EXPERT TUITION

Maths Questions By Topic:

Differentiation

A-Level Edexcel

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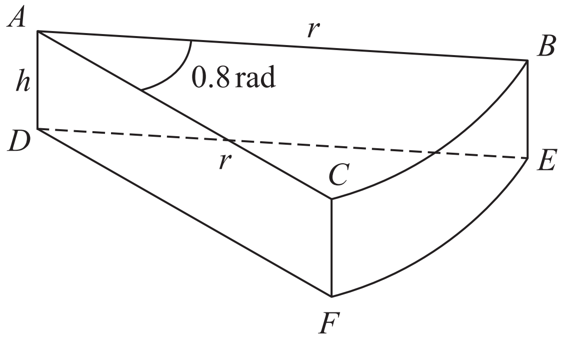

Figure 5

A company makes toys for children.

Figure 5 shows the design for a solid toy that looks like a piece of cheese.

The toy is modelled so that

- face *ABC* is a sector of a circle with radius r cm and centre *A*
- angle *BAC* = 0.8 radians
- faces *ABC* and *DEF* are congruent
- edges *AD*, *CF* and *BE* are perpendicular to faces *ABC* and *DEF*
- edges *AD*, *CF* and *BE* have length h cm

Given that the volume of the toy is $240\,\text{cm}^3$

(a) show that the surface area of the toy, $S\,\text{cm}^2$, is given by

$$S = 0.8r^2 + \frac{1680}{r}$$

making your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which S has a stationary point.

(4)

(c) Prove, by further differentiation, that this value of r gives the minimum surface area of the toy.

(2)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 10 marks)

2. A company makes drinks containers out of metal.

The containers are modelled as closed cylinders with base radius r cm and height h cm and the capacity of each container is $355\,\text{cm}^3$

The metal used

- for the circular base and the curved side costs 0.04 pence/cm^2
- for the circular top costs 0.09 pence/cm^2

Both metals used are of negligible thickness.

(a) Show that the total cost, C pence, of the metal for one container is given by

$$C = 0.13\pi r^2 + \frac{28.4}{r}$$

(4)

(b) Use calculus to find the value of r for which C is a minimum, giving your answer to 3 significant figures.

(4)

(c) Using $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$ prove that the cost is minimised for the value of r found in part (b).

(2)

(d) Hence find the minimum value of C, giving your answer to the nearest integer.

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 12 marks)

3.

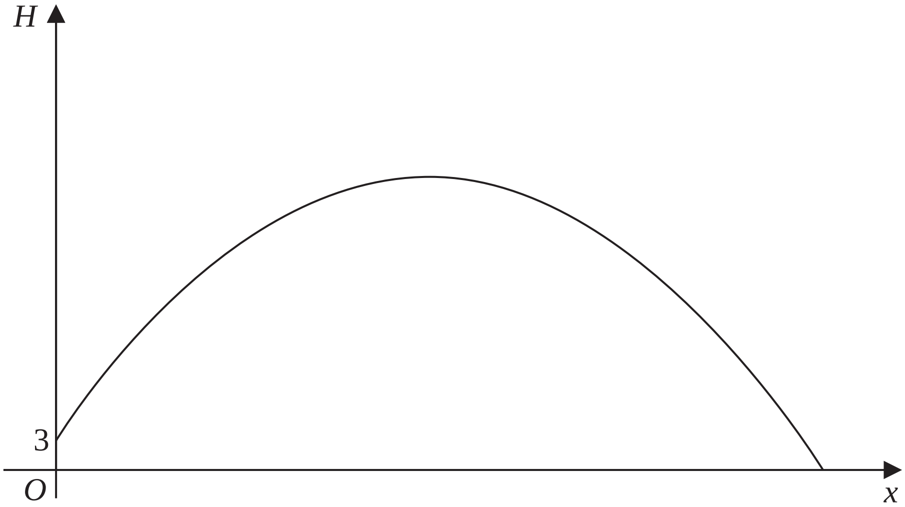

Figure 3

Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, H metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

- is hit from a point on the top of a platform of vertical height 3 m above the ground
- reaches its maximum vertical height after travelling a horizontal distance of 90 m
- is at a vertical height of 27 m above the ground after travelling a horizontal distance of 120 m

Given also that H is modelled as a **quadratic** function in x

(a) find H in terms of x

(5)

(b) Hence find, according to the model,

(i) the maximum vertical height of the ball above the ground,

(ii) the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre.

(3)

(c) The possible effects of wind or air resistance are two limitations of the model. Give one other limitation of this model.

(1)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4. Given that

$$y = \frac{x - 4}{2 + \sqrt{x}} \qquad x > 0$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{A\sqrt{x}} \qquad x > 0$$

where A is a constant to be found.

(4)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 4 marks)

5. A new smartphone was released by a company.

The company monitored the total number of phones sold, n, at time t days after the phone was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

Use this information to write down a suitable equation for n in terms of t.

(*You do not need to evaluate any unknown constants in your equation*.)

(2)

Question 5 continued

(Total for Question 5 is 2 marks)

6. A large spherical balloon is deflating.

At time t seconds the balloon has radius r cm and volume V cm^3

The volume of the balloon is modelled as decreasing at a constant rate.

(a) Using this model, show that

$$\frac{dr}{dt} = -\frac{k}{r^2}$$

where k is a positive constant.

(3)

Given that

- the initial radius of the balloon is 40 cm
- after 5 seconds the radius of the balloon is 20 cm
- the volume of the balloon continues to decrease at a constant rate until the balloon is empty

(b) solve the differential equation to find a complete equation linking r and t.

(5)

(c) Find the limitation on the values of t for which the equation in part (b) is valid.

(2)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 10 marks)

7. The curve C has equation

$$x^2 \tan y = 9 \qquad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$$

(4)

(b) Prove that C has a point of inflection at $x = \sqrt[4]{27}$

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 7 marks)

8. The temperature, θ°C, of a cup of tea t minutes after it was placed on a table in a room, is modelled by the equation

$$\theta = 18 + 65e^{-\frac{t}{8}} \qquad t \geqslant 0$$

Find, according to the model,

(a) the temperature of the cup of tea when it was placed on the table,

(1)

(b) the value of t, to one decimal place, when the temperature of the cup of tea was 35 °C.

(3)

(c) Explain why, according to this model, the temperature of the cup of tea could not fall to 15 °C.

(1)

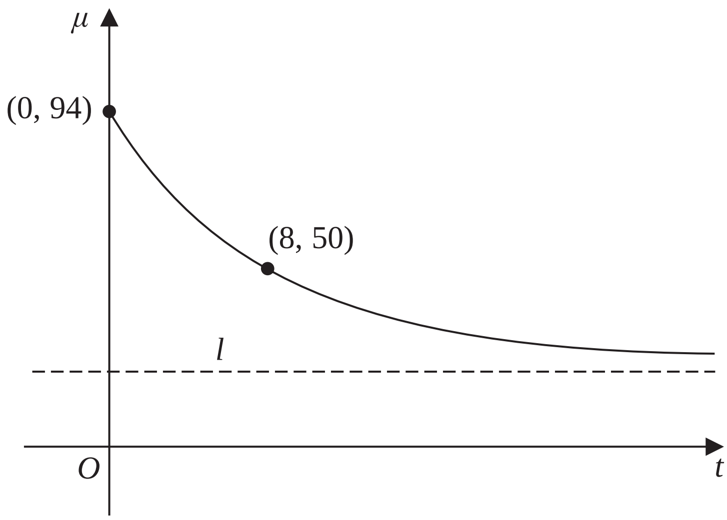

Figure 2

The temperature, μ°C, of a second cup of tea t minutes after it was placed on a table in a different room, is modelled by the equation

$$\mu = A + Be^{-\frac{t}{8}} \qquad t \geqslant 0$$

where A and B are constants.

Figure 2 shows a sketch of μ against t with two data points that lie on the curve.

The line l, also shown on Figure 2, is the asymptote to the curve.

Using the equation of this model and the information given in Figure 2

(d) find an equation for the asymptote l.

(4)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 9 marks)

9. $$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{A}{(x+1)^n}$ where A and n are constants to be found.

(4)

(b) Hence deduce the range of values for x for which $\dfrac{dy}{dx} < 0$

(1)

Question 9 continued

(Total for Question 9 is 5 marks)

10. $$f(x) = 10e^{-0.25x}\sin x, \qquad x \geqslant 0$$

(a) Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

(4)

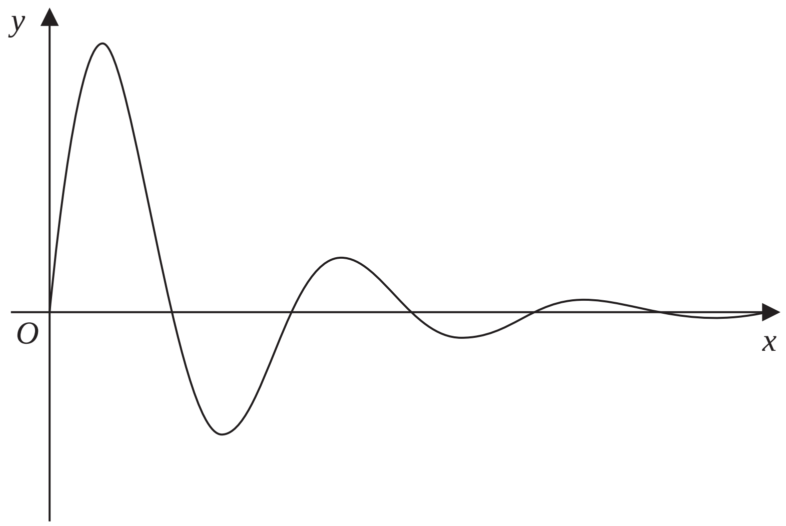

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$.

(b) Sketch the graph of H against t where

$$H(t) = \left|10e^{-0.25t}\sin t\right| \qquad t \geqslant 0$$

showing the long-term behaviour of this curve.

(2)

The function $H(t)$ is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

(c) the maximum height of the ball above the ground between the first and second bounce.

(3)

(d) Explain why this model should not be used to predict the time of each bounce.

(1)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 10 marks)

11. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

Question 11 continued

Question 11 continued

(Total for Question 11 is 7 marks)

12. A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

(b) Hence find the exact range of values of x for which the curve is increasing.

(2)

Question 12 continued

(Total for Question 12 is 5 marks)

13. A company started mining tin in Riverdale on 1st January 2019.

A model to find the total mass of tin that will be mined by the company in Riverdale is given by the equation

$$T = 1200 - 3(n - 20)^2$$

where T tonnes is the total mass of tin mined in the n years after the start of mining.

Using this model,

(a) calculate the mass of tin that will be mined up to 1st January 2020,

(1)

(b) deduce the maximum total mass of tin that could be mined,

(1)

(c) calculate the mass of tin that will be mined in 2023.

(2)

(d) State, giving reasons, the limitation on the values of n.

(2)

Question 13 continued

Question 13 continued

Question 13 continued

(Total for Question 13 is 6 marks)

14. A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \qquad x > 0$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 4$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

Question 14 continued

(Total for Question 14 is 7 marks)

15. Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}\theta} = \frac{A}{1 + \sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where A is a rational constant to be found.

(5)

Question 15 continued

(Total for Question 15 is 5 marks)

16. A lorry is driven between London and Newcastle.

In a simple model, the cost of the journey £C when the lorry is driven at a steady speed of v kilometres per hour is

$$C = \frac{1500}{v} + \frac{2v}{11} + 60$$

(a) Find, according to this model,

(i) the value of v that minimises the cost of the journey,

(ii) the minimum cost of the journey.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(6)

(b) Prove by using $\frac{d^2C}{dv^2}$ that the cost is minimised at the speed found in (a)(i).

(2)

(c) State one limitation of this model.

(1)

Question 16 continued

Question 16 continued

Question 16 continued

(Total for Question 16 is 9 marks)

17. Prove, from first principles, that the derivative of x^3 is $3x^2$

(4)

Question 17 continued

(Total for Question 17 is 4 marks)

18. A curve has equation

$$y = 2x^3 - 2x^2 - 2x + 8$$

(a) Find $\frac{dy}{dx}$

(2)

(b) Hence find the range of values of x for which y is increasing.
Write your answer in set notation.

(4)

Question 18 continued

(Total for Question 18 is 6 marks)

19. The growth of pond weed on the surface of a pond is being investigated.

The surface area of the pond covered by the weed, $A\,\text{m}^2$, can be modelled by the equation

$$A = 0.2\text{e}^{0.3t}$$

where t is the number of days after the start of the investigation.

(a) State the surface area of the pond covered by the weed at the start of the investigation.
(1)

(b) Find the rate of increase of the surface area of the pond covered by the weed, in m^2/day, exactly 5 days after the start of the investigation.
(2)

Given that the pond has a surface area of $100\,\text{m}^2$,

(c) find, to the nearest hour, the time taken, according to the model, for the surface of the pond to be fully covered by the weed.
(4)

The pond is observed for one month and by the end of the month 90% of the surface area of the pond was covered by the weed.

(d) Evaluate the model in light of this information, giving a reason for your answer.
(1)

Question 19 continued

(Total for Question 19 is 8 marks)

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20. The curve C has equation

$$y = 2x^3 + 5$$

The curve C passes through the point $P(1, 7)$.

Use differentiation from first principles to find the value of the gradient of the tangent to C at P.

(5)

Question 20 continued

(Total for Question 20 is 5 marks)

21.

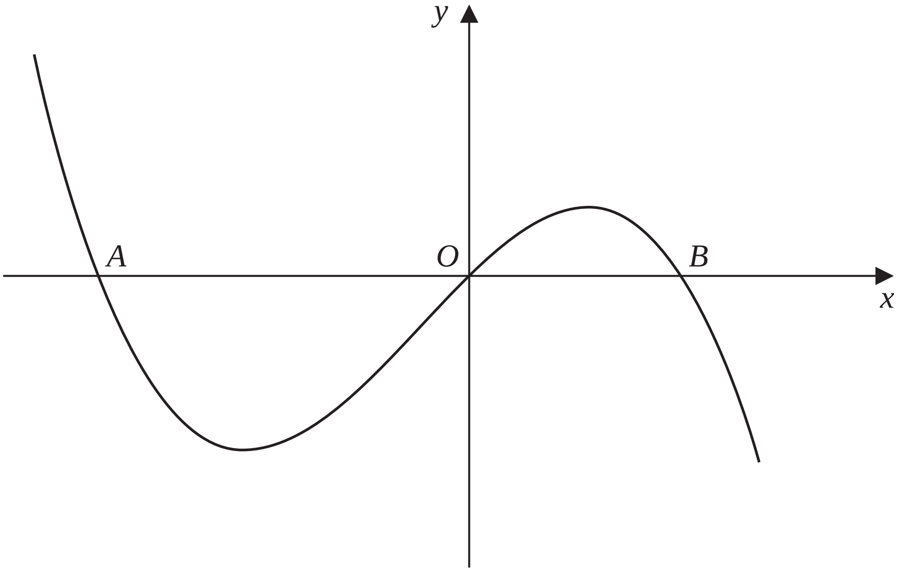

Figure 5

Figure 5 shows a sketch of the curve C with equation $y = f(x)$.
The curve C crosses the x-axis at the origin, O, and at the points A and B as shown in Figure 5.

Given that

$$f'(x) = k - 4x - 3x^2$$

where k is constant,

(a) show that C has a point of inflection at $x = -\frac{2}{3}$

(3)

Given also that the distance $AB = 4\sqrt{2}$

(b) find, showing your working, the integer value of k.

(7)

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Question 21 continued

(Total for Question 21 is 10 marks)

22. The curve C has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point P (5, 6).

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total for Question 22 is 4 marks)

23. Prove, from first principles, that the derivative of $3x^2$ is $6x$.

(4)

(Total for Question 23 is 4 marks)

24. The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

Question 24 continued

(Total for Question 24 is 7 marks)

25. Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that as $h \to 0$, $\dfrac{\sin h}{h} \to 1$ and $\dfrac{\cos h - 1}{h} \to 0$

(5)

(Total for Question 25 is 5 marks)

26.

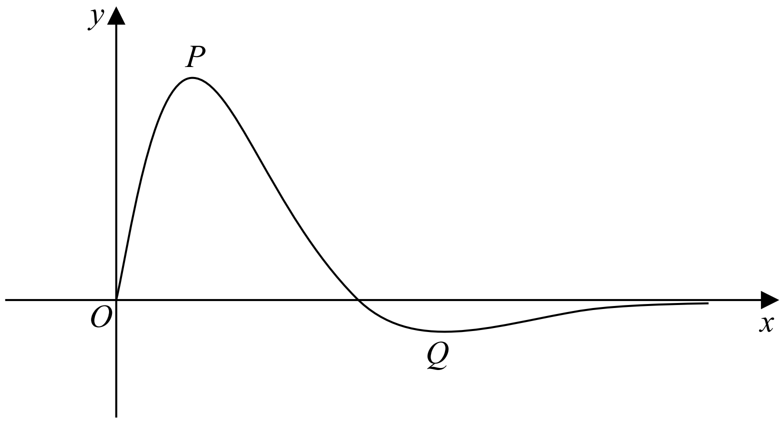


Figure 5

Figure 5 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = \frac{4\sin 2x}{e^{\sqrt{2}x-1}}, \quad 0 \leqslant x \leqslant \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q as shown in Figure 5.

(a) Show that the x coordinates of point P and point Q are solutions of the equation

$$\tan 2x = \sqrt{2}$$

(4)

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = f(2x)$.

(ii) $y = 3 - 2f(x)$.

(4)

Question 26 continued

Question 26 continued

(Total for Question 26 is 8 marks)

27. Given that

$$y = 2x^2$$

use differentiation from first principles to show that

$$\frac{dy}{dx} = 4x$$

(3)

Question 27 continued

(Total for Question 27 is 3 marks)

28.

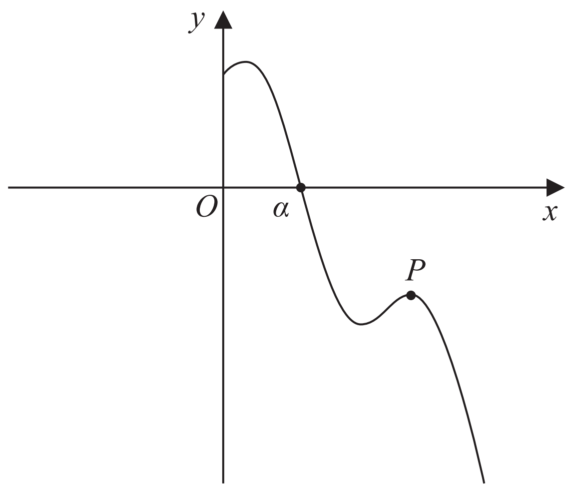


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = 8\sin\left(\frac{1}{2}x\right) - 3x + 9 \qquad x > 0$$

and x is measured in radians.

