	2.1
(b)(ii)	**Note that in (b)(ii) there are no marks for <u>just</u> evaluating** $\left(\frac{d^2y}{dx^2}\right)_{x=1}$		
	E.g. $\left(\frac{d^2y}{dx^2}\right)_{x=0.8} = \ldots \quad \left(\frac{d^2y}{dx^2}\right)_{x=1.2} = \ldots$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=0.8} > 0, \quad \left(\frac{d^2y}{dx^2}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
		(4)	
	Alternative 1 for (b)(ii)		
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 60x^2 - 144x + 84 = 0$ (is inconclusive) $\left(\frac{d^3y}{dx^3}\right) = 120x - 144 \Rightarrow \left(\frac{d^3y}{dx^3}\right)_{x=1} = \ldots$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 0$ and $\left(\frac{d^3y}{dx^3}\right)_{x=1} \neq 0$ Hence point of inflection	A1	2.2a
	Alternative 2 for (b)(ii)		
	E.g. $\left(\frac{dy}{dx}\right)_{x=0.8} = \ldots \quad \left(\frac{dy}{dx}\right)_{x=1.2} = \ldots$	M1	2.1
	$\left(\frac{dy}{dx}\right)_{x=0.8} < 0, \quad \left(\frac{dy}{dx}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
			(7 marks)

Notes

(a)(i)
M1: $x^n \to x^{n-1}$ for at least one power of x
A1: $\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$
(a)(ii)

A1ft: Achieves a correct $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ for their $\frac{\mathrm{d}y}{\mathrm{d}x} = 20x^3 - 72x^2 + 84x - 32$

(b)(i)

M1: Substitutes x = 1 into their $\frac{\mathrm{d}y}{\mathrm{d}x}$

A1: Obtains $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ following a correct derivative and makes a conclusion which can be minimal e.g. tick, QED etc. which may be in a preamble e.g. stationary point when $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and then shows $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

Alternative:

M1: Attempts to solve $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ by factorisation. This may be by using the factor of (x – 1) or possibly using a calculator to find the roots and showing the factorisation. Note that they may divide by 4 before factorising which is acceptable. Need to either see either $4(x-1)^2(5x-8)$ or $(x-1)^2(5x-8)$ for the factorisation or $x = \frac{8}{5}$ and x = 1 seen as the roots.

A1: Obtains x = 1 and makes a conclusion as above

(b)(ii)

M1: Considers the value of the second derivative either side of x = 1. Do not be too concerned with the interval for the method mark.

(NB $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = (x-1)(60x-84)$ so may use this factorised form when considering x < 1, x > 1 for sign change of second derivative)

A1: Fully correct work including a correct $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ with a reasoned conclusion indicating that the stationary point is a point of inflection. Sufficient reason is e.g. "sign change"/ "> 0, < 0". If values are given they should be correct (but be generous with accuracy) but also just allow "> 0" and "< 0" provided they are correctly paired. The interval must be where x < 1.4

Alternative 1 for (b)(ii)

M1: Shows that second derivative at x = 1 is zero and **then finds the third derivative at x = 1**

A1: Fully correct work including a correct $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is "≠ 0" but must follow a correct third derivative and a correct value if evaluated. For reference $\left(\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)_{x=1} = -24$

Alternative 2 for (b)(ii)

M1: Considers the value of the first derivative either side of x = 1. Do not be too concerned with the interval for the method mark.

A1: Fully correct work with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is e.g. "same sign"/"both negative"/"< 0, < 0". If values are given they should be correct (but be generous with accuracy). The interval must be where x < 1.4

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
f′(x)	-32	-24.3	-17.92	-12.74	-8.64	-5.5	-3.2	-1.62	-0.64	-0.14	0
f″(x)	84	70.2	57.6	46.2	36	27	19.2	12.6	7.2	3	0

x	1.1	1.2	1.3	1.4	1.5	1.6	1.7
f′(x)	-0.1	-0.32	-0.54	-0.64	-0.5	0	0.98
f″(x)	-1.8	-2.4	-1.8	0	3	7.2	12.6

Question	Scheme	Marks	AOs
31(a)	$y=x^3-10x^2+27x-23 \Rightarrow \frac{dy}{dx}=3x^2-20x+27$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=5}=3\times5^2-20\times5+27(=2)$	M1	1.1b
	$y+13=2(x-5)$	M1	2.1
	$y=2x-23$	A1	1.1b
		(4)	
(b)	Both C and l pass through (0, −23) and so C meets l again on the y-axis	B1	2.2a
		(1)	
(c)	$\pm\int\left(x^3-10x^2+27x-23-(2x-23)\right)\,dx$ $=\pm\left(\frac{x^4}{4}-\frac{10}{3}x^3+\frac{25}{2}x^2\right)$	M1 A1ft	1.1b 1.1b
	$\left[\frac{x^4}{4}-\frac{10}{3}x^3+\frac{25}{2}x^2\right]_0^5$ $=\left(\frac{625}{4}-\frac{1250}{3}+\frac{625}{2}\right)(-0)$	dM1	2.1
	$=\frac{625}{12}$	A1	1.1b
		(4)	
	(c) Alternative:		
	$\pm\int\left(x^3-10x^2+27x-23\right)\,dx$ $=\pm\left(\frac{x^4}{4}-\frac{10}{3}x^3+\frac{27}{2}x^2-23x\right)$	M1 A1	1.1b 1.1b
	$\left[\frac{x^4}{4}-\frac{10}{3}x^3+\frac{27}{2}x^2-23x\right]_0^5+\frac{1}{2}\times5(23+13)$ $=-\frac{455}{12}+90$	dM1	2.1
	$=\frac{625}{12}$	A1	1.1b
			(9 marks)

Notes
(a) B1: Correct derivative M1: Substitutes $x = 5$ into their derivative. This may be implied by their value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ M1: Fully correct straight line method using $(5, -13)$ and their $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 5$ A1: cao. Must see the full equation in the required form. (b) B1: Makes a suitable deduction. Alternative via equating l and C and factorising e.g. $x^3 - 10x^2 + 27x - 23 = 2x - 23$ $x^3 - 10x^2 + 25x = 0$ $x\left(x^2 - 10x + 25\right) = 0 \Rightarrow x = 0$ So they meet on the y-axis (c) M1: For an attempt to integrate $x^n \to x^{n+1}$ for ±" $C - l$" A1ft: Correct integration in any form which may be simplified or unsimplified. (follow through their equation from (a)) If they attempt as 2 separate integrals e.g. $\int \left(x^3 - 10x^2 + 27x - 23\right) \mathrm{d}x - \int \left(2x - 23\right) \mathrm{d}x$ then award this mark for the correct integration of the curve as in the alternative. If they combine the curve with the line first then the subsequent integration must be correct or a correct ft for their line and allow for ±" $C - l$" dM1: Fully correct strategy for the area. Award for use of 5 as the limit and condone the omission of the "– 0". **Depends on the first method mark.** A1: Correct exact value **Alternative:** M1: For an attempt to integrate $x^n \to x^{n+1}$ for $\pm C$ A1: Correct integration for $\pm C$ dM1: Fully correct strategy for the area e.g. correctly attempts the area of the trapezium and subtracts the area enclosed between the curve and the x-axis. Need to see the use of 5 as the limit condoning the omission of the "– 0" **and** a correct attempt at the trapezium **and** the subtraction. May see the trapezium area attempted as $\int \left(2x - 23\right) \mathrm{d}x$ in which case the integration and use of the limits needs to be correct or correct follow through for their straight line equation. **Depends on the first method mark.** A1: Correct exact value Note if they do $l - C$ rather than $C - l$ and the working is otherwise correct allow full marks if their final answer is given as a positive value. E.g. correct work with $l - C$ leading to $-\frac{625}{12}$ and then e.g. hence area is $\frac{625}{12}$ is acceptable for full marks. If the answer is left as $-\frac{625}{12}$ then score A0

Question	Scheme	Marks	AOs
32(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(3y^2\right)=6y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}}{\mathrm{d}x}\left(qxy\right)=qx\frac{\mathrm{d}y}{\mathrm{d}x}+qy$	M1	2.1
	$3px^2+qx\frac{\mathrm{d}y}{\mathrm{d}x}+qy+6y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	A1	1.1b
	$\left(qx+6y\right)\frac{\mathrm{d}y}{\mathrm{d}x}=-3px^2-qy\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\ldots$	dM1	2.1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3px^2-qy}{qx+6y}$	A1	1.1b
		(4)	
(b)	$p\left(-1\right)^3+q\left(-1\right)\left(-4\right)+3\left(-4\right)^2=26$	M1	1.1b
	$19x+26y+123=0\Rightarrow m=-\frac{19}{26}$	B1	2.2a
	$\frac{-3p\left(-1\right)^2-q\left(-4\right)}{q\left(-1\right)+6\left(-4\right)}=\frac{26}{19}$ or $\frac{q\left(-1\right)+6\left(-4\right)}{3p\left(-1\right)^2+q\left(-4\right)}=-\frac{19}{26}$	M1	3.1a
	$p-4q=22,\ 57p-102q=624\Rightarrow p=\ldots,q=\ldots$	dM1	1.1b
	$p=2,\ q=-5$	A1	1.1b
		(5)	
		(9 marks)	

Notes

(a)

M1: For selecting the appropriate method of differentiating:

Allow this mark for either $3y^2\to\alpha y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $qxy\to\alpha x\frac{\mathrm{d}y}{\mathrm{d}x}+\beta y$

A1: Fully correct differentiation. Ignore any spurious $\frac{\mathrm{d}y}{\mathrm{d}x}=\ldots$

dM1: A valid attempt to make $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject with 2 terms only in $\frac{\mathrm{d}y}{\mathrm{d}x}$ coming from qxy and $3y^2$

Depends on the first method mark.

A1: Fully correct expression

(b)

M1: Uses $x=-1$ and $y=-4$ in the equation of C to obtain an equation in p and q

B1: Deduces the correct gradient of the given normal.

This may be implied by e.g.

$$19x+26y+123=0\Rightarrow y=-\frac{19}{26}x+\ldots\Rightarrow\text{Tangent equation is } y=\frac{26}{19}x+\ldots$$

M1: Fully correct strategy to establish an equation connecting p and q using $x=-1$ and $y=-4$ in their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and the gradient of the normal. E.g. $(a)=-1\div\text{their}-\frac{19}{26}$ or $-1\div(a)=\text{their}-\frac{19}{26}$

dM1: Solves simultaneously to obtain values for p and q.

Depends on both previous method marks.

A1: Correct values

Alternative for (b):

$$\frac{dy}{dx}=\frac{-3p+4q}{-q-24}\Rightarrow y+4=\frac{q+24}{4q-3p}(x+1)$$

M1A1

$$\Rightarrow y(4q-3p)+4(4q-3p)=(q+24)x+q+24$$

M1

$$19x+26y+123=0\Rightarrow q+24=19\Rightarrow q=-5$$

$$3p-4q=26\Rightarrow 3p+20=26\Rightarrow p=2$$

M1A1

M1: Uses (–1, –4) in the tangent gradient and attempts to form normal equation

A1: Correct equation for normal

M1: Multiplies up so that coefficients can be compared

dM1: Full method comparing coefficients to find values for p and q

A1: Correct values

Question	Scheme	Marks	AOs
33(a)	$y=\text{cosec}^3\theta \Rightarrow \dfrac{dy}{d\theta}=-3\text{cosec}^2\theta\,\text{cosec}\,\theta\cot\theta$	B1	1.1b
	$\dfrac{dy}{dx}=\dfrac{dy}{d\theta}\div\dfrac{dx}{d\theta}$	M1	1.1b
	$\dfrac{dy}{dx}=\dfrac{-3\text{cosec}^3\theta\cot\theta}{2\cos 2\theta}$	A1	1.1b
		(3)	
(b)	$y=8\Rightarrow \text{cosec}^3\theta=8\Rightarrow \sin^3\theta=\dfrac{1}{8}\Rightarrow \sin\theta=\dfrac{1}{2}$	M1	3.1a
	$\theta=\dfrac{\pi}{6}\Rightarrow\dfrac{dy}{dx}=\dfrac{-3\text{cosec}^3\left(\dfrac{\pi}{6}\right)\cot\left(\dfrac{\pi}{6}\right)}{2\cos\left(\dfrac{2\pi}{6}\right)}=\ldots$ **or** $\sin\theta=\dfrac{1}{2}\Rightarrow\dfrac{dy}{dx}=\dfrac{\dfrac{-3}{\sin^3\theta}\times\dfrac{\cos\theta}{\sin\theta}}{2\left(1-2\sin^2\theta\right)}=\dfrac{-3\times8\times\dfrac{\sqrt{3}/2}{1/2}}{2\left(1-2\times\dfrac{1}{4}\right)}$	M1	2.1
	$=-24\sqrt{3}$	A1	2.2a
		(3)	
		(6 marks)	

Notes

(a)

B1: Correct expression for $\dfrac{dy}{d\theta}$ seen or implied in any form e.g. $\dfrac{-3\cos\theta}{\sin^4\theta}$

M1: Obtains $\dfrac{dx}{d\theta}=k\cos2\theta$ or $\alpha\cos^2\theta+\beta\sin^2\theta$ (from product rule on sinθcosθ)

and attempts $\dfrac{dy}{dx}=\dfrac{dy}{d\theta}\div\dfrac{dx}{d\theta}$

A1: Correct expression in any form.

May see e.g. $\dfrac{-3\cos\theta}{2\sin^4\theta\cos2\theta},\ -\dfrac{3}{4\sin^4\theta\cos\theta-2\sin^3\theta\tan\theta}$

(b)

M1: Recognises the need to find the value of $\sin\theta$ or θ when y = 8 and uses the y parameter to establish its value. This should be correct work leading to $\sin\theta=\dfrac{1}{2}$ or e.g. $\theta=\dfrac{\pi}{6}$ or 30°.

M1: Uses their value of $\sin\theta$ or θ in their $\dfrac{dy}{dx}$ from part (a) (working in exact form) in an attempt to obtain an exact value for $\dfrac{dy}{dx}$. May be implied by a correct exact answer.

If no working is shown but an exact answer is given you may need to check that this follows their $\dfrac{dy}{dx}$.

A1: Deduces the correct gradient

Question	Scheme	Marks	AOs
34(a)	$\frac{dV}{dt} = 0.48 - 0.1h$	B1	3.1b
	$V = 24h \Rightarrow \frac{dV}{dh} = 24$ or $\frac{dh}{dV} = \frac{1}{24}$	B1	3.1b
	$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV} = \frac{0.48 - 0.1h}{24}$ or e.g. $\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} \Rightarrow 0.48 - 0.1h = 24\frac{dh}{dt}$	M1	2.1
	$1200\frac{dh}{dt} = 24 - 5h$ *	A1*	1.1b
		(4)	
(b)	$1200\frac{dh}{dt} = 24 - 5h \Rightarrow \int \frac{1200}{24 - 5h}\,dh = \int dt$ $\Rightarrow$ *e.g.* $\alpha \ln(24 - 5h) = t(+c)$ oe or $1200\frac{dh}{dt} = 24 - 5h \Rightarrow \frac{dt}{dh} = \frac{1200}{24 - 5h}$ $\Rightarrow$ *e.g.* $t(+c) = \alpha \ln(24 - 5h)$ oe	M1	3.1a
	$t = -240\ln(24 - 5h)(+c)$ oe	A1	1.1b
	$t = 0, h = 2 \Rightarrow 0 = -240\ln(24 - 10) + c \Rightarrow c = ...(240\ln 14)$	M1	3.4
	$t = 240\ln(14) - 240\ln(24 - 5h)$	A1	1.1b
	$t = 240\ln\frac{14}{24 - 5h} \Rightarrow \frac{t}{240} = \ln\frac{14}{24 - 5h} \Rightarrow e^{\frac{t}{240}} = \frac{14}{24 - 5h}$ $\Rightarrow 14e^{-\frac{t}{240}} = 24 - 5h \Rightarrow h = ...$	ddM1	2.1
	$h = 4.8 - 2.8e^{-\frac{t}{240}}$ oe e.g. $h = \frac{24}{5} - \frac{14}{5}e^{-\frac{t}{240}}$	A1	3.3
		(6)	
(c)	**Examples:** • As $t \to \infty, e^{-\frac{t}{240}} \to 0$ • When $h > 4.8$, $\frac{dV}{dt} < 0$ • Flow in = flow out at max h so $0.1h = 4.8 \rightarrow h = 4.8$ • As $e^{-\frac{t}{240}} > 0,\ h < 4.8$ • $h = 5 \Rightarrow \frac{dV}{dt} = -0.02$ or $\frac{dh}{dt} = -\frac{1}{1200}$ • $\frac{dh}{dt} = 0 \Rightarrow h = 4.8$ • $h = 5 \Rightarrow 4.8 - 2.8e^{-\frac{t}{240}} = 5 \Rightarrow e^{-\frac{t}{240}} < 0$	M1	3.1b
	• The limit for h (according to the model) is 4.8m and the tank is 5m high so the tank will never become full • If $h = 5$ the tank would be emptying so can never be full • The equation can't be solved when $h = 5$	A1	3.2a

		(2)	
			(12 marks)

Notes

(a)

B1: Identifies the correct expression for $\frac{\mathrm{d}V}{\mathrm{d}t}$ according to the model

B1: Identifies the correct expression for $\frac{\mathrm{d}V}{\mathrm{d}h}$ according to the model

M1: Applies $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}h}{\mathrm{d}V}$ or equivalent correct formula with their $\frac{\mathrm{d}V}{\mathrm{d}t}$ and $\frac{\mathrm{d}V}{\mathrm{d}h}$ which may be implied by their working

A1*: Correct equation obtained with no errors

Note that: $\frac{\mathrm{d}V}{\mathrm{d}t}=0.48-0.1h\Rightarrow\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{0.48-0.1h}{24}\Rightarrow 1200\frac{\mathrm{d}h}{\mathrm{d}t}=24-5h*$ scores B1B0M0A0. There must be clear evidence where the "24" comes from and evidence of the correct chain rule being applied.

(b)

M1: Adopts a correct strategy by separating the variables correctly or rearranges to obtain $\frac{\mathrm{d}t}{\mathrm{d}h}$ correctly in terms of h **and** integrates to obtain $t=\alpha\ln\left(24-5h\right)\left(+c\right)$ or equivalent (condone missing brackets around the "24 – 5h") and + c not required for this mark.

A1: Correct equation in any form and + c not required. Do not condone missing brackets unless they are implied by subsequent work.

M1: Substitutes t = 0 and h = 2 to find their constant of integration (there must have been some attempt to integrate)

A1: Correct equation in any form

ddM1: Uses fully correct log work to obtain h in terms of t.

This depends on <u>both</u> previous method marks.

A1: Correct equation

Note that the marks may be earned in a different order e.g.:

$$t+c=-240\ln\left(24-5h\right)\Rightarrow-\frac{t}{240}+d=\ln\left(24-5h\right)\Rightarrow Ae^{-\frac{t}{240}}=24-5h$$

$$t=0,h=2\Rightarrow A=14\Rightarrow 14e^{-\frac{t}{240}}=24-5h\Rightarrow h=4.8-2.8e^{-\frac{t}{240}}$$

Score as M1 A1 as in main scheme then

M1: Correct work leading to $Ae^{\alpha t}=24-5h$ (must have a constant "A")

A1: $Ae^{-\frac{t}{240}}=24-5h$

ddM1: Uses t = 0, h = 2 in an expression of the form above to find A

A1: $h=4.8-2.8e^{-\frac{t}{240}}$

(c)

M1: See scheme for some examples

A1: Makes a correct interpretation for their method.

There must be no incorrect working or contradictory statements.

This is not a follow through mark and if their equation in (b) is used it must be correct.

Question	Scheme	Marks	AOs
35(a)	$\ln x \to \frac{1}{x}$	B1	1.1a
	Method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$ – see notes	M1	1.1b
	E.g. $2\times\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$	A1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=3\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{4}{x}=\frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ *	A1*	2.1
		(4)	
(b)	$12x^2+x-16\sqrt{x}=0 \Rightarrow 12x^{\frac{3}{2}}+x^{\frac{1}{2}}-16=0$	M1	1.1b
	E.g. $12x^{\frac{3}{2}}=16-\sqrt{x}$	dM1	1.1b
	$x^{\frac{3}{2}}=\frac{4}{3}-\frac{\sqrt{x}}{12} \Rightarrow x=\left(\frac{4}{3}-\frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$ *	A1*	2.1
		(3)	
(c)	$x_2=\sqrt[3]{\left(\frac{4}{3}-\frac{\sqrt{2}}{12}\right)^2}$	M1	1.1b
	x_2 = awrt 1.13894	A1	1.1b
	x = 1.15650	A1	2.2a
		(3)	
			(10 marks)

Notes:

(a)

B1: Differentiates $\ln x \to \frac{1}{x}$ seen or implied

M1: Correct method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$:

Look for $\frac{4x^2+x}{2\sqrt{x}} \to ...x^{\frac{3}{2}}+...x^{\frac{1}{2}}$ being then differentiated to $Px^{\frac{1}{2}}+...$ **or** $...+Qx^{-\frac{1}{2}}$

Alternatively uses the quotient rule on $\frac{4x^2+x}{2\sqrt{x}}$.

Condone slips but if rule is not quoted expect $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{2\sqrt{x}\left(Ax+B\right)-\left(4x^2+x\right)Cx^{-\frac{1}{2}}}{\left(2\sqrt{x}\right)^2}\left(A,B,C>0\right)$

But a correct rule may be implied by their *u*, *v*, *u′*, *v′* followed by applying $\frac{vu'-uv'}{v^2}$ etc.

Alternatively uses the product rule on $\left(4x^2+x\right)\left(2\sqrt{x}\right)^{-1}$

Condone slips but expect $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=Ax^{-\frac{1}{2}}\left(Bx+C\right)+D\left(4x^2+x\right)x^{-\frac{3}{2}}\left(A,B,C>0\right)$

In general condone missing brackets for the M mark. If they quote $u=4x^2+x$ and $v=2\sqrt{x}$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have *v* rather than v^2 in the denominator.

A1: Correct differentiation of $\frac{4x^2+x}{2\sqrt{x}}$ although may not be simplified.

Examples: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{2\sqrt{x}(8x+1)-\left(4x^2+x\right)x^{-\frac{1}{2}}}{\left(2\sqrt{x}\right)^2}$, $\frac{1}{2}x^{-\frac{1}{2}}(8x+1)-\frac{1}{4}\left(4x^2+x\right)x^{-\frac{3}{2}}$, $2\times\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$

A1*: Obtains $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ via $3\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{4}{x}$ or a correct application of the quotient or product rule and with sufficient working shown to reach the printed answer.
<u>**There must be no errors e.g. missing brackets.**</u>

(b)

M1: Sets $12x^2+x-16\sqrt{x}=0$ and divides by $\sqrt{x}$ or equivalent e.g. divides by x and multiplies by $\sqrt{x}$

dM1: Makes the term in $x^{\frac{3}{2}}$ the subject of the formula

A1*: A correct and rigorous argument leading to the given solution.

Alternative - working backwards:

$$x=\left(\frac{4}{3}-\frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}\Rightarrow x^{\frac{3}{2}}=\frac{4}{3}-\frac{\sqrt{x}}{12}\Rightarrow 12x^{\frac{3}{2}}=16-\sqrt{x}\Rightarrow 12x^2=16\sqrt{x}-x\Rightarrow 12x^2-16\sqrt{x}+x=0$$

M1: For raising to power of 3/2 both sides. dM1: Multiplies through by $\sqrt{x}$. A1: Achieves printed answer and makes a minimal comment e.g. tick, #, QED, true etc.

(c)

M1: Attempts to use the iterative formula with $x_1 = 2$. This is implied by sight of $x_2=\left(\frac{4}{3}-\frac{\sqrt{2}}{12}\right)^{\frac{2}{3}}$ or awrt 1.14

A1: x_2 = awrt 1.13894

A1: Deduces that $x = 1.15650$

Question	Scheme	Marks	AOs
36(a)	$k = e^2$ **or** $x \neq e^2$	B1	2.2a
		(1)	
(b)	$$g'(x) = \frac{(\ln x - 2) \times \frac{3}{x} - (3\ln x - 7) \times \frac{1}{x}}{(\ln x - 2)^2} = \frac{1}{x(\ln x - 2)^2}$$ or $$g'(x) = \frac{d}{dx}\left(3 - (\ln(x) - 2)^{-1}\right) = (\ln x - 2)^{-2} \times \frac{1}{x} = \frac{1}{x(\ln x - 2)^2}$$ or $$g'(x) = (\ln x - 2)^{-1} \times \frac{3}{x} - (3\ln x - 7)(\ln x - 2)^{-2} \times \frac{1}{x} = \frac{1}{x(\ln x - 2)^2}$$	M1 A1	1.1b 2.1
	As $x > 0$ (or $1/x > 0$) **AND** $\ln x - 2$ is squared so $g'(x) > 0$	A1cso	2.4
		(3)	
(c)	Attempts to solve either $3\ln x - 7 \ldots 0$ or $\ln x - 2 \ldots 0$ or $3\ln a - 7 \ldots 0$ or $\ln a - 2 \ldots 0$ where … is "=" or ">" to reach a value for x or a but may be seen as an inequality e.g. $x > \ldots$ or $a > \ldots$	M1	3.1a
	$0 < a < e^2, \quad a > e^{\frac{7}{3}}$	A1	2.2a
		(2)	
			(6 marks)

Notes:

(a)

B1: Deduces $k = e^2$ **or** $x \neq e^2$ Condone k = awrt 7.39 or $x \neq$ awrt 7.39

(b)

M1: Attempts to differentiate via the quotient rule and with $\ln x \to \frac{1}{x}$ so allow for:

$$\frac{d}{dx}(g(x)) = \frac{(\ln x - 2) \times \frac{\alpha}{x} - (3\ln x - 7) \times \frac{\beta}{x}}{(\ln x - 2)^2}, \ \beta > 0$$

But a correct rule may be implied by their *u*, *v*, *u′*, *v′* followed by applying $\frac{vu' - uv'}{v^2}$ etc.

Alternatively attempts to write $g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} = 3 - (\ln(x) - 2)^{-1}$ and attempts the chain rule so allow for:

$$3 - (\ln(x) - 2)^{-1} \to (\ln(x) - 2)^{-2} \times \frac{\alpha}{x}$$

Alternatively writes $g(x) = (3\ln(x) - 7)(\ln(x) - 2)^{-1}$ and attempts the product rule so allow for:

$$g'(x) = (\ln x - 2)^{-1} \times \frac{\alpha}{x} - (3\ln x - 7)(\ln x - 2)^{-2} \times \frac{\beta}{x}$$

In general condone missing brackets for the M mark. E.g. if they quote *u* = 3ln*x* - 7 and *v* = ln*x* – 2 and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have *v* rather than *v*² in the denominator.

A1: $\frac{1}{x(\ln x - 2)^2}$ **Allow** $\frac{\frac{1}{x}}{(\ln x - 2)^2}$ **i.e. we need to see the numerator simplified to 1/*x***

Note that some candidates establish the correct numerator and correct denominator independently and provided they obtain the correct expressions, this mark can be awarded.
But allow a correctly expanded denominator.

A1cso: States that as $x > 0$ **AND** $\ln x - 2$ is squared so $g'(x) > 0$

(c)

M1: Attempts to solve either $3\ln x - 7 = 0$ or $\ln x - 2 = 0$ or using inequalities e.g. $3\ln x - 7 > 0$

A1: $0 < a < e^2, \ a > e^{\frac{7}{3}}$

Question	Scheme	Marks	AOs
37 (a) **Way 1**	$\{y = x^x \Rightarrow\}\ \ \ln y = x \ln x$	B1	1.1a
	$\dfrac{1}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x$	M1	1.1b
		A1	2.1
	$\left\{\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ \ \dfrac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = \mathrm{e}^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	**(5)**	
(a) **Way 2**	$\{y = x^x \Rightarrow\}\ \ y = \mathrm{e}^{x \ln x}$	B1	1.1a
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \left(\dfrac{x}{x} + \ln x\right)\mathrm{e}^{x \ln x}$	M1	1.1b
		A1	2.1
	$\left\{\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ \ \dfrac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = \mathrm{e}^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	**(5)**	
(b) **Way 1**	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$1.8... < 2$ and $2.1... > 2$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
(c)	Attempts $x_{n+1} = 2x_n^{1-x_n}$ at least once with $x_1 = 1.5$ Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63	M1	1.1b
	$\{x_4 = 1.67313... \Rightarrow\}\ x_4 = 1.673$ (3 dp) **cao**	A1	1.1b
		(2)	
(d)	Give 1st B1 for any of • oscillates • periodic • non-convergent • divergent • fluctuates • goes up and down • 1, 2, 1, 2, 1, 2 • alternates (condone) Give B1 B1 for any of • periodic {sequence} with period 2 • oscillates between 1 and 2	B1	2.5
	Condone B1 B1 for any of • fluctuates between 1 and 2 • keep getting 1, 2 • alternates between 1 and 2 • goes up and down between 1 and 2 • 1, 2, 1, 2, 1, 2, …	B1	2.5
		(2)	
		(11 marks)	

Note	**A common solution** A maximum of 3 marks (i.e. B1 1st M1 and 2nd M1) can be given for the solution $\log y = x \log x \Rightarrow \dfrac{1}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 + \log x$ $\left\{\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ \ 1 + \log x = 0 \Rightarrow x = 10^{-1}$
	• 1st B1 for $\log y = x \log x$ • 1st M1 for $\log y \to \lambda \dfrac{1}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x};\ \lambda \neq 0$ or $x \log x \to 1 + \log x$ or $\dfrac{x}{x} + \log x$ • 2nd M1 can be given for $1 + \log x = 0 \Rightarrow \log x = k \Rightarrow x = ...;\ \ k \neq 0$

Question	Scheme	Marks	AOs
37 (b) Way 2	For $x^x - 2$, attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$-0.16... < 0$ and $0.12... > 0$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
37 (b) Way 3	For $\ln y = x\ln x$, attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$0.608... < 0.69...$ and $0.752... > 0.69...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
37 (b) Way 4	For $\log y = x\log x$, attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp	M1	1.1b
	$0.264... < 0.301...$ and $0.326... > 0.301...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	

Notes for Question 37

(a)	**Way 1**
B1:	$\ln y = x\ln x$. Condone $\log_x y = x\log_x x$ or $\log_x y = x$
M1:	For either $\ln y \to \frac{1}{y}\frac{dy}{dx}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{1}{y}\frac{dy}{dx} = 1 + \ln x$ or $\frac{1}{y}\frac{dy}{dx} = \frac{x}{x} + \ln x$ or $\frac{dy}{dx} = y(1+\ln x)$ or $\frac{dy}{dx} = x^x(1+\ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give no marks for no working leading to 0.368
Note:	Give M0 A0 M0 A0 for $\ln y = x\ln x \to x = 0.368$ with no intermediate working
(a)	**Way 2**
B1:	$y = e^{x\ln x}$
M1:	For either $y = e^{x\ln x} \Rightarrow \frac{dy}{dx} = f(\ln x)e^{x\ln x}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{dy}{dx} = \left(\frac{x}{x} + \ln x\right)e^{x\ln x}$ or $\frac{dy}{dx} = (1+\ln x)e^{x\ln x}$ or $\frac{dy}{dx} = x^x(1+\ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give B1 M1 A0 M1 A1 for the following solution: $\{y = x^x \Rightarrow\}$ $\ln y = x\ln x \Rightarrow \frac{dy}{dx} = 1 + \ln x \Rightarrow 1 + \ln x = 0 \Rightarrow x = e^{-1}$ or awrt 0.368

Notes for Question **37** Continued	
(b)	**Way 1**
M1:	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} =$ awrt $1.8...$ and $1.6^{1.6} =$ awrt $2.1...$, reason (e.g. $1.8... < 2$ and $2.1... > 2$ or states C cuts through $y = 2$), C continuous and conclusion
(b)	**Way 2**
M1:	Attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ correct to awrt 1 dp, reason (e.g. $-0.16... < 0$ and $0.12... > 0$, sign change or states C cuts through $y = 0$), C continuous and conclusion
(b)	**Way 3**
M1:	Attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ correct to awrt 1 dp, reason (e.g. $0.608... < 0.69...$ and $0.752... > 0.69...$ or states they are either side of $\ln 2$), C continuous and conclusion.
(b)	**Way 4**
M1:	Attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp
A1:	Both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ correct to awrt 2 dp, reason (e.g. $0.264... < 0.301...$ and $0.326... > 0.301...$ or states they are either side of $\log 2$), C continuous and conclusion.
(c)	
M1:	An attempt to use the given or their formula once. Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63
A1:	States $x_4 = 1.673$ **cao** (to 3 dp)
Note:	Give M1 A1 for stating $x_4 = 1.673$
Note:	M1 can be implied by stating their final answer $x_4 =$ awrt 1.673
Note:	$x_2 = 1.63299...$, $x_3 = 1.46626...$, $x_4 = 1.67313...$
(d)	
B1:	see scheme
B1:	see scheme
Note:	Only marks of B1B0 or B1B1 are possible in (d)
Note:	Give B0 B0 for "Converges in a cob-web pattern" or "Converges up and down to α"

Question	Scheme	Marks	AOs
38 (a)	States or uses $6 = \pi r^2 h + \frac{2}{3}\pi r^3$	B1	1.1a
	$\Rightarrow\ h = \frac{6}{\pi r^2} - \frac{2}{3}r,\ \pi h = \frac{6}{r^2} - \frac{2}{3}\pi r,\ \pi rh = \frac{6}{r} - \frac{2}{3}\pi r^2,\ rh = \frac{6}{\pi r} - \frac{2}{3}r^2$		
	$A = \pi r^2 + 2\pi rh + 2\pi r^2\ \{\Rightarrow A = 3\pi r^2 + 2\pi rh\}$		
	$A = 2\pi r^2 + 2\pi r\left(\frac{6}{\pi r^2} - \frac{2}{3}r\right) + \pi r^2$	M1	3.1a
		A1	1.1b
	$A = 3\pi r^2 + \frac{12}{r} - \frac{4}{3}\pi r^2\ \Rightarrow\ A = \frac{12}{r} + \frac{5}{3}\pi r^2$ *	A1*	2.1
		(4)	
(b)	$\left\{A = 12r^{-1} + \frac{5}{3}\pi r^2 \Rightarrow\right\}\ \frac{dA}{dr} = -12r^{-2} + \frac{10}{3}\pi r$	M1	3.4
		A1	1.1b
	$\left\{\frac{dA}{dr} = 0 \Rightarrow\right\}\ -\frac{12}{r^2} + \frac{10}{3}\pi r = 0\ \Rightarrow\ -36 + 10\pi r^3 = 0 \Rightarrow r^{\pm 3} = \ldots \left\{= \frac{18}{5\pi}\right\}$	M1	2.1
	$r = 1.046447736... \Rightarrow r =$ 1.05 (m) (3 sf) or awrt 1.05 (m)	A1	1.1b
	Note: Give final A1 for correct exact values for r	**(4)**	
(c)	$A_{\min} = \frac{12}{(1.046...)} + \frac{5}{3}\pi(1.046...)^2$	M1	3.4
	$\{A_{\min} = 17.20...\ \Rightarrow\}\ A = 17\ (\text{m}^2)$ or $A =$ awrt 17 (m^2)	A1ft	1.1b
		(2)	
			(10 marks)

Notes for Question 38

(a)	
B1:	See scheme
M1:	Complete process of substituting their $h = \ldots$ or $\pi h = \ldots$ or $\pi rh = \ldots$ or $rh = \ldots$, where $'\ldots' = f(r)$ into an expression for the surface area which is of the form $A = \lambda\pi r^2 + \mu\pi rh;\ \lambda, \mu \neq 0$
A1:	Obtains correct simplified or un-simplified $\{A =\}\ 2\pi r^2 + 2\pi r\left(\frac{6}{\pi r^2} - \frac{2}{3}r\right) + \pi r^2$
A1*:	Proceeds, using rigorous and careful reasoning, to $A = \frac{12}{r} + \frac{5}{3}\pi r^2$
Note:	Condone the lack of $A = \ldots$ or $S = \ldots$ for any one of the A marks or for both of the A marks
(b)	
M1:	Uses the model (or their model) and differentiates $\frac{\lambda}{r} + \mu r^2$ to give $\alpha r^{-2} + \beta r$; $\lambda, \mu, \alpha, \beta \neq 0$
A1:	$\left\{\frac{dA}{dr} =\right\}\ -12r^{-2} + \frac{10}{3}\pi r$ o.e.
M1:	Sets their $\frac{dA}{dr} = 0$ and rearranges to give $r^{\pm 3} = k,\ k \neq 0$ (**Note:** k can be positive or negative)
Note:	This mark can be implied. Give M1 (and A1) for $-36 + 10\pi r^3 = 0 \rightarrow r = \left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$
A1:	$r =$ awrt 1.05 (ignoring units) or $r =$ awrt 105 cm
Note:	Give M0 A0 M0 A0 where $r =$ 1.05 (m) (3 sf) or awrt 1.05 (m) is found from no working.
Note:	Give final A1 for correct exact values for r. E.g. $r = \left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$

Notes for Question **38** Continued	
Note:	Give final M0 A0 for $-\frac{12}{r^2}+\frac{10}{3}\pi r\ > 0 \Rightarrow r > 1.0464$
Note:	Give final M1 A1 for $-\frac{12}{r^2}+\frac{10}{3}\pi r\ > 0 \Rightarrow r > 1.0464... \Rightarrow r = 1.0464...$
(c)	
M1:	Substitutes their $r = 1.046...$, found from solving $\frac{dA}{dr}=0$ in part (b), into the model with equation $A=\frac{12}{r}+\frac{5}{3}\pi r^2$
Note:	Give M0 for substituting their r which has been found from solving $\frac{d^2A}{dr^2}=0$ or from using $\frac{d^2A}{dr^2}$ into the model with equation $A=\frac{12}{r}+\frac{5}{3}\pi r^2$
A1ft:	$\{A=\}$ 17 or $\{A=\}$ awrt 17 (ignoring units)
Note:	You can only follow through on values of r for $0.6 \leq$ their $r \leq 1.3$ (and where their r has been found from solving $\frac{dA}{dr}=0$ in part (b))
Note:	Give M1 A1 for $A = 17\ (\text{m}^2)$ or $A =$ awrt $17\ (\text{m}^2)$ from no working

r	***A***	***A* (nearest integer)**
0.6	21.88495…	awrt 22
0.7	19.70849…	awrt 20
0.8	18.35103…	awrt 18
0.9	17.57448…	awrt 18
1.0	17.23598…	awrt 17
1.1	17.24463…	awrt 17
1.2	17.53982…	awrt 18
1.3	18.07958…	awrt 18
1.05	17.20124…	awrt 17
1.04644…	17.20105…	awrt 17

Question	Scheme	Marks	AOs
39	$\dfrac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$; as $h \to 0$, $\dfrac{\sin h}{h} \to 1$ and $\dfrac{\cos h - 1}{h} \to 0$		
	$\dfrac{\cos(\theta+h) - \cos\theta}{h}$	B1	2.1
	$= \dfrac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$	M1 A1	1.1b 1.1b
	$= -\dfrac{\sin h}{h}\sin\theta + \left(\dfrac{\cos h - 1}{h}\right)\cos\theta$		
	As $h \to 0$, $-\dfrac{\sin h}{h}\sin\theta + \left(\dfrac{\cos h - 1}{h}\right)\cos\theta \to -1\sin\theta + 0\cos\theta$	dM1	2.1
	so $\dfrac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$ *	A1*	2.5
		(5)	
		(5 marks)	

Notes for Question 39

B1:	Gives the correct fraction such as $\dfrac{\cos(\theta+h) - \cos\theta}{h}$ or $\dfrac{\cos(\theta+\delta\theta) - \cos\theta}{\delta\theta}$ Allow $\dfrac{\cos(\theta+h) - \cos\theta}{(\theta+h) - \theta}$ o.e. **Note:** $\cos(\theta+h)$ or $\cos(\theta+\delta\theta)$ may be expanded
M1:	Uses the compound angle formula for $\cos(\theta+h)$ to give $\cos\theta\cos h \pm \sin\theta\sin h$
A1:	Achieves $\dfrac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$ or equivalent
dM1:	**dependent on both the B and M marks being awarded** Complete attempt to apply the given limits to the gradient of their chord
Note:	They must isolate $\dfrac{\sin h}{h}$ and $\left(\dfrac{\cos h - 1}{h}\right)$, and replace $\dfrac{\sin h}{h}$ with 1 and replace $\left(\dfrac{\cos h - 1}{h}\right)$ with 0
A1*:	cso. Uses correct mathematical language of limiting arguments to prove $\dfrac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$
Note:	Acceptable responses for the final A mark include: • $\dfrac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = \lim\limits_{h\to 0}\left(-\dfrac{\sin h}{h}\sin\theta + \left(\dfrac{\cos h - 1}{h}\right)\cos\theta\right) = -1\sin\theta + 0\cos\theta = -\sin\theta$ • Gradient of chord $= -\dfrac{\sin h}{h}\sin\theta + \left(\dfrac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of chord tends to the gradient of the curve, so derivative is $-\sin\theta$ • Gradient of chord $= -\dfrac{\sin h}{h}\sin\theta + \left(\dfrac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of ***curve*** is $-\sin\theta$
Note:	Give final A0 for the following example which shows ***no limiting arguments***: when $h = 0$, $\dfrac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\dfrac{\sin h}{h}\sin\theta + \left(\dfrac{\cos h - 1}{h}\right)\cos\theta = -1\sin\theta + 0\cos\theta = -\sin\theta$
Note:	Do not allow the final A1 for stating $\dfrac{\sin h}{h} = 1$ or $\left(\dfrac{\cos h - 1}{h}\right) = 0$ and attempting to apply these
Note:	In this question $\delta\theta$ may be used in place of h
Note:	Condone $\mathrm{f}'(\theta)$ where $\mathrm{f}(\theta) = \cos\theta$ or $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}$ where $y = \cos\theta$ used in place of $\dfrac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta)$

Notes for Question 39 Continued	
Note:	Condone x used in place of θ if this is done consistently
Note:	Give final A0 for • $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos x) = \lim_{h\to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right) = -1\sin\theta + 0\cos\theta = -\sin\theta$ • $\frac{\mathrm{d}}{\mathrm{d}\theta} = \ldots$ • Defining $\mathrm{f}(x) = \cos\theta$ and applying $\mathrm{f}'(x) = \ldots$ • $\frac{\mathrm{d}}{\mathrm{d}x}(\cos\theta)$
Note:	Give final A1 for a correct limiting argument in x, followed by $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$ e.g. $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos x) = \lim_{h\to 0}\left(-\frac{\sin h}{h}\sin x + \left(\frac{\cos h - 1}{h}\right)\cos x\right) = -1\sin x + 0\cos x = -\sin x$ $\Rightarrow \frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$
Note:	Applying $h\to 0,\ \sin h\to h,\ \cos h\to 1$ to give e.g. $\lim_{h\to 0}\left(\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}\right) = \left(\frac{\cos\theta(1) - \sin\theta(h) - \cos\theta}{h}\right) = \frac{-\sin\theta(h)}{h} = -\sin\theta$ is final M0 A0 for incorrect application of limits
Note:	$\lim_{h\to 0}\left(\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}\right) = \lim_{h\to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right)$ $= \underleftrightarrow{\lim_{h\to 0}}\ (-(1)\sin\theta + 0\cos\theta) = -\sin\theta$. So for not removing $\underleftrightarrow{\lim_{h\to 0}}$ when the limit was taken is final A0
Note:	<u>**Alternative Method:**</u> Considers $\frac{\cos(\theta+h) - \cos(\theta-h)}{(\theta+h)-(\theta-h)}$ which simplifies to $\frac{-2\sin\theta\sin h}{2h}$

EXPERT TUITION

Question	Scheme	Marks	AOs
40 (a)	$\dfrac{\mathrm{d}r}{\mathrm{d}t} \propto \pm\dfrac{1}{r^2}$ or $\dfrac{\mathrm{d}r}{\mathrm{d}t} = \pm\dfrac{k}{r^2}$ (for k or a numerical k)	M1	3.3
	$\int r^2\,\mathrm{d}r = \int \pm k\,\mathrm{d}t \Rightarrow \ldots$ (for k or a numerical k)	M1	2.1
	$\dfrac{1}{3}r^3 = \pm kt\ \{+c\}$	A1	1.1b
	$t=0,\ r=5$ and $t=4,\ r=3$ gives $\dfrac{1}{3}r^3 = -\dfrac{49}{6}t + \dfrac{125}{3}$, where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth **or** $t=0,\ r=5$ and $t=240,\ r=3$ gives $\dfrac{1}{3}r^3 = -\dfrac{49}{360}t + \dfrac{125}{3}$, where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth	M1 A1	3.1a 1.1b
		(5)	
(b)	$r = 0 \Rightarrow 0 = -\dfrac{49}{6}t + \dfrac{125}{3} \Rightarrow\ 0 = -49t + 250\ \Rightarrow t = \ldots$	M1	3.4
	time = 5 minutes 6 seconds	A1	1.1b
		(2)	
(c)	Suggests a suitable limitation of the model. E.g. • Model does not consider how the mint is sucked • Model does not consider whether the mint is bitten • Model is limited for times up to 5 minutes 6 seconds, o.e. • Not valid for times greater than 5 minutes 6 seconds, o.e. • Mint may not retain the shape of a sphere (or have uniform radius) as it is being sucked • The model indicates that the radius of the mint is negative after it dissolves • Model does not consider the temperature in the mouth • Model does not consider rate of saliva production • Mint could be swallowed before it dissolves in the mouth	B1	3.5b
		(1)	
			(8 marks)

Notes for Question 40	
(a)	
M1:	Translates the description of the model into mathematics. See scheme.
M1:	Separates the variables of their differential equation which is in the form $\frac{\mathrm{d}r}{\mathrm{d}t} = \mathrm{f}(r)$ and some attempt at integration. (e.g. attempts to integrate at least one side). e.g. $\int r^2\,\mathrm{d}r = \int \pm k\,\mathrm{d}t$ and some attempt at integration. Condone the lack of integral signs
Note:	You can imply the M1 mark for $r^2 dr = -k\,\mathrm{d}t \Rightarrow \frac{1}{3}r^3 = -kt$
Note:	A numerical value of k (e.g. $k = \pm 1$) is allowed for the first two M marks
A1:	Correct integration to give $\frac{1}{3}r^3 = \pm kt$ with or without a constant of integration, c
M1:	For a complete process of using the boundary conditions to find both their unknown constants and finds an equation linking r and t So applies either • $t = 0,\ r = 5$ and $t = 4,\ r = 3,$ or • $t = 0,\ r = 5$ and $t = 240,\ r = 3,$ ***on their integrated equation*** to find their constants k and c ***and*** obtains an equation linking r and t
A1:	Correct equation, with variables r and t fully defined including correct reference to units. • $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$, {or an equivalent equation,} where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth • $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$, {or an equivalent equation,} where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth
Note:	Allow correct equations such as • in minutes, $r = \sqrt[3]{\frac{250-49t}{2}}$, $r^3 = -\frac{49}{2}t + 125$ or $t = \frac{250-2r^3}{49}$ • in seconds, $r = \sqrt[3]{\frac{15000-49t}{120}}$, $r^3 = -\frac{49}{120}t + 125$ or $t = \frac{15000-120r^3}{49}$
Note:	t defined as "the time from the start" is not sufficient for the final A1
(b)	
M1:	Sets $r = 0$ in their part (a) equation which links r with t and rearranges to make $t = ...$
A1:	5 minutes 6 seconds cao (**Note:** 306 seconds with no reference to 5 minutes 6 seconds is A0)
Note:	Give M0 if their equation would solve to give a negative time or a negative time is found
Note:	You can mark part (a) and part (b) together
(c)	
B1:	See scheme
Note:	Do not accept by itself • mint may not dissolve at a constant rate • rate of decrease of mint must be constant • $0 \le t < \frac{250}{49}$, $r \ge 0$; without any written explanation • reference to a mint having $r > 5$

Question	Scheme	Marks	AOs
41	$\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A+\dfrac{B}{(x-3)}+\dfrac{C}{(1-2x)}$		
(a) **Way 1**	$1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using a correct identity	A1	1.1b
		(4)	
(a) **Way 2**	{long division gives} $\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3+\dfrac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using $-10x+10 \equiv B(1-2x)+C(x-3)$	A1	1.1b
		(4)	
(b)	$\mathrm{f}(x)=3+\dfrac{4}{(x-3)}-\dfrac{2}{(1-2x)}$ $\{=3+4(x-3)^{-1}-2(1-2x)^{-1}\}$; $x>3$		
	$\mathrm{f}'(x)=-4(x-3)^{-2}-4(1-2x)^{-2}$ $\left\{=-\dfrac{4}{(x-3)^2}-\dfrac{4}{(1-2x)^2}\right\}$	M1 A1ft	2.1 1.1b
	Correct $\mathrm{f}'(x)$ **and** as $(x-3)^2>0$ and $(1-2x)^2>0$, then $\mathrm{f}'(x)=-(+\text{ve})-(+\text{ve})<0$, so $\mathrm{f}(x)$ is a decreasing function	A1	2.4
		(3)	
			(7 marks)

Notes for Question 41

(a)	
M1:	**Way 1:** Uses a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ in a complete method to find values for *B* and *C*. **Note:** Allow one slip in copying $1+11x-6x^2$ **Way 2:** Uses a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ (which has been found from long division) in a complete method to find values for *B* and *C*
B1:	$A=3$
M1:	Attempts to find the value of either *B* or *C* from their identity This can be achieved by ***either*** substituting values into their identity ***or*** by comparing coefficients and solving the resulting equations simultaneously
A1:	See scheme
Note:	**Way 1:** Comparing terms: $x^2: \ -6=-2A$; $\quad x: \ 11=7A-2B+C$; $\quad$ constant: $1=-3A+B-3C$ **Way 1:** Substituting: $x=3: \ -20=-5B \Rightarrow B=4$; $x=\dfrac{1}{2}: \ 5=-\dfrac{5}{2}C \Rightarrow C=-2$
Note:	**Way 2:** Comparing terms: $x: \ -10=-2B+C$; $\quad$ constant: $10=B-3C$ **Way 2:** Substituting: $x=3: \ -20=-5B \Rightarrow B=4$; $x=\dfrac{1}{2}: \ 5=-\dfrac{5}{2}C \Rightarrow C=-2$

Note:	$A=3,\ B=4,\ C=-2$ from no working scores M1B1M1A1
Note:	The final A1 mark is effectively dependent upon both M marks

Notes for Question **4**1 Continued	
(a) ctd	
Note:	Writing $1+11x-6x^2 \equiv B(1-2x)+C(x-3) \Rightarrow B=4,\ C=-2$ will get 1st M0, 2nd M1, 1st A0
Note:	**Way 1:** You can imply a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ from seeing $\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \dfrac{A(1-2x)(x-3)+B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
Note:	**Way 2:** You can imply a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ from seeing $\dfrac{-10x+10}{(x-3)(1-2x)} \equiv \dfrac{B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
(b)	
M1:	Differentiates to give $\{\mathrm{f}'(x)=\}\ \pm\lambda(x-3)^{-2} \pm \mu(1-2x)^{-2};\ \lambda,\ \mu \neq 0$
A1ft:	$\mathrm{f}'(x) = -4(x-3)^{-2}-4(1-2x)^{-2}$, which can be simplified or un-simplified
Note:	Allow A1ft for $\mathrm{f}'(x) = -(\text{their } B)(x-3)^{-2}+(2)(\text{their } C)(1-2x)^{-2}$; $(\text{their } B), (\text{their } C) \neq 0$
A1:	$\mathrm{f}'(x) = -4(x-3)^{-2}-4(1-2x)^{-2}$ or $\mathrm{f}'(x) = -\dfrac{4}{(x-3)^2}-\dfrac{4}{(1-2x)^2}$ **and** a correct explanation e.g. $\mathrm{f}'(x) = -(+\text{ ve})-(+\text{ ve})<0$, so $\mathrm{f}(x)$ is a decreasing {function}
Note:	The final A mark can be scored in part (b) from an incorrect $A=\ldots$ or from $A=0$ or no value of A found in part (a)

Notes for Question 41 Continued - Alternatives	
(a)	
Note:	Be aware of the following alternative solutions, by initially dividing by "$(x-3)$" or "$(1-2x)$"

- $\dfrac{1+11x-6x^2}{"(x-3)"(1-2x)} \equiv \dfrac{-6x-7}{(1-2x)} - \dfrac{20}{(x-3)(1-2x)} \equiv 3 - \dfrac{10}{(1-2x)} - \dfrac{20}{(x-3)(1-2x)}$

  $\dfrac{20}{(x-3)(1-2x)} \equiv \dfrac{D}{(x-3)} + \dfrac{E}{(1-2x)} \Rightarrow 20 \equiv D(1-2x) + E(x-3) \Rightarrow D = -4,\ E = -8$

  $\Rightarrow 3 - \dfrac{10}{(1-2x)} - \left(\dfrac{-4}{(x-3)} + \dfrac{-8}{(1-2x)}\right) \equiv 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)};\ A = 3,\ B = 4,\ C = -2$

- $\dfrac{1+11x-6x^2}{(x-3)"(1-2x)"} \equiv \dfrac{3x-4}{(x-3)} + \dfrac{5}{(x-3)(1-2x)} \equiv 3 + \dfrac{5}{(x-3)} + \dfrac{5}{(x-3)(1-2x)}$

  $\dfrac{5}{(x-3)(1-2x)} \equiv \dfrac{D}{(x-3)} + \dfrac{E}{(1-2x)} \Rightarrow 5 \equiv D(1-2x) + E(x-3) \Rightarrow D = -1,\ E = -2$

  $\Rightarrow 3 + \dfrac{5}{(x-3)} + \left(\dfrac{-1}{(x-3)} + \dfrac{-2}{(1-2x)}\right) \equiv 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)};\ A = 3,\ B = 4,\ C = -2$

(b)

<u>Alternative Method 1:</u>

$\mathrm{f}(x) = \dfrac{1+11x-6x^2}{(x-3)(1-2x)},\ x > 3 \Rightarrow \mathrm{f}(x) = \dfrac{1+11x-6x^2}{-2x^2+7x-3};\ \begin{cases} u = 1+11x-6x^2 & v = -2x^2+7x-3 \\ u' = 11-12x & v' = -4x+7 \end{cases}$

$\mathrm{f}'(x) = \dfrac{(-2x^2+7x-3)(11-12x)-(1+11x-6x^2)(-4x+7)}{(-2x^2+7x-3)^2}$	Uses quotient rule to find $\mathrm{f}'(x)$	**M1**
	Correct differentiation	**A1**
$\mathrm{f}'(x) = \dfrac{-20((x-1)^2+1)}{(-2x^2+7x-3)^2}$ and a correct explanation, e.g. $\mathrm{f}'(x) = -\dfrac{(+\text{ ve})}{(+\text{ ve})} < 0$, so $\mathrm{f}(x)$ is a decreasing {function}		**A1**

<u>Alternative Method 2:</u>

Allow M1A1A1 for the following solution:

Given $\mathrm{f}(x) = 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)} = 3 + \dfrac{4}{(x-3)} + \dfrac{2}{(2x-1)}$

as $\dfrac{4}{(x-3)}$ decreases when $x > 3$ and $\dfrac{2}{(2x-1)}$ decreases when $x > 3$

then $\mathrm{f}(x)$ is a decreasing {function}

Question	Scheme	Marks	AOs
42	$N = \dfrac{900}{3+7e^{-0.25t}} = 900(3+7e^{-0.25t})^{-1},\ t \in \mathbb{R},\ t \geq 0\ ;\ \dfrac{dN}{dt} = \dfrac{N(300-N)}{1200}$		
(a)	90	B1	3.4
		(1)	
(b) Way 1	$\dfrac{dN}{dt} = -900(3+7e^{-0.25t})^{-2}\left(7(-0.25)e^{-0.25t}\right)\ \left\{= \dfrac{900(0.25)(7)e^{-0.25t}}{(3+7e^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\Rightarrow \dfrac{dN}{dt} = \dfrac{900(0.25)\left(\left(\dfrac{900}{N}\right) - 3\right)}{\left(\dfrac{900}{N}\right)^2}$	dM1	2.1
	correct algebra leading to $\dfrac{dN}{dt} = \dfrac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	
(b) Way 2	$\dfrac{dN}{dt} = -900(3+7e^{-0.25t})^{-2}\left(7(-0.25)e^{-0.25t}\right)\ \left\{= \dfrac{900(0.25)(7)e^{-0.25t}}{(3+7e^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\dfrac{N(300-N)}{1200} = \dfrac{\left(\dfrac{900}{3+7e^{-0.25t}}\right)\left(300 - \dfrac{900}{3+7e^{-0.25t}}\right)}{1200}$	dM1	2.1
	$\text{LHS} = \dfrac{1575e^{-0.25t}}{(3+7e^{-0.25t})^2}$ o.e., $\text{RHS} = \dfrac{900(300(3+7e^{-0.25t})-900)}{1200(3+7e^{-0.25t})^2} = \dfrac{1575e^{-0.25t}}{(3+7e^{-0.25t})^2}$ o.e. and states hence $\dfrac{dN}{dt} = \dfrac{N(300-N)}{1200}$ (or LHS = RHS) *	A1*	1.1b
		(4)	
(c)	Deduces $N = 150$ (can be implied)	B1	2.2a
	so $150 = \dfrac{900}{3+7e^{-0.25T}} \Rightarrow e^{-0.25T} = \dfrac{3}{7}$	M1	3.4
	$T = -4\ln\left(\dfrac{3}{7}\right)$ or $T =$ awrt 3.4 (months)	dM1	1.1b
		A1	1.1b
		(4)	
(d)	either one of 299 or 300	B1	3.4
		(1)	
			(10 marks)

Notes for Question **42**	
42 (b)	
M1:	Attempts to differentiate using • the chain rule to give $\frac{dN}{dt} = \pm Ae^{-0.25t}(3+7e^{-0.25t})^{-2}$ or $\frac{\pm Ae^{-0.25t}}{(3+7e^{-0.25t})^2}$ o.e. • the quotient rule to give $\frac{dN}{dt} = \frac{(3+7e^{-0.25t})(0) \pm Ae^{-0.25t}}{(3+7e^{-0.25t})^2}$ • implicit differentiation to give $N(3+7e^{-0.25t}) = 900 \Rightarrow (3+7e^{-0.25t})\frac{dN}{dt} \pm ANe^{-0.25t} = 0$, o.e. where $A \neq 0$
Note:	Condone a slip in copying $(3+7e^{-0.25t})$ for the M mark
A1:	A correct differentiation statement
Note:	Implicit differentiation gives $(3+7e^{-0.25t})\frac{dN}{dt} - 1.75Ne^{-0.25t} = 0$
dM1:	**Way 1:** Complete attempt, by eliminating t, to form an equation linking $\frac{dN}{dt}$ and N only **Way 2:** Complete substitution of $N = \frac{900}{3+7e^{-0.25t}}$ into $\frac{dN}{dt} = \frac{N(300-N)}{1200}$
Note:	**Way 1:** e.g. substitutes $3+7e^{-0.25t} = \frac{900}{N}$ and $e^{-0.25t} = \frac{900}{N}$ or substitutes $e^{-0.25t} = \frac{\frac{900}{N}-3}{7}$ into their $\frac{dN}{dt} = ...$ to form an equation linking $\frac{dN}{dt}$ and N
A1*:	**Way 1:** Correct algebra leading to $\frac{dN}{dt} = \frac{N(300-N)}{1200}$ * **Way 2:** See scheme
(c)	
B1:	Deduces or shows that $\frac{dN}{dt}$ is maximised when $N = 150$
M1:	Uses the model $N = \frac{900}{3+7e^{-0.25t}}$ with their $N = 150$ and proceeds as far as $e^{-0.25T} = k,\ k > 0$ or $e^{0.25T} = k,\ k > 0$. Condone $t \equiv T$
dM1:	Correct method of using logarithms to find a value for T. Condone $t \equiv T$
A1:	see scheme
Note:	$\frac{d^2N}{dt^2} = \frac{dN}{dt}\left(\frac{300}{1200} - \frac{2N}{1200}\right) = 0 \Rightarrow N = 150$ is acceptable for B1
Note:	Ignore units for T
Note:	Applying $300 = \frac{900}{3+7e^{-0.25t}} \Rightarrow t = ...$ or $0 = \frac{900}{3+7e^{-0.25t}} \Rightarrow t = ...$ is M0 dM0 A0
Note:	M1 dM1 can only be gained in (c) by using an N value in the range $90 < N < 300$
(d)	
B1:	300 (or accept 299)

Question	Scheme	Marks	AOs
42	$N = \dfrac{900}{3+7\mathrm{e}^{-0.25t}} = 900(3+7\mathrm{e}^{-0.25t})^{-1},\ t \in \mathbb{R},\ t \geq 0\ ;\quad \dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{N(300-N)}{1200}$		
(b) Way 3	$\displaystyle\int \frac{1}{N(300-N)}\,\mathrm{d}N = \int \frac{1}{1200}\,\mathrm{d}t$	M1	2.1
	$\displaystyle\int \frac{1}{300}\left(\frac{1}{N} + \frac{1}{300-N}\right)\mathrm{d}N = \int \frac{1}{1200}\,\mathrm{d}t$ $\dfrac{1}{300}\ln N - \dfrac{1}{300}\ln(300-N) = \dfrac{1}{1200}t\ \ \{+c\}$	A1	1.1b
	$\{t=0,\ N=90 \Rightarrow\}\ \ c = \dfrac{1}{300}\ln(90) - \dfrac{1}{300}\ln(210)\ \Rightarrow c = \dfrac{1}{300}\ln\left(\dfrac{3}{7}\right)$ $\dfrac{1}{300}\ln N - \dfrac{1}{300}\ln(300-N) = \dfrac{1}{1200}t + \dfrac{1}{300}\ln\left(\dfrac{3}{7}\right)$ $\ln N - \ln(300-N) = \dfrac{1}{4}t + \ln\left(\dfrac{3}{7}\right)$ $\ln\left(\dfrac{N}{300-N}\right) = \dfrac{1}{4}t + \ln\left(\dfrac{3}{7}\right)\ \Rightarrow\ \dfrac{N}{300-N} = \dfrac{3}{7}\mathrm{e}^{\frac{1}{4}t}$	dM1	2.1
	$7N = 3\mathrm{e}^{\frac{1}{4}t}(300-N)\ \Rightarrow 7N + 3N\mathrm{e}^{\frac{1}{4}t} = 900\mathrm{e}^{\frac{1}{4}t}$ $N(7+3\mathrm{e}^{\frac{1}{4}t}) = 900\mathrm{e}^{\frac{1}{4}t}\ \Rightarrow\ N = \dfrac{900\mathrm{e}^{\frac{1}{4}t}}{7+3\mathrm{e}^{\frac{1}{4}t}}\ \Rightarrow\ N = \dfrac{900}{3+7\mathrm{e}^{-0.25t}}\ \ *$	A1*	1.1b
		(4)	
(b) Way 4	$N(3+7\mathrm{e}^{-0.25t}) = 900\ \Rightarrow \mathrm{e}^{-0.25t} = \dfrac{1}{7}\left(\dfrac{900}{N} - 3\right)\ \Rightarrow\ \mathrm{e}^{-0.25t} = \dfrac{900-3N}{7N}$	M1	2.1
	$\Rightarrow t = -4\big(\ln(900-3N) - \ln(7N)\big)$ $\Rightarrow \dfrac{\mathrm{d}t}{\mathrm{d}N} = -4\left(\dfrac{-3}{900-3N} - \dfrac{7}{7N}\right)$	A1	1.1b
	$\dfrac{\mathrm{d}t}{\mathrm{d}N} = 4\left(\dfrac{1}{300-N} + \dfrac{1}{N}\right)\ \Rightarrow\ \dfrac{\mathrm{d}t}{\mathrm{d}N} = 4\left(\dfrac{N+300-N}{N(300-N)}\right)$	dM1	2.1
	$\dfrac{\mathrm{d}t}{\mathrm{d}N} = \left(\dfrac{1200}{N(300-N)}\right)\ \Rightarrow\ \dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{N(300-N)}{1200}\ \ *$	A1*	1.1b
		(4)	

Notes for Question **42** Continued	
(b) Way 3	
M1:	Separates the variables, an attempt to form and apply partial fractions and integrates to give $\ln$ terms $= kt\ \{+c\},\ \ k \neq 0$, with or without a constant of integration c
A1:	$\frac{1}{300}\ln N - \frac{1}{300}\ln(300-N) = \frac{1}{1200}t\ \ \{+c\}$ or equivalent with or without a constant of integration c
dM1:	Uses $t=0,\ N=90$ to find their constant of integration and obtains an expression of the form $\lambda e^{\frac{1}{4}t} = \mathrm{f}(N);\ \lambda \neq 0$ or $\lambda e^{-\frac{1}{4}t} = \mathrm{f}(N);\ \lambda \neq 0$
A1*:	Correct manipulation leading to $N = \frac{900}{3+7e^{-0.25t}}$ *
(b) Way 4	
M1:	Valid attempt to make t the subject, followed by an attempt to find two ln derivatives, condoning sign errors and constant errors.
A1:	$\frac{dt}{dN} = -4\left(\frac{-3}{900-3N} - \frac{7}{7N}\right)$ or equivalent
dM1:	Forms a common denominator to combine their fractions
A1*:	Correct algebra leading to $\frac{dN}{dt} = \frac{N(300-N)}{1200}$ *

Question	Scheme	Marks	AOs
43(a)	$\mathrm{f}(x) = (8 - x)\ln x,\ x > 0$		
	Crosses x-axis $\Rightarrow \mathrm{f}(x) = 0 \Rightarrow (8 - x)\ln x = 0$		
	x coordinates are 1 and 8	B1	1.1b
		(1)	
(b)	Complete strategy of setting $\mathrm{f}'(x) = 0$ and rearranges to make $x = \ldots$	M1	3.1a
	$\left\{\begin{matrix} u = (8 - x) & v = \ln x \\ \dfrac{\mathrm{d}u}{\mathrm{d}x} = -1 & \dfrac{\mathrm{d}v}{\mathrm{d}x} = \dfrac{1}{x} \end{matrix}\right\}$		
	$\mathrm{f}'(x) = -\ln x + \dfrac{8-x}{x}$	M1 A1	1.1b 1.1b
	$-\ln x + \dfrac{8-x}{x} = 0 \Rightarrow -\ln x + \dfrac{8}{x} - 1 = 0$ $\Rightarrow \dfrac{8}{x} = 1 + \ln x \Rightarrow x = \dfrac{8}{1 + \ln x}$ *	A1*	2.1
		(4)	
			(5 marks)

Question **43** Notes:	
(a)	
B1:	Either • 1 and 8 • on Figure 2, marks 1 next to A and 8 next to B
(b)	
M1:	Recognises that Q is a stationary point (and not a root) and applies a complete strategy of setting $f'(x) = 0$ and rearranges to make $x = ...$
M1:	Applies $vu' + uv'$, where $u = 8 - x$, $v = \ln x$ **Note:** This mark can be recovered for work in part (c)
A1:	$(8 - x)\ln x \to -\ln x + \dfrac{8-x}{x}$, or equivalent **Note:** This mark can be recovered for work in part (c)
A1*:	Correct proof with no errors seen in working.

Question	Scheme	Marks	AOs
44(a)	$\frac{\mathrm{d}V}{\mathrm{d}t} = 160\pi\,,\ V = \frac{1}{3}\pi h^2(75 - h) = 25\pi h^2 - \frac{1}{3}\pi h^3$		
	$\frac{\mathrm{d}V}{\mathrm{d}h} = 50\pi h - \pi h^2$	M1	1.1b
		A1	1.1b
	$\left\{\frac{\mathrm{d}V}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \Rightarrow\right\} \left(50\pi h - \pi h^2\right)\frac{\mathrm{d}h}{\mathrm{d}t} = 160\pi$	M1	3.1a
	When $h = 10$, $\left\{\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \div \frac{\mathrm{d}V}{\mathrm{d}h} \Rightarrow\right\} \frac{160\pi}{50\pi(10) - \pi(10)^2} \left\{= \frac{160\pi}{400\pi}\right\}$	dM1	3.4
	$\frac{\mathrm{d}h}{\mathrm{d}t} = 0.4\ (\text{cms}^{-1})$	A1	1.1b
		(5)	
(b)	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{300\pi}{50\pi(20) - \pi(20)^2}$	M1	3.4
	$\frac{\mathrm{d}h}{\mathrm{d}t} = 0.5\ (\text{cms}^{-1})$	A1	1.1b
		(2)	
			(7 marks)

Question **44** Notes:

(a)

M1: Differentiates V with respect to h to give $\pm\alpha h \pm \beta h^2$, $\alpha \neq 0$, $\beta \neq 0$

A1: $50\pi h - \pi h^2$

M1: Attempts to solve the problem by applying a complete method of $\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = 160\pi$

M1: Depends on the previous M mark.

