EXPERT TUITION

Maths Questions By Topic:

Exponentials & Logarithms
Mark Scheme

A-Level Edexcel

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Question	Scheme	Marks	AOs
1 (a)	265 thousand	B1	3.4
		(1)	
(b)	Attempts $\frac{dN_b}{dt} = 11e^{0.05t}$	M1	1.1b
	Substitutes $t = 10$ into their $\frac{dN_b}{dt}$	M1	3.4
	$\frac{dN_b}{dt} =$ awrt 18.1 which is approximately 18 thousand per year *	A1*	2.1
		(3)	
(c)	Sets $45 + 220e^{0.05t} = 10 + 800e^{-0.05t} \Rightarrow 220e^{0.05t} + 35 - 800e^{-0.05t} = 0$	M1	3.1b
	Correct quadratic equation $\Rightarrow 220\left(e^{0.05t}\right)^2 + 35e^{0.05t} - 800 = 0$	A1	1.1b
	$e^{0.05t} = 1.829, (-1.988) \Rightarrow 0.05t = \ln(1.829)$	M1	2.1
	$T = 12.08$	A1	1.1b
		(4)	
			(8 marks)
Notes:			

(a) May be seen in the question so watch out.
B1: Accept 265 thousand or 265 000 or equivalent such as 265 k but not just 265.
(b)
M1: Differentiates to a form $k\,e^{0.05t}, k > 0, k \neq 220$. Do not be too concerned about the lhs.
M1: Substitutes $t = 10$ into a changed function that was formed from an attempt at differentiation.
The left hand side must have implied differentiation. E.g. Rate = , $N', \frac{dN_b}{dt}, \frac{dN}{dt}$ or even $\frac{dy}{dx}$
A1*: Full and complete proof that requires

- some correct lhs seen at some point. E.g. "Rate = , " $\frac{dN_b}{dt}, \frac{dN}{dt}$ but condone N'.
- an intermediate line/answer of either $11e^{0.05\times 10}$ or awrt 18.1 before a minimal conclusion which must be referencing the 18 000 or 18 thousand

(c)

M1: Attempts to set both equations equal to each other and simplify the constant terms.

Look for $220\,e^{0.05t}+35=800\,e^{-0.05t}$ o.e but condone slips

It is also possible to set $\dfrac{N-45}{220}=\left(e^{0.05t}=\right)\dfrac{800}{N-10}$ and form an equation in N

A1: Correct quadratic form.

Look for $220\left(e^{0.05t}\right)^2+35e^{0.05t}-800=0$ or $220\,e^{0.1t}+35e^{0.05t}-800=0$ but allow with terms in different order such as $220\,e^{0.1t}+35e^{0.05t}=800$

FYI the equation in N is $N^2-55N-175550=0$

M1: Full attempt to find the value of t (or a constant multiple of t)

This involves the key step of recognising and solving a 3TQ in $e^{0.05t}$ followed by the use of lns.
If the answers to the quadratic just appear (from a calculator) you will need to check.
Accuracy should be to 3sf.
You may see different variables used such as x

$x=e^{0.05t},\ 220\,e^{0.1t}+35e^{0.05t}=800\Rightarrow 220x^2+35x=800\Rightarrow x=1.82...\Rightarrow t=20\ln 1.82...$

Allow use of calculator for solving the quadratic and for $e^{0.05t}=1.82..\Rightarrow t=12.08$

Via the N route it will involve substituting the positive solution to their quadratic into either equation to find a value for t/T using same rules as above.

A1: AWRT 12.08

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Answers with limited or no working in (b) and (c)

(b) A derivative in the correct form must be seen

(c) Candidates who state $45+220\,e^{0.05t}=10+800\,e^{-0.05t}$ followed by awrt 12.08 (presumably from using num-solv on their calculators) can score SC 1100. Rubric on the front of the paper states that "Answers without working may not gain full credit" so we demand a method in this part.