The point P, shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

(a) find the x coordinate of P, giving your answer to 3 significant figures.

(4)

The curve crosses the x-axis at $x = \alpha$, as shown in Figure 2.

Given that, to 3 decimal places, $f(4) = 4.274$ and $f(5) = -1.212$

(b) explain why α must lie in the interval $[4, 5]$

(1)

(c) Taking $x_0 = 5$ as a first approximation to α, apply the Newton-Raphson method once to $f(x)$ to obtain a second approximation to α.

Show your method and give your answer to 3 significant figures.

(2)

Question 28 continued

Question 28 continued

Question 28 continued

(Total for Question 28 is 7 marks)

29. The function f is defined by

$$f(x) = \frac{e^{3x}}{4x^2 + k}$$

where k is a positive constant.

(a) Show that

$$f'(x) = (12x^2 - 8x + 3k)\,g(x)$$

where $g(x)$ is a function to be found.

(3)

Given that the curve with equation $y = f(x)$ has at least one stationary point,

(b) find the range of possible values of k.

(3)

Question 29 continued

(Total for Question 29 is 6 marks)

30. The curve C has equation

$$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \qquad x \in \mathbb{R}$$

(a) Find

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) (i) Verify that C has a stationary point at $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

(4)

Question 30 continued

(Total for Question 30 is 7 marks)

31. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

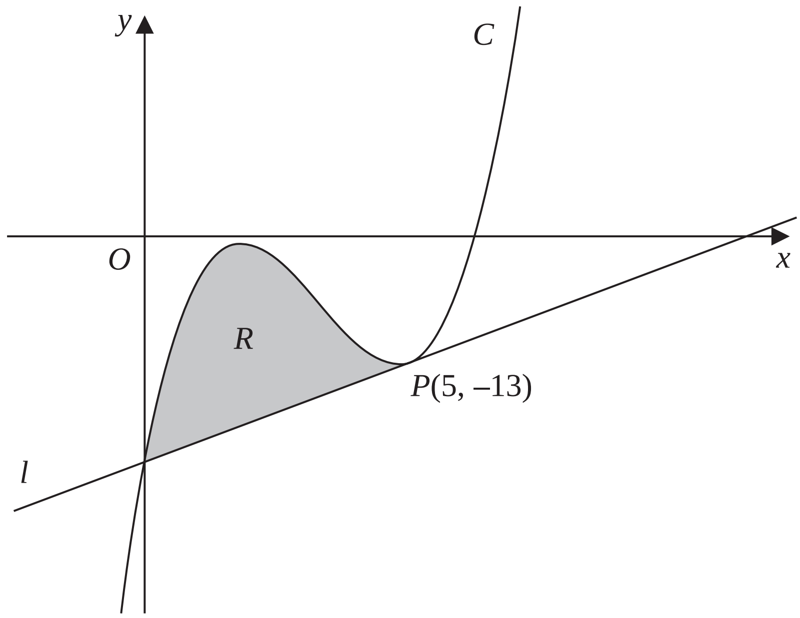


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on C

The line l is the tangent to C at P

(a) Use differentiation to find the equation of l, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(4)

(b) Hence verify that l meets C again on the y-axis.

(1)

The finite region R, shown shaded in Figure 2, is bounded by the curve C and the line l.

(c) Use algebraic integration to find the exact area of R.

(4)

Question 31 continued

Question 31 continued

Question 31 continued

(Total for Question 31 is 9 marks)

32. The curve C has equation

$$px^3 + qxy + 3y^2 = 26$$

where p and q are constants.

(a) Show that

$$\frac{dy}{dx} = \frac{apx^2 + bqy}{qx + cy}$$

where a, b and c are integers to be found.

(4)

Given that

- the point $P(-1, -4)$ lies on C
- the normal to C at P has equation $19x + 26y + 123 = 0$

(b) find the value of p and the value of q.

(5)

Question 32 continued

Question 32 continued

Question 32 continued

(Total for Question 32 is 9 marks)

33. The curve C has parametric equations

$$x = \sin 2\theta \qquad y = \operatorname{cosec}^3\theta \qquad 0 < \theta < \frac{\pi}{2}$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of θ

(3)

(b) Hence find the exact value of the gradient of the tangent to C at the point where $y = 8$

(3)

Question 33 continued

(Total for Question 33 is 6 marks)

34.

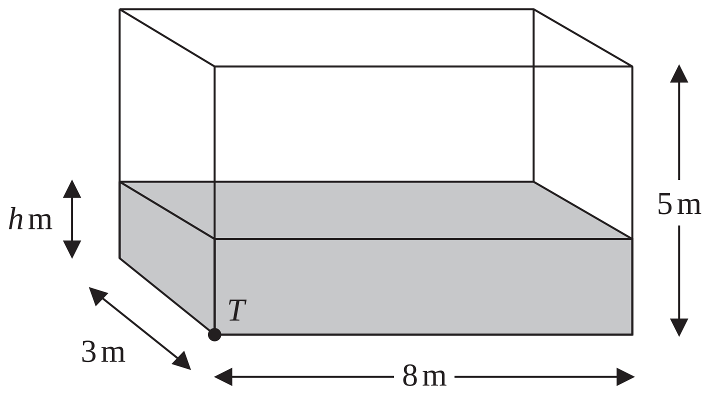


Figure 5

Water flows at a constant rate into a large tank.

The tank is a cuboid, with all sides of negligible thickness.

The base of the tank measures 8 m by 3 m and the height of the tank is 5 m.

There is a tap at a point T at the bottom of the tank, as shown in Figure 5.

At time t minutes after the tap has been opened

- the depth of water in the tank is h metres
- water is flowing into the tank at a constant rate of $0.48\,\text{m}^3$ per minute
- water is modelled as leaving the tank through the tap at a rate of $0.1h\,\text{m}^3$ per minute

(a) Show that, according to the model,

$$1200\frac{\mathrm{d}h}{\mathrm{d}t} = 24 - 5h$$

(4)

Given that when the tap was opened, the depth of water in the tank was 2 m,

(b) show that, according to the model,

$$h = A + B\mathrm{e}^{-kt}$$

where A, B and k are constants to be found.

(6)

Given that the tap remains open,

(c) determine, according to the model, whether the tank will ever become full, giving a reason for your answer.

(2)

Question 34 continued

Question 34 continued

Question 34 continued

(Total for Question 34 is 12 marks)

35.

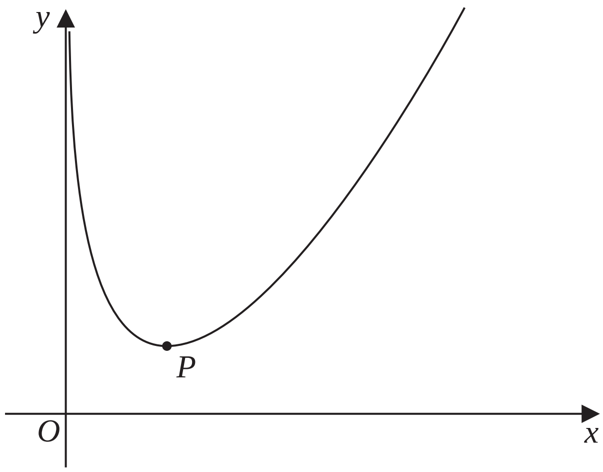


Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \qquad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

(4)

The point P, shown in Figure 1, is the minimum turning point on C.

(b) Show that the x coordinate of P is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$$

(3)

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12}\right)^{\frac{2}{3}} \qquad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,

(ii) the x coordinate of P to 5 decimal places.

(3)

Question 35 continued

Question 35 continued

Question 35 continued

(Total for Question 35 is 10 marks)

36. The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \qquad x > 0 \qquad x \neq k$$

where k is a constant.

(a) Deduce the value of k.

(1)

(b) Prove that

$$g'(x) > 0$$

for all values of x in the domain of g.

(3)

(c) Find the range of values of a for which

$$g(a) > 0$$

(2)

Question 36 continued

Question 36 continued

Question 36 continued

(Total for Question 36 is 6 marks)

37.

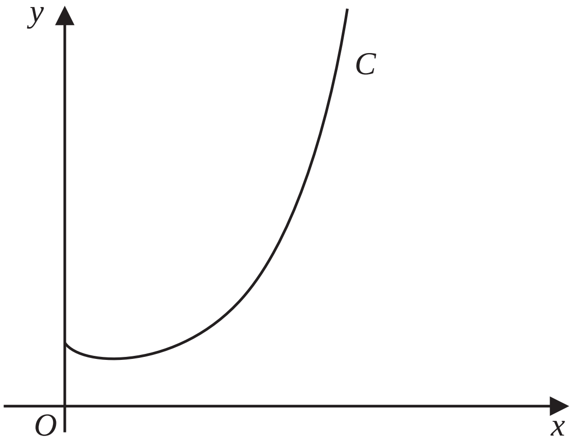


Figure 8

Figure 8 shows a sketch of the curve C with equation $y = x^x$, $x > 0$

(a) Find, by firstly taking logarithms, the x coordinate of the turning point of C.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

The point $P(\alpha, 2)$ lies on C.

(b) Show that $1.5 < \alpha < 1.6$

(2)

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{1-x_n}$$

Using this formula with $x_1 = 1.5$

(c) find x_4 to 3 decimal places,

(2)

(d) describe the long-term behaviour of x_n

(2)

Question 37 continued

Question 37 continued

Question 37 continued

(Total for Question 37 is 11 marks)

38.

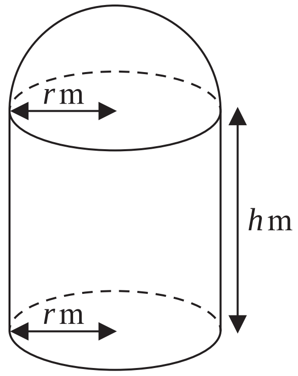


Figure 9

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is $6\,\text{m}^3$.

(a) Show that, according to the model, the surface area of the tank, in m^2, is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

(4)

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

(4)

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer.

(2)

Question 38 continued

Question 38 continued

Question 38 continued

(Total for Question 38 is 10 marks)

39. Given that θ is measured in radians, prove, from first principles, that

$$\frac{d}{d\theta}(\cos\theta) = -\sin\theta$$

You may assume the formula for $\cos(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

Question 39 continued

(Total for Question 39 is 5 marks)

40. A spherical mint of radius 5 mm is placed in the mouth and sucked.
Four minutes later, the radius of the mint is 3 mm.

In a simple model, the rate of decrease of the radius of the mint is inversely proportional to the square of the radius.

Using this model and all the information given,

(a) find an equation linking the radius of the mint and the time.
(You should define the variables that you use.)

(5)

(b) Hence find the total time taken for the mint to completely dissolve. Give your answer in minutes and seconds to the nearest second.

(2)

(c) Suggest a limitation of the model.

(1)

Question 40 continued

Question 40 continued

Question 40 continued

(Total for Question 40 is 8 marks)

41.

$$\frac{1 + 11x - 6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}$$

(a) Find the values of the constants A, B and C.

(4)

$$\mathrm{f}(x) = \frac{1 + 11x - 6x^2}{(x-3)(1-2x)} \qquad x > 3$$

(b) Prove that $\mathrm{f}(x)$ is a decreasing function.

(3)

Question 41 continued

Question 41 continued

Question 41 continued

(Total for Question 41 is 7 marks)

42. A scientist is studying a population of mice on an island.

The number of mice, N, in the population, t months after the start of the study, is modelled by the equation

$$N = \frac{900}{3 + 7e^{-0.25t}}, \quad t \in \mathbb{R}, \quad t \geqslant 0$$

(a) Find the number of mice in the population at the start of the study.

(1)

(b) Show that the rate of growth $\frac{dN}{dt}$ is given by $\frac{dN}{dt} = \frac{N(300 - N)}{1200}$

(4)

The rate of growth is a maximum after T months.

(c) Find, according to the model, the value of T.

(4)

According to the model, the maximum number of mice on the island is P.

(d) State the value of P.

(1)

Question 42 continued

Question 42 continued

(Total for Question 42 is 10 marks)

43.

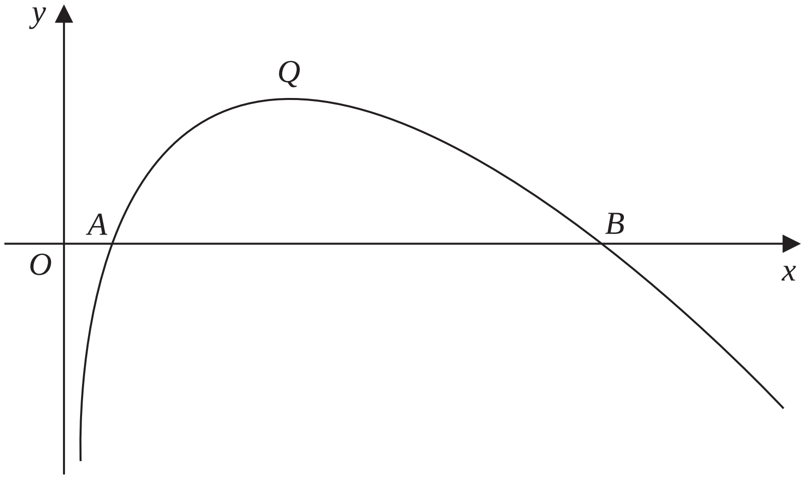


Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = (8 - x)\ln x, \qquad x > 0$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 2.

(a) Find the x coordinate of A and the x coordinate of B.

(1)

(b) Show that the x coordinate of Q satisfies

$$x = \frac{8}{1 + \ln x}$$

(4)

Question 43 continued

(Total for Question 43 is 5 marks)

44.

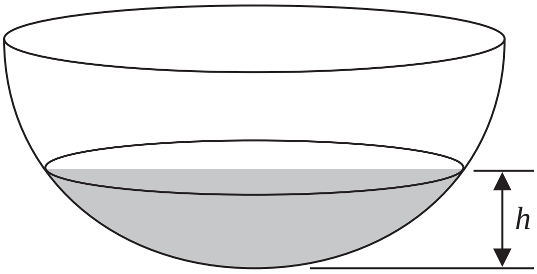


Figure 3

A bowl is modelled as a hemispherical shell as shown in Figure 3.

Initially the bowl is empty and water begins to flow into the bowl.

When the depth of the water is h cm, the volume of water, V cm^3, according to the model is given by

$$V = \frac{1}{3}\pi h^2(75 - h), \qquad 0 \leqslant h \leqslant 24$$

The flow of water into the bowl is at a constant rate of 160π cm^3 s^{-1} for $0 \leqslant h \leqslant 12$

(a) Find the rate of change of the depth of the water, in cm s^{-1}, when $h = 10$

(5)

Given that the flow of water into the bowl is increased to a constant rate of 300π cm^3 s^{-1} for $12 < h \leqslant 24$

(b) find the rate of change of the depth of the water, in cm s^{-1}, when $h = 20$

(2)

Question 44 continued

(Total for Question 44 is 7 marks)

45. A curve C is given by the equation

$$\sin x + \cos y = 0.5 \qquad -\frac{\pi}{2} \leqslant x < \frac{3\pi}{2}, -\pi < y < \pi$$

A point P lies on C.

The tangent to C at the point P is parallel to the x-axis.

Find the exact coordinates of all possible points P, justifying your answer.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(7)

EXPERT TUITION

Question 45 continued

(Total for Question 45 is 7 marks)

46. Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x + 1)^n (Ax + B)$$

where n, A and B are constants to be found.

(4)

(Total for Question 46 is 4 marks)

47. A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm.

In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r}$$

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

Question 47 continued

(Total for Question 47 is 9 marks)

Leave blank

48. Given

$$y = 3\sqrt{x} - 6x + 4, \qquad x > 0$$

(i) Find $\frac{dy}{dx}$

(ii) Hence find the value of x such that $\frac{dy}{dx} = 0$

(4)

EXPERT TUITION

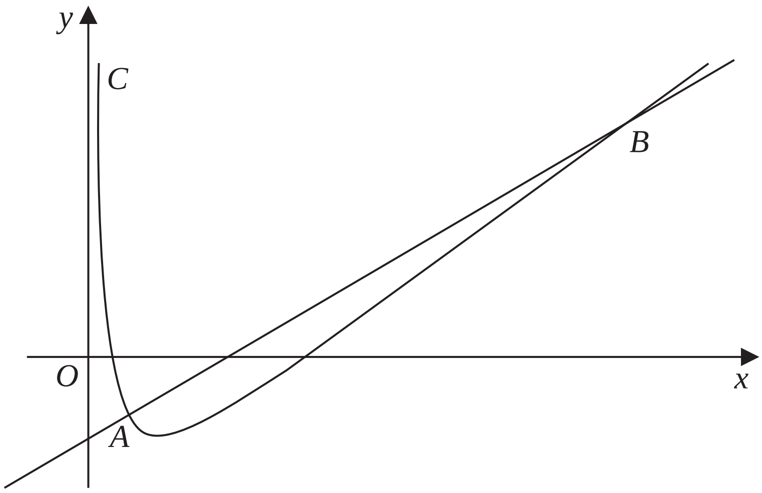
Leave blank

Question 48 continued

(Total 4 marks)

Leave blank

49.



Not to scale

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{1}{2}x + \frac{27}{x} - 12, \qquad x > 0$$

The point A lies on C and has coordinates $\left(3, -\frac{3}{2}\right)$.

(a) Show that the equation of the normal to C at A can be written as $10y = 4x - 27$ **(5)**

The normal to C at A meets C again at the point B, as shown in Figure 3.

(b) Use algebra to find the coordinates of B. **(5)**

Leave blank

Question 49 continued

Leave blank

Question 49 continued

EXPERT TUITION

Leave blank

Question 49 continued

Leave blank

Question 49 continued

(Total 10 marks)

Leave blank

50. Given

$$y = \sqrt{x} + \frac{4}{\sqrt{x}} + 4, \qquad x > 0$$

find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 8$, writing your answer in the form $a\sqrt{2}$, where a is a rational number.

(5)

Leave blank

Question 50 continued

(Total 5 marks)

Leave blank

51. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, where

$$\mathrm{f}'(x) = 30 + \frac{6 - 5x^2}{\sqrt{x}}$$

Given that the point $P(4, -8)$ lies on C,

find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Leave blank

Question 51 continued

(Total 4 marks)

EXPERT TUITION

Leave blank

52.