Substitutes $h = 10$ into their model for $\frac{\mathrm{d}h}{\mathrm{d}t}$ which is in the form $\frac{160\pi}{\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)}$

A1: Obtains the correct answer 0.4

(b)

M1: Realises that rate for of $160\pi\ \text{cm}^3\,\text{s}^{-1}$ for $0 \leqslant h \leqslant 12$ has no effect when the rate is increased to $300\pi\ \text{cm}^3\,\text{s}^{-1}$ for $12 < h \leqslant 24$ and so substitutes $h = 20$ into their model for $\frac{\mathrm{d}h}{\mathrm{d}t}$ which is in the form $\frac{300\pi}{\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)}$

A1: Obtains the correct answer 0.5

Question	Scheme	Marks	AOs
45	Complete process to find at least one set of coordinates for P. The process must include evidence of • differentiating • setting $\frac{dy}{dx} = 0$ to find $x = \ldots$ • substituting $x = \ldots$ into $\sin x + \cos y = 0.5$ to find $y = \ldots$	M1	3.1a
	$\left\{ \cancel{\frac{dy}{dx}} \cancel{\times} \right\}$ $\cos x - \sin y \frac{dy}{dx} = 0$	B1	1.1b
	Applies $\frac{dy}{dx} = 0$ (e.g. $\cos x = 0$ or $\frac{\cos x}{\sin y} = 0 \Rightarrow \cos x = 0$) $\Rightarrow x = \ldots$	M1	2.2a
	giving at least one of either $x = -\frac{\pi}{2}$ or $x = \frac{\pi}{2}$	A1	1.1b
	$x = \frac{\pi}{2} \Rightarrow \sin\left(\frac{\pi}{2}\right) + \cos y = 0.5 \Rightarrow \cos y = -\frac{1}{2} \Rightarrow y = \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$	M1	1.1b
	So in specified range, $(x, y) = \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)$, by cso	A1	1.1b
	$x = -\frac{\pi}{2} \Rightarrow \sin\left(-\frac{\pi}{2}\right) + \cos y = 0.5 \Rightarrow \cos y = 1.5$ has no solutions, and so there are exactly 2 possible points P.	B1	2.1
		(7)	
			(7 marks)

Question **45** Notes:

M1:	See scheme
B1:	Correct differentiated equation. E.g. $\cos x - \sin y \frac{dy}{dx} = 0$
M1:	Uses the information "the tangent to C at the point P is parallel to the x-axis" to deduce and apply $\frac{dy}{dx} = 0$ and finds $x = \ldots$
A1:	See scheme
M1:	For substituting one of their values from $\frac{dy}{dx} = 0$ into $\sin x + \cos y = 0.5$ and so finds $x = \ldots, y = \ldots$
A1:	Selects coordinates for P on C satisfying $\frac{dy}{dx} = 0$ and $-\frac{\pi}{2} \leqslant x < \frac{3\pi}{2}$, $-\pi < y < \pi$ i.e. finds $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)$ and no other points by correct solution only
B1:	Complete argument to show that there are exactly 2 possible points P.

Question	Scheme	Marks	AOs
46	Attempts the product and chain rule on $y = x(2x+1)^4$	M1	2.1
	$\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$	A1	1.1b
	Takes out a common factor $\frac{dy}{dx} = (2x+1)^3\{(2x+1)+8x\}$	M1	1.1b
	$\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$	A1	1.1b
			(4 marks)

Notes:

M1: Applies the product rule to reach $\frac{dy}{dx} = (2x+1)^4 + Bx(2x+1)^3$

A1: $\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$

M1: Takes out a common factor of $(2x+1)^3$

A1: The form of this answer is given. Look for $\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$

Question	Scheme	Marks	AOs
47(a)	Sets $500 = \pi r^2 h$	B1	2.1
	Substitute $h = \dfrac{500}{\pi r^2}$ into $S = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi r \times \dfrac{500}{\pi r^2}$	M1	2.1
	Simplifies to reach given answer $S = 2\pi r^2 + \dfrac{1000}{r}$ *	A1*	1.1b
		(3)	
(b)	Differentiates S with both indices correct in $\dfrac{dS}{dr}$	M1	3.4
	$\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$	A1	1.1b
	Sets $\dfrac{dS}{dr} = 0$ and proceeds to $r^3 = k,\ k$ is a constant	M1	2.1
	Radius $= 4.30\,\text{cm}$	A1	1.1b
	Substitutes their $r = 4.30$ into $h = \dfrac{500}{\pi r^2} \Rightarrow$ Height $= 8.60\,\text{cm}$	A1	1.1b
		(5)	
(c)	States a valid reason such as • The radius is too big for the size of our hands • If $r = 4.3\,\text{cm}$ and $h = 8.6\,\text{cm}$ the can is square in profile. All drinks cans are taller than they are wide • The radius is too big for us to drink from • They have different dimensions to other drinks cans and would be difficult to stack on shelves with other drinks cans	B1	3.2a
		(1)	
			9 marks

Notes:

(a)

B1: Uses the correct volume formula with V =500. Accept $500 = \pi r^2 h$

M1: Substitutes $h = \dfrac{500}{\pi r^2}$ or $rh = \dfrac{500}{\pi r}$ into $S = 2\pi r^2 + 2\pi rh$ to get S as a function of r

A1*: $S = 2\pi r^2 + \dfrac{1000}{r}$ Note that this is a given answer.

(b)

M1: Differentiates the given S to reach $\dfrac{dS}{dr} = Ar \pm Br^{-2}$

A1: $\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$ or exact equivalent

M1: Sets $\dfrac{dS}{dr} = 0$ and proceeds to $r^3 = k,\ k$ is a constant

A1: R = awrt 4.30cm

A1: H = awrt 8.60 cm

(c)

B1: Any valid reason. See scheme for alternatives

Question Number	Scheme		Marks
48(i)	**Mark (b)(i) and (ii) together and must be differentiating the original function not their answer to part (a)**		
	$\frac{3}{2}x^{-0.5}-6$	M1: For $x^n \rightarrow x^{n-1}$ i.e. $x^{0.5} \rightarrow x^{-0.5}$ or $6x \rightarrow 6$	M1A1
		A1: For $\frac{3}{2}x^{-0.5}-6$ or equivalent. May be un-simplified. Allow $\frac{3/2}{\sqrt{x}}-6$.	
			(2)
(ii)	$\frac{3}{2}x^{-0.5}-6=0 \Rightarrow x^n = \ldots$	Sets their $\frac{dy}{dx}=0$ (may be implied by their working) and reaches $x^n = C$ (including n = 1) **with correct processing allowing sign errors only** – this may be implied by e.g. $\sqrt{x}=\frac{1}{4}$ or $\frac{1}{\sqrt{x}}=4$.	M1
	$x=\frac{1}{16}$ cso	Allow equivalent fractions e.g. $\frac{9}{144}$ or 0.0625. If other solutions are given (e.g. likely to be x = 0 or x = -1/16) then this mark should be withheld.	A1
			(2)
			(4 marks)

Question Number	Scheme		Marks
49(a)	$\frac{dy}{dx}=\frac{1}{2}-\frac{27}{x^2}$	M1: $\frac{1}{2}$ or $-\frac{27}{x^2}$ A1: $\frac{dy}{dx}=\frac{1}{2}-\frac{27}{x^2}$ oe e.g. $\frac{1}{2}x^0-27x^{-2}$	M1A1
	$x=3\Rightarrow\frac{dy}{dx}=\frac{1}{2}-\frac{27}{9}=\left(-\frac{5}{2}\right)$	Substitutes $x = 3$ into their $\frac{dy}{dx}$ to obtain a numerical gradient	M1
	$m_T=-\frac{5}{2}\Rightarrow m_N=-1\div-\frac{5}{2}$ $\Rightarrow y-\left(-\frac{3}{2}\right)=\frac{2}{5}(x-3)$	The correct method to find the equation of a normal. Uses $-\frac{1}{m_T}$ with $\left(3,-\frac{3}{2}\right)$ where m_T has come from calculus. If using $y = mx + c$ must reach as far as $c = \ldots$	M1
	$10y=4x-27$*	Cso (correct equation must be seen in (a))	A1*
			(5)
(b)	$\frac{1}{2}x+\frac{27}{x}-12=\frac{4x-27}{10}$ or $y=\frac{10y+27}{8}+\frac{108}{10y+27}-12$	Equate equations to produce an equation just in x or just in y. Do not allow e.g. $\frac{1}{2}x^2+27-12x=\frac{4x-27}{10}$ Unless $\frac{1}{2}x+\frac{27}{x}-12=\frac{4x-27}{10}$ was seen previously. **Allow sign slips only.**	M1
	$x^2-93x+270=0$ or $20y^2-636y-999=0$	Correct 3 term quadratic equation (or any multiple of). Allow terms on both sides e.g. $x^2-93x=-270$ (The "= 0" may be implied by their attempt to solve)	A1
	$(x-90)(x-3)=0\Rightarrow x=\ldots$ or $x=\frac{93\pm\sqrt{93^2-4\times270}}{2}$ or $(10y-333)(2y+3)=0\Rightarrow y=\ldots$ or $y=\frac{636\pm\sqrt{636^2-4\times20\times(-999)}}{2\times20}$	Attempt to solve a 3TQ (see general guidance) leading to at least one for x or y. **Dependent on the first method mark.**	dM1
	$x=90$ **or** $y = 33.3$ oe	Cso. The x must be 90 and the y an equivalent number such as e.g. $\frac{333}{10}$	A1
	$x=90$ **and** $y = 33.3$ oe	Cso. The x must be 90 and the y an equivalent number such as e.g. $\frac{333}{10}$	A1
			(5)
			(10 marks)

Question Number	Scheme		Marks
50.	$y=\sqrt{x}+\dfrac{4}{\sqrt{x}}+4=x^{\frac{1}{2}}+4x^{-\frac{1}{2}}+4$		
	$x^n \to x^{n-1}$	Decreases any power by 1. Either $x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$ or $x^{-\frac{1}{2}} \to x^{-\frac{3}{2}}$ or $4 \to 0$ or $x^{\text{their } n} \to x^{\text{their } n-1}$ for fractional n.	M1
	$\left(\dfrac{dy}{dx}=\right)\dfrac{1}{2}x^{-\frac{1}{2}}+4\times-\dfrac{1}{2}x^{-\frac{3}{2}}$ $\left(=\dfrac{1}{2}x^{-\frac{1}{2}}-2x^{-\frac{3}{2}}\right)$	Correct derivative, simplified or un-simplified including indices. E.g. allow $\frac{1}{2}-1$ for $-\frac{1}{2}$ and allow $-\frac{1}{2}-1$ for $-\frac{3}{2}$	A1
	$x=8 \Rightarrow \dfrac{dy}{dx}=\dfrac{1}{2}8^{-\frac{1}{2}}+4\times-\dfrac{1}{2}8^{-\frac{3}{2}}$	Attempts to substitute x = 8 into their 'changed' (even integrated) expression that is clearly not y. If they attempt algebraic manipulation of their dy/dx before substitution, this mark is still available.	M1
	$=\dfrac{1}{2\sqrt{8}}-\dfrac{2}{\left(\sqrt{8}\right)^3}=\dfrac{1}{2\sqrt{8}}-\dfrac{2}{8\sqrt{8}}=\dfrac{1}{8\sqrt{2}}=\dfrac{1}{16}\sqrt{2}$	B1: $\sqrt{8}=2\sqrt{2}$ seen or implied anywhere, including from substituting x = 8 into y. May be seen explicitly or implied from e.g. $8^{\frac{3}{2}}=16\sqrt{2}$ or $8^{\frac{5}{2}}=128\sqrt{2}$ or $4\sqrt{8}=8\sqrt{2}$ A1: cso $\dfrac{1}{16}\sqrt{2}$ or $\dfrac{\sqrt{2}}{16}$ and allow rational equivalents for $\dfrac{1}{16}$ e.g. $\dfrac{32}{512}$ Apply isw so award this mark as soon as a correct answer is seen.	B1A1
			(5 marks)

Question Number	Scheme		Marks
51	$f'(4) = 30 + \frac{6 - 5\times 4^2}{\sqrt{4}}$	Attempts to substitutes $x = 4$ into $f'(x) = 30 + \frac{6-5x^2}{\sqrt{x}}$ or their algebraically manipulated	M1
	$f'(4) = -7$	Gradient = -7	A1
	$y-(-8) = "-7"\times(x-4)$ or $y = "-7"x + c \Rightarrow -8 = "-7"\times 4 + c$ $\Rightarrow c = ...$	Attempts an equation of a tangent using their numeric $f'(4)$ which has come from substituting x = 4 into the given or their algebraically manipulated and $(4,-8)$ with the 4 and -8 correctly placed. If using $y = mx + c$, must reach as far as $c = \ldots$	M1
	$y = -7x + 20$	Cao. Allow $y = 20 - 7x$ and allow the "y =" to become "detached" but it must be present at some stage. E.g. y = ... $= -7x + 20$	A1
			(4)
			(4 marks)

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="2">52(a)(i)</td><td rowspan="2">$k=(-5)^2\times3=75$</td><td>M1: Attempts to find the 'y' intercept. Accept as evidence $(-5)^2\times3$ with or without the bracket. If they expand f(x) to polynomial form here then they must then select their constant to score this mark. May be implied by sight of 75 on the diagram.</td><td rowspan="2">M1A1</td></tr>
<tr><td>A1: $k=75.$ Must clearly be identified as k. Allow this mark even from an incorrect or incomplete expansion as long as the constant k = 75 is obtained. Do not isw e.g. if 75 is seen followed by k = -75 score M1A0.</td></tr>
<tr><td rowspan="2">(ii)</td><td>$c=\frac{5}{2}$ only</td><td>$c=\frac{5}{2}$ oe (and no other values). Do not award just from the diagram – must be stated as the value of c.</td><td>B1</td></tr>
<tr><td colspan="2"></td><td>(3)</td></tr>
<tr><td rowspan="3">(b)</td><td colspan="2">$\text{f}(x)=(2x-5)^2(x+3)=\left(4x^2-20x+25\right)(x+3)=4x^3-8x^2-35x+75$
Attempts f(x) as a cubic polynomial by attempting to square the first bracket and multiply by the linear bracket or expands $(2x-5)(x+3)$ and then multiplies by $2x-5$
Must be seen or used in (b) but may be done in part (a).
Allow poor squaring e.g. $(2x-5)^2=4x^2\pm25$</td><td>M1</td></tr>
<tr><td>$(\text{f}'(x)=)12x^2-16x-35*$</td><td>M1: Reduces powers by 1 in all terms including any constant $\to0$
A1: Correct proof. Withhold this mark if there have been any errors including missing brackets earlier e.g. $(2x-5)^2(x+3)=4x^2-20x+25(x+3)=...$</td><td>M1A1*</td></tr>
<tr><td colspan="2"></td><td>(3)</td></tr>
</table>

(c)	$f'(3) = 12\times 3^2 - 16\times 3 - 35$	Substitutes x = 3 into **their** $f'(x)$ **or the given** $f'(x)$. Must be a changed function i.e. not into f(x).	M1
	$12x^2 - 16x - 35 = '25'$	Sets **their** $f'(x)$ **or the given** $f'(x) =$ their $f'(3)$ with a consistent f'. **Dependent on the previous method mark.**	**d**M1
	$12x^2 - 16x - 60 = 0$	$12x^2 - 16x - 60 = 0$ or equivalent 3 term quadratic e.g. $12x^2 - 16x = 60$. (A correct quadratic equation may be implied by later work). **This is cso so must come from correct work – i.e. they must be using the <u>given</u>** $f'(x)$.	A1 cso
	$(x-3)(12x+20) = 0 \Rightarrow x = ...$	Solves 3 term quadratic by suitable method – see General Principles. **Dependent on both previous method marks.**	**dd**M1
	$x = -\frac{5}{3}$	$x = -\frac{5}{3}$ oe clearly identified. If x = 3 is also given and not rejected, this mark is withheld. (allow -1.6 recurring as long as it is clear i.e. a dot above the 6). **This is cso and must come from correct work – i.e. they must be using the <u>given</u>** $f'(x)$.	A1 cso
			(5)
			(11 marks)
Alt (b) Product rule.	$f(x) = (2x-5)^2(x+3) \Rightarrow f'(x) = (2x-5)^2 \times 1 + (x+3)\times 4(2x-5)$ M1: Attempts product rule to give an expression of the form $p(2x-5)^2 + q(x+3)(2x-5)$ M1: Multiplies out and collects terms A1: $f'(x) = 12x^2 - 16x - 35$*		M1 M1A1*

Question Number	Scheme	Notes	Marks
53.	$y=3x^2+6x^{\frac{1}{3}}+\dfrac{2x^3-7}{3\sqrt{x}}$		
	$\dfrac{2x^3-7}{3\sqrt{x}}=\dfrac{2x^3}{3\sqrt{x}}-\dfrac{7}{3\sqrt{x}}=\dfrac{2}{3}x^{\frac{5}{2}}-\dfrac{7}{3}x^{-\frac{1}{2}}$	Attempts to split the fraction into 2 terms and obtains either $\alpha x^{\frac{5}{2}}$ or $\beta x^{-\frac{1}{2}}$. This may be implied by a correct power of x in their differentiation of one of these terms. But beware of $\beta x^{-\frac{1}{2}}$ coming from $\dfrac{2x^3-7}{3\sqrt{x}}=2x^3-7+3x^{-\frac{1}{2}}$	M1
	$x^n \to x^{n-1}$	Differentiates by reducing power by one for any of their powers of x	M1
	$\left(\dfrac{dy}{dx}=\right)6x+2x^{-\frac{2}{3}}+\dfrac{5}{3}x^{\frac{3}{2}}+\dfrac{7}{6}x^{-\frac{3}{2}}$	A1: $6x$. Do not accept $6x^1$. Depends on second M mark only. Award when first seen and isw.	A1A1A1A1
		A1: $2x^{-\frac{2}{3}}$. Must be simplified so do not accept e.g. $\dfrac{2}{1}x^{-\frac{2}{3}}$ but allow $\dfrac{2}{\sqrt[3]{x^2}}$. Depends on second M mark only. Award when first seen and isw.	
		A1: $\dfrac{5}{3}x^{\frac{3}{2}}$. Must be simplified but allow e.g. $1\dfrac{2}{3}x^{1.5}$ or e.g. $\dfrac{5}{3}\sqrt{x^3}$. Award when first seen and isw.	
		A1: $\dfrac{7}{6}x^{-\frac{3}{2}}$. Must be simplified but allow e.g. $1\dfrac{1}{6}x^{-1\frac{1}{2}}$ or e.g. $\dfrac{7}{6\sqrt{x^3}}$. Award when first seen and isw.	
	In an otherwise <u>fully correct solution</u>, penalise the presence of + c by deducting the final A1		
			[6]
	Use of Quotient Rule: First M1 and final A1A1 (Other marks as above)		
	$\dfrac{d\left(\dfrac{2x^3-7}{3\sqrt{x}}\right)}{dx}=\dfrac{3\sqrt{x}\left(6x^2\right)-\left(2x^3-7\right)\frac{3}{2}x^{-\frac{1}{2}}}{\left(3\sqrt{x}\right)^2}$	Uses <u>**correct**</u> quotient rule	M1
	$=\dfrac{10x^{\frac{5}{2}}+7x^{-\frac{1}{2}}}{6x}$	A1: Correct first term of numerator and correct denominator	A1A1
		A1: All correct as simplified as shown	
	So $\dfrac{dy}{dx}=6x+2x^{-\frac{2}{3}}+\dfrac{10x^{\frac{5}{2}}+7x^{-\frac{1}{2}}}{6x}$ scores full marks		
			6 marks

Question Number	Scheme		Marks
54. (a)	$y = 2x^3 + kx^2 + 5x + 6$		
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right) 6x^2 + 2kx + 5$	M1: $x^n \to x^{n-1}$ for one of the terms including $6 \to 0$ A1: Correct derivative	M1 A1
			[2]
(b)	Gradient of given line is $\frac{17}{2}$	Uses or states $\frac{17}{2}$ or equivalent e.g. 8.5. Must be stated or used in (b) and not just seen as part of $y = \frac{17}{2}x + \frac{1}{2}$.	B1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=-2} = 6(-2)^2 + 2k(-2) + 5$	Substitutes $x = -2$ into their derivative **(not the curve)**	M1
	$"24 - 4k + 5" = "\frac{17}{2}" \Rightarrow k = \frac{41}{8}$	dM1: Puts their expression = their $\frac{17}{2}$ (Allow BOD for 17 or -17 but **not** the normal gradient) and solves to obtain a value for k. **Dependent on the previous method mark**. A1: $\frac{41}{8}$ or $5\frac{1}{8}$ or 5.125	dM1 A1
	Note: $6x^2 + 2kx + 5 = \frac{17}{2}x + \frac{1}{2}$ scores no marks on its own but may score the first M mark if they substitute x = -2 into the lhs. If they rearrange this equation and then substitute x = -2, this scores no marks.		
			[4]
(c)	$y = -16 + 4k - 10 + 6 = 4"k" - 20 = \frac{1}{2}$	M1: Substitutes $x = -2$ and their numerical k into $y = \ldots$ A1: $y = \frac{1}{2}$	M1 A1
	Allow the marks for part (c) to be scored in part (b).		
			[2]
(d)	$y - "\frac{1}{2}" = "\frac{17}{2}"\left(x - {}^{-}2\right) \Rightarrow -17x + 2y - 35 = 0$ or $y = "\frac{17}{2}"x + c \Rightarrow c = \ldots \Rightarrow -17x + 2y - 35 = 0$ or $2y - 17x = 1 + 34 \Rightarrow -17x + 2y - 35 = 0$	M1: Correct attempt at linear equation with their 8.5 gradient (not the normal gradient) using x = -2 and their $\frac{1}{2}$ A1: cao (allow any integer multiple)	M1 A1
			[2]
			10 marks

Question Number	Scheme		Marks
55.	$y = 4x^3 - \dfrac{5}{x^2}$		
	$12x^2 + \dfrac{10}{x^3}$	M1: $x^n \to x^{n-1}$ e.g. Sight of x^2 or x^{-3} or $\dfrac{1}{x^3}$	M1A1A1
		A1: $3\times 4x^2$ or $-5\times -2x^{-3}$ (oe) (Ignore + c for this mark)	
		A1: $12x^2 + \dfrac{10}{x^3}$ or $12x^2 + 10x^{-3}$ <u>all on one line</u> and no + c	
	Apply ISW here and award marks when first seen.		
			(3) **(3 marks)**

EXPERT TUITION

Question Number	Scheme		Marks
56(a)	$(x^2+4)(x-3)=x^3-3x^2+4x-12$	Attempt to multiply out the numerator to get a cubic with 4 terms and at least 2 correct	M1
	$\dfrac{x^3-3x^2+4x-12}{2x}=\dfrac{x^2}{2}-\dfrac{3}{2}x+2-6x^{-1}$	M1: Attempt to divide each term by $2x$. The powers of x of at least two terms must follow from their expansion. Allow an attempt to multiply by $2x^{-1}$ A1: Correct expression. May be un-simplified but powers of x must be combined e.g. $\dfrac{x^2}{2}$ not $\dfrac{x^3}{2x}$	M1A1
	$\dfrac{dy}{dx}=x-\dfrac{3}{2}+\dfrac{6}{x^2}$ oe e.g. $\dfrac{2x^3-3x^2+12}{2x^2}$	ddM1: $x^n \to x^{n-1}$ or $2\to 0$ **Dependent on both previous method marks.** A1: $x-\dfrac{3}{2}+\dfrac{6}{x^2}$ oe and **isw** Accept $1x$ or even $1x^1$ but not $\dfrac{2x}{2}$ and not x^0. If they lose the previous A1 because of an **incorrect constant only** then allow recovery here and in part (b) for a correct derivative.	ddM1A1
			(5)
	See appendix for alternatives using product/quotient rule		
(b)	At $x=-1,\ y=10$	Correct value for y	B1
	$\left(\dfrac{dy}{dx}=\right)-1-\dfrac{3}{2}+\dfrac{6}{1}=3.5$	M1: Substitutes $x=-1$ into their expression for dy/dx A1: 3.5 oe cso	M1A1
	$y-{'10'}={'3.5'}(x--1)$	Uses their **tangent** gradient **which must come from calculus** with $x=-1$ and their numerical y with a correct straight line method. If using $y=mx+c$, this mark is awarded for correctly establishing a value for c.	M1
	$2y-7x-27=0$	$\pm k(2y-7x-27)=0$ cso	A1
			(5)
			(10 marks)

Appendix

56(a)

<table>
<tr><td rowspan="5">Way 2 Quotient</td><td>$(x^2+4)(x-3)=x^3-3x^2+4x-12$</td><td colspan="2">Attempt to multiply out the numerator to get a cubic with 4 terms and at least 2 correct</td><td>M1</td></tr>
<tr><td colspan="2" rowspan="2">$\frac{dy}{dx}=\frac{2x\left(3x^2-6x+4\right)-2\left(x^3-3x^2+4x-12\right)}{\left(2x\right)^2}$</td><td>M1: Correct application of quotient rule</td><td rowspan="2">M1A1</td></tr>
<tr><td>A1: Correct derivative</td></tr>
<tr><td rowspan="2">$=\frac{4x^3}{4x^2}-\frac{6x^2}{4x^2}+\frac{24}{4x^2}=x-\frac{3}{2}+\frac{6}{x^2}$
oe e.g. $\frac{2x^3-3x^2+12}{2x^2}$</td><td colspan="2">M1: Collects terms and divides by denominator. Dependent on both previous method marks.</td><td rowspan="2">ddM1A1</td></tr>
<tr><td colspan="2">A1: $x-\frac{3}{2}+\frac{6}{x^2}$ oe and isw
Accept $1x$ or even $1x^1$ but not $\frac{2x}{2}$ and not x^0.</td></tr>
<tr><td rowspan="5">Way 3 Product</td><td>$y=\left(\frac{x}{2}+\frac{2}{x}\right)\left(x-3\right)$ or $\left(x^2+4\right)\left(\frac{1}{2}-\frac{3}{2x}\right)$</td><td colspan="2">Divides one bracket by $2x$</td><td>M1</td></tr>
<tr><td rowspan="2">$\frac{dy}{dx}=\left(x-3\right)\left(\frac{1}{2}-\frac{2}{x^2}\right)+\left(\frac{x}{2}+\frac{2}{x}\right)$ or
$\frac{dy}{dx}=\left(x^2+4\right)\frac{3}{2x^2}+2x\left(\frac{1}{2}-\frac{3}{2x}\right)$</td><td colspan="2">M1: Correct application of product rule</td><td rowspan="2">M1A1</td></tr>
<tr><td colspan="2">A1: Correct derivative</td></tr>
<tr><td rowspan="2">$=\frac{3}{2}+\frac{6}{x^2}+x-3=x-\frac{3}{2}+\frac{6}{x^2}$
oe e.g.
$\frac{2x^3-3x^2+12}{2x^2}$</td><td colspan="2">M1: Expands and collects terms. Dependent on both previous method marks.</td><td>ddM1A1</td></tr>
<tr><td colspan="2">A1: $x-\frac{3}{2}+\frac{6}{x^2}$ oe and isw
Accept $1x$ or even $1x^1$ but not $\frac{2x}{2}$ and not x^0.</td><td></td></tr>
<tr><td rowspan="3">Way 4 Product</td><td>$(x^2+4)(x-3)=x^3-3x^2+4x-12$</td><td colspan="2">Attempt to multiply out the numerator to get a cubic with 4 terms and at least 2 correct</td><td>M1</td></tr>
<tr><td colspan="3">$\frac{dy}{dx}=\left(x^3-3x^2+4x-12\right)\times-\frac{1}{2}x^{-2}+\frac{1}{2}x^{-1}\left(3x^2-6x+4\right)$
M1: Correct application of product rule A1: Correct derivative</td><td>M1A1</td></tr>
<tr><td colspan="3">$\frac{dy}{dx}=-\frac{x}{2}+\frac{3}{2}-\frac{2}{x}+\frac{6}{x^2}+\frac{3x}{2}-3+\frac{2}{x}=x-\frac{3}{2}+\frac{6}{x^2}$
ddM1: Expands and collects terms Dependent on both previous method marks.
A1: $x-\frac{3}{2}+\frac{6}{x^2}$ oe e.g. $\frac{2x^3-3x^2+12}{2x^2}$ and isw. Accept $1x$ or even $1x^1$ but not $\frac{2x}{2}$ and not x^0.</td><td>ddM1A1</td></tr>
<tr><td></td><td></td><td colspan="2"></td><td></td></tr>
</table>

Way 5	$y=\left(\frac{x}{2}+\frac{2}{x}\right)(x-3)$ or $(x^2+4)\left(\frac{1}{2}-\frac{3}{2x}\right)$	Divides one bracket by $2x$	M1
	$=\frac{x^2}{2}-\frac{3}{2}x+2-6x^{-1}$	M1: Expands A1: Correct expression	M1A1
	$\frac{dy}{dx}=x-\frac{3}{2}+\frac{6}{x^2}$ oe e.g. $\frac{2x^3-3x^2+12}{2x^2}$	ddM1: $x^n \to x^{n-1}$ or $2 \to 0$ **Dependent on both previous method marks.** A1: $x-\frac{3}{2}+\frac{6}{x^2}$ oe and **isw** Accept $1x$ or even $1x^1$ but not $\frac{2x}{2}$ If they lose the previous A1 because of an **incorrect constant only** then allow recovery here for a correct derivative.	ddM1A1

Question Number	Scheme		Marks
57	Gradient of normal is $-\frac{1}{2} \Rightarrow$ Gradient of tangent = +2	M1: Gradient of $2y+x=0$ is $\pm\frac{1}{2}(m) \Rightarrow \frac{dy}{dx} = -\frac{1}{\pm\frac{1}{2}}$	M1A1
		A1: Gradient of tangent = +2 (May be implied)	
	The A1 may be implied by $\frac{-1}{\frac{3\sqrt{x}}{2}-\frac{9}{4\sqrt{x}}+2} = -\frac{1}{2}$		
	$\frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2 = 2 \Rightarrow \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} = 0$	Sets the given $f'(x)$ **or their** $f'(x)$ = their **changed** *m* and **not** their *m* where *m* has come from $2y + x = 0$	M1
	$\times 4\sqrt{x} \Rightarrow 6x - 9 = 0 \Rightarrow x = ..$	$\times 4\sqrt{x}$ or equivalent **correct algebraic** processing (**allow sign/arithmetic errors only**) and attempt to solve to obtain a value for *x*. If $f'(x) \neq 2$ they need to be solving a three term quadratic in $\sqrt{x}$ correctly and square to obtain a value for *x*. **Must be using the given** $f'(x)$ **for this mark.**	M1
	$x = 1.5$	$x = \frac{3}{2}$ (1.5) Accept equivalents e.g . $x = \frac{9}{6}$ **If any 'extra' values are not rejected, score A0.**	A1
			(5)
	Beware $\frac{-1}{\frac{3\sqrt{x}}{2}-\frac{9}{4\sqrt{x}}+2} = -\frac{1}{2} \Rightarrow \frac{-2}{3\sqrt{x}} + \frac{4\sqrt{x}}{9} - \frac{1}{2} = -\frac{1}{2}$ etc. leads to the correct answer and could score M1A1M1M0(incorrect processing)A0		
			(5 marks)

Question Number	Scheme	Marks
58.	(a) $(1-2x)^2 = 1-4x+4x^2$ $\frac{d}{dx}(1-2x)^2 = \frac{d}{dx}(1-4x+4x^2) = -4+8x$ o.e.	M1 M1A1 (3)
	Alternative method using chain rule: Answer of -4 (1 – 2 x)	M1M1A1 (3)
	(b) $\frac{x^5+6\sqrt{x}}{2x^2} = \frac{x^5}{2x^2} + 6\frac{\sqrt{x}}{2x^2}, = \frac{1}{2}x^3 + 3x^{-\frac{3}{2}}$ Attempts to differentiate $x^{-\frac{3}{2}}$ to give $k\ x^{-\frac{5}{2}}$ $= \frac{3}{2}x^2 - \frac{9}{2}x^{-\frac{5}{2}}$ o.e. Quotient Rule (May rarely appear) – See note below	M1,A1 M1 A1 (4) **(7 marks)**

Notes

(a) M1 Attempt to multiply out bracket. Must be 3 or 4 term quadratic and **must have constant term 1**

M1 $x^n \to x^{n-1}$. Follow through on any term in an incorrect expression. Accept a constant → 0

A1 $-4+8x$ Accept -4 (1 – 2x) or equivalent. This is not cso and may follow error in the constant term
Following correct answer by -2 + 4x – apply isw

Correct answer with no working – assume chain rule and give M1M1A1 i.e. 3/3

Common errors: $(1-2x)^2 = 2-4x+4x^2$ is M0, then allow M1A1 for -4 + 8x

$(1-2x)^2 = 1-4x^2$ is M0 then -8x earns M1A0 or $(1-2x)^2 = 1-2x^2$ is M0 then -4x earns M1A0

Use of Chain Rule:

M1M1: first M1 for complete method so $2\times(\pm 2)(1-2x)$ second M1 for (1 – 2x) (as power reduced)

Then A1 for -4 (1 – 2x) or for -4 + 8x

So (i) 2(1 – 2x) gets M0 M1A0 for reducing power and (ii) 2×2(1 – 2x) gets M1 M1A0

(b) M1 An attempt to divide by 2x^2 first. This can be implied by the sight of the following

Some correct working e.g. $\frac{x^5}{2x^2} + 6\frac{\sqrt{x}}{2x^2}$ or $(x^5+6\sqrt{x})(2x^2)^{-1}$ **leading to** $ax^p + bx^q$ in either case

or can be **implied by** $\frac{1}{2}x^3 + 3x^p$ (after no working) i.e. both coefficients correct and power 3 correct

Common error: $(x^5+6\sqrt{x})2x^{-2}$ is M0 (may earn next M mark for the differentiation $x^{-\frac{3}{2}} \to x^{-\frac{5}{2}}$)

A1 Writing the given expression as $\frac{1}{2}x^3 + 3x^{-\frac{3}{2}}$ or $0.5x^3 + \frac{6}{2}x^{-\frac{3}{2}}$ or $0.5x^3 + \frac{6}{2}x^{-1\frac{1}{2}}$ or etc…

M1 $x^{-\frac{3}{2}} \to x^{-\frac{5}{2}}$ A1 Cao $\frac{3}{2}x^2 - \frac{9}{2}x^{-\frac{5}{2}}$ o.e. e.g. $\frac{3}{2}x^2 - \frac{9}{2x^2\sqrt{x}}$ then isw. Allow factorised form. Do not

penalise $+-\frac{9}{2}x^{-\frac{5}{2}}$ used instead of $-\frac{9}{2}x^{-\frac{5}{2}}$

Use of Quotient Rule : M1,A1:Reaching $\frac{2x^2(5x^4 + 3x^{-\frac{1}{2}}) - 4x(x^5+6x^{\frac{1}{2}})}{4x^4}$, $= \frac{6x^6 - 18x^{\frac{3}{2}}}{4x^4}$

Send to review if doubtful M1A1: Simplifying (e.g.dividing numerator and denominator by 2) to reach $\frac{3x^6 - 9x^{\frac{3}{2}}}{2x^4}$ o.e.

Question Number	Scheme	Marks
59.	Sub x=4 into $f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1$	M1
	$\Rightarrow f'(4) = \frac{3}{8} \times 4^2 - 10 \times 4^{-\frac{1}{2}} + 1$	
	$\Rightarrow f'(4) = 2$	A1
	Gradient of tangent $= 2 \Rightarrow$ Gradient of normal is –1/2	dM1
	Substitute $x = 4, y = 25$ into line equation with their changed gradient e.g. $y - 25 = -\frac{1}{2}(x-4)$	dM1
	$\pm k(2y + x - 54) = 0$ o.e. (but must have integer coefficients)	A1cso
		(5)
		(5 Marks)

Notes

M1 Attempt to substitute $x = 4$ into $f'(x)$ must be in part (b)

A1 $f'(x) = 2$ at $x = 4$

dM1 (Dependent on first method mark in part (b)) Using $m_1 \times m_2 = -1$ to find the gradient of the normal from their tangent gradient (Give mark if gradient of 1 becomes -1 as they will lose accuracy)

dM1 (Dependent on first method mark in part (b)) Attempt to find the equation of the normal (not tangent). Eg use x=4, y=25 in y= '–1/2'x+c to find a value of c or use $'-\frac{1}{2}' = \frac{y-25}{x-4}$ with their adapted gradient.

A1 cso $\pm k(2y + x - 54) = 0$ (where k is any integer)

Question Number	Scheme	Marks
60.	(a) Discriminant = $b^2-4ac=8^2-4\times2\times3,=40$	M1, A1
		(2)
	(b) $2x^2+8x+3=2(x^2+............)$ or p=2	B1
	$=2((x+2)^2\pm...)$ or $q=2$	M1
	$=2(x+2)^2-5$ or $p=2$, $q=2$ and r = -5	A1
		(3)
	(c) Method 1A: Sets derivative $"4x+8"=4\Rightarrow\ x=,\quad x=-1$	M1, A1
	Substitute $x=-1$ in $y=2x^2+8x+3\quad(\Rightarrow y=-3)$	dM1
	Substitute $x=-1$ and y= -3 in $y=4x+c$ or into $(y+3)=4(x+1)$ and expand	dM1
	c = 1 or writing $y=4x+1$	A1cso
		(5)
	Method 1B: Sets derivative $"4x+8"=4\Rightarrow\ x=,\quad x=-1$	M1, A1
	Substitute $x=-1$ in $2x^2+8x+3\ =\ 4x+c$	dM1
	Attempts to find value of c	dM1
	c = 1 or writing $y=4x+1$	A1cso
		(5)
	Method 2: Sets $2x^2+8x+3=4x+c$ and collects x terms together	M1
	Obtains $2x^2+4x+3-c=0$ or equivalent	A1
	States that $b^2-4ac=0$	dM1
	$4^2-4\times2\times(3-c)=0$ and so c =	dM1
	$c=1$	A1cso
		(5)
	Method 3: Sets $2x^2+8x+3=4x+c$ and collects x terms together	M1
	Obtains $2x^2+4x+3-c=0$ or equivalent	A1
	Uses $2(x+1)^2-2+3-c=0$ or equivalent	dM1
	Writes -2 +3 –c = 0	dM1
	So c = 1	A1cso
		(5)
	Also see special case for using a perpendicular gradient (overleaf)	**(10 marks)**

Notes

(a) M1 Attempts to calculate b^2-4ac using $8^2-4\times2\times3$ - must be correct – not just part of a quadratic formula

A1 Cao 40

(b) B1 See 2(….) or p = 2

M1 $..((x+2)^2\pm...)$ is sufficient evidence or obtaining q = 2

A1 Fully correct values. $2(x+2)^2-5$ or p = 2, q = 2, r = –5 cso.

Ignore inclusion of "=0".

[In many respects these marks are similar to three B marks.
p = 2 is B1; q = 2 is B1 and p = 2, q = 2 and r = -5 is final B1 but they must be entered on epen as **B1 M1 A1]**

Special case: Obtains $2x^2+8x+3=2(x+2)-1$ This may have first B1, for p = 2 then M0A0 (c) Method 1A (Differentiates and puts gradient equal to 4. Needs both x and y to find c)

M1 Attempts to solve their $\frac{dy}{dx} = 4$. They must reach $x = ...$ (Just differentiating is M0 A0)

A1 $x = -1$ (If this follows $\frac{dy}{dx} = 4x + 8$, then give M1 A1 by implication)

dM1 (Depends on previous M mark) Substitutes **their** $x = -1$ into f(x) or into "their f(x) from (b)" to find y

dM1 (Depends on both previous M marks) Substitutes **their** $x = -1$ and **their** $y = -3$ values into $y = 4x + c$ to find c
or uses equation of line is $(y + \text{"3"}) = 4(x + \text{"1"})$ and rearranges to $y = mx + c$

A1 $c = 1$ or allow for $y = 4x + 1$ cso

(c) Method 1B (Differentiates and puts gradient equal to 4. Also equates equations and uses x to find c)

M1A1 Exactly as in Method 1A above

dM1 (Depends on previous M mark) Substitutes **their** $x = -1$ into $2x^2 + 8x + 3 = 4x + c$

dM1 Attempts to find value of c then A1 as before

(c) Method 2 (uses repeated root to find c by discriminant)

M1 Sets $2x^2 + 8x + 3 = 4x + c$ and tries to collect x terms together

A1 Collects terms e.g. $2x^2 + 4x + 3 - c = 0$ or $-2x^2 - 4x - 3 + c = 0$ or $2x^2 + 4x + 3 = c$ or even
$2x^2 + 4x = c - 3$ Allow "=0" to be missing on RHS.

dM1 (If the line is a tangent it meets the curve at just one point so repeated root and $b^2 - 4ac = 0$)
Stating that $b^2 - 4ac = 0$ is enough

dM1 Using $b^2 - 4ac = 0$ to obtain equation in terms of c
(Eg. $4^2 - 4 \times 2 \times (3 - c) = 0$) AND leading to a solution for c

A1 $c = 1$ or allow for $y = 4x + 1$ cso

(c) Method 3 (Similar to method 2 but uses completion of the square on the quadratic to find repeated root)

M1 Sets $2x^2 + 8x + 3 = 4x + c$ and tries to collect x terms together. May be implied by $2x^2 + 8x + 3 - 4x \pm c$ on one side

A1 Collects terms e.g. $2x^2 + 4x + 3 - c = 0$ or $-2x^2 - 4x - 3 + c = 0$ or $2x^2 + 4x + 3 = c$
or even $2x^2 + 4x = c - 3$ Allow "=0" to be missing on RHS.

dM1 Then use completion of square $2(x+1)^2 - 2 + 3 - c = 0$ (Allow $2(x+1)^2 - k + 3 - c = 0$)
where k is non zero. It is enough to give the correct or almost correct (with k) completion of the square

dM1 $-2 + 3 - c = 0$ AND leading to a solution for c (Allow $-1 + 3 - c = 0$) ($x = -1$ has been used)

A1 $c = 1$ cso

In Method 1 they may use part (b) and differentiate their f(x) and put it equal to 4
They can earn M1, but do not follow through errors.

In Methods 2 and 3 they may use part (b) to write
their $2(x+2)^2 - 5 = 4x + c$. They need to expand and collect x terms together for M1
Then expanding gives $2x^2 + 4x + 3 - c = 0$ for A1 – do not follow through errors
Then the scheme is as before

If they just state $c = 1$ with little or no working – please send to review,

PTO for special case

Special case uses perpendicular gradient (maximum of 2/5)

Sets $4x+8=-\frac{1}{4} \Rightarrow x=,\quad x=-\frac{33}{16}$ M1 A0

Substitute $x=-\frac{33}{16}$ in $y=2x^2+8x+3 \quad (\Rightarrow y=-\frac{639}{128})$ M0

Substitute $x=-\frac{33}{16}$ and $y=-\frac{639}{128}$ into $y=4x+c$ or into $(y+\frac{639}{128})=4(x+\frac{33}{16})$ and expand M1 A0

Question Number	Scheme	Marks
61	$y = 2x^5 + \dfrac{6}{\sqrt{x}}$	
	$x^n \to x^{n-1}$	M1
	$\dfrac{dy}{dx} = 10x^4 - 3x^{-\frac{3}{2}}$ oe	A1A1
		(3)
		(3 marks)

M1 For $x^n \to x^{n-1}$. ie. x^4 or $x^{-\frac{3}{2}}$ or $\left(\dfrac{1}{x^{\frac{3}{2}}}\right)$ seen

A1 For $2 \times 5x^4$ **or** $6 \times -\dfrac{1}{2}x^{-\frac{3}{2}}$ (oe). (Ignore $+c$ for this mark)

A1 For simplified expression $10x^4 - 3x^{-\frac{3}{2}}$ or $10x^4 - \dfrac{3}{x^{\frac{3}{2}}}$ o.e. and no $+c$

Apply ISW here and award marks when first seen.

Question Number	Scheme	Marks
62(a)	Substitutes $x = 2$ into $y = 20 - 4 \times 2 - \frac{18}{2}$ and gets 3	B1
	$\frac{dy}{dx} = -4 + \frac{18}{x^2}$	M1 A1
	Substitute $x = 2 \Rightarrow \frac{dy}{dx} = \left(\frac{1}{2}\right)$ then finds negative reciprocal (-2)	dM1
	Method 1: States or uses $y - 3 = -2(x-2)$ or $y = -2x + c$ with their (2, 3) Method 2: Or: Check that (2, 3) lies on the line $y = -2x + 7$	dM1
	Method 1: to deduce that $y = -2x + 7$ * Method 2: Deduce equation of normal as it has the **same gradient** and passes through **a common point**	A1*
		(6)
(b)	Put $20 - 4x - \frac{18}{x} = -2x + 7$ and simplify to give $2x^2 - 13x + 18 = 0$	M1 A1
	Or put $y = 20 - 4\left(\frac{7-y}{2}\right) - \frac{18}{\left(\frac{7-y}{2}\right)}$ to give $y^2 - y - 6 = 0$	
	$(2x-9)(x-2) = 0$ so $x =$ or $(y-3)(y+2) = 0$ so $y =$	dM1
	$x = \frac{9}{2}, y = -2$	A1, A1
		(5) (11 marks)

PTO for notes on this question.

(a)	B1	Substitutes $x = 2$ into expression for y and gets 3 cao (must be in part (a) and **must use curve equation – not line equation) This must be seen to be substituted.**
	M1	For an attempt to differentiate the negative power with $x^{-1} \to x^{-2}$.
	A1	Correct expression for $\frac{dy}{dx} = -4 + \frac{18}{x^2}$, accept equivalents
	dM1	Dependent on **first** M1 Substitutes $x = 2$ into their derivative to obtain a numerical gradient and find negative reciprocal or states that $-2 \times \frac{1}{2} = -1$

(Method 1)

	dM1	Dependent on **first** M1 Finds equation of line using changed gradient (not their ½ but -1/2, 2 or -2) e.g. $y - "3" = -"2"(x-2)$ or y = "-2"$x + c$ and use of (2, "3") to find c=
	A1*	CSO. This is a given answer $y = -2x + 7$ obtained with no errors seen and equation should be stated

(Method 2)- checking given answer

	dM1	Uses given equation of line and checks that (2, 3) lies on the line
	A1*	CSO. This is a given answer $y = -2x + 7$ so statement that normal and line **have the same gradient** and **pass through the same point** must be stated

(b)	M1	Equate the **two given** expressions, collect terms and simplify to a 3TQ. There may be sign errors when collecting terms But putting for example $20x - 4x^2 - 18 = -2x + 7$ is M0 here
	A1	Correct 3TQ = 0 (need = 0 for A mark) $2x^2 - 13x + 18 = 0$
	dM1	Attempt to solve an appropriate quadratic by factorisation, use of formula, or completion of the square (see general instructions).
	A1	$x = \frac{9}{2}$ oe or y = -2 (allow second answers for this mark so ignore x = 2 or y = 3)
	A1	Correct solution only so both $x = \frac{9}{2}, y = -2$ or $\left(\frac{9}{2}, -2\right)$
		If x = 2, y = 3 is included as an answer and point B is not identified then last mark is A0 Answer only – with no working – send to review. The question stated "use algebra"

Question Number	Scheme		Marks
63 (a)	$(3-x^2)^2 = 9-6x^2+x^4$	An attempt to expand the numerator obtaining an expression of the form $9 \pm px^2 \pm qx^4, \quad p,q \neq 0$	M1
	$9x^{-2}+x^2$	Must come from $\dfrac{9+x^4}{x^2}$	A1
	-6	Must come from $\dfrac{-6x^2}{x^2}$	A1
	Alternative 1: Writes $\dfrac{(3-x^2)^2}{x^2}$ as $(3x^{-1}-x)^2$ and attempts to expand = M1 then A1A1 as in the scheme.		
	Alternative 2: Sets $(3-x^2)^2 = 9+Ax^2+Bx^4$, expands $(3-x^2)^2$ and compares coefficients = M1 then A1A1 as in the scheme.		
			(3)
	$(f'(x) = 9x^{-2}-6+x^2)$		
(b)	$-18x^{-3}+2x$	M1: $x^n \to x^{n-1}$ on separate terms at least once. Do not award for $A \to 0$ (Integrating is M0) A1ft: $-18x^{-3}+2\text{"}B\text{"}x$ with a numerical B and no extra terms. (A may have been incorrect or even zero)	M1 A1ft
			(2)
(c)	$f(x) = -9x^{-1}-6x+\dfrac{x^3}{3}(+c)$	M1: $x^n \to x^{n+1}$ on separate terms at least once. (Differentiating is M0) A1ft: $-9x^{-1}+Ax+\dfrac{Bx^3}{3}(+c)$ with numerical A and B, $A,B \neq 0$	M1A1ft
	$10 = \dfrac{-9}{-3}-6(-3)+\dfrac{(-3)^3}{3}+c$ so c = ...	Uses x = -3 and y =10 in what they think is f(x) (They may have differentiated here) but it must be a changed function i.e. not the original $f'(x)$, to form a linear equation in c and attempts to find c. No + c gets M0 and A0 unless their method implies that they are correctly finding a constant.	M1
	c = -2	cso	A1
	$(f(x) =) -9x^{-1}-6x+\dfrac{x^3}{3}$ + their c	Follow through their c in an otherwise (possibly un-simplified) **correct expression**. Allow $-\dfrac{9}{x}$ for $-9x^{-1}$ or even $\dfrac{9x^{-1}}{-1}$.	A1ft
	Note that if they integrate in (b), no marks there but if they then go on to use their integration in (c), the marks for integration are available.		
			(5)
			[10]

Question Number	Scheme		Marks
64 (a)	$\left(-\dfrac{3}{4}, 0\right)$. Accept $x = -\frac{3}{4}$		B1
			(1)
(b)	$y = 4$	B1: One correct asymptote	B1B1
	$x = 0$ or 'y-axis'	B1: Both correct asymptotes and no extra ones.	
	Special case $x \neq 0$ **and** $y \neq 4$ scores B1B0		
			(2)
(c)	$\dfrac{dy}{dx} = -3x^{-2}$	$\dfrac{dy}{dx} = kx^{-2}$ (Allow $\dfrac{dy}{dx} = kx^{-2} + 4$)	M1
	At $x = -3$, gradient of curve = $-\frac{1}{3}$	Cao (may be un-simplified but must be a fraction with no powers) e.g. $-3(-3)^{-2}$ scores A0 unless evaluated as e.g. $\dfrac{-3}{9}$ or is implied by their normal gradient.	A1
	Gradient of normal = -1/m	Correct perpendicular gradient rule applied to a numerical gradient that must have come from substituting x = -3 into their derivative. **Dependent on the previous M1.**	dM1
	Normal at P is $(y-3) = 3(x+3)$	M1: Correct straight line method using (-3, 3) and a "changed" gradient. A wrong equation with no formula quoted is M0. **Also dependent on the first M1.**	dM1A1
		A1: Any correct equation	
			(5)
(d)	(-4, 0) and (0, 12).	Both correct (May be seen on a sketch)	B1
	So AB has length $\sqrt{160}$ or AB^2 has length 160	M1: Correct use of Pythagoras for their A and B one of which lies on the x-axis and the other on the y-axis, obtained from their equation in (c). A correct method for AB^2 or AB.	M1 A1cso
		A1: $\sqrt{160}$ or better e.g. $4\sqrt{10}$ **with no errors seen**	
			(3)
			[11]

Question Number	Scheme	Notes	Marks
65.	$y = x^3 + 4x + 1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 4(+0)$	M1: $x^n \to x^{n-1}$ including 1→0 A1: Correct differentiation (Do not allow $4x^0$ unless $x^0 = 1$ is implied by later work)	M1A1
	substitute $x = 3 \Rightarrow$ gradient = 31	M1: Substitutes x = 3 into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ (not y) Substitutes x = 3 into a "changed" function. They may even have integrated. A1: cao	M1A1
			[4]

Question Number	Scheme	Notes	Marks
66.(a)	$\mathrm{f}'(x)=\frac{x+9}{\sqrt{x}}=\frac{x}{\sqrt{x}}+\frac{9}{\sqrt{x}}=x^{\frac{1}{2}}+9x^{-\frac{1}{2}}$	M1: Correct attempt to split into 2 separate terms or fractions. May be implied by one correct term. Divides by $x^{\frac{1}{2}}$ or multiplies by $x^{-\frac{1}{2}}$.	M1A1
		A1: $x^{\frac{1}{2}}+9x^{-\frac{1}{2}}$ or equivalent	
	$\mathrm{f}(x)=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+9\frac{x^{\frac{1}{2}}}{\frac{1}{2}}(+c)$	M1: Independent method mark for $x^n \to x^{n+1}$ **on separate terms**	M1A1
		A1: Allow un-simplified answers. No requirement for + c here	
	$\frac{(9)^{\frac{3}{2}}}{\frac{3}{2}}+9\frac{(9)^{\frac{1}{2}}}{\frac{1}{2}}+c=0\Rightarrow c=...$	Substitutes $x=9$ and $y=0$ into their integrated expression leading to a value for c. If no c at this stage M0A0 follows unless their method implies that they are correctly finding a constant of integration.	M1
	$\mathrm{f}(x)=\frac{2}{3}x^{\frac{3}{2}}+18x^{\frac{1}{2}}-72$	There is no requirement to simplify their f(x) so accept any correct un-simplified form.	A1
			(6)
(b)	$\mathrm{f}'(x)=\frac{x+9}{\sqrt{x}}=10\Rightarrow x+9=10\sqrt{x}$	Sets $\mathrm{f}'(x)=\frac{x+9}{\sqrt{x}}=10$ and multiplies by $\sqrt{x}$. The terms in x must be in the numerator. E.g. allow $\frac{x+9}{10}=\sqrt{x}$	M1
	They must be setting either the **original** $\mathrm{f}'(x)=10$ or an equivalent **correct** expression = 10		
	$(\sqrt{x}-9)(\sqrt{x}-1)=0\Rightarrow\sqrt{x}=...$	Correct attempt to solve a relevant 3TQ in $\sqrt{x}$ leading to solution for $\sqrt{x}$. **Dependent on the previous M1.**	dM1
	$x=81$, $x=1$	Note that the $x=1$ solution could be just written down and is B1but must come from a correct equation.	A1, B1
			(4)
			[10]
Alternative to part (b)	$(\frac{x+9}{\sqrt{x}})^2=10^2\Rightarrow x^2+18x+81=100x$	Sets $\frac{x+9}{\sqrt{x}}=10$, squares and multiplies by x. They must be setting either the original $\mathrm{f}'(x)=10$ or an equivalent **correct** expression = 10	M1
	$(x-81)(x-1)=0\Rightarrow x=..$	Correct attempt to solve a relevant 3TQ leading to solution for x. **Dependent on the previous M1.**	dM1
	$x=81$, $x=1$	Note that the $x=1$ solution could be just written down and is B1but must come from a correct equation.	A1, B1

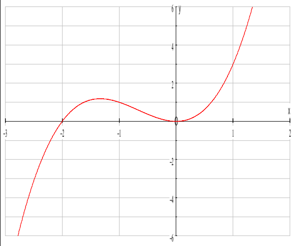
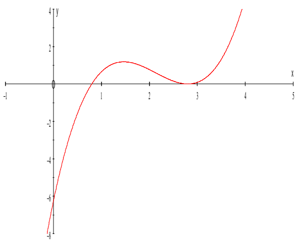
Question Number	Scheme	Marks
67.	$C: y = 2x - 8\sqrt{x} + 5, \quad x \geqslant 0$	
(a)	So, $y = 2x - 8x^{\frac{1}{2}} + 5$	
	$\frac{dy}{dx} = 2 - 4x^{-\frac{1}{2}} + \{0\} \qquad (x > 0)$	M1 A1 A1
		[3]
(b)	(When $x = \frac{1}{4}$, $y = 2\left(\frac{1}{4}\right) - 8\sqrt{\left(\frac{1}{4}\right)} + 5$ so) $y = \frac{3}{2}$	B1
	(gradient $= \frac{dy}{dx} =$) $2 - \frac{4}{\sqrt{\left(\frac{1}{4}\right)}}$ $\{= -6\}$	M1
	Either : $y - "\tfrac{3}{2}" = "-6"\left(x - \frac{1}{4}\right)$ **or**: $y = "-6"x + c$ and $"\tfrac{3}{2}" = "-6"\left(\frac{1}{4}\right) + c \Rightarrow c = "3"$	dM1
	So $\underline{y = -6x + 3}$	A1
		[4]
(c)	Tangent at Q is parallel to $2x - 3y + 18 = 0$	
	$(y = \frac{2}{3}x + 6 \Rightarrow)$ Gradient $= \frac{2}{3}$. so tangent gradient is $\frac{2}{3}$	B1
	So, $"2 - \frac{4}{\sqrt{x}}" = "\frac{2}{3}"$ Sets their gradient function = their numerical gradient.	M1
	$\Rightarrow \frac{4}{3} = \frac{4}{\sqrt{x}} \Rightarrow x = 9$ Ignore extra answer x = -9	A1
	When $x = 9$, $y = 2(9) - 8\sqrt{9} + 5 = -1$ Substitutes their found x into equation of curve.	dM1
	$y = -1$.	A1
		[5]
		12 marks
	Notes	
(a)	**M1:** Evidence of differentiation, so $x^n \to x^{n-1}$ at least once so $x^1 \to 1$ or x^0 or $x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$ not just $5 \to 0$ **A1:** Any two of the three terms correct – do not need to see zero – the 5 disappearing is sufficient; need not be simplified. **A1:** $2 - 4x^{-\frac{1}{2}}$ Both terms correct, and simplified. Do not need to include domain $x > 0$	
(b)	**B1:** Obtaining y = 3/2 or fractional or decimal equivalent (no working need be seen) **M1:** An attempt to substitute $x = \frac{1}{4}$ into $\frac{dy}{dx}$. to establish **gradient**. This may be implied by –6 or m = -6 but not y = **-** 6. Can earn this M mark if they go on to use $m = \frac{1}{6}$ or use their numerical value of $\frac{dy}{dx}$. **dM1: This depends on previous method mark.** Complete method for obtaining the equation of the tangent**,** using their **tangent** gradient and their value for y_1 (obtained from x = ¼, allow slip) i.e. $y - y_1 = m_T\left(x - \frac{1}{4}\right)$ with their tangent gradient and their y_1 or uses $y = mx + c$ with $\left(\frac{1}{4}, \text{their } y_1\right)$ and their tangent gradient. **A1:** y = -6x + 3 or y= 3 – 6x or a = -6 and b = 3	
(c)	**B1:** For the value 2/3 **not** 2/3 x **not** -3/2 **M1:** Sets their gradient function dy/dx = their numerical gradient **A1:** Obtains x = 9 **dM1:** Substitutes their x (from gradient equation) into original equation of curve C i.e. original expression y = **A1:** (9, -1) or x = 9, y = -1, or just y = -1	
	Special Cases: In (b) Finds normal could get B1 M1 M0 A0 i.e. max of 2/4	
	In (c) Uses **perpendicular** instead of **parallel** then award B0 M1 A0 M1 A0 i.e max 2/5 – see over	

Question Number	Scheme	Marks
68. (a)	$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$ $\left\{\frac{dy}{dx} =\right\}\ 5(3)x^2 - 6\left(\frac{4}{3}\right)x^{\frac{1}{3}} + 2$ $= 15x^2 - 8x^{\frac{1}{3}} + 2$	M1 A1 A1 A1 **[4]**
(b)	$\left\{\frac{d^2y}{dx^2} =\right\}\ 30x - \frac{8}{3}x^{-\frac{2}{3}}$	M1 A1 **[2]** **6**
	Notes	
(a) (b)	**M1**: for an attempt to differentiate $x^n \to x^{n-1}$ to one of the first three terms of $y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$. So seeing either $5x^3 \to \pm\lambda x^2$ or $-6x^{\frac{4}{3}} \to \pm\mu x^{\frac{1}{3}}$ or $2x \to 2$ is M1. **1st A1**: for $15x^2$ only. **2nd A1:** for $-8x^{\frac{1}{3}}$ or $-8\sqrt[3]{x}$ only. **3rd A1:** for $+2$ ($+c$ included in part (a) loses this mark). **Note:** $2x^0$ is A0 unless simplified to 2. **M1:** For differentiating $\frac{dy}{dx}$ again to give **either** • a correct follow through differentiation of their x^2 term • **or** for $\pm\alpha x^{\frac{1}{3}} \to \pm\beta x^{-\frac{2}{3}}$. **A1:** for any ***correct*** expression ***on the same line*** (accept un-simplified coefficients). For powers: $30x^{2-1} - \frac{8}{3}x^{\frac{1}{3}-1}$ is A0, but writing powers as one term eg: $(15\times 2x) - \frac{8}{3}x^{-\frac{4}{6}}$ is ok for A1. **Note:** Candidates leaving their answers as $\left\{\frac{dy}{dx} =\right\} 15x^2 - \frac{24}{3}x^{\frac{1}{3}} + 2$ and $\left(\frac{d^2y}{dx^2} =\right)\ 30x - \frac{24}{9}x^{-\frac{2}{3}}$ are awarded M1A1A0A1 in part (a) and M1A1 in part (b). **Be careful:** $30x - \frac{8}{3}x^{-\frac{1}{3}}$ will be A0. **Note:** For an extra term appearing in part (b) on the same line, ie $30x - \frac{8}{3}x^{-\frac{2}{3}} + 2$ is M1A0 **Note:** If a candidate writes in part (a) $15x^2 - 8x^{\frac{1}{3}} + 2 + c$ and in part (b) $30x - \frac{8}{3}x^{-\frac{2}{3}} + c$ then award (a) M1A1A1A0 (b) M1A1	

Question Number	Scheme	Marks
	$P(4, -1)$ lies on C where $\mathrm{f}'(x) = \dfrac{1}{2}x - \dfrac{6}{\sqrt{x}} + 3, \;\; x > 0$	
69. (a)	$\mathrm{f}'(4) = \dfrac{1}{2}(4) - \dfrac{6}{\sqrt{4}} + 3\,; = 2$	M1; A1
	T: $y - -1 = 2(x - 4)$	dM1
	T: $y = 2x - 9$	A1
		[4]
(b)	$\mathrm{f}(x) = \dfrac{x^{1+1}}{2(2)} - \dfrac{6x^{-\frac{1}{2}+1}}{(\frac{1}{2})} + 3x\,(+c)$ or equivalent.	M1 A1
	$\{\mathrm{f}(4) = -1 \Rightarrow\}\ \dfrac{16}{4} - 12(2) + 3(4) + c = -1$	dM1
	$\{\,4 - 24 + 12 + c = -1 \quad \Rightarrow \quad c = 7\,\}$	
	So, $\{\mathrm{f}(x) =\}\ \dfrac{x^2}{2(2)} - \dfrac{6x^{\frac{1}{2}}}{(\frac{1}{2})} + 3x + 7$	A1 cso
	$\left\{\text{NB: } \mathrm{f}(x) = \dfrac{x^2}{4} - 12\sqrt{x} + 3x + 7\right\}$	**[4]**
		8

	Notes	
(a)	**1st M1:** for clear attempt at $\mathrm{f}'(4)$. **1st A1:** for obtaining 2 from $\mathrm{f}'(4)$. **2nd dM1:** for $y - -1 = (\text{their } \mathrm{f}'(4))(x-4)$ or $\dfrac{y - -1}{x - 4} = (\text{their } \mathrm{f}'(4))$ or full method of $y = mx + c$, with $x = 4$, $y = -1$ and their $\mathrm{f}'(4)$ to find a value for c. **Note:** this method mark is dependent on the first method mark being awarded. **2nd A1:** for $y = 2x - 9$ or $y = -9 + 2x$ **Note:** This work needs to be contained in part (a) only.	
(b)	**1st M1**: for a clear attempt to integrate $\mathrm{f}'(x)$ with at least one correct application of $x^n \to x^{n+1}$ on $\mathrm{f}'(x) = \dfrac{1}{2}x - \dfrac{6}{\sqrt{x}} + 3$. So seeing either $\dfrac{1}{2}x \to \pm\lambda x^{1+1}$ or $-\dfrac{6}{\sqrt{x}} \to \pm\mu x^{-\frac{1}{2}+1}$ or $3 \to 3x^{0+1}$ is M1. **1st A1:** for correct un-simplified coefficients and powers (or equivalent) with or without $+c$. **2nd dM1:** for use of $x = 4$ and $y = -1$ in an integrated equation to form a linear equation in c equal to -1. ie: applying $\mathrm{f}(4) = -1$. This mark is dependent on the first method mark being awarded. **A1:** For $\{\mathrm{f}(x) = \}\ \dfrac{x^2}{2(2)} - \dfrac{6x^{\frac{1}{2}}}{(\frac{1}{2})} + 3x + 7$ stated on one line where coefficients can be un-simplified or simplified, but must contain one term powers. Note this mark is for **correct solution only.** **<u>Note: For a candidate attempting to find f(x) in part (a)</u>** If it is clear that they understand that they are finding $\mathrm{f}(x)$ in part (a); ie. by writing $\mathrm{f}(x) = \ldots$ or $y = \ldots$ then you can give credit for this working in part (b).	

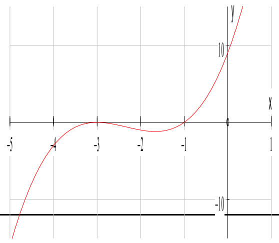
Question	Scheme	Marks
70.	$4x^3+3x^{-\frac{1}{2}}$	M1A1A1 (3) **3 marks**
	Notes	
	M1 for $x^n \to x^{n-1}$ i.e. x^3 or $x^{-\frac{1}{2}}$ seen 1st A1 for $4x^3$ <u>or</u> $6\times\frac{1}{2}\times x^{-\frac{1}{2}}$ (o.e.) (ignore any + c for this mark) 2nd A1 for simplified terms i.e. <u>both</u> $4x^3$ <u>and</u> $3x^{-\frac{1}{2}}$ or $\frac{3}{\sqrt{x}}$ and no $+c$ $\left[\frac{3}{1}x^{-\frac{1}{2}} \text{ is A0}\right]$ Apply ISW here and award marks when first seen	

Question	Scheme	Marks
71. (a)	$[\,y = x^3 + 2x^2\,]$ so $\dfrac{dy}{dx} = 3x^2 + 4x$	M1A1 (2)
(b)	 Shape 	B1
	Touching x-axis at origin	B1
	Through and not touching or stopping at -2 on x –axis. Ignore extra intersections.	B1 (3)
(c)	At $x = -2$: $\dfrac{dy}{dx} = 3(-2)^2 + 4(-2) = 4$	M1
	At $x = 0$: $\dfrac{dy}{dx} = 0$ (Both values correct)	A1 (2)
(d)	 Horizontal translation (touches x-axis still)	M1
	$k-2$ and k marked on positive x-axis	B1
	$k^2(2-k)$ (o.e) marked on negative y-axis	B1 (3)
		10 marks

	Notes	
(a)	M1 for attempt to multiply out and then some attempt to differentiate $x^n \to x^{n-1}$ Do not award for $2x(x + 2)$ or $2x\,(1 + 2)$ etc	
Prod Rule	Award M1 for a correct attempt: 2 products with a + and at least one product correct A1 for both terms correct. (If $+c$ or extra term is included score A0)	
(b)	1st B1 for correct shape (anywhere). Must have 2 clear turning points. 2nd B1 for graph touching at origin (not crossing or ending) 3rd B1 for graph passing through (not stopping or touching at) -2 on x axis and -2 marked on axis	
SC	B0B0B1 for $y = x^3$ <u>or</u> cubic with straight line between $(-2, 0)$ and $(0, 0)$	
(c)	M1 for attempt at $y'(0)$ or $y'(-2)$. Follow through their 0 or – 2 and their $y'(x)$ <u>or</u> for a <u>correct</u> statement of zero gradient for an identified point on their curve that touches x-axis A1 for both correct answers	
(d)	For the M1 in part (d) ignore any coordinates marked – just mark the shape. M1 for a horizontal translation of their (b). Should still touch x – axis if it did in (b) <u>Or</u> for a graph of correct shape with min. and intersection in correct order on +ve x-axis 1st B1 for k and $k - 2$ on the positive x-axis. Curve must pass through $k - 2$ and touch at k 2nd B1 for a correct intercept on negative y-axis in terms of k. Allow $(0, 2k^2 - k^3)$ (o.e.) seen in script if curve passes through –ve y-axis	

Question	Scheme	Marks
72. (a)	$\left(\frac{1}{2}, 0\right)$	B1 (1)
(b)	$\frac{dy}{dx} = x^{-2}$	M1A1
	At $x = \frac{1}{2}$, $\frac{dy}{dx} = \left(\frac{1}{2}\right)^{-2} = 4$ $(= m)$	A1
	Gradient of normal $= -\frac{1}{m}$ $\left(= -\frac{1}{4}\right)$	M1
	Equation of normal: $y - 0 = -\frac{1}{4}\left(x - \frac{1}{2}\right)$	M1
	$2x + 8y - 1 = 0$ **(*)**	A1cso (6)
(c)	$2 - \frac{1}{x} = -\frac{1}{4}x + \frac{1}{8}$	M1
	$[= 2x^2 + 15x - 8 = 0]$ <u>or</u> $[8y^2 - 17y = 0]$	
	$(2x-1)(x+8) = 0$ leading to $x = \ldots$	M1
	$x = \left[\frac{1}{2}\right]$ or -8	A1
	$y = \frac{17}{8}$ (or exact equivalent)	A1ft (4)
		11 marks
	Notes	
(a)	B1 accept $x = \frac{1}{2}$ if evidence that $y = 0$ has been used. Can be written on graph. Use ISW	
(b)	1st M1 for kx^{-2} even if the '2' is not differentiated to zero. If no evidence of $\frac{dy}{dx}$ seen then 0/6 1st A1 for x^{-2} (o.e.) only 2nd A1 for using $x = 0.5$ to get $m = 4$ (correctly) (or $m = 1/0.25$) To score final A1cso must see at least one intermediate equation for the line after $m = 4$ 2nd M1 for using the perpendicular gradient rule on their m coming from their $\frac{dy}{dx}$ Their m must be a value not a letter. 3rd M1 for using a changed gradient (based on y') and their A to find equation of line 3rd A1cso for reaching printed answer with no incorrect working seen. Accept $2x + 8y = 1$ or equivalent equations with $\pm 2x$ and $\pm 8y$	
(c)	Trial and improvement requires sight of first equation. 1st M1 for attempt to form a suitable equation in one variable. Do not penalise poor use of brackets etc. 2nd M1 for simplifying their equation to a 3TQ and attempting to solve. May be $\Rightarrow$ by $x = -8$ 1st A1 for $x = -8$ (ignore a second value). If found y first allow ft for x if $x < 0$ 2nd A1ft for $y = \frac{17}{8}$ Follow through their x value in line or curve provided answer is > 0 This second A1 is dependent on <u>both</u> M marks	

Question Number	Scheme	Marks
73.	$\dfrac{dy}{dx} = 10x^4 - 3x^{-4}$ or $10x^4 - \dfrac{3}{x^4}$	M1 A1 A1 (3)
	<u>Notes</u> M1: Attempt to differentiate $x^n \to x^{n-1}$ (for any of the 3 terms) i.e. ax^4 or ax^{-4}, where a is any non-zero constant or the 7 differentiated to give 0 is sufficient evidence for M1 1st A1: One correct (non-zero) term, possibly unsimplified. 2nd A1: Fully correct **simplified** answer.	