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Question	Scheme	Marks	AOs
2(a)	$p=10^{0.5}\left(\text{or } \log_{10} p=0.5\right)$ or $q=10^{0.03}\left(\text{or } \log_{10} q=0.03\right)$	M1	1.1b
	$p=$ awrt 3.162 or $q=$ awrt 1.072	A1	1.1b
	$p=10^{0.5}\left(\text{or } \log_{10} p=0.5\right)$ and $q=10^{0.03}\left(\text{or } \log_{10} q=0.03\right)$	dM1	3.1a
	$A=3.162\times1.072^{t}$	A1	3.3
		(4)	
(b)(i)	The initial mass (in kg) of algae (in the pond).	B1	3.4
(b)(ii)	The ratio of algae from one week to the next.	B1	3.4
		(2)	
(c)(i)	5.5 kg	B1	2.2a
(c)(ii)	$4=\text{"}3.162\text{"}\times\text{"}1.072\text{"}^{\,t}$ or $\log_{10}4=0.03\,t+0.5$	M1	3.4
	awrt 3.4 (weeks)	A1	1.1b
		(3)	
(d)	• The model predicts unlimited growth. • The weather may affect the rate of growth	B1	3.5b
		(1)	
			(10 marks)

Notes

(a)

M1: A correct equation in p **or** q. May be implied by a correct value for p or q.
Also score for rearranging the equation to the form $A=10^{0.5}...10^{0.03t}$

A1: For p = awrt 3.162 or q = awrt 1.072. May be embedded within the equation.

dM1: Correct equations in p **and** q. Also score for rearranging the equation to the form $A=10^{0.5}\times10^{0.03t}$

A1: Complete equation with p = awrt 3.162 and q = awrt 1.072. **Must be seen in (a)**
If p and q are just stated but the equation is not written with the values embedded then withhold this mark.
Withhold the final mark if the correct values for p and q result from incorrect working such as $A=10^{0.5}+10^{0.03t}\Rightarrow A=3.162\times1.072^{t}$.
If p and q are stated the wrong way round, take the stated equation as their final answer and isw.

(b)
(i)
B1: Must reference mass of algae and relating to initially/at the start/beginning
Examples of acceptable answers:
The mass of algae originally (in the pond)
p is the mass of algae when t = 0
Examples of answers we would not accept
p is how much algae there is at the beginning
The relationship between algae and number of weeks

(ii)

B1: Must reference the rate of change/multiplier and the time frame eg per week/every week/each week.

Examples of acceptable answers:

q is the rate at which the mass of algae increases for every week

The amount of algae increases by 7.2% each week (condone amount for mass in ii)

The proportional increase in mass of the algae each week

Examples of answers we would not accept:

q is how much algae will increase when t increases by 1

The amount that grows per unit of time

The rate at which the mass of algae in the small pond increases after t number of weeks

The rate in which the algae mass increases

(c)

B1: cao (including units)

M1: Setting up a correct equation to find t using the given equation or their part (a)
Substitution of A = 4 into their equation for A or the given equation is sufficient for this mark.

A1: awrt 3.4 (weeks). Accept any acceptable method (including trial and improvement)
Condone lack of units. isw if they subsequently convert to weeks and days. Allow awrt 3.5 (weeks) following p = awrt 3.16 and q = awrt 1.07.
An answer of only awrt 3.4 is M1A1, but an answer of 4 (weeks) with no working is M0A0

(d)

B1: Any reason why the rate of change, growth or the mass of algae might change or why the model in not realistic.
Be generous with the awarding of this mark as long as the answer has engaged with the context of the problem or the model

Examples of acceptable answers:

Seasonal changes (which would affect the growth rate)

Overcrowding (as it is a small pond)

Algae may stop growing (the model predicts unlimited growth)

Algae may die / be removed / eaten (so the rate of growth may not continue at the same rate)

Examples of answers we would not accept:

There could be other factors that affect the amount of algae (too vague)