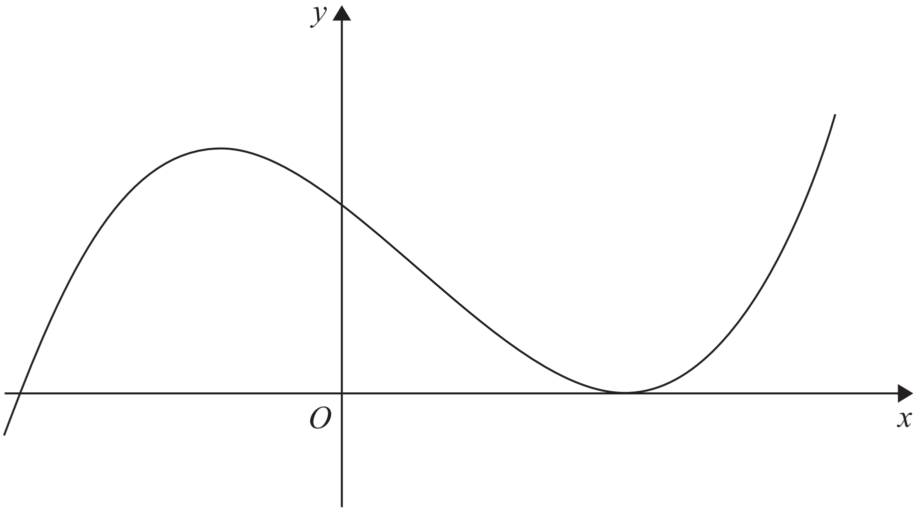


Figure 2

Figure 2 shows a sketch of part of the curve $y = \mathrm{f}(x)$, $x \in \mathbb{R}$, where

$$\mathrm{f}(x) = (2x - 5)^2(x + 3)$$

(a) Given that

(i) the curve with equation $y = \mathrm{f}(x) - k$, $x \in \mathbb{R}$, passes through the origin, find the value of the constant k,

(ii) the curve with equation $y = \mathrm{f}(x + c)$, $x \in \mathbb{R}$, has a minimum point at the origin, find the value of the constant c.

(3)

(b) Show that $\mathrm{f}'(x) = 12x^2 - 16x - 35$

(3)

Points A and B are distinct points that lie on the curve $y = \mathrm{f}(x)$.

The gradient of the curve at A is equal to the gradient of the curve at B.

Given that point A has x coordinate 3

(c) find the x coordinate of point B.

(5)

Leave blank

Question 52 continued

Leave blank

Question 52 continued

Leave blank

Question 52 continued

Leave blank

Question 52 continued

(Total 11 marks)

Leave blank

53. Given that

$$y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3\sqrt{x}}, \quad x > 0$$

find $\frac{dy}{dx}$. Give each term in your answer in its simplified form.

(6)

(Total 6 marks)

EXPERT TUITION

Leave blank

54. The curve C has equation $y = 2x^3 + kx^2 + 5x + 6$, where k is a constant.

(a) Find $\dfrac{dy}{dx}$

(2)

The point P, where $x = -2$, lies on C.

The tangent to C at the point P is parallel to the line with equation $2y - 17x - 1 = 0$

Find

(b) the value of k,

(4)

(c) the value of the y coordinate of P,

(2)

(d) the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

Leave blank

Question 54 continued

(Total 10 marks)

Leave blank

55. Given that $y = 4x^3 - \frac{5}{x^2}$, $x \neq 0$, find in their simplest form

$\frac{dy}{dx}$

(3)

(Total 3 marks)

Leave blank

56. The curve C has equation

$$y = \frac{(x^2 + 4)(x - 3)}{2x}, \quad x \neq 0$$

(a) Find $\frac{dy}{dx}$ in its simplest form.

(5)

(b) Find an equation of the tangent to C at the point where $x = -1$

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

Leave blank

Question 56 continued

(Total 10 marks)

Leave blank

57. A curve with equation $y = f(x)$ passes through the point (4, 9).

Given that

$$f'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2, \quad x > 0$$

Point P lies on the curve.

The normal to the curve at P is parallel to the line $2y + x = 0$

Find the x coordinate of P.

(5)

Leave blank

Question 57 continued

(Total 5 marks)

Leave blank

58. Differentiate with respect to x, giving each answer in its simplest form.

(a) $(1 - 2x)^2$

(3)

(b) $\dfrac{x^5 + 6\sqrt{x}}{2x^2}$

(4)

(Total 7 marks)

Leave blank

59. A curve with equation $y = f(x)$ passes through the point (4, 25).

Given that

$$f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1, \qquad x > 0$$

Find an equation of the normal to the curve at the point (4, 25).

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

Leave blank

Question 59 continued

(Total 5 marks)

Leave blank

60. Given that

$$f(x) = 2x^2 + 8x + 3$$

(a) find the value of the discriminant of $f(x)$.

(2)

(b) Express $f(x)$ in the form $p(x + q)^2 + r$ where p, q and r are integers to be found.

(3)

The line $y = 4x + c$, where c is a constant, is a tangent to the curve with equation $y = f(x)$.

(c) Calculate the value of c.

(5)

Leave blank

Question 60 continued

(Total 10 marks)

Leave blank

61. Given that $y = 2x^5 + \frac{6}{\sqrt{x}}$, $x > 0$, find in their simplest form

$\frac{dy}{dx}$

(3)

(Total 3 marks)

Leave blank

62.

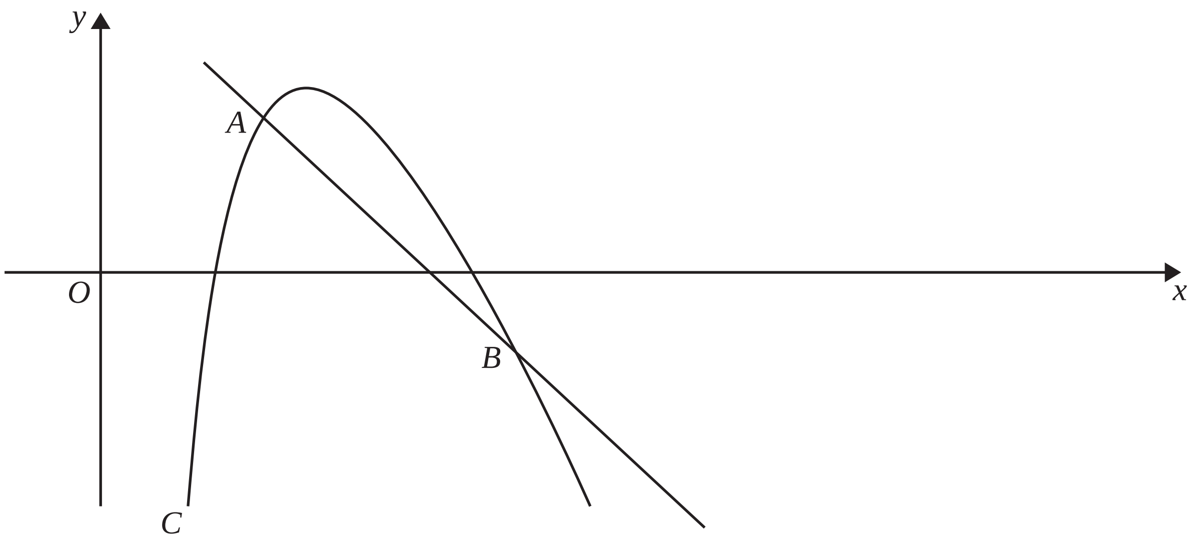


Figure 3

A sketch of part of the curve *C* with equation

$$y = 20 - 4x - \frac{18}{x}, \quad x > 0$$

is shown in Figure 3.

Point *A* lies on *C* and has an *x* coordinate equal to 2

(a) Show that the equation of the normal to *C* at *A* is $y = -2x + 7$

(6)

The normal to *C* at *A* meets *C* again at the point *B*, as shown in Figure 3.

(b) Use algebra to find the coordinates of *B*.

(5)

Leave blank

Question 62 continued

(Total 11 marks)

Leave blank

63.

$$f'(x) = \frac{(3 - x^2)^2}{x^2}, \quad x \neq 0$$

(a) Show that $\quad f'(x) = 9x^{-2} + A + Bx^2,$

where A and B are constants to be found.

(3)

(b) Find $f''(x)$.

(2)

Given that the point (–3, 10) lies on the curve with equation $y = f(x)$,

(c) find $f(x)$.

(5)

Leave blank

Question 63 continued

(Total 10 marks)

Leave blank

64.

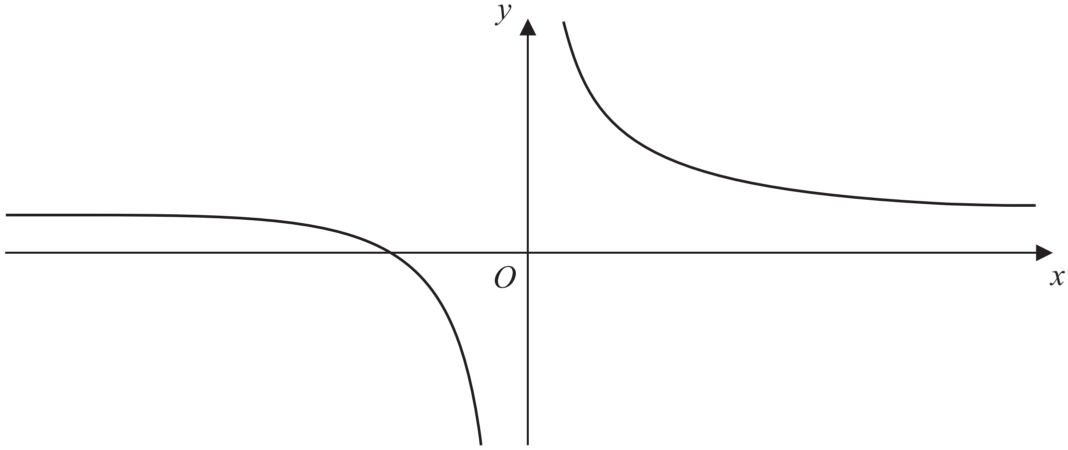


Figure 2

Figure 2 shows a sketch of the curve H with equation $y = \dfrac{3}{x} + 4$, $x \neq 0$.

(a) Give the coordinates of the point where H crosses the x-axis.
(1)

(b) Give the equations of the asymptotes to H.
(2)

(c) Find an equation for the normal to H at the point $P(-3, 3)$.
(5)

This normal crosses the x-axis at A and the y-axis at B.

(d) Find the length of the line segment AB. Give your answer as a surd.
(3)

Leave blank

Question 64 continued

(Total 11 marks)

Leave blank

65. Given $y = x^3 + 4x + 1$, find the value of $\frac{dy}{dx}$ when $x = 3$

(4)

(Total 4 marks)

Leave blank

66. A curve has equation $y = f(x)$. The point P with coordinates (9, 0) lies on the curve.

Given that

$$f'(x) = \frac{x+9}{\sqrt{x}}, \qquad x > 0$$

(a) find $f(x)$.

(6)

(b) Find the x-coordinates of the two points on $y = f(x)$ where the gradient of the curve is equal to 10

(4)

(Total 10 marks)

Leave blank

67. The curve C has equation

$$y = 2x - 8\sqrt{x} + 5, \quad x \geqslant 0$$

(a) Find $\frac{dy}{dx}$, giving each term in its simplest form.

(3)

The point P on C has x-coordinate equal to $\frac{1}{4}$

(b) Find the equation of the tangent to C at the point P, giving your answer in the form $y = ax + b$, where a and b are constants.

(4)

The tangent to C at the point Q is parallel to the line with equation $2x - 3y + 18 = 0$

(c) Find the coordinates of Q.

(5)

Leave blank

Question 67 continued

(Total 12 marks)

Leave blank

68. $$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$$

(a) Find $\frac{dy}{dx}$ giving each term in its simplest form.

(4)

(b) Find $\frac{d^2y}{dx^2}$

(2)

(Total 6 marks)

Leave blank

69. The point P (4, –1) lies on the curve C with equation $y = \mathrm{f}(x)$, $x > 0$, and

$$\mathrm{f}'(x) = \frac{1}{2}x - \frac{6}{\sqrt{x}} + 3$$

(a) Find the equation of the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers.

(4)

(b) Find $\mathrm{f}(x)$.

(4)

(Total 8 marks)

Leave blank

70. Given that $y = x^4 + 6x^{\frac{1}{2}}$, find in their simplest form

$\dfrac{dy}{dx}$

(3)

(Total 3 marks)

Leave blank

71. The curve C_1 has equation

$$y = x^2(x+2)$$

(a) Find $\frac{dy}{dx}$

(2)

(b) Sketch C_1, showing the coordinates of the points where C_1 meets the x-axis.

(3)

(c) Find the gradient of C_1 at each point where C_1 meets the x-axis.

(2)

The curve C_2 has equation

$$y = (x-k)^2(x-k+2)$$

where k is a constant and $k > 2$

(d) Sketch C_2, showing the coordinates of the points where C_2 meets the x and y axes.

(3)

Leave blank

Question 71 continued

(Total 10 marks)

Leave blank

72.

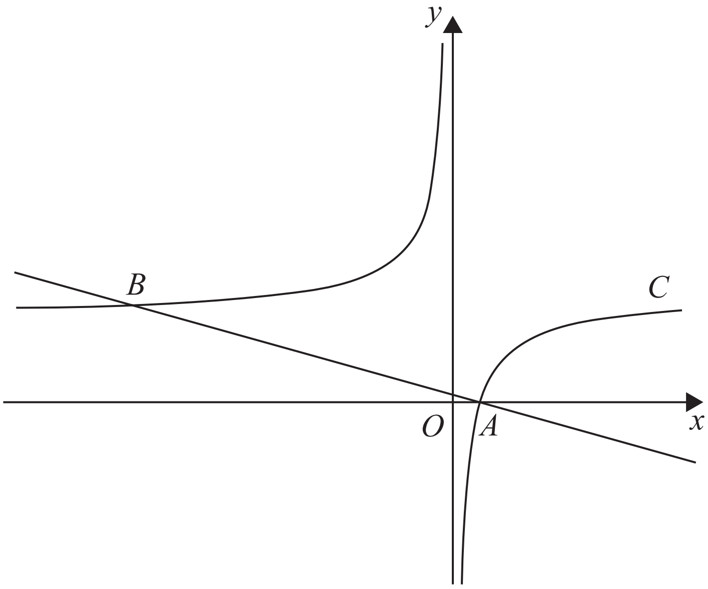


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y = 2 - \frac{1}{x}, \quad x \neq 0$$

The curve crosses the x-axis at the point A.

(a) Find the coordinates of A.

(1)

(b) Show that the equation of the normal to C at A can be written as

$$2x + 8y - 1 = 0$$

(6)

The normal to C at A meets C again at the point B, as shown in Figure 2.

(c) Find the coordinates of B.

(4)

Leave blank

Question 72 continued

(Total 11 marks)

EXPERT TUITION

Leave blank

73. Given that $y = 2x^5 + 7 + \frac{1}{x^3}$, $x \neq 0$, find, in their simplest form,

$\frac{dy}{dx}$

(3)

(Total 3 marks)

Leave blank

74. The curve C has equation

$$y = (x+1)(x+3)^2$$

(a) Sketch C, showing the coordinates of the points at which C meets the axes.

(4)

(b) Show that $\frac{dy}{dx} = 3x^2 + 14x + 15$.

(3)

The point A, with x-coordinate -5, lies on C.

(c) Find the equation of the tangent to C at A, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Another point B also lies on C. The tangents to C at A and B are parallel.

(d) Find the x-coordinate of B.

(3)

Leave blank

Question 74 continued

(Total 14 marks)

Leave blank

75. The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \qquad x > 0$$

(a) Find $\frac{dy}{dx}$.

(4)

(b) Show that the point $P(4, -8)$ lies on C.

(2)

(c) Find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

Leave blank

Question 75 continued

(Total 12 marks)

Leave blank

76. Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \quad x > 0$$

find $\frac{dy}{dx}$.

(6)

(Total 6 marks)

Leave blank

77. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, where

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x - \frac{5}{\sqrt{x}} - 2$$

Given that the point $P(4, 5)$ lies on C, find

(a) $\mathrm{f}(x)$,

(5)

(b) an equation of the tangent to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Leave blank

Question 77 continued

(Total 9 marks)

Leave blank

78. Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\dfrac{dy}{dx}$.

(3)

(Total 3 marks)

Leave blank

79. The curve C has equation

$$y = \frac{(x+3)(x-8)}{x}, \quad x > 0$$

(a) Find $\frac{dy}{dx}$ in its simplest form.

(4)

(b) Find an equation of the tangent to C at the point where $x = 2$

(4)

(Total 8 marks)

Leave blank

80. Given that $y = 2x^3 + \dfrac{3}{x^2}$, $x \neq 0$, find

$$\frac{\mathrm{d}y}{\mathrm{d}x}$$

(3)

(Total 3 marks)

Leave blank

81.

$$f(x) = \frac{(3 - 4\sqrt{x})^2}{\sqrt{x}}, \quad x > 0$$

(a) Show that $f(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where A and B are constants to be found.

(3)

(b) Find $f'(x)$.

(3)

(c) Evaluate $f'(9)$.

(2)

(Total 8 marks)

Leave blank

82. The curve C has equation

$$y = x^3 - 2x^2 - x + 9, \qquad x > 0$$

The point P has coordinates (2, 7).

(a) Show that P lies on C.

(1)

(b) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(5)

The point Q also lies on C.

Given that the tangent to C at Q is perpendicular to the tangent to C at P,

(c) show that the x-coordinate of Q is $\frac{1}{3}\left(2+\sqrt{6}\right)$.

(5)

Leave blank

Question 82 continued

(Total 11 marks)

EXPERT TUITION

Leave blank

83. Given that $\dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of p and the value of q.

(2)

Given that $y = 5x^4 - 3 + \dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(b) find $\dfrac{dy}{dx}$, simplifying the coefficient of each term.

(4)

(Total 6 marks)

Leave blank

84. The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \qquad x > 0 .$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$.

(6)

(b) Find an equation of the normal to C at the point P.

(3)

The tangent at P meets the x-axis at A and the normal at P meets the x-axis at B.

(c) Find the area of triangle APB.

(4)

Leave blank

Question 84 continued

(Total 13 marks)

Leave blank

85. $f(x) = 3x + x^3, \qquad x > 0.$

(a) Differentiate to find $f'(x)$.

(2)

Given that $f'(x) = 15$,

(b) find the value of x.

(3)

(Total 5 marks)

EXPERT TUITION

Leave blank

86. The curve C has equation $y = kx^3 - x^2 + x - 5$, where k is a constant.

(a) Find $\dfrac{dy}{dx}$.

(2)

The point A with x-coordinate $-\frac{1}{2}$ lies on C. The tangent to C at A is parallel to the line with equation $2y - 7x + 1 = 0$.

Find

(b) the value of k,

(4)

(c) the value of the y-coordinate of A.

(2)

Leave blank

Question 86 continued

(Total 8 marks)

Leave blank

87. (a) Write $\dfrac{2\sqrt{x}+3}{x}$ in the form $2x^p+3x^q$ where p and q are constants.