Question Number	Scheme	Marks
74. **(a)**	 Shape (cubic in this orientation) **Touching** x-axis at **–3** **Crossing** at **–1** on x-axis Intersection at **9** on y-axis	B1 B1 B1 B1 (4)
(b)	$y=(x+1)(x^2+6x+9)=x^3+7x^2+15x+9$ or equiv. (possibly unsimplified) Differentiates their polynomial correctly – may be unsimplified $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2+14x+15$ **(*)**	B1 M1 A1 cso (3)
(c)	At $x=-5$: $\frac{\mathrm{d}y}{\mathrm{d}x}=75-70+15=20$ At $x=-5$: $y=-16$ $y-("-16")="20"(x-(-5))$ or y = "20x" + c with (-5, -"16") used to find c $y=20x+84$	B1 B1 M1 A1 (4)
(d)	Parallel: $3x^2+14x+15="20"$ $(3x-1)(x+5)=0$ $x=\ldots$ $x=\frac{1}{3}$	M1 M1 A1 (3) **14**
	Notes (a) Crossing at –3 is B0. Touching at –1 is B0 (b) M: This needs to be correct differentiation here A1: Fully correct simplified answer. (c) M: If the –5 and "-16" are the wrong way round or – omitted the M mark can still be given if a correct formula is seen, (e.g. $y-y_1=m(x-x_1)$) otherwise M0. m should be numerical and not 0 or infinity and should not have involved negative reciprocal. (d) 1st M: Putting the derivative expression equal to their value for gradient 2nd M: Attempt to solve quadratic (see notes) This may be implied by correct answer.	

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Question Number	Scheme	Marks
75.		
(a)	$\left(\frac{dy}{dx}=\right)\frac{3}{2}x^2-\frac{27}{2}x^{\frac{1}{2}}-8x^{-2}$	M1A1A1A1 (4)
(b)	$x=4 \Rightarrow y=\frac{1}{2}\times 64-9\times 2^3+\frac{8}{4}+30$	M1
	$= 32 - 72 + 2 + 30 \qquad = \underline{-8}$ *	A1cso (2)
(c)	$x=4 \Rightarrow y'=\frac{3}{2}\times 4^2-\frac{27}{2}\times 2-\frac{8}{16}$	M1
	$= 24-27-\frac{1}{2} = \qquad -\frac{7}{2}$	A1
	Gradient of the normal $= -1\div"\frac{7}{2}"$	M1
	Equation of normal: $y--8=\frac{2}{7}(x-4)$	M1A1ft
	$\underline{7y-2x+64=0}$	A1 (6) **12**

Question Number	Scheme	Marks
	Notes	
(a)	1st M1 for an attempt to differentiate $x^n \to x^{n-1}$ 1st A1 for one correct term in x 2nd A1 for 2 terms in x correct 3rd A1 for all correct x terms. No 30 term and no $+c$.	
(b)	M1 for substituting x = 4 into y = and attempting $4^{\frac{3}{2}}$ A1 note this is a printed answer	
(c)	1st M1 Substitute x = 4 into y' (allow slips) A1 Obtains –3.5 or equivalent 2nd M1 for correct use of the perpendicular gradient rule using their gradient. (May be slip doing the division) Their gradient must have come from y' 3rd M1 for an attempt at equation of tangent or normal at *P* 2nd A1ft for correct use of their changed gradient to find **normal** at *P*. Depends on 1st, 2nd and 3rd Ms 3rd A1 for any equivalent form with integer coefficients	

Question Number	Scheme		Marks
76.	$\dfrac{3x^2+2}{x}=3x+2x^{-1}$ $(y'=)24x^2, -2x^{-\frac{1}{2}}, +3-2x^{-2}$ $\left[24x^2-2x^{-\frac{1}{2}}+3-2x^{-2}\right]$		M1 A1 M1 A1 A1A1 **6**
	Notes		
	1st M1 for attempting to divide(one term correct) 1st A1 for both terms correct on the same line, accept $3x^1$ for $3x$ or $\dfrac{2}{x}$ for $2x^{-1}$ These first two marks may be implied by a correct differentiation at the end. 2nd M1 for an attempt to differentiate $x^n \to x^{n-1}$ for at least one term of their expression "Differentiating" $\dfrac{3x^2+2}{x}$ and getting $\dfrac{6x}{1}$ is M0 2nd A1 for $24x^2$ only 3rd A1 for $-2x^{-\frac{1}{2}}$ allow $\dfrac{-2}{\sqrt{x}}$. Must be simplified to this, not e.g. $\dfrac{-4}{2}x^{-\frac{1}{2}}$ 4th A1 for $3-2x^{-2}$ allow $\dfrac{-2}{x^2}$. Both terms needed. Condone $3+(-2)x^{-2}$ If "$+c$" is included then they lose this final mark They do not need one line with all terms correct for full marks. Award marks when first seen in this question and apply ISW. Condone a mixed line of some differentiation and some division e.g. $24x^2-4x^{\frac{1}{2}}+3x+2x^{-1}$ can score 1st M1A1 and 2nd M1A1		
Quotient /Product Rule	$\dfrac{x(6x)-(3x^2+2)\times 1}{x^2}$ or $6x(x^{-1})+(3x^2+2)(-x^{-2})$ $\dfrac{3x^2-2}{x^2}$ or $3-\dfrac{2}{x^2}$ (o.e.)	1st M1 for an attempt: $\frac{P-Q}{x^2}$ or $R+(-S)$ with one of P,Q or R,S correct. 1st A1 for a correct expression 4th A1 same rules as above	

Question Number	Scheme	Marks
77. **(a)**	$(y=)\frac{3x^2}{2}-\frac{5x^{\frac{1}{2}}}{\frac{1}{2}}-2x\quad(+c)$	M1A1A1
	$\mathrm{f}(4)=5\quad\Rightarrow 5=\frac{3}{2}\times16-10\times2-8+c$	M1
	$\underline{c=9}$	A1 (5)
	$\left[\mathrm{f}(x)=\frac{3}{2}x^2-10x^{\frac{1}{2}}-2x+9\right]$	
(b)	$m=3\times4-\frac{5}{2}-2\quad\left(=7.5\text{ or }\frac{15}{2}\right)$	M1
	Equation is: $y-5=\frac{15}{2}(x-4)$	M1A1
	$\underline{2y-15x+50=0}$ o.e.	A1 (4) **(9marks)**
(a)	1st M1 for an attempt to integrate $x^n\to x^{n+1}$ 1st A1 for at least 2 correct terms in x (unsimplified) 2nd A1 for all 3 terms in x correct (condone missing $+c$ at this point). Needn't be simplified 2nd M1 for using the point (4, 5) to form a linear equation for c. Must use x = 4 and y = 5 and have no x term and the function must have "changed". 3rd A1 for c = 9. The final expression is not required.	
(b)	1st M1 for an attempt to evaluate $\mathrm{f}'(4)$. Some correct use of x = 4 in $\mathrm{f}'(x)$ but condone slips. They must therefore have at least 3×4 or $-\frac{5}{2}$ and clearly be using $\mathrm{f}'(x)$ with x = 4. Award this mark wherever it is seen. 2nd M1 for using their value of m [or their $-\frac{1}{m}$] (provided it clearly comes from using x = 4 in f'(x)) to form an equation of the line through (4,5)). Allow this mark for an attempt at a normal or tangent. Their m must be numerical. Use of $y = mx + c$ scores this mark when c is found. 1st A1 for any correct expression for the equation of the line 2nd A1 for any correct equation with integer coefficients. An "=" is required. e.g. $2y = 15x - 50$ etc as long as the equation is correct and has integer coefficients.	
Normal	Attempt at normal can score both M marks in (b) but A0A0	

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Question number	Scheme	Marks
78	$x^4 \to kx^3$ or $x^{1/3} \to kx^{-2/3}$ or $3 \to 0$ (k a non-zero constant)	M1
	$\left(\frac{dy}{dx} =\right)$ $4x^3$.......... , with '3' differentiated to zero (or 'vanishing')	A1
	$\left(\frac{dy}{dx} =\right)$ .... $+\frac{1}{3}x^{-2/3}$ or equivalent, e.g. $\frac{1}{3\sqrt[3]{x^2}}$ or $\frac{1}{3\left(\sqrt[3]{x}\right)^2}$	A1
		[3]
	1st A1 requires $4x^3$, <u>and</u> 3 differentiated to zero. Having '$+C$' loses the 1st A mark. Terms not added, but otherwise correct, e.g. $4x^3$, $\frac{1}{3}x^{-2/3}$ loses the 2nd A mark.	

Question number	Scheme	Marks
79	(a) $y=\dfrac{x^2-5x-24}{x}=x-5-24x^{-1}$ (or equiv., e.g. $x+3-8-\dfrac{24}{x}$)	M1 A1
	$\dfrac{dy}{dx}=1+24x^{-2}$ or $\dfrac{dy}{dx}=1+\dfrac{24}{x^2}$	M1 A1 (4)
	(b) $x=2$: $y=-15$ Allow if seen in part (a).	B1
	$\left(\dfrac{dy}{dx}=\right)$ $1+\dfrac{24}{4}=7$ Follow-through from candidate's <u>non-constant</u> $\dfrac{dy}{dx}$. This must be simplified to a "single value".	B1ft
	$y+15=7(x-2)$ (or equiv., e.g. $y=7x-29$) Allow $\dfrac{y+15}{x-2}=7$	M1 A1 (4) **[8]**
	(a) 1st M: Mult. out to get $x^2+bx+c,\ b\neq 0,\ c\neq 0$ <u>and</u> dividing by x (<u>not</u> x^2). Obtaining one correct term, e.g. x…….. is sufficient evidence of a division attempt. 2nd M: <u>Dependent on the 1st M</u>: Evidence of $x^n \to kx^{n-1}$ for one x term (i.e. not just the constant term) is sufficient). Note that mark is <u>not</u> given if, for example, the numerator and denominator are differentiated separately. A mistake in the 'middle term', e.g. $x+5-24x^{-1}$, does not invalidate the 2nd A mark, so M1 A0 M1 A1 is possible. (b) B1ft: For evaluation, using $x=2$, of their $\dfrac{dy}{dx}$, even if unlabelled or called y. M: For the equation, in any form, of a straight line through (2, '−15') with candidate's $\dfrac{dy}{dx}$ value as gradient. Alternative is to use (2, '−15') in $y=mx+c$ to <u>find a value</u> for c, in which case $y=7x+c$ leading to $c=-29$ is sufficient for the A1). . (See general principles for straight line equations at the end of the scheme). Final A: 'Unsimplified' forms are acceptable, but… $y-(-15)=7(x-2)$ is A0 (unresolved 'minus minus').	

Question Number	Scheme	Marks
80	$\frac{dy}{dx} = 6x^2 - 6x^{-3}$	M1 A1 A1 (3) **[3]**
	M1 for an attempt to differentiate $x^n \to x^{n-1}$ 1st A1 for $6x^2$ 2nd A1 for $-6x^{-3}$ or $-\frac{6}{x^3}$ Condone $+ -6x^{-3}$ here. Inclusion of $+c$ scores A0 here.	

Question Number	Scheme	Marks
81 (a)	$\left[(3-4\sqrt{x})^2=\right]9-12\sqrt{x}-12\sqrt{x}+(-4)^2x$	M1
	$9x^{-\frac{1}{2}}+16x^{\frac{1}{2}}-24$	A1, A1 (3)
(b)	$f'(x)=-\frac{9}{2}x^{-\frac{3}{2}},\ +\frac{16}{2}x^{-\frac{1}{2}}$	M1 A1, A1ft
		(3)
(c)	$f'(9)=-\frac{9}{2}\times\frac{1}{27}+\frac{16}{2}\times\frac{1}{3}=-\frac{1}{6}+\frac{16}{6}=\frac{5}{2}$	M1 A1 (2)
		[8]
(a)	M1 for an attempt to expand $(3-4\sqrt{x})^2$ with at least 3 terms correct- as printed or better Or $9-k\sqrt{x}+16x$ $(k\neq 0)$. See also the MR rule below 1st A1 for their coefficient of $\sqrt{x}$ = 16. Condone writing $(\pm)9x^{(\pm)\frac{1}{2}}$ instead of $9x^{-\frac{1}{2}}$ 2nd A1 for B = - 24 or their constant term = - 24	
(b)	M1 for an attempt to differentiate an x term $x^n\to x^{n-1}$ 1st A1 for $-\frac{9}{2}x^{-\frac{3}{2}}$ and their constant B differentiated to zero. NB $-\frac{1}{2}\times 9x^{-\frac{3}{2}}$ is A0 2nd A1ft follow through their $Ax^{\frac{1}{2}}$ but can be scored without a value for A, i.e. for $\frac{A}{2}x^{-\frac{1}{2}}$	
(c)	M1 for some correct substitution of x = 9 in their expression for $f'(x)$ including an attempt at $(9)^{\pm\frac{k}{2}}$ (k odd) somewhere that leads to some appropriate multiples of $\frac{1}{3}$ or 3 A1 accept $\frac{15}{6}$ or any exact equivalent of 2.5 e.g. $\frac{45}{18},\frac{135}{54}$ or even $\frac{67.5}{27}$ Misread (MR) Only allow MR of the form $\frac{(3-k\sqrt{x})^2}{\sqrt{x}}$ N.B. Leads to answer in (c) of $\frac{k^2-1}{6}$ Score as M1A0A0, M1A1A1ft, M1A1ft	

Question Number	Scheme	Marks
82 (a)	$x = 2$: $y = 8 - 8 - 2 + 9 = 7$ (*)	B1 (1)
(b)	$\dfrac{dy}{dx} = 3x^2 - 4x - 1$	M1 A1
	$x = 2$: $\dfrac{dy}{dx} = 12 - 8 - 1 (= 3)$	A1ft
	$y - 7 = 3(x - 2)$, $\underline{y = 3x + 1}$	M1, <u>A1</u> (5)
(c)	$m = -\dfrac{1}{3}$ (for $-\dfrac{1}{m}$ with their m)	B1ft
	$3x^2 - 4x - 1 = -\dfrac{1}{3}$, $9x^2 - 12x - 2 = 0$ or $x^2 - \dfrac{4}{3}x - \dfrac{2}{9} = 0$ (o.e.)	M1, A1
	$\left(x = \dfrac{12 + \sqrt{144 + 72}}{18}\right)\left(\sqrt{216} = \sqrt{36}\sqrt{6} = 6\sqrt{6}\right)$ or $(3x - 2)^2 = 6 \rightarrow 3x = 2 \pm \sqrt{6}$	M1
	$x = \dfrac{1}{3}\left(2 + \sqrt{6}\right)$ (*)	A1cso (5)
		[11]
(a)	B1 there must be a clear attempt to substitute $x = 2$ leading to 7 e.g. $2^3 - 2 \times 2^2 - 2 + 9 = 7$	
(b)	1^{st} M1 for an attempt to differentiate with at least one of the given terms fully correct. 1^{st} A1 for a fully correct expression 2^{nd} A1ft for sub. $x = 2$ in <u>their</u> $\dfrac{dy}{dx}(\neq y)$ accept for a correct expression e.g. $3 \times (2)^2 - 4 \times 2 - 1$ 2^{nd} M1 for use of their "3" (provided it comes from their $\dfrac{dy}{dx}(\neq y)$ and x=2) to find equation of tangent. Alternative is to use (2, 7) in $y = mx + c$ to <u>find a value</u> for c. Award when $c = \ldots$ is seen. **No attempted use of $\dfrac{dy}{dx}$ in (b) scores 0/5**	
(c)	1^{st} M1 for forming an equation from their $\dfrac{dy}{dx}(\neq y)$ and their $-\dfrac{1}{m}$ (must be changed from m) 1^{st} A1 for a correct 3TQ all terms on LHS (condone missing =0) 2^{nd} M1 for proceeding to x =… or $3x$ = … by formula or completing the square for a 3TQ. Not factorising. Condone <u>+</u> 2^{nd} A1 for proceeding to given answer with no incorrect working seen. Can still have <u>+</u>.	
ALT	<u>Verify (for M1A1M1A1)</u> 1^{st} M1 for attempting to square need $\geq$ 3 correct values in $\frac{4+6+4\sqrt{6}}{9}$, 1^{st} A1 for $\frac{10+4\sqrt{6}}{9}$ 2^{nd} M1 Dependent on 1^{st} M1 in this case for substituting in all terms of their $\dfrac{dy}{dx}$ 2^{nd} A1cso for cso <u>with a full comment</u> e.g. "the x co-ord of Q is ..."	

Question Number	Scheme	Marks
83 (a)	$2x^{3/2}$ or $p=\frac{3}{2}$ (<u>Not</u> $2x\sqrt{x}$)	B1
	$-x$ or $-x^1$ or $q=1$	B1 (2)
(b)	$\left(\frac{dy}{dx}=\right)$ $20x^3+2\times\frac{3}{2}x^{1/2}-1$	M1
	$=\underline{20x^3+3x^{\frac{1}{2}}-1}$	A1A1ftA1ft (4) **[6]**
(a)	1st B1 for $p = 1.5$ or exact equivalent 2nd B1 for $q = 1$	
(b)	M1 for an attempt to differentiate $x^n \to x^{n-1}$ (for any of the 4 terms) 1st A1 for $20x^3$ (the -3 must 'disappear') 2nd A1ft for $3x^{\frac{1}{2}}$ or $3\sqrt{x}$. Follow through their p but they must be differentiating $2x^p$, where p is a <u>fraction</u>, and the coefficient must be simplified if necessary. 3rd A1ft for -1 (<u>not</u> the unsimplified $-x^0$), or follow through for correct differentiation of their $-x^q$ (i.e. coefficient of x^q is -1). If ft is applied, the coefficient must be simplified if necessary. 'Simplified' coefficient means $\frac{a}{b}$ where a and b are integers with no common factors. Only a single + or – sign is allowed (e.g. – – must be replaced by +). If there is a 'restart' in part (b) it can be marked independently of part (a), but marks for part (a) cannot be scored for work seen in (b). <u>Multiplying</u> by $\sqrt{x}$: (assuming this is a restart) e.g. $y=5x^4\sqrt{x}-3\sqrt{x}+2x^2-x^{3/2}$ $\left(\frac{dy}{dx}=\right)\frac{45}{2}x^{7/2}-\frac{3}{2}x^{-1/2}+4x-\frac{3}{2}x^{1/2}$ scores M1 A0 A0 (p not a fraction) A1ft. <u>Extra term</u> included: This invalidates the final mark. e.g. $y=5x^4-3+2x^2-x^{3/2}-x^{1/2}$ $\left(\frac{dy}{dx}=\right)20x^3+4x-\frac{3}{2}x^{1/2}-\frac{1}{2}x^{-1/2}$ scores M1 A1 A0 (p not a fraction) A0. <u>Numerator and denominator differentiated separately</u>: For this, neither of the last two (ft) marks should be awarded. <u>Quotient/product rule</u>: Last two terms must be correct to score the last 2 marks. (If the M mark has not already been earned, it can be given for the quotient/product rule attempt.)	

Question Number	Scheme	Marks
84 (a)	$\left(\frac{dy}{dx}=\right) -4+8x^{-2}$ (4 or $8x^{-2}$ for M1… sign can be wrong)	M1A1
	$x=2 \Rightarrow \quad m=-4+2=-2$	M1
	$y=9-8-\frac{8}{2}=-3$ The first 4 marks <u>could</u> be earned in part (b)	B1
	Equation of tangent is: $y+3=-2(x-2) \rightarrow y=1-2x$ (*)	M1 A1cso (6)
(b)	Gradient of normal = $\frac{1}{2}$	B1ft
	Equation is: $\frac{y+3}{x-2}=\frac{1}{2}$ or better equivalent, e.g. $y=\frac{1}{2}x-4$	M1A1 (3)
(c)	$(A:)\ \frac{1}{2}$, $(B:)\ 8$	B1, B1
	Area of triangle is: $\frac{1}{2}(x_B \pm x_A)\times y_P$ with values for all of x_B, x_A and y_P	M1
	$\frac{1}{2}\left(8-\frac{1}{2}\right)\times 3 = \frac{45}{4}$ or 11.25	A1 (4) **[13]**

(a)

1st M1 for 4 or $8x^{-2}$ (ignore the signs).

1st A1 for both terms correct (including signs).

2nd M1 for substituting $x = 2$ into their $\frac{dy}{dx}$ (must be different from their y)

B1 for $y_P=-3$, but not if clearly found from the given equation of the <u>tangent</u>.

3rd M1 for attempt to find the equation of tangent at P, follow through their m and y_P.
Apply general principles for straight line equations (see end of scheme).
<u>NO DIFFERENTIATION ATTEMPTED</u>: Just assuming $m=-2$ at this stage is M0

2nd A1cso for correct work leading to printed answer (allow equivalents with $2x$, y, and 1 terms… such as $2x+y-1=0$).

(b)

B1ft for correct use of the perpendicular gradient rule. Follow through their m, but if $m \neq -2$ there must be clear evidence that the m is thought to be the gradient of the tangent.

M1 for an attempt to find normal at P using their changed gradient and their y_P.
Apply general principles for straight line equations (see end of scheme).

A1 for any correct form as specified above (correct answer only).

(c)

1st B1 for $\frac{1}{2}$ and 2nd B1 for 8.

M1 for a full method for the area of triangle ABP. Follow through their x_A, x_B and their y_P, but the mark is to be awarded 'generously', condoning sign errors..
The final answer must be positive for A1, with negatives in the working condoned.

<u>Determinant</u>: Area = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & -3 & 1 \\ 0.5 & 0 & 1 \\ 8 & 0 & 1 \end{vmatrix} = \ldots$ (Attempt to multiply out required for M1)

<u>Alternative</u>: $AP=\sqrt{(2-0.5)^2+(-3)^2}$, $BP=\sqrt{(2-8)^2+(-3)^2}$, Area = $\frac{1}{2}AP\times BP=\ldots$ M1

<u>Intersections with y-axis instead of x-axis</u>: Only the M mark is available B0 B0 M1 A0.

Question number	Scheme	Marks
85. (a)	$[\,f'(x) = \,]\; 3 + 3x^2$	M1A1 (2)
(b)	$3 + 3x^2 = 15$ and start to try and simplify	M1
	$x^2 = k \to x = \sqrt{k}$ (ignore $\pm$)	M1
	$x = 2$ (ignore $x = -2$)	A1 (3)
		5
(a)	M1 for attempting to differentiate $x^n \to x^{n-1}$. Just one term will do. A poor integration attempt that gives $3x^2 + ...$ (or similar) scores M0A0 A1 for a fully correct expression. Must be 3 not $3x^0$. If there is a + c they score A0.	
(b)	1st M1 for forming a correct equation and trying to rearrange their $f'(x) = 15$ e.g. collect terms. e.g. $3x^2 = 15 - 3$ or $1 + x^2 = 5$ or even $3+3x^2 \to 3x^2 = \frac{15}{3}$ or $3x^{-1} + 3x^2 = 15 \to 6x = 15$ (i.e algebra can be awful as long as they try to collect terms in their $f'(x) = 15$ equation) 2nd M1 this is dependent upon their $f'(x)$ being of the form $a + bx^2$ and attempting to solve $a + bx^2 = 15$ For correct processing leading to $x = \ldots$ Can condone arithmetic slips but processes should be correct so e.g. $3 + 3x^2 = 15 \to 3x^2 = \frac{15}{3} \to x = \frac{\sqrt{15}}{3}$ scores M1M0A0 $3 + 3x^2 = 15 \to 3x^2 = 12 \to x^2 = 9 \to x = 3$ scores M1M0A0 $3 + 3x^2 = 15 \to 3x^2 = 12 \to 3x = \sqrt{12} \to x = \frac{\sqrt{12}}{3}$ scores M1M0A0	

Question number	Scheme	Marks
86. (a)	$\left[\frac{dy}{dx}=\right]3kx^2-2x+1$	M1A1 (2)
(b)	Gradient of line is $\frac{7}{2}$	B1
	When $x=-\frac{1}{2}$: $3k\times\left(\frac{1}{4}\right)-2\times\left(-\frac{1}{2}\right)+1,=\frac{7}{2}$	M1, M1
	$\frac{3k}{4}=\frac{3}{2}\Rightarrow k=2$	A1 (4)
(c)	$x=-\frac{1}{2}\Rightarrow y=k\times\left(-\frac{1}{8}\right)-\left(\frac{1}{4}\right)-\frac{1}{2}-5,=-6$	M1, A1 (2)
		8

(a) M1 for attempting to differentiate $x^n\to x^{n-1}$ (or -5 going to 0 will do)

A1 all correct. A "+ c" scores A0

(b) B1 for $m=\frac{7}{2}$. Rearranging the line into $y=\frac{7}{2}x+c$ does not score this mark until you are sure they are using $\frac{7}{2}$ as the gradient of the line or state $m=\frac{7}{2}$

1st M1 for substituting $x=-\frac{1}{2}$ into their $\frac{dy}{dx}$, some correct substitution seen

2nd M1 for forming a suitable equation in k and attempting to solve leading to $k=\ldots$

Equation must use their $\frac{dy}{dx}$ and <u>their gradient of line</u>. Assuming the gradient is 0 or 7 scores M0 unless they have clearly stated that this is the gradient of the line.

A1 for k = 2

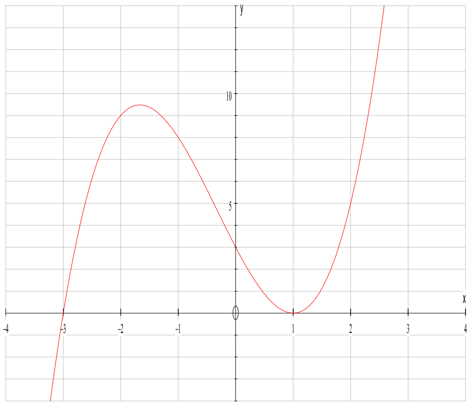
(c) M1 for attempting to substitute their k (however it was found or can still be a letter) and $x=-\frac{1}{2}$ into y (some correct substitution)

A1 for - 6

Question number	Scheme	Marks
87.	(a) $\left(2x^{-\frac{1}{2}}+3x^{-1}\right)$ $\quad p=-\frac{1}{2}, \quad q=-1$	B1, B1 (2)
	(b) $\left(y=5x-7+2x^{-\frac{1}{2}}+3x^{-1}\right)$	
	$\left(\frac{dy}{dx}=\right)$ $\quad 5 \quad \left(\text{or } 5x^0\right)$ $\quad$ ($5x-7$ correctly differentiated)	B1
	Attempt to differentiate either $2x^p$ with a fractional p, giving kx^{p-1} $(k\neq 0)$, (the fraction p could be in decimal form) or $3x^q$ with a negative q, giving kx^{q-1} $(k\neq 0)$.	M1
	$\left(-\frac{1}{2}\times 2x^{-\frac{3}{2}}-1\times 3x^{-2}=\right)$ $\quad -x^{-\frac{3}{2}},\ -3x^{-2}$	A1ft, A1ft (4)
		6
	(b): N.B. It is possible to 'start again' in (b), so the p and q may be different from those seen in (a), but note that the M mark is for the attempt to differentiate $\underline{\underline{2}}x^p$ or $\underline{\underline{3}}x^q$. However, marks for part (a) <u>cannot</u> be earned in part (b). 1st A1ft: ft their $2x^p$, but p must be a fraction and coefficient must be simplified (the fraction p could be in decimal form). 2nd A1ft: ft their $3x^q$, but q must be negative and coefficient must be simplified. 'Simplified' coefficient means $\frac{a}{b}$ where a and b are integers with no common factors. Only a single + or – sign is allowed (e.g. – – must be replaced by +). Having $+C$ loses the B mark.	

EXPERT TUITION

Question number	Scheme	Marks
88.	(a) $4x \to kx^2$ or $6\sqrt{x} \to kx^{3/2}$ or $\frac{8}{x^2} \to kx^{-1}$ (k a non-zero constant)	M1
	$f(x) = 2x^2, \ -4x^{3/2}, \ -8x^{-1}$ (+ C) (+ C not required)	A1, A1, A1
	At x = 4, y = 1: $1 = (2\times16) - \left(4\times4^{3/2}\right) - \left(8\times4^{-1}\right) + C$ <u>Must be in part (a)</u>	M1
	$C = 3$	A1 (6)
	(b) $f'(4) = 16 - (6\times2) + \frac{8}{16} = \frac{9}{2} \ (= m)$ M: Attempt $f'(4)$ with the <u>given</u> f'. <u>Must be in part (b)</u>	M1
	Gradient of normal is $-\frac{2}{9}\left(= -\frac{1}{m}\right)$ M: Attempt perp. grad. rule. Dependent on the use of their $f'(x)$	M1
	Eqn. of normal: $y - 1 = -\frac{2}{9}(x-4)$ (or any equiv. form, e.g. $\frac{y-1}{x-4} = -\frac{2}{9}$)	M1 A1 (4)
	Typical answers for A1: $\left(y = -\frac{2}{9}x + \frac{17}{9}\right)$ $(2x + 9y - 17 = 0)$ $(y = -0.\dot{2}x + 1.\dot{8})$	
	Final answer: gradient $-\frac{1}{(9/2)}$ or $-\frac{1}{4.5}$ is A0 (but all M marks are available).	
		10
	(a) The first 3 A marks are awarded in the order shown, and the terms must be simplified. 'Simplified' coefficient means $\frac{a}{b}$ where a and b are integers with no common factors. Only a single + or – sign is allowed (e.g. + – must be replaced by –). 2nd M: Using x = 4 <u>and</u> y = 1 (<u>not</u> y = 0) to form an eqn in C. (No C is M0) (b) 2nd M: Dependent upon use of their $f'(x)$. 3rd M: eqn. of a straight line through (4, 1) with any gradient except 0 or ∞. <u>Alternative for 3rd M:</u> Using (4, 1) in $y = mx + c$ to <u>find a value</u> of c, but an equation (general or specific) must be seen. Having coords the <u>wrong way round</u>, e.g. $y - 4 = -\frac{2}{9}(x-1)$, loses the 3rd M mark <u>unless</u> a correct general formula is seen, e.g. $y - y_1 = m(x - x_1)$. N.B. The A mark is scored for <u>any</u> form of the correct equation… be prepared to apply isw if necessary.	

Question number	Scheme	Marks
89.	(a) 	
	Shape (drawn anywhere)	B1
	Minimum at (1, 0) (perhaps labelled 1 on x-axis)	B1
	$(-3, 0)$ (or -3 shown on –ve x-axis)	B1
	(0, 3) (or 3 shown on +ve y-axis)	B1 (4)
	N.B. The max. can be anywhere.	
	(b) $y = (x+3)(x^2 - 2x + 1)$ — Marks can be awarded if this is seen in part (a)	M1
	$= x^3 + x^2 - 5x + 3$ $(k = 3)$	A1cso (2)
	(c) $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2x - 5$	M1 A1
	$3x^2 + 2x - 5 = 3$ or $3x^2 + 2x - 8 = 0$	M1
	$(3x-4)(x+2) = 0$ $x = \ldots$	M1
	$x = \frac{4}{3}$ (or exact equiv.) , $x = -2$	A1, A1 (6)
		12
	(a) The individual marks are independent, <u>but</u> the 2nd, 3rd and 4th B's are dependent upon a sketch having been attempted. B marks for coordinates: Allow (0, 1), etc. (coordinates the wrong way round) <u>if</u> marked in the correct place on the sketch. (b) M: Attempt to multiply out $(x-1)^2$ and write as a product with $(x+3)$, or attempt to multiply out $(x+3)(x-1)$ and write as a product with $(x-1)$, or attempt to expand $(x+3)(x-1)(x-1)$ directly (at least 7 terms). The $(x-1)^2$ or $(x+3)(x-1)$ expansion must have 3 (or 4) terms, so should not, for example, be just $x^2 + 1$. A: It is not necessary to state explicitly '$k = 3$'. Condone missing brackets if the intention seems clear and a fully correct expansion is seen. (c) 1st M: Attempt to differentiate (correct power of x in at least one term). 2nd M: Setting their derivative equal to 3. 3rd M: Attempt to solve a 3-term quadratic based on their derivative. The equation <u>could</u> come from $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. N.B. After an incorrect k value in (b), full marks are still possible in (c).	

EXPERT TUITION

Question Number	Scheme	Marks
90. (a)	$\frac{dy}{dx} = 70x - 35x^{\frac{3}{2}}$	M1A1
	Put $\frac{dy}{dx} = 0$ to give $70x - 35x^{\frac{3}{2}} = 0$ *so* $x^{\frac{1}{2}} = 2$	M1
	$x = 4$	A1
	$y = 112$	A1 **(5)**
(b) (Way 1)	When $y = 0$, $35x^2 = 14x^{\frac{5}{2}}$ *and* $x^{\frac{1}{2}} = \frac{35}{14}$ *or* $5 = 2\sqrt{x}$ *so* $\sqrt{x} = \frac{5}{2}$	M1
	$x = \frac{25}{4}$	A1 **(2)**
(b) (Way 2)	When $y = 0$, $35x^2 = 14x^{\frac{5}{2}}$ *so* $1225x^4 = 196x^5$ *or* $5 = 2\sqrt{x}$ *so* $25 = 4x$	M1
	$x = \frac{25}{4}$ or $x = \frac{1225}{196}$	A1 **(2)**
		[7]

Notes

(a)

M1: Attempt at differentiation after multiplying out - may be awarded for $70x$ term correct

(If product rule is used it must be of correct form i.e. $\frac{\mathrm{d}y}{\mathrm{d}x} = 7x^2\left(-2kx^{k-1}\right) + 14x(5-2x^k)$)

A1: the derivative must be completely correct but may be unsimplified

For product rule this is $\frac{\mathrm{d}y}{\mathrm{d}x} = 7x^2\left(-x^{-\frac{1}{2}}\right) + 14x(5-2\sqrt{x})$

M1: uses derivative = 0 to find $x^k =$ or $x =$ with correct work for their equation (even without fractional powers)

A1: obtains $x = 4$ then

A1: for $y = 112$ (may be credited if seen in part (a) or in part(c))

(b)

Way 1 (Dividing first)

M1: Puts $y = 0$ and obtains expression of the form $x^k = A$ (where k is not equal to 1) after correct algebra for their equation (may be a sign slip)

A1: Obtains $x = 6.25$ or equivalent correct answer

(b)

Way 2 (dealing with fractional power first i.e. Squaring)

M1: Puts $y = 0$ and squares each term correctly for their equation obtaining expression of the form $A^2 x^m = B^2 x^n$ after correct algebra

A1: Obtains $x = 6.25$ or equivalent correct answer

Question Number	Scheme		Marks
91.	$\frac{dy}{dx} = 12x^2 + 18x - 30$		M1
	Either	Or	
	Substitute $x = 1$ to give $\frac{dy}{dx} = 12 + 18 - 30 = 0$	Solve $\frac{dy}{dx} = 12x^2 + 18x - 30 = 0$ to give $x =$	A1
	So turning point (all correct work so far)	Deduce $x = 1$ from correct work	A1cso (3)

	Notes
	M1: Attempt at differentiation - all powers reduced by 1 with $8 \to 0$. A1: the derivative must be correct and uses derivative = 0 to find x or substitutes $x = 1$ to give 0. Ignore any reference to the other root (−5/2) for this mark. A1cso: obtains $x = 1$ from correct work, or deduces turning point (if substitution used – may be implied by a preamble e.g. dy/dx =0 at T.P.) **N.B**. If their factorisation or their second root is incorrect then award A0cso. If however their factorisation/roots are correct, it is not necessary for them to comment that −2.5 is outside the range given.

Question Number	Scheme		Marks
92. (a)	Area$(FEA) = \frac{1}{2}x^2\left(\frac{2\pi}{3}\right); = \frac{\pi x^2}{3}$	$\frac{1}{2}x^2 \times \left(\frac{2\pi}{3}\right)$ or $\frac{120}{360}\times \pi x^2$ simplified or un-simplified	M1
		$\frac{\pi x^2}{3}$	A1
			[2]
	Parts (b) and (c) may be marked together		
(b)	$\{A =\}\ \frac{1}{2}x^2\sin 60^\circ + \frac{1}{3}\pi x^2 + 2xy$	Attempt to sum 3 areas (at least one correct)	M1
		Correct expression for at least two terms of A	A1
	$1000 = \frac{\sqrt{3}x^2}{4} + \frac{\pi x^2}{3} + 2xy \Rightarrow y = \frac{500}{x} - \frac{\sqrt{3}x}{8} - \frac{\pi x}{6}$ $\Rightarrow \underline{y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)}$ *	Correct proof.	A1 *
			[3]
(c)	$\{P =\}\ x + x\theta + y + 2x + y\ \left\{= 3x + \frac{2\pi x}{3} + 2y\right\}$	Correct expression in x and y for their θ measured in rads	B1ft
	$...2y = \quad + 2\left(\frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)\right)$	Substitutes expression from (b) into y term.	M1
	$P = 3x + \frac{2\pi x}{3} + \frac{1000}{x} - \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x \Rightarrow P = \frac{1000}{x} + 3x + \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x$		
	$\Rightarrow \underline{P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)}$ *	Correct proof.	A1 *
			[3]
	Parts (d) and (e) should be marked together		
(d)	$\frac{dP}{dx} = -1000x^{-2} + \frac{4\pi + 36 - 3\sqrt{3}}{12}; = 0$	$\frac{1000}{x} \to \frac{\pm\lambda}{x^2}$	M1
		Correct differentiation (need not be simplified).	A1;
		Their $P' = 0$	M1
	$\Rightarrow\ x = \sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}\ (= 16.63392808...)$	$\sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ or awrt 17 (may be implied)	A1
	$\left\{P = \frac{1000}{(16.63...)} + \frac{(16.63...)}{12}\left(4\pi + 36 - 3\sqrt{3}\right)\right\} \Rightarrow P = 120.236..\ (\text{m})$	awrt 120	A1
			[5]
(e)	$\frac{d^2P}{dx^2} = \frac{2000}{x^3} > 0 \Rightarrow$ Minimum	Finds P'' **and** considers sign.	M1
		$\frac{2000}{x^3}$ (need not be simplified) and > 0 and conclusion. Only follow through on a correct P'' and x in range $10 < x < 25$.	A1ft
			[2]
			15

		Question 92 Notes
(a)	**M1**	Attempts to use Area$(FEA) = \frac{1}{2}x^2 \times \frac{2\pi}{3}$ (using radian angle) or $\frac{120}{360} \times \pi x^2$ (using angle in degrees)
	A1	$\frac{\pi x^2}{3}$ cao (Must be simplified and be their answer in part (a)) Answer only implies M1A1. N.B. Area$(FEA) = \frac{1}{2}x^2 \times 120$ is awarded M0A0
(b)	**M1**	An attempt to sum 3 " areas" consisting of rectangle, triangle and sector (allow slips even in dimensions) but **one area** should be correct
	1st A1	Correct expression for **two** of the **three** areas listed above. Accept any correct equivalents e.g. two correct from $\frac{1}{2}x^2 \sin\left(\frac{\pi}{3}\right)$ or $\frac{1}{4}x^2\sqrt{3}$, $\frac{1}{2} \times \frac{2}{3}\pi x^2$, $2xy$
	2nd A1*	This is a given answer which should be stated and should be achieved without error so all three areas must have been correct and their sum put equal to 1000 and an intermediate step of rearrangement should be present.
(c)	**B1ft**	Correct expression for P from arc length, length AB and three sides of rectangle in terms of both x and y with $2y$ (or $y + y$), $3x$ (or $x + 2x$) (or $x + x + x$), and $x\theta$ clearly listed . Allow addition after substitution of y. NB $\theta = \frac{2\pi}{3}$ but allow use of their consistent θ in radians (usually $\theta = \frac{\pi}{3}$) from parts (a) and (b) for this mark. $120x$ or $60x$ do not get this mark.
	M1	Substitutes $y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$ or their unsimplified attempt at y from earlier (allow slips e.g. sign slips) into $2y$ term.
	A1*	This is a given answer which should be stated and should be achieved without error
(d)	**1st M1**	Need to see at least $\frac{1000}{x} \to \frac{\pm\lambda}{x^2}$
	1st A1	Correct differentiation of both terms (need not be simplified) Not follow through. Allow any correct equivalent. e.g. $\frac{\mathrm{d}P}{\mathrm{d}x} = -1000x^{-2} + \frac{\pi}{3} + 3 - \frac{\sqrt{3}}{4}$ Also allow $\frac{\mathrm{d}P}{\mathrm{d}x} = -1000x^{-2} + awrt\ 3.61$ Check carefully as there are many correct equivalents and some have two terms in $x\pi$ to differentiate obtaining for example $\frac{2\pi}{3} - \frac{8\pi}{24}$ instead of $\frac{\pi}{3}$
	2nd M1	Setting their $\frac{\mathrm{d}P}{\mathrm{d}x} = 0$. Do not need to find x, but if inequalities are used this mark cannot be gained until candidate states or uses a value of x without inequalities. May not be explicit but may be implied by correct working and value or expression for x . May result in $x^2 < 0$ so M1A0
	2nd A1	There is no requirement to write down a value for x, so this mark may be implied by a correct value for P. It may be given for a correct expression or value for x of 16.6, 16.7 or 17
	3rd A1	Allow answers wrt 120 but not 121
(e)	**M1**	Finds P'' **and** considers sign. Follow through **correct** differentiation of their P' (not just reduction of power)
	A1ft	Need $\frac{2000}{x^3}$ and > 0 (or positive value) and conclusion. Only follow through on a correct P'' and a value for x in the range $10 < x < 25$ (need not see x substituted but an x should have been found) If P is substituted then this is awarded M1 A0

	Special case	(d) Some candidates m they write $\frac{dP}{dx} = -12000x^{-2} + 4\pi + 36 - 3\sqrt{3}\ ;\ = 0$ then solve they will get the correct x and P They should be awarded M1A0M1A1A1 in part (d). If they then do part (e) writing $\frac{d^2P}{dx^2} = \frac{24000}{x^3} > 0 \Rightarrow$ Minimum They should be awarded M1A0 (so lose 2 marks in all) If they wrote $\frac{d(12P)}{dx} = -12000x^{-2} + 4\pi + 36 - 3\sqrt{3}\ ;\ = 0$ etc they could get full marks.

Question Number	Scheme	Marks
93. (a)	Either: (Cost of polishing top and bottom (two circles) is)$3\times 2\pi r^2$ **or (**Cost of polishing curved surface area is) $2\times 2\pi rh$ or both - just need to see at least one of these products	B1
	Uses volume to give $(h=)\dfrac{75\pi}{\pi r^2}$ or $(h=)\dfrac{75}{r^2}$ (simplified) (if V is misread – see below)	B1ft
	$(C)=6\pi r^2+4\pi r\left(\dfrac{75}{r^2}\right)$ — Substitutes expression for h into area or cost expression of form Ar^2+Brh	M1
	$C=6\pi r^2+\dfrac{300\pi}{r}$ *	A1* (4)
(b)	$\left\{\dfrac{dC}{dr}=\right\}12\pi r-\dfrac{300\pi}{r^2}$ or $12\pi r-300\pi r^{-2}$ (then isw)	M1 A1 ft
	$12\pi r-\dfrac{300\pi}{r^2}=0$ **so** r^k = value where $k=\pm 2,\pm 3,\pm 4$	dM1
	Use **cube** root to obtain $r=\left(their\,\frac{300}{12}\right)^{\frac{1}{3}}$ $(=2.92)$- allow $r=3$, and thus $C=$	ddM1
	Then C = awrt 483 or 484	A1cao (5)
(c)	$\left\{\dfrac{d^2C}{dr^2}=\right\}12\pi+\dfrac{600\pi}{r^3}>0$ **so minimum**	B1ft (1) **[10]**

Notes

(a) **B1:** States $3\times 2\pi r^2$ or states $2\times 2\pi rh$

B1ft: Obtains a **correct** expression for h in terms of r **(**ft only follows misread of V)

M1: Substitutes their expression for h into **area or cost** expression of form Ar^2+Brh

A1*: Had correct expression for C and achieves **given** answer in part (a) including "C =" or "Cost=" and **no errors seen** such as C = area expression without multiples of (£)3 and (£)2 at any point. Cost and area must be perfectly distinguished at all stages for this A mark.

N.B. Candidates using Curved Surface Area = $\dfrac{2V}{r}$ - please send to review

(b) **M1:** Attempts to differentiate as evidenced by at least one term differentiated correctly

A1ft: Correct derivative – allow $12\pi r-300\pi r^{-2}$ then isw if the power is misinterpreted (ft only for misread)

dM1: Sets their $\dfrac{dC}{dr}$ to 0, and obtains r^k = value where k = 2, 3 or 4 (needs correct collection of powers of r from their original derivative expression – allow errors dividing by 12π)

ddM1: Uses **cube** root to find r **or** see r = awrt 3 as evidence of cube root and substitutes into correct expression for C to obtain value for C

A1: Accept awrt 483 or 484

(c) **B1ft: Finds** correct expression for $\dfrac{d^2C}{dr^2}$ and deduces value of $\dfrac{d^2C}{dr^2}>0$ so minimum (r may have been wrong)

OR checks gradient to left and right of 2.92 and shows gradient goes from negative to zero to positive so minimum

OR checks value of C to left and right of 2.92 and shows that $C>483$ so deduces minimum (i.e. uses shape of graph) Only ft on misread of V for each ft mark (see below)

N..B. Some candidates have **misread** the volume as 75 instead of 75π. PTO for marking instruction.

Following this misread candidates cannot legitimately obtain the printed answer in part (a). Either they obtain $C = 6\pi r^2 + \dfrac{300}{r}$ or they "fudge" their working to appear to give the printed answer.

The policy for a misread is **to subtract 2 marks from A or B marks**. In this case the A mark is to be subtracted from part (a) and the final A mark is to be subtracted from part (b)
The maximum mark for part (a) following this misread is 3 marks. The award is B1 B1 M1 A0 as a maximum.

(a) B1: as before

B1: Uses volume to give $(h =)\dfrac{75}{\pi r^2}$

M1: $(C) = 6\pi r^2 + 4\pi r\left(\dfrac{75}{\pi r^2}\right)$

A0: Printed answer is not obtained without error

Most Candidates may then adopt the printed answer and gain up to full marks for the rest of the question so 9 of the 10 marks maximum in all.

Any candidate who proceeds with **their** answer $C = 6\pi r^2 + \dfrac{300}{r}$ may be awarded up to 4 marks in part (b). These are M1A1dM1ddM1A0 and then the candidate may also be awarded the B1 mark in part (c). So 8 of the 10 marks maximum in all.

(b) M1 A1: $\left\{\dfrac{dC}{dr} =\right\}\ 12\pi r - \dfrac{300}{r^2}$ or $12\pi r - 300r^{-2}$ (then isw)

dM1: $12\pi r - \dfrac{300}{r^2} = 0$ **so** r^k = value where k = 2, 3 or 4 or $12\pi r - \dfrac{300}{r^2} = 0$ **so** r^k = value

ddM1: Use **cube** root to obtain $r = \left(\textit{their}\ \frac{300}{12\pi}\right)^{\frac{1}{3}}$ $(=1.996)$ - allow r = 2, and thus $C = \ldots$ must use $C = 6\pi r^2 + \dfrac{300}{r}$

A0: Cannot obtain C = 483 or 484

(c) B1: $\left\{\dfrac{d^2C}{dr^2} =\right\}\ 12\pi + \dfrac{600}{r^3} > 0$ **so minimum** OR checks gradient to left and right of 1.966 and shows gradient goes from negative to zero to positive so minimum
OR checks value of C to left and right of 1.966 and shows that $C > 225.4$ so deduces minimum (i.e. uses shape of graph)
There is an example in Practice of this misread.

Question Number	Scheme		Marks
94. (a)	$\frac{1}{2}(9x+6x)4x$ or $2x\times15x$ or $\left(\frac{1}{2}4x\times(9x-6x)+6x\times4x\right)$ or $6x^2+24x^2$ or $\left(9x\times4x-\frac{1}{2}4x\times(9x-6x)\right)$ or $36x^2-6x^2$	M1: Correct attempt at the area of a trapezium. Note that $30x^2$ on its own or $30x^2$ from incorrect work e.g. $5x\times6x$ is M0. If there is a clear intention to find the area of the trapezium correctly allow the M1 but the A1 can be withheld if there are any slips.	M1A1**cso**
	$\Rightarrow 30x^2y=9600 \Rightarrow y=\dfrac{9600}{30x^2} \Rightarrow y=\dfrac{320}{x^2}$ *	A1: Correct proof with at least one intermediate step and no errors seen. **" *y* =" is required.**	
			[2]
(b)	$(S=)\ \frac{1}{2}(9x+6x)4x+\frac{1}{2}(9x+6x)4x+6xy+9xy+5xy+4xy$		M1A1
	M1: An attempt to find the area of **six** faces of the prism. The 2 trapezia may be combined as $(9x+6x)4x$ or $60x^2$ and the 4 other faces may be combined as $24xy$ but all six faces must be included. There must be attempt at the areas of two trapezia that are dimensionally correct. A1: Correct expression in any form. Allow just $(S=)\ 60x^2+24xy$ for M1A1		
	$y=\dfrac{320}{x^2} \Rightarrow (S=)\ 30x^2+30x^2+24x\left(\dfrac{320}{x^2}\right)$		M1
	Substitutes $y=\dfrac{320}{x^2}$ into their expression for S (may be done earlier). S should have at least one x^2 term and one xy term but there may be other terms which may be dimensionally incorrect.		
	So, $(S=)\ 60x^2+\dfrac{7680}{x}$ *	Correct solution only. "S = " is **not** required here.	A1* **cso**
			[4]

94(c)	$\frac{dS}{dx} = 120x - 7680x^{-2}$ $\left\{= 120x - \frac{7680}{x^2}\right\}$	M1: Either $60x^2 \to 120x$ or $\frac{7680}{x} \to \frac{\pm\lambda}{x^2}$	M1
		A1: Correct differentiation (need not be simplified).	A1 aef
	$120x - \frac{7680}{x^2} = 0$ $\Rightarrow x^3 = \frac{7680}{120}; = 64 \Rightarrow x = 4$	M1: $S' = 0$ and "their $x^3 = \pm\text{value}$" or "their $x^{-3} = \pm\text{value}$" Setting their $\frac{dS}{dx} = 0$ and "candidate's ft ***correct*** power of x = a value". **The power of x must be consistent with their differentiation**. If inequalities are used this mark cannot be gained until candidate states value of x or S from their x without inequalities. $S' = 0$ can be implied by $120x = \frac{7680}{x^2}$. Some may spot that $x = 4$ gives $S' = 0$ and provided they clearly show $S'(4) = 0$ allow this mark as long as S' is correct. (If S' is incorrect this method is allowed if their derivative is clearly zero for their value of x)	M1A1**cso**
		A1: $x = 4$ **only** ($x^3 = 64 \Rightarrow x = \pm 4$ scores A0) Note that the value of x is not explicitly required so the use of $x = \sqrt[3]{64}$ to give $S = 2880$ would imply this mark.	
	Note some candidates stop here and do not go on to find S – maximum mark is 4/6		
	$\{x = 4,\}$ $S = 60(4)^2 + \frac{7680}{4} = 2880 \text{ (cm}^2\text{)}$	Substitute candidate's value of $x\,(\neq 0)$ into a formula for S. **Dependent on both previous M marks.**	**dd**M1
		2880 **cso (Must come from correct work)**	A1 **cao and cso**
			[6]

94(d)	$\dfrac{d^2S}{dx^2} = 120 + \dfrac{15360}{x^3} > 0$ $\Rightarrow$ Minimum	M1: Attempt $S''\ \left(x^n \to x^{n-1}\right)$ **and** considers sign. This mark requires an attempt at the second derivative and **some consideration of its sign**. There does not necessarily need to be any substitution. An attempt to solve $S''=0$ is M0 A1: $120 + \dfrac{15360}{x^3}$ and >0 **and** conclusion. Requires a **correct** second derivative of $120 + \dfrac{15360}{x^3}$ (need not be simplified) **and** a valid reason (e.g. > 0), **and** conclusion. Only follow through a correct second derivative i.e. x may be incorrect **but must be positive** and/or S'' may have been evaluated incorrectly.	M1A1ft
	A correct S'' followed by $S''("4")="360"$ therefore minimum would score no marks in (d) **A correct S'' followed by $S''("4")="360"$ which is positive therefore minimum would score both marks**		
			[2]
	Note parts (c) and (d) can be marked together.		
			Total 14

Question Number	Scheme		Marks
95. (a)	$\{A=\}\ xy+\frac{\pi}{2}\left(\frac{x}{2}\right)^2+\frac{1}{2}x^2\sin 60^\circ$	M1: An attempt to find 3 areas of the form: $xy,\quad p\pi x^2$ and qx^2 A1: Correct expression for A (terms must be added)	M1A1
	$50=\ xy+\frac{\pi x^2}{8}+\frac{\sqrt{3}x^2}{4}\ \Rightarrow\ y=\frac{50}{x}-\frac{\pi x}{8}-\frac{\sqrt{3}x}{4}\ \Rightarrow\underline{y=\frac{50}{x}-\frac{x}{8}\left(\pi+2\sqrt{3}\right)}$ * Correct proof with no errors seen		A1 *
			[3]
(b)	$\{P=\}\ \frac{\pi x}{2}+2x+2y$	Correct expression for P in terms of x and y	B1
	$P=\ \frac{\pi x}{2}+2x+2\left(\frac{50}{x}-\frac{x}{8}\left(\pi+2\sqrt{3}\right)\right)$	Substitutes the given expression for y into an expression for P where P is at least of the form $\alpha x+\beta y$	M1
	$P=\ \frac{\pi x}{2}+2x+\frac{100}{x}-\frac{\pi x}{4}-\frac{\sqrt{3}}{2}x\ \Rightarrow\ P=\frac{100}{x}+\frac{\pi x}{4}+2x-\frac{\sqrt{3}}{2}x$		
	$\Rightarrow\ \underline{P=\frac{100}{x}+\frac{x}{4}\left(\pi+8-2\sqrt{3}\right)}$	Correct proof with no errors seen	A1 *
			[3]
	(Note $\frac{\pi+8-2\sqrt{3}}{4}=1.919\ldots\ldots$)		
(c) and (d) can be marked together	$\frac{\mathrm{d}P}{\mathrm{d}x}=-100x^{-2}+\frac{\pi+8-2\sqrt{3}}{4}$	M1: Either $\mu x\to\mu$ or $\frac{100}{x}\to\frac{\pm\lambda}{x^2}$ A1: Correct differentiation (need not be simplified). Allow $-100x^{-2}+(\text{awrt}1\cdot 92)$	M1A1
	$-100x^{-2}+\frac{\pi+8-2\sqrt{3}}{4}=0\Rightarrow x=\ldots$	Their $P'=0$ and attempt to solve as far as $x=\ldots.$ (ignore poor manipulation)	M1
	$\Rightarrow\ x=\sqrt{\frac{400}{\pi+8-2\sqrt{3}}}\ =\ 7.2180574\ldots$	$\sqrt{\frac{400}{\pi+8-2\sqrt{3}}}$ or awrt 7.2 **and no other values**	A1
	$\{x=7.218\ldots,\}\ \Rightarrow P=27.708\ldots$ (m)	awrt 27.7	A1
			[5]
	$\frac{\mathrm{d}^2P}{\mathrm{d}x^2}=\frac{200}{x^3}>0\ \Rightarrow$ Minimum	M1: Finds P'' ($x^n\to x^{n-1}$ allow for constant $\to 0$) **and** considers sign A1ft: $\frac{200}{x^3}$ (need not be simplified) and >0 and conclusion. Only follow through on a correct P'' and a single positive value of x found earlier.	M1A1ft
			[2]
			Total 13

Question Number	Scheme	Marks
96. (a)	$\left\{\frac{dy}{dx}=\right\}\ 2x-16x^{-\frac{1}{2}}$	M1 A1
	$2x-16x^{-\frac{1}{2}}=0 \Rightarrow\ x^{\frac{3}{2}}=,\ x^{-\frac{3}{2}}=$,or $2x-=16x^{-\frac{1}{2}}$ then squared then obtain $x^3=$ [or $2x-16x^{-\frac{1}{2}}=0 \Rightarrow x=4$ (no wrong work seen)]	M1
	$(x^{\frac{3}{2}}=8 \Rightarrow)\,x=\ 4$	A1
	$x=4,\ y=\ 4^2-32\sqrt{4}+20=-28$ (ignore y = 100 as second answer)	M1 A1
		(6)
(b)	$\left\{\frac{d^2y}{dx^2}=\right\}\ 2+8x^{-\frac{3}{2}}$	M1 A1
	$(\frac{d^2y}{dx^2}>0 \Rightarrow)\,y$ is a minimum (there should be no wrong reasoning)	A1
		(3) **[9]**
(b)	**<u>Alternative Method: Gradient Test:</u>** M1 for finding the gradient either side of their x-value from part (a). A1 for <u>both gradients calculated correctly to 1 significant figure</u>, then <u>using < 0 and > 0 respectively *maybe by use of sketch or table*</u>. (See appendix for gradient values. This is **not ft their *x***) A1 states minimum needs M1A1 to have been awarded.	
	Notes for Question 96	
(a)	1st M1: At least one term differentiated correctly, so $x^2 \to 2x$, or $32\sqrt{x} \to 16x^{-\frac{1}{2}}$, or $20 \to 0$ A1: This answer or equivalent e.g. $2x-\frac{16}{\sqrt{x}}$ 2nd M1: Sets their $\frac{dy}{dx}$ to 0, and solves to give $x^{\frac{3}{2}}=,\ x^{-\frac{3}{2}}=or\ \ x^3=$ after correct squaring or spots x = 4 (NB $\left\{\frac{d^2y}{dx^2}=0\right\}$ *so* $2+8x^{-\frac{3}{2}}=0$ **is M0)** N.B. Common error: Putting derivative = 0 and merely obtaining x = 0 is M0A0, then M0A0 for next two marks. (The first two marks in (a) and marks for second derivative may be earned in part (b).) A1: x = 4 cao [x = -4 is A0 and $x=\pm4$ is also A0] 3rd M1: Substitutes **their positive** found x **(NOT zero)** into $y=x^2-32\sqrt{x}+20,\ \ x>0$.**Should follow attempting to set** $\frac{dy}{dx}$ **= 0 and not setting** $\frac{d^2y}{dx^2}=0$ A1: -28 cao (Does not need to be written as coordinates)	
(b)	M1: Attempts differentiation of their first derivative with at least one term differentiated correctly. Should be seen or referred to (in part (b)) in determining the nature of the stationary point. A1: Answer in scheme or equivalent A1: States minimum (Second derivative should be correct- can follow incorrect positive x. Needs M1A1 to have been awarded- should not follow incorrect reasoning – (need not say $\frac{d^2y}{dx^2}>0$ but should not have said $\frac{d^2y}{dx^2}=0$ for example)	

Question Number	Scheme	Marks
97.	$\frac{dy}{dx} = 2 - 16x^{-3}$	M1 A1
	$2 - 16x^{-3} = 0$ so $x^{-3} =$ or $x^3 =$, or $2 - 16x^{-3} = 0$ so $x = 2$	M1
	$x = 2$ only (after correct derivative)	A1
	$y = 2 \times "2" + 3 + \frac{8}{"2^2"}$	M1
	= 9	A1
		(6)
		Total 6
	Notes for Question 97	
	1st M1: At least one term **differentiated (not integrated)** correctly, so $2x \to 2$, or $\frac{8}{x^2} \to -16x^{-3}$, or $3 \to 0$ A1: This answer or equivalent e.g. $2 - \frac{16}{x^3}$ 2nd M1: Sets $\frac{dy}{dx}$ to 0, and solves to give $x^3 =$ value or $x^{-3} =$ value (or states x =2 with no working following correctly stated $2 - 16x^{-3} = 0$) A1: $x = 2$ cso (if $x = -2$ is included this is A0 here) 3rd M1: Attempts to substitutes **their positive** x (found from attempt to differentiate) into $y = 2x + 3 + \frac{8}{x^2}$, $x > 0$ Or may be implied by $y = 9$ or correct follow through from their positive x A1: 9 cao (Does not need to be written as coordinates) (ignore the extra (-2,1) here)	

Question Number	Scheme	Marks
98(a)	$x^2+2x+2=10 \Rightarrow x^2+2x-8=0$ (so $(x+4)(x-2)=0$) $\Rightarrow x=$........	M1
	$x=-4,\ 2$	A1 (2)
(b) Way 1	$\int\left(x^2+2x+2\right)\mathrm{d}x=\frac{x^3}{3}+\frac{2x^2}{2}+2x(+C)$	M1A1A1
	$\left[\frac{x^3}{3}+\frac{2x^2}{2}+2x\right]_{"-4"}^{"2"}=\left(\frac{8}{3}+\frac{8}{2}+4\right)-\left(-\frac{64}{3}+\frac{32}{2}-8\right)\quad (=24)$	M1
	Rectangle: $10\times(2--4)=60$	B1 cao
	$R="60"-"24"$	M1
	$=36$	A1 (7)
		Total 9
(b) Way 2	$\int\left(8-x^2-2x\right)\mathrm{d}x=8x-\frac{x^3}{3}-\frac{2x^2}{2}(+C)$	M1 A1ft A1
	$\left[8x-\frac{x^3}{3}-\frac{2x^2}{2}\right]_{"-4"}^{"2"}=\left(16-\frac{8}{3}-4\right)-\left(-32+\frac{64}{3}-16\right)$ =(9.3 – (- 26.7))	M1
	Implied by final answer of 36 after correct work	B1
	$10-\left(x^2+2x+2\right)=8-x^2-2x,\ =36$	M1, A1
	Notes for Question 98	
(a)	M1 Set the curve equation equal to 10 and collect terms. Solves quadratic to $x=$ ..... A1 cao : Both values correct – allow A = -4, B = 2	
(b)	M1: One correct integration A1: Two correct integrations(ft slips subtracting in Way 2) A1: All 3 terms **correct (penalise subtraction errors here in Way 2)** M1: Substitute their limits from (a) into the **integrated** function and subtract (either way round) B1: Way 1:Find area under the line by integration or area of rectangle – should be 60 here (no follow through) Way 2: (implied by final correct answer in second method) M1: Subtract one area from the other (implied by subtraction of functions in second method)- award even after differentiation A1: Must be 36 not –36.	
	Special case 1: Combines both methods. Uses Way 2 integration, but continues after reaching "36"to subtract "36" from rectangle giving answer as "24" This loses final M1 A1 *Special case* 2: Integrates $\left(x^2+2x-8\right)$ between limits –4 and 2 to get -36 and then changes sign and obtains 36. Do not award final A mark – so M1A1A1M1B1M1A0 If the answer is left as -36, then M1A1A1M1B0M1A0 N.B. Allow full marks for modulus used earlier in working e.g. $\left\vert\int_{-4}^{2}x^2+2x-2\mathrm{d}x-\int_{-4}^{2}10\mathrm{d}x\right\vert$	

EXPERT TUITION

99.	$y = 6 - 3x - \dfrac{4}{x^3}$		
(a)	$\dfrac{dy}{dx} = -3 + \dfrac{12}{x^4}$ *or* $-3 + 12x^{-4}$	M1: $x^n \to x^{n-1}$ $(x^1 \to x^0$ or $x^{-3} \to x^{-4}$ or $6 \to 0)$ A1: Correct derivative	M1 A1
	$\dfrac{dy}{dx} = 0 \Rightarrow -3 + \dfrac{12}{x^4} = 0 \Rightarrow x = ...$ **or** $\dfrac{dy}{dx} = -3 + \dfrac{12}{\sqrt{2}^4}$	$y' = 0$ and attempt to solve for x May be implied by $\dfrac{dy}{dx} = -3 + \dfrac{12}{x^4} = 0 \Rightarrow \dfrac{12}{x^4} = 3 \Rightarrow x = ...$ **or** Substitutes $x = \sqrt{2}$ into their y'	M1
	So $x^4 = 4$ and $x = \sqrt{2}$ **or** $\dfrac{dy}{dx} = -3 + \dfrac{12}{\left(\sqrt{2}\right)^4}$ or $-3 + 12\left(\sqrt{2}\right)^{-4} = 0$	Correct completion to answer with no errors by solving their $y' = 0$ or substituting $x = \sqrt{2}$ into their y'	A1
			(4)
(b)	$x = -\sqrt{2}$	Awrt -1.41	B1
			(1)
(c)	$\dfrac{d^2y}{dx^2} = \dfrac{-48}{x^5}$ or $-48x^{-5}$	Follow through their first derivative from part (a)	B1ft
			(1)
(d)	An appreciation that either $y'' > 0 \Rightarrow$ a minimum or $y'' < 0 \Rightarrow$ a maximum		B1
	Maximum at P as $y'' < 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct and there must be reference to P or to $\sqrt{2}$ and negative or < 0 and maximum. There must be no incorrect or contradictory statements (NB allow y'' = awrt-8 or -9)		
	Minimum at Q as $y'' > 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct **and part (b) must be correct** and there must be reference to P or to $-\sqrt{2}$ and positive or > 0 and minimum. There must be no incorrect or contradictory statements (NB allow y'' = awrt 8 or 9)		
			(3)
			[9]
	Other methods for identifying the nature of the turning points are acceptable. The first B1 is for finding values of y or dy/dx either side of $\sqrt{2}$ or their x at Q and the second and third B1's for fully correct solutions to identify the maximum/minimum.		