The mass of algae might change

Question	Scheme	Marks	AOs
3(a)	$(k=)\ 0.8$	B1	1.1b
		(1)	
(b)	$1=0.8+1.4\mathrm{e}^{-0.5t} \Rightarrow 1.4\mathrm{e}^{-0.5t}=0.2$	M1	3.1b
	$-0.5t=\ln\left(\dfrac{0.2}{1.4}\right) \Rightarrow t=...$	M1	1.1b
	awrt 3.9 minutes	A1	1.1b
		(3)	
(c)	$\left(\dfrac{\mathrm{d}P}{\mathrm{d}t}=\right)\ -0.7\mathrm{e}^{-0.5t}$ $\left(\dfrac{\mathrm{d}P}{\mathrm{d}t}\right)_{t=2}=-0.7\mathrm{e}^{-0.5\times 2}$	M1	3.1b
	= awrt 0.258 (kg/cm^2 per minute)	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)

B1: Completes the equation for the model by obtaining (k =) 0.8 or equivalent.

(b) ***Be aware this could be solved entirely using a calculator which is not acceptable***

M1: For using the model with P = 1 and their value for k from (a) and proceeding to $A\mathrm{e}^{\pm 0.5t}=B$. Condone if A or B are negative for this mark.

M1: Uses correct log work to solve an equation of the form $A\mathrm{e}^{\pm 0.5t}=B$ leading to a value for t. They cannot proceed directly to awrt 3.9 without some intermediate working seen.

Eg $t=2\ln 7$ or $-2\ln\left(\dfrac{1}{7}\right)$ is acceptable.

Also allow $1.4\mathrm{e}^{-0.5t}=0.2 \Rightarrow -0.5t=-1.9459... \Rightarrow t=...$

This cannot be scored from an unsolvable equation (eg when their $k \geq 1$ so that $\mathrm{e}^{\pm 0.5t} \leq 0$).

A1: Accept awrt 3.9 minutes or t = awrt 3.9 with correct working seen.

eg $1.4\mathrm{e}^{-0.5t}=0.2 \Rightarrow t=3.9$ would be M1M0A0

(c) ***Be aware this can be solved entirely using a calculator which is not acceptable***

M1: Links rate of change to gradient and differentiates to obtain an expression of the form $A\mathrm{e}^{-0.5t}$ and substitutes t = 2. Do not accept $At\mathrm{e}^{-0.5t}$ as the derivative.

Beware that substituting t = 2 and proceeding from e^{-1} to e^{-2} is M0A0

A1: Obtains awrt 0.258 with differentiation seen. (Units not required) Condone awrt -0.258

Awrt ±0.258 with no working is M0A0. Isw after a correct answer is seen.

(Ignore in (c) any spurious notation on the LHS when differentiating such as P = …. or $\dfrac{\mathrm{d}y}{\mathrm{d}x}=...$)

Question	Scheme	Marks	AOs
4(a)(i)	$\log_3\left(\frac{x}{9}\right) = \log_3 x - \log_3 9 = p - 2$	B1	1.2
(ii)	$\log_3\left(\sqrt{x}\right) = \frac{1}{2}p$	B1	1.1b
		(2)	
(b)	$2\log_3\left(\frac{x}{9}\right) + 3\log_3\left(\sqrt{x}\right) = -11 \Rightarrow 2p - 4 + \frac{3}{2}p = -11 \Rightarrow p = ...$	M1	1.1b
	$p = -2$	A1	1.1b
	$\log_3 x = -2 \Rightarrow x = 3^{-2}$	M1	1.1b
	$x = \frac{1}{9}$	A1	1.1b
		(4)	
	Alternative for (b) not using (a):		
	$2\log_3\left(\frac{x}{9}\right) + 3\log_3\left(\sqrt{x}\right) = -11 \Rightarrow \log_3\left(\frac{x}{9}\right)^2 + \log_3\left(\sqrt{x}\right)^3 = -11$ $\Rightarrow \log_3 \frac{x^{\frac{7}{2}}}{81} = -11$	M1	1.1b
	$\Rightarrow \frac{x^{\frac{7}{2}}}{81} = 3^{-11}$ or equivalent eg $x^{\frac{7}{2}} = 3^{-7}$	A1	1.1b
	$x^{\frac{7}{2}} = 81 \times 3^{-11} \Rightarrow x^{\frac{7}{2}} = 3^4 \times 3^{-11} = 3^{-7} \Rightarrow x = \left(3^{-7}\right)^{\frac{2}{7}} = 3^{-2}$	M1	1.1b
	$x = \frac{1}{9}$	A1	1.1b
			(6 marks)