(2)

Given that $y = 5x - 7 + \dfrac{2\sqrt{x}+3}{x}$, $x > 0$,

(b) find $\dfrac{dy}{dx}$, simplifying the coefficient of each term.

(4)

(Total 6 marks)

EXPERT TUITION

Leave blank

88. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, and $\mathrm{f}'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2}$.

Given that the point $P(4, 1)$ lies on C,

(a) find $\mathrm{f}(x)$ and simplify your answer.

(6)

(b) Find an equation of the normal to C at the point $P(4, 1)$.

(4)

Leave blank

Question 88 continued

(Total 10 marks)

Leave blank

89. The curve C has equation

$$y = (x+3)(x-1)^2 .$$

(a) Sketch C showing clearly the coordinates of the points where the curve meets the coordinate axes.

(4)

(b) Show that the equation of C can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where k is a positive integer, and state the value of k.

(2)

There are two points on C where the gradient of the tangent to C is equal to 3.

(c) Find the x-coordinates of these two points.

(6)

Leave blank

Question 89 continued

(Total 12 marks)

EXPERT TUITION

Leave blank

90.

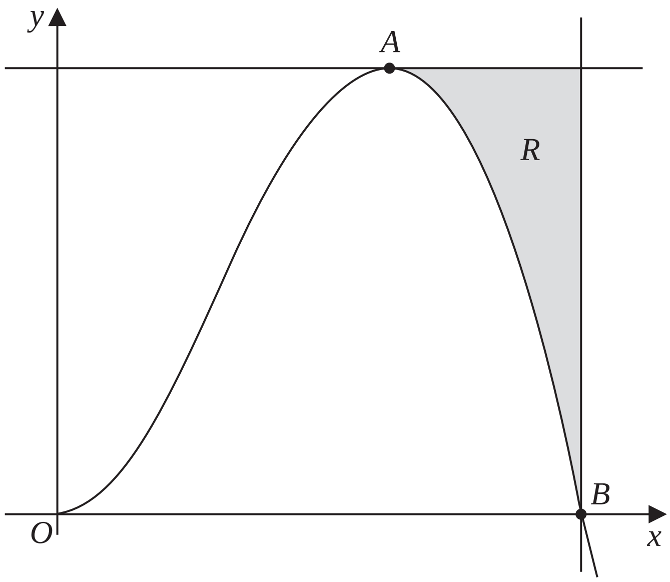


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 7x^2(5 - 2\sqrt{x}), \qquad x \geqslant 0$$

The curve has a turning point at the point A, where $x > 0$, as shown in Figure 3.

(a) Using calculus, find the coordinates of the point A.

(5)

The curve crosses the x-axis at the point B, as shown in Figure 3.

(b) Use algebra to find the x coordinate of the point B.

(2)

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Question 90 continued

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Question 90 continued

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Question 90 continued

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Question 90 continued

(Total 7 marks)

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91.

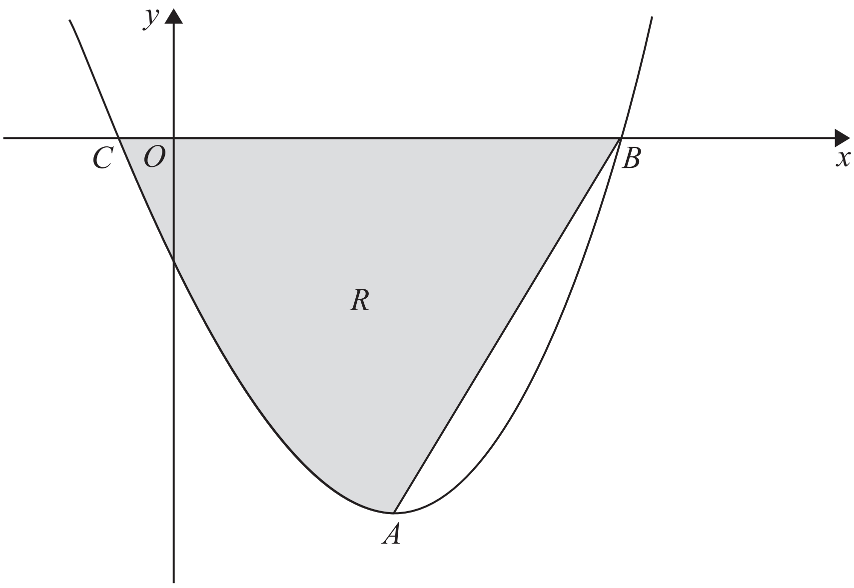

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 4x^3 + 9x^2 - 30x - 8, \quad -0.5 \leqslant x \leqslant 2.2$$

The curve has a turning point at the point A.

Using calculus, show that the x coordinate of A is 1

(3)

(Total 3 marks)

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92.

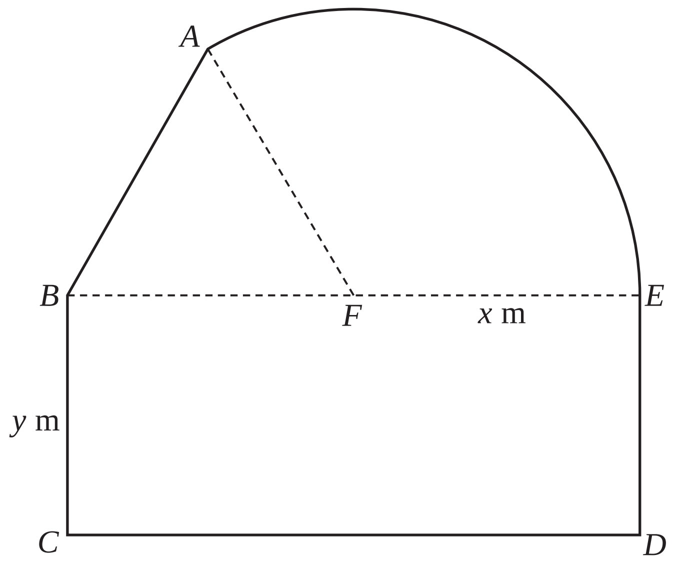

Figure 4

Figure 4 shows a plan view of a sheep enclosure.

The enclosure *ABCDEA*, as shown in Figure 4, consists of a rectangle *BCDE* joined to an equilateral triangle *BFA* and a sector *FEA* of a circle with radius x metres and centre *F*.

The points *B*, *F* and *E* lie on a straight line with $FE = x$ metres and $10 \leqslant x \leqslant 25$

(a) Find, in m^2, the exact area of the sector *FEA*, giving your answer in terms of x, in its simplest form.
(2)

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is $1000\,m^2$,

(b) show that

$$y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$$

(3)

(c) Hence show that the perimeter P metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)$$

(3)

(d) Use calculus to find the minimum value of P, giving your answer to the nearest metre.
(5)

(e) Justify, by further differentiation, that the value of P you have found is a minimum.
(2)

Leave blank

Question 92 continued

(Total 15 marks)

Leave blank

93. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of 75π cm^3.
The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(a) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}$$

(4)

(b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.

(5)

(c) Justify that the answer that you have obtained in part (b) is a minimum.

(1)

Leave blank

Question 93 continued

(Total 10 marks)

EXPERT TUITION

Leave blank

94.

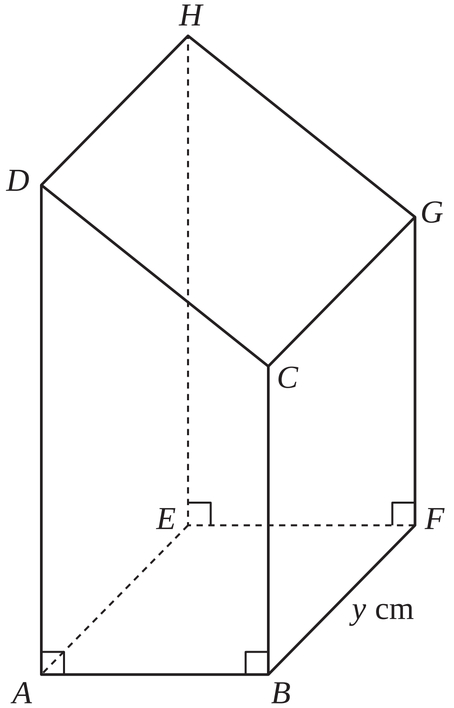

Figure 4

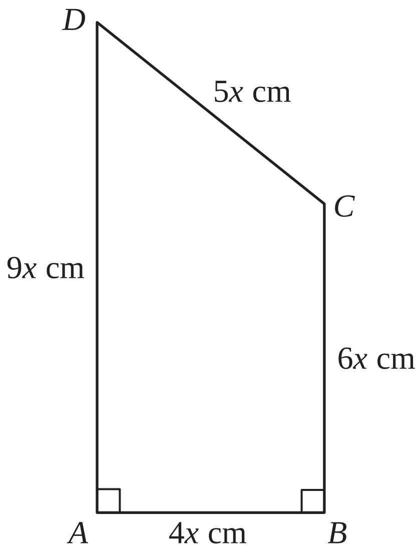

Figure 5

Figure 4 shows a closed letter box *ABFEHGCD*, which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base *ABFE* of the prism is a rectangle. The total surface area of the six faces of the prism is S cm^2.

The cross section *ABCD* of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5.
The angle $DAB = 90°$ and the angle $ABC = 90°$.

The volume of the letter box is 9600 cm^3.

(a) Show that

$$y = \frac{320}{x^2}$$

(2)

(b) Hence show that the surface area of the letter box, S cm^2, is given by

$$S = 60x^2 + \frac{7680}{x}$$

(4)

(c) Use calculus to find the minimum value of S.

(6)

(d) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)

Leave blank

Question 94 continued

(Total 14 marks)

Leave blank

95.

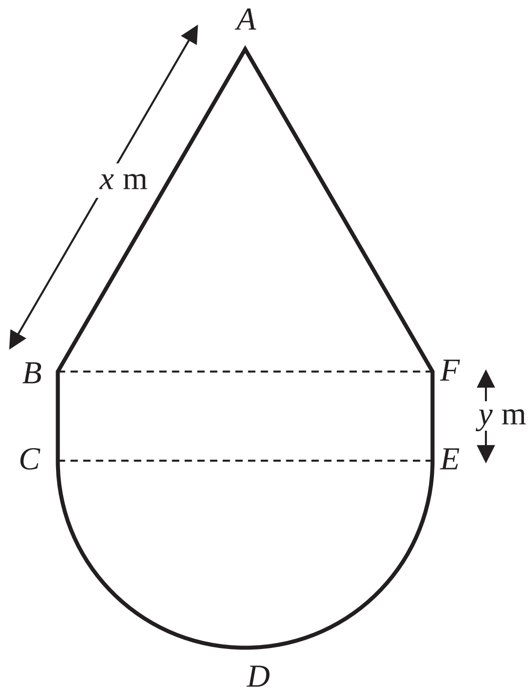

Figure 4

Figure 4 shows the plan of a pool.

The shape of the pool *ABCDEFA* consists of a rectangle *BCEF* joined to an equilateral triangle *BFA* and a semi-circle *CDE*, as shown in Figure 4.

Given that $AB = x$ metres, $EF = y$ metres, and the area of the pool is 50 m^2,

(a) show that

$$y = \frac{50}{x} - \frac{x}{8}(\pi + 2\sqrt{3})$$

(3)

(b) Hence show that the perimeter, P metres, of the pool is given by

$$P = \frac{100}{x} + \frac{x}{4}(\pi + 8 - 2\sqrt{3})$$

(3)

(c) Use calculus to find the minimum value of P, giving your answer to 3 significant figures.

(5)

(d) Justify, by further differentiation, that the value of P that you have found is a minimum.

(2)

Leave blank

Question 95 continued

(Total 13 marks)

Leave blank

96. The curve with equation

$$y = x^2 - 32\sqrt{(x)} + 20, \qquad x > 0$$

has a stationary point P.

Use calculus

(a) to find the coordinates of P,

(6)

(b) to determine the nature of the stationary point P.

(3)

(Total 9 marks)

EXPERT TUITION

Leave blank

97. Using calculus, find the coordinates of the stationary point on the curve with equation

$$y = 2x + 3 + \frac{8}{x^2}, \quad x > 0$$

(6)

(Total 6 marks)

EXPERT TUITION

Leave blank

98.

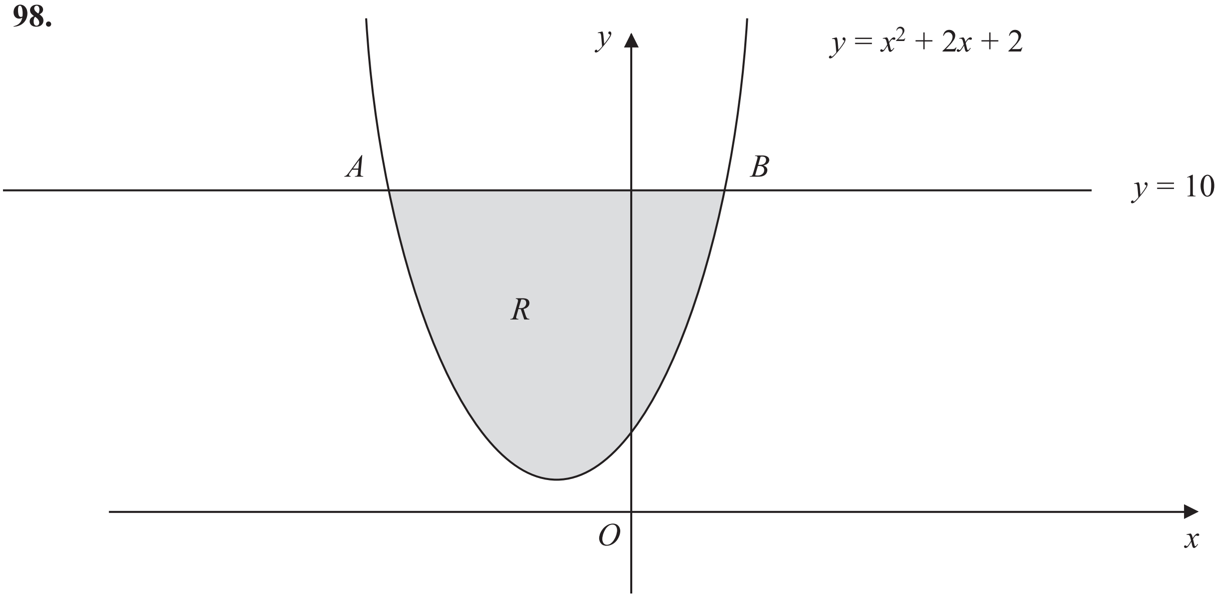

Figure 1

The line with equation $y = 10$ cuts the curve with equation $y = x^2 + 2x + 2$ at the points A and B as shown in Figure 1. The figure is not drawn to scale.

(a) Find by calculation the x-coordinate of A and the x-coordinate of B.

(2)

The shaded region R is bounded by the line with equation $y = 10$ and the curve as shown in Figure 1.

(b) Use calculus to find the exact area of R.

(7)

Leave blank

Question 98 continued

(Total 9 marks)

Leave blank

99. The curve C has equation $y = 6 - 3x - \frac{4}{x^3}$, $x \neq 0$

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$

(4)

(b) Find the x-coordinate of the other turning point Q on the curve.

(1)

(c) Find $\frac{d^2y}{dx^2}$.

(1)

(d) Hence or otherwise, state with justification, the nature of each of these turning points P and Q.

(3)

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Question 99 continued

(Total 9 marks)

Leave blank

100.

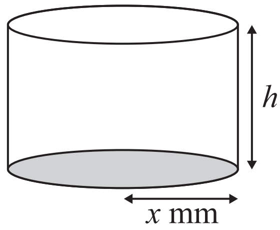

Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm^3,

(a) express h in terms of x,

(1)

(b) show that the surface area, A mm^2, of a tablet is given by $A = 2\pi x^2 + \dfrac{120}{x}$

(3)

The manufacturer needs to minimise the surface area A mm^2, of a tablet.

(c) Use calculus to find the value of x for which A is a minimum.

(5)

(d) Calculate the minimum value of A, giving your answer to the nearest integer.

(2)

(e) Show that this value of A is a minimum.

(2)

Leave blank

Question 100 continued

(Total 13 marks)

Leave blank

101.

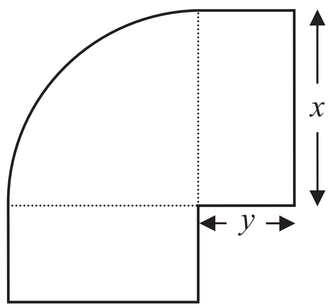

Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4 m^2,

(a) show that

$$y = \frac{16 - \pi x^2}{8x}$$

(3)

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x$$

(3)

(c) Use calculus to find the minimum value of P.

(5)

(d) Find the width of each rectangle when the perimeter is a minimum. Give your answer to the nearest centimetre.

(2)

Leave blank

Question 101 continued

(Total 13 marks)

EXPERT TUITION

Leave blank

102.

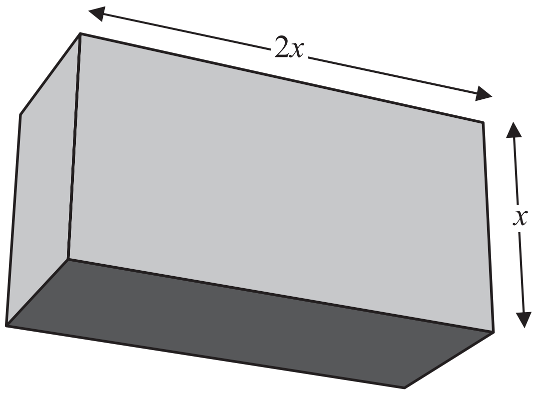

Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.
The volume of the cuboid is 81 cubic centimetres.

(a) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}$$

(3)

(b) Use calculus to find the minimum value of L.

(6)

(c) Justify, by further differentiation, that the value of L that you have found is a minimum.

(2)

Leave blank

Question 102 continued

(Total 11 marks)

Leave blank

103. The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5-x)^2, \quad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum.

(2)

Leave blank

Question 103 continued

(Total 10 marks)

Leave blank

104.

$$y = x^2 - k\sqrt{x}, \quad \text{where } k \text{ is a constant.}$$

(a) Find $\dfrac{dy}{dx}$.

(2)

(b) Given that y is decreasing at $x = 4$, find the set of possible values of k.

(2)

(Total 4 marks)

Leave blank

105.