Question number	Scheme	Marks
100 (a)	$(h=)\dfrac{60}{\pi x^2}$ or **equivalent exact** (not decimal) expression e.g. $(h=)60\div\pi x^2$	B1 (1)
(b)	$(A=)2\pi x^2+2\pi xh$ or $(A=)2\pi r^2+2\pi rh$ or $(A=)2\pi r^2+\pi dh$ may not be simplified and may appear on separate lines	B1
	Either $(A)=2\pi x^2+2\pi x\left(\dfrac{60}{\pi x^2}\right)$ or As $\pi xh=\dfrac{60}{x}$ then $(A=)2\pi x^2+2\left(\dfrac{60}{x}\right)$	M1
	$A=2\pi x^2+\left(\dfrac{120}{x}\right)$ *	A1 cso (3)
(c)	$\left(\dfrac{dA}{dx}\right)=4\pi x-\dfrac{120}{x^2}$ or $=4\pi x-120x^{-2}$	M1 A1
	$4\pi x-\dfrac{120}{x^2}=0$ implies $x^3=$ (Use of >0 or <0 is M0 then M0A0)	M1
	$x=\sqrt[3]{\dfrac{120}{4\pi}}$ or answers which round to 2.12 (-2.12 is A0)	dM1 A1 (5)
(d)	$A=2\pi(2.12)^2+\dfrac{120}{2.12}$, $=85$ (only ft x = 2 or 2.1 – both give 85)	M1, A1 (2)
(e)	**Either** $\dfrac{d^2A}{dx^2}=4\pi+\dfrac{240}{x^3}$ and sign considered (May appear in (c)) **Or** *(method 2)* considers gradient to left and right of their 2.12 (e.g at 2 and 2.5) **Or** (*method 3*) considers value of A either side	M1
	which is >0 and therefore minimum (most substitute 2.12 but it is not essential to see a substitution) (may appear in (c)) Finds numerical values for gradients and observes gradients go from negative to zero to positive so concludes minimum **OR** finds numerical values of A , observing greater than minimum value and draws conclusion	A1 (2)
		13 marks
Notes	(a) **B1**: This expression must be correct and in part (a) $\dfrac{60}{\pi r^2}$ is B0 (b) **B1:** Accept any equivalent correct form – may be on two or more lines. **M1 :** substitute their expression for h in terms of x into Area formula **of the form** kx^2+cxh **A1**: There should have been no errors in part (b) in obtaining this printed answer (c) **M1:** At least one power of x decreased by 1 **A1** accept any equivalent correct answer **M1:** Setting $\dfrac{dA}{dx}$=0 and finding a value for x^3 (x^3= may be implied by answer). Allow $\dfrac{dy}{dx}=0$ **dM1:** Using **cube root** to find x **A1 :** For any equivalent correct answer (need 3sf or more) Correct answer implies previous M mark (d) **M1 :** Substitute the (+ve) x value found in (c) into equation for A and evaluate . **A1** is for **85 only** (e) **M1**: Complete method, usually one of the three listed in the scheme. For first method $A''(x)$ must be attempted and sign considered **A1**: Clear statements and conclusion. (numerical substitution of x is not necessary in first method shown, and x or calculation could be wrong but $A''(x)$ **must be correct** . Must **not** see 85 substituted)	

Question number	Scheme	Marks
101 (a)	$kr^2 + cxy = 4$ or $kr^2 + c[(x+y)^2 - x^2 - y^2] = 4$	M1
	$\frac{1}{4}\pi x^2 + 2xy = 4$	A1
	$y = \dfrac{4 - \frac{1}{4}\pi x^2}{2x} = \dfrac{16 - \pi x^2}{8x}$ *	B1 cso (3)
(b)	$P = 2x + cy + k\,\pi r$ where $c = 2$ or 4 and $k = ¼$ or $½$	M1
	$P = \dfrac{\pi x}{2} + 2x + 4\left(\dfrac{4 - \frac{1}{4}\pi x^2}{2x}\right)$ or $P = \dfrac{\pi x}{2} + 2x + 4\left(\dfrac{16 - \pi x^2}{8x}\right)$ o.e.	A1
	$P = \dfrac{\pi x}{2} + 2x + \dfrac{8}{x} - \dfrac{\pi x}{2}$ so $P = \dfrac{8}{x} + 2x$ *	A1 (3)
(c)	$\left(\dfrac{dP}{dx} = \right) - \dfrac{8}{x^2} + 2$	M1 A1
	$-\dfrac{8}{x^2} + 2 = 0 \Rightarrow x^2 = ..$	M1
	and so $x = 2$ o.e. (ignore extra answer $x = -2$)	A1
	$P = 4 + 4 = 8$ (m)	B1 (5)
(d)	$y = \dfrac{4 - \pi}{4}$, (and so width) = 21 (cm)	M1, A1 (2) **13**
Notes	(a) **M1**: Putting sum of one or two xy terms and one $k\,r^2$ term equal to 4 (**k and c may be wrong**) **A1:** For any correct form of this equation with x for radius (may be unsimplified) **B1 :** Making y the subject of their formula to give this printed answer with no errors (b) **M1** : Uses Perimeter formula of the form $2x + cy + k\,\pi r$ where $c = 2$ or 4 and $k = ¼$ or $½$ **A1:** Correct unsimplified formula with y substituted as shown, i.e. $c = 4$, $k = ½$, $r = x$ and $y = \dfrac{16 - \pi x^2}{8x}$ or $y = \left(\dfrac{4 - \frac{1}{4}\pi x^2}{2x}\right)$ **A1:** obtains printed answer with at least one line of correct simplification or expansion before giving printed answer or stating result has been shown or equivalent (c) **M1:** At least one power of x decreased by 1 (Allow $2x$ becomes 2) **A1:** accept any equivalent correct answer **M1:** Setting $\dfrac{dP}{dx}$ =0 and finding a value for correct power of x for candidate **A1 :** For $x = 2$. (This mark may be given for equivalent and may be implied by correct P) **B1:** 8 (cao) N.B. This may be awarded if seen in part (d) (d) **M1 :** Substitute x value found in (c) into equation for y from (a) (or substitute x and P into equation for P from (b)) and evaluate (may see 0.2146 and correct answer implies M1 or need to see substitution if x value was wrong.) **A1** is for 21 or 21cm or 0.21m as this is to nearest cm	

Question Number	Scheme		Marks
102. (a)	$\{V =\}\ 2x^2y = 81$	$2x^2y = 81$	B1 oe
	$\{L = 2(2x + x + 2x + x) + 4y \Rightarrow L = 12x + 4y\}$		
	$y = \dfrac{81}{2x^2} \Rightarrow L = 12x + 4\left(\dfrac{81}{2x^2}\right)$	Making y the subject of their expression and substitute this into the correct L formula.	M1
	So, $L = 12x + \dfrac{162}{x^2}$ **AG**	Correct solution only. **AG.**	A1 **cso**
			[3]
(b)	$\dfrac{dL}{dx} = 12 - \dfrac{324}{x^3}\quad \{= 12 - 324x^{-3}\}$	Either $12x \to 12$ or $\dfrac{162}{x^2} \to \dfrac{\pm\lambda}{x^3}$	M1
		Correct differentiation (need not be simplified).	A1 aef
	$\left\{\dfrac{dL}{dx} =\right\} 12 - \dfrac{324}{x^3} = 0 \Rightarrow x^3 = \dfrac{324}{12}; = 27 \Rightarrow x = 3$	$L' = 0$ and "their $x^3 = \pm$ value" or "their $x^{-3} = \pm$ value"	M1;
		$x = \sqrt[3]{27}$ or $x = 3$	A1 **cso**
	$\{x = 3,\}\quad L = 12(3) + \dfrac{162}{3^2} = 54$ (cm)	Substitute candidate's value of $x\,(\neq 0)$ into a formula for L.	ddM1
		54	A1 **cao**
			[6]
(c)	$\{\text{For } x = 3\}$, $\dfrac{d^2L}{dx^2} = \dfrac{972}{x^4} > 0 \Rightarrow$ Minimum	Correct ft L'' and considering sign.	M1
		$\dfrac{972}{x^4}$ and > 0 and conclusion.	A1 **[2]**
			11

(a)	B1: For any correct form of $2x^2y = 81$. (may be unsimplified). Note that $2x^3 = 81$ is B0. Otherwise, candidates can use any symbol or letter in place of y. M1: Making y the subject of their formula and substituting this into a correct expression for L. A1: Correct solution only. Note that the answer is given.
(b)	**Note you can mark parts (b) and (c) together.** 2nd M1: Setting their $\dfrac{dL}{dx} = 0$ and "candidate's ft ***correct*** power of x = a value". The power of x must be consistent with their differentiation. If inequalities are used this mark cannot be gained until candidate states value of x or L from their x without inequalities. $L' = 0$ can be implied by $12 = \frac{324}{x^3}$. 2nd A1: $x^3 = 27 \Rightarrow x = \pm 3$ scores A0. 2nd A1: can be given for no value of x given but followed through by correct working leading to $L = 54$. 3rd M1: Note that this method mark is dependent upon the two previous method marks being awarded.
(c)	M1: for attempting correct ft second derivative and considering its sign. A1: Correct second derivative of $\dfrac{972}{x^4}$ (need not be simplified) and a valid reason (e.g. > 0), and conclusion. Need to conclude minimum (allow x and not L is a minimum) or indicate by a tick that it is a minimum. The actual value of the second derivative, if found, can be ignored, although substituting their L and not x into L'' is A0. **Note:** 2 marks can be scored from a wrong value of x, no value of x found or from not substituting in the value of their x into L''. Gradient test or testing values either side of their x scores M0A0 in part (c). **Throughout this question allow confused notation such as $\dfrac{dy}{dx}$ for $\dfrac{dL}{dx}$.**

Question Number	Scheme		Marks
103.			
(a)	$V = 4x(5-x)^2 = 4x(25-10x+x^2)$		
	So, $V = 100x - 40x^2 + 4x^3$	$\pm\alpha x \pm \beta x^2 \pm \gamma x^3$, where $\alpha, \beta, \gamma \neq 0$	M1
		$V = 100x - 40x^2 + 4x^3$	A1
	$\frac{dV}{dx} = 100 - 80x + 12x^2$	At least two of their expanded terms differentiated correctly.	M1
		$100 - 80x + 12x^2$	A1 cao
			(4)
(b)	$100 - 80x + 12x^2 = 0$	Sets their $\frac{dV}{dx}$ from part (a) = 0	M1
	$\{\Rightarrow 4(3x^2 - 20x + 25) = 0 \Rightarrow 4(3x-5)(x-5) = 0\}$		
	$\{\text{As } 0 < x < 5\}\ x = \frac{5}{3}$	$x = \frac{5}{3}$ or $x =$ awrt 1.67	A1
	$x = \frac{5}{3},\quad V = 4\left(\frac{5}{3}\right)\left(5 - \frac{5}{3}\right)^2$	Substitute candidate's value of x **where** $0 < x < 5$ into a formula for V.	dM1
	So, $V = \frac{2000}{27} = 74\frac{2}{27} = 74.074...$	Either $\frac{2000}{27}$ or $74\frac{2}{27}$ or awrt 74.1	A1
			(4)
(c)	$\frac{d^2V}{dx^2} = -80 + 24x$	Differentiates their $\frac{dV}{dx}$ **correctly** to give $\frac{d^2V}{dx^2}$.	M1
	When $x = \frac{5}{3}$, $\frac{d^2V}{dx^2} = -80 + 24\left(\frac{5}{3}\right)$		
	$\frac{d^2V}{dx^2} = -40 < 0 \Rightarrow V$ is a maximum	$\underline{\frac{d^2V}{dx^2} = -40}$ and $\underline{< 0 \text{ or negative}}$ and $\underline{\text{maximum}}$.	A1 cso
			(2)
			[10]
	Notes		
(a)	1st M1 for a three term cubic in the form $\pm\alpha x \pm \beta x^2 \pm \gamma x^3$. Note that an un-combined $\pm\alpha x \pm \lambda x^2 \pm \mu x^2 \pm \gamma x^3$, $\alpha, \lambda, \mu, \gamma \neq 0$ is fine for the 1st M1. 1st A1 for either $100x - 40x^2 + 4x^3$ or $100x - 20x^2 - 20x^2 + 4x^3$. 2nd M1 for any two of their expanded terms differentiated correctly. NB: If expanded expression is divided by a constant, then the 2nd M1 can be awarded for at least two terms are correct. Note for un-combined $\pm\lambda x^2 \pm \mu x^2$, $\pm 2\lambda x \pm 2\mu x$ counts as one term differentiated correctly. 2nd A1 for $100 - 80x + 12x^2$, **cao**. **Note:** See appendix for those candidates who apply the product rule of differentiation.		

Question Number	Scheme	Marks
(b)	**Note you can mark parts (b) and (c) together.** Ignore the extra solution of $x = 5$ (and $V = 0$). Any extra solutions for V inside found for values inside the range of x, then award the final A0.	
(c)	M1 is for their $\frac{dV}{dx}$ differentiated **correctly (follow through)** to give $\frac{d^2V}{dx^2}$. A1 for all three of $\underline{\frac{d^2V}{dx^2} = -40}$ and $\underline{< 0 \text{ or negative}}$ and <u>maximum</u>. Ignore any second derivative testing on $x = 5$ for the final accuracy mark. **<u>Alternative Method: Gradient Test:</u>** M1 for finding the gradient either side of their x-value from part (b) where $0 < x < 5$. A1 for <u>both gradients calculated correctly to the near integer, using > 0 and < 0 respectively or a correct sketch</u> and <u>maximum</u>. (See appendix for gradient values.)	

Question Number	Scheme	Marks
Aliter **103 (a)** Way2	**Product Rule Method:**	
	$\begin{cases} u = 4x & v = (5-x)^2 \\ \dfrac{du}{dx} = 4 & \dfrac{dv}{dx} = 2(5-x)^1(-1) \end{cases}$	
	$\pm(\text{their } u')(5-x)^2 \pm (4x)(\text{their } v')$	M1
	$\dfrac{dy}{dx} = 4(5-x)^2 + 4x(2)(5-x)^1(-1)$ — A correct attempt at differentiating any one of either u or v correctly.	dM1
	Both $\dfrac{du}{dx}$ and $\dfrac{dv}{dx}$ correct	A1
	$\dfrac{dy}{dx} = 4(5-x)^2 - 8x(5-x)$ — $4(5-x)^2 - 8x(5-x)$	A1
		(4)
Aliter **103 (a)** Way3	$\begin{cases} u = 4x & v = 25 - 10x + x^2 \\ \dfrac{du}{dx} = 4 & \dfrac{dv}{dx} = -10 + 2x \end{cases}$	
	$\pm(\text{their } u')\left(\text{their}(5-x)^2\right) \pm (4x)(\text{their } v')$	M1
	$\dfrac{dy}{dx} = 4(25 - 10x + x^2) + 4x(-10 + 2x)$ — A correct attempt at differentiating any one of either u or **their** v correctly.	dM1
	Both $\dfrac{du}{dx}$ and $\dfrac{dv}{dx}$ correct	A1
	$\dfrac{dV}{dx} = 100 - 80x + 12x^2$ — $100 - 80x + 12x^2$	A1
		(4)
	Note: The candidate needs to use a complete product rule method in order for you to award the the first M1 mark here. The second method mark is dependent on the first method mark awarded.	

Question Number	Scheme	Marks
Aliter **103 (c)** *Way 2*	**Gradient Test Method:** $\frac{\mathrm{d}V}{\mathrm{d}x}=100-80x+12x^2$ ***Helpful table!***	

x	$\frac{\mathrm{d}V}{\mathrm{d}x}$
0.8	43.68
0.9	37.72
1	32
1.1	26.52
1.2	21.28
1.3	16.28
1.4	11.52
1.429	10.204
1.5	7
1.6	2.72
1.7	-1.32
1.8	-5.12
1.9	-8.68
2	-12
2.1	-15.08
2.2	-17.92
2.3	-20.52
2.4	-22.88
2.5	-25

Question Number	Scheme	Marks
104	(a) $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\ 2x-\frac{1}{2}kx^{-\frac{1}{2}}$ (Having an extra term, e.g. $+C$, is A0)	M1 A1 (2)
	(b) Substituting $x = 4$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and 'compare with zero' (The mark is allowed for : $<, >, =, \leq, \geq$) $8-\frac{k}{4}<0$ $k > 32$ (or $32 < k$) Correct inequality needed	M1 A1 (2) **4**
Notes	(a) M: $x^2 \to cx$ or $k\sqrt{x} \to cx^{-\frac{1}{2}}$ (c constant, $c \neq 0$) (b) Substitution of $x = 4$ into y scores M0. However, $\frac{\mathrm{d}y}{\mathrm{d}x}$ is sometimes called y, and in this case the M mark can be given. $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ may be 'implied' for M1, when, for example, a value of k or an inequality solution for k is found. Working must be seen to justify marks in (b), i.e. $k > 32$ alone is M0 A0.	

Question Number	Scheme	Marks
105	$\frac{dy}{dx} = 3x^2 - 20x + k$ (Differentiation is required)	M1 A1
	At $x = 2$, $\frac{dy}{dx} = 0$, so $12 - 40 + k = 0$ $k = 28$ (*)	A1 cso
	N.B. The '= 0' must be seen at some stage to score the final mark.	
	Alternatively: (using $k = 28$)	
	$\frac{dy}{dx} = 3x^2 - 20x + 28$ (M1 A1)	
	'Assuming' $k = 28$ only scores the final cso mark if there is justification that $\frac{dy}{dx} = 0$ at $x = 2$ represents the maximum turning point.	(3)
	M: $x^n \to cx^{n-1}$ (c constant, $c \neq 0$) for one term,	

Question Number	Scheme	Marks
106 (a)	$\left[y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}} - 10\right]$	
	$[y' =]$ $6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$	M1 A1
	Puts their $\frac{6}{x^{\frac{1}{2}}} - \frac{3}{2}x^{\frac{1}{2}} = 0$	M1
	So $x =$ $, \frac{12}{3} = 4$ (If $x = 0$ appears also as solution then lose A1)	M1, A1
	$x = 4, \Rightarrow y = 12 \times 2 - 4^{\frac{3}{2}} - 10$, so $y = 6$	dM1,A1 (7)
(b)	$y'' = -3x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$	M1A1 (2)
(c)	[Since x >0] It is a maximum	B1 (1) **[10]**
(a)	1st M1 for an attempt to differentiate a fractional power $x^n \to x^{n-1}$ A1 a.e.f – can be unsimplified 2nd M1 for forming a suitable equation using their y'=0 3rd M1 for correct processing of fractional powers leading to x = … (Can be implied by x = 4) A1 is for x = 4 only. If x = 0 also seen and not discarded they lose this mark only. 4th M1 for substituting their value of x back into y to find y value. Dependent on three previous M marks. Must see evidence of the substitution with attempt at fractional powers to give M1A0, but y = 6 can imply M1A1	
(b)	M1 for differentiating their y' again A1 should be simplified	
(c)	B1 . Clear conclusion needed and must follow correct y'' It is dependent on previous A mark (Do not need to have found x earlier). (Treat parts (a),(b) and (c) together for award of marks)	

Question Number	Scheme	Marks
107 (a)	(Arc length =) $r\theta = r\times 1 = r$. Can be awarded by implication from later work, e.g. $3rh$ or $(2rh + rh)$ in the S formula. (Requires use of $\theta = 1$).	B1
	(Sector area =) $\frac{1}{2}r^2\theta = \frac{1}{2}r^2\times 1 = \frac{r^2}{2}$. Can be awarded by implication from later work, e.g. the correct volume formula. (Requires use of $\theta = 1$).	B1
	Surface area = 2 sectors + 2 rectangles + curved face $(= r^2 + 3rh)$ (See notes below for what is allowed here)	M1
	Volume = 300 = $\frac{1}{2}r^2h$	B1
	Sub for h: $S = r^2 + 3\times\frac{600}{r} = r^2 + \frac{1800}{r}$ (*)	A1cso (5)
(b)	$\frac{\mathrm{d}S}{\mathrm{d}r} = 2r - \frac{1800}{r^2}$ or $2r - 1800r^{-2}$ or $2r + -1800r^{-2}$	M1A1
	$\frac{\mathrm{d}S}{\mathrm{d}r} = 0 \Rightarrow r^3 = \ldots,\quad r = \sqrt[3]{900}$, or AWRT 9.7 (NOT -9.7 or ± 9.7)	M1, A1 (4)
(c)	$\frac{\mathrm{d}^2S}{\mathrm{d}r^2} = \ldots.$ and consider sign, $\frac{\mathrm{d}^2S}{\mathrm{d}r^2} = 2 + \frac{3600}{r^3} > 0$ so point is a minimum	M1, A1ft (2)
(d)	$S_{\min} = (9.65\ldots)^2 + \frac{1800}{9.65\ldots}$ (Using their value of r, however found, in the <u>given</u> S formula)	M1
	= 279.65… (AWRT: 280) (Dependent on full marks in part (b))	A1 (2) **[13]**
(a)	M1 for attempting a formula (with terms added) for surface area. May be incomplete or wrong and may have extra term(s), but must have an r^2 (or $r^2\theta$) term and an rh (or $rh\theta$) term.	
(b)	<u>In parts (b), (c) and (d), ignore labelling of parts</u> 1st M1 for attempt at differentiation (one term is sufficient) $r^n \to kr^{n-1}$ 2nd M1 for setting their derivative (a 'changed function') = 0 and solving as far as $r^3 = \ldots$ (depending upon their 'changed function', this could be $r = \ldots$ or $r^2 = \ldots$, etc., but the algebra <u>must deal with a negative power</u> of r and should be sound apart from possible <u>sign</u> errors, so that $r^n = \ldots$ is consistent with their derivative).	
(c)	M1 for attempting second derivative (one term is sufficient) $r^n \to kr^{n-1}$, <u>and considering its sign</u>. Substitution of a value of r is not required. (<u>Equating it to zero is M0</u>). A1ft for a correct second derivative (or correct ft from their first derivative) <u>and</u> a valid reason (e.g. > 0), <u>and</u> conclusion. The actual <u>value</u> of the second derivative, if found, can be ignored. To score this mark as ft, their second derivative must indicate a minimum. <u>Alternative</u>: M1: Find <u>value</u> of $\frac{\mathrm{d}S}{\mathrm{d}r}$ on each side of their value of r and consider sign. A1ft: Indicate sign change of negative to positive for $\frac{\mathrm{d}S}{\mathrm{d}r}$, and conclude minimum. <u>Alternative</u>: M1: Find <u>value</u> of S on each side of their value of r and compare with their 279.65. A1ft: Indicate that both values are more than 279.65, and conclude minimum.	

Question Number	Scheme	Marks
108 (a)	$2\pi rh + 2\pi r^2 = 800$	B1
	$h = \dfrac{400 - \pi r^2}{\pi r}$, $\quad V = \pi r^2\left(\dfrac{400 - \pi r^2}{\pi r}\right) = 400r - \pi r^3$ $\quad$ (*)	M1, M1 A1 (4)
(b)	$\dfrac{\mathrm{d}V}{\mathrm{d}r} = 400 - 3\pi r^2$	M1 A1
	$400 - 3\pi \quad r^2 = 0 \qquad r^2 = \ldots, \qquad r = \sqrt{\dfrac{400}{3\pi}} \qquad$ (= 6.5 (2 s.f.))	M1 A1
	$V = 400r - \pi r^3 = 1737 = \dfrac{800}{3}\sqrt{\dfrac{400}{3\pi}}\ \left(\text{cm}^3\right)$ (accept awrt 1737 or exact answer)	M1 A1 (6)
(c)	$\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2} = -6\pi r$, Negative, $\therefore$ maximum (Parts (b) and (c) should be considered together when marking)	M1 A1 (2) **[12]**
Other methods for part (c):	Either: M: Find value of $\dfrac{\mathrm{d}V}{\mathrm{d}r}$ on each side of "$r = \sqrt{\dfrac{400}{3\pi}}$" and consider sign. A: Indicate sign change of positive to negative for $\dfrac{\mathrm{d}V}{\mathrm{d}r}$, and conclude max. Or: M: Find value of V on each side of "$r = \sqrt{\dfrac{400}{3\pi}}$" and compare with "1737". A: Indicate that both values are less than 1737 or 1737.25, and conclude max.	
Notes (a)	**B1:** For any correct form of this equation (may be unsimplified, may be implied by 1st M1) **M1 :** Making h the subject of their three or four term formula **M1:** Substituting expression for h into $\pi r^2 h$ (independent mark) Must now be expression in r only. **A1:** cso	
(b)	**M1:** At least one power of r decreased by 1 **A1:** cao **M1:** Setting $\dfrac{\mathrm{d}V}{\mathrm{d}r}$ =0 and finding a value for correct power of r for candidate **A1 :** This mark may be credited if the value of V is correct. Otherwise answers should round to 6.5 (allow ± 6.5)or be exact answer **M1:** Substitute a positive value of r to give V **A1:** 1737 or 1737.25….. or exact answer	

(c)	**M1:** needs complete method **e.g.**attempts differentiation (power reduced) of their first derivative and considers its sign **A1(first method)** should be $-6\pi r$ (do not need to substitute r and can condone wrong r if found in (b)) Need to conclude maximum or indicate by a tick that it is maximum. Throughout allow confused notation such as dy/dx for dV/dr
Alternative for (a)	$A = 2\pi r^2 + 2\pi rh$, $\frac{A}{2}\times r = \pi r^3 + \pi r^2 h$ is **M1** Equate to 400r **B1** Then $V = 400r - \pi r^3$ is **M1 A1**

Question number	Scheme	Marks
109	$\left(\frac{dy}{dx}=\right) 8+2x-3x^2$ (M: $x^n \to x^{n-1}$ for one of the terms, <u>not</u> just $10 \to 0$)	M1 A1
	$3x^2-2x-8=0$ $(3x+4)(x-2)=0$ $x=2$ (Ignore other solution) (*)	A1cso (3)
	The final mark may also be scored by <u>verifying</u> that $\frac{dy}{dx}=0$ at $x=2$.	

110 (a)	(Total area) = $3xy + 2x^2$	B1
	(Vol:) $x^2y = 100$ $\left(y = \frac{100}{x^2}, xy = \frac{100}{x}\right)$	B1
	Deriving expression for area in terms of x only	M1
	(Substitution, or clear use of, y or xy into expression for area)	
	(Area =) $\frac{300}{x} + 2x^2$ **AG**	A1 cso (4)
(b)	$\frac{dA}{dx} = -\frac{300}{x^2} + 4x$	M1A1
	Setting $\frac{dA}{dx} = 0$ and finding a value for correct power of x, for cand. M1	
	[$x^3 = 75$]	
	$x = 4.2172$ awrt 4.22 (allow exact $\sqrt[3]{75}$)	A1 (4)
(c)	$\frac{d^2A}{dx^2} = \frac{600}{x^3} + 4$ = positive therefore minimum	M1A1 (2)
(d)	Substituting found value of x into (a)	M1
	(Or finding y for found x and substituting both in $3xy + 2x^2$)	
	[$y = \frac{100}{4.2172^2} = 5.6228$]	
	Area = 106.707 awrt 107	A1 (2) **[12]**
Notes	(a) First B1: Earned for correct unsimplified expression, isw. (c) For M1: Find $\frac{d^2A}{dx^2}$ and explicitly consider its sign, state > 0 or "positive" A1: Candidate's $\frac{d^2A}{dx^2}$ must be correct for their $\frac{dA}{dx}$, sign must be + ve and conclusion "so minimum", (allow QED, √). (may be wrong x, or even no value of x found) <u>Alternative:</u> M1: Find value of $\frac{dA}{dx}$ on either side of "$x = \sqrt[3]{75}$" and consider sign A1: Indicate sign change of negative to positive for $\frac{dA}{dx}$, and conclude minimum. OR M1: Consider values of A on either side of "$x = \sqrt[3]{75}$" and compare with"107" A1: Both values greater than "$x = 107$" and conclude minimum. Allow marks for (c) and (d) where seen; even if part labelling confused.	

EXPERT TUITION

Question Number	Scheme	Marks
111.(a)	$y=2x(3x-1)^5 \Rightarrow \frac{dy}{dx}=2(3x-1)^5+30x(3x-1)^4$	M1A1
	$\Rightarrow\left(\frac{dy}{dx}\right)=2(3x-1)^4\{(3x-1)+15x\}=2(3x-1)^4(18x-1)$	M1A1
		(4)
(b)	$\frac{dy}{dx}\leqslant 0 \Rightarrow 2(3x-1)^4(18x-1)\leqslant 0 \Rightarrow x\leqslant\frac{1}{18}\quad x=\frac{1}{3}$	B1ft, B1
		(2)
		(6 marks)

This may be marked as one complete question

(a)

M1: Uses the product rule $vu'+uv'$ with $u=2x$ and $v=(3x-1)^5$ or vice versa to achieve an expression of the form $A(3x-1)^5+Bx(3x-1)^4,\quad A,B>0$

Condone slips on the $(3x-1)$ and $2x$ terms but misreads on the question must be of equivalent difficulty. If in doubt use review.

Eg: $y=2x(3x+1)^5 \Rightarrow \frac{dy}{dx}=2(3x+1)^5+30x(3x+1)^4$ can potentially score 1010 in (a) and 11 in (b)

Eg: $y=2x(3x+1)^{15} \Rightarrow \frac{dy}{dx}=2(3x+1)^{15}+90x(3x+1)^{14}$ can potentially score 1010 in (a) and 11 in (b)

Eg: $y=2(3x+1)^5 \Rightarrow \frac{dy}{dx}=30(3x+1)^4$ is 0000 even if attempted using the product rule (as it is easier)

A1: A correct un-simplified expression. You may never see the lhs which is fine for all marks.

M1: Scored for taking a common factor of $(3x-1)^4$ out of $A(3x-1)^5 \pm Bx^n(3x-1)^4$ where $n=1$ or 2,to reach a form $(3x-1)^4\{........\}$ You may condone one slip in the $\{........\}$

Alternatively they take out a common factor of $2(3x-1)^4$ which can be scored in the same way

Example of one slip $2(3x-1)^5+30x(3x-1)^4=(3x-1)^4\{(3x-1)+30x\}$

If a different form is reached, see examples above, it is for equivalent work.

A1: Achieves a fully factorised simplified form $2(3x-1)^4(18x-1)$ which may be awarded in (b)

(b)

B1ft: For a final answer of either $x\leqslant\frac{1}{18}$ or $x=\frac{1}{3}$ Condone $x\leqslant\frac{2}{36}\quad x\leq 0.0\dot{5}\quad x=0.\dot{3}$

Do not allow $x=\frac{1}{3}$ if followed by $x\leqslant\frac{1}{3}$ Follow through on a linear factor of $(Ax+B)\leqslant 0\Rightarrow x...$ where $A,B\neq 0$. Watch for negative A's where the inequality would reverse.

It may be awarded within an equality such as $\frac{1}{3}\leqslant x\leqslant\frac{1}{18}$

B1: For a final answer of $x\leqslant\frac{1}{18}$ oe (and) $x=\frac{1}{3}$ oe with no other solutions. Ignore any references to and/or here. Misreads can score these marks

Question Number	Scheme	Marks
112	$\frac{dy}{dx}=-2e^{-2x}+2x$	M1A1
	At $x=0$ $\frac{dy}{dx}=-2\Rightarrow\frac{dx}{dy}=\frac{1}{2}$	M1
	Equation of normal is $y-(-2)=\frac{1}{2}(x-0)\Rightarrow y=\frac{1}{2}x-2$	M1 A1
		(5)
		(5 marks)

M1: Attempts to differentiate with $e^{-2x}\to Ae^{-2x}$ with any non -zero A, even 1.

Watch for $e^{-2x}\to Ae^{2x}$ which is M0 A0

A1: $\frac{dy}{dx}=-2e^{-2x}+2x$

M1: A correct method of finding the gradient of the normal at $x=0$

To score this the candidate must find the negative reciprocal of $\left.\frac{dy}{dx}\right|_{x=0}$

So for example candidates who find $\frac{dy}{dx}=e^{-2x}+2x$ should be using a gradient of -1

Candidates who write down $\frac{dy}{dx}=-2$ (from their calculators?) have an opportunity to score this mark and the next.

M1: An attempt at the equation of the normal at $(0,-2)$

To score this mark the candidate must be using the point $(0,-2)$ and a gradient that has been **changed from** $\left.\frac{dy}{dx}\right|_{x=0}$

Look for $y-(-2)=changed\left|\frac{dy}{dx}\right|_{x=0}(x-0)$ or $y=mx-2$ where $m=changed\left|\frac{dy}{dx}\right|_{x=0}$

If there is an attempt using $y=mx+c$ then it must proceed using $(0,-2)$ with $m=$ $changed\left|\frac{dy}{dx}\right|_{x=0}$

Question Number	Scheme	Marks
113(a)	Applies $\dfrac{vu'-uv'}{v^2}$ to $y=\dfrac{\ln\left(x^2+1\right)}{x^2+1}$ with $u=\ln\left(x^2+1\right)$ and $v=x^2+1$	
	$\dfrac{dy}{dx}=\dfrac{\left(x^2+1\right)\times\dfrac{2x}{x^2+1}-2x\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$	M1 A1
	$\dfrac{dy}{dx}=\dfrac{2x-2x\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$	A1
		(3)
(b)	Sets $2x-2x\ln\left(x^2+1\right)=0$	M1
	$2x\left(1-\ln\left(x^2+1\right)\right)=0\Rightarrow x=\pm\sqrt{e-1},$	M1,A1
	Sub $x=\pm\sqrt{e-1},0$ into $f(x)=\dfrac{\ln\left(x^2+1\right)}{x^2+1}$	dM1
	Stationary points $\left(\sqrt{e-1},\dfrac{1}{e}\right),\left(-\sqrt{e-1},\dfrac{1}{e}\right),\underline{\underline{(0,0)}}$	A1 $\underline{\underline{\text{B1}}}$
		(6)
		(9 marks)

(a)

M1: Attempts the quotient or product rule to achieve an expression in the correct form

Using the quotient rule achieves an expression of the form $\dfrac{dy}{dx}=\dfrac{\left(x^2+1\right)\times\dfrac{...}{x^2+1}-2x\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$

or the form $\dfrac{dy}{dx}=\dfrac{...-2x\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$ where $...=A$ or Ax

or using the product rule achieves and an expression $\dfrac{dy}{dx}=\left(x^2+1\right)^{-1}\times\dfrac{...}{x^2+1}-2x\left(x^2+1\right)^{-2}\ln\left(x^2+1\right)$

You may condone the omission of brackets ....especially on the denominator

A1: A correct un-simplified expression for $\dfrac{dy}{dx}$

$$\frac{dy}{dx}=\frac{\left(x^2+1\right)\times\dfrac{2x}{x^2+1}-2x\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}\text{ or }\frac{dy}{dx}=\left(x^2+1\right)^{-1}\times\frac{2x}{x^2+1}-2x\left(x^2+1\right)^{-2}\ln\left(x^2+1\right)$$

A1: $\dfrac{dy}{dx}=\dfrac{2x-2x\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$ or exact simplified equivalent such as $\dfrac{dy}{dx}=\dfrac{2x\left(1-\ln\left(x^2+1\right)\right)}{\left(x^2+1\right)^2}$.

Condone $\frac{dy}{dx}=\frac{2x-\ln\left(x^2+1\right)2x}{\left(x^2+1\right)^2}$ which may be a little ambiguous. The lhs $\frac{dy}{dx}=$ does not need to be seen. You may assume from the demand in the question that is what they are finding.
ISW can be applied here.

(b)

M1: Sets the numerator of their $\frac{dy}{dx}$, which must contain at least two terms, equal to 0

M1: For solving an equation of the form $\ln(x^2+1)=k,\quad k>0$ to get at least one non- zero value of x. Accept decimal answers. $x = awrt \pm 1.31$ The equation must be legitimately obtained from a numerator = 0

A1: Both $x=\pm\sqrt{e-1}$ scored from ± a correct numerator Condone $x=\pm\sqrt{e^1-1}$

dM1: Substitutes any of their non zero solutions to $\frac{dy}{dx}=0$ into $f(x)=\frac{\ln\left(x^2+1\right)}{x^2+1}$ to find at least one 'y' value. It is dependent upon both previous M's

A1: Both $\left(\sqrt{e-1},\frac{1}{e}\right),\left(-\sqrt{e-1},\frac{1}{e}\right)$ oe or the equivalent with $x=...,y=...$ ln e must be simplified

Condone $\left(\sqrt{e^1-1},\frac{1}{e^1}\right),\left(-\sqrt{e^1-1},\frac{1}{e^1}\right)$ but the y coordinates must be simplified as shown.

Condone $\left(\pm\sqrt{e-1},\frac{1}{e}\right)$ Withhold this mark if there are extra solutions to these apart from (0,0)

It can only be awarded from ± a correct numerator

B1: $(0,0)$ or the equivalent $x=0, y=0$

Notes:

(1) A candidate can "recover" and score all marks in (b) when they have an incorrect denominator in part (a) or a numerator the wrong way around in (a)

(2) A candidate who differentiates $\ln\left(x^2+1\right)\to\frac{1}{x^2+1}$ will probably only score (a) 100 (b) 100000

(3) A candidate who has $\frac{vu'+uv'}{v^2}$ cannot score anything more than (a) 000 (b)100001 as they would have $k<0$

(4) A candidate who attempts the product rule to get $\frac{dy}{dx}=\left(x^2+1\right)^{-1}\times\frac{1}{x^2+1}-\left(x^2+1\right)^{-2}\ln\left(x^2+1\right)=\frac{1-\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$

can score (a) 000 (b) 110100 even though they may obtain the correct non zero coordinates.

Question Number	Scheme	Marks
114(a)	$\frac{d}{d\theta}(\sec\theta)=\frac{d}{d\theta}(\cos\theta)^{-1}=-1\times(\cos\theta)^{-2}\times-\sin\theta$	M1
	$=\frac{1}{\cos\theta}\times\frac{\sin\theta}{\cos\theta}$	
	$=\sec\theta\tan\theta$	A1*
		(2)
(b)	$x=e^{\sec y}\Rightarrow\frac{dx}{dy}=e^{\sec y}\times\sec y\tan y$ oe	M1A1
	$\Rightarrow\frac{dy}{dx}=\frac{1}{e^{\sec y}\times\sec y\tan y}$	M1
	Uses $1+\tan^2 y=\sec^2 y$ with $\sec y=\ln x$ $\Rightarrow\tan y=\sqrt{(\ln x)^2-1}$	M1
	$\Rightarrow\frac{dy}{dx}=\frac{1}{x\times\ln x\times\sqrt{(\ln x)^2-1}}=\frac{1}{x\sqrt{(\ln x)^4-(\ln x)^2}}$ oe	A1
		(5)
		(7 marks)
Alt (b)	$\ln x=\sec y\Rightarrow\frac{1}{x}\frac{dx}{dy}=\sec y\tan y$	**M1A1**
	$\Rightarrow\frac{dy}{dx}=\frac{1}{x\times\sec y\tan y}$	**M1**
	Uses $1+\tan^2 y=\sec^2 y$ and $\sec y=\ln x$ $\Rightarrow\tan y=\sqrt{(\ln x)^2-1}$	**M1**
	$\Rightarrow\frac{dy}{dx}=\frac{1}{x\times\ln x\times\sqrt{(\ln x)^2-1}}=\frac{1}{x\sqrt{(\ln x)^4-(\ln x)^2}}$ oe	**A1**
		(5)

(a)

M1: Uses the chain rule to get $\pm 1\times(\cos\theta)^{-2}\times\sin\theta$

Alternatively uses the quotient rule to get $\dfrac{\cos\theta\times 0\pm 1\times\sin\theta}{\cos^2\theta}$ condoning the denominator as $\cos\theta^2$

When applying the quotient rule it is very difficult to see if the correct rule has been used. So only withhold this mark if an incorrect rule is quoted.

A1*: Completes proof with no errors (see below *) and shows line $\dfrac{1}{\cos\theta}\times\dfrac{\sin\theta}{\cos\theta}$, $\dfrac{\tan\theta}{\cos\theta}$ or $\dfrac{\sin\theta}{\cos\theta\times\cos\theta}$ before the given answer. The notation should be correct so do not allow if they start $y=\sec\theta\Rightarrow\dfrac{dy}{dx}=\sec\theta\tan\theta$

* You do not need to see $\dfrac{d}{d\theta}(\sec\theta)=...$ or $\dfrac{dy}{d\theta}$ anywhere in the solution

(b)

M1 Differentiates to get the rhs as $e^{\sec y}\times...$

A1 Completely correct differential inc the lhs $\dfrac{dx}{dy}=e^{\sec y}\times\sec y\tan y$

M1 Inverts **their** $\dfrac{dx}{dy}$ to get $\dfrac{dy}{dx}$.

The variable used **must be** consistent. Eg $\dfrac{dx}{dy}=e^{\sec y}\Rightarrow\dfrac{dy}{dx}=\dfrac{1}{e^{\sec x}}$ is M0

M1 For attempting to use $1+\tan^2 y=\sec^2 y$ with $\sec y=\ln x$

(You may condone $\ln x^2\to 2\ln x$ for the method mark)

It may be implied by $\tan y=\sqrt{\pm(\ln x)^2\pm 1}$ They must have a term in $\tan y$ to score this.

A valid alternative would be attempting to use $1+\cot^2 y=\operatorname{cosec}^2 y$ with $\operatorname{cosec} y=\dfrac{1}{\sqrt{1-\frac{1}{\ln^2 x}}}$ oe

A1 $\dfrac{dy}{dx}=\dfrac{1}{x\sqrt{(\ln x)^4-(\ln x)^2}}$ or exact equivalents such as $\dfrac{dy}{dx}=\dfrac{1}{x\sqrt{\ln^4 x-\ln^2 x}}$

Do not isw here. Withhold this mark if candidate then writes down $\dfrac{dy}{dx}=\dfrac{1}{x\sqrt{4(\ln x)-2(\ln x)}}$

Also watch for candidates who write $\dfrac{dy}{dx}=\dfrac{1}{x\sqrt{\ln x^4-\ln x^2}}$ which is incorrect (without the brackets)

Question Number	Scheme	Marks
115.	At P $x=-2 \Rightarrow y=3$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4}{2x+5}-\frac{3}{2}$	M1, A1
	$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\vert_{x=-2}=\frac{5}{2} \Rightarrow$ Equation of normal is $y-'3'=-\frac{2}{5}\left(x-(-2)\right)$	M1
	$\Rightarrow 2x+5y=11$	A1
		(5)
		(5 marks)

B1 $y = 3$ at point *P*. This may be seen embedded within their equation which may be a tangent

M1 Differentiates $\ln(2x+5) \rightarrow \frac{A}{2x+5}$ or equivalent. You may see $\ln(2x+5)^2 \rightarrow \frac{A(2x+5)}{(2x+5)^2}$

A1 $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{2x+5} - \frac{3}{2}$ oe. It need not be simplified.

M1 For using a correct method of finding the equation of the normal using their numerical value of $-\left.\frac{\mathrm{d}x}{\mathrm{d}y}\right|_{x=-2}$ as the gradient. Allow for $(y - '3') = -\left.\frac{\mathrm{d}x}{\mathrm{d}y}\right|_{x=-2}(x - -2)$, oe.

At least one bracket must be correct for their $(-2, 3)$

If the form $y = mx + c$ is used it is scored for proceeding as far as *c* = ..

A1 $\pm k(5y + 2x = 11)$ It must be in the form $ax + by = c$ as stated in the question

Score this mark once it is seen. Do not withhold it if they proceed to another form, $y = mx + c$ for example

If a candidate uses a graphical calculator to find the gradient they can score a maximum of B1 M0 A0 M1 A1

Question Number	Scheme	Marks
116(i) (a)	$y=2x\left(x^2-1\right)^5 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2-1\right)^5\times 2+2x\times 10x\left(x^2-1\right)^4$ $\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2-1\right)^4\left(2x^2-2+20x^2\right)=\left(x^2-1\right)^4\left(22x^2-2\right)$	M1A1 M1 A1 (4)
(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}\ldots 0\Rightarrow\left(22x^2-2\right)\ldots 0\Rightarrow$ critical values of $\pm\dfrac{1}{\sqrt{11}}$ $x\ldots\dfrac{1}{\sqrt{11}}\quad x,,\ -\dfrac{1}{\sqrt{11}}$	M1 A1 (2)
(ii)	$x=\ln(\sec 2y)\Rightarrow\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{1}{\sec 2y}\times 2\sec 2y\tan 2y$ $\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{2\tan 2y}=\dfrac{1}{2\sqrt{\sec^2 2y-1}}=\dfrac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$	B1 M1 M1 A1 (4) **10 marks**
Alt 1 (ii)	$x=\ln(\sec 2y)\Rightarrow\sec 2y=\mathrm{e}^x$ $\Rightarrow 2\sec 2y\tan 2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^x$ $\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{e}^x}{2\sec 2y\tan 2y}=\dfrac{\mathrm{e}^x}{2\mathrm{e}^x\sqrt{\sec^2 2y-1}}=\dfrac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$	B1 M1M1A1 **(4)**
Alt 2 (ii)	$y=\dfrac{1}{2}\arccos\left(\mathrm{e}^{-x}\right)\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{1}{2}\times\dfrac{1}{\sqrt{1-\left(\mathrm{e}^{-x}\right)^2}}\times-\mathrm{e}^{-x}$ $\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$	B1M1M1 A1 **(4)**

(i)(a)

M1 Attempts the product rule to differentiate $2x(x^2-1)^5$ to a form $A\left(x^2-1\right)^5+Bx^n\left(x^2-1\right)^4$ where $n=1$ or 2. and A, B >0 If the rule is stated it must be correct, and not with a "$-$" sign.

A1 Any unsimplified but correct form $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=2\left(x^2-1\right)^5+20x^2\left(x^2-1\right)^4$

M1 For taking a common factor of $\left(x^2-1\right)^4$ out of a suitable expression

Look for $A\left(x^2-1\right)^5 \pm Bx^n\left(x^2-1\right)^4=\left(x^2-1\right)^4\left\{A\left(x^2-1\right)\pm Bx^n\right\}$ but you may condone missing brackets

It can be scored from a *vu*'-*uv*' or similar.

A1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\left(x^2-1\right)^4\left(22x^2-2\right)$ Expect $\mathrm{g}(x)$ to be simplified but accept $\frac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2-1\right)^4 2\left(11x^2-1\right)$

There is no need to state g(x) and remember to isw after a correct answer. This must be in part (a).

(i)(b)

M1 Sets their $\frac{\mathrm{d}y}{\mathrm{d}x}\geqslant 0, >0$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and proceeds to find one of the critical values for **their** $\mathrm{g}(x)$ or their $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ rearranged and $\div\left(x^2-1\right)^4$ if g(x) not found. $\mathrm{g}(x)$ should be at least a 2TQ with real roots. If g(x) is factorised, the usual rules apply. The M cannot be awarded from work **just** on $\left(x^2-1\right)^4 \geqslant 0$ ie $x=\pm 1$

You may see and accept decimals for the M.

A1 cao $x\geqslant\frac{1}{\sqrt{11}}$ $x\leqslant-\frac{1}{\sqrt{11}}$ or exact equivalent only. Condone $x\geqslant\frac{1}{\sqrt{11}}$ $x\leqslant-\frac{1}{\sqrt{11}}$, with $x\geqslant 1, x\leqslant -1$

Accept exact equivalents such as $x\geqslant\frac{\sqrt{11}}{11}$ $x\leqslant-\frac{\sqrt{11}}{11}$; $|x|\geqslant\frac{1}{\sqrt{11}}$; $\left\{\left(-\infty,-\frac{\sqrt{11}}{11}\right]\cup\left[\frac{\sqrt{11}}{11},\infty\right)\right\}$

Condone the word "and" appearing between the two sets of values.

Withhold the final mark if $x\geqslant\frac{1}{\sqrt{11}}$ $x\leqslant-\frac{1}{\sqrt{11}}$, appears with values not in this region eg $x\leqslant 1, x\geqslant -1$

(ii)

B1 Differentiates and achieves a correct line involving $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$

Accept $\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{1}{\sec 2y}\times 2\sec 2y\tan 2y$, $\frac{\mathrm{d}x}{\mathrm{d}y}=-\frac{1}{\cos 2y}\times -2\sin 2y$ $2\sec 2y\tan 2y\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^x$

M1 For inverting their expression for $\frac{\mathrm{d}x}{\mathrm{d}y}$ to achieve an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The variables (on the rhs) must be consistent, you may condone slips on the coefficients but not the terms.
In the alternative method it is for correctly changing the subject

M1 Scored for using $\tan^2 2y=\pm 1\pm\sec^2 2y$ **and** $\sec 2y=\mathrm{e}^x$ to achieve $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x

Alternatively they could use $\sin^2 2y+\cos^2 2y=1$ with $\cos 2y=\mathrm{e}^{-x}$ to achieve $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x

For the M mark you may condone $\sec^2 2y=\left(\mathrm{e}^x\right)^2$ appearing as e^{x^2}

A1 cso $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$ Final answer, do not allow if students then simplify this to eg. $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\mathrm{e}^x-1}$

Condone $\frac{\mathrm{d}y}{\mathrm{d}x}=\pm\frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$ but do not allow $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$

Allow a misread on $x=\ln(\sec y)$ for the two method marks only

Question Number	Scheme	Marks
117 (a)	$P_0 = \dfrac{100}{1+3} + 40 = 65$	B1
		(1)
	$\dfrac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{kt} = C\mathrm{e}^{kt}$	M1
(b)	$\dfrac{\mathrm{d}P}{\mathrm{d}t} = \dfrac{\left(1+3\mathrm{e}^{-0.9t}\right)\times -10\mathrm{e}^{-0.1t} - 100\mathrm{e}^{-0.1t}\times -2.7\mathrm{e}^{-0.9t}}{\left(1+3\mathrm{e}^{-0.9t}\right)^2}$	M1 A1
		(3)
(c)(i)	At maximum $-10\mathrm{e}^{-0.1t} - 30\mathrm{e}^{-0.1t}\times \mathrm{e}^{-0.9t} + 270\mathrm{e}^{-0.1t}\times\mathrm{e}^{-0.9t} = 0$	
	$\mathrm{e}^{-0.1t}\left(-10+240\mathrm{e}^{-0.9t}\right) = 0$	
	$\mathrm{e}^{-0.9t} = \dfrac{10}{240}$ oe $\mathrm{e}^{0.9t} = 24$	M1
	$-0.9t = \ln\left(\dfrac{1}{24}\right) \Rightarrow t = \dfrac{10}{9}\ln(24) = 3.53$	M1, A1
(c) (ii)	Sub $t = 3.53 \Rightarrow P_T = 102$	A1
		(4)
(d)	40	B1
		(1)
		9 marks

(a)

B1 $\left(P_0 =\right)65$

(b)

M1 For sight of $\dfrac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{kt} = C\mathrm{e}^{kt}$ (Allow C =1)This may be within an incorrect product or quotient rule

M1 Scored for a full application of the quotient rule. If the formula is quoted it should be correct.

The denominator should be present even when the correct formula has been quoted.

In cases where a formula has not been quoted it is very difficult to judge that a correct formula has been used (due to the signs between the terms). So…………….

if the formula has not been quoted look for the **order** of the terms $\dfrac{(1+3\mathrm{e}^{-0.9t})\times p\mathrm{e}^{-0.1t} - q\mathrm{e}^{-0.1t}\times\mathrm{e}^{-0.9t}}{(1+3\mathrm{e}^{-0.9t})^2}$

$$\dfrac{(1+3\mathrm{e}^{-0.9t})\times p\mathrm{e}^{-0.1t} + q\mathrm{e}^{-0.1t}\times\mathrm{e}^{-0.9t}}{(1+3\mathrm{e}^{-0.9t})^2}$$

For the product rule. Look for $a\mathrm{e}^{-0.1t}\left(1+3\mathrm{e}^{-0.9t}\right)^{-1} \pm b\mathrm{e}^{-0.1t}\mathrm{e}^{-0.9t}\left(1+3\mathrm{e}^{-0.9t}\right)^{-2}$ either way around

Penalise if an incorrect formula is quoted . Condone missing brackets in both cases.

A1 A correct **unsimplified** answer.

Eg using quotient rule $\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right) = \frac{-10\mathrm{e}^{-0.1t}\left(1+3\mathrm{e}^{-0.9t}\right)+270\mathrm{e}^{-0.1t}\mathrm{e}^{-0.9t}}{\left(1+3\mathrm{e}^{-0.9t}\right)^2}$ oe $\frac{-10\mathrm{e}^{-0.1t}+240\mathrm{e}^{-1t}}{\left(1+3\mathrm{e}^{-0.9t}\right)^2}$ simplified

Eg using product rule $\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right) = -10\mathrm{e}^{-0.1t}\left(1+3\mathrm{e}^{-0.9t}\right)^{-1} + 270\mathrm{e}^{-0.1t}\mathrm{e}^{-0.9t}\left(1+3\mathrm{e}^{-0.9t}\right)^{-2}$ oe

Remember to isw after a correct (unsimplified) answer.

There is no need to have the $\frac{\mathrm{d}P}{\mathrm{d}t}$ and it could be called $\frac{\mathrm{d}y}{\mathrm{d}x}$

(c)(i) Do NOT allow any marks in here without sight/implication of $\frac{\mathrm{d}P}{\mathrm{d}t} = 0, \frac{\mathrm{d}P}{\mathrm{d}t} < 0$ OR $\frac{\mathrm{d}P}{\mathrm{d}t} > 0$

The question requires the candidate to find t using part (b) so it is possible to do this part using inequalities using the same criteria as we apply for the equality. All marks in (c) can be scored from an incorrect denominator (most likely v), no denominator, or using a numerator the wrong way around ie $uv' - u'v$

M1 Sets their $\frac{\mathrm{d}P}{\mathrm{d}t} = 0$ or the numerator of their $\frac{\mathrm{d}P}{\mathrm{d}t} = 0$, factorises out or cancels a term in $\mathrm{e}^{-0.1t}$ to reach a form $A\mathrm{e}^{\pm 0.9t} = B$ oe. Alternatively they could combine terms to reach $A\mathrm{e}^{-t} = B\mathrm{e}^{-0.1t}$ or equivalent

Condone a double error on $\mathrm{e}^{-0.1t} \times \mathrm{e}^{-0.9t} = \mathrm{e}^{-0.1t \times -0.9t}$ or similar before factorising. **Look for correct indices**.

If they use the product rule then expect to see their $\frac{\mathrm{d}P}{\mathrm{d}t} = 0$ followed by multiplication of $\left(1+3\mathrm{e}^{-0.9t}\right)^2$ before similar work to the quotient rule leads to a form $A\mathrm{e}^{\pm 0.9t} = B$

M1 Having set the numerator of their $\frac{\mathrm{d}P}{\mathrm{d}t} = 0$ and obtained either $\mathrm{e}^{\pm kt} = C$ (k may be incorrect) or $A\mathrm{e}^{-t} = B\mathrm{e}^{-0.1t}$ it is awarded for the correct order of operations, taking ln's leading to $t = ..$

It cannot be awarded from impossible equations Eg $\mathrm{e}^{\pm 0.9t} = -0.3$

A1 cso $t =$ awrt 3.53 Accept $t = \frac{10}{9}\ln(24)$ or exact equivalent.

(c)(ii)

A1 awrt 102 following 3.53 The M's must have been awarded. This is not a B mark.

(d)

B1 Sight of 40

Condone statements such as $P \to 40$ $k\ldots 40$ or likewise

Question	Scheme	Marks
118(a)	$y=\dfrac{4x}{\left(x^2+5\right)}\Rightarrow\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=\dfrac{4\left(x^2+5\right)-4x\times 2x}{\left(x^2+5\right)^2}$	M1A1
	$\Rightarrow\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=\dfrac{20-4x^2}{\left(x^2+5\right)^2}$	M1A1
		(4)
(b)	$\dfrac{20-4x^2}{\left(x^2+5\right)^2}<0\Rightarrow x^2>\dfrac{20}{4}$ Critical values of $\pm\sqrt{5}$	M1
	$x<-\sqrt{5},\ x>\sqrt{5}$ or equivalent	dM1A1
		(3)
		7 marks

(a)M1 Attempt to use the **quotient rule** $\dfrac{vu'-uv'}{v^2}$ with $u=4x$ and $v=x^2+5$. If the rule is quoted it must be correct. It may be implied by their $u=4x, u'=A,\ v=x^2+5, v'=Bx$ followed by their $\dfrac{vu'-uv'}{v^2}$

If the rule is neither quoted nor implied only accept expressions of the form

$\dfrac{A\left(x^2+5\right)-4x\times Bx}{\left(x^2+5\right)^2},\ A,B>0$ You may condone missing (invisible) brackets

Alternatively uses the **product rule** with $u(/v)=4x$ and $v(/u)=\left(x^2+5\right)^{-1}$. If the rule is quoted it must be correct. It may be implied by their $u=4x, u'=A,\ v=x^2+5, v'=Bx\left(x^2+5\right)^{-2}$ followed by their $vu'+uv'$. If the rule is neither quoted nor implied only accept expressions of the form $A\left(x^2+5\right)^{-1}\pm 4x\times Bx\left(x^2+5\right)^{-2}$

A1 $\mathrm{f}'(x)$ correct (unsimplified). For the product rule look for versions of $4\left(x^2+5\right)^{-1}-4x\times 2x\left(x^2+5\right)^{-2}$

M1 Simplifies to the form $\mathrm{f}'(x)=\dfrac{A+Bx^2}{\left(x^2+5\right)^2}$ oe. This is not dependent so could be scored from $\dfrac{v'u-u'v}{v^2}$

When the product rule has been used the A of $A\left(x^2+5\right)^{-1}$ must be adapted.

A1 CAO. Accept exact equivalents such as $(\mathrm{f}'(x))=\dfrac{4\left(5-x^2\right)}{\left(x^2+5\right)^2}$, $-\dfrac{4x^2-20}{\left(x^2+5\right)^2}$ or $\dfrac{-4\left(x^2-5\right)}{x^4+10x^2+25}$

Remember to isw after a correct answer

(b)

M1 Sets their numerator either $=0$, <0, $\leqslant 0$, >0, $\geqslant 0$ and proceeds to at least **one** value for x

For example $20-4x^2\geqslant 0\Rightarrow x\geqslant\sqrt{5}$ will be M1 dM0 A0.

It cannot be scored from a numerator such as 4 or indeed $20+4x^2$

dM1 Achieves **two** critical values for their numerator $=0$ and chooses the outside region

Look for $x<$ smaller root, $x>$ bigger root. Allow decimals for the roots.

Condone $x\leqslant-\sqrt{5}, x\geqslant\sqrt{5}$ and expressions like $-\sqrt{5}>x>\sqrt{5}$

If they have $4x^2-20<0$ following an incorrect derivative they should be choosing the inside region

A1 Allow $x<-\sqrt{5}, x>\sqrt{5}$ $\quad x<-\sqrt{5}$ or $x>\sqrt{5}$ $\quad \{x:-\infty<x<-\sqrt{5}\cup\sqrt{5}<x<\infty\}$ $\quad |x|>\sqrt{5}$

Do not allow for the A1 $x<-\sqrt{5}$ and $x>\sqrt{5}$. $\sqrt{5}<x<-\sqrt{5}$ or $\{x:-\infty<x<-\sqrt{5}\cap\sqrt{5}<x<\infty\}$ but you may isw following a correct answer.

Question	Scheme	Marks
119 (i)	$y = e^{3x}\cos 4x \Rightarrow \left(\frac{dy}{dx}\right) = \cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x$	M1A1
	Sets $\cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x = 0 \Rightarrow 3\cos 4x - 4\sin 4x = 0$	M1
	$\Rightarrow x = \frac{1}{4}\arctan\frac{3}{4}$	M1
	$\Rightarrow x =$ awrt 0.9463 4dp	A1
		(5)
(ii)	$x = \sin^2 2y \Rightarrow \frac{dx}{dy} = 2\sin 2y \times 2\cos 2y$	M1A1
	Uses $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\frac{dx}{dy} = 2\sin 4y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5) **(10 marks)**
(ii) Alt I	$x = \sin^2 2y \Rightarrow x = \frac{1}{2} - \frac{1}{2}\cos 4y$	2nd M1
	$\frac{dx}{dy} = 2\sin 4y$	1st M1 A1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5)
(ii) Alt II	$x^{\frac{1}{2}} = \sin 2y \Rightarrow \frac{1}{2}x^{-\frac{1}{2}} = 2\cos 2y\frac{dy}{dx}$	M1A1
	Uses $x^{\frac{1}{2}} = \sin 2y$ AND $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5)
(ii) Alt III	$x^{\frac{1}{2}} = \sin 2y \Rightarrow 2y = \text{invsin}\, x^{\frac{1}{2}} \Rightarrow 2\frac{dy}{dx} = \frac{1}{\sqrt{1-x}} \times \frac{1}{2}x^{-\frac{1}{2}}$	M1A1
	Uses $x^{\frac{1}{2}} = \sin 2y$, $\sqrt{1-x} = \cos 2y$ and $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5)

(i)

M1 Uses the product rule $uv' + vu'$ to achieve $\left(\frac{dy}{dx}\right) = Ae^{3x}\cos 4x \pm Be^{3x}\sin 4x \quad A, B \neq 0$

The product rule if stated must be correct

A1 Correct (unsimplified) $\frac{dy}{dx} = \cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x$

M1 Sets/implies their $\frac{dy}{dx}=0$ factorises/cancels)by e^{3x} to form a trig equation in just $\sin 4x$ and $\cos 4x$

M1 Uses the identity $\frac{\sin 4x}{\cos 4x}\equiv\tan 4x$, moves from $\tan 4x=C, C\neq 0$ using correct order of operations to $x=...$ Accept $x=$ awrt 0.16 (radians) $x=$ awrt 9.22 (degrees) for this mark.
If a candidate elects to pursue a more difficult method using $R\cos(\theta+\alpha)$, for example, the minimum expectation will be that they get (1) the identity correct, and (2) the values of R and α correct to 2dp. So for the correct equation you would only accept $5\cos(4x+\text{awrt } 0.93)$ or $5\sin(4x-\text{awrt } 0.64)$ before using the correct order of operations to $x=...$
Similarly candidates who square $3\cos 4x-4\sin 4x=0$ then use a Pythagorean identity should proceed from either $\sin 4x=\frac{3}{5}$ or $\cos 4x=\frac{4}{5}$ before using the correct order of operations …

A1 $\Rightarrow x=$ awrt 0.9463.
Ignore any answers outside the domain. Withhold mark for additional answers inside the domain

(ii)

M1 Uses chain rule (or product rule) to achieve $\pm P\sin 2y\cos 2y$ as a derivative.
There is no need for lhs to be seen/ correct
If the product rule is used look for $\cancel{\frac{dx}{dy}}=\pm A\sin 2y\cos 2y\pm B\sin 2y\cos 2y$,

A1 Both lhs and rhs correct (unsimplified) . $\frac{dx}{dy}=2\sin 2y\times 2\cos 2y=(4\sin 2y\cos 2y)$ or

$1=2\sin 2y\times 2\cos 2y\frac{dy}{dx}$

M1 Uses $\sin 4y=2\sin 2y\cos 2y$ in their expression.
You may just see a statement such as $4\sin 2y\cos 2y=2\sin 4y$ which is fine.
Candidates who write $\cancel{\frac{dx}{dy}}=A\sin 2x\cos 2x$ can score this for $\cancel{\frac{dx}{dy}}=\frac{A}{2}\sin 4x$

M1 Uses $\frac{dy}{dx}=1\Big/\frac{dx}{dy}$ **for their expression in *y***. Concentrate on the trig identity rather than the coefficient in awarding this. Eg $\frac{dx}{dy}=2\sin 4y\Rightarrow\frac{dy}{dx}=2\operatorname{cosec}4y$ is condoned for the M1

If $\frac{dx}{dy}=a+b$ do not allow $\frac{dy}{dx}=\frac{1}{a}+\frac{1}{b}$

A1 $\frac{dy}{dx}=\frac{1}{2}\operatorname{cosec}4y$ If a candidate then proceeds to write down incorrect values of p and q then do not withhold the mark.

NB: See the three alternatives which may be less common but mark in exactly the same way. If you are uncertain as how to mark these please consult your team leader.

In **Alt I** the second M is for writing $x=\sin^2 2y\Rightarrow x=\pm\frac{1}{2}\pm\frac{1}{2}\cos 4y$ from $\cos 4y=\pm 1\pm 2\sin^2 2y$

In **Alt II** the first M is for writing $x^{\frac{1}{2}}=\sin 2y$ and differentiating both sides to $Px^{-\frac{1}{2}}=Q\cos 2y\cancel{\frac{dy}{dx}}$ oe

In **Alt 111** the first M is for writing $2y=\text{invsin}\left(x^{0.5}\right)$ oe and differentiating to $M\cancel{\frac{dy}{dx}}=N\frac{1}{\sqrt{1-\left(x^{0.5}\right)^2}}\times x^{-0.5}$

Question	Scheme	Marks
120(a)	$$\begin{array}{r} x^2+3 \\ x^2+x-6\overline{\big)x^4+x^3-3x^2+7x-6} \\ \underline{x^4+x^3-6x^2} \\ 3x^2+7x-6 \\ \underline{3x^2+3x-18} \\ 4x+12 \end{array}$$	M1 A1
	$\dfrac{x^4+x^3-3x^2+7x-6}{x^2+x-6} \equiv x^2+3+\dfrac{4(x+3)}{(x+3)(x-2)}$	M1
	$\equiv x^2+3+\dfrac{4}{(x-2)}$	A1
		(4)
(b)	$\mathrm{f}'(x) = 2x - \dfrac{4}{(x-2)^2}$	M1A1ft
	Subs $x=3$ into $\mathrm{f}'(x=3) = 2\times 3 - \dfrac{4}{(3-2)^2} = (2)$	M1
	Uses $m = -\dfrac{1}{\mathrm{f}'(3)} = \left(-\dfrac{1}{2}\right)$ with $(3, \mathrm{f}(3)) = (3,16)$ to form eqn of normal	
	$y-16 = -\dfrac{1}{2}(x-3)$ or equivalent cso	M1A1
		(5) **(9 marks)**

(a)

M1 Divides $x^4+x^3-3x^2+7x-6$ by x^2+x-6 to get a quadratic quotient and a linear or constant remainder. To award this look for a minimum of the following

$$\begin{array}{r} \boldsymbol{x^2(+..x)+A} \\ x^2+x-6\overline{\big)x^4+x^3-3x^2+7x-6} \\ \underline{\boldsymbol{x^4+x^3-6x^2}} \\ \\ \overline{\boldsymbol{(Cx)+D}} \end{array}$$

If they divide by $(x+3)$ first they must then divide their by result by $(x-2)$ before they score this method mark. Look for a cubic quotient with a constant remainder followed by a quadratic quotient and a constant remainder

Note: FYI Dividing by $(x+3)$ gives x^3-2x^2+3x-2 and $\left(x^3-2x^2+3x-2\right)\div\left(x-2\right) = x^2+3$ with a remainder of 4.

Division by $(x-2)$ first is possible but difficult.....please send to review any you feel deserves credit.

A1 Quotient = x^2+3 and Remainder = $4x+12$

M1 Factorises x^2+x-6 and writes their expression in the appropriate form.

$$\left(\frac{x^4+x^3-3x^2+7x-6}{x^2+x-6}\right) \equiv \text{Their Quadratic Quotient} + \frac{\text{Their Linear Remainder}}{(x+3)(x-2)}$$

It is possible to do this part by partial fractions. To score M1 under this method the terms must be correct and it must be a full method to find both "numerators"

A1 $x^2+3+\frac{4}{(x-2)}$ or $A=3, B=4$ but don't penalise after a correct statement.

(b)

M1 $x^2+A+\frac{B}{x-2} \to 2x \pm \frac{B}{(x-2)^2}$

If they fail in part (a) to get a function in the form $x^2+A+\frac{B}{x-2}$ allow candidates to pick up this method mark for differentiating a function of the form $x^2+Px+Q+\frac{Rx+S}{x \pm T}$ using the quotient rule oe.

A1ft $x^2+A+\frac{B}{x-2} \to 2x - \frac{B}{(x-2)^2}$ oe. FT on their numerical A, B for for $x^2+A+\frac{B}{x-2}$ only

M1 Subs $x=3$ into their $\mathrm{f}'(x)$ in an attempt to find a numerical gradient

M1 For the correct method of finding an equation of a normal. The gradient must be $-\frac{1}{\text{their f}'(3)}$ and the point must be $(3, \mathrm{f}(3))$. Don't be overly concerned about how they found their f(3), ie accept x=3 y =.

Look for $y-\mathrm{f}(3) = -\frac{1}{\mathrm{f}'(3)}(x-3)$ or $\left(y-\mathrm{f}(3)\right) \times -\mathrm{f}'(3) = (x-3)$

If the form $y=mx+c$ is used they must proceed as far as c =

A1 cso $y-16=-\frac{1}{2}(x-3)$ oe such as $2y+x-35=0$ but remember to isw after a correct answer.

Alt (a) attempted by equating terms.

Alt (a)	$x^4+x^3-3x^2+7x-6 \equiv (x^2+A)(x^2+x-6)+B(x+3)$ Compare 2 terms (or substitute 2 values) AND solve simultaneously ie $x^2 \Rightarrow A-6=-3, \quad x \Rightarrow A+B=7, \quad$ const $\Rightarrow$ −6A+3B=− 6 $A=3, B=4$	M1 M1 A1,A1

1st Mark M1 Scored for multiplying by (x^2+x-6) and cancelling/dividing to achieve $x^4+x^3-3x^2+7x-6 \equiv (x^2+A)(x^2+x-6)+B(x \pm 3)$

3rd Mark M1 Scored for comparing two terms (or for substituting two values) **AND** solving simultaneously to get values of ***A*** and ***B***.

2nd Mark A1 Either $A=3$ or $B=4$. One value may be correct by substitution of say $x=-3$

4th Mark A1 Both $A=3$ and $B=4$

Alt (b) is attempted by the quotient (or product rule)

ALT (b) 1st 3 marks	$\mathrm{f}'(x) = \frac{\left(x^2+x-6\right)\left(4x^3+3x^2-6x+7\right)-\left(x^4+x^3-3x^2+7x-6\right)(2x+1)}{\left(x^2+x-6\right)^2}$ Subs $x=3$ into	M1A1 M1

M1 Attempt to use the **quotient rule** $\frac{vu'-uv'}{v^2}$ with $u=x^4+x^3-3x^2+7x-6$ and $v=x^2+x-6$ and achieves an expression of the form $\mathrm{f}'(x) = \frac{\left(x^2+x-6\right)\left(..x^3........\right)-\left(x^4+x^3-3x^2+7x-6\right)(..x..)}{\left(x^2+x-6\right)^2}$.