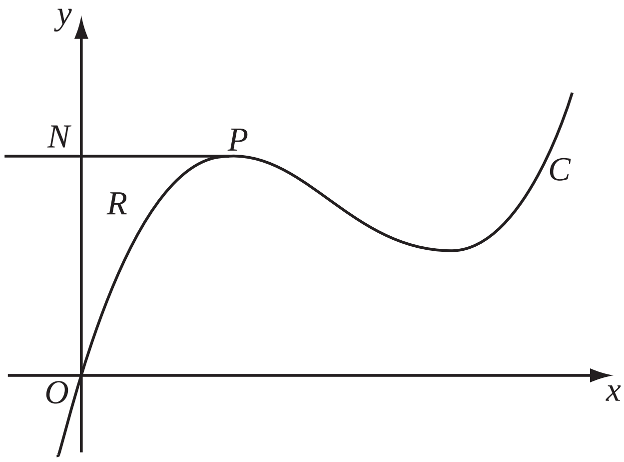

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

show that $k = 28$.

(3)

EXPERT TUITION

Leave blank

Question 105 continued

(Total 3 marks)

Leave blank

106. The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10, \quad x > 0$

(a) Use calculus to find the coordinates of the turning point on C.

(7)

(b) Find $\dfrac{d^2y}{dx^2}$.

(2)

(c) State the nature of the turning point.

(1)

Leave blank

Question 106 continued

(Total 10 marks)

Leave blank

107.

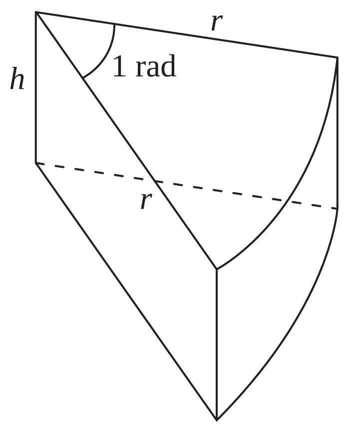

Figure 2

Figure 2 shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height h cm. The cross section is a sector of a circle. The sector has radius r cm and angle 1 radian.

The volume of the box is 300 cm^3.

(a) Show that the surface area of the box, S cm^2, is given by

$$S = r^2 + \frac{1800}{r}$$

(5)

(b) Use calculus to find the value of r for which S is stationary.

(4)

(c) Prove that this value of r gives a minimum value of S.

(2)

(d) Find, to the nearest cm^2, this minimum value of S.

(2)

Leave blank

Question 107 continued

(Total 13 marks)

Leave blank

108. A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2 .

(a) Show that the volume, V cm^3 , of the cylinder is given by

$$V = 400r - \pi r^3.$$

(4)

Given that r varies,

(b) use calculus to find the maximum value of V, to the nearest cm^3.

(6)

(c) Justify that the value of V you have found is a maximum.

(2)

Leave blank

Question 108 continued

(Total 12 marks)

EXPERT TUITION

Leave blank

109.

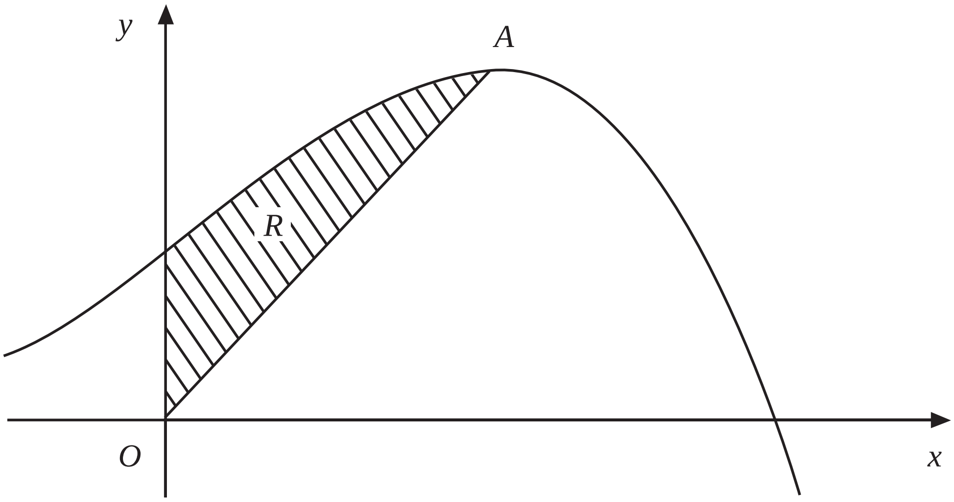

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$.

The curve has a maximum turning point *A*.

Using calculus, show that the *x*-coordinate of *A* is 2.

(3)

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Question 109 continued

(Total 3 marks)

Leave blank

110. **Figure 4**

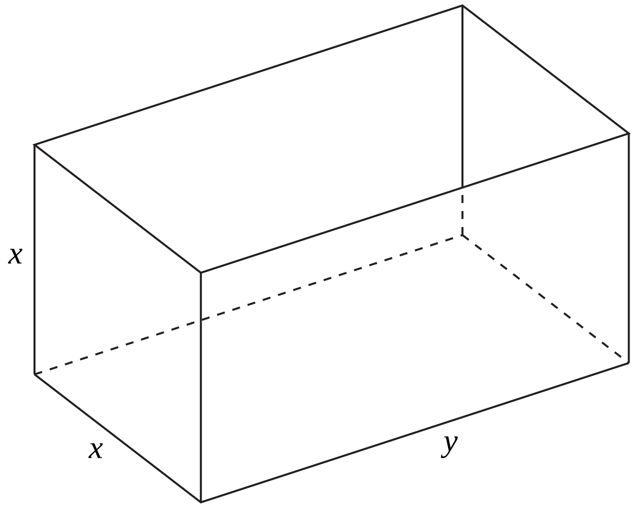

Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is $100\,\text{m}^3$.

(a) Show that the area A m^2 of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2 .$$

(4)

(b) Use calculus to find the value of x for which A is stationary.

(4)

(c) Prove that this value of x gives a minimum value of A.

(2)

(d) Calculate the minimum area of sheet metal needed to make the tank.

(2)

EXPERT TUITION

Leave blank

Question 110 continued

(Total 12 marks)

Leave blank

111. Given $y = 2x(3x-1)^5$,

(a) find $\frac{dy}{dx}$, giving your answer as a single fully factorised expression.

(4)

(b) Hence find the set of values of x for which $\frac{dy}{dx} \leqslant 0$

(2)

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Question 111 continued

(Total 6 marks)

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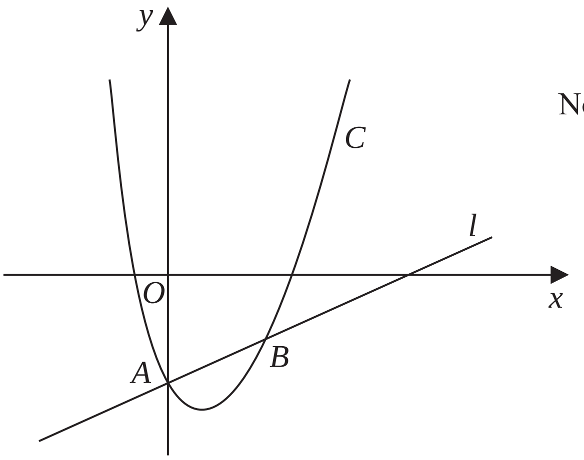
112.



Not to scale

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = e^{-2x} + x^2 - 3$$

The curve C crosses the y-axis at the point A.

The line l is the normal to C at the point A.

Find the equation of l, writing your answer in the form $y = mx + c$, where m and c are constants.

(5)

Leave blank

Question 112 continued

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Question 112 continued

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Question 112 continued

(Total 5 marks)

Leave blank

113. The curve C has equation $y = \dfrac{\ln(x^2 + 1)}{x^2 + 1}, \quad x \in \mathbb{R}$

(a) Find $\dfrac{dy}{dx}$ as a single fraction, simplifying your answer.

(3)

(b) Hence find the exact coordinates of the stationary points of C.

(6)

Leave blank

Question 113 continued

Leave blank

Question 113 continued

Leave blank

Question 113 continued

(Total 9 marks)

Leave blank

114. (a) By writing $\sec\theta = \dfrac{1}{\cos\theta}$, show that $\dfrac{d}{d\theta}(\sec\theta) = \sec\theta\tan\theta$

(2)

(b) Given that

$$x = e^{\sec y} \qquad x > e, \quad 0 < y < \frac{\pi}{2}$$

show that

$$\frac{dy}{dx} = \frac{1}{x\sqrt{g(x)}}, \qquad x > e$$

where $g(x)$ is a function of $\ln x$.

(5)

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Question 114 continued

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Question 114 continued

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Question 114 continued

(Total 7 marks)

Leave blank

115.

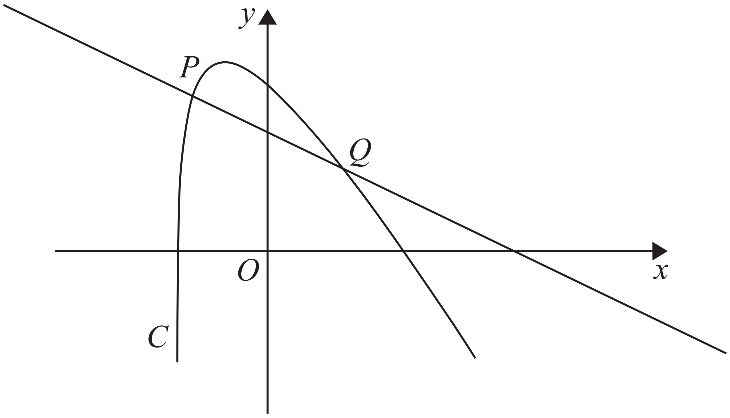

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 2\ln(2x + 5) - \frac{3x}{2}, \qquad x > -2.5$$

The point P with x coordinate -2 lies on C.

Find an equation of the normal to C at P. Write your answer in the form $ax + by = c$, where a, b and c are integers.

(5)

EXPERT TUITION

Leave blank

Question 115 continued

Leave blank

Question 115 continued

Leave blank

Question 115 continued

(Total 5 marks)

EXPERT TUITION

Leave blank

116. (i) Given $y = 2x(x^2 - 1)^5$, show that

(a) $\dfrac{dy}{dx} = g(x)(x^2 - 1)^4$ where $g(x)$ is a function to be determined.

(4)

(b) Hence find the set of values of x for which $\dfrac{dy}{dx} \geqslant 0$

(2)

(ii) Given

$$x = \ln(\sec 2y), \qquad 0 < y < \frac{\pi}{4}$$

find $\dfrac{dy}{dx}$ as a function of x in its simplest form.

(4)

Leave blank

Question 116 continued

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Question 116 continued

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Question 116 continued

(Total 10 marks)

Leave blank

117.

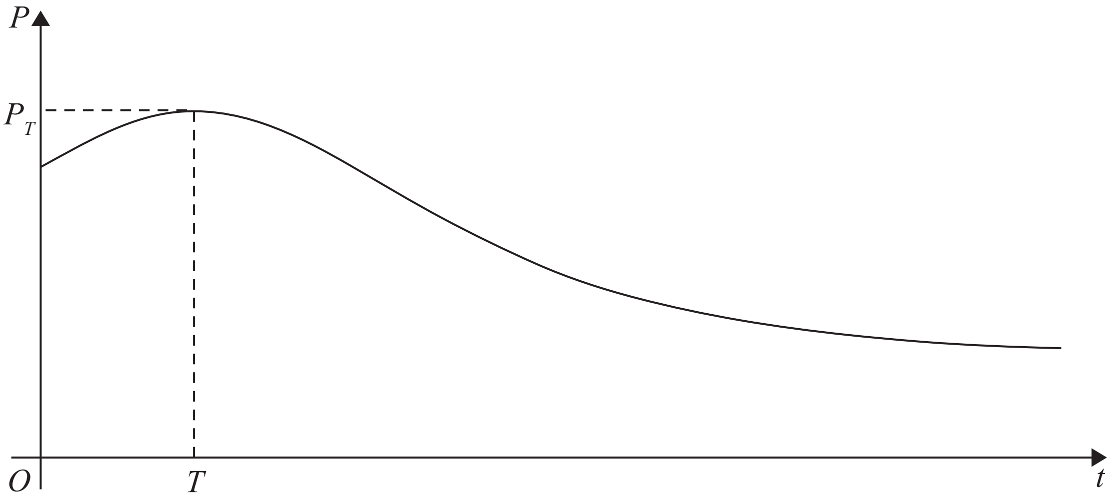

Figure 3

The number of rabbits on an island is modelled by the equation

$$P = \frac{100e^{-0.1t}}{1 + 3e^{-0.9t}} + 40, \qquad t \in \mathbb{R}, t \geqslant 0$$

where P is the number of rabbits, t years after they were introduced onto the island.

A sketch of the graph of P against t is shown in Figure 3.

(a) Calculate the number of rabbits that were introduced onto the island.

(1)

(b) Find $\dfrac{dP}{dt}$

(3)

The number of rabbits initially increases, reaching a maximum value P_T when $t = T$

(c) Using your answer from part (b), calculate

(i) the value of T to 2 decimal places,

(ii) the value of P_T to the nearest integer.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

For $t > T$, the number of rabbits decreases, as shown in Figure 3, but never falls below k, where k is a positive constant.

(d) Use the model to state the maximum value of k.

(1)

Leave blank

Question 117 continued

Leave blank

Question 117 continued

EXPERT TUITION

Leave blank

Question 117 continued

(Total 9 marks)

Leave blank

118. $$y = \frac{4x}{x^2 + 5}$$

(a) Find $\frac{dy}{dx}$, writing your answer as a single fraction in its simplest form.

(4)

(b) Hence find the set of values of x for which $\frac{dy}{dx} < 0$

(3)

(Total 7 marks)

Leave blank

119. (i) Find, using calculus, the x coordinate of the turning point of the curve with equation

$$y = e^{3x} \cos 4x, \quad \frac{\pi}{4} \leqslant x < \frac{\pi}{2}$$

Give your answer to 4 decimal places.

(5)

(ii) Given $x = \sin^2 2y$, $\quad 0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of y.

Write your answer in the form

$$\frac{dy}{dx} = p \operatorname{cosec}(qy), \qquad 0 < y < \frac{\pi}{4}$$

where p and q are constants to be determined.

(5)

EXPERT TUITION

Leave blank

Question 119 continued

(Total 10 marks)

Leave blank

120.

$$f(x) = \frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6}, \qquad x > 2, x \in \mathbb{R}$$

(a) Given that

$$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + A + \frac{B}{x - 2}$$

find the values of the constants A and B.

(4)

(b) Hence or otherwise, using calculus, find an equation of the normal to the curve with equation $y = f(x)$ at the point where $x = 3$

(5)

Leave blank

Question 120 continued

(Total 9 marks)

EXPERT TUITION

Leave blank

121. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

(a) find the exact value of p.

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(b) Use calculus to find the coordinates of A.

(6)

Leave blank

Question 121 continued

(Total 7 marks)

EXPERT TUITION

Leave blank

122. Given that k is a **negative** constant and that the function $f(x)$ is defined by

$$f(x) = 2 - \frac{(x-5k)(x-k)}{x^2 - 3kx + 2k^2}, \quad x \geqslant 0$$

(a) show that $f(x) = \dfrac{x+k}{x-2k}$

(3)

(b) Hence find $f'(x)$, giving your answer in its simplest form.

(3)

(c) State, with a reason, whether $f(x)$ is an increasing or a decreasing function.

Justify your answer.

(2)

EXPERT TUITION

Leave blank

Question 122 continued

(Total 8 marks)

Leave blank

123. The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{4x+1}{x-2}, \quad x > 2$$

(a) Show that

$$\mathrm{f}'(x) = \frac{-9}{(x-2)^2}$$

(3)

Given that P is a point on C such that $\mathrm{f}'(x) = -1$,

(b) find the coordinates of P.

(3)

(Total 6 marks)

EXPERT TUITION

Leave blank

124. The curve C has equation $x = 8y \tan 2y$

The point P has coordinates $\left(\pi, \frac{\pi}{8}\right)$

(a) Verify that P lies on C.

(1)

(b) Find the equation of the tangent to C at P in the form $ay = x + b$, where the constants a and b are to be found in terms of π.

(7)

(Total 8 marks)

Leave blank

125. A rare species of primrose is being studied. The population, P, of primroses at time t years after the study started is modelled by the equation

$$P = \frac{800e^{0.1t}}{1 + 3e^{0.1t}}, \qquad t \geqslant 0, \quad t \in \mathbb{R}$$

(a) Calculate the number of primroses at the start of the study.

(2)

(b) Find the exact value of t when $P = 250$, giving your answer in the form $a \ln(b)$ where a and b are integers.

(4)

(c) Find the exact value of $\frac{dP}{dt}$ when $t = 10$. Give your answer in its simplest form.

(4)

(d) Explain why the population of primroses can never be 270

(1)

Leave blank

Question 125 continued

(Total 11 marks)

EXPERT TUITION

Leave blank

126. A curve C has equation $y = e^{4x} + x^4 + 8x + 5$

Show that the x coordinate of any turning point of C satisfies the equation

$$x^3 = -2 - e^{4x}$$

(3)

(Total 3 marks)

Leave blank

127. (i) Given that

$$x = \sec^2 2y, \qquad 0 < y < \frac{\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{1}{4x\sqrt{(x-1)}}$$

(4)

(ii) Given that

$$y = (x^2 + x^3)\ln 2x$$

find the exact value of $\frac{dy}{dx}$ at $x = \frac{e}{2}$, giving your answer in its simplest form.

(5)

(iii) Given that

$$f(x) = \frac{3\cos x}{(x+1)^{\frac{1}{3}}}, \qquad x \neq -1$$

show that

$$f'(x) = \frac{g(x)}{(x+1)^{\frac{4}{3}}}, \qquad x \neq -1$$

where g(x) is an expression to be found.

(3)

Leave blank

Question 127 continued

(Total 12 marks)

Leave blank

128. $f(x) = 25x^2e^{2x} - 16, \qquad x \in \mathbb{R}$

Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$.

(5)

(Total 5 marks)

Leave blank

129. Given that

$$x = \sec^2 3y, \qquad 0 < y < \frac{\pi}{6}$$

(a) find $\dfrac{dx}{dy}$ in terms of y.

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

(4)

(c) Find an expression for $\dfrac{d^2y}{dx^2}$ in terms of x. Give your answer in its simplest form.

(4)

(Total 10 marks)

EXPERT TUITION

Leave blank

130. (i) (a) Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^{\frac{1}{2}}\ln x\right) = \dfrac{\ln x}{2\sqrt{x}} + \dfrac{1}{\sqrt{x}}$

(3)

The curve with equation $y = x^{\frac{1}{2}}\ln x, \ \ x > 0$ has one turning point at the point P.

(b) Find the exact coordinates of P. Give your answer in its simplest form.

(4)

(ii) A curve C has equation $y = \dfrac{x-k}{x+k}$, where k is a positive constant.

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, and show that C has no turning points.

(4)

Leave blank

Question 130 continued

(Total 11 marks)

EXPERT TUITION

Leave blank

131.

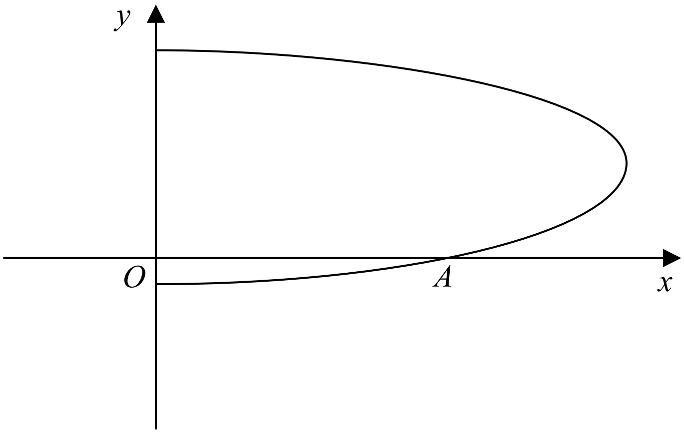

Figure 4

Figure 4 shows a sketch of the curve with equation $x = (9 + 16y - 2y^2)^{\frac{1}{2}}$.

The curve crosses the x-axis at the point A.

(a) State the coordinates of A.

(1)

(b) Find an expression for $\frac{dx}{dy}$, in terms of y.

(3)

(c) Find an equation of the tangent to the curve at A.

(4)

Leave blank

Question 131 continued

(Total 8 marks)

EXPERT TUITION

Leave blank

132. (a) Differentiate

$$\frac{\cos 2x}{\sqrt{x}}$$

with respect to x.

(3)

(b) Show that $\frac{d}{dx}\left(\sec^2 3x\right)$ can be written in the form

$$\mu(\tan 3x + \tan^3 3x)$$

where μ is a constant.

(3)

(c) Given $x = 2\sin\left(\frac{y}{3}\right)$, find $\frac{dy}{dx}$ in terms of x, simplifying your answer.

(4)

Leave blank

Question 132 continued

(Total 10 marks)

Leave blank

133. (i) Use an appropriate double angle formula to show that

$$\operatorname{cosec}2x = \lambda \operatorname{cosec} x \sec x,$$

and state the value of the constant λ.

(3)

(ii) Solve, for $0 \leqslant \theta < 2\pi$, the equation

$$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta$$

You must show all your working. Give your answers in terms of π.

(6)

(Total 9 marks)

Leave blank

134.

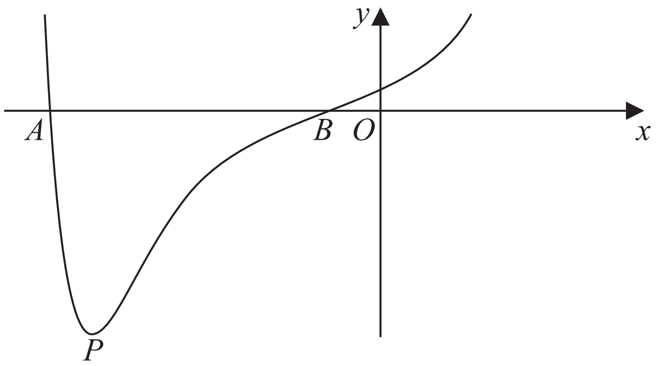

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (x^2 + 3x + 1)\,\mathrm{e}^{x^2}$$

The curve cuts the x-axis at points A and B as shown in Figure 2.

(a) Calculate the x coordinate of A and the x coordinate of B, giving your answers to 3 decimal places.

(2)

(b) Find $\mathrm{f}'(x)$.

(3)

The curve has a minimum turning point at the point P as shown in Figure 2.

Leave blank

Question 134 continued

(Total 5 marks)

Leave blank

135. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(a) the value of w,

(2)

(b) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

(Total 7 marks)

Leave blank

136. (i) Differentiate with respect to x

(a) $y = x^3 \ln 2x$

(b) $y = (x + \sin 2x)^3$

(6)

Given that $x = \cot y$,

(ii) show that $\frac{dy}{dx} = \frac{-1}{1+x^2}$

(5)

(Total 11 marks)

Leave blank

137. $$\mathrm{h}(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \qquad x \geqslant 0$$

(a) Show that $\mathrm{h}(x) = \dfrac{2x}{x^2+5}$ **(4)**

(b) Hence, or otherwise, find $\mathrm{h}'(x)$ in its simplest form. **(3)**

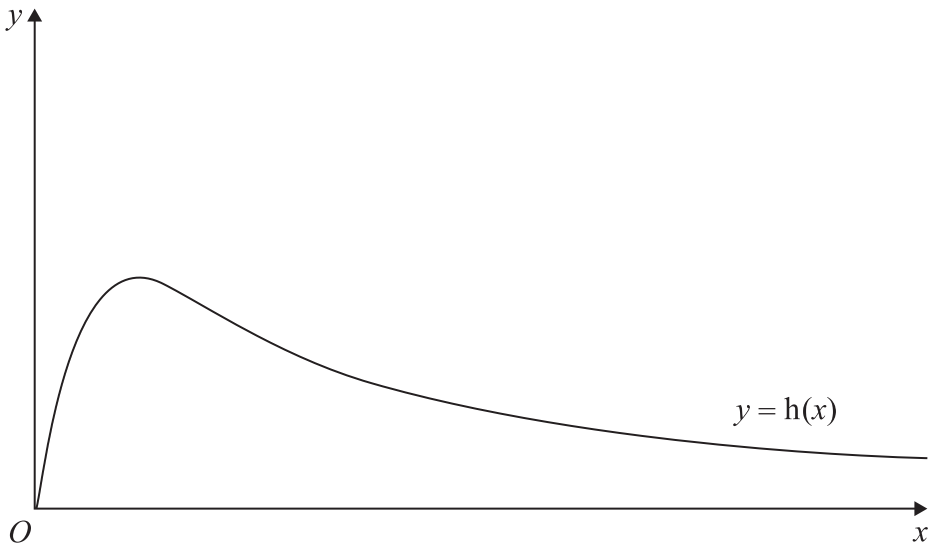

Figure 2

Figure 2 shows a graph of the curve with equation $y = \mathrm{h}(x)$.

(c) Calculate the range of $\mathrm{h}(x)$. **(5)**

Leave blank

Question 137 continued

(Total 12 marks)

Leave blank

138.

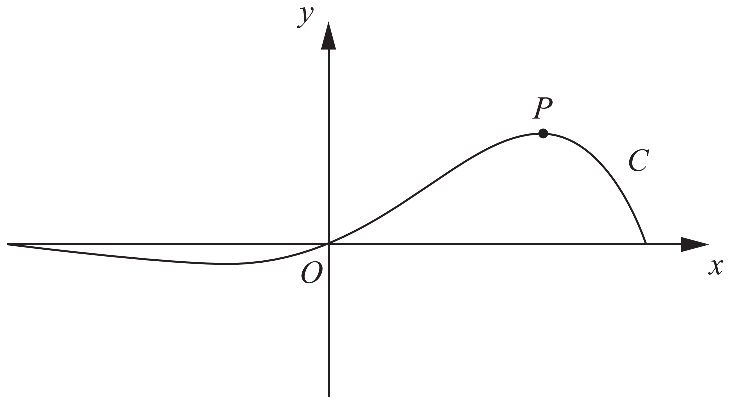

Figure 1

Figure 1 shows a sketch of the curve C which has equation

$$y = e^{x\sqrt{3}} \sin 3x \,, \quad -\frac{\pi}{3} \leqslant x \leqslant \frac{\pi}{3}$$

(a) Find the x coordinate of the turning point P on C, for which $x > 0$
Give your answer as a multiple of π.

(6)

(b) Find an equation of the normal to C at the point where $x = 0$

(3)

Leave blank

Question 138 continued

(Total 9 marks)

Leave blank

139. (a) Differentiate with respect to x,

(i) $x^{\frac{1}{2}} \ln(3x)$

(ii) $\dfrac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form.

(6)

(b) Given that $x = 3\tan 2y$ find $\dfrac{dy}{dx}$ in terms of x.

(5)

(Total 11 marks)

EXPERT TUITION

Leave blank

140. Differentiate with respect to x, giving your answer in its simplest form,

(a) $x^2 \ln(3x)$

(4)

(b) $\dfrac{\sin 4x}{x^3}$

(5)

(Total 9 marks)

Leave blank

141. The point P is the point on the curve $x = 2\tan\left(y + \frac{\pi}{12}\right)$ with y-coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P.

(7)

(Total 7 marks)

Leave blank

142. Differentiate with respect to x

(a) $\ln\left(x^2 + 3x + 5\right)$

(2)

(b) $\dfrac{\cos x}{x^2}$

(3)

(Total 5 marks)

Leave blank

143. $$f(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9}, \qquad x \neq \pm 3,\ x \neq -\frac{1}{2}$$

(a) Show that

$$f(x) = \frac{5}{(2x+1)(x+3)}$$

(5)

The curve C has equation $y = f(x)$. The point $P\left(-1, -\frac{5}{2}\right)$ lies on C.

(b) Find an equation of the normal to C at P.

(8)

Leave blank

Question 143 continued

(Total 13 marks)

Leave blank

144. (a) Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form.

(4)

Given that

$$\mathrm{f}(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \qquad x > 1,$$

(b) show that

$$\mathrm{f}(x) = \frac{3}{2x-1}$$

(2)

(c) Hence differentiate $\mathrm{f}(x)$ and find $\mathrm{f}'(2)$.

(3)

Leave blank

Question 144 continued

(Total 9 marks)

Leave blank

145. The curve C has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}$$

(a) Show that

$$\frac{dy}{dx} = \frac{6\sin 2x + 4\cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(b) Find an equation of the tangent to C at the point on C where $x = \frac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where a and b are exact constants.

(4)

(Total 8 marks)

Leave blank

146. (a) Given that

$$\frac{d}{dx}(\cos x) = -\sin x$$

show that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

(3)

Given that

$$x = \sec 2y$$

(b) find $\frac{dx}{dy}$ in terms of y.

(2)

(c) Hence find $\frac{dy}{dx}$ in terms of x.

(4)

Leave blank

Question 146 continued

(Total 9 marks)

Leave blank

147. A curve C has equation

$$y = \frac{3}{(5-3x)^2}, \quad x \neq \frac{5}{3}$$

The point P on C has x-coordinate 2. Find an equation of the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

(Total 7 marks)

Leave blank

148.

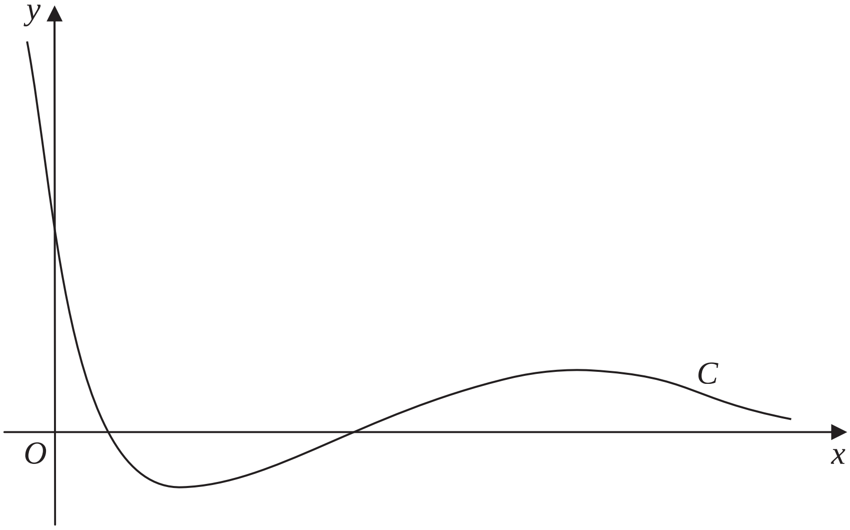

Figure 1

Figure 1 shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(a) Find the coordinates of the point where C crosses the y-axis.

(1)

(b) Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis.

(3)

(c) Find $\dfrac{dy}{dx}$.

(3)

(d) Hence find the exact coordinates of the turning points of C.

(5)

Leave blank

Question 148 continued

(Total 12 marks)

Leave blank

149. (i) Given that $y = \frac{\ln(x^2+1)}{x}$, find $\frac{dy}{dx}$.

(4)

(ii) Given that $x = \tan y$, show that $\frac{dy}{dx} = \frac{1}{1+x^2}$.

(5)

(Total 9 marks)

Leave blank

150. (a) By writing $\sec x$ as $\dfrac{1}{\cos x}$, show that $\dfrac{\mathrm{d}(\sec x)}{\mathrm{d}x} = \sec x \tan x$.

(3)

Given that $y = \mathrm{e}^{2x} \sec 3x$,

(b) find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

The curve with equation $y = \mathrm{e}^{2x} \sec 3x$, $-\frac{\pi}{6} < x < \frac{\pi}{6}$, has a minimum turning point at (a, b).

(c) Find the values of the constants a and b, giving your answers to 3 significant figures.

(4)

Leave blank

Question 150 continued

(Total 11 marks)

Leave blank

151. (i) Differentiate with respect to x

(a) $x^2 \cos 3x$

(3)

(b) $\dfrac{\ln(x^2+1)}{x^2+1}$

(4)

(ii) A curve C has the equation

$$y = \sqrt{(4x+1)}\,, \quad x > -\tfrac{1}{4}, \ y > 0$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

Leave blank

Question 151 continued

(Total 13 marks)

Leave blank

152. The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R},\ x \neq -4,\ x \neq 2$$

(a) Show that $f(x) = \dfrac{x-3}{x-2}$ **(5)**

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R},\ x \neq \ln 2$$

(b) Differentiate $g(x)$ to show that $g'(x) = \dfrac{e^x}{(e^x - 2)^2}$ **(3)**

(c) Find the exact values of x for which $g'(x) = 1$ **(4)**

Leave blank

Question 152 continued

(Total 12 marks)

Leave blank

153. (a) Find the value of $\frac{dy}{dx}$ at the point where $x = 2$ on the curve with equation

$$y = x^2\sqrt{(5x - 1)}.$$

(6)

(b) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x.

(4)

(Total 10 marks)

EXPERT TUITION

Leave blank

154. $$f(x) = \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3}$$

(a) Express $f(x)$ as a single fraction in its simplest form.

(4)

(b) Hence show that $f'(x) = \dfrac{2}{(x-3)^2}$

(3)

(Total 7 marks)

Leave blank

155. $$\mathrm{f}(x) = 3x\mathrm{e}^{x} - 1$$

The curve with equation $y = \mathrm{f}(x)$ has a turning point P.

Find the exact coordinates of P.

(5)

(Total 5 marks)

Leave blank

156. The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y-coordinate of P is 8.

(a) Find, in terms of ln 2, the x-coordinate of P.

(2)

(b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found.

(4)

(Total 6 marks)

EXPERT TUITION

Leave blank

157. (a) Differentiate with respect to x,

(i) $e^{3x}(\sin x + 2\cos x)$,

(3)

(ii) $x^3 \ln(5x+2)$.

(3)

Given that $y = \dfrac{3x^2+6x-7}{(x+1)^2}$, $\quad x \neq -1$,

(b) show that $\dfrac{dy}{dx} = \dfrac{20}{(x+1)^3}$.

(5)

(c) Hence find $\dfrac{d^2y}{dx^2}$ and the real values of x for which $\dfrac{d^2y}{dx^2} = -\dfrac{15}{4}$.

(3)

Leave blank

Question 157 continued

(Total 14 marks)

EXPERT TUITION

Leave blank

158. A curve C has equation

$$y = e^{2x} \tan x, \quad x \neq (2n+1)\frac{\pi}{2}.$$

(a) Show that the turning points on C occur where $\tan x = -1$.

(6)

(b) Find an equation of the tangent to C at the point where $x = 0$.

(2)

(Total 8 marks)

Leave blank

159. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x, \; -\pi \leqslant x \leqslant \pi .$$

The point $A(0, 4)$ lies on C.

Find an equation of the normal to the curve C at A.

(5)

(Total 5 marks)

Leave blank

160. The functions f and g are defined by

$$f : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}$$

$$g : x \mapsto \frac{3}{x} - 4, \quad x > 0, \quad x \in \mathbb{R}$$

(a) Find the inverse function f^{-1}.

(2)

(b) Show that the composite function gf is

$$gf : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(c) Solve $gf(x) = 0$.

(2)

(d) Use calculus to find the coordinates of the stationary point on the graph of $y = gf(x)$.

(5)

Leave blank

Question 160 continued

(Total 13 marks)

Leave blank

161. The curve C has equation

$$x^2 + xy + y^2 - 4x - 5y + 1 = 0$$

(a) Use implicit differentiation to find $\dfrac{dy}{dx}$ in terms of x and y.

(5)

(b) Find the x coordinates of the two points on C where $\dfrac{dy}{dx} = 0$

Give exact answers in their simplest form.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Leave blank

Question 161 continued

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Question 161 continued

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Question 161 continued

(Total 10 marks)

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162.

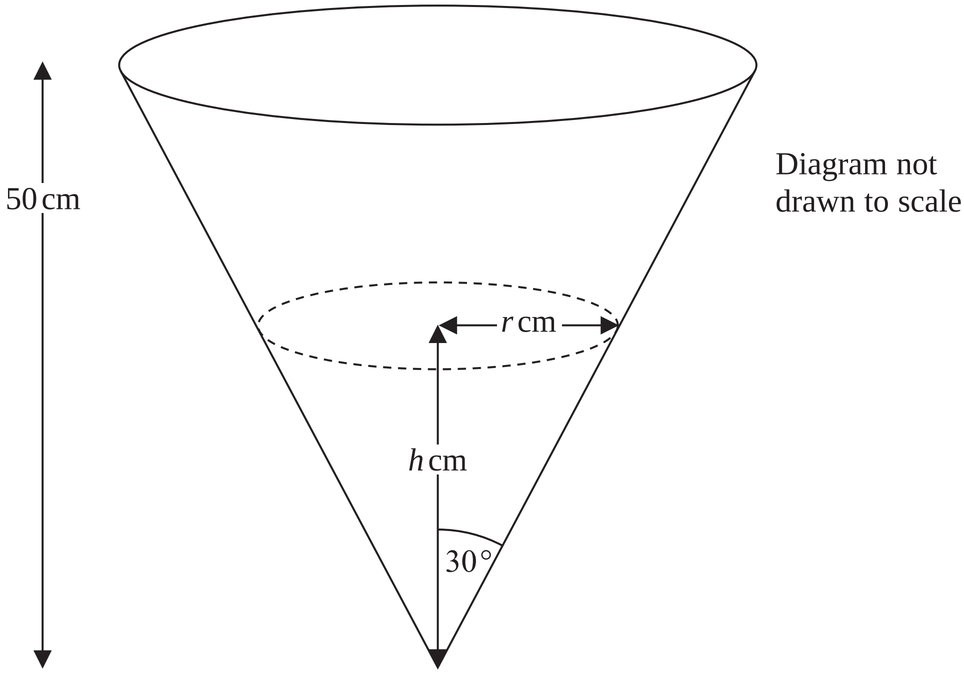

Figure 1

A water container is made in the shape of a hollow inverted right circular cone with semi-vertical angle of 30°, as shown in Figure 1. The height of the container is 50 cm.