Use a similar approach to the product rule with $u=x^4+x^3-3x^2+7x-6$ and $v=\left(x^2+x-6\right)^{-1}$

Note that this can score full marks from a partially solved part (a) where $\mathrm{f}(x) \equiv x^2+3+\frac{4x+12}{x^2+x-6}$

Question Number	Scheme	Marks
121.(a)	$p = 4\pi^2$ or $(2\pi)^2$	B1
		(1)
(b)	$x = (4y - \sin 2y)^2 \Rightarrow \dfrac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y)$	M1A1
	Sub $y = \dfrac{\pi}{2}$ into	
	Þ $\dfrac{dx}{dy} = 24\pi$ $(= 75.4)$ / $\dfrac{dy}{dx} = \dfrac{1}{24\pi} (= 0.013)$	M1
	Equation of tangent	M1
	Using with $x = 0$ Þ $y = \dfrac{\pi}{3}$ cso	M1, A1
		(6)
		(7 marks)
Alt (b) I	$x = (4y - \sin 2y)^2 \Rightarrow x^{0.5} = 4y - \sin 2y$ $\Rightarrow 0.5x^{-0.5} \dfrac{dx}{dy} = 4 - 2\cos 2y$	M1A1
Alt (b) II	$x = (16y^2 - 8y \sin 2y + \sin^2 2y)$ $\Rightarrow 1 = 32y \dfrac{dy}{dx} - 8\sin 2y \dfrac{dy}{dx} - 16y\cos 2y \dfrac{dy}{dx} + 4\sin 2y \cos 2y \dfrac{dy}{dx}$ Or $1dx = 32y\, dy - 8\sin 2y\, dy - 16y\cos 2y\, dy + 4\sin 2y \cos 2y\, dy$	M1A1

(a)

B1 $p = 4\pi^2$ or exact equivalent $(2\pi)^2$

Also allow $x = 4\pi^2$

(b)

M1 Uses the chain rule of differentiation to get a form

$A(4y - \sin 2y)(B \pm C\cos 2y), \quad A, B, C \neq 0$ on the right hand side

Alternatively attempts to expand and then differentiate using product rule and chain rule to a form $x = \left(16y^2 - 8y\sin 2y + \sin^2 2y\right) \Rightarrow \dfrac{dx}{dy} = Py \pm Q\sin 2y \pm Ry\cos 2y \pm S\sin 2y\cos 2y \quad P, Q, R, S \neq 0$

A second method is to take the square root first. To score the method look for a differentiated expression of the form $Px^{-0.5}... = 4 - Q\cos 2y$

A third method is to multiply out and use implicit differentiation. Look for the correct terms, condoning errors on just the constants.

A1 $\dfrac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y)$ or $\dfrac{dy}{dx} = \dfrac{1}{2(4y - \sin 2y)(4 - 2\cos 2y)}$ with both sides correct. The lhs may be seen elsewhere if clearly linked to the rhs.

In the alternative $\dfrac{dx}{dy} = 32y - 8\sin 2y - 16y\cos 2y + 4\sin 2y\cos 2y$

M1 Sub $y = \dfrac{\pi}{2}$ into their $\dfrac{dx}{dy}$ or inverted $\dfrac{dx}{dy}$. Evidence could be minimal, eg $y = \dfrac{\pi}{2} \Rightarrow \dfrac{dx}{dy} = ...$

It is not dependent upon the previous M1 but it must be a changed $x = (4y - \sin 2y)^2$

M1 Score for a correct method for finding the equation of the tangent at $\left('4\pi^2', \dfrac{\pi}{2}\right)$.

Allow for $y - \dfrac{\pi}{2} = \dfrac{1}{\text{their numerical } \left(\frac{dx}{dy}\right)}\left(x - \text{their } 4\pi^2\right)$

Allow for $\left(y - \dfrac{\pi}{2}\right) \times \text{their numerical } \left(\frac{dx}{dy}\right) = \left(x - \text{their } 4\pi^2\right)$

Even allow for $y - \dfrac{\pi}{2} = \dfrac{1}{\text{their numerical } \left(\frac{dx}{dy}\right)}(x - p)$

It is possible to score this by stating the equation $y = \dfrac{1}{24\pi}x + c$ as long as $\left('4\pi^2', \dfrac{\pi}{2}\right)$ is used in a subsequent line.

M1 Score for writing their equation in the form $y = mx + c$ and stating the value of 'c'

Or setting $x = 0$ in their $y - \dfrac{\pi}{2} = \dfrac{1}{24\pi}\left(x - 4\pi^2\right)$ and solving for y.

Alternatively using the gradient of the line segment AP = gradient of tangent.

Look for $\dfrac{\frac{\pi}{2} - y}{4\pi^2} = \dfrac{1}{24\pi} \Rightarrow y = ..$ Such a method scores the previous M mark as well.

At this stage all of the constants must be numerical. It is not dependent and it is possible to score this using the "incorrect" gradient.

A1 cso $y = \dfrac{\pi}{3}$. You do not have to see $\left(0, \dfrac{\pi}{3}\right)$

Question Number	Scheme	Marks
122.(a)	$x^2-3kx+2k^2=(x-2k)(x-k)$	B1
	$2-\dfrac{(x-5k)(x-k)}{(x-2k)(x-k)}=2-\dfrac{(x-5k)}{(x-2k)}=\dfrac{2(x-2k)-(x-5k)}{(x-2k)}$	M1
	$=\dfrac{x+k}{(x-2k)}$	A1*
		(3)
(b)	Applies $\dfrac{vu'-uv'}{v^2}$ to $y=\dfrac{x+k}{x-2k}$ with $u=x+k$ and $v=x-2k$	
	$\Rightarrow \mathrm{f}'(x)=\dfrac{(x-2k)\times1-(x+k)\times1}{(x-2k)^2}$	M1, A1
	$\Rightarrow \mathrm{f}'(x)=\dfrac{-3k}{(x-2k)^2}$	A1
		(3)
(c)	If $\mathrm{f}'(x)=\dfrac{-Ck}{(x-2k)^2}\Rightarrow \mathrm{f}(x)$ is an increasing function as $\mathrm{f}'(x)>0$,	M1
	$\mathrm{f}'(x)=\dfrac{-3k}{(x-2k)^2}>0$ for all values of x as $\dfrac{\text{negative}\times\text{negative}}{\text{positive}}=\text{positive}$	A1
		(2)
		(8 marks)

(a)

B1 For seeing $x^2-3kx+2k^2=(x-2k)(x-k)$ anywhere in the solution

M1 For writing as a single term or two terms with the same denominator

Score for $2-\dfrac{(x-5k)}{(x-2k)}=\dfrac{2(x-2k)-(x-5k)}{(x-2k)}$ or

$$2-\frac{(x-5k)(x-k)}{(x-2k)(x-k)}=\frac{2(x-2k)(x-k)-(x-5k)(x-k)}{(x-2k)(x-k)} \qquad \left(=\frac{x^2-k^2}{x^2-3kx+2k^2}\right)$$

A1* Proceeds without any errors (including bracketing) to $=\dfrac{x+k}{(x-2k)}$

(b)

M1 Applies $\frac{vu'-uv'}{v^2}$ to $y=\frac{x+k}{x-2k}$ with $u=x+k$ and $v=x-2k$.

If the rule it is stated it must be correct. It can be implied by $u=x+k$ and $v=x-2k$ with their u', v' *and* $\frac{vu'-uv'}{v^2}$

If it is neither stated nor implied only accept expressions of the form $f'(x)=\frac{x-2k-x\pm k}{(x-2k)^2}$

The mark can be scored for applying the product rule to $y=(x+k)(x-2k)^{-1}$ If the rule it is stated it must be correct. It can be implied by $u=x+k$ and $v=(x-2k)^{-1}$ with their u', v' and $vu'+uv'$

If it is neither stated nor implied only accept expressions of the form

$f'(x)=(x-2k)^{-1}\pm(x+k)(x-2k)^{-2}$

Alternatively writes $y=\frac{x+k}{x-2k}$ as $y=1+\frac{3k}{x-2k}$ and differentiates to $\frac{dy}{dx}=\frac{A}{(x-2k)^2}$

A1 Any correct form (unsimplified) form of $f'(x)$.

$f'(x)=\frac{(x-2k)\times1-(x+k)\times1}{(x-2k)^2}$ by quotient rule

$f'(x)=(x-2k)^{-1}-(x+k)(x-2k)^{-2}$ by product rule

and $f'(x)=\frac{-3k}{(x-2k)^2}$ by the third method

A1 cao $f'(x)=\frac{-3k}{(x-2k)^2}$. Allow $f'(x)=\frac{-3k}{x^2-4kx+4k^2}$

As this answer is not given candidates you may allow recovery from missing brackets

(c) Note that this is B1 B1 on e pen. We are scoring it M1 A1

M1 If in part (b) $f'(x)=\frac{-Ck}{(x-2k)^2}$, look for f($x$) is an increasing function as $f'(x)$ / gradient >0

Accept a version that states as $k<0\Rightarrow -Ck>0$ hence increasing

If in part (b) $f'(x)=\frac{(+)Ck}{(x-2k)^2}$, look for f($x$) is an decreasing function as $f'(x)$ / gradient<0

Similarly accept a version that states as $k<0\Rightarrow(+)Ck<0$ hence decreasing

A1 Must have $f'(x)=\frac{-3k}{(x-2k)^2}$ and give a reason that links the gradient with its sign.

There must have been reference to the sign of both numerator and denominator to justify the overall positive sign.

Question Number	Scheme	Marks
123.(a)	$\mathrm{f}(x)=\dfrac{4x+1}{x-2}, \quad x>2$	
	Applies $\dfrac{vu'-uv'}{v^2}$ to get $\dfrac{(x-2)\times 4-(4x+1)\times 1}{(x-2)^2}$	M1A1
	$=\dfrac{-9}{(x-2)^2}$	A1*
		(3)
(b)	$\dfrac{-9}{(x-2)^2}=-1\Rightarrow x=..$	M1
	$(5,7)$	A1,A1
		(3)
		6 marks
Alt 1.(a)	$\mathrm{f}(x)=\dfrac{4x+1}{x-2}=4+\dfrac{9}{x-2}$	
	Applies chain rule to get $\mathrm{f}'(x)=A(x-2)^{-2}$	M1
	$=-9(x-2)^{-2}=\dfrac{-9}{(x-2)^2}$	A1, A1*
		(3)

(a)

M1 Applies the quotient rule to $\mathrm{f}(x)=\frac{4x+1}{x-2}$ with $u=4x+1$ and $v=x-2$. If the rule is quoted it must be correct. It may be implied by their $u=4x+1, v=x-2, u'=.., v'=..$ followed by $\frac{vu'-uv'}{v^2}$.

If neither quoted nor implied only accept expressions of the form $\frac{(x-2)\times A-(4x+1)\times B}{(x-2)^2}$ $A, B>0$ allowing for a sign slip inside the brackets.

Condone missing brackets for the method mark but not the final answer mark.

Alternatively they could apply the product rule with $u=4x+1$ and $v=(x-2)^{-1}$. If the rule is quoted it must be correct. It may be implied by their $u=4x+1, v=(x-2)^{-1}, u'=.., v'=..$ followed by $vu'+uv'$.

If it is neither quoted nor implied only accept expressions of the form/ or equivalent to the form

$$(x-2)^{-1}\times C+(4x+1)\times D(x-2)^{-2}$$

A third alternative is to use the Chain rule. For this to score there must have been some attempt to divide first to achieve $\mathrm{f}(x)=\frac{4x+1}{x-2}=..+\frac{..}{x-2}$ before applying the chain rule to get $\mathrm{f}'(x)=A(x-2)^{-2}$

A1 A correct and unsimplified form of the answer.

Accept $\frac{(x-2)\times 4-(4x+1)\times 1}{(x-2)^2}$ from the quotient rule

Accept $\frac{4x-8-4x-1}{(x-2)^2}$ from the quotient rule even if the brackets were missing in line 1

Accept $(x-2)^{-1}\times 4+(4x+1)\times -1(x-2)^{-2}$ or equivalent from the product rule

Accept $9\times -1(x-2)^{-2}$ from the chain rule

A1* Proceeds to achieve the given answer $=\frac{-9}{(x-2)^2}$. Accept $-9(x-2)^{-2}$

All aspects must be correct including the bracketing.

If they differentiated using the product rule the intermediate lines must be seen.

Eg. $(x-2)^{-1}\times 4+(4x+1)\times -1(x-2)^{-2}=\frac{4}{(x-2)}-\frac{4x+1}{(x-2)^2}=\frac{4(x-2)-(4x+1)}{(x-2)^2}=\frac{-9}{(x-2)^2}$

(b)

M1 Sets $\frac{-9}{(x-2)^2}=-1$ and proceeds to $x=\ldots$.

The minimum expectation is that they multiply by $(x-2)^2$ and then either, divide by -1 before square rooting or multiply out before solving a 3TQ equation.

A correct answer of $x = 5$ would also score this mark following $\frac{-9}{(x-2)^2}=-1$ as long as no incorrect work is seen.

A1 $x = 5$

A1 (5, 7) or $x = 5$, $y = 7$. Ignore any reference to $x = -1$ (and $y = 1$). Do not accept 21/3 for 7

If there is an extra solution, $x>2$, then withhold this final mark.

Question Number	Scheme	Marks
124(a)	$x = 8\frac{\pi}{8}\tan\left(2\times\frac{\pi}{8}\right) = \pi$	B1*
		(1)
(b)	$\frac{\mathrm{d}x}{\mathrm{d}y} = 8\tan 2y + 16y\sec^2(2y)$	M1A1A1
	At P $\frac{\mathrm{d}x}{\mathrm{d}y} = 8\tan 2\frac{\pi}{8} + 16\frac{\pi}{8}\sec^2\left(2\times\frac{\pi}{8}\right) = \{8+4\pi\}$	M1
	$\frac{y-\frac{\pi}{8}}{x-\pi} = \frac{1}{8+4\pi}$, accept $y-\frac{\pi}{8} = 0.049(x-\pi)$	M1A1
	$\Rightarrow (8+4\pi)y = x + \frac{\pi^2}{2}$	A1
		(7)
		(8 marks)

(a)

B1* Either sub $y = \frac{\pi}{8}$ into $x = 8y\tan(2y) \Rightarrow x = 8\times\frac{\pi}{8}\tan\left(2\times\frac{\pi}{8}\right) = \pi$

Or sub $x = \pi$, $y = \frac{\pi}{8}$ into $x = 8y\tan(2y) \Rightarrow \pi = 8\times\frac{\pi}{8}\tan\left(2\times\frac{\pi}{8}\right) = \pi\times 1 = \pi$

This is a proof and therefore an expectation that at least one intermediate line must be seen, including a term in tangent.

Accept as a minimum $y = \frac{\pi}{8} \Rightarrow x = \pi\tan\left(\frac{\pi}{4}\right) = \pi$

Or $\pi = \pi\times\tan\left(\frac{\pi}{4}\right) = \pi$ ✓

This is a given answer however, and as such there can be no errors.

(b)

M1 Applies the product rule to $8y\tan 2y$ achieving $A\tan 2y + By\sec^2(2y)$

A1 One term correct. Either $8\tan 2y$ or $+16y\sec^2(2y)$. There is no requirement for $\frac{dx}{dy} =$

A1 Both lhs and rhs correct. $\frac{dx}{dy} = 8\tan 2y + 16y\sec^2(2y)$

It is an intermediate line and the expression does not need to be simplified.

Accept $\frac{dx}{dy} = \tan 2y \times 8 + 8y \times 2\sec^2(2y)$ or $\frac{dy}{dx} = \frac{1}{\tan 2y \times 8 + 8y \times 2\sec^2(2y)}$ or using implicit differentiation $1 = \tan 2y \times 8\frac{dy}{dx} + 8y \times 2\sec^2(2y)\frac{dy}{dx}$

M1 For fully substituting $y = \frac{\pi}{8}$ into their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ to find a 'numerical' value

Accept $\frac{dx}{dy} =$ awrt 20.6 or $\frac{dy}{dx} =$ awrt 0.05 as evidence

M1 For a correct attempt at an equation of the tangent at the point $\left(\pi, \frac{\pi}{8}\right)$.

The gradient must be an inverted numerical value of their $\frac{dx}{dy}$

Look for $\frac{y - \frac{\pi}{8}}{x - \pi} = \frac{1}{\text{numerical }\frac{dx}{dy}}$,

Watch for negative reciprocals which is M0
If the form $y = mx + c$ is used it must be a full method to find a ‘numerical’ value to c.

A1 A correct equation of the tangent.

Accept $\frac{y - \frac{\pi}{8}}{x - \pi} = \frac{1}{8 + 4\pi}$ or if $y = mx + c$ is used accept $m = \frac{1}{8 + 4\pi}$ and $c = \frac{\pi}{8} - \frac{\pi}{8 + 4\pi}$

Watch for answers like this which are correct $x - \pi = (8 + 4\pi)\left(y - \frac{\pi}{8}\right)$

Accept the decimal answers awrt 2sf $y = 0.049x + 0.24$, awrt 2sf $21y = x + 4.9$, $\frac{y - 0.39}{x - 3.1} = 0.049$

Accept a mixture of decimals and $\pi's$ for example $20.6\left(y - \frac{\pi}{8}\right) = x - \pi$

A1 Correct answer and solution only. $(8 + 4\pi)y = x + \frac{\pi^2}{2}$

Accept exact alternatives such as $4(2 + \pi)y = x + 0.5\pi^2$ and because the question does not ask for a and b to be simplified in the form $ay = x + b$, accept versions like

$(8 + 4\pi)y = x + \frac{\pi}{8}(8 + 4\pi) - \pi$ and $(8 + 4\pi)y = x + (8 + 4\pi)\left(\frac{\pi}{8} - \frac{\pi}{8 + 4\pi}\right)$

Question Number	Scheme	Marks
125.(a)	$P=\dfrac{800e^{0}}{1+3e^{0}},=\dfrac{800}{1+3}=200$	M1,A1
		(2)
(b)	$250=\dfrac{800e^{0.1t}}{1+3e^{0.1t}}$	
	$250(1+3e^{0.1t})=800e^{0.1t}\Rightarrow 50e^{0.1t}=250,\ \Rightarrow e^{0.1t}=5$	M1,A1
	$t=\dfrac{1}{0.1}\ln(5)$	M1
	$t=10\ln(5)$	A1
		(4)
(c)	$P=\dfrac{800e^{0.1t}}{1+3e^{0.1t}}\Rightarrow\dfrac{dP}{dt}=\dfrac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t}-800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^{2}}$	M1,A1
	At t=10	
	$\dfrac{dP}{dt}=\dfrac{(1+3e)\times 80e-240e^{2}}{(1+3e)^{2}}=\dfrac{80e}{(1+3e)^{2}}$	M1,A1
		(4)
(d)	$P=\dfrac{800e^{0.1t}}{1+3e^{0.1t}}=\dfrac{800}{e^{-0.1t}+3}\Rightarrow P_{max}=\dfrac{800}{3}=266$. Hence P cannot be 270	B1
		(1)
		(11 marks)

(a)

M1 Sub t = 0 into P **and** use $e^{0}=1$ in at least one of the two cases. Accept $P=\dfrac{800}{1+3}$ as evidence

A1 200. Accept this for both marks as long as no incorrect working is seen.

(b)

M1 Sub P=250 into $P=\dfrac{800e^{0.1t}}{1+3e^{0.1t}}$, cross multiply, collect terms in $e^{0.1t}$ **and** proceed to $Ae^{0.1t}=B$

Condone bracketing issues and slips in arithmetic.

If they divide terms by $e^{0.1t}$ you should expect to see $Ce^{-0.1t}=D$

A1 $e^{0.1t}=5$ or $e^{-0.1t}=0.2$

M1 Dependent upon gaining $e^{0.1t} = E$, for taking ln's of both sides and proceeding to t=…

Accept $e^{0.1t} = E \Rightarrow 0.1t = \ln E \Rightarrow t = ...$ It could be implied by $t = \text{awrt } 16.1$

A1 $t = 10\ln(5)$

Accept exact equivalents of this as long as a and b are integers. Eg. $t = 5\ln(25)$ is fine.

(c)

M1 Scored for a full application of the quotient rule and knowing that $\frac{d}{dt}e^{0.1t} = ke^{0.1t}$ and NOT $kte^{0.1t}$

If the rule is quoted it must be correct.

It may be implied by their $u = 800e^{0.1t}, v = 1+3e^{0.1t}, u' = pe^{0.1t}, v' = qe^{0.1t}$ followed by $\frac{vu'-uv'}{v^2}$.

If it is neither quoted nor implied only accept expressions of the form

$$\frac{(1+3e^{0.1t})\times pe^{0.1t} - 800e^{0.1t}\times qe^{0.1t}}{(1+3e^{0.1t})^2}$$

Condone missing brackets.

You may see the chain or product rule applied to

For applying the product rule see question 1 but still insist on $\frac{d}{dt}e^{0.1t} = ke^{0.1t}$

For the chain rule look for

$$P = \frac{800e^{0.1t}}{1+3e^{0.1t}} = \frac{800}{e^{-0.1t}+3} \Rightarrow \frac{dP}{dt} = 800\times\left(e^{-0.1t}+3\right)^{-2}\times -0.1e^{-0.1t}$$

A1 A correct unsimplified answer to

$$\frac{dP}{dt} = \frac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t} - 800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^2}$$

M1 For substituting $t = 10$ into their $\frac{dP}{dt}$, NOT P

Accept numerical answers for this. 2.59 is the numerical value if $\frac{dP}{dt}$ was correct

A1 $\frac{dP}{dt} = \frac{80e}{(1+3e)^2}$ or equivalent such as $\frac{dP}{dt} = 80e(1+3e)^{-2}, \frac{80e}{1+6e+9e^2}$

Note that candidates who substitute $t = 10$ before differentiation will score 0 marks

(d)

B1 Accept solutions from substituting P=270 and showing that you get an unsolvable equation

Eg. $270 = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow -27 = e^{0.1t} \Rightarrow 0.1t = \ln(-27)$ which has no answers.

Eg. $270 = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow -27 = e^{0.1t} \Rightarrow e^{0.1t} / e^x$ is never negative

Accept solutions where it implies the max value is 266.6 or 267. For example accept sight of $\frac{800}{3}$, with a comment 'so it cannot reach 270', or a large value of t ($t > 99$) being substituted in to get 266.6 or 267 with a similar statement, or a graph drawn with an asymptote marked at 266.6 or 267

Do not accept exp's cannot be negative or you cannot ln a negative number without numerical evidence.

Look for both a statement and a comment

Question Number	Scheme	Marks
126.	$\dfrac{dy}{dx} = 4e^{4x} + 4x^3 + 8$	M1, A1
	Puts $\dfrac{dy}{dx} = 0$ to give $x^3 = -2 - e^{4x}$	A1 *
		(3)

M1 Two (of the four) terms differentiated correctly

A1 All correct $\dfrac{dy}{dx} = 4e^{4x} + 4x^3 + 8$

A1* States or sets $\dfrac{dy}{dx} = 0$, and proceeds correctly to achieve printed answer $x^3 = -2 - e^{4x}$.

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Question Number	Scheme	Marks
127. (i)	$\dfrac{dx}{dy} = 4\sec^2 2y \tan 2y$	B1
	Use $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$	M1
	Uses $\tan^2 2y = \sec^2 2y - 1$ and $\sec 2y = \sqrt{x}$ to get $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of just x	M1
	$\dfrac{dy}{dx} = \dfrac{1}{4x(x-1)^{\frac{1}{2}}}$ (conclusion stated with no errors previously)	A1*
		(4)
(ii)	$\dfrac{dy}{dx} = (x^2 + x^3) \times \dfrac{2}{2x} + (2x + 3x^2)\ln 2x$	M1 A1 A1
	When $x = \dfrac{e}{2}$, $\dfrac{dy}{dx} = 3(\frac{e}{2}) + 4(\frac{e}{2})^2 = 3(\frac{e}{2}) + e^2$	dM1 A1
		(5)
(iii)	$f'(x) = \dfrac{(x+1)^{\frac{1}{3}}(-3\sin x) - 3\cos x(\frac{1}{3}(x+1)^{-\frac{2}{3}})}{(x+1)^{\frac{2}{3}}}$	M1 A1
	$f'(x) = \dfrac{-3(x+1)(\sin x) - \cos x}{(x+1)^{\frac{4}{3}}}$	A1
		(3)
		12 marks

(i)

B1 $\frac{dx}{dy} = 4\sec^2 2y \tan 2y$ or equivalent such as $\frac{dx}{dy} = 4\frac{\sin 2y \cos 2y}{\cos^4 2y}$

Accept $\frac{dx}{dy} = 2\sec 2y \tan 2y \times \sec 2y + 2\sec 2y \tan 2y \times \sec 2y$, $1 = 4\sec^2 2y \tan 2y \frac{dy}{dx}$

M1 Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to get an expression for $\frac{dy}{dx}$ in terms of y.

It may be scored following the award of the next M1 if $\frac{dx}{dy}$ has been written in terms of x.

Follow through on their expression but condone errors on the coefficient.

For example $\frac{dx}{dy} = 2\sec^2 2y \tan 2y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sec^2 2y \tan 2y}$ is OK as is $\frac{dy}{dx} = \frac{2}{\sec^2 2y \tan 2y}$

Do not accept y's going to x's. So for example $\frac{dx}{dy} = 2\sec^2 2y \tan 2y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sec^2 2x \tan 2x}$ is M0

M1 Uses $\tan^2 2y = \sec^2 2y - 1$ **and** $x = \sec^2 2y$ to get their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of just x

$\frac{dx}{dy} = 2\sec^2 2y \tan 2y \Rightarrow \frac{dx}{dy} = 2x\sqrt{(\sec^2 2y - 1)} = 2x\sqrt{x-1}$ is incorrect but scores M1

$\frac{dx}{dy} = 2\sec 2y \tan 2y \Rightarrow \frac{dx}{dy} = 2\sec 2y\sqrt{(\sec^2 2y - 1)} = 2\sqrt{x}\sqrt{x-1}$ is incorrect but scores M1

The stating and use $1 + \tan^2 x = \sec^2 x$ is unlikely to score this mark.

Accept $1 + \tan^2 2y = \sec^2 2y \Rightarrow 1 + \tan^2 2y = x \Rightarrow \tan 2y = \sqrt{x-1}$. So $\frac{dy}{dx} = \frac{1}{4\sec^2 2y \tan 2y} = \frac{1}{4x\sqrt{x-1}}$

Condone examples where the candidate adapts something to get the given answer

Eg. $\frac{dy}{dx} = \frac{1}{4\sec^2 2y \tan^2 2y} = \frac{1}{4\sec^2 2y\left(\sec^2 2y - 1\right)} = \frac{1}{4x\sqrt{(x-1)}}$

A1* Completely correct solution. This is a ‘show that’ question and it is a requirement that all elements are seen.

(ii)

M1 Uses the product rule to differentiate $(x^2 + x^3)\ln 2x$. If the rule is stated it must be correct. It may be implied by their $u = .., u' = .., v = .., v' = ..$ followed by $vu' + uv'$. If the rule is neither stated nor implied only accept expressions of the form $\ln 2x \times (ax + bx^2) + (x^2 + x^3) \times \frac{C}{x}$

It is acceptable to multiply out the expression to get $x^2 \ln 2x + x^3 \ln 2x$ but the product rule must be applied to both terms

A1 One term correct (unsimplified). Either $(x^2+x^3)\times\dfrac{2}{2x}$ or $(2x+3x^2)\ln 2x$

If they have multiplied out before differentiating the equivalent would be two of the four terms correct.

A1 A completely correct (unsimplified) expression $\dfrac{dy}{dx}=(x^2+x^3)\times\dfrac{2}{2x}+(2x+3x^2)\ln 2x$

dM1 Fully substitutes $x=\dfrac{e}{2}$ (dependent on previous M mark) into their expression for $\dfrac{dy}{dx}=....$ Implied by awrt 11.5

A1 $\dfrac{dy}{dx}=3(\frac{e}{2})+e^2$ Accept equivalent simplified forms such as $\dfrac{dy}{dx}=1.5e+e^2$, $\dfrac{dy}{dx}=e(1.5+e)$, $\dfrac{dy}{dx}=\dfrac{e(2e+3)}{2}$

(iii)

M1 Uses quotient rule with $u=3\cos x$, $v=(x+1)^{\frac{1}{3}}$, $u'=\pm A\sin x$ and $v'=B(x+1)^{-\frac{2}{3}}$.

If the rule is quoted it must be correct. It may be implied by their $u=3\cos x$, $v=(x+1)^{\frac{1}{3}}$, $u'=\pm A\sin x$ and $v'=B(x+1)^{-\frac{2}{3}}$ followed by $\dfrac{vu'-uv'}{v^2}$

Additionally this could be scored by using the product rule with $u=3\cos x$, $v=(x+1)^{-\frac{1}{3}}$ $u'=\pm A\sin x$ and $v'=B(x+1)^{-\frac{4}{3}}$. If the rule is quoted it must be correct. It may be implied by their $u=3\cos x$, $v=(x+1)^{-\frac{1}{3}}$ $u'=\pm A\sin x$ and $v'=B(x+1)^{-\frac{4}{3}}$ followed by $vu'+uv'$

If it is not quoted nor implied only accept either of the two expressions

1) Using quotient form $\dfrac{(x+1)^{\frac{1}{3}}\times\pm A\sin x-3\cos x\times B(x+1)^{-\frac{2}{3}}}{\left((x+1)^{\frac{1}{3}}\right)^2}$ or $\dfrac{(x+1)^{\frac{1}{3}}\times\pm A\sin x-3\cos x\times B(x+1)^{-\frac{2}{3}}}{(x+1)^{\frac{1}{9}}}$

2) Using product form $(x+1)^{-\frac{1}{3}}\times\pm A\sin x+3\cos x\times B(x+1)^{-\frac{4}{3}}$

A1 A correct gradient. Accept $f'(x)=\dfrac{(x+1)^{\frac{1}{3}}(-3\sin x)-3\cos x(\frac{1}{3}(x+1)^{-\frac{2}{3}})}{\left((x+1)^{\frac{1}{3}}\right)^2}$

or $f'(x)=(x+1)^{-\frac{1}{3}}\times-3\sin x+3\cos x\times-\dfrac{1}{3}(x+1)^{-\frac{4}{3}}$

A1 $f'(x)=\dfrac{-3(x+1)(\sin x)-\cos x}{(x+1)^{\frac{4}{3}}}$ oe. or a statement that $g(x)=-3(x+1)(\sin x)-\cos x$ oe.

Question Number	Scheme	Marks
128	$f'(x)=50x^2e^{2x}+50xe^{2x}$ oe.	M1A1
	Puts $f'(x)=0$ to give x = -1 and x = 0 or one coordinate	dM1A1
	Obtains $(0,-16)$ and $(-1, 25e^{-2}-16)$ CSO	A1
		(5)
		(5 marks)

Notes for Question 128

No marks can be scored in part (a) unless you see differentiation as required by the question.

(a)

M1 Uses $vu'+uv'$. If the rule is quoted it must be correct.
It can be implied by their $u=..,v=...,u'=...,v'=...$ followed by their $vu'+uv'$
If the rule is not quoted nor implied only accept answers of the form $Ax^2e^{2x}+Bxe^{2x}$

A1 $f'(x)=50x^2e^{2x}+50xe^{2x}$.
Allow un simplified forms such as $f'(x)=25x^2\times 2e^{2x}+50x\times e^{2x}$

dM1 Sets $f'(x)=0$, factorises out/ or cancels the e^{2x} leading to at least one solution of x
This is dependent upon the first M1 being scored.

A1 Both x = -1 and x = 0 or one complete coordinate . Accept $(0,-16)$ and $(-1, 25e^{-2}-16)$ or $(-1, awrt -12.6)$

A1 CSO. Obtains both solutions from differentiation. Coordinates can be given in any way.
x= - 1, 0 $y=\frac{25}{e^2}-16, -16$ or linked together by coordinate pairs $(0,-16)$ and $(-1, 25e^{-2}-16)$ but the ‘pairs’ must be correct and exact.

Question Number	Scheme	Marks
129(a)	$\frac{dx}{dy} = 2\times 3\sec 3y \sec 3y \tan 3y = \left(6\sec^2 3y \tan 3y\right)$ $\left(\text{oe } \frac{6\sin 3y}{\cos^3 3y}\right)$	M1A1 **(2)**
(b)	Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to obtain $\frac{dy}{dx} = \frac{1}{6\sec^2 3y \tan 3y}$	M1
	$\tan^2 3y = \sec^2 3y - 1 = x - 1$	B1
	Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x-1$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in just x.	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$ CSO	A1* **(4)**
(c)	$\frac{d^2y}{dx^2} = \frac{0-[6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$	M1A1
	$\frac{d^2y}{dx^2} = \frac{6-9x}{36x^2(x-1)^{\frac{3}{2}}} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$	dM1A1
		(4)
		(10 marks)
Alt 1 to 5(a)	$x = (\cos 3y)^{-2} \Rightarrow \frac{dx}{dy} = -2(\cos 3y)^{-3} \times -3\sin 3y$	M1A1
Alt 2 to 5 (a)	$x = \sec 3y \times \sec 3y \Rightarrow \frac{dx}{dy} = \sec 3y \times 3\sec 3y \tan 3y + \sec 3y \times 3\sec 3y \tan 3y$	M1A1
Alt 1 To 5 (c)	$\frac{d^2y}{dx^2} = \tfrac{1}{6}[x^{-1}(-\tfrac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$	M1A1
	$= \tfrac{1}{6}x^{-2}(x-1)^{-\frac{3}{2}}[x(-\tfrac{1}{2}) + (-1)(x-1)]$	dM1
	$= \tfrac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ oe	A1
		(4)

EXPERT TUITION

Notes for Question 129

(a)

M1 Uses the chain rule to get $A\sec 3y\sec 3y\tan 3y = \left(A\sec^2 3y\tan 3y\right)$.

There is no need to get the lhs of the expression. Alternatively could use the chain rule on $(\cos 3y)^{-2} \Rightarrow A(\cos 3y)^{-3}\sin 3y$

or the quotient rule on $\dfrac{1}{(\cos 3y)^2} \Rightarrow \dfrac{\pm A\cos 3y\sin 3y}{(\cos 3y)^4}$

A1 $\dfrac{dx}{dy} = 2\times 3\sec 3y\sec 3y\tan 3y$ or equivalent. There is no need to simplify the rhs but both sides must be correct.

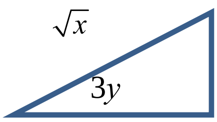
(b)

M1 Uses $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$ to get an expression for $\dfrac{dy}{dx}$. Follow through on their $\dfrac{dx}{dy}$

Allow slips on the coefficient but not trig expression.

B1 Writes $\tan^2 3y = \sec^2 3y - 1$ or an equivalent such as $\tan 3y = \sqrt{\sec^2 3y - 1}$ and uses $x = \sec^2 3y$ to obtain either $\tan^2 3y = x - 1$ or $\tan 3y = (x-1)^{\frac{1}{2}}$

All elements **must be present.**

Accept 

$\sqrt{x-1}$ $\cos 3y = \dfrac{1}{\sqrt{x}} \Rightarrow \tan 3y = \sqrt{x-1}$

If the differential was in terms of $\sin 3y, \cos 3y$ it is awarded for $\sin 3y = \dfrac{\sqrt{x-1}}{\sqrt{x}}$

M1 Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ or equivalent to get $\dfrac{dy}{dx}$ in just x. Allow slips on the signs in $\tan^2 3y = \sec^2 3y - 1$.

It may be implied- see below

A1* CSO. This is a given solution and you must be convinced that all steps are shown.

Note that the two method marks may occur the other way around

Eg. $\dfrac{dx}{dy} = 6\sec^2 3y\tan 3y = 6x(x-1)^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{6x(x-1)^{\frac{1}{2}}}$

Scores the 2nd method

Scores the 1st method

The above solution will score M1, B0, M1, A0

Notes for Question 129 Continued

Example 1- Scores 0 marks in part (b)

$$\frac{\mathrm{d}x}{\mathrm{d}y}=6\sec^2 3y\tan 3y \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{6\sec^2 3x\tan 3x}=\frac{1}{6\sec^2 3x\sqrt{\sec^2 3x-1}}=\frac{1}{6x(x-1)^{\frac{1}{2}}}$$

Example 2- Scores M1B1M1A0

$$\frac{\mathrm{d}x}{\mathrm{d}y}=2\sec^2 3y\tan 3y \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sec^2 3y\tan 3y}=\frac{1}{2\sec^2 3y\sqrt{\sec^2 3y-1}}=\frac{1}{2x(x-1)^{\frac{1}{2}}}$$

(c) **Using Quotient and Product Rules**

M1 Uses the quotient rule $\frac{vu'-uv'}{v^2}$ with $u=1$ and $v=6x(x-1)^{\frac{1}{2}}$ **and** achieving

$u'=0$ and $v'=A(x-1)^{\frac{1}{2}}+Bx(x-1)^{-\frac{1}{2}}$.

If the formulae are quoted, **both** must be correct. If they are not quoted nor implied by their working allow expressions of the form

$$\cancel{\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)}=\frac{0-[A(x-1)^{\frac{1}{2}}+Bx(x-1)^{-\frac{1}{2}}]}{\left(6x(x-1)^{\frac{1}{2}}\right)^2} \quad \text{or} \quad \cancel{\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)}=\frac{0-A(x-1)^{\frac{1}{2}}\pm Bx(x-1)^{-\frac{1}{2}}}{Cx^2(x-1)}$$

A1 Correct un simplified expression $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{0-[6(x-1)^{\frac{1}{2}}+3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$ oe

dM1 Multiply numerator and denominator by $(x-1)^{\frac{1}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{1}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe

Notes for Question 129 Continued

(c) Using Product and Chain Rules

M1 Writes $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6x(x-1)^{\frac{1}{2}}} = Ax^{-1}(x-1)^{-\frac{1}{2}}$ and uses the product rule with u or $v = Ax^{-1}$ and

v or $u = (x-1)^{-\frac{1}{2}}$. If any rule is quoted it must be correct.

If the rules are not quoted nor implied then award if you see an expression of the form

$(x-1)^{-\frac{3}{2}} \times Bx^{-1} \pm C(x-1)^{-\frac{1}{2}} \times x^{-2}$

A1 $\cancel{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}} = \frac{1}{6}[x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$

dM1 Factorises out / uses a common denominator of $x^{-2}(x-1)^{-\frac{3}{2}}$ producing a linear factor/numerator which must be simplified by collecting like terms. Need a single fraction.

A1 Correct simplified expression $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} == \frac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ *oe*

(c) **Using Quotient and Chain rules Rules**

M1 Uses the quotient rule $\frac{vu'-uv'}{v^2}$ with $u = (x-1)^{-\frac{1}{2}}$ and $v = 6x$ **and** achieving

$u' = A(x-1)^{-\frac{3}{2}}$ and $v' = B$.

If the formulae is quoted, it must be correct. If it is not quoted nor implied by their working allow an expression of the form

$$\cancel{\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)} = \frac{Cx(x-1)^{-\frac{3}{2}} - D(x-1)^{-\frac{1}{2}}}{Ex^2}$$

A1 Correct un simplified expression $\cancel{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}} = \frac{6x \times -\frac{1}{2}(x-1)^{-\frac{3}{2}} - (x-1)^{-\frac{1}{2}} \times 6}{(6x)^2}$

dM1 Multiply numerator and denominator by $(x-1)^{\frac{3}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{3}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{(2-3x)x^{-2}(x-1)^{-\frac{3}{2}}}{12}$

Notes for Question 129 Continued	
(c)	**Using just the chain rule**
M1	Writes $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6x(x-1)^{\frac{1}{2}}} = \frac{1}{(36x^3 - 36x^2)^{\frac{1}{2}}} = (36x^3 - 36x^2)^{-\frac{1}{2}}$ and proceeds by the chain rule to $A(36x^3 - 36x^2)^{-\frac{3}{2}}(Bx^2 - Cx)$.
M1	Would automatically follow under this method if the first M has been scored

Question Number	Scheme	Marks
130.	(a) (i) Applies $vu'+uv'$ to $x^{\frac{1}{2}}\ln x$	M1
	$=\ln x\times\frac{1}{2}x^{-\frac{1}{2}}+x^{\frac{1}{2}}\times\frac{1}{x}$	A1
	$=\frac{\ln x}{2\sqrt{x}}+\frac{1}{\sqrt{x}}$	A1*
		(3)
	(ii) Sets $\frac{\ln x}{2\sqrt{x}}+\frac{1}{\sqrt{x}}=0$	M1
	Multiplies (or factorises) by $\sqrt{x}$, with correct *ln* work leading to x=	M1
	$P=(e^{-2},-2e^{-1})$ oe.	A1,A1
		(4)
	(b) Applies $\frac{vu'-uv'}{v^2}$ to $y=\frac{x-k}{x+k}$ with $u=x-k$ and $v=x+k$	M1
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+k)\times1-(x-k)\times1}{(x+k)^2}$	A1
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2k}{(x+k)^2}$	A1
	As $k>0\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}>0\Rightarrow C$ has no turning points	B1
		(4)
		(11 marks)

Question Number	Scheme	Marks
131.	(a) $x=3$ or $(3,0)$	B1
		(1)
	(b) $\frac{dx}{dy}=\frac{1}{2}(9+16y-2y^2)^{-\frac{1}{2}}(16-4y)$ oe	M1M1A1
		(3)
	(c) Substitute y=0 into their $\frac{dx}{dy}$ or $\frac{dy}{dx}$	M1
	$\Rightarrow \frac{dx}{dy}=\frac{8}{3}$ or $\frac{dy}{dx}=\frac{3}{8}$	A1
	Uses their numerical $\frac{dy}{dx}$ and their 3 from (3,0) to find equation of tangent	
	$\frac{y-0}{x-3}=\frac{3}{8}$ or $y-0=\frac{3}{8}(x-3)$	M1A1
		(4)
		(8 marks)

EXPERT TUITION

Question Number	Scheme	Marks
132.(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(\cos 2x) = -2\sin 2x$	B1
	Applies $\frac{vu'-uv'}{v^2}$ to $\frac{\cos 2x}{\sqrt{x}}$ $\quad = \frac{\sqrt{x}\times -2\sin 2x - \cos 2x \times \frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x})^2}$	M1A1
	$= \frac{-2\sqrt{x}\sin 2x - \frac{1}{2}x^{-\frac{1}{2}}\cos 2x}{x}$	
		(3)
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}(\sec^2 3x) = 2\sec 3x \times 3\sec 3x \tan 3x \, (= 6\sec^2 3x \tan 3x)$	M1
	$= 6(1+\tan^2 3x)\tan 3x$	dM1
	$= 6(\tan 3x + \tan^3 3x)$	A1
		(3)
(c)	$x = 2\sin\left(\frac{y}{3}\right) \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = \frac{2}{3}\cos\left(\frac{y}{3}\right)$	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\frac{2}{3}\cos\left(\frac{y}{3}\right)} = \frac{1}{\frac{2}{3}\sqrt{\left(1-\sin^2\left(\frac{y}{3}\right)\right)}} = \frac{1}{\frac{2}{3}\sqrt{1-\left(\frac{x}{2}\right)^2}}$	dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{\sqrt{4-x^2}}$ cao	A1
		(4)
		(10 marks)
Alt 132(c)	$y = 3\arcsin\left(\frac{x}{2}\right) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \times \frac{1}{2}$	M1dM1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{\sqrt{4-x^2}}$	A1
	M1 Rearranging to $y = A\arcsin Bx$ and differentiating to $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{A}{\sqrt{1-Bx^2}}$	
	dM1 As above, but form of the rhs must be correct $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{C}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$	**(4)**
	A1 Correct but un simplified answer	

Notes for Question 132

(a)

B1 Award for the sight of $\frac{d}{dx}(\cos 2x) = -2\sin 2x$. This could be seen in their differential.

M1 Applies $\frac{vu'-uv'}{v^2}$ to $\frac{\cos 2x}{\sqrt{x}}$

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{\sqrt{x}\times\pm A\sin 2x - \cos 2x\times Bx^{-\frac{1}{2}}}{(\sqrt{x}\,)^2 \text{ or } x^{\frac{1}{4}}}$$

A1 Award for a correct answer. This does not need to be simplified.

Alt (a) using the product rule

B1 Award for the sight of $\frac{d}{dx}(\cos 2x) = -2\sin 2x$. This could be seen in their differential.

M1 Applies $vu'+uv'$ to $x^{-\frac{1}{2}}\cos 2x$. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their $vu'+uv'$) then only accept answers of the form

$$\pm Ax^{-\frac{1}{2}}\sin 2x - Bx^{-\frac{3}{2}}\cos 2x$$

A1 Award for a correct answer. This does not need to be simplified.

$$-2x^{-\frac{1}{2}}\sin 2x - \frac{1}{2}x^{-\frac{3}{2}}\cos 2x$$

(b)

M1 Award for a correct application of the chain rule on $\sec^2 3x$

Sight of $C\sec 3x\sec 3x\tan 3x$ is sufficient

dM1 Replacing $\sec^2 3x = 1+\tan^2 3x$ in their derivative to create an expression in just $\tan 3x$. It is dependent upon the first M being scored.

A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

Alt (b) using the product rule

M1 Writes $\sec^2 3x$ as $\sec 3x\times\sec 3x$ and uses the product rule with $u' = A\sec 3x\tan 3x$ and $v' = B\sec 3x\tan 3x$ to produce a derivative of the form $A\sec 3x\tan 3x\sec 3x + B\sec 3x\tan 3x\sec 3x$

dM1 Replaces $\sec^2 3x$ with $1+\tan^2 3x$ to produce an expression in just $\tan 3x$. It is dependent upon the first M being scored.

Notes for Question 132 Continued
A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

Alt (b) using $\sec 3x = \dfrac{1}{\cos 3x}$ **and proceeding by the chain or quotient rule**

M1 Writes $\sec^2 3x$ as $(\cos 3x)^{-2}$ and differentiates to $A(\cos 3x)^{-3}\sin 3x$

Alternatively writes $\sec^2 3x$ as $\dfrac{1}{(\cos 3x)^2}$ and achieves $\dfrac{(\cos 3x)^2 \times 0 - 1 \times A\cos 3x \sin 3x}{\left(\cos^2 3x\right)^2}$

dM1 Uses $\dfrac{\sin 3x}{\cos 3x} = \tan 3x$ and $\dfrac{1}{\cos^2 3x} = \sec^2 3x$ and $\sec^2 3x = 1 + \tan^2 3x$ in their derivative to create an expression in just $\tan 3x$. It is dependent upon the first M being scored.

A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

Alt (b) using $\sec^2 3x = 1 + \tan^2 3x$

M1 Writes $\sec^2 3x$ as $1 + \tan^2 3x$ and
uses chain rule to produce a derivative of the form $A\tan 3x \sec^2 3x$
or the product rule to produce a derivative of the form $C\tan 3x \sec^2 3x + D\tan 3x \sec^2 3x$

dM1 Replaces $\sec^2 3x = 1 + \tan^2 3x$ to produce an expression in just $\tan 3x$. It is dependent upon the first M being scored.

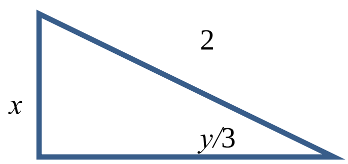
A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

(c)

M1 Award for knowing the method that $\sin\left(\dfrac{y}{3}\right)$ differentiates to $\cos\left(\dfrac{y}{3}\right)$ The lhs does not need to be correct/present. Award for $2\sin\left(\dfrac{y}{3}\right) \to A\cos\left(\dfrac{y}{3}\right)$

A1 $x = 2\sin\left(\dfrac{y}{3}\right) \Rightarrow \dfrac{dx}{dy} = \dfrac{2}{3}\cos\left(\dfrac{y}{3}\right)$. Both sides must be correct

dM1 Award for inverting their $\dfrac{dx}{dy}$ and using $\sin^2\dfrac{y}{3} + \cos^2\dfrac{y}{3} = 1$ to produce an expression for $\dfrac{dy}{dx}$ in terms of x only. It is dependent upon the first M 1 being scored.
An alternative to Pythagoras is a triangle.



$$\sin\left(\frac{y}{3}\right) = \frac{x}{2} \Rightarrow \cos\left(\frac{y}{3}\right) = \frac{\sqrt{4 - x^2}}{2}$$

Notes for Question 132 Continued
Candidates who write $\dfrac{dy}{dx}=\dfrac{3}{2\cos\left(\arcsin\left(\frac{x}{2}\right)\right)}$ do not score the mark. BUT $\dfrac{dy}{dx}=\dfrac{3}{2\sqrt{1-\sin^2\left(\arcsin\left(\frac{x}{2}\right)\right)}}$ does score M1 as they clearly use a correct Pythagorean identity as required by the notes. A1 $\dfrac{dy}{dx}=\dfrac{3}{\sqrt{4-x^2}}$. Expression must be in its simplest form. Do not accept $\dfrac{dy}{dx}=\dfrac{3}{2\sqrt{1-\frac{1}{4}x^2}}$ or $\dfrac{dy}{dx}=\dfrac{1}{\frac{1}{3}\sqrt{4-x^2}}$ for the final A1

Question Number	Scheme	Marks
133.(i)	$\text{cosec}2x = \dfrac{1}{\sin 2x}$	M1
	$= \dfrac{1}{2\sin x \cos x}$	M1
	$= \dfrac{1}{2}\text{cosec}x \sec x \Rightarrow \quad \lambda = \dfrac{1}{2}$	A1
		(3)
(ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\sec^2\theta + 3\sec\theta = 2(\sec^2\theta - 1)$	M1
	$\sec^2\theta + 3\sec\theta + 2 = 0$	
	$(\sec\theta + 2)(\sec\theta + 1) = 0$	M1
	$\sec\theta = -2, -1$	A1
	$\cos\theta = -0.5, -1$	M1
	$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$	A1A1
		(6)
		(9 marks)
ALT (ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\times\dfrac{1}{\cos^2\theta} + 3\times\dfrac{1}{\cos\theta} = 2\times\dfrac{\sin^2\theta}{\cos^2\theta}$	
	$3 + 3\cos\theta = 2\sin^2\theta$	
	$3 + 3\cos\theta = 2(1 - \cos^2\theta)$	M1
	$2\cos^2\theta + 3\cos\theta + 1 = 0$	
	$(2\cos\theta + 1)(\cos\theta + 1) = 0 \Rightarrow \cos\theta = -0.5, -1$	M1A1
	$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$	M1,A1,A1
		(6)
		(9 marks)

Notes for Question 133
(i) M1 Uses the identity $\operatorname{cosec}2x = \dfrac{1}{\sin 2x}$ M1 Uses the correct identity for $\sin 2x = 2\sin x\cos x$ in their expression. Accept $\sin 2x = \sin x\cos x + \cos x\sin x$ A1 $\lambda = \dfrac{1}{2}$ following correct working (ii) M1 Replaces $\tan^2\theta$ by $\pm\sec^2\theta \pm 1$ to produce an equation in just $\sec\theta$ M1 Award for a forming a 3TQ=0 in $\sec\theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\sec\theta$ If they replace $\sec\theta = \dfrac{1}{\cos\theta}$ it is for forming a 3TQ in $\cos\theta$ and applying a correct method for finding two answers to $\cos\theta$ A1 Correct answers to $\sec\theta = -2, -1$ or $\cos\theta = -\dfrac{1}{2}, -1$ M1 Award for using the identity $\sec\theta = \dfrac{1}{\cos\theta}$ and proceeding to find at least one value for θ. If the 3TQ was in cosine then it is for finding at least one value of θ. A1 Two correct values of θ. All method marks must have been scored. Accept two of $120°, 180°, 240°$ or two of $\dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19 A1 All three answers correct. They must be given in terms of π as stated in the question. Accept $0.\dot{6}\pi, 1.\dot{3}\pi, \pi$ Withhold this mark if further values in the range are given. All method marks must have been scored. Ignore any answers outside the range. Alt (ii) M1 Award for replacing $\sec^2\theta$ with $\dfrac{1}{\cos^2\theta}$, $\sec\theta$ with $\dfrac{1}{\cos\theta}$, $\tan^2\theta$ with $\dfrac{\sin^2\theta}{\cos^2\theta}$ multiplying through by $\cos^2\theta$ (seen in at least 2 terms) and replacing $\sin^2\theta$ with $\pm 1 \pm \cos^2\theta$ to produce an equation in just $\cos\theta$ M1 Award for a forming a 3TQ=0 in $\cos\theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\cos\theta$ A1 $\cos\theta = -\dfrac{1}{2}, -1$ M1 Proceeding to finding at least one value of θ from an equation in $\cos\theta$. A1 Two correct values of θ. All method marks must have been scored Accept two of $120°, 180°, 240°$ or two of $\dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19 A1 All three answers correct. They must be given in terms of π as stated in the question.

Notes for Question 133 Continued
Accept $0.\dot{6}\pi, 1.\dot{3}\pi, \pi$ All method marks must have been scored. Withhold this mark if further values in the range are given. Ignore any answers outside the range

Question Number	Scheme	Marks
134.(a)	$f(x)=0 \Rightarrow x^2+3x+1=0$	
	$\Rightarrow x=\dfrac{-3\pm\sqrt{5}}{2}=$ awrt -0.382, -2.618	M1A1
		(2)
(b)	Uses $vu'+uv'$ $\quad$ $\text{f}'(x)=e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x$	M1A1A1
		(3)
		(5 marks)

Notes for Question 134

(a)

M1 Solves $x^2+3x+1=0$ by completing the square or the formula, producing two 'non integer answers. **Do not accept factorisation here** . Accept awrt -0.4 and -2.6 for this mark

A1 Answers correct. Accept awrt -0.382, -2.618.

Accept just the answers for both marks. Don't withhold the marks for incorrect labelling.

(b)

M1 Applies the product rule $vu'+uv'$ to $(x^2+3x+1)e^{x^2}$.

If the rule is quoted it must be correct and there must have been some attempt to differentiate both terms.
If the rule is not quoted (nor implied by their working, ie. terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') only accept answers of the form

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\mathrm{f}'(x)=e^{x^2}(Ax+B)+(x^2+3x+1)Cxe^{x^2}$$

A1 One term of $\mathrm{f}'(x) = e^{x^2}(2x+3) + (x^2+3x+1)e^{x^2}\times 2x$ correct.

There is no need to simplify

A1 A fully correct (un simplified) answer $\mathrm{f}'(x)=e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2$

Question Number	Scheme	Marks
135.	(a) $-32=(2w-3)^5 \Rightarrow w=\frac{1}{2}$ oe	M1A1
		(2)
	(b) $\frac{dy}{dx}=5\times(2x-3)^4\times 2$ or $10(2x-3)^4$	M1A1
	When $x=\frac{1}{2}$, Gradient = 160	M1
	Equation of tangent is $'160'=\frac{y-(-32)}{x-'\frac{1}{2}'}$ oe	dM1
	$y=160x-112$ cso	A1
		(5)
		(7 marks)

(a) M1 Substitute ***y***=-32 into $y=(2w-3)^5$ **and** proceed to w=….. [Accept positive sign used of y, ie y=+32]

A1 Obtains w or $x=\frac{1}{2}$ oe with no incorrect working seen. Accept alternatives such as 0.5.

Sight of just the answer would score both marks as long as no incorrect working is seen.

(b) M1 Attempts to differentiate $y=(2x-3)^5$ using the chain rule.

Sight of $\pm A(2x-3)^4$ where A is a non- zero constant is sufficient for the method mark.

A1 A correct (un simplified) form of the differential.

Accept $\frac{dy}{dx}=5\times(2x-3)^4\times 2$ or $\frac{dy}{dx}=10(2x-3)^4$

M1 This is awarded for an attempt to find the gradient of the tangent to the curve at P

Award for substituting their numerical value to part (a) into their differential to find the numerical gradient of the tangent

dM1 Award for a correct method to find an equation of the tangent to the curve at P. It is dependent upon the previous M mark being awarded.

Award for $'their\,160'=\frac{y-(-32)}{x-their\,'\frac{1}{2}'}$

If they use $y=mx+c$ it must be a full method, using m= 'their 160', their '$\frac{1}{2}$' and -32.

An attempt must be seen to find c=…

A1 cso $y = 160x - 112$. The question is specific and requires the answer in this form. You may isw in this question after a correct answer.

Question Number	Scheme	Marks
136.	(i)(a) $\frac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$	M1A1A1
	$= 3x^2 \ln 2x + x^2$	
		(3)
	(i)(b) $\frac{dy}{dx} = 3(\boldsymbol{x + \sin 2x})^2 \times (1 + 2\cos 2x)$	**B1**M1A1
		(3)
	(ii) $\frac{dx}{dy} = -\text{cosec}^2 y$	M1A1
	$\frac{dy}{dx} = -\frac{1}{\text{cosec}^2 y}$	M1
	Uses $\text{cosec}^2 y = 1 + \cot^2 y$ and $x = \cot y$ in $\frac{dy}{dx}$ or $\frac{dx}{dy}$ to get an expression in x	
	$\frac{dy}{dx} = -\frac{1}{\text{cosec}^2 y} = -\frac{1}{1 + \cot^2 y} = -\frac{1}{1 + x^2}$ cso	M1, A1*
		(5)
		(11 marks)

(i)(a) M1 Applies the product rule vu'+uv' to $x^3 \ln 2x$.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$Ax^2 \times \ln 2x + x^3 \times \frac{B}{x} \qquad \text{where } A, B \text{ are constants} \neq 0$$

A1 One term correct, either $3x^2 \times \ln 2x$ or $x^3 \times \frac{1}{2x} \times 2$

A1 Cao. $\frac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$. The answer does not need to be simplified.

For reference the simplified answer is $\frac{dy}{dx} = 3x^2 \ln 2x + x^2 = x^2(3\ln 2x + 1)$

(i)(b) B1 Sight of $(x + \sin 2x)^2$

M1 For applying the chain rule to $(x + \sin 2x)^3$. If the rule is quoted it must be correct. If it is not quoted possible forms of evidence could be sight of $C(x + \sin 2x)^2 \times (1 \pm D\cos 2x)$ where C and D are non- zero constants.

Alternatively accept $u = x + \sin 2x$, u'= followed by $Cu^2 \times$ their u'

Do not accept $C(x + \sin 2x)^2 \times 2\cos 2x$ unless you have evidence that this is their u'

Allow 'invisible' brackets for this mark, ie. $C(x + \sin 2x)^2 \times 1 \pm D\cos 2x$

A1 Cao $\frac{dy}{dx} = 3(x + \sin 2x)^2 \times (1 + 2\cos 2x)$. There is no requirement to simplify this.

You may ignore subsequent working (isw) after a correct answer in part (i)(a) and (b)

(ii) M1 Writing the derivative of cot***y*** as -cosec^2***y***. It must be in terms of y

A1 $\frac{\mathrm{d}x}{\mathrm{d}y} = -\mathrm{cosec}^2 y$ or $1 = -\mathrm{cosec}^2 y \frac{\mathrm{d}y}{\mathrm{d}x}$. Both lhs and rhs must be correct.

M1 Using $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\mathrm{d}x/\mathrm{d}y}$

M1 Using $\mathrm{cosec}^2 y = 1 + \cot^2 y$ **and** $x = \cot y$ to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ just in terms of *x*.

A1 cso $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{1+x^2}$

Alternative to (a)(i) when ln(2x) is written lnx+ln2

M1 Writes $x^3 \ln 2x$ as $x^3 \ln 2 + x^3 \ln x$.
Achieves Ax^2 for differential of $x^3 \ln 2$ and applies the product rule vu'+uv' to $x^3 \ln x$.

A1 Either $3x^2 \times \ln 2 + 3x^2 \ln x$ or $x^3 \times \frac{1}{x}$

A1 A correct (un simplified) answer. Eg $3x^2 \times \ln 2 + 3x^2 \ln x + x^3 \times \frac{1}{x}$

Alternative to 5(ii) using quotient rule

M1 Writes $\cot y$ as $\frac{\cos y}{\sin y}$ and applies the quotient rule, a form of which appears in the formula book. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) only accept answers of the form $\frac{\sin y \times \pm \sin y - \cos y \times \pm \cos y}{(\sin y)^2}$

A1 Correct un simplified answer with both lhs and rhs correct.

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{\sin y \times -\sin y - \cos y \times \cos y}{(\sin y)^2} = \left\{-1 - \cot^2 y\right\}$$

M1 Using $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\mathrm{d}x/\mathrm{d}y}$

M1 Using $\sin^2 y + \cos^2 y = 1$, $\frac{1}{\sin^2 y} = \mathrm{cosec}^2 y$ and $\mathrm{cosec}^2 y = 1 + \cot^2 y$ to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ in x

A1 cso $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{1+x^2}$

Alternative to 5(ii) using the chain rule, first two marks

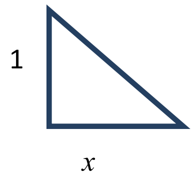
M1 Writes $\cot y$ as $(\tan y)^{-1}$ and applies the chain rule (or quotient rule).

Accept answers of the form $-(\tan y)^{-2} \times \sec^2 y$

A1 Correct un simplified answer with both lhs and rhs correct.

$$\frac{dx}{dy} = -(\tan y)^{-2} \times \sec^2 y$$

$x = \cot y \Rightarrow \tan y = \dfrac{1}{x}$



$\sin y = \dfrac{1}{\sqrt{1+x^2}}$

Alternative to 5(ii) using a triangle – last M1

M1 Uses triangle with $\tan y = \dfrac{1}{x}$ to find siny

and get $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ just in terms of x

Question Number	Scheme	Marks
137.	(a) $\dfrac{2}{x+2}+\dfrac{4}{x^2+5}-\dfrac{18}{(x+2)(x^2+5)}=\dfrac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$	M1A1
	$=\dfrac{2x(x+2)}{(x+2)(x^2+5)}$	M1
	$=\dfrac{2x}{(x^2+5)}$	A1*
		(4)
	(b) $\mathrm{h}'(x)=\dfrac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$	M1A1
	$\mathrm{h}'(x)=\dfrac{10-2x^2}{(x^2+5)^2}$ cso	A1
		(3)
	(c) Maximum occurs when $\mathrm{h}'(x)=0\Rightarrow 10-2x^2=0\Rightarrow x=..$	M1
	$\Rightarrow x=\sqrt{5}$	A1
	When $x=\sqrt{5}\Rightarrow \mathrm{h}(x)=\dfrac{\sqrt{5}}{5}$	M1,A1
	Range of $\mathrm{h}(x)$ is $0\le h(x)\le\dfrac{\sqrt{5}}{5}$	A1ft
		(5) **(12 marks)**

(a) M1 Combines the three fractions to form a single fraction with a common denominator.
Allow errors on the numerator but at least one must have been adapted.
Condone 'invisible' brackets for this mark.
Accept three separate fractions with the same denominator.
Amongst possible options allowed for this method are

$\dfrac{2x^2+5+4x+2-18}{(x+2)(x^2+5)}$ Eg 1 An example of 'invisible' brackets

$\dfrac{2(x^2+5)}{(x+2)(x^2+5)}+\dfrac{4}{(x+2)(x^2+5)}-\dfrac{18}{(x+2)(x^2+5)}$ Eg 2An example of an error (on middle term), 1st term has been adapted

$\dfrac{2(x^2+5)^2(x+2)+4(x+2)^2(x^2+5)-18(x^2+5)(x+2)}{(x+2)^2(x^2+5)^2}$ Eg 3 An example of a correct fraction with a different denominator

A1 Award for a correct un simplified fraction with the correct (lowest) common denominator.

$\dfrac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$

Accept if there are three separate fractions with the correct (lowest) common denominator.

Eg $\dfrac{2(x^2+5)}{(x+2)(x^2+5)}+\dfrac{4(x+2)}{(x+2)(x^2+5)}-\dfrac{18}{(x+2)(x^2+5)}$

Note, Example 3 would score M1A0 as it does not have the correct lowest common denominator

M1 There must be a single denominator. Terms must be collected on the numerator. A factor of (x+2) must be taken out of the numerator and then cancelled with one in the denominator. The cancelling may be assumed if the term 'disappears'

A1* Cso $\frac{2x}{(x^2+5)}$ This is a given solution and this mark should be withheld if there are any errors

(b) M1 Applies the quotient rule to $\frac{2x}{(x^2+5)}$, a form of which appears in the formula book.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{(x^2+5)\times A-2x\times Bx}{(x^2+5)^2} \quad \text{where } A,B>0$$

A1 Correct unsimplified answer $\mathrm{h}'(x)=\frac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$

A1 $\mathrm{h}'(x)=\frac{10-2x^2}{(x^2+5)^2}$ The correct simplified answer. Accept $\frac{2(5-x^2)}{(x^2+5)^2}$ $\frac{-2(x^2-5)}{(x^2+5)^2}$, $\frac{10-2x^2}{(x^4+10x^2+25)}$

DO NOT ISW FOR PART (b). INCORRECT SIMPLIFICATION IS A0

(c) M1 Sets their h'(x)=0 and proceeds with a correct method to find x. There must have been an attempt to differentiate. Allow numerical errors but do not allow solutions from 'unsolvable' equations.

A1 Finds the correct x value of the maximum point x=√5. Ignore the solution x=-√5 but withhold this mark if other positive values found.

M1 Substitutes their answer into their h'(x)=0 in h(x) to determine the maximum value

A1 Cso-the maximum value of h(x) = $\frac{\sqrt{5}}{5}$. Accept equivalents such as $\frac{2\sqrt{5}}{10}$ **but not** 0.447

A1ft Range of h(x) is $0\le \mathrm{h}(x)\le\frac{\sqrt{5}}{5}$. Follow through on their maximum value if the M's have been scored. Allow $0\le y\le\frac{\sqrt{5}}{5}$, $0\le Range\le\frac{\sqrt{5}}{5}$, $\left[0,\frac{\sqrt{5}}{5}\right]$ but not $0\le x\le\frac{\sqrt{5}}{5}$, $\left(0,\frac{\sqrt{5}}{5}\right)$

If a candidate attempts to work out $h^{-1}(x)$ in (b) and does all that is required for (b) in (c), then allow. Do not allow $h^{-1}(x)$ to be used for h'(x) in part (c). For this question (b) and (c) can be scored together.

Alternative to (b) using the product rule

M1 Sets $\mathrm{h}(x)=2x(x^2+5)^{-1}$ and applies the product rule vu'+uv' with terms being 2x and $(x^2+5)^{-1}$ If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$(x^2+5)^{-1}\times A+2x\times\pm Bx(x^2+5)^{-2}$$

A1 Correct un simplified answer $(x^2+5)^{-1}\times 2+2x\times -2x(x^2+5)^{-2}$

A1 The question asks for h'(x) to be put in its simplest form. Hence in this method the terms need to be combined to form a single correct expression.

For a correct simplified answer accept

$$h'(x) = \frac{10-2x^2}{(x^2+5)^2} = \frac{2(5-x^2)}{(x^2+5)^2} = \frac{-2(x^2-5)}{(x^2+5)^2} = (10-2x^2)(x^2+5)^{-2}$$

Question Number	Scheme	Marks
138.	(a) $\dfrac{dy}{dx}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\dfrac{dy}{dx}=0 \qquad e^{x\sqrt{3}}(\sqrt{3}\sin 3x+3\cos 3x)=0$	M1
	$\tan 3x=-\sqrt{3}$	A1
	$3x=\dfrac{2\pi}{3}\Rightarrow x=\dfrac{2\pi}{9}$	M1A1
		(6)
	(b) At x=0 $\dfrac{dy}{dx}=3$	B1
	Equation of normal is $-\dfrac{1}{3}=\dfrac{y-0}{x-0}$ or any equivalent $y=-\dfrac{1}{3}x$	M1A1
		(3) **(9 marks)**

(a) M1 Applies the product rule vu'+uv' to $e^{x\sqrt{3}}\sin 3x$. If the rule is quoted it must be correct and there must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, ie. terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') only accept answers of the form $\dfrac{dy}{dx}=Ae^{x\sqrt{3}}\sin 3x+e^{x\sqrt{3}}\times\pm B\cos 3x$

A1 Correct expression for $\dfrac{dy}{dx}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$

M1 Sets **their** $\dfrac{dy}{dx}=0$, factorises out or divides by $e^{x\sqrt{3}}$ producing an equation in sin3x and cos3x

A1 Achieves either $\tan 3x=-\sqrt{3}$ or $\tan 3x=-\dfrac{3}{\sqrt{3}}$

M1 Correct order of arctan, followed by ÷3.

Accept $3x=\dfrac{5\pi}{3}\Rightarrow x=\dfrac{5\pi}{9}$ or $3x=\dfrac{-\pi}{3}\Rightarrow x=\dfrac{-\pi}{9}$ but not $x=\arctan(\dfrac{-\sqrt{3}}{3})$

A1 CS0 $x=\dfrac{2\pi}{9}$ Ignore extra solutions outside the range. Withhold mark for extra inside the range.

(b) B1 Sight of **3** for the gradient

M1 A full method for finding an equation of the normal.

Their tangent gradient m must be modified to $-\dfrac{1}{m}$ and used together with (0, 0).

Eg $-\dfrac{1}{their\ 'm'}=\dfrac{y-0}{x-0}$ or equivalent is acceptable

A1 $y=-\dfrac{1}{3}x$ or any correct equivalent including $-\dfrac{1}{3}=\dfrac{y-0}{x-0}$.