When the depth of the water in the container is h cm, the surface of the water has radius r cm and the volume of water is V cm^3.

(a) Show that $V = \frac{1}{9}\pi h^3$

[*You may assume the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

(2)

Given that the volume of water in the container increases at a constant rate of 200 cm^3 s^{-1},

(b) find the rate of change of the depth of the water, in cm s^{-1}, when $h = 15$
Give your answer in its simplest form in terms of π.

(4)

Leave blank

Question 162 continued

Leave blank

Question 162 continued

Leave blank

Question 162 continued

(Total 6 marks)

Leave blank

163. The curve C has parametric equations

$$x = 3t - 4, \quad y = 5 - \frac{6}{t}, \qquad t > 0$$

(a) Find $\frac{dy}{dx}$ in terms of t

(2)

The point P lies on C where $t = \frac{1}{2}$

(b) Find the equation of the tangent to C at the point P. Give your answer in the form $y = px + q$, where p and q are integers to be determined.

(3)

(c) Show that the cartesian equation for C can be written in the form

$$y = \frac{ax + b}{x + 4}, \qquad x > -4$$

where a and b are integers to be determined.

(3)

Leave blank

Question 163 continued

Leave blank

Question 163 continued

Leave blank

Question 163 continued

(Total 8 marks)

Leave blank

164. The curve C has equation

$$4x^2 - y^3 - 4xy + 2^y = 0$$

The point P with coordinates $(-2, 4)$ lies on C.

(a) Find the exact value of $\frac{dy}{dx}$ at the point P.

(6)

The normal to C at P meets the y-axis at the point A.

(b) Find the y coordinate of A, giving your answer in the form $p + q\ln 2$, where p and q are constants to be determined.

(3)

Leave blank

Question 164 continued

Leave blank

Question 164 continued

Leave blank

Question 164 continued

(Total 9 marks)

EXPERT TUITION

Leave blank

165. The curve C has equation

$$2x^2y + 2x + 4y - \cos(\pi y) = 17$$

(a) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P with coordinates $\left(3, \frac{1}{2}\right)$ lies on C.

The normal to C at P meets the x-axis at the point A.

(b) Find the x coordinate of A, giving your answer in the form $\frac{a\pi + b}{c\pi + d}$, where a, b, c and d are integers to be determined.

(4)

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Question 165 continued

(Total 9 marks)

Leave blank

166.

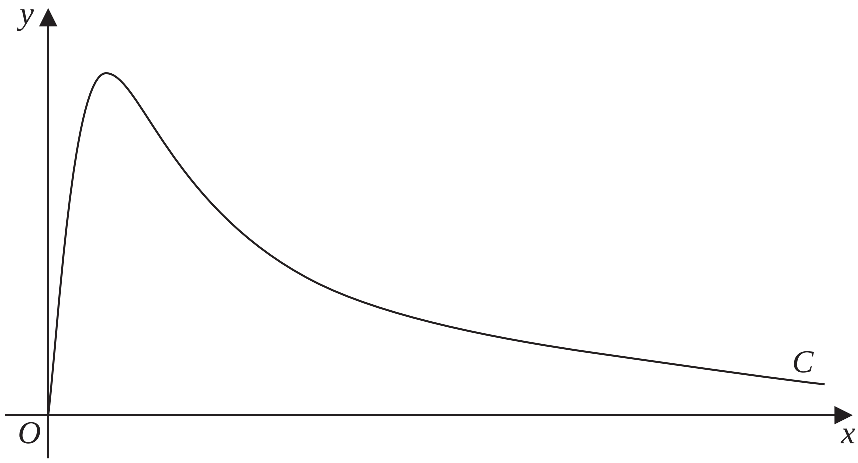

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\tan t, \qquad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

(a) Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point P.

Give your answer as a simplified surd.

(4)

The point Q lies on the curve C, where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

(b) Find the exact coordinates of the point Q.

(2)

Leave blank

Question 166 continued

(Total 6 marks)

Leave blank

167. The curve C has equation

$$x^2 - 3xy - 4y^2 + 64 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find the coordinates of the points on C where $\frac{dy}{dx} = 0$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

Leave blank

Question 167 continued

(Total 11 marks)

Leave blank

168. A curve C has parametric equations

$$x = 4t + 3, \quad y = 4t + 8 + \frac{5}{2t}, \quad t \neq 0$$

(a) Find the value of $\frac{dy}{dx}$ at the point on C where $t = 2$, giving your answer as a fraction in its simplest form.

(3)

(b) Show that the cartesian equation of the curve C can be written in the form

$$y = \frac{x^2 + ax + b}{x - 3}, \quad x \neq 3$$

where a and b are integers to be determined.

(3)

Leave blank

Question 168 continued

(Total 6 marks)

Leave blank

169. A curve C has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find an equation of the tangent to C at the point $(3, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

(Total 7 marks)

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170.

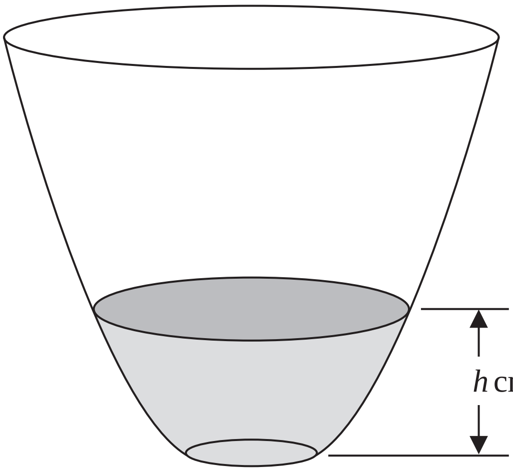

Figure 2

A vase with a circular cross-section is shown in Figure 2. Water is flowing into the vase.

When the depth of the water is h cm, the volume of water V cm^3 is given by

$$V = 4\pi h(h + 4), \quad 0 \leqslant h \leqslant 25$$

Water flows into the vase at a constant rate of 80π cm^{3}s^{-1}

Find the rate of change of the depth of the water, in cm s^{-1}, when $h = 6$

(5)

(Total 5 marks)

Leave blank

171.

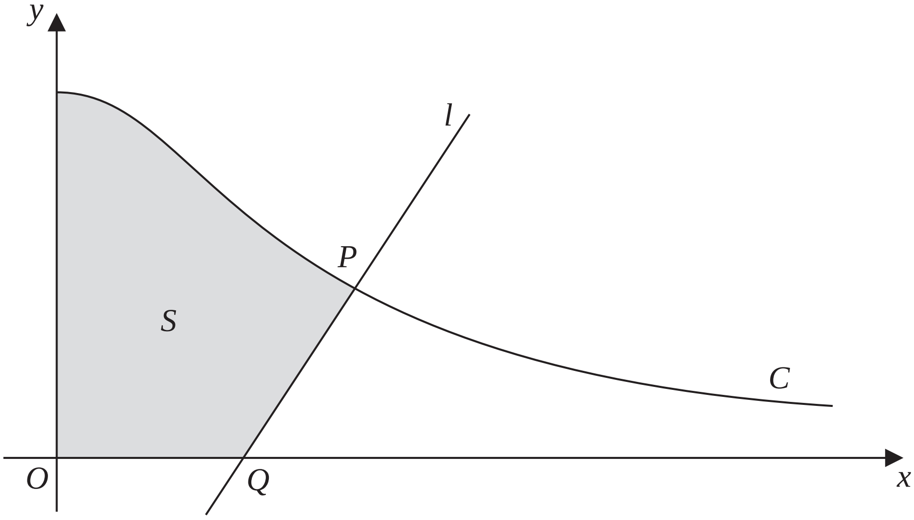

Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$$x = 3\tan\theta, \quad y = 4\cos^2\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates (3, 2).

The line l is the normal to C at P. The normal cuts the x-axis at the point Q.

Find the x coordinate of the point Q.

(6)

Leave blank

Question 171 continued

(Total 6 marks)

Leave blank

172. $$x^2 + y^2 + 10x + 2y - 4xy = 10$$

(a) Find $\frac{dy}{dx}$ in terms of x and y, fully simplifying your answer.

(5)

(b) Find the values of y for which $\frac{dy}{dx} = 0$

(5)

(Total 10 marks)

Leave blank

173. At time t seconds the radius of a sphere is r cm, its volume is V cm^3 and its surface area is S cm^2.

[*You are given that* $V = \frac{4}{3}\pi r^3$ *and that* $S = 4\pi r^2$]

The volume of the sphere is increasing uniformly at a constant rate of 3 cm^3 s^{-1}.

(a) Find $\frac{\mathrm{d}r}{\mathrm{d}t}$ when the radius of the sphere is 4 cm, giving your answer to 3 significant figures.

(4)

(b) Find the rate at which the surface area of the sphere is increasing when the radius is 4 cm.

(2)

(Total 6 marks)

Leave blank

174.

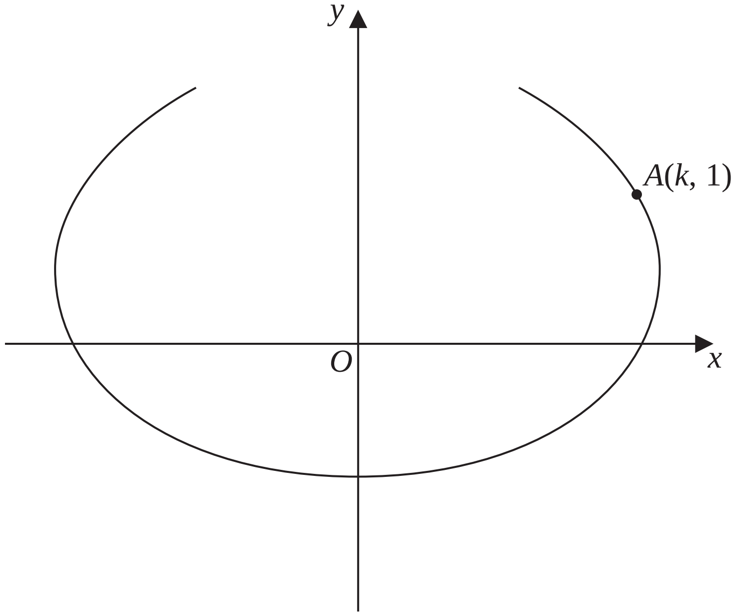

Figure 3

The curve shown in Figure 3 has parametric equations

$$x = t - 4\sin t, \quad y = 1 - 2\cos t, \qquad -\frac{2\pi}{3} \leqslant t \leqslant \frac{2\pi}{3}$$

The point A, with coordinates $(k, 1)$, lies on the curve.

Given that $k > 0$

(a) find the exact value of k,

(2)

(b) find the gradient of the curve at the point A.

(4)

There is one point on the curve where the gradient is equal to $-\frac{1}{2}$

(c) Find the value of t at this point, showing each step in your working and giving your answer to 4 decimal places.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

Leave blank

Question 174 continued

(Total 12 marks)

Leave blank

175. A curve C has parametric equations

$$x = 2\sin t, \quad y = 1 - \cos 2t, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

Find $\dfrac{dy}{dx}$ at the point where $t = \dfrac{\pi}{6}$

(4)

(Total 4 marks)

Leave blank

176. A curve is described by the equation

$$x^2 + 4xy + y^2 + 27 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

A point Q lies on the curve.

The tangent to the curve at Q is parallel to the y-axis.

Given that the x coordinate of Q is negative,

(b) use your answer to part (a) to find the coordinates of Q.

(7)

Leave blank

Question 176 continued

(Total 12 marks)

Leave blank

177. The curve C has equation

$$3^{x-1} + xy - y^2 + 5 = 0$$

Show that $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point (1, 3) on the curve C can be written in the form $\frac{1}{\lambda}\ln(\mu e^3)$, where λ and μ are integers to be found.

(7)

(Total 7 marks)

Leave blank

178.

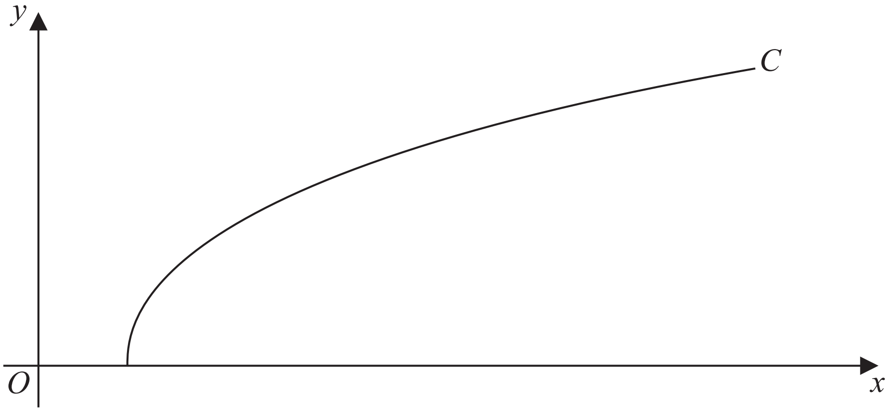

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 27\sec^3 t,\ \ y = 3\tan t, \qquad 0 \leqslant t \leqslant \frac{\pi}{3}$$

Find the gradient of the curve C at the point where $t = \dfrac{\pi}{6}$

(4)

(Total 4 marks)

Leave blank

179. A curve C has parametric equations

$$x = 2t + 5, \quad y = 3 + \frac{4}{t}, \qquad t \neq 0$$

(a) Find the value of $\frac{dy}{dx}$ at the point on C with coordinates (9, 5).

(4)

(b) Find a cartesian equation of the curve in the form

$$y = \frac{ax + b}{cx + d}$$

where a, b, c and d are integers.

(3)

(Total 7 marks)

Leave blank

180. The curve C has the equation

$$\sin(\pi y) - y - x^2y = -5, \qquad x > 0$$

(a) Find $\dfrac{dy}{dx}$ in terms of x and y.

(5)

The point P with coordinates (2, 1) lies on C.
The tangent to C at P meets the x-axis at the point A.

(b) Find the exact value of the x-coordinate of A.

(4)

(Total 9 marks)

181.

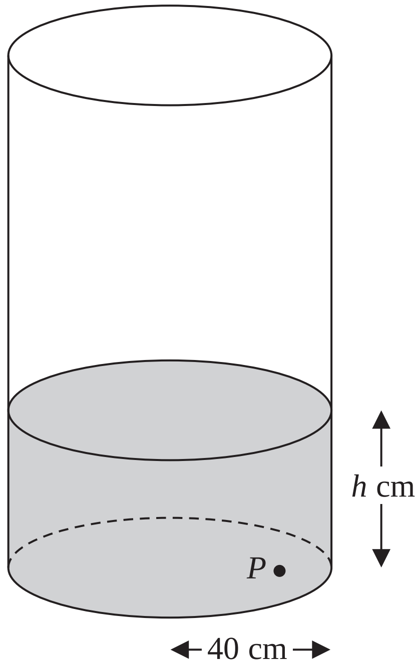

Figure 3

Figure 3 shows a large vertical cylindrical tank containing a liquid. The radius of the circular cross-section of the tank is 40 cm. At time t minutes, the depth of liquid in the tank is h centimetres. The liquid leaks from a hole P at the bottom of the tank.

The liquid leaks from the tank at a rate of $32\pi\sqrt{h}$ cm^3 min^{-1}.

Show that at time t minutes, the height h cm of liquid in the tank satisfies the differential equation

$$\frac{dh}{dt} = -0.02\sqrt{h}$$

(4)

Leave blank

Leave blank

Question 181 continued

(Total 4 marks)

Leave blank

182.

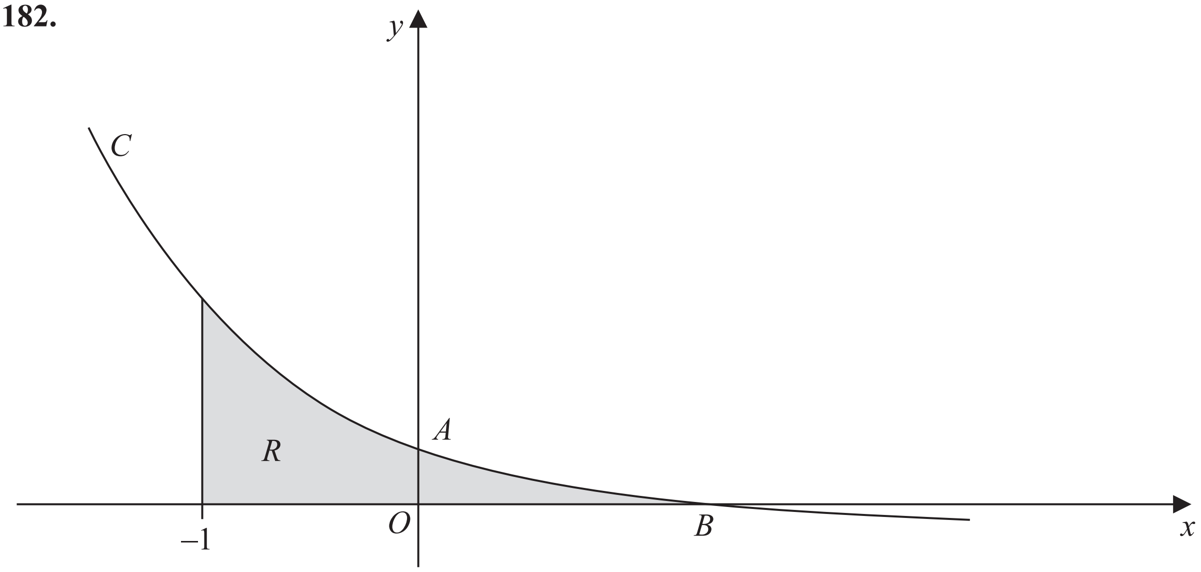

Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the y-axis at the point A and crosses the x-axis at the point B.

(a) Show that A has coordinates (0, 3).

(2)

(b) Find the x coordinate of the point B.

(2)

(c) Find an equation of the normal to C at the point A.

(5)

Leave blank

Question 182 continued

(Total 9 marks)

Leave blank

183.