Alternative in part (a) using the form $R\sin(3x+\alpha)$ JUST LAST 3 MARKS

Question Number	Scheme	Marks
138.	(a) $\dfrac{dy}{dx}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\dfrac{dy}{dx}=0$ $\quad e^{x\sqrt{3}}(\sqrt{3}\sin 3x+3\cos 3x)=0$	M1
	$(\sqrt{12})\sin(3x+\dfrac{\pi}{3})=0$	A1
	$3x=\dfrac{2\pi}{3}\Rightarrow x=\dfrac{2\pi}{9}$	M1A1
		(6)

A1 Achieves either $(\sqrt{12})\sin(3x+\dfrac{\pi}{3})=0$ or $(\sqrt{12})\cos(3x-\dfrac{\pi}{6})=0$

M1 Correct order of arcsin or arcos, etc to produce a value of x

Eg accept $3x+\dfrac{\pi}{3}=0$ or π or $2\pi\Rightarrow x=....$

A1 Cao $x=\dfrac{2\pi}{9}$ Ignore extra solutions outside the range. Withhold mark for extra inside the range.

Alternative to part (a) squaring both sides JUST LAST 3 MARKS

Question Number	Scheme	Marks
138.	(a) $\dfrac{dy}{dx}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\dfrac{dy}{dx}=0$ $\quad e^{x\sqrt{3}}(\sqrt{3}\sin 3x+3\cos 3x)=0$	M1
	$\sqrt{3}\sin 3x=-3\cos 3x\Rightarrow\cos^2(3x)=\dfrac{1}{4}$ or $\sin^2(3x)=\dfrac{3}{4}$	A1
	$x=\dfrac{1}{3}\arccos(\pm\sqrt{\dfrac{1}{4}})$ oe	M1
	$x=\dfrac{2\pi}{9}$	A1

Question Number	Scheme	Marks
139.	(a)(i) $\frac{d}{dx}(\ln(3x)) = \frac{3}{3x}$	M1
	$\frac{d}{dx}(x^{\frac{1}{2}}\ln(3x)) = \ln(3x)\times\frac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\frac{3}{3x}$	M1A1
		(3)
	(ii) $\frac{dy}{dx} = \frac{(2x-1)^5\times -10-(1-10x)\times 5(2x-1)^4\times 2}{(2x-1)^{10}}$	M1A1
	$\frac{dy}{dx} = \frac{80x}{(2x-1)^6}$	A1
		(3)
	(b) $x = 3\tan 2y \quad \Rightarrow \frac{dx}{dy} = 6\sec^2 2y$	M1A1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6\sec^2 2y}$	M1
	Uses $\sec^2 2y = 1+\tan^2 2y$ and uses $\tan 2y = \frac{x}{3}$	
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6(1+(\frac{x}{3})^2)} = (\frac{3}{18+2x^2})$	M1A1
		(5)
		(11 marks)

Note that this is marked B1M1A1 on EPEN

(a)(i) M1 Attempts to differentiate $\ln(3x)$ to $\frac{B}{x}$. Note that $\frac{1}{3x}$ is fine.

M1 Attempts the product rule for $x^{\frac{1}{2}}\ln(3x)$. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the rule is not quoted nor implied from their stating of u, u', v, v' and their subsequent expression, only accept answers of the form

$$\ln(3x)\times Ax^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\frac{B}{x}, \quad A, B > 0$$

A1 Any correct (un simplified) form of the answer. Remember to isw any incorrect further work

$$\frac{d}{dx}(x^{\frac{1}{2}}\ln(3x)) = \ln(3x)\times\frac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\frac{3}{3x} = (\frac{\ln(3x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}}) = x^{-\frac{1}{2}}(\frac{1}{2}\ln 3x + 1)$$

Note that this part does not require the answer to be in its simplest form

(ii) M1 Applies the quotient rule, a version of which appears in the formula booklet. If the formula is quoted it must be correct. There must have been an attempt to differentiate both terms. If the formula is not quoted nor implied from their stating of u, u', v, v' and their subsequent expression, only accept answers of the form

$$\frac{(2x-1)^5\times\pm10-(1-10x)\times C(2x-1)^4}{(2x-1)^{10\text{ or }7\text{ or }25}}$$

A1 Any un simplified form of the answer. Eg $\frac{dy}{dx}=\frac{(2x-1)^5\times-10-(1-10x)\times5(2x-1)^4\times2}{((2x-1)^5)^2}$

A1 Cao. It must be simplified as required in the question $\frac{dy}{dx}=\frac{80x}{(2x-1)^6}$

(b) M1 Knows that $3\tan 2y$ differentiates to $C\sec^2 2y$. The lhs can be ignored for this mark. If they write $3\tan 2y$ as $\frac{3\sin 2y}{\cos 2y}$ this mark is awarded for a correct attempt of the quotient rule.

A1 Writes down $\frac{dx}{dy}=6\sec^2 2y$ or implicitly to get $1=6\sec^2 2y\frac{dy}{dx}$

Accept from the quotient rule $\frac{6}{\cos^2 2y}$ or even $\frac{\cos 2y\times6\cos 2y-3\sin 2y\times-2\sin 2y}{\cos^2 2y}$

M1 An attempt to invert 'their' $\frac{dx}{dy}$ to reach $\frac{dy}{dx}=f(y)$, or changes the subject of their implicit

. differential to achieve a similar result $\frac{dy}{dx}=f(y)$

M1 Replaces an expression for $\sec^2 2y$ in their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ with x by attempting to use $\sec^2 2y=1+\tan^2 2y$. Alternatively, replaces an expression for y in $\frac{dx}{dy}$ or $\frac{dy}{dx}$ with $\frac{1}{2}\arctan(\frac{x}{3})$

A1 Any correct form of $\frac{dy}{dx}$ in terms of x. $\frac{dy}{dx}=\frac{1}{6(1+(\frac{x}{3})^2)}$ $\frac{dy}{dx}=\frac{3}{18+2x^2}$ *or* $\frac{1}{6\sec^2(\arctan(\frac{x}{3}))}$

Question Number	Scheme	Marks
139.	**(a)(ii) Alt using the product rule** Writes $\frac{1-10x}{(2x-1)^5}$ as $(1-10x)(2x-1)^{-5}$ and applies vu'+uv'. See (a)(i) for rules on how to apply $(2x-1)^{-5}\times-10+(1-10x)\times-5(2x-1)^{-6}\times2$ Simplifies as main scheme to $80x(2x-1)^{-6}$ or equivalent	 M1A1 A1 (3)
	(b) Alternative using arctan. They must attempt to differentiate to score any marks. Technically this is M1A1M1A2 Rearrange $x=3\tan 2y$ to $y=\frac{1}{2}\arctan(\frac{x}{3})$ **and attempt to differentiate** Differentiates to a form $\frac{A}{1+(\frac{x}{3})^2}$, $=\frac{1}{2}\times\frac{1}{(1+(\frac{x}{3})^2)}\times\frac{1}{3}$ or $\frac{1}{6(1+(\frac{x}{3})^2)}$ oe	 M1A1 M1, A2 (5)

Question No		Marks
140	(a) $\frac{d}{dx}(\ln(3x)) \to \frac{B}{x}$ for any constant B Applying vu'+uv' , $\ln(3x) \times 2x + x$ (b) Applying $\frac{vu' - uv\prime}{v^2}$ $\frac{x^3 \times 4cos(4x) - \sin(4x) \times 3x^2}{x^6}$ $= \frac{4xcos(4x) - 3\sin(4x)}{x^4}$	M1 M1, A1 A1 (4) M1 A1+A1 A1 A1 (5) (9 MARKS)

(a) M1 Differentiates the $\ln(3x)$ term to $\frac{B}{x}$. Note that $\frac{1}{3x}$ is fine for this mark.

M1 Applies the product rule to $x^2\ln(3x)$. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is **not quoted (or implied by their working)** only accept answers of the form $\ln(3x) \times Ax + x^2 \times \frac{B}{x}$ where A and B are non- zero constants

A1 One term correct and simplified, either $2xln(3x)$or x. $\ln 3x^{2x}$ and $\ln(3x)\, 2x$ are acceptable forms

A1 Both terms correct and simplified on the same line. $2xln(3x) + x$, $\ln(3x) \times 2x + x$, $x(2ln3x + 1)$ oe

(b) M1 Applies the quotient rule. A version of this appears in the formula booklet. If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the formula is **not quoted (nor implied by their working)** only accept answers of the form

$$\frac{x^3 \times \pm Acos(4x) - \sin(4x) \times Bx^2}{(x^3)^2 \text{ or } x^6 \text{ or } x^5 \text{ or } x^9} \text{ with } B > 0$$

A1 Correct first term on numerator $x^3 \times 4cos(4x)$

A1 Correct second term on numerator $-\sin(4x) \times 3x^2$

A1 Correct denominator x^6, the $(x^3)^2$ needs to be simplified

A1 Fully correct simplified expression $\frac{4xcos(4x) - 3\sin(4x)}{x^4}$, $\frac{cos(4x)4x - \sin(4x)3}{x^4}$ oe .

Accept $4x^{-3}\cos(4x) - 3x^{-4}\sin(4x)$ oe

Alternative method using the product rule.

M1,A1 **Writes** $\frac{sin(4x)}{x^3}$ **as** $\sin(4x) \times x^{-3}$ and applies the product rule. They will score both of these marks or neither of them. If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the formula is **not quoted (nor implied by their working)** only accept answers of the form $x^{-3} \times Acos(4x) + \sin(4x) \times \pm Bx^{-4}$

A1 One term correct, either $x^{-3} \times 4cos(4x)$ or $\sin(4x) \times -3x^{-4}$

A1 Both terms correct,Eg. $x^{-3} \times 4cos(4x) + \sin(4x) \times -3x^{-4}$.

A1 Fully correct expression. $4x^{-3}cos(4x) - 3x^{-4}\sin(4x)$ or $4cos(4x)x^{-3} - 3\sin(4x)x^{-4}$ oe

The negative must have been dealt with for the final mark.

Question No	Scheme	Marks
141	$\left(\frac{dx}{dy}\right) = 2sec^2\left(y + \frac{\pi}{12}\right)$	M1,A1
	substitute $y = \frac{\pi}{4}$ into their $\frac{dx}{dy} = 2sec^2\left(\frac{\pi}{4} + \frac{\pi}{12}\right) = 8$	M1, A1
	When $y=\frac{\pi}{4}$. $x=2\sqrt{3}$ awrt 3.46	B1
	$\left(y - \frac{\pi}{4}\right) = their\ m(x - their\ 2\sqrt{3})$	M1
	$\left(y - \frac{\pi}{4}\right) = -8(x - 2\sqrt{3})$ oe	A1 (7 marks)

M1 For differentiation of $2\tan\left(\boldsymbol{y} + \frac{\pi}{12}\right) \rightarrow 2sec^2\left(\boldsymbol{y} + \frac{\pi}{12}\right)$. There is no need to identify this with $\frac{dx}{dy}$

A1 For correctly writing $\frac{dx}{dy} = 2sec^2\left(y + \frac{\pi}{12}\right)$ or $\frac{dy}{dx} = \frac{1}{2sec^2\left(y+\frac{\pi}{12}\right)}$

M1 Substitute $y = \frac{\pi}{4}$ into their $\frac{dx}{dy}$. Accept if $\frac{dx}{dy}$ is inverted and $y = \frac{\pi}{4}$ substituted into $\frac{dy}{dx}$.

A1 $\frac{dx}{dy} = 8$ or $\frac{dy}{dx} = \frac{1}{8}$ oe

B1 Obtains the value of $x=2\sqrt{3}$ corresponding to $y=\frac{\pi}{4}$. Accept awrt 3.46

M1 This mark requires **all of the necessary elements for** finding **a numerical equation** of the **normal.**

Either Invert their value of $\frac{dx}{dy}$, to find $\frac{dy}{dx}$, then use $m_1 \times m_2=-1$ to find the numerical gradient of the normal

Or use their numerical value of $-\frac{dx}{dy}$

Having done this then use $\left(y - \frac{\pi}{4}\right) = their\ m(x - their\ 2\sqrt{3})$

The $2\sqrt{3}$ could appear as awrt 3.46, the $\frac{\pi}{4}$ as awrt 0.79,

This cannot be awarded for finding the equation of a tangent.

Watch for candidates who correctly use $\left(x - their\ 2\sqrt{3}\right) = -their\ numerical\ \frac{dy}{dx}\left(y - \frac{\pi}{4}\right)$

If they use 'y=mx+c' it must be a full method to find c.

A1 Any correct form of the answer. It does not need to be simplified and the question does not ask for an exact answer.

$\left(y - \frac{\pi}{4}\right) = -8(x - 2\sqrt{3})$, $\frac{y-\frac{\pi}{4}}{x-2\sqrt{3}} = -8$, $y = -8x + \frac{\pi}{4} + 16\sqrt{3}$, y=-8x+ (awrt) 28.5

Alternatives using arctan (first 3 marks)

M1 Differentiates $y = \arctan\left(\frac{x}{2}\right) - \frac{\pi}{12}$ to get $\frac{1}{1+(\frac{x}{2})^2} \times constant$. Don't worry about the lhs

A1 Achieves $\frac{dy}{dx} = \frac{1}{1+(\frac{x}{2})^2} \times \frac{1}{2}$

M1 This method mark requires *x* to be found, which then needs to be substituted into $\frac{dy}{dx}$

The rest of the marks are then the same.

Or implicitly (first 2 marks)

M1 Differentiates implicitly to get $1 = 2\,sec^2\left(y + \frac{\pi}{12}\right) \times \frac{dy}{dx}$

A1 Rearranges to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of y

The rest of the marks are the same

Or by compound angle identities

$x = 2\tan\left(y + \frac{\pi}{12}\right) = \frac{2tany+2\tan\left(\frac{\pi}{12}\right)}{1-tany\; tan\frac{\pi}{12}}$ oe

M1 Differentiates using quotient rule-see question 1 in applying this. Additionally the tany **must** have been differentiated to $sec^2 y$. There is no need to assign to $\frac{dx}{dy}$

A1 The correct answer for $\frac{dx}{dy} = \frac{\left(1-tany\; tan\frac{\pi}{12}\right)\times 2sec^2 y - \left(2tany+2\tan\left(\frac{\pi}{12}\right)\right)\times -sec^2 y tan\frac{\pi}{12}}{(1-tany\; tan\frac{\pi}{12})^2}$

The rest of the marks are as the main scheme

Question Number	Scheme	Marks
142. (a)	$\frac{1}{(x^2+3x+5)} \times \ldots , = \frac{2x+3}{(x^2+3x+5)}$	M1,A1 (2)
(b)	Applying $\frac{vu'-uv'}{v^2}$	M1,
	$\frac{x^2\times -\sin x - \cos x \times 2x}{(x^2)^2} = \frac{-x^2\sin x - 2x\cos x}{x^4} = \frac{-x\sin x - 2\cos x}{x^3}$ oe	A2,1,0 (3)
		5 Marks

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Question Number	Scheme	Marks
143. (a)	$x^2-9=(x+3)(x-3)$	B1
	$\frac{4x-5}{(2x+1)(x-3)}-\frac{2x}{(x+3)(x-3)}$	
	$=\frac{(4x-5)(x+3)}{(2x+1)(x-3)(x+3)}-\frac{2x(2x+1)}{(2x+1)(x+3)(x-3)}$	M1
	$=\frac{5x-15}{(2x+1)(x-3)(x+3)}$	M1A1
	$=\frac{5\cancel{(x-3)}}{(2x+1)\cancel{(x-3)}(x+3)}=\frac{5}{(2x+1)(x+3)}$	A1*
		(5)
(b)	$f(x)=\frac{5}{2x^2+7x+3}$	
	$f'(x)=\frac{-5(\mathbf{4x+7})}{(2x^2+7x+3)^2}$	M1M1A1
	$f'(-1)=-\frac{15}{4}$	M1A1
	Uses $m_1m_2=-1$ to give gradient of normal$=\frac{4}{15}$	M1
	$\frac{y-(-\frac{5}{2})}{(x--1)}=their\ \frac{4}{15}$	M1
	$y+\frac{5}{2}=\frac{4}{15}(x+1)$ or any equivalent form	A1
		(8)
		13 Marks

Question Number	Scheme		Marks
144. **(a)**	$\dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)}$		
	$= \dfrac{(4x-1)(2x-1)-3}{2(x-1)(2x-1)}$	An attempt to form a single fraction	M1
	$= \dfrac{8x^2-6x-2}{\{2(x-1)(2x-1)\}}$	Simplifies to give a correct quadratic numerator over a correct quadratic denominator	A1 aef
	$= \dfrac{2(x-1)(4x+1)}{\{2(x-1)(2x-1)\}}$	An attempt to factorise a 3 term quadratic numerator	M1
	$= \dfrac{4x+1}{2x-1}$		A1 (4)
(b)	$\mathrm{f}(x) = \dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)} - 2, \quad x > 1$		
	$\mathrm{f}(x) = \dfrac{(4x+1)}{(2x-1)} - 2$		
	$= \dfrac{(4x+1)-2(2x-1)}{(2x-1)}$	An attempt to form a single fraction	M1
	$= \dfrac{4x+1-4x+2}{(2x-1)}$		
	$= \dfrac{3}{(2x-1)}$	Correct result	A1 * (2)
(c)	$\mathrm{f}(x) = \dfrac{3}{(2x-1)} = 3(2x-1)^{-1}$		
	$\mathrm{f}'(x) = 3(-1)(2x-1)^{-2}(2)$	$\pm k(2x-1)^{-2}$	M1
			A1 aef
	$\mathrm{f}'(2) = \dfrac{-6}{9} = -\dfrac{2}{3}$	Either $\dfrac{-6}{9}$ or $-\dfrac{2}{3}$	A1
			(3) **[9]**

Question Number	Scheme		Marks
145			
(a)	$y = \dfrac{3+\sin 2x}{2+\cos 2x}$		
	Apply quotient rule: $\begin{cases} u = 3+\sin 2x & v = 2+\cos 2x \\ \dfrac{du}{dx} = 2\cos 2x & \dfrac{dv}{dx} = -2\sin 2x \end{cases}$		
	$\dfrac{dy}{dx} = \dfrac{2\cos 2x(2+\cos 2x) - -2\sin 2x(3+\sin 2x)}{(2+\cos 2x)^2}$	Applying $\dfrac{vu'-uv'}{v^2}$	M1
		Any one term correct on the numerator	A1
		Fully correct (unsimplified).	A1
	$= \dfrac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2+\cos 2x)^2}$		
	$= \dfrac{4\cos 2x + 6\sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2+\cos 2x)^2}$		
	$= \dfrac{4\cos 2x + 6\sin 2x + 2}{(2+\cos 2x)^2}$ (as required)	For correct proof with an understanding that $\cos^2 2x + \sin^2 2x = 1$. No errors seen in working.	A1*
			(4)
(b)	When $x = \frac{\pi}{2}$, $y = \dfrac{3+\sin\pi}{2+\cos\pi} = \dfrac{3}{1} = 3$	$y = 3$	B1
	At $(\frac{\pi}{2}, 3)$, $\text{m}(\mathbf{T}) = \dfrac{6\sin\pi + 4\cos\pi + 2}{(2+\cos\pi)^2} = \dfrac{-4+2}{1^2} = -2$	$\text{m}(\mathbf{T}) = -2$	B1
	Either **T**: $y-3=-2(x-\frac{\pi}{2})$ or $y=-2x+c$ and $3=-2(\frac{\pi}{2})+c \Rightarrow c = 3+\pi$;	$y-y_1 = m(x-\frac{\pi}{2})$ with 'their TANGENT gradient' and their y_1; or uses $y=mx+c$ with 'their TANGENT gradient';	M1
	T: $y = -2x + (\pi+3)$	$y = -2x+\pi+3$	A1
			(4)
			[8]

Question Number	Scheme		Marks
146.			
(a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$		
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$	Writes $\sec x$ as $(\cos x)^{-1}$ and gives $\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$	M1
		$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$	Convincing proof. Must see both $\underline{\underline{\text{underlined steps.}}}$	A1 **AG**
			(3)
(b)	$x = \sec 2y, \quad y \neq (2n+1)\frac{\pi}{4},\ n \in \mathbb{Z}.$		
	$\dfrac{dx}{dy} = 2\sec 2y \tan 2y$	$K\sec 2y \tan 2y$	M1
		$2\sec 2y \tan 2y$	A1
			(2)
(c)	$\dfrac{dy}{dx} = \dfrac{1}{2\sec 2y \tan 2y}$	Applies $\dfrac{dy}{dx} = \dfrac{1}{\left(\frac{dx}{dy}\right)}$	M1
	$\dfrac{dy}{dx} = \dfrac{1}{2x \tan 2y}$	Substitutes x for $\sec 2y$.	M1
	$1 + \tan^2 A = \sec^2 A \Rightarrow \tan^2 2y = \sec^2 2y - 1$	Attempts to use the identity $1 + \tan^2 A = \sec^2 A$	M1
	So $\tan^2 2y = x^2 - 1$		
	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	A1
			(4)
			[9]

Question Number	Scheme	Marks
147.	At P, $y = \underline{3}$	B1
	$\dfrac{dy}{dx} = \underline{3(-2)(5-3x)^{-3}(-3)}$ $\left\{\text{or } \dfrac{18}{(5-3x)^3}\right\}$	M1$\underline{\text{A1}}$
	$\dfrac{dy}{dx} = \dfrac{18}{(5-3(2))^3}$ $\{= -18\}$	M1
	$\text{m}(\mathbf{N}) = \dfrac{-1}{-18}$ or $\dfrac{1}{18}$	M1
	$\mathbf{N}$: $y - 3 = \frac{1}{18}(x-2)$	M1
	$\mathbf{N}$: $\underline{x - 18y + 52 = 0}$	A1
		[7]
	1st M1: $\pm k(5-3x)^{-3}$ can be implied. See appendix for application of the quotient rule. 2nd M1: Substituting x = 2 into an equation involving their $\frac{dy}{dx}$; 3rd M1: Uses $\text{m}(\mathbf{N}) = -\dfrac{1}{\text{their m}(\mathbf{T})}$. 4th M1: $y - y_1 = m(x-2)$ with 'their NORMAL gradient' or a "changed" tangent gradient and their y_1. Or uses a complete method to express the equation of the tangent in the form $y = mx + c$ with 'their NORMAL ("changed" **numerical**) gradient', their y_1 and x = 2. Note: To gain the final A1 mark all the previous 6 marks in this question need to be earned. Also there must be a completely correct solution given.	

Question Number	Scheme	Marks
148. (a)	Either $y = 2$ or $(0,\ 2)$	B1
		(1)
(b)	When $x = 2,\ y = (8 - 10 + 2)e^{-2} = 0e^{-2} = 0$	B1
	$(2x^2 - 5x + 2) = 0 \Rightarrow (x-2)(2x-1) = 0$	M1
	Either $x = 2$ (for possibly B1 above) or $\underline{x = \frac{1}{2}}$.	A1
		(3)
(c)	$\frac{dy}{dx} = (4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x}$	M1A1A1
		(3)
(d)	$(4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x} = 0$	M1
	$2x^2 - 9x + 7 = 0 \Rightarrow (2x - 7)(x - 1) = 0$	M1
	$x = \frac{7}{2}, 1$	A1
	When $x = \frac{7}{2},\ y = 9e^{-\frac{7}{2}}$, when $x = 1,\ y = -e^{-1}$	ddM1A1
		(5)
		[12]
	(b) If the candidate believes that $e^{-x} = 0$ solves to $x = 0$ or gives an extra solution of $x = 0$, then withhold the final accuracy mark. (c) M1: (their u')e^{-x} + $(2x^2 - 5x + 2)$(their v') A1: Any one term correct. A1: Both terms correct. (d) 1st M1: For setting their $\frac{dy}{dx}$ found in part (c) equal to 0. 2nd M1: Factorise or eliminate out e^{-x} correctly and an attempt to factorise a 3-term quadratic or apply the formula to candidate's $ax^2 + bx + c$. See rules for solving a three term quadratic equation on page 1 of this Appendix. 3rd ddM1: An attempt to use at least one x-coordinate on $y = (2x^2 - 5x + 2)e^{-x}$. Note that this method mark is dependent on the award of the two previous method marks in this part. Some candidates write down corresponding y-coordinates without any working. It may be necessary on some occasions to use your calculator to check that at least one of the two y-coordinates found is correct to awrt 2 sf. Final A1: Both $\{x = 1\},\ y = -e^{-1}$ and $\{x = \frac{7}{2}\},\ y = 9e^{-\frac{7}{2}}$. **cao** Note that both exact values of y are required.	

Question Number	Scheme		Marks
149. (i)	$y = \dfrac{\ln(x^2+1)}{x}$		
	$u = \ln(x^2+1) \Rightarrow \dfrac{du}{dx} = \dfrac{2x}{x^2+1}$	$\ln(x^2+1) \to \dfrac{\text{something}}{x^2+1}$	M1
		$\ln(x^2+1) \to \dfrac{2x}{x^2+1}$	A1
	Apply quotient rule: $\left\{\begin{matrix} u = \ln(x^2+1) & v = x \\ \dfrac{du}{dx} = \dfrac{2x}{x^2+1} & \dfrac{dv}{dx} = 1 \end{matrix}\right\}$		
	$\dfrac{dy}{dx} = \dfrac{\left(\dfrac{2x}{x^2+1}\right)(x) - \ln(x^2+1)}{x^2}$	Applying $\dfrac{xu' - \ln(x^2+1)v'}{x^2}$ correctly.	M1
		Correct differentiation with correct bracketing but allow recovery.	A1
			(4)
	$\left\{\dfrac{dy}{dx} = \dfrac{2}{(x^2+1)} - \dfrac{1}{x^2}\ln(x^2+1)\right\}$	{Ignore subsequent working.}	
(ii)	$x = \tan y$		
	$\dfrac{dx}{dy} = \sec^2 y$	$\tan y \to \sec^2 y$ or an attempt to differentiate $\dfrac{\sin y}{\cos y}$ using either the quotient rule or product rule.	M1*
		$\dfrac{dx}{dy} = \sec^2 y$	A1
	$\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y} \quad \left\{= \cos^2 y\right\}$	Finding $\dfrac{dy}{dx}$ by reciprocating $\dfrac{dx}{dy}$.	dM1*
	$\dfrac{dy}{dx} = \dfrac{1}{1+\tan^2 y}$	For writing down or applying the identity $\sec^2 y = 1 + \tan^2 y$, which must be applied/stated completely in y.	dM1*
	Hence, $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$, (as required)	For the correct proof, leading on from the previous line of working.	A1 **AG**
			(5)
			[9]

Question Number	Scheme		Marks
150 (a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$		
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$	$\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$	M1
		$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$	Convincing proof. Must see both underlined steps.	A1 **AG**
			(3)
(b)	$y = e^{2x}\sec 3x$		
	$\left\{\begin{matrix} u = e^{2x} & v = \sec 3x \\ \dfrac{du}{dx} = 2e^{2x} & \dfrac{dv}{dx} = 3\sec 3x \tan 3x \end{matrix}\right\}$	Seen or implied. Either $e^{2x} \to 2e^{2x}$ or $\sec 3x \to 3\sec 3x \tan 3x$	M1
		Both $e^{2x} \to 2e^{2x}$ and $\sec 3x \to 3\sec 3x \tan 3x$	A1
	$\dfrac{dy}{dx} = 2e^{2x}\sec 3x + 3e^{2x}\sec 3x \tan 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v'	M1
		$2e^{2x}\sec 3x + 3e^{2x}\sec 3x \tan 3x$	A1 **isw**
			(4)
(c)	Turning point $\Rightarrow \dfrac{dy}{dx} = 0$		
	Hence, $e^{2x}\sec 3x(2 + 3\tan 3x) = 0$	Sets their $\dfrac{dy}{dx} = 0$ and factorises (or cancels) out at least e^{2x} from at least two terms.	M1
	{Note $e^{2x} \neq 0$, $\sec 3x \neq 0$, so $2 + 3\tan 3x = 0$, }		
	giving $\tan 3x = -\frac{2}{3}$	$\tan 3x = \pm k$; $k \neq 0$	M1
	$\Rightarrow 3x = -0.58800 \Rightarrow x = \{a\} = -0.19600...$	Either awrt -0.196^c or awrt -11.2°	A1
	Hence, $y = \{b\} = e^{2(-0.196)}\sec(3 \times -0.196)$		
	$= 0.812093... = 0.812$ (3sf)	0.812	A1 **cao**
			(4)
			[11]

Part (c): If there are any EXTRA solutions for x (or a) inside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$ or ANY EXTRA solutions for y (or b)**,** (for these values of x) then withhold the final accuracy mark.
Also ignore EXTRA solutions outside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$.

Question Number	Scheme		Marks
151(i)(a)	$y = x^2\cos 3x$		
	Apply product rule: $\left\{\begin{array}{ll} u = x^2 & v = \cos 3x \\ \dfrac{du}{dx} = 2x & \dfrac{dv}{dx} = -3\sin 3x \end{array}\right\}$		
	$\dfrac{dy}{dx} = 2x\cos 3x - 3x^2\sin 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v' AND gives an expression of the form $\alpha x\cos 3x \pm \beta x^2 \sin 3x$	M1
		Any one term correct	A1
		Both terms correct and no further simplification to terms in $\cos\alpha x^2$ or $\sin\beta x^3$.	A1
			(3)
(b)	$y = \dfrac{\ln(x^2+1)}{x^2+1}$		
	$u = \ln(x^2+1) \Rightarrow \dfrac{du}{dx} = \dfrac{2x}{x^2+1}$	$\ln(x^2+1) \rightarrow \dfrac{\text{something}}{x^2+1}$	M1
		$\ln(x^2+1) \rightarrow \dfrac{2x}{x^2+1}$	A1
	Apply quotient rule: $\left\{\begin{array}{ll} u = \ln(x^2+1) & v = x^2+1 \\ \dfrac{du}{dx} = \dfrac{2x}{x^2+1} & \dfrac{dv}{dx} = 2x \end{array}\right\}$		
	$\dfrac{dy}{dx} = \dfrac{\left(\dfrac{2x}{x^2+1}\right)\left(x^2+1\right) - 2x\ln(x^2+1)}{\left(x^2+1\right)^2}$	Applying $\dfrac{vu' - uv'}{v^2}$	M1
		Correct differentiation with correct bracketing but allow recovery.	A1
			(4)
	$\left\{\dfrac{dy}{dx} = \dfrac{2x - 2x\ln(x^2+1)}{\left(x^2+1\right)^2}\right\}$	{Ignore subsequent working.}	

Question Number	Scheme		Marks
(ii)	$y = \sqrt{4x+1},\ \ x > -\frac{1}{4}$		
	At P, $y = \sqrt{4(2)+1} = \underline{\sqrt{9}} = \underline{3}$	At P, $y = \underline{\sqrt{9}}$ or $\underline{3}$	B1
	$\frac{dy}{dx} = \underline{\frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)}$	$\pm k(4x+1)^{-\frac{1}{2}}$ $2(4x+1)^{-\frac{1}{2}}$	M1* A1 aef
	$\frac{dy}{dx} = \frac{2}{(4x+1)^{\frac{1}{2}}}$		
	At P, $\frac{dy}{dx} = \frac{2}{\left(4(2)+1\right)^{\frac{1}{2}}}$	Substituting $x = 2$ into an equation involving $\frac{dy}{dx}$;	M1
	Hence m(**T**) $= \frac{2}{3}$		
	Either **T**: $y - 3 = \frac{2}{3}(x-2)$; or $y = \frac{2}{3}x + c$ and $3 = \frac{2}{3}(2) + c \Rightarrow c = 3 - \frac{4}{3} = \frac{5}{3}$;	$y - y_1 = m(x-2)$ or $y - y_1 = m(x - \text{their stated } x)$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient', their x and their y_1.	dM1*;
	Either **T**: $3y - 9 = 2(x-2)$;		
	T: $3y - 9 = 2x - 4$		
	T: $\underline{2x - 3y + 5 = 0}$	$\underline{2x - 3y + 5 = 0}$ Tangent must be stated in the form $ax + by + c = 0$, where a, b and c are integers.	A1
			(6)
	or **T**: $y = \frac{2}{3}x + \frac{5}{3}$		
	T: $3y = 2x + 5$		
	T: $\underline{2x - 3y + 5 = 0}$		
			[13]

Question Number	Scheme		Marks
152	$\mathrm{f}(x) = 1 - \dfrac{2}{(x+4)} + \dfrac{x-8}{(x-2)(x+4)}$ $x \in \mathbb{R},\ x \neq -4,\ x \neq 2.$		
(a)	$\mathrm{f}(x) = \dfrac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$	An attempt to combine to one fraction Correct result of combining all three fractions	M1 A1
	$= \dfrac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$		
	$= \dfrac{x^2 + x - 12}{[(x+4)(x-2)]}$	Simplifies to give the correct numerator. Ignore omission of denominator	A1
	$= \dfrac{(x+4)(x-3)}{[(x+4)(x-2)]}$	An attempt to factorise the numerator.	dM1
	$= \dfrac{(x-3)}{(x-2)}$	Correct result	A1 cso **AG**
			(5)
(b)	$\mathrm{g}(x) = \dfrac{\mathrm{e}^x - 3}{\mathrm{e}^x - 2} \quad x \in \mathbb{R},\ x \neq \ln 2.$		
	Apply quotient rule: $\left\{\begin{array}{ll} u = \mathrm{e}^x - 3 & v = \mathrm{e}^x - 2 \\ \dfrac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x & \dfrac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^x \end{array}\right\}$		
	$\mathrm{g}'(x) = \dfrac{\mathrm{e}^x(\mathrm{e}^x - 2) - \mathrm{e}^x(\mathrm{e}^x - 3)}{(\mathrm{e}^x - 2)^2}$	Applying $\dfrac{vu' - uv'}{v^2}$ Correct differentiation	M1 A1
	$= \dfrac{\mathrm{e}^{2x} - 2\mathrm{e}^x - \mathrm{e}^{2x} + 3\mathrm{e}^x}{(\mathrm{e}^x - 2)^2}$		
	$= \dfrac{\mathrm{e}^x}{(\mathrm{e}^x - 2)^2}$	Correct result	A1 **AG** cso
			(3)

Question Number	Scheme		Marks
(c)	$g'(x) = 1 \Rightarrow \dfrac{e^x}{(e^x - 2)^2} = 1$		
	$e^x = (e^x - 2)^2$	Puts their differentiated numerator equal to their denominator.	M1
	$e^x = e^{2x} - 2e^x - 2e^x + 4$		
	$\underline{e^{2x} - 5e^x + 4} = 0$	$\underline{e^{2x} - 5e^x + 4}$	A1
	$(e^x - 4)(e^x - 1) = 0$	Attempt to factorise or solve quadratic in e^x	M1
	$e^x = 4$ or $e^x = 1$		
	$x = \ln 4$ or $x = 0$	both $x = 0,\ \ln 4$	A1
			(4)
			[12]

Question Number	Scheme	Marks
153.	(a) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{(5x-1)}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left((5x-1)^{\frac{1}{2}}\right)$	
	$=5\times\frac{1}{2}(5x-1)^{-\frac{1}{2}}$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2x\sqrt{(5x-1)}+\frac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$	M1 A1ft
	At $x=2$, $\frac{\mathrm{d}y}{\mathrm{d}x}=4\sqrt{9}+\frac{10}{\sqrt{9}}=12+\frac{10}{3}$	M1
	$=\frac{46}{3}$ Accept awrt 15.3	A1 **(6)**
	(b) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\sin 2x}{x^2}\right)=\frac{2x^2\cos 2x-2x\sin 2x}{x^4}$	M1 $\frac{\text{A1+A1}}{\text{A1}}$
		(4)
		[10]
	Alternative to (b)	
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin 2x\times x^{-2}\right)=2\cos 2x\times x^{-2}+\sin 2x\times(-2)x^{-3}$	M1 A1 + A1
	$=2x^{-2}\cos 2x-2x^{-3}\sin 2x \quad \left(=\frac{2\cos 2x}{x^2}-\frac{2\sin 2x}{x^3}\right)$	A1 **(4)**

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Question Number	Scheme	Marks
154.	(a) $\dfrac{2x+2}{x^2-2x-3}-\dfrac{x+1}{x-3}=\dfrac{2x+2}{(x-3)(x+1)}-\dfrac{x+1}{x-3}$	
	$=\dfrac{2x+2-(x+1)(x+1)}{(x-3)(x+1)}$	M1 A1
	$=\dfrac{(x+1)(1-x)}{(x-3)(x+1)}$	M1
	$=\dfrac{1-x}{x-3}$ Accept $-\dfrac{x-1}{x-3},\ \dfrac{x-1}{3-x}$	A1 **(4)**
	(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1-x}{x-3}\right)=\dfrac{(x-3)(-1)-(1-x)1}{(x-3)^2}$	M1 A1
	$=\dfrac{-x+3-1+x}{(x-3)^2}=\dfrac{2}{(x-3)^2}$ * cso	A1 **(3)**
		[7]
	Alternative to (a)	
	$\dfrac{2x+2}{x^2-2x-3}=\dfrac{2(x+1)}{(x-3)(x+1)}=\dfrac{2}{x-3}$	M1 A1
	$\dfrac{2}{x-3}-\dfrac{x+1}{x-3}=\dfrac{2-(x+1)}{x-3}$	M1
	$=\dfrac{1-x}{x-3}$	A1 **(4)**
	Alternatives to (b)	
	① $\mathrm{f}(x)=\dfrac{1-x}{x-3}=-1-\dfrac{2}{x-3}=-1-2(x-3)^{-1}$	
	$\mathrm{f}'(x)=(-1)(-2)(x-3)^{-2}$	M1 A1
	$=\dfrac{2}{(x-3)^2}$ * cso	A1 **(3)**
	② $\mathrm{f}(x)=(1-x)(x-3)^{-1}$	
	$\mathrm{f}'(x)=(-1)(x-3)+(1-x)(-1)(x-3)^{-2}$	M1
	$=-\dfrac{1}{x-3}-\dfrac{1-x}{(x-3)^2}=\dfrac{-(x-3)-(1-x)}{(x-3)^2}$	A1
	$=\dfrac{2}{(x-3)^2}$ *	A1 **(3)**

Question Number	Scheme	Marks
155.	$\mathrm{f}'(x) = 3\mathrm{e}^x + 3x\mathrm{e}^x$	M1 A1
	$3\mathrm{e}^x + 3x\mathrm{e}^x = 3\mathrm{e}^x(1+x) = 0$	
	$x = -1$	M1 A1
	$\mathrm{f}(-1) = -3\mathrm{e}^{-1} - 1$	B1 **(5)**

Question Number	Scheme	Marks
156.	(a) $e^{2x+1} = 2$	
	$2x+1 = \ln 2$	M1
	$x = \frac{1}{2}(\ln 2 - 1)$	A1 (2)
	(b) $\frac{dy}{dx} = 8e^{2x+1}$	B1
	$x = \frac{1}{2}(\ln 2 - 1) \Rightarrow \frac{dy}{dx} = 16$	B1
	$y - 8 = 16\left(x - \frac{1}{2}(\ln 2 - 1)\right)$	M1
	$y = 16x + 16 - 8\ln 2$	A1 (4) **[6]**

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>157.</td><td>(a)(i) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3x}(\sin x+2\cos x)\right)=3\mathrm{e}^{3x}(\sin x+2\cos x)+\mathrm{e}^{3x}(\cos x-2\sin x)$
$\left(=\mathrm{e}^{3x}(\sin x+7\cos x)\right)$</td><td>M1 A1 A1 (3)</td></tr>
<tr><td></td><td>(ii) $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3\ln(5x+2)\right)=3x^2\ln(5x+2)+\frac{5x^3}{5x+2}$</td><td>M1 A1 A1 (3)</td></tr>
<tr><td></td><td>(b) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+1)^2(6x+6)-2(x+1)(3x^2+6x-7)}{(x+1)^4}$</td><td>M1 $\frac{\text{A1}}{\text{A1}}$</td></tr>
<tr><td></td><td>$=\frac{(x+1)(6x^2+12x+6-6x^2-12x+14)}{(x+1)^4}$</td><td>M1</td></tr>
<tr><td></td><td>$=\frac{20}{(x+1)^3}$ * cso</td><td>A1 (5)</td></tr>
<tr><td></td><td>(c) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{60}{(x+1)^4}=-\frac{15}{4}$</td><td>M1</td></tr>
<tr><td></td><td>$(x+1)^4=16$</td><td>M1</td></tr>
<tr><td></td><td>$x=1,-3$ both</td><td>A1 (3)
[14]</td></tr>
<tr><td></td><td>Note: The simplification in part (b) can be carried out as follows
$\frac{(x+1)^2(6x+6)-2(x+1)(3x^2+6x-7)}{(x+1)^4}$
$=\frac{(6x^3+18x^2+18x+6)-(6x^3+18x^2-2x-14)}{(x+1)^4}$
$=\frac{20x+20}{(x+1)^4}=\frac{20(x+1)}{(x+1)^4}=\frac{20}{(x+1)^3}$</td><td>M1 A1</td></tr>
</table>

Question Number	Scheme		Marks	
158.	(a)			
	$\dfrac{dy}{dx} = 2e^{2x}\tan x + e^{2x}\sec^2 x$		M1 A1+A1	
	$\dfrac{dy}{dx} = 0 \quad \Rightarrow \quad 2e^{2x}\tan x + e^{2x}\sec^2 x = 0$		M1	
	$2\tan x + 1 + \tan^2 x = 0$		A1	
	$(\tan x + 1)^2 = 0$			
	$\tan x = -1$ *	cso	A1	(6)
	(b) $\left(\dfrac{dy}{dx}\right)_0 = 1$		M1	
	Equation of tangent at $(0, 0)$ is $y = x$		A1	(2)
				[8]

EXPERT TUITION

Question Number	Scheme		Marks
159.	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6\cos 2x - 8\sin 2x$		M1 A1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = 6$		B1
	$y - 4 = -\frac{1}{6}x$	or equivalent	M1 A1 (5)

Question Number	Scheme	Marks
160.	(a) $x=1-2y^3 \Rightarrow y=\left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ or $\sqrt[3]{\frac{1-x}{2}}$	M1 A1 (2)
	$\mathrm{f}^{-1}: x \mapsto \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ Ignore domain	
	(b) $\mathrm{gf}(x)=\frac{3}{1-2x^3}-4$	M1 A1
	$=\frac{3-4(1-2x^3)}{1-2x^3}$	M1
	$=\frac{8x^3-1}{1-2x^3}$ ✱ cso	A1 (4)
	$\mathrm{gf}: x \mapsto \frac{8x^3-1}{1-2x^3}$ Ignore domain	
	(c) $8x^3-1=0$ Attempting solution of numerator = 0	M1
	$x=\frac{1}{2}$ Correct answer and no additional answers	A1 (2)
	(d) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(1-2x^3)\times 24x^2+(8x^3-1)\times 6x^2}{(1-2x^3)^2}$	M1 A1
	$=\frac{18x^2}{(1-2x^3)^2}$	A1
	Solving their numerator = 0 and substituting to find y.	M1
	$x=0,\ y=-1$	A1 (5) **[13]**

Question Number	Scheme	Notes	Marks
161.	$x^2 + xy + y^2 - 4x - 5y + 1 = 0$		
(a)	$\left\{\cancel{\frac{dy}{dx}} \times\right\} \quad \underline{2x} + \underline{\underline{\left(y + x\frac{dy}{dx}\right)}} \underline{+ 2y\frac{dy}{dx} - 4 - 5\frac{dy}{dx}} = \underline{0}$		M1<u>A1</u> $\underline{\underline{\text{B1}}}$
	$2x + y - 4 + (x + 2y - 5)\frac{dy}{dx} = 0$		dM1
	$\frac{dy}{dx} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{4 - 2x - y}{x + 2y - 5}$	**o.e.**	A1 **cso**
			[5]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\} \; 2x + y - 4 = 0$		M1
	$\{y = 4 - 2x \Rightarrow\} \; x^2 + x(4 - 2x) + (4 - 2x)^2 - 4x - 5(4 - 2x) + 1 = 0$		dM1
	$x^2 + 4x - 2x^2 + 16 - 16x + 4x^2 - 4x - 20 + 10x + 1 = 0$		
	gives $3x^2 - 6x - 3 = 0$ **or** $3x^2 - 6x = 3$ **or** $x^2 - 2x - 1 = 0$	Correct 3TQ in terms of x	A1
	$(x - 1)^2 - 1 - 1 = 0$ and $x = ...$	Method mark for solving a 3TQ in x	ddM1
	$x = 1 + \sqrt{2},\; 1 - \sqrt{2}$	$x = 1 + \sqrt{2},\; 1 - \sqrt{2}$ only	A1
			[5]
(b) **Alt 1**	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\} \; 2x + y - 4 = 0$		M1
	$\left\{x = \frac{4 - y}{2} \Rightarrow\right\} \; \left(\frac{4 - y}{2}\right)^2 + \left(\frac{4 - y}{2}\right)y + y^2 - 4\left(\frac{4 - y}{2}\right) - 5y + 1 = 0$		dM1
	$\left(\frac{16 - 8y + y^2}{2}\right) + \left(\frac{4y - y^2}{2}\right) + y^2 - 2(4 - y) - 5y + 1 = 0$		
	gives $3y^2 - 12y - 12 = 0$ **or** $3y^2 - 12y = 12$ **or** $y^2 - 4y - 4 = 0$	Correct 3TQ in terms of y	A1
	$(y - 2)^2 - 4 - 4 = 0$ and $y = ...$ $x = \frac{4 - (2 + 2\sqrt{2})}{2},\; x = \frac{4 - (2 - 2\sqrt{2})}{2}$	Solves a 3TQ in y **and finds at least one value for x**	ddM1
	$x = 1 + \sqrt{2},\; 1 - \sqrt{2}$	$x = 1 + \sqrt{2},\; 1 - \sqrt{2}$ only	A1
			[5]
			10
(a) **Alt 1**	$\left\{\cancel{\frac{dx}{dy}} \times\right\} \quad \underline{2x\frac{dx}{dy}} + \underline{\underline{\left(y\frac{dx}{dy} + x\right)}} \underline{+ 2y - 4\frac{dx}{dy} - 5} = \underline{0}$		M1<u>A1</u> $\underline{\underline{\text{B1}}}$
	$x + 2y - 5 + (2x + y - 4)\frac{dx}{dy} = 0$		dM1
	$\frac{dy}{dx} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{4 - 2x - y}{x + 2y - 5}$	**o.e.**	A1 **cso**
			[5]

		Question 161 Notes
161. (a)	**M1**	Differentiates implicitly to include either $x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $y^2 \to 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $-5y \to -5\frac{\mathrm{d}y}{\mathrm{d}x}$. $\left(\text{Ignore } \frac{\mathrm{d}y}{\mathrm{d}x} = \ldots\right)$
	A1	$x^2 \to 2x$ **and** $y^2 - 4x - 5y + 1 = 0 \to 2y\frac{\mathrm{d}y}{\mathrm{d}x} - 4 - 5\frac{\mathrm{d}y}{\mathrm{d}x} = 0$
	B1	$xy \to y + x\frac{\mathrm{d}y}{\mathrm{d}x}$
	Note	If an extra term appears then award 1st A0
	Note	$2x + y + x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\frac{\mathrm{d}y}{\mathrm{d}x} - 4 - 5\frac{\mathrm{d}y}{\mathrm{d}x} \;\to\; 2x + y - 4 = -x\frac{\mathrm{d}y}{\mathrm{d}x} - 2y\frac{\mathrm{d}y}{\mathrm{d}x} + 5\frac{\mathrm{d}y}{\mathrm{d}x}$ will get 1st A1 (implied) as the $"=0"$ can be implied the rearrangement of their equation.
	dM1	**dependent on the previous M mark** An attempt to factorise out **all the terms in** $\frac{\mathrm{d}y}{\mathrm{d}x}$ as long as there are **at least two terms** in $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	A1 **cso**	$\frac{2x + y - 4}{5 - x - 2y}$ or $\frac{4 - 2x - y}{x + 2y - 5}$ If the candidate's solution is not completely correct, then do not give the final A mark
(b)	**M1** **Note** **Note**	Sets the numerator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero (or the denominator of their $\frac{\mathrm{d}x}{\mathrm{d}y}$ equal to zero) o.e. This mark can also be gained by setting $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero in their differentiated equation from (a) **If the numerator involves one variable only then *only* the 1st M1 mark is possible in part (b).**
	dM1	**dependent on the previous M mark** Substitutes their x or their y (from their numerator = 0) into the printed equation to give an equation in one variable only
	A1 **Note**	For obtaining the correct 3TQ. E.g.: either $3x^2 - 6x - 3\ \{=0\}$ or $-3x^2 + 6x + 3\ \{=0\}$ This mark can also be awarded for a correct 3 term equation. E.g. either $3x^2 - 6x = 3$ $x^2 - 2x - 1 = 0$ or $x^2 = 2x + 1$ are all fine for A1
	ddM1	**dependent on the previous 2 M marks** See page 6: Method mark for solving THEIR 3-term quadratic in one variable <u>**Quadratic Equation to solve:**</u> $3x^2 - 6x - 3 = 0$ <u>**Way 1:**</u> $x = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(-3)}}{2(3)}$ <u>**Way 2:**</u> $x^2 - 2x - 1 = 0 \Rightarrow (x-1)^2 - 1 - 1 = 0 \Rightarrow x = \ldots$ <u>**Way 3:**</u> Or writes down at least one ***exact*** correct x-root **(*or one correct x-root to 2 dp*)** from ***their*** quadratic equation. This is usually found on their calculator. <u>**Way 4:**</u> **(Only allowed if their 3TQ can be factorised)** • $(x^2 + bx + c) = (x+p)(x+q)$, where $\lvert pq \rvert = \lvert c \rvert$, leading to $x = \ldots$ • $(ax^2 + bx + c) = (mx+p)(nx+q)$, where $\lvert pq \rvert = \lvert c \rvert$ and $\lvert mn \rvert = a$, leading to $x = \ldots$
	Note	If a candidate applies ***the alternative method*** then they also need to use their $x = \frac{4-y}{2}$ to find **at least one value** for x in order to gain the final M mark.
	A1	Exact values of $x = 1 + \sqrt{2},\ 1 - \sqrt{2}$ (or $1 \pm \sqrt{2}$) , **cao** Apply isw if y-values are also found.
	Note	It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\frac{\mathrm{d}y}{\mathrm{d}x}$) to gain all 5 marks in part (b)

		Question 161 Notes
161. (a) **Alt 1**	**M1**	Differentiates implicitly to include either $y\frac{\mathrm{d}x}{\mathrm{d}y}$ or $x^2 \to 2x\frac{\mathrm{d}x}{\mathrm{d}y}$ or $-4x \to -4\frac{\mathrm{d}x}{\mathrm{d}y}$. $\left(\text{Ignore } \frac{\mathrm{d}x}{\mathrm{d}y} = ...\right)$
	A1	$x^2 \to 2x\frac{\mathrm{d}x}{\mathrm{d}y}$ **and** $y^2 - 4x - 5y + 1 = 0 \to 2y - 4\frac{\mathrm{d}x}{\mathrm{d}y} - 5 = 0$
	B1	$xy \to y\frac{\mathrm{d}x}{\mathrm{d}y} + x$
	Note	If an extra term appears then award 1st A0
	Note	$2x\frac{\mathrm{d}x}{\mathrm{d}y} + y\frac{\mathrm{d}x}{\mathrm{d}y} + x + 2y - 4\frac{\mathrm{d}x}{\mathrm{d}y} - 5 \ \to\ x + 2y - 5 = -2x\frac{\mathrm{d}x}{\mathrm{d}y} - y\frac{\mathrm{d}x}{\mathrm{d}y} + 4\frac{\mathrm{d}x}{\mathrm{d}y}$ will get 1st A1 (implied) as the $"=0"$ can be implied the rearrangement of their equation.
	dM1	**dependent on the previous M mark** An attempt to factorise out **all the terms in** $\frac{\mathrm{d}x}{\mathrm{d}y}$ as long as there are **at least two terms** in $\frac{\mathrm{d}x}{\mathrm{d}y}$
	A1 **cso**	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4 - 2x - y}{x + 2y - 5}$ If the candidate's solution is not completely correct, then do not give the final A mark
(a)	**Note**	Writing down ***from no working*** • $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4 - 2x - y}{x + 2y - 5}$ scores M1 A1 B1 M1 A1 • $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4 - 2x - y}{5 - x - 2y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + y - 4}{x + 2y - 5}$ scores M1 A0 B1 M1 A0
	Note	Writing $2x\mathrm{d}x + y\mathrm{d}x + x\mathrm{d}y + 2y\mathrm{d}y - 4\mathrm{d}x - 5\mathrm{d}y = 0$ scores M1 A1 B1

Question Number	Scheme	Notes	Marks
162. (a)	$\frac{r}{h} = \tan 30 \Rightarrow r = h\tan 30 \left\{\Rightarrow r = \frac{h}{\sqrt{3}} \text{ or } r = \frac{\sqrt{3}}{3}h\right\}$ **or** $\frac{h}{r} = \tan 60 \Rightarrow r = \frac{h}{\tan 60} \left\{\Rightarrow r = \frac{h}{\sqrt{3}} \text{ or } r = \frac{\sqrt{3}}{3}h\right\}$ **or** $\frac{r}{\sin 30} = \frac{h}{\sin 60} \Rightarrow r = \frac{h\sin 30}{\sin 60} \left\{\Rightarrow r = \frac{h}{\sqrt{3}} \text{ or } r = \frac{\sqrt{3}}{3}h\right\}$ **or** $h^2 + r^2 = (2r)^2 \Rightarrow r^2 = \frac{1}{3}h^2$	Correct use of trigonometry to find r in terms of h **or** correct use of Pythagoras to find r^2 in terms of h^2	M1
	$\left\{V = \frac{1}{3}\pi r^2 h \Rightarrow\right\} V = \frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h \Rightarrow V = \frac{1}{9}\pi h^3$ *	Correct proof of $V = \frac{1}{9}\pi h^3$ or $V = \frac{1}{9}h^3\pi$ Or shows $\frac{1}{9}\pi h^3$ or $\frac{1}{9}h^3\pi$ with some reference to $V =$ in their solution	A1 *
			[2]
(b) **Way 1**	$\frac{dV}{dt} = 200$		
	$\frac{dV}{dh} = \frac{1}{3}\pi h^2$	$\frac{1}{3}\pi h^2$ o.e.	B1
	Either • $\left\{\frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\} \left(\frac{1}{3}\pi h^2\right)\frac{dh}{dt} = 200$ • $\left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow\right\} \frac{dh}{dt} = 200 \times \frac{1}{\frac{1}{3}\pi h^2}$	**either** $\left(\text{their } \frac{dV}{dh}\right) \times \frac{dh}{dt} = 200$ **or** $200 \div \left(\text{their } \frac{dV}{dh}\right)$	M1
	When $h = 15,\ \frac{dh}{dt} = 200 \times \frac{1}{\frac{1}{3}\pi(15)^2}\ \left\{= \frac{200}{75\pi} = \frac{600}{225\pi}\right\}$	**dependent on the previous M mark**	dM1
	$\frac{dh}{dt} = \frac{8}{3p}\ (\text{cm s}^{-1})$	$\frac{8}{3p}$	A1 **cao**
			[4]
			6
(b) **Way 2**	$\frac{dV}{dt} = 200 \Rightarrow V = 200t + c \Rightarrow \frac{1}{9}\pi h^3 = 200t + c$		
	$\left(\frac{1}{3}\pi h^2\right)\frac{dh}{dt} = 200$	$\frac{1}{3}\pi h^2$ o.e.	B1
		as in Way 1	M1
	When $h = 15,\ \frac{dh}{dt} = 200 \times \frac{1}{\frac{1}{3}\pi(15)^2}\ \left\{= \frac{200}{75\pi} = \frac{600}{225\pi}\right\}$	**dependent on the previous M mark**	dM1
	$\frac{dh}{dt} = \frac{8}{3p}\ (\text{cm s}^{-1})$	$\frac{8}{3p}$	A1 **cao**
			[4]

		Question 162 Notes
162. (a)	**Note**	Allow M1 for writing down $r = h\tan 30$
	Note	Give M0 A0 for writing down $r = \frac{h\sqrt{3}}{3}$ or $r = \frac{h}{\sqrt{3}}$ with no evidence of using trigonometry on r and h or Pythagoras on r and h
	Note	Give M0 (unless recovered) for evidence of $\frac{1}{3}\pi r^2 h = \frac{1}{9}\pi h^3$ leading to either $r^2 = \frac{1}{3}h^2$ or $r = \frac{h\sqrt{3}}{3}$ or $r = \frac{h}{\sqrt{3}}$
(b)	**B1**	Correct simplified or un-simplified differentiation of V. E.g. $\frac{1}{3}\pi h^2$ or $\frac{3}{9}\pi h^2$
	Note	$\frac{\mathrm{d}V}{\mathrm{d}h}$ does not have to be explicitly stated, but it should be clear that they are differentiating their V
	M1	$\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = 200$ or $200 \div \left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)$
	dM1	**dependent on the previous M mark** Substitutes $h = 15$ into an expression ***which is a result*** of either $200 \div \left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)$ or $200 \times \frac{1}{\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)}$
	A1	$\frac{8}{3p}$ (units are not required)
	Note	Give final A0 for using $\frac{\mathrm{d}V}{\mathrm{d}t} = -200$ to give $\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{8}{3\pi}$, unless recovered to $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{8}{3\pi}$

Question Number	Scheme	Notes	Marks
163.	$x = 3t - 4,\ \ y = 5 - \dfrac{6}{t},\ \ \ t > 0$		
(a)	$\dfrac{dx}{dt} = 3,\ \ \dfrac{dy}{dt} = 6t^{-2}$		
	$\dfrac{dy}{dx} = \dfrac{6t^{-2}}{3}\ \left\{= \dfrac{6}{3t^2} = 2t^{-2} = \dfrac{2}{t^2}\right\}$	their $\dfrac{dy}{dt}$ divided by their $\dfrac{dx}{dt}$ to give $\dfrac{dy}{dx}$ in terms of t **or** their $\dfrac{dy}{dt}$ multiplied by their $\dfrac{dt}{dx}$ to give $\dfrac{dy}{dx}$ in terms of t	M1
		$\dfrac{6t^{-2}}{3}$, simplified or un-simplified, in terms of t. **See note.**	A1 isw
	Award **Special Case 1st M1** if **both** $\dfrac{dx}{dt}$ **and** $\dfrac{dy}{dt}$ are stated **correctly** and **explicitly**.		**[2]**
	Note: You can recover the work for part (a) in part (b).		
(a) **Way 2**	$y = 5 - \dfrac{18}{x+4} \Rightarrow \dfrac{dy}{dx} = \dfrac{18}{(x+4)^2} = \dfrac{18}{(3t)^2}$	Writes $\dfrac{dy}{dx}$ in the form $\dfrac{\pm\lambda}{(x+4)^2}$, ***and*** writes $\dfrac{dy}{dx}$ as a function of t.	M1
		Correct un-simplified or simplified answer, in terms of t. **See note.**	A1 isw
			[2]
(b)	$\left\{t = \dfrac{1}{2} \Rightarrow\right\} P\left(-\dfrac{5}{2}, -7\right)$	$x = -\dfrac{5}{2},\ y = -7$ **or** $P\left(-\dfrac{5}{2}, -7\right)$ seen or implied.	B1
	$\dfrac{dy}{dx} = \dfrac{2}{\left(\frac{1}{2}\right)^2}$ **and** either • $y - \text{"}-7\text{"} = \text{"}8\text{"}\left(x - \text{"}-\frac{5}{2}\text{"}\right)$ • $\text{"}-7\text{"} = (\text{"}8\text{"})(\text{"}-\frac{5}{2}\text{"}) + c$ So, $y = (\text{their } m_T)x + \text{"}c\text{"}$	**Some** attempt to substitute $t = 0.5$ into their $\dfrac{dy}{dx}$ which contains t in order to find m_T **and either** applies $y - (\text{their } y_P) = (\text{their } m_T)(x - \text{their } x_P)$ **or** finds c from $(\text{their } y_P) = (\text{their } m_T)(\text{their } x_P) + c$ and uses their numerical c in $y = (\text{their } m_T)x + c$	M1
	T: $y = 8x + 13$	$y = 8x + 13$ or $y = 13 + 8x$	A1 cso
	Note: their x_P, their y_P and their m_T must be numerical values in order to award M1		**[3]**
(c) **Way 1**	$\left\{t = \dfrac{x+4}{3} \Rightarrow\right\} y = 5 - \dfrac{6}{\left(\dfrac{x+4}{3}\right)}$	An attempt to eliminate t. **See notes.**	M1
		Achieves a correct equation in x and y only	A1 o.e.
	$\Rightarrow\ y = 5 - \dfrac{18}{x+4}\ \Rightarrow\ y = \dfrac{5(x+4) - 18}{x+4}$		
	So, $y = \dfrac{5x+2}{x+4},\ \{x > -4\}$	$y = \dfrac{5x+2}{x+4}$ (or implied equation)	A1 cso
			[3]
(c) **Way 2**	$\left\{t = \dfrac{6}{5-y} \Rightarrow\right\} x = \dfrac{18}{5-y} - 4$	An attempt to eliminate t. **See notes.**	M1
		Achieves a correct equation in x and y only	A1 o.e.
	$\Rightarrow\ (x+4)(5-y) = 18 \Rightarrow 5x - xy + 20 - 4y = 18$		
	$\{\Rightarrow 5x + 2 = y(x+4)\}$ So, $y = \dfrac{5x+2}{x+4},\ \{x > -4\}$	$y = \dfrac{5x+2}{x+4}$ (or implied equation)	A1 cso
			[3]
	Note: Some or all of the work for part (c) can be recovered in part (a) or part (b)		**8**

Question Number	Scheme	Notes	Marks
163. (c) **Way 3**	$y = \dfrac{3at - 4a + b}{3t - 4 + 4} = \dfrac{3at}{3t} - \dfrac{4a - b}{3t} = a - \dfrac{4a - b}{3t} \Rightarrow a = 5$	A full method leading to the value of a being found	M1
		$y = a - \dfrac{4a-b}{3t}$ and $a = 5$	A1
	$\dfrac{4a-b}{3} = 6 \Rightarrow b = 4(5) - 6(3) = 2$	**Both** $a = 5$ **and** $b = 2$	A1
			[3]

		Question 163 Notes
1. (a)	**Note**	Condone $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\left(\frac{6}{t^2}\right)}{3}$ for A1
	Note	You can ignore subsequent working following on from a correct expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.
(b)	**Note**	Using a changed gradient (i.e. applying $\dfrac{-1}{\text{their } \frac{\mathrm{d}y}{\mathrm{d}x}}$ or $\dfrac{1}{\text{their } \frac{\mathrm{d}y}{\mathrm{d}x}}$ or $-\left(\text{their } \frac{\mathrm{d}y}{\mathrm{d}x}\right)$) is M0.
	Note	**Final A1:** A correct solution is required from a correct $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.
	Note	**Final A1:** You can ignore subsequent working following on from a correct solution.
(c)	**Note**	**1st M1:** A full attempt to eliminate t is defined as either • rearranging one of the parametric equations to make t the subject and substituting for t in the other parametric equation (only the RHS of the equation required for M mark) • rearranging both parametric equations to make t the subject and putting the results equal to each other.
	Note	Award M1A1 for $\dfrac{6}{5-y} = \dfrac{x+4}{3}$ or equivalent.

Question Number	Scheme	Notes	Marks
164	$4x^2 - y^3 - 4xy + 2^y = 0$		
(a) **Way 1**	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \cancel{=}\right\}\ \underline{8x - 3y^2\frac{dy}{dx}}\ \underline{\underline{-4y - 4x\frac{dy}{dx}}}\ + \overline{\overline{2^y \ln 2\frac{dy}{dx}}}\ \underline{=0}$		M1 $\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$ $\overline{\overline{\text{B1}}}$
	$8(-2) - 3(4)^2\frac{dy}{dx} - 4(4) - 4(-2)\frac{dy}{dx} + 2^4\ln 2\frac{dy}{dx} = 0$	**dependent on the first M mark**	dM1
	$-16 - 48\frac{dy}{dx} - 16 + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} = 0$		
	$\frac{dy}{dx} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$	or exact equivalent	A1 **cso**
	NOTE: You can recover work for part (a) in part (b)		**[6]**
(b)	e.g. $m_N = \frac{-40+16\ln 2}{-32}$ or $\frac{40-16\ln 2}{32}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical m_N **Can be implied by later working**	M1
	• $y - 4 = \left(\frac{40-16\ln 2}{32}\right)(x - -2)$ Cuts y-axis $\Rightarrow x = 0 \Rightarrow\ y - 4 = \left(\frac{40-16\ln 2}{32}\right)(2)$ • $4 = \left(\frac{40-16\ln 2}{32}\right)(-2) + c$	Using a numerical m_N $(\neq m_T)$, either $y - 4 = m_N(x - -2)$ **and** sets $x = 0$ in their normal equation **or** $4 = (\text{their } m_N)(-2) + c$	M1
	$\left\{\Rightarrow\ c = 4 + \frac{40-16\ln 2}{16}, \text{ so } y = \frac{104-16\ln 2}{16} \Rightarrow\right\}$		
	y (or c) $= \frac{13}{2} - \ln 2$	$\frac{104}{16} - \ln 2$ **or** $\frac{13}{2} - \ln 2$ **or** $-\ln 2 + \frac{13}{2}$	A1 **cso isw**
	Note: Allow exact equivalents in the form $p - \ln 2$ for the final A mark		**[3]**
			9
(a) **Way 2**	$\left\{\frac{\cancel{dx}}{\cancel{dy}} \cancel{=}\right\}\ \underline{8x\frac{dx}{dy} - 3y^2}\ \underline{\underline{-4y\frac{dx}{dy} - 4x}} + \overline{\overline{2^y \ln 2}}\ \underline{=0}$		M1 $\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$ $\overline{\overline{\text{B1}}}$
	$8(-2)\frac{dx}{dy} - 3(4)^2 - 4(4)\frac{dx}{dy} - 4(-2) + 2^4\ln 2 = 0$	**dependent on the first M mark**	dM1
	$\frac{dy}{dx} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$	or exact equivalent	A1 **cso**
	Note: You must be clear that Way 2 is being applied before you use this scheme		**[6]**

		Question 164 Notes
164. (a)	**Note**	**<u>For the first four marks</u>**
		Writing down ***from no working*** • $\frac{dy}{dx} = \frac{4y - 8x}{-3y^2 - 4x + 2^y\ln 2}$ or $\frac{8x - 4y}{3y^2 + 4x - 2^y\ln 2}$ scores M1A1M1B1 • $\frac{dy}{dx} = \frac{8x - 4y}{-3y^2 - 4x + 2^y\ln 2}$ or $\frac{4y - 8x}{3y^2 + 4x - 2^y\ln 2}$ scores M1A0M1B1
		Writing $8x\,dx - 3y^2\,dy - 4y\,dx - 4x\,dy + 2^y\ln 2\,dy = 0$ scores M1A1M1B1

		Question 164 Notes Continued
164. (a)	**1st M1**	Differentiates implicitly to include ***either*** $\pm 4x\frac{dy}{dx}$ ***or*** $-y^3 \to \pm \lambda y^2 \frac{dy}{dx}$ ***or*** $2^y \to \pm \mu 2^y \frac{dy}{dx}$ (Ignore $\left(\frac{dy}{dx} = \right)$). λ , μ are constants which can be 1
	1st <u>A1</u>	**Both** $4x^2 - y^3 \to 8x - 3y^2\frac{dy}{dx}$ **and** $= 0 \to = 0$
	Note	e.g. $8x - 3y^2\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} + 2^y\ln 2\frac{dy}{dx} \to -3y^2\frac{dy}{dx} - 4x\frac{dy}{dx} + 2^y \ln 2\frac{dy}{dx} = 4y - 8x$ **or** e.g. $-16 - 48\frac{dy}{dx} - 16 + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} \to -48\frac{dy}{dx} + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} = 32$ will get 1st A1 (implied) as the "$=0$" can be implied by the rearrangement of their equation.
	2nd <u>M1</u>	$-4xy \to -4y - 4x\frac{dy}{dx}$ or $4y - 4x\frac{dy}{dx}$ or $-4y + 4x\frac{dy}{dx}$ or $4y + 4x\frac{dy}{dx}$
	$\overline{\overline{\textbf{B1}}}$	$2^y \to 2^y \ln 2\frac{dy}{dx}$ **or** $2^y \to e^{y\ln 2}\ln 2\frac{dy}{dx}$
	Note	If an extra term appears then award 1st A0
	3rd dM1	**dependent on the first M mark** For substituting $x = -2$ and $y = 4$ into an equation involving $\frac{dy}{dx}$
	Note	M1 can be gained by seeing at least one example of substituting $x = -2$ and at least one example of substituting $y = 4$ **unless** it is clear that they are instead applying $x = 4$ and $y = -2$ Otherwise, you will NEED to check (with your calculator) that $x = -2$, $y = 4$ that has been substituted into their equation involving $\frac{dy}{dx}$
	Note	**A1 cso:** If the candidate's solution is not completely correct, then do not give this mark.
	Note	**isw:** You can, however, ignore subsequent working following on from correct solution.
(b)	**Note**	The 2nd M1 mark can be implied by later working. **Eg. Award 1st M1 and 2nd M1** for $\frac{y-4}{2} = \frac{-1}{\text{their } m_T \text{ evaluated at } x=-2 \text{ and } y = 4}$
	Note	**A1:** Allow the alternative answer $\{y =\} \ln\left(\frac{1}{2}\right) + \frac{13}{2\ln 2}(\ln 2)$ which is in the form $p + q\ln 2$
164. (a) **Way 2**	**1st M1**	Differentiates implicitly to include ***either*** $\pm 4y\frac{dx}{dy}$ ***or*** $4x^2 \to \pm \lambda x\frac{dx}{dy}$ (Ignore $\left(\frac{dx}{dy} = \right)$). λ is a constant which can be 1
	1st <u>A1</u>	**Both** $4x^2 - y^3 \to 8x\frac{dx}{dy} - 3y^2$ **and** $= 0 \to = 0$
	2nd <u>M1</u>	$-4xy \to -4y\frac{dx}{dy} - 4x$ or $4y\frac{dx}{dy} - 4x$ or $-4y\frac{dx}{dy} + 4x$ or $4y\frac{dx}{dy} + 4x$
	$\overline{\overline{\textbf{B1}}}$	$2^y \to 2^y \ln 2$
	3rd dM1	**dependent on the first M mark** For substituting $x = -2$ and $y = 4$ into an equation involving $\frac{dx}{dy}$

Question Number		Notes	Marks
165.	$2x^2y + 2x + 4y - \cos(\pi y) = 17$		
(a) **Way 1**	$\left\{\frac{\cancel{dx}}{\cancel{dx}} \times\right\} \quad \underline{\underline{\left(4xy + 2x^2\frac{dy}{dx}\right)}} \; \underline{+ 2 + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} = 0}$		M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	$\frac{dy}{dx}\left(2x^2 + 4 + \pi\sin(\pi y)\right) + 4xy + 2 = 0$		dM1
	$\left\{\frac{dy}{dx} = \right\} \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$	Correct answer or equivalent	A1 **cso**
			[5]
(b)	At $\left(3, \frac{1}{2}\right)$, $m_T = \frac{dy}{dx} = \frac{-4(3)(\frac{1}{2}) - 2}{2(3)^2 + 4 + \pi\sin\left(\frac{1}{2}\pi\right)} \quad \left\{= \frac{-8}{22 + \pi}\right\}$	Substituting $x = 3$ & $y = \frac{1}{2}$ into an equation involving $\frac{dy}{dx}$	M1
	$m_N = \frac{22 + \pi}{8}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical m_N **Can be implied by later working**	M1
	• $y - \frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$ • $\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(3) + c \Rightarrow c = \frac{1}{2} - \frac{66 + 3\pi}{8}$ $\Rightarrow y = \left(\frac{22 + \pi}{8}\right)x + \frac{1}{2} - \frac{66 + 3\pi}{8}$ Cuts x-axis $\Rightarrow y = 0$ $\Rightarrow -\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$	$y - \frac{1}{2} = m_N(x - 3)$ or $y = m_N x + c$ where $\frac{1}{2} = (\text{their } m_N)3 + c$ with a numerical m_N $(\neq m_T)$ where m_N is in terms of π and sets $y = 0$ in their normal equation.	dM1
	So, $\left\{x = \frac{-4}{22 + \pi} + 3 \Rightarrow\right\} x = \frac{3\pi + 62}{\pi + 22}$	$\frac{3\pi + 62}{\pi + 22}$ or $\frac{6\pi + 124}{2\pi + 44}$ or $\frac{62 + 3\pi}{22 + \pi}$	A1 o.e.
			[4]
			9
(a) **Way 2**	$\left\{\frac{\cancel{dx}}{\cancel{dy}} \times\right\} \quad \underline{\underline{\left(4xy\frac{dx}{dy} + 2x^2\right)}} \; \underline{+ 2\frac{dx}{dy} + 4 + \pi\sin(\pi y) = 0}$		M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	$\frac{dx}{dy}\left(4xy + 2\right) + 2x^2 + 4 + \pi\sin(\pi y) = 0$		dM1
	$\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$	Correct answer or equivalent	A1 **cso**
			[5]

Question 165 Notes

165. (a)

Note Writing down ***from no working***

- $\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$ scores M1A1B1M1A1
- $\frac{dy}{dx} = \frac{4xy + 2}{2x^2 + 4 + \pi\sin(\pi y)}$ scores M1A0B1M1A0

Note Few candidates will write $4xy\,dx + 2x^2dy + 2dx + 4dy + \pi\sin(\pi y)dy = 0$ leading to $\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or equivalent. This should get full marks.

	Question 165 Notes Continued	
165. (a) **Way 1**	**M1**	Differentiates implicitly to include either $2x^2\dfrac{dy}{dx}$ or $4y \to 4\dfrac{dy}{dx}$ or $-\cos(\pi y) \to \pm\lambda\sin(\pi y)\dfrac{dy}{dx}$ (Ignore $\left(\dfrac{dy}{dx} = \right)$). λ is a constant which can be 1.
	1st A1	$2x + 4y - \cos(\pi y) = 17 \;\; \to \;\; 2 + 4\dfrac{dy}{dx} + \pi\sin(\pi y)\dfrac{dy}{dx} = 0$
	Note	$4xy + 2x^2\dfrac{dy}{dx} + 2 + 4\dfrac{dy}{dx} + \pi\sin(\pi y)\dfrac{dy}{dx} \;\to\; 2x^2\dfrac{dy}{dx} + 4\dfrac{dy}{dx} + \pi\sin(\pi y)\dfrac{dy}{dx} = -4xy - 2$ will get 1st A1 (implied) as the $"=0"$ can be implied by the rearrangement of their equation.
	B1	$2x^2y \to 4xy + 2x^2\dfrac{dy}{dx}$
	Note	If an extra term appears then award 1st A0.
	dM1	**Dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\dfrac{dy}{dx}$ as long as there are ***at least two terms*** in $\dfrac{dy}{dx}$. ie. $\dfrac{dy}{dx}\left(2x^2 + 4 + \pi\sin(\pi y)\right) + \; ... \; = \; ...$
	Note	Writing down an extra $\dfrac{dy}{dx} = ...$ and then including it in their factorisation is fine for dM1.
	Note	**Final A1 cso:** If the candidate's solution is not completely correct, then do not give this mark.
	Note	**Final A1 isw:** You can, however, ignore subsequent working following on from correct solution.
(a)	**Way 2**	Apply the mark scheme for Way 2 in the same way as Way 1.
(b)	**1st M1**	M1 can be gained by seeing at least one example of substituting $x = 3$ and at least one example of substituting $y = \dfrac{1}{2}$. E.g. $"-4xy" \to "-6"$ in their $\dfrac{dy}{dx}$ would be sufficient for M1, unless it is clear that they are instead applying $x = \dfrac{1}{2}$, $y = 3$.
	3rd M1	***is dependent on the first M1.***
	Note	The 2nd M1 mark can be implied by later working. **Eg. Award 2nd M1 3rd M1** for $\dfrac{\frac{1}{2}}{3-x} = \dfrac{-1}{\text{their } m_T}$
	Note	We can accept $\sin\pi$ or $\sin\left(\dfrac{\pi}{2}\right)$ as a numerical value for the 2nd M1 mark. But, $\sin\pi$ by itself or $\sin\left(\dfrac{\pi}{2}\right)$ by itself are not allowed as being in terms of π for the 3rd M1 mark. The 3rd M1 can be accessed for terms containing $\pi\sin\left(\dfrac{\pi}{2}\right)$.

Question Number	Scheme	Notes	Marks
166.	$x = 4\tan t, \quad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 1**	$\frac{\mathrm{d}x}{\mathrm{d}t} = 4\sec^2 t,\ \frac{\mathrm{d}y}{\mathrm{d}t} = 10\sqrt{3}\cos 2t$	**Either both** x and y are differentiated correctly with respect to t **or** their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$ **or** applies $\frac{\mathrm{d}y}{\mathrm{d}t}$ multiplied by their $\frac{\mathrm{d}t}{\mathrm{d}x}$	M1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{10\sqrt{3}\cos 2t}{4\sec^2 t} \quad \left\{= \frac{5}{2}\sqrt{3}\cos 2t\cos^2 t\right\}$	Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ (Can be implied)	A1 oe
	$\left\{\text{At } P\left(4\sqrt{3}, \frac{15}{2}\right),\ t = \frac{\pi}{3}\right\}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{10\sqrt{3}\cos\left(\frac{2\pi}{3}\right)}{4\sec^2\left(\frac{\pi}{3}\right)}$	**dependent on the previous M mark** ***Some evidence*** of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$	dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]
(b)	$\left\{10\sqrt{3}\cos 2t = 0 \Rightarrow t = \frac{\pi}{4}\right\}$		
	So $x = 4\tan\left(\frac{\pi}{4}\right),\ y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$	At least one of either $x = 4\tan\left(\frac{\pi}{4}\right)$ **or** $y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$ **or** $x = 4$ **or** $y = 5\sqrt{3}$ **or** $y =$ awrt 8.7	M1
	Coordinates are $\left(4, 5\sqrt{3}\right)$	$\left(4, 5\sqrt{3}\right)$ **or** $x = 4,\ y = 5\sqrt{3}$	A1
			[2]
			6

Question 166 Notes

166. (a)	**1st A1**	Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$. E.g. $\frac{10\sqrt{3}\cos 2t}{4\sec^2 t}$ or $\frac{5}{2}\sqrt{3}\cos 2t\cos^2 t$ or $\frac{5}{2}\sqrt{3}\cos^2 t(\cos^2 t - \sin^2 t)$ or any equivalent form.
	Note	Give the final A0 for a final answer of $-\frac{10}{32}\sqrt{3}$ **without reference to** $-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$
	Note	Give the final A0 for more than one value stated for $\frac{\mathrm{d}y}{\mathrm{d}x}$
(b)	**Note**	Also allow M1 for either $x = 4\tan(45)$ or $y = 5\sqrt{3}\sin\left(2(45)\right)$
	Note	M1 can be gained by ignoring previous working in part (a) and/or part (b)
	Note	Give A0 for stating more than one set of coordinates for Q.
	Note	Writing $x = 4,\ y = 5\sqrt{3}$ followed by $\left(5\sqrt{3}, 4\right)$ is A0.

Question Number	Scheme	Notes	Marks
166.	$x = 4\tan t, \quad y = 5\sqrt{3}\sin 2t, \quad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 2**	$\tan t = \frac{x}{4} \Rightarrow \sin t = \frac{x}{\sqrt{(x^2+16)}}, \cos t = \frac{4}{\sqrt{(x^2+16)}} \Rightarrow y = \frac{40\sqrt{3}x}{x^2+16}$		
	$\left\{\begin{array}{ll} u = 40\sqrt{3}x & v = x^2+16 \\ \frac{du}{dx} = 40\sqrt{3} & \frac{dv}{dx} = 2x \end{array}\right\}$		
	$\frac{dy}{dx} = \frac{40\sqrt{3}(x^2+16) - 2x(40\sqrt{3}x)}{(x^2+16)^2} \left\{= \frac{40\sqrt{3}(16-x^2)}{(x^2+16)^2}\right\}$	$\frac{\pm A(x^2+16) \pm Bx^2}{(x^2+16)^2}$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified	A1
	$\frac{dy}{dx} = \frac{40\sqrt{3}(48+16) - 80\sqrt{3}(48)}{(48+16)^2}$	**dependent on the previous M mark** ***Some evidence*** of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]
(a) **Way 3**	$y = 5\sqrt{3}\sin\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)$		
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{2}{1+\left(\frac{x}{4}\right)^2}\right)\left(\frac{1}{4}\right)$	$\frac{dy}{dx} = \pm A\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{1}{1+x^2}\right)$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified.	A1
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\sqrt{3}\right)\right)\left(\frac{2}{1+3}\right)\left(\frac{1}{4}\right) \left\{= 5\sqrt{3}\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\right\}$	**dependent on the previous M mark** ***Some evidence*** of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]

Question Number	Scheme	Marks
167.	$x^2 - 3xy - 4y^2 + 64 = 0$	
(a)	$\left\{\cancel{\frac{dy}{dx}} \cancel{=}\right\}$ $\underline{2x} - \left(\underline{\underline{3y + 3x\frac{dy}{dx}}}\right) \underline{- 8y\frac{dy}{dx}} = \underline{0}$	M1A1 M1
	$2x - 3y + (-3x - 8y)\frac{dy}{dx} = 0$	dM1
	$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ **o.e.**	A1 **cso**
		[5]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}$ $2x - 3y = 0$	M1
	$y = \frac{2}{3}x$ | $x = \frac{3}{2}y$	A1ft
	$x^2 - 3x\left(\frac{2}{3}x\right) - 4\left(\frac{2}{3}x\right)^2 + 64 = 0$ | $\left(\frac{3}{2}y\right)^2 - 3\left(\frac{3}{2}y\right)y - 4y^2 + 64 = 0$	dM1
	$x^2 - 2x^2 - \frac{16}{9}x^2 + 64 = 0 \Rightarrow -\frac{25}{9}x^2 + 64 = 0$ | $\frac{9}{4}y^2 - \frac{9}{2}y^2 - 4y^2 + 64 = 0 \Rightarrow -\frac{25}{4}y^2 + 64 = 0$	
	$\left\{\Rightarrow x^2 = \frac{576}{25} \Rightarrow\right\}$ $x = \frac{24}{5}$ or $-\frac{24}{5}$ | $\left\{\Rightarrow y^2 = \frac{256}{25} \Rightarrow\right\}$ $y = \frac{16}{5}$ or $-\frac{16}{5}$	A1 **cso**
	When $x = \pm\frac{24}{5}$, $y = \frac{2}{3}\left(\frac{24}{5}\right)$ and $-\frac{2}{3}\left(\frac{24}{5}\right)$ | When $y = \pm\frac{16}{5}$, $x = \frac{3}{2}\left(\frac{16}{5}\right)$ and $-\frac{3}{2}\left(\frac{16}{5}\right)$	
	$\left(\frac{24}{5}, \frac{16}{5}\right)$ **and** $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ **or** $x = \frac{24}{5}$, $y = \frac{16}{5}$ **and** $x = -\frac{24}{5}$, $y = -\frac{16}{5}$ **cso**	ddM1 A1
		[6] **11**
	Alternative method for part (a)	
(a)	$\left\{\cancel{\frac{dx}{dy}} \cancel{=}\right\}$ $\underline{2x\frac{dx}{dy}} - \left(\underline{\underline{3y\frac{dx}{dy} + 3x}}\right) \underline{- 8y} = \underline{0}$	M1A1 M1
	$(2x - 3y)\frac{dx}{dy} - 3x - 8y = 0$	dM1
	$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ **o.e.**	A1 **cso**
		[5]

		Question 167 Notes
167. (a) **General**	**Note**	Writing down $\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ from **no working** is full marks
	Note	Writing down $\frac{dy}{dx} = \frac{2x - 3y}{-3x - 8y}$ or $\frac{3y - 2x}{3x + 8y}$ from **no working** is M1A0B1M1A0
	Note	Few candidates will write $2x\,dx - 3y\,dx - 3x\,dy - 8y\,dy = 0$ leading to $\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$, o.e. This should get full marks.

167. (a)	**M1**	Differentiates implicitly to include either $\pm 3x\frac{dy}{dx}$ or $-4y^2 \to \pm ky\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} =\right)$).
	A1	**Both** $x^2 \to \underline{2x}$ **and** $... - 4y^2 + 64 = 0 \to \underline{-8y\frac{dy}{dx} = 0}$
	Note	If an extra term appears then award A0.
	M1	$-3xy \to -3x\frac{dy}{dx} - 3y$ or $-3x\frac{dy}{dx} + 3y$ or $3x\frac{dy}{dx} - 3y$ or $3x\frac{dy}{dx} + 3y$
	Note	$2x - 3y - 3x\frac{dy}{dx} - 8y\frac{dy}{dx} \to 2x - 3y = 3x\frac{dy}{dx} + 8y\frac{dy}{dx}$ will get 1st A1 (implied) as the $"=0"$ can be implied by the rearrangement of their equation.
	dM1	**dependent on the FIRST method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{dy}{dx}$ as long as there are ***at least two terms*** in $\frac{dy}{dx}$. i.e. $... + (-3x - 8y)\frac{dy}{dx} = ...$ or $... = (3x + 8y)\frac{dy}{dx}$. (Allow combining in 1 variable).
	A1	$\frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ or equivalent.
	Note	**cso** If the candidate's solution is not completely correct, then do not give this mark.
	Note	You cannot recover work for part (a) in part (b).
167. (b)	**M1**	Sets their numerator of their $\frac{dy}{dx}$ equal to zero (or the denominator of their $\frac{dx}{dy}$ equal to zero) o.e.
	Note	1st M1 can also be gained by setting $\frac{dy}{dx}$ equal to zero in their " $2x - 3y - 3x\frac{dy}{dx} - 8y\frac{dy}{dx} = 0$ "
	Note	**If their numerator involves one variable only then only the 1st M1 mark is possible in part (b).**
	Note	**If their numerator is a constant then no marks are available in part (b)**
	Note	**If their numerator is in the form** $\pm ax^2 \pm by = 0$ or $\pm ax \pm by^2 = 0$ **then the first 3 marks are possible in part (b).**
	Note	$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y} = 0$ is not sufficient for M1.
	A1ft	Either • Sets $2x - 3y$ to zero and obtains either $y = \frac{2}{3}x$ or $x = \frac{3}{2}y$ • the follow through result of making either y or x the subject from setting their numerator of their $\frac{dy}{dx}$ equal to zero
	dM1	**dependent on the first method mark being awarded.** Substitutes ***either*** their $y = \frac{2}{3}x$ ***or*** their $x = \frac{3}{2}y$ into the original equation to give an equation in one variable only.
	A1	Obtains either $x = \frac{24}{5}$ or $-\frac{24}{5}$ or $y = \frac{16}{5}$ or $-\frac{16}{5}$, (or equivalent) ***by correct solution only***. i.e. You can allow for example $x = \frac{48}{10}$ or 4.8, etc.
	Note	$x = \sqrt{\frac{576}{25}}$ (not simplified) or $y = \sqrt{\frac{256}{25}}$ (not simplified) is not sufficient for A1.

167. (b) ctd	**ddM1**	**dependent on both previous method marks being awarded in this part.** **<u>Method 1</u>** **Either:** • substitutes their x into their $y = \frac{2}{3}x$ **or** substitutes their y into their $x = \frac{3}{2}y$, **or** • substitutes ***the other of*** their $y = \frac{2}{3}x$ ***or*** their $x = \frac{3}{2}y$ into the original equation, **and achieves either:** • exactly two sets of two coordinates **or** • exactly two distinct values for x and exactly two distinct values for y. **<u>Method 2</u>** **Either:** • substitutes their first x-value, x_1 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain one y-value, y_1 and substitutes their second x-value, x_2 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain 1 y-value y_2 **or** • substitutes their first y-value, y_1 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain one x-value x_1 and substitutes their second y-value, y_2 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain one x-value x_2.
	Note	Three or more sets of coordinates given (without identification of two sets of coordinates) is ddM0.
	A1	Both $\left(\frac{24}{5}, \frac{16}{5}\right)$ **and** $\left(-\frac{24}{5}, -\frac{16}{5}\right)$, only **by cso.** Note that decimal equivalents are fine.
	Note	Also allow $x = \frac{24}{5}$, $y = \frac{16}{5}$ **and** $x = -\frac{24}{5}$, $y = -\frac{16}{5}$ all seen in their working to part (b).
	Note	Allow $x = \pm\frac{24}{5}$, $y = \pm\frac{16}{5}$ for 3rd A1.
	Note	$x = \pm\frac{24}{5}$, $y = \pm\frac{16}{5}$ followed by eg. $\left(\frac{16}{5}, \frac{24}{5}\right)$ **and** $\left(-\frac{16}{5}, -\frac{24}{5}\right)$ (eg. coordinates stated the wrong way round) is 3rd A0.
	Note	It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\frac{dy}{dx}$) to gain all 6 marks in part (b).
	Note	Decimal equivalents to fractions are fine in part (b). i.e. $(4.8, 3.2)$ **and** $(-4.8, -3.2)$.
	Note	$\left(\frac{24}{5}, \frac{16}{5}\right)$ **and** $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ from no working is M0A0M0A0M0A0.
	Note	Candidates could potentially lose the final 2 marks for setting both their numerator and denominator to zero.
	Note	No credit in this part can be gained by only setting the denominator to zero.

Question Number	Scheme		Marks
168.	**Note: You can mark parts (a) and (b) together.**		
(a)	$x = 4t + 3,\ \ y = 4t + 8 + \dfrac{5}{2t}$		
	$\dfrac{dx}{dt} = 4,\ \ \dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	Both $\dfrac{dx}{dt} = 4$ or $\dfrac{dt}{dx} = \dfrac{1}{4}$ and $\dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	B1
	So, $\dfrac{dy}{dx} = \dfrac{4 - \frac{5}{2}t^{-2}}{4}\ \left\{= 1 - \dfrac{5}{8}t^{-2} = 1 - \dfrac{5}{8t^2}\right\}$	Candidate's $\dfrac{dy}{dt}$ divided by a candidate's $\dfrac{dx}{dt}$	M1 **o.e.**
	$\{\text{When } t = 2,\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 **cao**	A1
			[3]
	Way 2: Cartesian Method		
	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$, simplified or un-simplifed.	B1
		$\dfrac{dy}{dx} = \pm\lambda \pm \dfrac{\mu}{(x-3)^2}$, $\lambda \neq 0, \mu \neq 0$	M1
	$\{\text{When } t = 2, x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 **cao**	A1
			[3]
	Way 3: Cartesian Method		
	$\dfrac{dy}{dx} = \dfrac{(2x+2)(x-3) - (x^2+2x-5)}{(x-3)^2}$	Correct expression for $\dfrac{dy}{dx}$, simplified or un-simplified.	B1
	$\left\{= \dfrac{x^2 - 6x - 1}{(x-3)^2}\right\}$	$\dfrac{dy}{dx} = \dfrac{f'(x)(x-3) - 1f(x)}{(x-3)^2}$, where $f(x) = \text{their } "x^2 + ax + b", g(x) = x - 3$	M1
	$\{\text{When } t = 2, x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 **cao**	A1
			[3]
(b)	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\}\ y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\dfrac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \dfrac{10}{x-3}$		
	$y = \dfrac{(x-3)(x-3) + 8(x-3) + 10}{x-3}$ ***or*** $y(x-3) = (x-3)(x-3) + 8(x-3) + 10$ ***or*** $y = \dfrac{(x+5)(x-3) + 10}{x-3}$ ***or*** $y = \dfrac{(x+5)(x-3)}{x-3} + \dfrac{10}{x-3}$	See notes	dM1
	$\Rightarrow y = \dfrac{x^2 + 2x - 5}{x-3}$, $\{a = 2 \text{ and } b = -5\}$	Correct algebra leading to $y = \dfrac{x^2 + 2x - 5}{x-3}$ or $a = 2$ and $b = -5$	A1 **cso**
			[3] **6**

EXPERT TUITION

Question Number	Scheme	Marks
168. (b)	**Alternative Method 1 of Equating Coefficients**	
	$y = \dfrac{x^2 + ax + b}{x - 3} \Rightarrow y(x-3) = x^2 + ax + b$	
	$y(x-3) = (4t+3)^2 + 2(4t+3) - 5 = 16t^2 + 32t + 10$	
	$x^2 + ax + b = (4t+3)^2 + a(4t+3) + b$	
	$(4t+3)^2 + a(4t+3) + b = 16t^2 + 32t + 10$ — Correct method of obtaining an equation in only t, a and b	M1
	t: $24 + 4a = 32 \Rightarrow a = 2$; constant: $9 + 3a + b = 10 \Rightarrow b = -5$ — Equates their coefficients in t and finds both $a = ...$ and $b = ...$	dM1
	$a = 2$ and $b = -5$	A1
		[3]
168. (b)	**Alternative Method 2 of Equating Coefficients**	
	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\}\ y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\dfrac{x-3}{4}\right)}$ — Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \dfrac{10}{x-3} \Rightarrow y = x + 5 + \dfrac{10}{(x-3)}$	
	$\underline{\underline{y(x-3)}} = (x+5)(x-3) + 10 \Rightarrow x^2 + ax + b = \underline{\underline{(x+5)(x-3) + 10}}$	dM1
	$\Rightarrow y = \dfrac{x^2 + 2x - 5}{x - 3}$ **or** equating coefficients to give $a = 2$ and $b = -5$ — Correct algebra leading to $y = \dfrac{x^2 + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$	A1 **cso**
		[3]

	Question 168 Notes	
168. (a)	**B1**	$\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$ or $\frac{dy}{dt} = \frac{8t^2 - 5}{2t^2}$ or $\frac{dy}{dt} = 4 - 5(2t)^{-2}(2)$, etc.
	Note	$\frac{dy}{dt}$ can be simplified or un-simplified.
	Note	**You can imply the B1 mark by later working.**
	M1	Candidate's $\frac{dy}{dt}$ divided by a candidate's $\frac{dx}{dt}$ **or** $\frac{dy}{dt}$ multiplied by a candidate's $\frac{dt}{dx}$
	Note	M1 can be also be obtained by substituting $t = 2$ into both their $\frac{dy}{dt}$ and their $\frac{dx}{dt}$ and then dividing their values the correct way round.
	A1	$\frac{27}{32}$ or 0.84375 **cao**
(b)	**M1**	Eliminates t to achieve an equation in only x and y.
	dM1	**dependent on the first method mark being awarded.** **Either: (ignoring sign slips or constant slips, noting that *k* can be 1)** • Combining all three parts of their $\underline{x - 3} + \overset{=}{8} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ to form a single fraction with a common denominator of $\pm k(x-3)$. Accept three separate fractions with the same denominator. • Combining both parts of their $\underline{x + 5} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$, (where $\underline{x+5}$ is their $4\left(\frac{x-3}{4}\right) + 8$), to form a single fraction with a common denominator of $\pm k(x-3)$. Accept two separate fractions with the same denominator. • Multiplies both sides of their $y = \underline{x - 3} + \overset{=}{8} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ or their $y = \underline{x + 5} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ by $\pm k(x-3)$. Note that all terms in their equation must be multiplied by $\pm k(x-3)$.
	Note	Condone "invisible" brackets for dM1.
	A1	Correct algebra with no incorrect working leading to $y = \frac{x^2 + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$
	Note	**Some examples for the award of dM1 in (b):** **dM0** for $y = x - 3 + 8 + \frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3) + 8 + 10}{x-3}$. Should be $... + 8(x-3) + ...$ **dM0** for $y = x - 3 + \frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3) + 10}{x-3}$. The "8" part has been omitted. **dM0** for $y = x + 5 + \frac{10}{x-3} \rightarrow y = \frac{x(x-3) + 5 + 10}{x-3}$. Should be $... + 5(x-3) + ...$ **dM0** for $y = x + 5 + \frac{10}{x-3} \rightarrow y(x-3) = x(x-3) + 5(x-3) + 10(x-3)$. Should be just 10.
	Note	$y = x + 5 + \frac{10}{x-3} \rightarrow y = \frac{x^2 + 2x - 5}{x-3}$ with no intermediate working is dM1A1.

Question Number	Scheme	Marks
169.	$x^3 + 2xy - x - y^3 - 20 = 0$	
(a)	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \cancel{\times}\right\}$ $\underline{3x^2} + \left(\underline{\underline{2y + 2x\frac{dy}{dx}}}\right) \underline{-1 - 3y^2\frac{dy}{dx} = 0}$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	$3x^2 + 2y - 1 + \left(2x - 3y^2\right)\frac{dy}{dx} = 0$	dM1
	$\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$	A1 **cso**
		[5]
(b)	At $P(3, -2)$, $m(\mathbf{T}) = \frac{dy}{dx} = \frac{3(3)^2 + 2(-2) - 1}{3(-2)^2 - 2(3)}; = \frac{22}{6}$ or $\frac{11}{3}$	
	and either T: $y - -2 = \text{"}\frac{11}{3}\text{"}(x - 3)$ **see notes**	M1
	or $(-2) = \left(\frac{11}{3}\right)(3) + c \Rightarrow c = \dots,$	
	T: $11x - 3y - 39 = 0$ or $K(11x - 3y - 39) = 0$	A1 **cso**
		[2] **7**
(a)	**<u>Alternative method for part (a)</u>**	
	$\left\{\frac{\cancel{dx}}{\cancel{dy}} \cancel{\times}\right\}$ $\underline{3x^2\frac{dx}{dy}} + \left(\underline{\underline{2y\frac{dx}{dy} + 2x}}\right) \underline{- \frac{dx}{dy} - 3y^2 = 0}$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	$2x - 3y^2 + \left(3x^2 + 2y - 1\right)\frac{dx}{dy} = 0$	dM1
	$\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$	A1 **cso**
		[5]

		Question 169 Notes
(a) **General**	**Note**	Writing down $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$ from **no working** is full marks.
	Note	Writing down $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{2x - 3y^2}$ or $\frac{1 - 3x^2 - 2y}{3y^2 - 2x}$ from **no working is** M1A0B0M1A0
	Note	Few candidates will write $3x^2 + 2y + 2x\,dy - 1 - 3y^2 dy = 0$ leading to $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$, o.e. This should get full marks.
1. (a)	**M1**	Differentiates implicitly to include either $2x\frac{dy}{dx}$ or $-y^3 \to \pm ky^2\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).
	A1	$x^3 \to 3x^2$ **and** $-x - y^3 - 20 = 0 \to -1 - 3y^2\frac{dy}{dx} = 0$
	B1	$2xy \to 2y + 2x\frac{dy}{dx}$
	Note	If an extra term appears then award 1st A0.

169. (a) ctd	**Note**	$3x^2 + 2y + 2x\frac{dy}{dx} - 1 - 3y^2\frac{dy}{dx} \rightarrow 3x^2 + 2y - 1 = 3y^2\frac{dy}{dx} - 2x\frac{dy}{dx}$ will get 1st A1 (implied) as the "$= 0$" can be implied by rearrangement of their equation.
	dM1	**dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{dy}{dx}$ as long as there are ***at least two terms*** in $\frac{dy}{dx}$. ie. $... + (2x - 3y^2)\frac{dy}{dx} = ...$
	Note	Placing an extra $\frac{dy}{dx}$ at the beginning and then including it in their factorisation is fine for dM1.
	A1	For $\frac{1 - 2y - 3x^2}{2x - 3y^2}$ or equivalent. Eg: $\frac{3x^2 + 2y - 1}{3y^2 - 2x}$ **cso:** If the candidate's solution is not completely correct, then do not give this mark. **isw:** You can, however, ignore subsequent working following on from correct solution.
169. (b)	**M1**	**Some** attempt to substitute **both** $x = 3$ **and** $y = -2$ into their $\frac{dy}{dx}$ which contains both x and y to find m_T **and** • **either** applies $y - -2 = (\text{their } m_T)(x - 3)$, where m_T is a numerical value. • **or** finds c by solving $(-2) = (\text{their } m_T)(3) + c$, where m_T is a numerical value.
	Note	Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{dy}{dx}}$ or $\frac{1}{\text{their } \frac{dy}{dx}}$ is M0).
	A1	Accept any integer multiple of $11x - 3y - 39 = 0$ or $11x - 39 - 3y = 0$ or $-11x + 3y + 39 = 0$, where their tangent equation is equal to 0.
	cso	A correct solution is required from a correct $\frac{dy}{dx}$.
	isw	You can ignore subsequent working following a correct solution.
	<u>Alternative method for part (a): Differentiating with respect to *y*</u>	
169. (a)	**M1**	Differentiates implicitly to include either $2y\frac{dx}{dy}$ or $x^3 \rightarrow \pm kx^2\frac{dx}{dy}$ or $-x \rightarrow -\frac{dx}{dy}$ (Ignore $\left(\frac{dx}{dy} = \right)$).
	A1	$x^3 \rightarrow 3x^2\frac{dx}{dy}$ **and** $-x - y^3 - 20 = 0 \rightarrow -\frac{dx}{dy} - 3y^2 = 0$
	B1	$2xy \rightarrow 2y\frac{dx}{dy} + 2x$
	dM1	**dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{dx}{dy}$ as long as there are ***at least two terms*** in $\frac{dx}{dy}$.
	A1	For $\frac{1 - 2y - 3x^2}{2x - 3y^2}$ or equivalent. Eg: $\frac{3x^2 + 2y - 1}{3y^2 - 2x}$ **cso:** If the candidate's solution is not completely correct, then do not give this mark.

Question Number	Scheme	Marks
170.	$\frac{dV}{dt} = 80\pi$, $V = 4\pi h(h+4) = 4\pi h^2 + 16\pi h$,	
	$\frac{dV}{dh} = 8\pi h + 16\pi$ $\pm\alpha h \pm \beta,\ \alpha \neq 0,\ \beta \neq 0$ $8\pi h + 16\pi$	M1 A1
	$\left\{\frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\}$ $(8\pi h + 16\pi)\frac{dh}{dt} = 80\pi$ $\left(\text{Candidate's } \frac{dV}{dh}\right) \times \frac{dh}{dt} = 80\pi$ $\left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow\right\}$ $\frac{dh}{dt} = 80\pi \times \frac{1}{8\pi h + 16\pi}$ or $80\pi \div \text{Candidate's } \frac{dV}{dh}$	M1 oe
	When $h = 6$, $\left\{\frac{dh}{dt} =\right\} \frac{1}{8\pi(6) + 16\pi} \times 80\pi$ $\left\{= \frac{80\pi}{64\pi}\right\}$ **dependent on the previous M1 see notes**	dM1
	$\frac{dh}{dt} = \underline{1.25}$ (cm s^{-1}) 1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$	A1 oe
		[5] **5**
	<u>Alternative Method for the first M1A1</u> Product rule: $\left\{\begin{array}{ll} u = 4\pi h & v = h + 4 \\ \frac{du}{dh} = 4\pi & \frac{dv}{dh} = 1 \end{array}\right\}$	
	$\frac{dV}{dh} = 4\pi(h+4) + 4\pi h$ $\pm\alpha h \pm \beta,\ \alpha \neq 0,\ \beta \neq 0$ $4\pi(h+4) + 4\pi h$	M1 A1

	Question 170 Notes	
	M1	An expression of the form $\pm\alpha h \pm \beta,\ \alpha \neq 0,\ \beta \neq 0$. **Can be simplified or un-simplified.**
	A1	Correct simplified or un-simplified differentiation of V. eg. $8\pi h + 16\pi$ or $4\pi(h+4) + 4\pi h$ or $8\pi(h+2)$ or equivalent.
	Note	Some candidates will use the product rule to differentiate V with respect to h. **(See Alt Method 1).**
	Note	$\frac{dV}{dh}$ does not have to be explicitly stated, but it should be clear that they are differentiating their V.
	M1	$\left(\text{Candidate's } \frac{dV}{dh}\right) \times \frac{dh}{dt} = 80\pi$ or $80\pi \div \text{Candidate's } \frac{dV}{dh}$
	Note	Also allow 2nd M1 for $\left(\text{Candidate's } \frac{dV}{dh}\right) \times \frac{dh}{dt} = \mathbf{80}$ or $\mathbf{80} \div \text{Candidate's } \frac{dV}{dh}$
	Note	Give 2nd M0 for $\left(\text{Candidate's } \frac{dV}{dh}\right) \times \frac{dh}{dt} = \mathbf{80\pi t}$ or $\mathbf{80k}$ or $\mathbf{80\pi t}$ or $\mathbf{80k} \div \text{Candidate's } \frac{dV}{dh}$
	dM1	**which is dependent on the previous M1 mark.** Substitutes $h = 6$ into an expression which is a result of a quotient of their $\frac{dV}{dh}$ and 80π (or 80)
	A1	1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$ (units are not required).
	Note	$\frac{80\pi}{64\pi}$ as a final answer is A0.
	Note	Substituting $h = 6$ into a correct $\frac{dV}{dh}$ gives 64π but the final M1 mark can only be awarded if this is used as a quotient with 80π (or 80)

Question Number	Scheme		Marks
171.	$x = 3\tan\theta$, $y = 4\cos^2\theta$ or $y = 2 + 2\cos 2\theta$, $0 \leqslant \theta < \frac{\pi}{2}$. $\frac{dx}{d\theta} = 3\sec^2\theta$, $\frac{dy}{d\theta} = -8\cos\theta\sin\theta$ **or** $\frac{dy}{d\theta} = -4\sin 2\theta$		
	$\frac{dy}{dx} = \frac{-8\cos\theta\sin\theta}{3\sec^2\theta}$ $\left\{= -\frac{8}{3}\cos^3\theta\sin\theta = -\frac{4}{3}\sin 2\theta\cos^2\theta\right\}$	their $\frac{dy}{d\theta}$ divided by their $\frac{dx}{d\theta}$ Correct $\frac{dy}{dx}$	M1 A1 oe
	At $P(3, 2)$, $\theta = \frac{\pi}{4}$, $\frac{dy}{dx} = -\frac{8}{3}\cos^3\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)$ $\left\{= -\frac{2}{3}\right\}$ So, $m(\mathbf{N}) = \frac{3}{2}$	Some evidence of substituting $\theta = \frac{\pi}{4}$ into their $\frac{dy}{dx}$ applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1 M1
	Either **N:** $y - 2 = \frac{"3"}{2}(x - 3)$ **or** $2 = \left(\frac{"3"}{2}\right)(3) + c$	**see notes**	M1
	$\{$At Q, $y = 0$, so, $-2 = \frac{3}{2}(x - 3)\}$ **giving** $\underline{x = \frac{5}{3}}$	$x = \frac{5}{3}$ or $1\frac{2}{3}$ or awrt 1.67	A1 **cso**
			[6]

	Question 171 Notes	
171. (a)	**1st M1**	Applies their $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ **or** applies $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ multiplied by their $\frac{\mathrm{d}\theta}{\mathrm{d}x}$
	SC	Award **Special Case 1st M1** if **both** $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ **and** $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ are both correct.
	1st A1	Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ i.e. $\frac{-8\cos\theta\sin\theta}{3\sec^2\theta}$ or $-\frac{8}{3}\cos^3\theta\sin\theta$ or $-\frac{4}{3}\sin 2\theta\cos^2\theta$ or any equivalent form.
	2nd M1	***Some evidence*** of substituting $\theta = \frac{\pi}{4}$ or $\theta = 45^\circ$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$
	Note	For 3rd M1 and 4th M1, $m(\mathbf{T})$ must be found by using $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	3rd M1	applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$. Numerical value for $m(\mathbf{N})$ is required here.
	4th M1	• Applies $y - 2 = (\text{their } m_N)(x-3)$, where $\mathrm{m}(\mathbf{N})$ is a numerical value, • or ***finds c*** by solving $2 = (\text{their } m_N)3 + c$, where $\mathrm{m}(\mathbf{N})$ is a numerical value, and $m_N = -\frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$ or $m_N = \frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$ or $m_N = -\text{their } \mathrm{m}(\mathbf{T})$.
	Note	This mark can be implied by subsequent working.
	2nd A1	$x = \frac{5}{3}$ or $1\frac{2}{3}$ or awrt 1.67 **from a correct solution only.**
(b)	**1st M1**	Applying $\int y^2 \mathrm{d}x$ as $y^2 \frac{\mathrm{d}x}{\mathrm{d}\theta}$ with their $\frac{\mathrm{d}x}{\mathrm{d}\theta}$. Ignore π or $\frac{1}{3}\pi$ outside integral.
	Note	You can ignore the omission of an integral sign and/or $\mathrm{d}\theta$ for the 1st M1.
	Note	Allow 1st M1 for $\int(\cos^2\theta)^2 \times \text{"their } 3\sec^2\theta\text{" } \mathrm{d}\theta$ or $\int 4(\cos^2\theta)^2 \times \text{"their } 3\sec^2\theta\text{ " } \mathrm{d}\theta$
	1st A1	Correct expression $\left\{\pi\int y^2 \mathrm{d}x\right\} = \pi\int(4\cos^2\theta)^2\, 3\sec^2\theta\,\{\mathrm{d}\theta\}$ (Allow the omission of $\mathrm{d}\theta$)
	Note	**IMPORTANT:** The π can be recovered later, **but as a correct statement only**.
	2nd A1	$\left\{\int y^2 \mathrm{d}x\right\} = \int 48\cos^2\theta\{\mathrm{d}\theta\}$. (Ignore $\mathrm{d}\theta$). **Note**: 48 can be written as 24(2) for example.
	2nd M1	Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral. (Seen or **implied**.)
	3rd dM1*	**which is dependent on the 1st M1 mark.** Integrating $\cos^2\theta$ to give $\pm\alpha\theta \pm \beta\sin 2\theta$, $\alpha \neq 0, \beta \neq 0$, un-simplified or simplified.
	3rd A1	**which is dependent on the 3rd M1 mark and the 1st M1 mark.** Integrating $\cos^2\theta$ to give $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$, un-simplified or simplified. This can be implied by $k\cos^2\theta$ giving $\frac{k}{2}\theta + \frac{k}{4}\sin 2\theta$, un-simplified or simplified.
	4th dM1	**which is dependent on the 3rd M1 mark and the 1st M1 mark.** Some evidence of applying limits of $\frac{\pi}{4}$ and 0 (0 can be implied) to an integrated function in θ
	5th M1	Applies $V_{\text{cone}} = \frac{1}{3}\pi(2)^2(3 - \text{their part } (a) \text{ answer})$.
	Note	Also allow the 5th M1 for $V_{\text{cone}} = \pi\int_{\text{their } \frac{5}{3}}^{3}\left(\frac{3}{2}x - \frac{5}{2}\right)^2\{dx\}$, which includes the correct limits.
	4th A1	$\frac{92}{9}\pi + 6\pi^2$ or $10\frac{2}{9}\pi + 6\pi^2$
	Note	A decimal answer of 91.33168464... (without a correct **exact** answer) is A0.
	Note	The π in the volume formula is only needed for the 1st A1 mark and the final accuracy mark.

171.		<u>**Working with a Cartesian Equation**</u> A cartesian equation for C is $y = \dfrac{36}{x^2+9}$
(a)	**1st M1** **1st A1** **2nd dM1**	$\dfrac{dy}{dx} = \pm\lambda x\left(\pm\alpha x^2 \pm \beta\right)^{-2}$ **or** $\dfrac{dy}{dx} = \dfrac{\pm\lambda x}{\left(\pm\alpha x^2\pm\beta\right)^2}$ $\dfrac{dy}{dx} = -36(x^2+9)^{-2}(2x)$ **or** $\dfrac{dy}{dx} = \dfrac{-72x}{(x^2+9)^2}$ un-simplified or simplified. **Dependent on the 1st M1 mark if a candidate uses this method** For substituting $x = 3$ into their $\dfrac{dy}{dx}$ i.e. at $P(3, 2)$, $\dfrac{dy}{dx} = \dfrac{-72(3)}{(3^2+9)^2}\left\{= -\dfrac{2}{3}\right\}$ From this point onwards the original scheme can be applied.
(b)	**1st M1** **A1**	For $\displaystyle\int\left(\frac{\pm\lambda}{\pm\alpha x^2\pm\beta}\right)^2\{dx\}$ (π not required for this mark) For $\displaystyle\pi\int\left(\frac{36}{x^2+9}\right)^2\{dx\}$ (π required for this mark) To integrate, a substitution of $x = 3\tan\theta$ is required which will lead to $\displaystyle\int 48\cos^2\theta\,d\theta$ and so from this point onwards the original scheme can be applied.
(a)	**1st M1** **1st A1** **2nd dM1**	Another cartesian equation for C is $x^2 = \dfrac{36}{y} - 9$ $\pm\alpha x = \pm\dfrac{\beta}{y^2}\dfrac{dy}{dx}$ or $\pm\alpha x\dfrac{dx}{dy} = \pm\dfrac{\beta}{y^2}$ $2x = -\dfrac{36}{y^2}\dfrac{dy}{dx}$ or $2x\dfrac{dx}{dy} = -\dfrac{36}{y^2}$ **Dependent on the 1st M1 mark if a candidate uses this method** For substituting $x = 3$ to find $\dfrac{dy}{dx}$ i.e. at $P(3, 2)$, $2(3) = -\dfrac{36}{4}\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \ldots$ From this point onwards the original scheme can be applied.

Question Number	Scheme		Marks
172.	$x^2 + y^2 + 10x + 2y - 4xy = 10$		
(a)	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \cancel{=}\right\}\quad \underline{2x + 2y\frac{dy}{dx} + 10 + 2\frac{dy}{dx}} - \underline{\underline{\left(4y + 4x\frac{dy}{dx}\right)}} = \underline{0}$	**See notes**	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$
	$2x + 10 - 4y + (2y + 2 - 4x)\frac{dy}{dx} = 0$	**Dependent on the first M1 mark.**	dM1
	$\frac{dy}{dx} = \frac{2x + 10 - 4y}{4x - 2y - 2}$		
	Simplifying gives $\frac{dy}{dx} = \frac{x + 5 - 2y}{2x - y - 1}$ $\left\{= \frac{-x - 5 + 2y}{-2x + y + 1}\right\}$		A1 **cso oe**
			[5]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}\quad x + 5 - 2y = 0$		M1
	So $x = 2y - 5$,		
	$(2y - 5)^2 + y^2 + 10(2y - 5) + 2y - 4(2y - 5)y = 10$		M1
	$4y^2 - 20y + 25 + y^2 + 20y - 50 + 2y - 8y^2 + 20y = 10$		
	gives $-3y^2 + 22y - 35 = 0$ or $3y^2 - 22y + 35 = 0$	$3y^2 - 22y + 35\ \{= 0\}$ **see notes**	A1 oe
	$(3y - 7)(y - 5) = 0$ and $y = ...$	Method mark for solving a quadratic equation.	ddM1
	$y = \frac{7}{3}, 5$	$\{y =\}\ \frac{7}{3}, 5$	A1 **cao**
			[5]
	<u>Alternative method for part (b)</u>		
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}\quad x + 5 - 2y = 0$		M1
	So $y = \frac{x + 5}{2}$,		
	$x^2 + \left(\frac{x + 5}{2}\right)^2 + 10x + 2\left(\frac{x + 5}{2}\right) - 4x\left(\frac{x + 5}{2}\right) = 10$		M1
	$x^2 + \frac{x^2 + 10x + 25}{4} + 10x + x + 5 - 2x^2 - 10x = 10$		
	$4x^2 + x^2 + 10x + 25 + 40x + 4x + 20 - 8x^2 - 40x = 40$		
	gives $-3x^2 + 14x + 5 = 0$ or $3x^2 - 14x - 5 = 0$	$3x^2 - 14x - 5\ \{= 0\}$ **see notes**	A1 oe
	$(3x + 1)(x - 5) = 0,\ x = ...$		
	$y = \frac{-\frac{1}{3} + 5}{2}, \frac{5 + 5}{2}$	Solves a quadratic **and finds at least one value for *y*.**	ddM1
	$y = \frac{7}{3}, 5$	$\{y =\}\ \frac{7}{3}, 5$	A1 **cao**
			[5]
			10

		Question 172 Notes
172. (a)	**M1**	Differentiates implicitly to include either $\pm 4x\frac{dy}{dx}$ or $y^2 \to 2y\frac{dy}{dx}$ or $2y \to 2\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).
	A1	$x^2 + y^2 + 10x + 2y \to 2x + 2y\frac{dy}{dx} + 10 + 2\frac{dy}{dx}$ **and** $10 \to 0$
	M1	$-4xy \to \pm 4y \pm 4x\frac{dy}{dx}$
	Note	If an extra term appears then award 1st A0.
	Note	$2x + 2y\frac{dy}{dx} + 10 + 2\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} \to 2x + 10 - 4y = -2y\frac{dy}{dx} - 2\frac{dy}{dx} + 4x\frac{dy}{dx}$ will get 1st A1 (implied) as the $"=0"$ can be implied by rearrangement of their equation.
	dM1	**dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{dy}{dx}$ as long as there are ***at least two terms*** in $\frac{dy}{dx}$.
	A1	$\frac{x+5-2y}{2x-y-1}$ or $\frac{-x-5+2y}{-2x+y+1}$ (must be simplified).
	cso:	If the candidate's solution is not completely correct, then do not give this mark.
(b)	**M1**	Sets the numerator of their $\frac{dy}{dx}$ equal to zero (or the denominator of their $\frac{dx}{dy}$ equal to zero) oe.
	NOTE	**If the numerator involves one variable only then *only* the 1st M1 mark is possible in part (b).**
	M1	Substitutes their x or their y into the printed equation to give an equation in one variable only.
	A1	For obtaining either $-3y^2 + 22y - 35\{=0\}$ or $3y^2 - 22y + 35\{=0\}$
	Note	This mark can also awarded for a correct three term equation, eg. either $-3y^2 + 22y = 35$ $3y^2 - 22y = -35$ or $3y^2 + 35 = 22y$ are all fine for A1.
	ddM1	**Dependent on the previous 2 M marks.** **See notes at the beginning of the mark scheme: Method mark for solving a 3 term quadratic** • $(3y-7)(y-5) = 0 \Rightarrow y = ...$ • $y = \frac{22 \pm \sqrt{(-22)^2 - 4(3)(35)}}{2(3)}$ • $y^2 - \frac{22}{3}y - \frac{35}{3} = 0 \Rightarrow \left(y - \frac{11}{3}\right)^2 - \frac{121}{9} + \frac{35}{3} = 0 \Rightarrow y = ...$ • Or writes down at least one correct y- root from their quadratic equation. This is usually found from their calculator.
	Note	If a candidate applies ***the alternative method*** then they also need to use their $y = \frac{x+5}{2}$ in order to find **at least one value** for y in order to gain the final M1.
	A1	$y = \frac{7}{3}, 5$. **cao.** (2.33 or 2.3 without reference to $\frac{7}{3}$ or $2\frac{1}{3}$ is not allowed for this mark.)
	Note	It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\frac{dy}{dx}$) to gain all 5 marks in part (b).

Question Number	Scheme	Marks
173. (a)	From question, $V=\frac{4}{3}\pi r^3,\ S=4\pi r^2,\ \frac{\mathrm{d}V}{\mathrm{d}t}=3$	
	$\left\{V=\frac{4}{3}\pi r^3 \Rightarrow\right\}\ \frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$ $\frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$ (Can be implied)	B1 oe
	$\left\{\frac{\mathrm{d}V}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\Rightarrow\right\}\ \left(4\pi r^2\right)\frac{\mathrm{d}r}{\mathrm{d}t}=3$ $\left(\text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}r}\right)\times\frac{\mathrm{d}r}{\mathrm{d}t}=3$ $\left\{\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\div\frac{\mathrm{d}V}{\mathrm{d}r}\Rightarrow\right\}\ \frac{\mathrm{d}r}{\mathrm{d}t}=(3)\frac{1}{4\pi r^2};\ \left\{=\frac{3}{4\pi r^2}\right\}$ or $3\div\text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}r}$;	M1 oe
	When $r=4\text{cm}$, $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{3}{4\pi(4)^2}\ \left\{=\frac{3}{64\pi}\right\}$ **dependent on previous M1. see notes**	dM1
	Hence, $\frac{\mathrm{d}r}{\mathrm{d}t}=0.01492077591...(\text{cm}^2\ \text{s}^{-1})$ anything that rounds to 0.0149	A1
		[4]
(b)	$\left\{\frac{\mathrm{d}S}{\mathrm{d}t}=\frac{\mathrm{d}S}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}=\right\}\Rightarrow\frac{\mathrm{d}S}{\mathrm{d}t}=\ 8\pi r\times\frac{3}{4\pi r^2}\ \left\{\text{or } \frac{6}{r} \text{ or } 8\pi r\times 0.0149...\right\}$ $8\pi r\times\text{Candidate's } \frac{\mathrm{d}r}{\mathrm{d}t}$	M1; oe
	When $r=4\text{cm}$, $\frac{\mathrm{d}r}{\mathrm{d}t}=8\pi(4)\times\frac{3}{4\pi(4)^2}$ or $\frac{6}{4}$ or $8\pi(4)\times 0.0149...$	
	Hence, $\frac{\mathrm{d}S}{\mathrm{d}t}=\ 1.5\ (\text{cm}^2\ \text{s}^{-1})$ anything that rounds to 1.5	A1 **cso**
		[2] **6**

Question 173 Notes

(a)	**B1**	$\frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$ Can be implied by later working.
	M1	$\left(\text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}r}\right)\times\frac{\mathrm{d}r}{\mathrm{d}t}=3$ or $3\div\text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}r}$
	dM1	**(dependent on the previous method mark)** Substitutes $r=4$ into an expression which is a result of a quotient of "3" and their $\frac{\mathrm{d}V}{\mathrm{d}r}$.
	A1	anything that rounds to 0.0149 (units are not required)
(b)	**M1**	$8\pi r\times\text{Candidate's } \frac{\mathrm{d}r}{\mathrm{d}t}$
	A1	anything that rounds to 1.5 (units are not required). **Correct solution only.**
	Note	Using $\frac{\mathrm{d}r}{\mathrm{d}t}=0.0149$ gives $\frac{\mathrm{d}S}{\mathrm{d}t}=\ 1.4979...$ which is fine for A1.

Question Number	Scheme		Marks
174.	$x = t - 4\sin t, \quad y = 1 - 2\cos t, \quad -\dfrac{2\pi}{3} \leqslant t \leqslant \dfrac{2\pi}{3} \quad A(k, 1)$ lies on the curve, $k > 0$		
(a)	$\{\text{When } y = 1,\}\ 1 = 1 - 2\cos t \Rightarrow t = -\dfrac{\pi}{2}, \dfrac{\pi}{2}$ k (or x) $= \dfrac{\pi}{2} - 4\sin\left(\dfrac{\pi}{2}\right)$ **or** $x = -\dfrac{\pi}{2} - 4\sin\left(-\dfrac{\pi}{2}\right)$	Sets $y = 1$ to find t and uses their t to find x.	M1
	$\left\{\text{When } t = -\dfrac{\pi}{2}, k > 0,\right\}$ so $k = 4 - \dfrac{\pi}{2}$ or $\dfrac{8-\pi}{2}$	x or $k = 4 - \dfrac{\pi}{2}$	A1
			[2]
(b)	$\dfrac{dx}{dt} = 1 - 4\cos t, \quad \dfrac{dy}{dt} = 2\sin t$	At least one of $\dfrac{dx}{dt}$ or $\dfrac{dy}{dt}$ correct.	B1
		Both $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$ are correct.	B1
	So, $\dfrac{dy}{dx} = \dfrac{2\sin t}{1 - 4\cos t}$	Applies their $\dfrac{dy}{dt}$ divided by their $\dfrac{dx}{dt}$ and substitutes their t into their $\dfrac{dy}{dx}$.	M1;
	At $t = -\dfrac{\pi}{2}$, $\dfrac{dy}{dx} = \dfrac{2\sin\left(-\dfrac{\pi}{2}\right)}{1 - 4\cos\left(-\dfrac{\pi}{2}\right)}$; $= -2$	Correct value for $\dfrac{dy}{dx}$ of -2	A1 **cao cso**
			[4]
(c)	$\dfrac{2\sin t}{1 - 4\cos t} = -\dfrac{1}{2}$	Sets their $\dfrac{dy}{dx} = -\dfrac{1}{2}$	M1
	gives $4\sin t - 4\cos t = -1$	**See notes**	A1
	So $4\sqrt{2}\sin\left(t - \dfrac{\pi}{4}\right)$; $= -1$ or $-4\sqrt{2}\cos\left(t + \dfrac{\pi}{4}\right)$; $= -1$	**See notes**	M1; A1
	$t = \sin^{-1}\left(\dfrac{-1}{4\sqrt{2}}\right) + \dfrac{\pi}{4}$ or $t = \cos^{-1}\left(\dfrac{1}{4\sqrt{2}}\right) - \dfrac{\pi}{4}$	**See notes**	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6] **12**

	Question 174 Notes	
(c)	**NOTE**	**<u>VERY IMPORTANT NOTE FOR PART (c)</u>** Candidates who state $t = 0.6077$ with no intermediate working from $4\sin t - 4\cos t = -1$ will get 2nd M0, 2nd A0, 3rd M0, 3rd A0. They will not express $4\sin t - 4\cos t$ as either $4\sqrt{2}\sin\left(t - \dfrac{\pi}{4}\right)$ or $-4\sqrt{2}\cos\left(t + \dfrac{\pi}{4}\right)$. OR use any acceptable alternative method to achieve $t = 0.6077$
	NOTE	Alternative methods for part (c) are given on the next page.

	Question 174: Alternative Methods for Part (c)		
174. (c)	**Alternative Method 1:**		
	$\frac{2\sin t}{1-4\cos t} = -\frac{1}{2}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	M1
	eg. $\left(\frac{2\sin t}{1-4\cos t}\right)^2 = \frac{1}{4}$ or $(4\sin t)^2 = (4\cos t - 1)^2$ or $(4\sin t + 1)^2 = (4\cos t)^2$ etc.	Squaring to give a correct equation. This mark can be implied by a "squared" correct equation.	A1
		Note: You can also give 1st A1 in this method for $4\sin t - 4\cos t = -1$ as in the main scheme.	
	Squares their equation, applies $\sin^2 t + \cos^2 t = 1$ and achieves a **three term quadratic equation** of the form $\pm a\cos^2 t \pm b\cos t \pm c = 0$ or $\pm a\sin^2 t \pm b\sin t \pm c = 0$ or eg. $\pm a\cos^2 t \pm b\cos t = \pm c$ where $a \neq 0, b \neq 0$ and $c \neq 0$.		M1
	• **Either** $32\cos^2 t - 8\cos t - 15 = 0$ • **or** $32\sin^2 t + 8\sin t - 15 = 0$	For a correct **three term quadratic equation.**	A1
	• **Either** $\cos t = \frac{8 \pm \sqrt{1984}}{64} = \frac{1+\sqrt{31}}{8} \Rightarrow t = \cos^{-1}(...)$ • **or** $\sin t = \frac{-8 \pm \sqrt{1984}}{64} = \frac{-1 \pm \sqrt{31}}{8} \Rightarrow t = \sin^{-1}(...)$	**which is dependent on the 2nd M1 mark.** Uses correct algebraic processes to give $t = ...$	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6]
174. (c)	**Alternative Method 2:**		
	$\frac{2\sin t}{1-4\cos t} = -\frac{1}{2}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	M1
	eg. $(4\sin t - 4\cos t)^2 = (-1)^2$	Squaring to give a correct equation. This mark can be implied by a correct equation. **Note:** You can also give 1st A1 in this method for $4\sin t - 4\cos t = -1$ as in the main scheme.	A1
	So $16\sin^2 t - 32\sin t\cos t + 16\cos^2 t = 1$		
	leading to $16 - 16\sin 2t = 1$	Squares their equation, applies both $\sin^2 t + \cos^2 t = 1$ and $\sin 2t = 2\sin t\cos t$ and then achieves an equation of the form $\pm a \pm b\sin 2t = \pm c$	M1
		$16 - 16\sin 2t = 1$ or equivalent.	A1
	$\left\{\sin 2t = \frac{15}{16} \Rightarrow\right\} t = \frac{\sin^{-1}(....)}{2}$	**which is dependent on the 2nd M1 mark.** Uses correct algebraic processes to give $t = ...$	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6]

		Question 174 Notes
174. (a)	**M1**	Sets $y=1$ to find t and uses their t to find x.
	Note	M1 can be implied by either x or $k = 4-\frac{\pi}{2}$ or 2.429... or $\frac{\pi}{2}-4$ or $-2.429...$
	A1	x or $k = 4-\frac{\pi}{2}$ or $\frac{8-\pi}{2}$
	Note	A decimal answer of 2.429... (without a correct **exact** answer) is A0.
	Note	Allow A1 for a candidate using $t=\frac{\pi}{2}$ to find $x=\frac{\pi}{2}-4$ **and** then stating that k must be $4-\frac{\pi}{2}$ o.e.
(b)	**B1**	At least one of $\frac{\mathrm{d}x}{\mathrm{d}t}$ or $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct. **Note:** that this mark can be implied from their working.
	B1	Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ are correct. **Note:** that this mark can be implied from their working.
	M1	Applies their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$ **and** attempts to substitute their t into their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	Note	This mark may be implied by their final answer. i.e. $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\sin t}{1-4\cos t}$ followed by an answer of -2 (from $t=-\frac{\pi}{2}$) or 2 (from $t=\frac{\pi}{2}$)
	Note	**Applying** $\frac{\mathrm{d}x}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}y}{\mathrm{d}t}$ is M0, even if they state $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\div\frac{\mathrm{d}x}{\mathrm{d}t}$.
	A1	Using $t=-\frac{\pi}{2}$ $\left(\text{and not } t=\frac{3\pi}{2}\right)$ to find a correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ of -2 ***by correct solution only.***
(c)		
	NOTE	If a candidate uses an incorrect $\frac{\mathrm{d}y}{\mathrm{d}x}$ expression in part (c) then the accuracy marks are not obtainable.
	1st M1	Sets their $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}$
	1st A1	Rearranges to give the correct equation with $\sin t$ and $\cos t$ **on the same side.** eg. $4\sin t-4\cos t=-1$ or $4\cos t-4\sin t=1$ or $\sin t-\cos t=-\frac{1}{4}$ or $\cos t-\sin t=\frac{1}{4}$ or $4\sin t-4\cos t+1=0$ or $4\cos t-4\sin t-1=0$ or $\sin t-\cos t+\frac{1}{4}=0$ etc. are fine for A1.
	2nd M1	Rewrites $\pm\lambda\sin t\pm\mu\cos t$ in the form of either $R\cos(t\pm\alpha)$ or $R\sin(t\pm\alpha)$ where $R\neq 1$ or 0 and $\alpha\neq 0$
	2nd A1	Correct equation. Eg. $4\sqrt{2}\sin\left(t-\frac{\pi}{4}\right)=-1$ or $-4\sqrt{2}\cos\left(t+\frac{\pi}{4}\right)=-1$ or $\sqrt{2}\sin\left(t-\frac{\pi}{4}\right)=-\frac{1}{4}$ or $\sqrt{2}\cos\left(t+\frac{\pi}{4}\right)=\frac{1}{4}$, etc.
	Note	Unless recovered, give A0 for $4\sqrt{2}\sin\left(t-45^\circ\right)=-1$ or $-4\sqrt{2}\cos\left(t+45^\circ\right)=-1$, etc.
	3rd M1	**which is dependent on the 2nd M1 mark.** Uses correct algebraic processes to give $t=...$
	4th A1	anything that rounds to 0.6077
	Note	**Do not give** the final A1 mark in (c) if there any extra solutions given in the range $-\frac{2\pi}{3}\leqslant t\leqslant\frac{2\pi}{3}$.
	Note	You can give the final A1 mark in (c) if extra solutions are given outside of $-\frac{2\pi}{3}\leqslant t\leqslant\frac{2\pi}{3}$.

Question Number	Scheme		Marks
175.	$x = 2\sin t,\quad y = 1 - \cos 2t \quad \{= 2\sin^2 t\},\quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$		
	$\frac{dx}{dt} = 2\cos t,\quad \frac{dy}{dt} = 2\sin 2t$ or $\frac{dy}{dt} = 4\sin t\cos t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\frac{dy}{dx} = \frac{2\sin 2t}{2\cos t}\left\{= \frac{4\cos t\sin t}{2\cos t} = 2\sin t\right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{2\sin\left(\frac{2\pi}{6}\right)}{2\cos\left(\frac{\pi}{6}\right)}$; $= 1$	and substitutes $t = \frac{\pi}{6}$ into their $\frac{dy}{dx}$.	M1;
		Correct value for $\frac{dy}{dx}$ of 1	A1 **cao cso**
			[4]
	Notes for Question 175		

B1: At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. **Note:** that this mark can be implied from their working.

B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. **Note:** that this mark can be implied from their working.

M1: Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **and** attempts to substitute $t = \frac{\pi}{6}$ into their expression for $\frac{dy}{dx}$.

This mark may be implied by their final answer.

Ie. $\frac{dy}{dx} = \frac{\sin 2t}{2\cos t}$ followed by an answer of $\frac{1}{2}$ would be M1 (implied).

A1: For an answer of 1 ***by correct solution only.***

Note: Don't just look at the answer! A number of candidates are finding $\frac{dy}{dx} = 1$ from incorrect methods.

Note: Applying $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ is M0, even if they state $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$.

Special Case: Award SC: B0B0M1A1 for $\frac{dx}{dt} = -2\cos t$, $\frac{dy}{dt} = -2\sin 2t$ leading to $\frac{dy}{dx} = \frac{-2\sin 2t}{-2\cos t}$

which after substitution of $t = \frac{\pi}{6}$, yields $\frac{dy}{dx} = 1$

Note: It is possible for you to mark part(a), part (b) and part (c) together. Ignore labelling!

	Notes for Question 175 Continued	
Aliter 175. (a) Way 2	$\dfrac{dx}{dt} = 2\cos t$, $\dfrac{dy}{dt} = 2\sin 2t$,	So B1, B1.
	At $t = \dfrac{\pi}{6}$, $\dfrac{dx}{dt} = 2\cos\left(\dfrac{\pi}{6}\right) = \sqrt{3}$, $\dfrac{dy}{dt} = 2\sin\left(\dfrac{2\pi}{6}\right) = \sqrt{3}$	
	Hence $\dfrac{dy}{dx} = 1$	So implied M1, A1.

Aliter 175. (a) Way 3	$y = \dfrac{1}{2}x^2 \Rightarrow \dfrac{dy}{dx} = x$	Correct differentiation of their Cartesian equation.	B1ft
		Finds $\dfrac{dy}{dx} = x$, using the correct Cartesian equation only.	B1
	At $t = \dfrac{\pi}{6}$, $\dfrac{dy}{dx} = 2\sin\left(\dfrac{\pi}{6}\right)$	Finds the value of "x" when $t = \dfrac{\pi}{6}$ and substitutes this into their $\dfrac{dy}{dx}$	M1
	$= 1$	Correct value for $\dfrac{dy}{dx}$ of 1	A1

Question Number	Scheme	Marks
176.	$x^2 + 4xy + y^2 + 27 = 0$	
(a)	$\left\{\cancel{\frac{\mathrm{d}y}{\mathrm{d}x}} \cancel{=}\right\} \quad \underline{2x} + \left(\underline{\underline{4y + 4x\frac{\mathrm{d}y}{\mathrm{d}x}}}\right) \underline{+ 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0}$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	$2x + 4y + (4x + 2y)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2x - 4y}{4x + 2y} \quad \left\{= \frac{-x - 2y}{2x + y}\right\}$	A1 **cso oe**
		[5]
(b)	$4x + 2y = 0$	M1
	$y = -2x$ | $x = -\frac{1}{2}y$	A1
	$x^2 + 4x(-2x) + (-2x)^2 + 27 = 0$ | $\left(-\frac{1}{2}y\right)^2 + 4\left(-\frac{1}{2}y\right)y + y^2 + 27 = 0$	M1*
	$-3x^2 + 27 = 0$ | $-\frac{3}{4}y^2 + 27 = 0$	
	$x^2 = 9$ | $y^2 = 36$	dM1*
	$x = -3$ | $y = 6$	A1
	When $x = -3$, $y = -2(-3)$ | When $y = 6$, $x = -\frac{1}{2}(6)$	ddM1*
	$y = 6$ | $x = -3$	A1 **cso**
		[7]
		12

Notes for Question 176	
(a)	**M1**: Differentiates implicitly to include either $4x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$).
	A1: $\left(x^2\right) \to \left(\underline{2x}\right)$ and $\left(\ldots + y^2 + 27 = 0 \to \underline{+ 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0}\right)$.
	Note: If an extra term appears then award A0.
	Note: The $"= 0"$ can be implied by rearrangement of their equation.
	i.e.: $2x + 4y + 4x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ leading to $4x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = -2x - 4y$ will get A1 (implied).
	B1: $4y + 4x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $4\left(y + x\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ or equivalent
	dM1: An attempt to factorise out $\frac{\mathrm{d}y}{\mathrm{d}x}$ as long as there are at least two terms in $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	ie. $\ldots + (4x + 2y)\frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$ or $\ldots + 2(2x + y)\frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$
	Note: This mark is dependent on the previous method mark being awarded.
	A1: For $\frac{-2x - 4y}{4x + 2y}$ or equivalent. Eg: $\frac{+2x + 4y}{-4x - 2y}$ or $\frac{-2(x + 2y)}{4x + 2y}$ or $\frac{-x - 2y}{2x + y}$
	cso: If the candidate's solution is not completely correct, then do not give this mark.