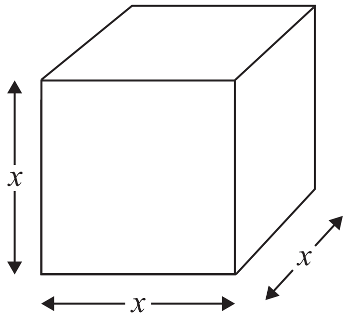

Figure 1

Figure 1 shows a metal cube which is expanding uniformly as it is heated.
At time t seconds, the length of each edge of the cube is x cm,
and the volume of the cube is V cm^3.

(a) Show that $\frac{dV}{dx} = 3x^2$ **(1)**

Given that the volume, V cm^3, increases at a constant rate of 0.048 cm^{3}s^{-1},

(b) find $\frac{dx}{dt}$, when $x = 8$ **(2)**

(c) find the rate of increase of the total surface area of the cube, in cm^{2}s^{-1}, when $x = 8$ **(3)**

Leave blank

Question 183 continued

(Total 6 marks)

Leave blank

184. The curve C has equation

$$16y^3 + 9x^2y - 54x = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find the coordinates of the points on C where $\frac{dy}{dx} = 0$.

(7)

(Total 12 marks)

Leave blank

185.

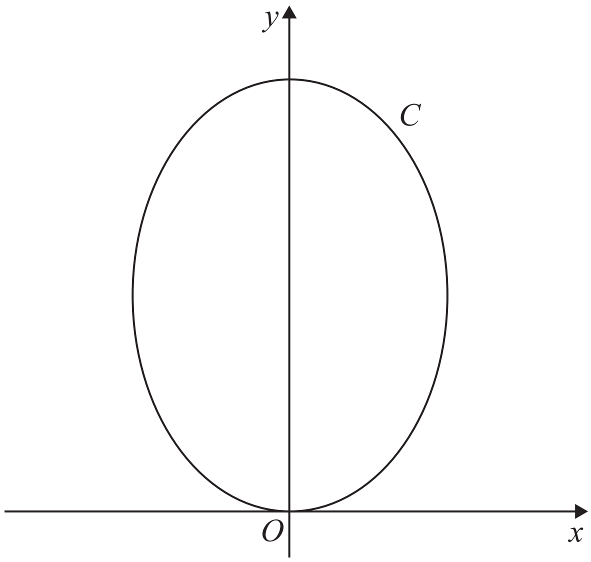

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = (\sqrt{3})\sin 2t, \quad y = 4\cos^2 t, \quad 0 \leqslant t \leqslant \pi$$

(a) Show that $\dfrac{dy}{dx} = k(\sqrt{3})\tan 2t$, where k is a constant to be determined.

(5)

(b) Find an equation of the tangent to C at the point where $t = \dfrac{\pi}{3}$.

Give your answer in the form $y = ax + b$, where a and b are constants.

(4)

Leave blank

Question 185 continued

(Total 9 marks)

EXPERT TUITION

Leave blank

186. The curve C has the equation $2x + 3y^2 + 3x^2y = 4x^2$.
The point P on the curve has coordinates $(-1, 1)$.

(a) Find the gradient of the curve at P.

(5)

(b) Hence find the equation of the normal to C at P, giving your answer in the form $ax+by+c = 0$, where a, b and c are integers.

(3)

(Total 8 marks)

Leave blank

187.

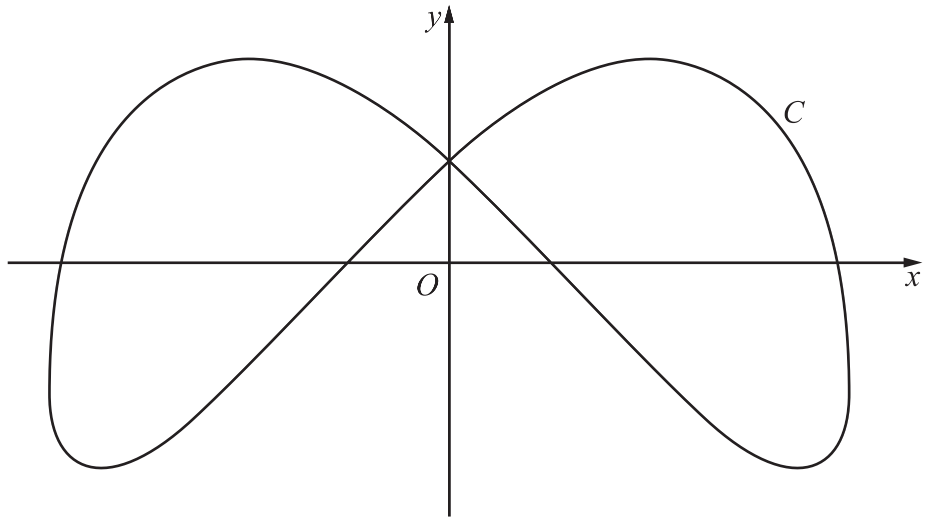

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\sin\left(t + \frac{\pi}{6}\right), \qquad y = 3\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

(3)

(b) Find the coordinates of all the points on C where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

(5)

Leave blank

Question 187 continued

(Total 8 marks)

Leave blank

188.

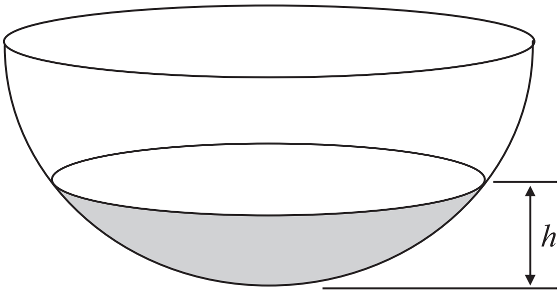

Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl. When the depth of the water is h m, the volume $V\,\text{m}^3$ is given by

$$V = \frac{1}{12}\pi h^2(3 - 4h), \qquad 0 \leqslant h \leqslant 0.25$$

(a) Find, in terms of π, $\frac{dV}{dh}$ when $h = 0.1$

(4)

Water flows into the bowl at a rate of $\frac{\pi}{800}\ \text{m}^3\text{s}^{-1}$.

(b) Find the rate of change of h, in $\text{m}\,\text{s}^{-1}$, when $h = 0.1$

(2)

(Total 6 marks)

EXPERT TUITION

Leave blank

189. Find the gradient of the curve with equation

$$\ln y = 2x \ln x, \quad x > 0,\ y > 0$$

at the point on the curve where $x = 2$. Give your answer as an exact value.

(7)

(Total 7 marks)

190.

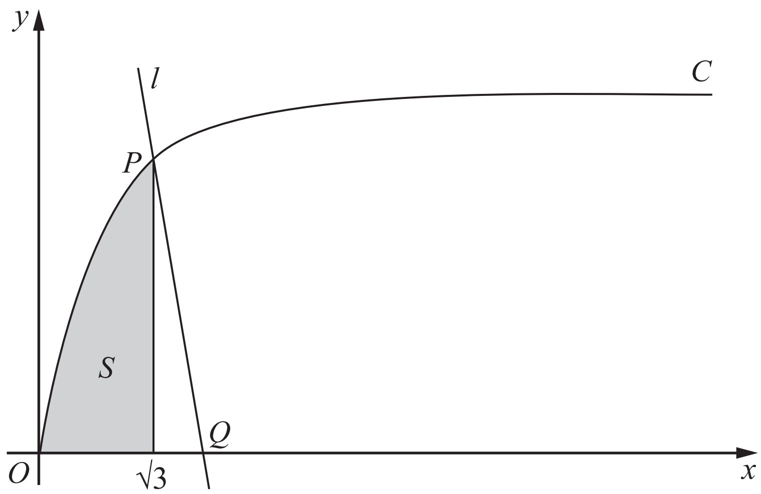

Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan\theta\,, \qquad y = \sin\theta\,, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$.

(a) Find the value of θ at the point P.

(2)

The line l is a normal to C at P. The normal cuts the x-axis at the point Q.

(b) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k.

(6)

Leave blank

Question 190 continued

(Total 8 marks)

Leave blank

191. The current, I amps, in an electric circuit at time t seconds is given by

$$I = 16 - 16(0.5)^t, \qquad t \geqslant 0$$

Use differentiation to find the value of $\frac{dI}{dt}$ when $t = 3$.

Give your answer in the form $\ln a$, where a is a constant.

(5)

(Total 5 marks)

Leave blank

192. A curve C has equation

$$2^x + y^2 = 2xy$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates $(3, 2)$.

(7)

(Total 7 marks)

Leave blank

193. A curve C has parametric equations

$$x = \sin^2 t, \quad y = 2\tan t, \quad 0 \leqslant t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t.

(4)

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x-axis at the point P.

(b) Find the x-coordinate of P.

(6)

Leave blank

Question 193 continued

(Total 10 marks)

Leave blank

194.

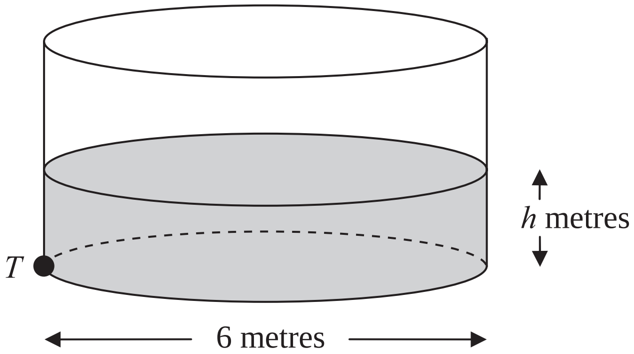

Figure 2

Figure 2 shows a cylindrical water tank. The diameter of a circular cross-section of the tank is 6 m. Water is flowing into the tank at a constant rate of 0.48π m^3 min^{-1}. At time t minutes, the depth of the water in the tank is h metres. There is a tap at a point T at the bottom of the tank. When the tap is open, water leaves the tank at a rate of $0.6\pi h$ m^3 min^{-1}.

Show that t minutes after the tap has been opened

$$75\frac{dh}{dt} = (4 - 5h)$$

(5)

Leave blank

Question 194 continued

(Total 5 marks)

Leave blank

195. The curve C has the equation

$$\cos 2x + \cos 3y = 1, \quad -\frac{\pi}{4} \leqslant x \leqslant \frac{\pi}{4}, \quad 0 \leqslant y \leqslant \frac{\pi}{6}$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(3)

The point P lies on C where $x = \frac{\pi}{6}$.

(b) Find the value of y at P.

(3)

(c) Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c\pi = 0$, where a, b and c are integers.

(3)

Leave blank

Question 195 continued

(Total 9 marks)

Leave blank

196. The area A of a circle is increasing at a constant rate of $1.5\,\text{cm}^2\,\text{s}^{-1}$. Find, to 3 significant figures, the rate at which the radius r of the circle is increasing when the area of the circle is $2\,\text{cm}^2$.

(5)

(Total 5 marks)

Leave blank

197. The curve C has the equation $ye^{-2x} = 2x + y^2$.

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P on C has coordinates $(0, 1)$.

(b) Find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Leave blank

Question 197 continued

(Total 9 marks)

Leave blank

198.

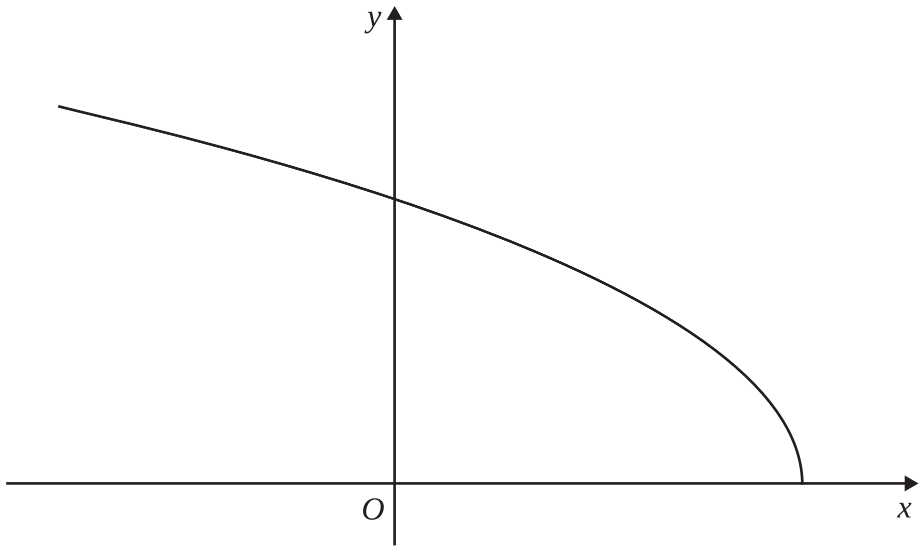

Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \quad y = 6\sin t, \quad 0 \leqslant t \leqslant \frac{\pi}{2}$$

Find the gradient of the curve at the point where $t = \frac{\pi}{3}$.

(4)

Leave blank

Question 198 continued

(Total 4 marks)

Leave blank

199. A curve C has the equation $y^2 - 3y = x^3 + 8$.

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(4)

(b) Hence find the gradient of C at the point where $y = 3$.

(3)

(Total 7 marks)

Leave blank

200.

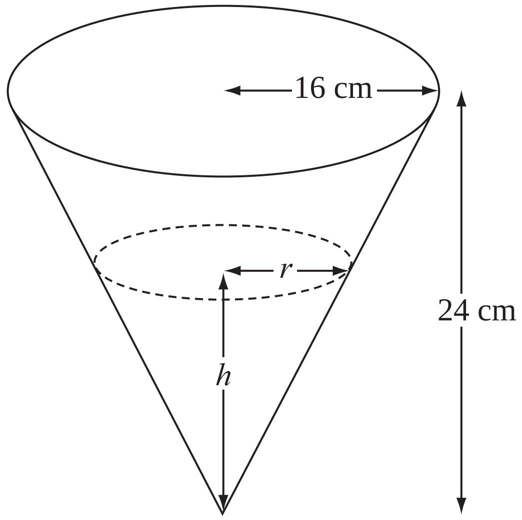

Figure 2

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in Figure 2. Water is flowing into the container. When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm^3.

(a) Show that $V = \dfrac{4\pi h^3}{27}$.

(2)

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \dfrac{1}{3}\pi r^2 h$.]

Water flows into the container at a rate of 8 cm^3 s^{-1}.

(b) Find, in terms of π, the rate of change of h when $h = 12$.

(5)

Leave blank

Question 200 continued

(Total 7 marks)

EXPERT TUITION

Leave blank

201.

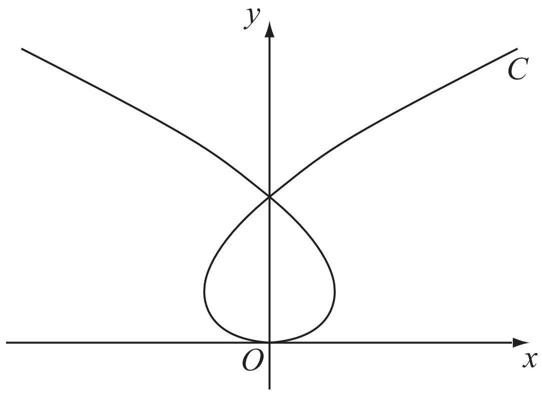

Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \quad y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,

(a) find the coordinates of A.

(1)

The line l is the tangent to C at A.

(b) Show that an equation for l is $2x - 5y - 9 = 0$.

(5)

The line l also intersects the curve at the point B.

(c) Find the coordinates of B.

(6)

Leave blank

Question 201 continued

(Total 12 marks)

EXPERT TUITION

Leave blank

202.

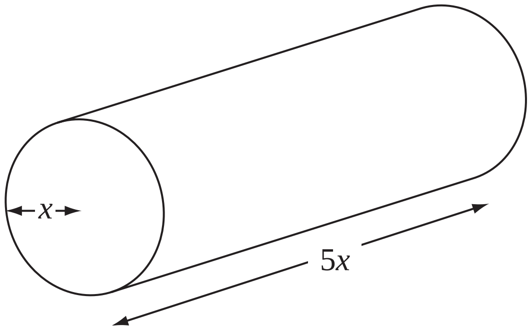

Figure 2

Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After t seconds the radius of the rod is x cm and the length of the rod is $5x$ cm.
The cross-sectional area of the rod is increasing at the constant rate of 0.032 $\text{cm}^2\ \text{s}^{-1}$.

(a) Find $\frac{\mathrm{d}x}{\mathrm{d}t}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures.

(4)

(b) Find the rate of increase of the volume of the rod when $x = 2$.

(4)

Leave blank

Question 202 continued

(Total 8 marks)

Leave blank

203. A curve has equation $3x^2 - y^2 + xy = 4$. The points P and Q lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q.

(a) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q.

(6)

(b) Find the coordinates of P and Q.

(3)

(Total 9 marks)

204.

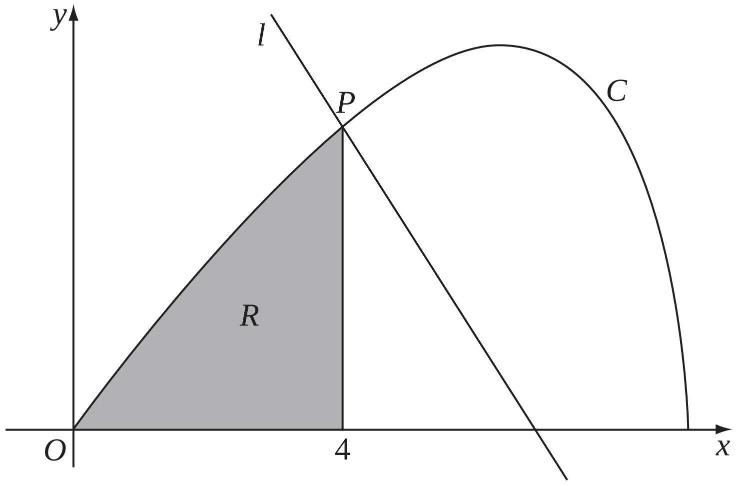

Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

(2)

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

Leave blank

Question 204 continued

(Total 8 marks)

EXPERT TUITION

Leave blank

205. A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where $x = -8$.

(3)

(b) Find the gradient of the curve at each of these points.

(6)

(Total 9 marks)

Leave blank

206. Liquid is pouring into a large vertical circular cylinder at a constant rate of 1600 $cm^3 s^{-1}$ and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 cm^2.

(a) Show that at time t seconds, the height h cm of liquid in the cylinder satisfies the differential equation

$$\frac{dh}{dt} = 0.4 - k\sqrt{h}, \text{ where } k \text{ is a positive constant.}$$

(3)

When $h = 25$, water is leaking out of the hole at 400 $cm^3 s^{-1}$.

(b) Show that $k = 0.02$

(1)

Leave blank

Question 206 continued

(Total 4 marks)

EXPERT TUITION