Notes for Question 176 Continued	
(b)	**M1:** Sets the denominator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero (or the numerator of their $\frac{\mathrm{d}x}{\mathrm{d}y}$ equal to zero) oe. **A1:** Rearranges to give either $y = -2x$ or $x = -\frac{1}{2}y$. (correct solution only). The first two marks can be implied from later working, i.e. for a correct substitution of either $y = -2x$ into y^2 or for $x = -\frac{1}{2}y$ into $4xy$. **M1*:** Substitutes $y = \pm\lambda x$ or or $x = \pm\mu y$ or $y = \pm\lambda x \pm a$ or $x = \pm\mu y \pm b$ $(\lambda \neq 0,\ \mu \neq 0)$ into $x^2 + 4xy + y^2 + 27 = 0$ to form an equation in one variable. **dM1*:** leading to at least either $x^2 = A,\ A > 0$ or $y^2 = B,\ B > 0$ **Note:** This mark is dependent on the previous method mark (M1*) being awarded. **A1:** For $x = -3$ (ignore $x = 3$) or if *y* was found first, $y = 6$ (ignore $y = -6$) (correct solution only). **ddM1*** Substitutes their value of *x* into $y = \pm\lambda x$ to give $y =$ value or substitutes their value of *x* into $x^2 + 4xy + y^2 + 27 = 0$ to give $y =$ value. **Alternatively, s**ubstitutes their value of *y* into $x = \pm\mu y$ to give $x =$ value or substitutes their value of *y* into $x^2 + 4xy + y^2 + 27 = 0$ to give $x =$ value **Note:** This mark is dependent on the two previous method marks (M1* and dM1*) being awarded. **A1:** $(-3, 6)$ **cso**. **Note:** If a candidate offers two sets of coordinates without either rejecting the incorrect set or accepting the correct set then award A0. **DO NOT APPLY ISW ON THIS OCCASION.** **Note:** $x = -3$ followed later in working by $y = 6$ is fine for A1. **Note:** $y = 6$ followed later in working by $x = -3$ is fine for A1. **Note:** $x = -3, 3$ followed later in working by $y = 6$ is A0, unless candidate indicates that they are rejecting $x = 3$ **Note:** Candidates who set the numerator of $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to 0 (or the denominator of their $\frac{\mathrm{d}x}{\mathrm{d}y}$ equal to zero) can ***only achieve a maximum of 3 marks*** in this part. They can only achieve the 2nd, 3rd and 4th Method marks to give a maximum marking profile of M0A0M1M1A0M1A0. They will usually find $(-6, 3)$ **{** or even $(6, -3)$ **}.** **Note:** Candidates who set ***the numerator*** or ***the denominator*** of $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to $\pm k$ (usually $k = 1$) can ***only achieve a maximum of 3 marks*** in this part. They can only achieve the 2nd, 3rd and 4th Method marks to give a marking profile of M0A0M1M1A0M1A0. **Special Case:** It is possible for a candidate who does not achieve full marks in part (a), (but has a correct denominator for $\frac{\mathrm{d}y}{\mathrm{d}x}$) to gain all 7 marks in part (b). Eg: An incorrect part (a) answer of $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x - 4y}{4x + 2y}$ can lead to a correct $(-3, 6)$ in part (b) and 7 marks.

Question Number	Scheme		Marks
177.	$3^{x-1} + xy - y^2 + 5 = 0$		
		$3^{x-1} \to 3^{x-1}\ln 3$	B1 oe
	$\left\{\cancel{\frac{\mathrm{d}y}{\mathrm{d}x}} \cancel{\times}\right\}$ $3^{x-1}\ln 3 + \left(y + x\frac{\mathrm{d}y}{\mathrm{d}x}\right) - 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ (ignore)	Differentiates implicitly to include either $\pm\lambda x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$.	M1*
		$xy \to + y + x\frac{\mathrm{d}y}{\mathrm{d}x}$	B1
		$... + y + x\frac{\mathrm{d}y}{\mathrm{d}x} - 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	A1
	$\{(1, 3) \Rightarrow\}$ $3^{(1-1)}\ln 3 + 3 + (1)\frac{\mathrm{d}y}{\mathrm{d}x} - 2(3)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	Substitutes $x = 1,\ y = 3$ into their differentiated equation or expression.	dM1*
	$\ln 3 + 3 + \frac{\mathrm{d}y}{\mathrm{d}x} - 6\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow 3 + \ln 3 = 5\frac{\mathrm{d}y}{\mathrm{d}x}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3 + \ln 3}{5}$		dM1*
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{5}\left(\ln \mathrm{e}^3 + \ln 3\right) = \frac{1}{5}\ln\left(3\mathrm{e}^3\right)$	**Uses** $3 = \ln \mathrm{e}^3$ to achieve $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{5}\ln\left(3\mathrm{e}^3\right)$	A1 **cso**
			[7] **7**

Notes for Question 177

B1: Correct differentiation of 3^{x-1}. I.e. $3^{x-1} \to 3^{x-1}\ln 3$ or $3^{x-1} = \frac{1}{3}\left(3^x\right) \to \frac{1}{3}\left(3^x\right)\ln 3$

or $3^{x-1} = \mathrm{e}^{(x-1)\ln 3} \to \ln 3\ \mathrm{e}^{(x-1)\ln 3}$ or $3^{x-1} = \frac{1}{3}\left(3^x\right) = \frac{1}{3}\mathrm{e}^{x\ln 3} \to \frac{1}{3}(\ln 3)\mathrm{e}^{x\ln 3}$

M1: Differentiates implicitly to include either $\pm\lambda x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$).

B1: $xy \to + y + x\frac{\mathrm{d}y}{\mathrm{d}x}$

1st A1: $... + y + x\frac{\mathrm{d}y}{\mathrm{d}x} - 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ **Note:** The 1st A0 follows from an award of the 2nd B0.

Note: The "$= 0$" can be implied by rearrangement of their equation.

ie: $3^{x-1}\ln 3 + y + x\frac{\mathrm{d}y}{\mathrm{d}x} - 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ leading to $3^{x-1}\ln 3 + y = 2y\frac{\mathrm{d}y}{\mathrm{d}x} - x\frac{\mathrm{d}y}{\mathrm{d}x}$ will get A1 (implied).

2nd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.
Substitutes $x = 1,\ y = 3$ into their differentiated equation or expression. Allow one slip.

3rd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.

Candidate has two differentiated terms in $\frac{\mathrm{d}y}{\mathrm{d}x}$ and rearranges to make $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject.

Note: It is possible to gain the 3rd M1 mark before the 2nd M1 mark.

Eg: Candidate may write $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y + 3^{x-1}\ln 3}{2y - x}$ before substituting in $x = 1$ and $y = 3$

2nd A1: **cso.** Uses $3 = \ln \mathrm{e}^3$ to achieve $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{5}\ln\left(3\mathrm{e}^3\right)$, $\left(= \frac{1}{\lambda}\ln\left(\mu \mathrm{e}^3\right),\ \lambda = 5 \text{ and } \mu = 3\right)$

Note: $3 = \ln \mathrm{e}^3$ needs to be seen in their proof.

177.	**Alternative Method:** Multiplying both sides by 3		
	$3^{x-1} + xy - y^2 + 5 = 0$		
	$3^x + 3xy - 3y^2 + 15 = 0$		
		$3^x \to 3^x \ln 3$	B1
***Aliter* Way 2**	$\left\{\dfrac{\cancel{\mathrm{d}y}}{\cancel{\mathrm{d}x}} \cancel{=}\right\}$ $3^x \ln 3 + \left(3y + 3x\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) - 6y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ (ignore)	Differentiates implicitly to include either $\pm\lambda x\dfrac{\mathrm{d}y}{\mathrm{d}x}$ or $\pm ky\dfrac{\mathrm{d}y}{\mathrm{d}x}$.	M1*
		$3xy \to +3y + 3x\dfrac{\mathrm{d}y}{\mathrm{d}x}$	B1
		$\ldots + 3y + 3x\dfrac{\mathrm{d}y}{\mathrm{d}x} - 6y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	A1
	$\{(1, 3) \Rightarrow\}$ $3^1 \ln 3 + 3(3) + (3)(1)\dfrac{\mathrm{d}y}{\mathrm{d}x} - 6(3)\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	Substitutes $x = 1$, $y = 3$ into their differentiated equation or expression.	dM1*
	$3\ln 3 + 9 + 3\dfrac{\mathrm{d}y}{\mathrm{d}x} - 18\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow 9 + 3\ln 3 = 15\dfrac{\mathrm{d}y}{\mathrm{d}x}$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{9 + 3\ln 3}{15}$ $\left\{= \dfrac{3 + \ln 3}{5}\right\}$		dM1*
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{5}\left(\ln e^3 + \ln 3\right)$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{5}\left(\ln e^3 + \ln 3\right) = \dfrac{1}{5}\ln\left(3e^3\right)$	**Uses** $3 = \ln e^3$ to achieve $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{5}\ln\left(3e^3\right)$	A1 **cso**
			[7] **7**
	NOTE: Only apply this scheme if the candidate has multiplied both sides of their equation by 3. **NOTE:** For reference, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3y + 3^x \ln 3}{6y - 3x}$ **NOTE:** If the candidate applies this method then $3xy \to +3y + 3x\dfrac{\mathrm{d}y}{\mathrm{d}x}$ must be seen for the 2nd B1 mark.		

Question Number	Scheme	Marks
178.	$x = 27\sec^3 t, \quad y = 3\tan t, \quad 0 \leqslant t \leqslant \frac{\pi}{3}$	
	$\frac{\mathrm{d}x}{\mathrm{d}t} = 81\sec^2 t\sec t\tan t, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = 3\sec^2 t$ At least one of $\frac{\mathrm{d}x}{\mathrm{d}t}$ or $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct.	B1
	Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ are correct.	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\sec^2 t}{81\sec^3 t\tan t}\left\{= \frac{1}{27\sec t\tan t} = \frac{\cos t}{27\tan t} = \frac{\cos^2 t}{27\sin t}\right\}$ Applies their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$	M1;
	At $t = \frac{\pi}{6}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\sec^2\left(\frac{\pi}{6}\right)}{81\sec^3\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{6}\right)} = \frac{4}{72}\left\{= \frac{3}{54} = \frac{1}{18}\right\}$ $\frac{4}{72}$	A1 **cao cso**
		[4]

Notes for Question 178

B1: At least one of $\frac{\mathrm{d}x}{\mathrm{d}t}$ or $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct. **Note:** that this mark can be implied from their working.

B1: Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ are correct. **Note:** that this mark can be implied from their working.

M1: Applies their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$, where both $\frac{\mathrm{d}y}{\mathrm{d}t}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}$ are trigonometric functions of t.

A1: $\frac{4}{72}$ or any equivalent correct rational answer not involving surds.

Allow $0.0\dot{5}$ with the recurring symbol.

Way 2	***Alternative response using the Cartesian equation in part (a)***	
	$\left\{y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} \Rightarrow\right\} \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}x^{-\frac{1}{3}}\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \pm K x^{-\frac{1}{3}}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}x^{-\frac{1}{3}}\right)$ oe	A1
	At $t = \frac{\pi}{6}$, $x = 27\sec^3\left(\frac{\pi}{6}\right) = 24\sqrt{3}$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(\left(24\sqrt{3}\right)^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}\left(24\sqrt{3}\right)^{-\frac{1}{3}}\right)$ Uses $t = \frac{\pi}{6}$ to find x and substitutes their x into an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.	dM1
	So, $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{3\sqrt{3}}\right) = \frac{1}{18}$ $\frac{1}{18}$	A1 **cao cso**
	Note: Way 2 is marked as M1 A1 dM1 A1 **Note:** For way 2 the second M1 mark is dependent on the first M1 being gained.	

Question Number	Scheme		Marks
179. (a)	$x = 2t + 5,\ \ y = 3 + \dfrac{4}{t}$		
	$\dfrac{dx}{dt} = 2,\ \ \dfrac{dy}{dt} = -4t^{-2}$		
	So, $\dfrac{dy}{dx} = \dfrac{-4t^{-2}}{2}\ \left\{= -2t^{-2} = -\dfrac{2}{t^2}\right\}$	Candidate's $\dfrac{dy}{dt}$ divided by candidate's $\dfrac{dx}{dt}$	M1
		Correct $\dfrac{dy}{dx}$	A1
	At $(9, 5)$, $t = 2$		
	When $t = 2,\ \dfrac{dy}{dx} = \dfrac{-4(2)^{-2}}{2}\ \left\{= -2(2)^{-2} = -\dfrac{2}{2^2}\right\}$	Substitutes their **found** t into their $\dfrac{dy}{dx}$	M1
	So, $\dfrac{dy}{dx} = -\dfrac{1}{2}$	$\dfrac{dy}{dx} = -\dfrac{1}{2}$	A1 cso
			[4]
(b)	$t = \dfrac{x-5}{2} \Rightarrow y = 3 + \dfrac{4}{\left(\dfrac{x-5}{2}\right)}$	An attempt to eliminate t.	M1
		Achieves a correct equation in x and y only.	A1oe
	$\Rightarrow y = 3 + \dfrac{8}{x-5}$		
	$\Rightarrow y = \dfrac{3(x-5)+8}{x-5}$		
	$\Rightarrow y = \dfrac{3x-7}{x-5} \qquad x \neq 5$	$\underline{a = 3}$, $\underline{b = -7}$, $\underline{c = 1}$ and $\underline{d = -5}$ or $\dfrac{3x-7}{x-5}$	A1 oe
			[3] **7**
	Notes on Question 179		
(a)	**Note:** Part (a) and part (b) can be marked together.		
	<u>Alternative Method for part (a)</u>		
	$y = 3 + \dfrac{8}{x-5} = 3 + 8(x-5)^{-1} \Rightarrow \dfrac{dy}{dx} = -8(x-5)^{-2}$	M1 for $\pm\lambda(x-5)^{-2}$ where $\lambda \neq 0$ A1 for $-8(x-5)^{-2}$	
	At $(9, 5)$, $\dfrac{dy}{dx} = -8(9-5)^{-2}$	M1 for substituting $x = 9$ into their $\dfrac{dy}{dx}$	
	So, $\dfrac{dy}{dx} = -\dfrac{1}{2}$	A1 for $\dfrac{dy}{dx} = -\dfrac{1}{2}$ by correct solution only	
(b)	Award M1A1 for either $x = \dfrac{8}{y-3} + 5$ or $\dfrac{4}{y-3} = \dfrac{x-5}{2}$ or equivalent.		

Question Number	Scheme		Marks
180.	$\sin(\pi y) - y - x^2 y = -5$		
		Differentiates implicitly to include either $\pm k\cos(\pi y)\frac{dy}{dx}$ or $-\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$)	M1
(a)	$\underline{\pi\cos(\pi y)\frac{dy}{dx}} - \underline{\frac{dy}{dx}} - \left(\underline{\underline{2xy + x^2\frac{dy}{dx}}}\right) = \underline{0}$	$(\sin(\pi y)) \to \left(\pi\cos(\pi y)\frac{dy}{dx}\right)$, $(-y) \to \left(-\frac{dy}{dx}\right)$ and $(-5 \to 0)$	A1
		$\pm 2xy \pm x^2\frac{dy}{dx}$	M1
	$\frac{dy}{dx}\left(\pi\cos(\pi y) - 1 - x^2\right) = 2xy$	Grouping terms and factorising out $\frac{dy}{dx}$.	dM1
	$\frac{dy}{dx} = \frac{2xy}{\left(\pi\cos(\pi y) - 1 - x^2\right)}$	$\frac{2xy}{\left(\pi\cos(\pi y) - 1 - x^2\right)}$	A1 oe
			[5]
(b)	At $(2, 1)$, $\frac{dy}{dx} = \frac{2(2)(1)}{\left(\pi\cos(\pi(1)) - 1 - (2)^2\right)}$; $\left(= \frac{4}{-\pi - 5}\right)$	Substituting $x = 2$ & $y = 1$ into an equation involving $\frac{dy}{dx}$;	M1;
	T: $y - 1 = \frac{4}{-\pi - 5}(x - 2)$	$y - 1 = m_T(x - 2)$ with 'their TANGENT gradient';	M1
	Cuts x-axis $\Rightarrow y = 0 \Rightarrow -1 = \frac{4}{-\pi - 5}(x - 2)$	Setting $y = 0$ in their tangent equation.	M1
	So, $x = \frac{\pi + 5}{4} + 2 \left\{= \frac{\pi + 13}{4}\right\}$	$\frac{\pi + 5}{4} + 2$	A1 oe cso
			[4] **9**
	Notes on Question 180		
(b)	**Note:** 2nd M1 can be implied for $-1 = \frac{4}{-\pi - 5}(x - 2)$ or $\frac{-1}{x - 2} = \frac{-4}{\pi + 5}$ if no equation of tangent is given. **Note:** Award 2nd M0 where m in $y - 1 = m(x - 2)$ is either a changed tangent gradient or a normal gradient.		

Question Number	Scheme		Marks
181.	$\dfrac{\mathrm{d}V}{\mathrm{d}t} = -\,32\pi\sqrt{h}$		
	$V = \pi(40)^2 h\ \{= 1600\pi h\}$	$V = \pi(40)^2 h$	B1
	$\dfrac{\mathrm{d}V}{\mathrm{d}h} = 1600\pi$	$\dfrac{\mathrm{d}V}{\mathrm{d}h} = 1600\pi$	B1ft
	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{\mathrm{d}h}{\mathrm{d}V} \times \dfrac{\mathrm{d}V}{\mathrm{d}t}$		
	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{1}{1600\pi} \times -32\pi\sqrt{h}$	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = \left(\pm\,32\pi\sqrt{h}\right) \div \left(\text{their } \dfrac{\mathrm{d}V}{\mathrm{d}h}\right)$	M1
	So, $\dfrac{\mathrm{d}h}{\mathrm{d}t} = -\,0.02\sqrt{h}$	Correct proof.	A1 ***** cso
			[4]
	Notes on Question 181		
	Note: Use of $V = \pi r^2 h$ is 1st B0 until $r = 40$ is substituted.		

	Notes on Question 181 continued	
	<u>**Alternative Method**</u>	
	$\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\pi 40^2 h\right) = -\,32\pi\sqrt{h}$	**B1B1:** $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\pi 40^2 h\right) = -\,32\pi\sqrt{h}$
	$\Rightarrow \dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{-\,32\pi\sqrt{h}}{\pi 40^2}$	**M1:** Simplifies to give an expression for $\dfrac{\mathrm{d}h}{\mathrm{d}t}$.
	So, $\dfrac{\mathrm{d}h}{\mathrm{d}t} = -\,0.02\sqrt{h}$ *	**A1:** Correct proof.

Question Number	Scheme		Marks
182.	***Working parametrically:***		
	$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$ or $y = e^{t\ln 2} - 1$		
(a)	$\{x = 0 \Rightarrow\}\ 0 = 1 - \frac{1}{2}t \Rightarrow t = 2$	Applies $x = 0$ to obtain a value for t.	M1
	When $t = 2$, $y = 2^2 - 1 = 3$	Correct value for y.	A1
			[2]
(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^t - 1 \Rightarrow t = 0$	Applies $y = 0$ to obtain a value for t. (Must be seen in part (b)).	M1
	When $t = 0$, $x = 1 - \frac{1}{2}(0) = 1$	$x = 1$	A1
			[2]
(c)	$\frac{dx}{dt} = -\frac{1}{2}$ and either $\frac{dy}{dt} = 2^t \ln 2$ or $\frac{dy}{dt} = e^{t\ln 2}\ln 2$		B1
	$\frac{dy}{dx} = \frac{2^t \ln 2}{-\frac{1}{2}}$	Attempts their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$.	M1
	At A, $t = "2"$, so $m(\mathbf{T}) = -8\ln 2 \Rightarrow m(\mathbf{N}) = \frac{1}{8\ln 2}$	Applies $t = "2"$ and $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.	See notes.	M1 A1 oe cso
			[5]

182. (a)	**M1:** Applies $x = 0$ and obtains a value of t.
	A1: For $y = 2^2 - 1 = 3$ or $y = 4 - 1 = 3$
	Alternative Solution 1:
	M1: For substituting $t = 2$ into either x or y.
	A1: $x = 1 - \frac{1}{2}(2) = 0$ **and** $y = 2^2 - 1 = 3$
	Alternative Solution 2:
	M1: Applies $y = 3$ and obtains a value of t.
	A1: For $x = 1 - \frac{1}{2}(2) = 0$ or $x = 1 - 1 = 0$.
	Alternative Solution 3:
	M1: Applies $y = 3$ or $x = 0$ and obtains a value of t.
	A1: Shows that $t = 2$ for both $y = 3$ and $x = 0$.
(b)	**M1:** Applies $y = 0$ and obtains a value of t. Working must be seen in part (b).
	A1: For finding $x = 1$.
	Note: Award M1A1 for $x = 1$.
(c)	**B1:** Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct. This mark can be implied by later working.
	M1: Their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$ or their $\frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{1}{\text{their}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)}$. **Note:** their $\frac{\mathrm{d}y}{\mathrm{d}t}$ must be a function of t.
	M1: Uses their value of t found in part (a) and applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$.
	M1: $y - 3 = (\text{their normal gradient})x$ **or** $y = (\text{their normal gradient})x + 3$ or equivalent.
	A1: $y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or $y - 3 = \frac{1}{\ln 256}(x - 0)$ or $(8\ln 2)y - 24\ln 2 = x$ or $\frac{y-3}{(x-0)} = \frac{1}{8\ln 2}$. You can apply isw here.
	Working in decimals is ok for the three method marks. B1, A1 require exact values.

Question Number	Scheme		Marks
182.	***Alternative: Converting to a Cartesian equation:***		
	$t = 2 - 2x \Rightarrow \quad y = 2^{2-2x} - 1$		
(a)	$\{x = 0 \Rightarrow\}\ y = 2^2 - 1$	Applies $x = 0$ in their Cartesian equation...	M1
	$y = 3$	... to arrive at a correct answer of 3.	A1
			[2]
(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^{2-2x} - 1 \Rightarrow 0 = 2 - 2x \Rightarrow x = ...$	Applies $y = 0$ to obtain a value for x. (Must be seen in part (b)).	M1
	$x = 1$	$x = 1$	A1
			[2]
(c)	$\dfrac{dy}{dx} = -2\left(2^{2-2x}\right)\ln 2$	$\pm\lambda 2^{2-2x},\ \lambda \neq 1$	M1
		$-2\left(2^{2-2x}\right)\ln 2$ or equivalent	A1
	At A, $x = 0$, so $m(\mathbf{T}) = -8\ln 2 \Rightarrow m(\mathbf{N}) = \dfrac{1}{8\ln 2}$	Applies $x = 0$ and $m(\mathbf{N}) = \dfrac{-1}{m(\mathbf{T})}$	M1
	$y - 3 = \dfrac{1}{8\ln 2}(x - 0)$ or $y = 3 + \dfrac{1}{8\ln 2}x$ or equivalent.	***As in the original scheme.***	M1 A1 oe
			[5]

Question Number	Scheme	Marks
183.	(a) $V = x^3 \Rightarrow \dfrac{\mathrm{d}V}{\mathrm{d}x} = 3x^2$ ∗ cso	B1 **(1)**
	(b) $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{\mathrm{d}x}{\mathrm{d}V} \times \dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{0.048}{3x^2}$	M1
	At $x = 8$	
	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{0.048}{3(8^2)} = 0.00025 \quad (\mathrm{cm\,s^{-1}})$ 2.5×10^{-4}	A1 **(2)**
	(c) $S = 6x^2 \Rightarrow \dfrac{\mathrm{d}S}{\mathrm{d}x} = 12x$	B1
	$\dfrac{\mathrm{d}S}{\mathrm{d}t} = \dfrac{\mathrm{d}S}{\mathrm{d}x} \times \dfrac{\mathrm{d}x}{\mathrm{d}t} = 12x\left(\dfrac{0.048}{3x^2}\right)$	M1
	At $x = 8$	
	$\dfrac{\mathrm{d}S}{\mathrm{d}t} = 0.024 \quad (\mathrm{cm^2\,s^{-1}})$	A1 **(3)**
		[6]

EXPERT TUITION

Question Number	Scheme	Marks	
184.	(a) Differentiating implicitly to obtain $\pm ay^2\frac{\mathrm{d}y}{\mathrm{d}x}$ and/or $\pm bx^2\frac{\mathrm{d}y}{\mathrm{d}x}$	M1	
	$48y^2\frac{\mathrm{d}y}{\mathrm{d}x}+\ \ldots\ -54\ \ldots$	A1	
	$9x^2y \to 9x^2\frac{\mathrm{d}y}{\mathrm{d}x}+18xy$ or equivalent	B1	
	$\left(48y^2+9x^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}+18xy-54=0$	M1	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{54-18xy}{48y^2+9x^2}\quad\left(=\frac{18-6xy}{16y^2+3x^2}\right)$	A1	**(5)**
	(b) $18-6xy=0$	M1	
	Using $x=\frac{3}{y}$ or $y=\frac{3}{x}$		
	$16y^3+9\left(\frac{3}{y}\right)^2y-54\left(\frac{3}{y}\right)=0$ or $16\left(\frac{3}{x}\right)^3+9x^2\left(\frac{3}{x}\right)-54x=0$	M1	
	Leading to		
	$16y^4+81-162=0$ or $16+x^4-2x^4=0$	M1	
	$y^4=\frac{81}{16}$ or $x^4=16$		
	$y=\frac{3}{2},-\frac{3}{2}$ or $x=2,-2$	A1 A1	
	Substituting either of their values into $xy=3$ to obtain a value of the other variable.	M1	
	$\left(2,\frac{3}{2}\right),\ \left(-2,-\frac{3}{2}\right)$ both	A1	**(7)**
			[12]

Question Number	Scheme	Marks
185.	(a) $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 2\sqrt{3}\cos 2t$	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}t} = -8\cos t \sin t$	M1 A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-8\cos t \sin t}{2\sqrt{3}\cos 2t}$	M1
	$= -\dfrac{4\sin 2t}{2\sqrt{3}\cos 2t}$	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2}{3}\sqrt{3}\tan 2t \qquad \left(k = -\dfrac{2}{3}\right)$	A1 **(5)**
	(b) When $t = \dfrac{\pi}{3}$ $\qquad x = \dfrac{3}{2},\ y = 1$ can be implied	B1
	$m = -\dfrac{2}{3}\sqrt{3}\tan\left(\dfrac{2\pi}{3}\right) \quad (=2)$	M1
	$y - 1 = 2\left(x - \dfrac{3}{2}\right)$	M1
	$y = 2x - 2$	A1 **(4)**

Question Number	Scheme	Marks
186. (a)	$\left\{\frac{\cancel{\mathrm{d}y}}{\cancel{\mathrm{d}x}} \cancel{=}\right\}$ $\underline{2+6y\frac{\mathrm{d}y}{\mathrm{d}x}} + \left(\underline{\underline{6xy+3x^2\frac{\mathrm{d}y}{\mathrm{d}x}}}\right) = \underline{8x}$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8x-2-6xy}{6y+3x^2}\right\}$ *not necessarily required.*	
	At $P(-1,1)$, $\mathrm{m}(\mathbf{T}) = \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8(-1)-2-6(-1)(1)}{6(1)+3(-1)^2} = -\frac{4}{9}$	dM1 A1 **cso**
		[5]
(b)	So, $\mathrm{m}(\mathbf{N}) = \frac{-1}{-\frac{4}{9}}$ $\left\{= \frac{9}{4}\right\}$	M1
	N: $y-1 = \frac{9}{4}(x+1)$	M1
	N: $9x - 4y + 13 = 0$	A1
		[3] **8**
(a)	**M1**: Differentiates implicitly to include either $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$ or $3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$). **A1**: $(2x+3y^2) \to \left(\underline{2+6y\frac{\mathrm{d}y}{\mathrm{d}x}}\right)$ and $(4x^2 \to \underline{8x})$. **Note:** If an extra "sixth" term appears then award A0. **B1**: $6xy + 3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$. **dM1**: Substituting $x=-1$ and $y=1$ into an equation involving $\frac{\mathrm{d}y}{\mathrm{d}x}$. Allow this mark if either the numerator or denominator of $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8x-2-6xy}{6y+3x^2}$ is substituted into or evaluated correctly. If it is clear, however, that the candidate is intending to substitute $x=1$ and $y=-1$, then award M0. Candidates who substitute $x=1$ and $y=-1$, will usually achieve $\mathrm{m}(\mathbf{T}) = -4$ Note that this mark is dependent on the previous method mark being awarded. **A1**: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or $-0.\dot{4}$ or awrt -0.44 If the candidate's solution is not completely correct, then do not give this mark.	
(b)	**M1:** Applies $\mathrm{m}(\mathbf{N}) = -\frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$. **M1:** Uses $y-1=(m_N)(x--1)$ or finds c using $x=-1$ and $y=1$ and uses $y=(\mathrm{m}_N)x+"c"$, Where $m_N = -\frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$ or $m_N = \frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$ or $m_N = -\text{their } \mathrm{m}(\mathbf{T})$. **A1:** $9x-4y+13=0$ or $-9x+4y-13=0$ or $4y-9x-13=0$ or $18x-8y+26=0$ etc. Must be " $=0$". So do not allow $9x+13=4y$ etc. **Note**: $m_N = -\left(\frac{6y+3x^2}{8x-2-6xy}\right)$ is M0M0 unless a numerical value is then found for m_N.	

Alternative method for part (a): Differentiating with respect to y

$$\left\{\cancel{\frac{\mathrm{d}x}{\mathrm{d}y}\times}\right\}\quad \underline{2\frac{\mathrm{d}x}{\mathrm{d}y}+6y}+\left(\underline{\underline{6xy\frac{\mathrm{d}x}{\mathrm{d}y}+3x^2}}\right)=\underline{8x\frac{\mathrm{d}x}{\mathrm{d}y}}$$

M1: Differentiates implicitly to include either $2\frac{\mathrm{d}x}{\mathrm{d}y}$ or $6xy\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\pm kx\frac{\mathrm{d}x}{\mathrm{d}y}$. (Ignore $\left(\frac{\mathrm{d}x}{\mathrm{d}y}=\right)$).

A1: $\left(2x+3y^2\right)\rightarrow\left(\underline{2\frac{\mathrm{d}x}{\mathrm{d}y}+6y}\right)$ and $\left(4x^2\rightarrow\underline{8x\frac{\mathrm{d}x}{\mathrm{d}y}}\right)$. **Note:** If an extra "sixth" term appears then award A0.

B1: $6xy+3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$.

dM1: Substituting $x=-1$ and $y=1$ into an equation involving $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$. Allow this mark if either the numerator or denominator of $\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{6y+3x^2}{8x-2-6xy}$ is substituted into or evaluated correctly.

If it is clear, however, that the candidate is intending to substitute $x=1$ and $y=-1$, then award M0.

Candidates who substitute $x=1$ and $y=-1$, will usually achieve $\mathrm{m}(\mathbf{T})=-4$

Note that this mark is dependent on the previous method mark being awarded.

A1: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or $-0.\dot{4}$ or awrt -0.44

If the candidate's solution is not completely correct, then do not give this mark.

Question Number	Scheme	Marks
187.	$x = 4\sin\left(t + \frac{\pi}{6}\right), \quad y = 3\cos 2t, \quad 0 \leq t < 2\pi$	
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 4\cos\left(t + \frac{\pi}{6}\right), \quad \frac{\mathrm{d}y}{\mathrm{d}t} = -6\sin 2t$	B1 B1
	So, $\frac{\mathrm{d}y}{\mathrm{d}x} = \underline{\frac{-6\sin 2t}{4\cos\left(t + \frac{\pi}{6}\right)}}$	B1 $\sqrt{}$ oe
		[3]
(b)	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\} -6\sin 2t = 0$	M1 oe
	@ $t = 0, \quad x = 4\sin\left(\frac{\pi}{6}\right) = 2, \quad y = 3\cos 0 = 3 \quad \rightarrow (2, 3)$	M1
	@ $t = \frac{\pi}{2}, \quad x = 4\sin\left(\frac{2\pi}{3}\right) = \frac{4\sqrt{3}}{2}, \quad y = 3\cos\pi = -3 \rightarrow (2\sqrt{3}, -3)$	
	@ $t = \pi, \quad x = 4\sin\left(\frac{7\pi}{6}\right) = -2, \quad y = 3\cos 2\pi = 3 \rightarrow (-2, 3)$	
	@ $t = \frac{3\pi}{2}, x = 4\sin\left(\frac{5\pi}{3}\right) = \frac{4(-\sqrt{3})}{2}, \quad y = 3\cos 3\pi = -3 \rightarrow (-2\sqrt{3}, -3)$	A1A1A1
		[5] **8**
(a)	**B1:** Either one of $\frac{\mathrm{d}x}{\mathrm{d}t} = 4\cos\left(t + \frac{\pi}{6}\right)$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = -6\sin 2t$. They do not have to be simplified. **B1:** Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct. They do not have to be simplified. Any or both of the first two marks can be implied. Don't worry too much about their notation for the first two B1 marks. **B1:** Their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$ or their $\frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{1}{\text{their}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)}$. **Note:** This is a follow through mark. *__Alternative differentiation in part (a)__* $x = 2\sqrt{3}\sin t + 2\cos t \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = 2\sqrt{3}\cos t - 2\sin t$ $y = 3(2\cos^2 t - 1) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = 3(-4\cos t\sin t)$ or $y = 3\cos^2 t - 3\sin^2 t \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = -6\cos t\sin t - 6\sin t\cos t$ or $y = 3(1 - 2\sin^2 t) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = 3(-4\cos t\sin t)$	

187. (b)	**M1:** Candidate sets their numerator from part (a) or their $\frac{dy}{dt}$ equal to 0. Note that their numerator must be a trig function. Ignore $\frac{dx}{dt}$ equal to 0 at this stage. **M1:** Candidate substitutes a found value of *t*, to attempt to find either one of *x* or *y*. The first two method marks can be implied by ONE correct set of coordinates for (x, y) or (y, x) interchanged. A correct point coming from NO WORKING can be awarded M1M1. **A1:** At least TWO sets of coordinates. **A1:** At least THREE sets of coordinates. **A1:** ONLY FOUR correct sets of coordinates. If there are more than 4 sets of coordinates then award A0. **Note:** Candidate can use the diagram's symmetry to write down some of their coordinates. **Note:** When $x = 4\sin\left(\frac{\pi}{6}\right) = 2$, $y = 3\cos 0 = 3$ is acceptable for a pair of coordinates. Also it is fine for candidates to display their coordinates on a table of values. **Note:** The coordinates must be exact for the accuracy marks. Ie (3.46..., -3) or (-3.46..., -3) is A0. **Note:** $\frac{dy}{dx} = 0 \Rightarrow \sin t = 0$ ONLY is fine for the first M1, and potentially the following M1A1A0A0. **Note:** $\frac{dy}{dx} = 0 \Rightarrow \cos t = 0$ ONLY is fine for the first M1 and potentially the following M1A1A0A0. **Note:** $\frac{dy}{dx} = 0 \Rightarrow \sin t = 0$ & $\cos t = 0$ has the potential to achieve all five marks. **Note:** It is possible for a candidate to gain full marks in part (b) if they make sign errors in part (a). *(b) An alternative method for finding the coordinates of the two maximum points.* Some candidates may use $y = 3\cos 2t$ to write down that the *y*-coordinate of a maximum point is 3. They will then deduce that $t = 0$ or π and proceed to find the *x*-coordinate of their maximum point. These candidates will receive no credit until they attempt to find one of the *x*-coordinates for the maximum point. **M1M1:** Candidate states $y = 3$ and attempts to substitute $t = 0$ or π into $x = 4\sin\left(t + \frac{\pi}{6}\right)$. M1M1 can be implied by candidate stating either $(2, 3)$ or $(2, -3)$. **Note:** these marks can only be awarded together for a candidate using this method. **A1:** For both $(2, 3)$ or $(-2, 3)$. **A0A0**: Candidate cannot achieve the final two marks by using this method. They can, however, achieve these marks by subsequently solving their numerator equal to 0.

Question Number	Scheme		Marks	
188.	(a) $\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{1}{2}\pi h - \pi h^2$	or equivalent	M1 A1	
	At $h = 0.1$, $\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{1}{2}\pi(0.1) - \pi(0.1)^2 = 0.04\pi$	$\frac{\pi}{25}$	M1 A1	**(4)**
	(b) $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \div \frac{\mathrm{d}V}{\mathrm{d}h} = \frac{\pi}{800} \times \frac{1}{\frac{1}{2}\pi h - \pi h^2}$	or $\frac{\pi}{800} \div$ their (a)	M1	
	At $h = 0.1$, $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\pi}{800} \times \frac{25}{\pi} = \frac{1}{32}$	awrt 0.031	A1	**(2)**
				[6]

Question Number	Scheme		Marks	
189.	$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \dots$		B1	
	$\dots = 2\ln x + 2x\left(\frac{1}{x}\right)$		M1 A1	
	At $x = 2$, $\ln y = 2(2)\ln 2$		M1	
	leading to $y = 16$	Accept $y = e^{4\ln 2}$	A1	
	At $(2, 16)$ $\frac{1}{16}\frac{\mathrm{d}y}{\mathrm{d}x} = 2\ln 2 + 2$		M1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 16(2 + 2\ln 2)$		A1	**(7)**
				[7]
	Alternative			
	$y = e^{2x\ln x}$		B1	
	$\frac{\mathrm{d}}{\mathrm{d}x}(2x\ln x) = 2\ln x + 2x\left(\frac{1}{x}\right)$		M1 A1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(2\ln x + 2x\left(\frac{1}{x}\right)\right)e^{2x\ln x}$		M1 A1	
	At $x = 2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = (2\ln 2 + 2)e^{4\ln 2}$		M1	
	$= 16(2 + 2\ln 2)$		A1	**(7)**

Question Number	Scheme		Marks	
190.	(a) $\tan\theta = \sqrt{3}$ *or* $\sin\theta = \dfrac{\sqrt{3}}{2}$		M1	
	$\theta = \dfrac{\pi}{3}$	awrt 1.05	A1	**(2)**
	(b) $\dfrac{dx}{d\theta} = \sec^2\theta, \quad \dfrac{dy}{d\theta} = \cos\theta$			
	$\dfrac{dy}{dx} = \dfrac{\cos\theta}{\sec^2\theta} \quad \left(= \cos^3\theta\right)$		M1 A1	
	At P, $m = \cos^3\left(\dfrac{\pi}{3}\right) = \dfrac{1}{8}$	Can be implied	A1	
	Using $mm' = -1$, $m' = -8$		M1	
	For normal $y - \frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$		M1	
	At Q, $y = 0$ $-\frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$			
	leading to $x = \frac{17}{16}\sqrt{3}$ $\left(k = \frac{17}{16}\right)$	1.0625	A1	**(6)**

Question Number	Scheme	Marks
191.	$\frac{\mathrm{d}I}{\mathrm{d}t} = -16\ln(0.5)0.5^t$	M1 A1
	At $t = 3$ $\quad \frac{\mathrm{d}I}{\mathrm{d}t} = -16\ln(0.5)0.5^3$	M1
	$= -2\ln 0.5 = \ln 4$	M1 A1
		[5]

Question Number	Scheme	Marks
192.	$\frac{\mathrm{d}}{\mathrm{d}x}\left(2^x\right) = \ln 2.2^x$	B1
	$\ln 2.2^x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 2y + 2x\frac{\mathrm{d}y}{\mathrm{d}x}$	M1 A1= A1
	Substituting $(3, 2)$	
	$8\ln 2 + 4\frac{\mathrm{d}y}{\mathrm{d}x} = 4 + 6\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4\ln 2 - 2$ Accept exact equivalents	M1 A1 **(7)**
		[7]

Question Number	Scheme		Marks
193.	(a) $\dfrac{dx}{dt} = 2\sin t\cos t,\ \dfrac{dy}{dt} = 2\sec^2 t$		B1 B1
	$\dfrac{dy}{dx} = \dfrac{\sec^2 t}{\sin t\cos t}\left(= \dfrac{1}{\sin t\cos^3 t}\right)$	or equivalent	M1 A1 **(4)**
	(b) At $t = \dfrac{\pi}{3},\ x = \dfrac{3}{4},\ y = 2\sqrt{3}$		B1
	$\dfrac{dy}{dx} = \dfrac{\sec^2 \frac{\pi}{3}}{\sin\frac{\pi}{3}\cos\frac{\pi}{3}} = \dfrac{16}{\sqrt{3}}$		M1 A1
	$y - 2\sqrt{3} = \dfrac{16}{\sqrt{3}}\left(x - \dfrac{3}{4}\right)$		M1
	$y = 0 \quad \Rightarrow \quad x = \dfrac{3}{8}$		M1 A1 **(6)**
			[10]

Question Number	Scheme		Marks
194.	$\dfrac{dV}{dt} = 0.48\pi - 0.6\pi h$		M1 A1
	$V = 9\pi h \quad \Rightarrow \dfrac{dV}{dt} = 9\pi \dfrac{dh}{dt}$		B1
	$9\pi \dfrac{dh}{dt} = 0.48\pi - 0.6\pi h$		M1
	Leading to $75\dfrac{dh}{dt} = 4 - 5h$ ∗	cso	A1 **(5)**

Question Number	Scheme	Marks
195	(a) $-2\sin 2x - 3\sin 3y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1 A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2\sin 2x}{3\sin 3y}$ Accept $\dfrac{2\sin 2x}{-3\sin 3y}, \dfrac{-2\sin 2x}{3\sin 3y}$	A1 (3)
	(b) At $x = \dfrac{\pi}{6}$, $\cos\left(\dfrac{2\pi}{6}\right) + \cos 3y = 1$	M1
	$\cos 3y = \dfrac{1}{2}$	A1
	$3y = \dfrac{\pi}{3} \Rightarrow y = \dfrac{\pi}{9}$ awrt 0.349	A1 (3)
	(c) At $\left(\dfrac{\pi}{6}, \dfrac{\pi}{9}\right)$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2\sin 2\left(\frac{\pi}{6}\right)}{3\sin 3\left(\frac{\pi}{9}\right)} = -\dfrac{2\sin\frac{\pi}{3}}{3\sin\frac{\pi}{3}} = -\dfrac{2}{3}$	M1
	$y - \dfrac{\pi}{9} = -\dfrac{2}{3}\left(x - \dfrac{\pi}{6}\right)$	M1
	Leading to $6x + 9y - 2\pi = 0$	A1 (3) **[9]**

Question Number	Scheme	Marks
196	$\frac{dA}{dt} = 1.5$	B1
	$A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$	B1
	When $A = 2$	
	$2 = \pi r^2 \Rightarrow r = \sqrt{\frac{2}{\pi}} \ (= 0.797\,884\ \ldots)$	M1
	$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$	
	$1.5 = 2\pi r \frac{dr}{dt}$	M1
	$\frac{dr}{dt} = \frac{1.5}{2\pi\sqrt{\frac{2}{\pi}}} \approx 0.299$ awrt 0.299	A1
		[5]

Question Number	Scheme	Marks
197 (a)	$\mathrm{e}^{-2x}\dfrac{\mathrm{d}y}{\mathrm{d}x}-2y\,\mathrm{e}^{-2x}=2+2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$ A1 correct RHS	M1 A1
	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(y\,\mathrm{e}^{-2x}\right)=\mathrm{e}^{-2x}\dfrac{\mathrm{d}y}{\mathrm{d}x}-2y\,\mathrm{e}^{-2x}$	B1
	$\left(\mathrm{e}^{-2x}-2y\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}=2+2y\,\mathrm{e}^{-2x}$	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2+2y\,\mathrm{e}^{-2x}}{\mathrm{e}^{-2x}-2y}$	A1 (5)
(b)	At P , $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2+2\mathrm{e}^{0}}{\mathrm{e}^{0}-2}=-4$	M1
	Using $mm'=-1$	
	$m'=\dfrac{1}{4}$	M1
	$y-1=\dfrac{1}{4}(x-0)$	M1
	$x-4y+4=0$ or any integer multiple	A1 (4)
		[9]
	Alternative for (a) *differentiating implicitly with respect to y*.	
	$\mathrm{e}^{-2x}-2y\,\mathrm{e}^{-2x}\dfrac{\mathrm{d}x}{\mathrm{d}y}=2\dfrac{\mathrm{d}x}{\mathrm{d}y}+2y$ A1 correct RHS	M1 A1
	$\dfrac{\mathrm{d}}{\mathrm{d}y}\left(y\,\mathrm{e}^{-2x}\right)=\mathrm{e}^{-2x}-2y\,\mathrm{e}^{-2x}\dfrac{\mathrm{d}x}{\mathrm{d}y}$	B1
	$\left(2+2y\,\mathrm{e}^{-2x}\right)\dfrac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{e}^{-2x}-2y$	M1
	$\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{\mathrm{e}^{-2x}-2y}{2+2y\,\mathrm{e}^{-2x}}$	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2+2y\,\mathrm{e}^{-2x}}{\mathrm{e}^{-2x}-2y}$	A1 (5)

Question Number	Scheme	Marks
198	$\frac{dx}{dt} = -4\sin 2t$, $\frac{dy}{dt} = 6\cos t$	B1, B1
	$\frac{dy}{dx} = -\frac{6\cos t}{4\sin 2t}$ $\left(= -\frac{3}{4\sin t}\right)$	M1
	At $t = \frac{\pi}{3}$, $m = -\frac{3}{4\times\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87	A1 (4)

EXPERT TUITION

Question Number	Scheme		Marks
199. (a)	C: $y^2 - 3y = x^3 + 8$		
	$\left\{\cancel{\frac{dy}{dx}} = \right\}$ $2y\frac{dy}{dx} - 3\frac{dy}{dx} = 3x^2$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $\pm 3\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.)	M1
		Correct equation.	A1
	$(2y-3)\frac{dy}{dx} = 3x^2$	A correct (condoning sign error) attempt to combine or factorise their '$2y\frac{dy}{dx} - 3\frac{dy}{dx}$'. Can be implied.	M1
	$\frac{dy}{dx} = \frac{3x^2}{2y-3}$	$\underline{\frac{3x^2}{2y-3}}$	A1 oe
			[4]
(b)	$y = 3 \Rightarrow 9 - 3(3) = x^3 + 8$	Substitutes $y = 3$ into C.	M1
	$x^3 = -8 \Rightarrow \underline{x = -2}$	Only $\underline{x = -2}$	A1
	$(-2, 3) \Rightarrow \frac{dy}{dx} = \frac{3(4)}{6-3} \Rightarrow \frac{dy}{dx} = 4$	$\frac{dy}{dx} = 4$ from correct working. Also can be ft using their 'x' value and $y = 3$ in the correct part (a) of $\frac{dy}{dx} = \frac{3x^2}{2y-3}$	A1 $\sqrt{\ }$
			[3]
			7 marks

199(b) final A1 $\sqrt{\ }$ **.** Note if the candidate inserts their x value and $y = 3$ into $\frac{dy}{dx} = \frac{3x^2}{2y-3}$, then an answer of $\frac{dy}{dx} =$ their x^2, ***may*** indicate a correct follow through.

Question Number	Scheme		Marks
200. (a)	Similar triangles $\Rightarrow \dfrac{r}{h}=\dfrac{16}{24} \Rightarrow r=\dfrac{2h}{3}$	Uses similar triangles, ratios or trigonometry to find either one of these two expressions oe.	M1
	$V=\dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi\left(\dfrac{2h}{3}\right)^2 h = \dfrac{4\pi h^3}{27}$ **AG**	Substitutes $r=\frac{2h}{3}$ into the formula for the volume of water V.	A1
			[2]
(b)	From the question, $\dfrac{\mathrm{d}V}{\mathrm{d}t}=8$	$\dfrac{\mathrm{d}V}{\mathrm{d}t}=8$	B1
	$\dfrac{\mathrm{d}V}{\mathrm{d}h}=\dfrac{12\pi h^2}{27}=\dfrac{4\pi h^2}{9}$	$\dfrac{\mathrm{d}V}{\mathrm{d}h}=\dfrac{12\pi h^2}{27}$ or $\dfrac{4\pi h^2}{9}$	B1
	$\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{\mathrm{d}V}{\mathrm{d}t}\div\dfrac{\mathrm{d}V}{dh} = 8\times\dfrac{9}{4\pi h^2} = \dfrac{18}{\pi h^2}$	Candidate's $\dfrac{\mathrm{d}V}{\mathrm{d}t}\div\dfrac{\mathrm{d}V}{\mathrm{d}h}$;	M1;
		$8\div\left(\dfrac{12\pi h^2}{27}\right)$ or $8\times\dfrac{9}{4\pi h^2}$ or $\dfrac{18}{\pi h^2}$ oe	A1
	When $h=12$, $\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{18}{144\pi}=\dfrac{1}{8\pi}$	$\dfrac{18}{144\pi}$ or $\dfrac{1}{8\pi}$	A1 oe isw
			[5]
			7 marks

Note the answer must be a one term exact value.

Note, also you can ignore subsequent working after $\dfrac{18}{144\pi}$.

Question 200 Alernative

Question Number	Scheme		Marks
200. (a)	Similar shapes $\Rightarrow$ either		
	$\underline{\dfrac{\frac{1}{3}\pi(16)^2 24}{V} = \left(\dfrac{24}{h}\right)^3}$ or $\underline{\dfrac{V}{\frac{1}{3}\pi(16)^2 24} = \left(\dfrac{h}{24}\right)^3}$ $\underline{\dfrac{\frac{1}{3}\pi r^2(24)}{V} = \left(\dfrac{24}{h}\right)^3}$ or $\underline{\dfrac{V}{\frac{1}{3}\pi r^2(24)} = \left(\dfrac{h}{24}\right)^3}$	Uses similar shapes to find either one of these two expressions oe.	M1
	$V = 2048\pi \times \left(\dfrac{h}{24}\right)^3 = \dfrac{4\pi h^3}{27}$ **AG**	Substitutes their equation to give the correct formula for the volume of water V.	A1
			[2]
200. (a)	Candidates simply writing:		
	$V = \dfrac{4}{9} \times \dfrac{1}{3}\pi h^3$ or $V = \dfrac{1}{3}\pi\left(\dfrac{16}{24}\right)^2 h^3$	would be awarded M0A0.	
(b)	From question, $\dfrac{dV}{dt} = 8 \Rightarrow V = 8t\ (+c)$	$\dfrac{dV}{dt} = 8$ or $V = 8t$	B1
	$h = \left(\dfrac{27V}{4\pi}\right)^{\frac{1}{3}} \Rightarrow h = \underline{\left(\dfrac{27(8t)}{4\pi}\right)^{\frac{1}{3}}} = \underline{\left(\dfrac{54t}{\pi}\right)^{\frac{1}{3}}} = \underline{3\left(\dfrac{2t}{\pi}\right)^{\frac{1}{3}}}$	$\underline{\left(\dfrac{27(8t)}{4\pi}\right)^{\frac{1}{3}}}$ or $\underline{\left(\dfrac{54t}{\pi}\right)^{\frac{1}{3}}}$ or $\underline{3\left(\dfrac{2t}{\pi}\right)^{\frac{1}{3}}}$	B1
	$\dfrac{dh}{dt} = 3\left(\dfrac{2}{\pi}\right)^{\frac{1}{3}}\dfrac{1}{3}t^{-\frac{2}{3}}$	$\dfrac{dh}{dt} = \pm k t^{-\frac{2}{3}}$; $\dfrac{dh}{dt} = 3\left(\dfrac{2}{\pi}\right)^{\frac{1}{3}}\dfrac{1}{3}t^{-\frac{2}{3}}$	M1; A1 oe
	When $h = 12$, $t = \left(\dfrac{12}{3}\right)^3 \times \dfrac{\pi}{2} = 32\pi$		
	So when $h = 12$, $\dfrac{dh}{dt} = \left(\dfrac{2}{\pi}\right)^{\frac{1}{3}}\left(\dfrac{1}{32\pi}\right)^{\frac{2}{3}} = \left(\dfrac{2}{1024\pi^3}\right)^{\frac{1}{3}} = \underline{\dfrac{1}{8\pi}}$	$\underline{\dfrac{1}{8\pi}}$	A1 oe
			[5]

EXPERT TUITION

Question Number	Scheme		Marks
201. (a)	At A, $x = -1 + 8 = 7$ & $y = (-1)^2 = 1 \Rightarrow A(7,1)$	$A(7,1)$	B1 **[1]**
(b)	$x = t^3 - 8t$, $y = t^2$,		
	$\frac{dx}{dt} = 3t^2 - 8$, $\frac{dy}{dt} = 2t$		
	$\therefore \frac{dy}{dx} = \frac{2t}{3t^2 - 8}$	Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ Correct $\frac{dy}{dx}$	M1 A1
	At A, $m(\mathbf{T}) = \underline{\frac{2(-1)}{3(-1)^2 - 8}} = \underline{\frac{-2}{3-8}} = \underline{\frac{-2}{-5}} = \underline{\frac{2}{5}}$	Substitutes for t to give any of the four underlined oe:	<u>A1</u>
	$\mathbf{T}$: $y - (\text{their } 1) = m_T(x - (\text{their } 7))$ or $1 = \frac{2}{5}(7) + c \Rightarrow c = 1 - \frac{14}{5} = -\frac{9}{5}$ Hence $\mathbf{T}$: $y = \frac{2}{5}x - \frac{9}{5}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$.	dM1
	gives $\mathbf{T}$: $\underline{2x - 5y - 9 = 0}$ **AG**	$\underline{2x - 5y - 9 = 0}$	A1 **cso** **[5]**
(c)	$2(t^3 - 8t) - 5t^2 - 9 = 0$	Substitution of both $x = t^3 - 8t$ and $y = t^2$ into $\mathbf{T}$	M1
	$2t^3 - 5t^2 - 16t - 9 = 0$		
	$(t+1)\{(2t^2 - 7t - 9) = 0\}$ $(t+1)\{(t+1)(2t-9) = 0\}$	A realisation that $(t+1)$ is a factor.	dM1
	$\{t = -1 \text{ (at } A)\}$ $t = \frac{9}{2}$ at B	$t = \frac{9}{2}$	A1
	$x = \left(\frac{9}{2}\right)^2 - 8\left(\frac{9}{2}\right) = \frac{729}{8} - 36 = \frac{441}{8} = 55.125$ or awrt 55.1 $y = \left(\frac{9}{2}\right)^2 = \frac{81}{4} = 20.25$ or awrt 20.3 Hence $B\left(\frac{441}{8}, \frac{81}{4}\right)$	Candidate uses their value of t to find either the x or y coordinate One of either x or y correct. Both x and y correct. awrt	ddM1 A1 A1 **[6]**
			12 marks

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark.
ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
Oe or equivalent.

Question 201 Alternative

Question Number	Scheme		Marks
201. (a)	It is acceptable for a candidate to write $x = 7,\ y = 1,$ to gain B1.	$A(7,1)$	B1
			[1]
***Aliter* (c) Way 2**	$x = t^3 - 8t = t(t^2 - 8) = t(y - 8)$		
	So, $x^2 = t^2(y-8)^2 = y(y-8)^2$		
	$2x - 5y - 9 = 0 \Rightarrow 2x = 5y + 9 \Rightarrow 4x^2 = (5y+9)^2$		
	Hence, $4y(y-8)^2 = (5y+9)^2$	Forming an equation in terms of y only.	M1
	$4y(y^2 - 16y + 64) = 25y^2 + 90y + 81$		
	$4y^3 - 64y^2 + 256y = 25y^2 + 90y + 81$		
	$4y^3 - 89y^2 + 166y - 81 = 0$		
	$(y-1)(y-1)(4y-81) = 0$	A realisation that $(y-1)$ is a factor.	dM1
		Correct factorisation	A1
	$y = \frac{81}{4} = 20.25$ (or awrt 20.3)	***Correct y-coordinate (see below!)***	
	$x^2 = \frac{81}{4}(\frac{81}{4} - 8)^2$	Candidate uses their y-coordinate to find their x-coordinate.	ddM1
		Decide to award A1 here for correct y-coordinate.	A1
	$x = \frac{441}{8} = 55.125$ (or awrt 55.1)	Correct x-coordinate	A1
	Hence $B\left(\frac{441}{8}, \frac{81}{4}\right)$		**[6]**

Question Number	Scheme		Marks
Aliter 201. (c) **Way 3**	$t=\sqrt{y}$		
	So $x=\left(\sqrt{y}\right)^3-8\left(\sqrt{y}\right)$		
	$2x-5y-9=0$ yields		
	$2\left(\sqrt{y}\right)^3-16\left(\sqrt{y}\right)-5y-9=0$	Forming an equation in terms of y only.	M1
	$\Rightarrow 2\left(\sqrt{y}\right)^3-5y-16\left(\sqrt{y}\right)-9=0$		
	$\left(\sqrt{y}+1\right)\left\{\left(2y-7\sqrt{y}-9\right)=0\right\}$	A realisation that $\left(\sqrt{y}+1\right)$ is a factor.	dM1
	$\left(\sqrt{y}+1\right)\left\{\left(\sqrt{y}+1\right)\left(2\sqrt{y}-9\right)=0\right\}$	Correct factorisation.	A1
	$y=\frac{81}{4}=20.25$ (or awrt 20.3)	***Correct y-coordinate (see below!)***	
	$x=\left(\sqrt{\frac{81}{4}}\right)^3-8\left(\sqrt{\frac{81}{4}}\right)$	Candidate uses their y-coordinate to find their x-coordinate.	ddM1
		Decide to award A1 here for correct y-coordinate.	A1
	$x=\frac{441}{8}=55.125$ (or awrt 55.1)	Correct x-coordinate	A1
	Hence $B\left(\frac{441}{8},\frac{81}{4}\right)$		**[6]**

Question Number	Scheme		Marks
202. (a)	From question, $\dfrac{dA}{dt} = 0.032$	$\dfrac{dA}{dt} = 0.032$ seen or implied from working.	B1
	$\left\{A = \pi x^2 \Rightarrow \dfrac{dA}{dx} = \right\} 2\pi x$	$2\pi x$ by itself seen or implied from working	B1
	$\dfrac{dx}{dt} = \dfrac{dA}{dt} \div \dfrac{dA}{dx} = (0.032)\dfrac{1}{2\pi x};\ \left\{= \dfrac{0.016}{\pi x}\right\}$	$0.032 \div \text{Candidate's } \dfrac{dA}{dx}$;	M1;
	When $x = 2\text{cm}$, $\dfrac{dx}{dt} = \dfrac{0.016}{2\pi}$		
	Hence, $\dfrac{dx}{dt} = 0.002546479...$ (cm s^{-1})	awrt 0.00255	A1 **cso**
			[4]
(b)	$V = \underline{\pi x^2(5x)} = \underline{5\pi x^3}$	$V = \underline{\pi x^2(5x)}$ or $\underline{5\pi x^3}$	B1
	$\dfrac{dV}{dx} = 15\pi x^2$	$\dfrac{dV}{dx} = 15\pi x^2$ or ft from candidate's V **in one variable**	B1 $\sqrt{}$
	$\dfrac{dV}{dt} = \dfrac{dV}{dx} \times \dfrac{dx}{dt} = 15\pi x^2.\left(\dfrac{0.016}{\pi x}\right);\ \{= 0.24x\}$	Candidate's $\dfrac{dV}{dx} \times \dfrac{dx}{dt}$;	M1 $\sqrt{}$
	When $x = 2\text{cm}$, $\dfrac{dV}{dt} = 0.24(2) = \underline{0.48}$ (cm^3s^{-1})	$\underline{0.48}$ or $\underline{\text{awrt } 0.48}$	A1 **cso**
			[4]
			8 marks

Question Number	Scheme		Marks
203. (a)	$3x^2 - y^2 + xy = 4 \quad (\text{ eqn } *\ )$		
	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \cancel{=}\right\}\quad \underline{6x - 2y\frac{dy}{dx}} + \left(\underline{\underline{y + x\frac{dy}{dx}}}\right) = \underline{0}$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $x\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$)	M1
		Correct application $\underline{\underline{(\ )}}$ of product rule	B1
		$(3x^2 - y^2) \to \left(\underline{6x - 2y\frac{dy}{dx}}\right)$ and $(4 \to \underline{0})$	<u>A1</u>
	$\left\{\frac{dy}{dx} = \frac{-6x - y}{x - 2y}\right\}$ or $\left\{\frac{dy}{dx} = \frac{6x + y}{2y - x}\right\}$	*not necessarily required.*	
	$\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x - y}{x - 2y} = \frac{8}{3}$	Substituting $\frac{dy}{dx} = \frac{8}{3}$ into their equation.	M1$*$
	giving $-18x - 3y = 8x - 16y$		
	giving $13y = 26x$	Attempt to combine either terms in *x* or terms in *y* together to give either ax or by.	dM1$*$
	Hence, $y = 2x \Rightarrow \underline{y - 2x = 0}$	simplifying to give $\underline{y - 2x = 0}$ **AG**	A1 **cso**
			[6]
(b)	At *P* & *Q*, $y = 2x$. Substituting into eqn $*$		
	gives $3x^2 - (2x)^2 + x(2x) = 4$	Attempt replacing y by $2x$ **in at least one** of the *y* terms in eqn$*$	M1
	Simplifying gives, $x^2 = 4 \Rightarrow \underline{x = \pm 2}$	Either $x = 2$ or $x = -2$	<u>A1</u>
	$y = 2x \Rightarrow y = \pm 4$		
	Hence coordinates are $\underline{(2,4)}$ and $\underline{(-2,-4)}$	Both $\underline{(2,4)}$ and $\underline{(-2,-4)}$	<u>A1</u>
			[3]
			9 marks

Question Number	Scheme		Marks
204. (a)	At $P(4, 2\sqrt{3})$ either $\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	$\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	M1
	$\Rightarrow$ only solution is $\underline{t = \frac{\pi}{3}}$ where $0 \leqslant t \leqslant \frac{\pi}{2}$	$\underline{t = \frac{\pi}{3}}$ or $\underline{\text{awrt } 1.05}$ (radians) only stated in the range $0 \leqslant t \leqslant \frac{\pi}{2}$	A1
			[2]
(b)	$x = 8\cos t$, $y = 4\sin 2t$		
	$\frac{dx}{dt} = -8\sin t$, $\frac{dy}{dt} = 8\cos 2t$	Attempt to differentiate both x and y wrt t to give $\pm p\sin t$ and $\pm q\cos 2t$ respectively	M1
		Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	At P, $\frac{dy}{dx} = \underline{\frac{8\cos\left(\frac{2\pi}{3}\right)}{-8\sin\left(\frac{\pi}{3}\right)}}$	Divides in correct way round and attempts to substitute their value of t (in degrees or radians) into their $\frac{dy}{dx}$ expression.	M1*
	$\left\{= \frac{8\left(-\frac{1}{2}\right)}{(-8)\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58\right\}$	You may need to check candidate's substitutions for M1* **Note the next two method marks are dependent on M1***	
	Hence $m(\mathbf{N}) = -\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	Uses $m(\mathbf{N}) = -\frac{1}{\text{their } m(\mathbf{T})}$.	dM1*
	N: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$	Uses $y - 2\sqrt{3} = (\text{their } m_N)(x - 4)$ or finds c using $x = 4$ and $y = 2\sqrt{3}$ and uses $y = (\text{their } m_N)x + "c"$.	dM1*
	N: $\underline{y = -\sqrt{3}x + 6\sqrt{3}}$ **AG**	$\underline{y = -\sqrt{3}x + 6\sqrt{3}}$	A1 **cso AG**
	or $2\sqrt{3} = -\sqrt{3}(4) + c \Rightarrow c = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$ so **N**: $\left[\underline{y = -\sqrt{3}x + 6\sqrt{3}}\right]$		
			[6]

Question Number	Scheme		Marks
205. (a)	$x^3 - 4y^2 = 12xy \quad (\text{ eqn } *)$		
	$x = -8 \Rightarrow \; -512 - 4y^2 = 12(-8)y$ $-512 - 4y^2 = -96y$	Substitutes $x = -8$ (at least once) into * to obtain a three term quadratic in y. Condone the loss of $= 0$.	M1
	$4y^2 - 96y + 512 = 0$ $y^2 - 24y + 128 = 0$		
	$(y-16)(y-8) = 0$ $y = \dfrac{24 \pm \sqrt{576 - 4(128)}}{2}$	An attempt to solve the quadratic in y by either factorising or by the formula or by ***completing the square***.	dM1
	$y = 16$ or $y = 8$.	Both $\underline{y = 16}$ and $\underline{y = 8}$. or $\underline{(-8, 8)}$ and $\underline{(-8, 16)}$.	A1
			[3]
(b)	$\left\{ \dfrac{\cancel{dy}}{\cancel{dx}} \cancel{\times} \right\} \quad 3x^2 - 8y\dfrac{dy}{dx}; = \left(\underline{12y + 12x\dfrac{dy}{dx}} \right)$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $12x\frac{dy}{dx}$. Ignore $\frac{dy}{dx} = \ldots$ Correct LHS equation; <u>Correct application of product rule</u>	M1 A1; (B1)
	$\left\{ \dfrac{dy}{dx} = \dfrac{3x^2 - 12y}{12x + 8y} \right\}$	*not necessarily required.*	
	$@\,(-8, 8), \quad \dfrac{dy}{dx} = \dfrac{3(64) - 12(8)}{12(-8) + 8(8)} = \dfrac{96}{-32} = \underline{-3},$ $@\,(-8, 16), \quad \dfrac{dy}{dx} = \dfrac{3(64) - 12(16)}{12(-8) + 8(16)} = \dfrac{0}{32} = \underline{0}.$	Substitutes $x = -8$ and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$. One gradient found. Both gradients of $\underline{-3}$ and $\underline{0}$ ***correctly*** found.	dM1 A1 A1 **cso**
			[6]
			9 marks

Question Number	Scheme		Marks
***Aliter* 205. (b) Way 2**	$\left\{\frac{\cancel{d}x}{\cancel{d}y}\cancel{=}\right\}\quad 3x^2\frac{dx}{dy} - 8y;\ = \left(\underline{12y\frac{dx}{dy} + 12x}\right)$	Differentiates implicitly to include either $\pm kx^2\frac{dx}{dy}$ or $12y\frac{dx}{dy}$. Ignore $\frac{dx}{dy} = \ldots$	M1
		Correct LHS equation	A1;
		<u>Correct application of product rule</u>	(B1)
	$\left\{\frac{dy}{dx} = \frac{3x^2 - 12y}{12x + 8y}\right\}$	*not necessarily required.*	
	@ $(-8, 8)$, $\quad \frac{dy}{dx} = \frac{3(64) - 12(8)}{12(-8) + 8(8)} = \frac{96}{-32} = \underline{-3}$,	Substitutes $x = -8$ and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$ or $\frac{dx}{dy}$.	dM1
	@ $(-8, 16)$, $\quad \frac{dy}{dx} = \frac{3(64) - 12(16)}{12(-8) + 8(16)} = \frac{0}{32} = \underline{0}$.	One gradient found.	A1
		Both gradients of <u>-3</u> and <u>0</u> ***correctly*** found.	A1 **cso**
			[6]

Question Number	Scheme		Marks
Aliter **205.** (b) **Way 3**	$x^3 - 4y^2 = 12xy$ (eqn $*$)		
	$4y^2 + 12xy - x^3 = 0$		
	$y = \dfrac{-12x \pm \sqrt{144x^2 - 4(4)(-x^3)}}{8}$		
	$y = \dfrac{-12x \pm \sqrt{144x^2 + 16x^3}}{8}$		
	$y = \dfrac{-12x \pm 4\sqrt{9x^2 + x^3}}{8}$		
	$y = -\frac{3}{2}x \pm \frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$		
	$\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$	A credible attempt to make y the subject and an attempt to differentiate either $-\frac{3}{2}x$ or $\frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$.	M1
	$\dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18x + 3x^2}{4(9x^2 + x^3)^{\frac{1}{2}}}$	$\dfrac{dy}{dx} = -\frac{3}{2} \pm k\left(9x^2 + x^3\right)^{-\frac{1}{2}}\left(g(x)\right)$	A1
		$\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$	A1
	@ $x = -8$ $\quad \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18(-8) + 3(64)}{4(9(64) + (-512))^{\frac{1}{2}}}$	Substitutes $x = -8$ find any one of $\frac{dy}{dx}$.	dM1
	$= -\dfrac{3}{2} \pm \dfrac{48}{4\sqrt{(64)}} = -\dfrac{3}{2} \pm \dfrac{48}{32}$		
	$\therefore \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{3}{2} = \underline{-3}, \underline{0}.$	One gradient correctly found. Both gradients of $\underline{-3}$ and $\underline{0}$ correctly found.	A1 A1
			[6]

Question Number	Scheme		Marks
206. (a)	$\dfrac{dV}{dt} = 1600 - c\sqrt{h}$ or $\dfrac{dV}{dt} = 1600 - k\sqrt{h}$,	Either of these statements	M1
	$\left(V = 4000h \;\Rightarrow\right) \dfrac{dV}{dh} = 4000$	$\dfrac{dV}{dh} = 4000$ or $\dfrac{dh}{dV} = \dfrac{1}{4000}$	M1
	$\dfrac{dh}{dt} = \dfrac{dh}{dV} \times \dfrac{dV}{dt} = \dfrac{\frac{dV}{dt}}{\frac{dV}{dh}}$		
	Either, $\dfrac{dh}{dt} = \dfrac{1600 - c\sqrt{h}}{4000} = \dfrac{1600}{4000} - \dfrac{c\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$ or $\dfrac{dh}{dt} = \dfrac{1600 - k\sqrt{h}}{4000} = \dfrac{1600}{4000} - \dfrac{k\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$	Convincing proof of $\dfrac{dh}{dt}$	A1 **AG**
			[3]
(b)	When $h = 25$ water ***leaks out such that*** $\dfrac{dV}{dt} = 400$		
	$400 = c\sqrt{h} \Rightarrow 400 = c\sqrt{25} \Rightarrow 400 = c(5) \Rightarrow c = 80$		
	From above; $k = \dfrac{c}{4000} = \dfrac{80}{4000} = 0.02$ as required	Proof that $k = 0.02$	B1 **AG**
			[1]
Aliter (b) **Way 2**	$400 = 4000k\sqrt{h}$		
	$\Rightarrow 400 = 4000k\sqrt{25}$		
	$\Rightarrow 400 = k(20000) \Rightarrow k = \frac{400}{20000} = 0.02$	Using 400, 4000 and $h = 25$ or $\sqrt{h} = 5$. Proof that $k = 0.02$	B1 **AG**
			[1]