Notes

(a)(i)
B1: Recalls the subtraction law of logs and so obtains $p - 2$

(a)(ii)
B1: $\frac{1}{2}p$ oe

(b) ***Be aware this should be solved by non-calculator methods***

M1: Uses their results from part (a) to form a linear equation in p and attempts to solve leading to a value for p. Allow slips in their rearrangement when solving. Allow a misread forming the equation equal to 11 instead of -11

A1: Correct value for p

M1: Uses $\log_3 x = p \Rightarrow x = 3^p$ following through on what they consider to be their p. It must be a value rather than p

A1: $\left(x=\right)\frac{1}{9}$ cao with correct working seen. Must be this fraction. Do not penalise invisible brackets as long as the intention is clear.

Alternative:

M1: Correct use of log rules to achieve an equation of the form $\log_3 ... = \log_3 ...$ or $\log_3 ... = \text{a number}$ (typically -11). Condone arithmetical slips.

A1: Correct equation with logs removed.

M1: Uses inverse operations to find x. Condone slips but look for proceeding from $x^{\frac{a}{b}} = ... \Rightarrow x = ...^{\frac{b}{a}}$ where they have to deal with a fractional power.

A1: $\left(x=\right)\frac{1}{9}$ cao with correct working seen. Must be this fraction. Do not penalise invisible brackets as long as the intention is clear.

Question	Scheme	Marks	AOs
5(a)	$A = 1000$	B1	3.4
	$2000 = 1000\mathrm{e}^{5k}$ or $\mathrm{e}^{5k} = 2$	M1	1.1b
	$\mathrm{e}^{5k} = 2 \Rightarrow 5k = \ln 2 \Rightarrow k = \ldots$	M1	2.1
	$N = 1000\mathrm{e}^{\left(\frac{1}{5}\ln 2\right)t}$ or $N = 1000\mathrm{e}^{0\ 139t}$	A1	3.3
		(4)	
(b)	$\dfrac{\mathrm{d}N}{\mathrm{d}t} = 1000 \times \left(\dfrac{1}{5}\ln 2\right)\mathrm{e}^{\left(\frac{1}{5}\ln 2\right)t}$ or $\dfrac{\mathrm{d}N}{\mathrm{d}t} = 1000 \times 0.139\mathrm{e}^{0\ 139t}$ $\left(\dfrac{\mathrm{d}N}{\mathrm{d}t}\right)_{t=8} = 1000 \times \left(\dfrac{1}{5}\ln 2\right)\mathrm{e}^{8\times\frac{1}{5}\ln 2}$ or $\left(\dfrac{\mathrm{d}N}{\mathrm{d}t}\right)_{t=8} = 1000 \times 0.139\mathrm{e}^{0\ 139\times 8}$	M1	3.1b
	= awrt 420	A1	1.1b
		(2)	
(c)	$500\mathrm{e}^{1\ 4\times\left(\frac{1}{5}\ln 2\right)T} = 1000\mathrm{e}^{\left(\frac{1}{5}\ln 2\right)T}$ or $500\mathrm{e}^{1\ 4\times\text{"}0\ 139\text{"}t} = 1000\mathrm{e}^{\text{"}0\ 139\text{"}t}$	M1	3.4
	Correct method of getting a linear equation in T E.g. $0.08T\ln 2 = \ln 2$ or $1.4 \times \text{"}0.339\text{"}T = \ln 2 + \text{"}0.339\text{"}t$	M1	2.1
	$T = 12.5$ hours	A1	1.1b
		(3)	
			(9 marks)

Notes

Mark as one complete question. Marks in (a) can be awarded from (b)

(a)

B1: Correct value of A for the model. Award if equation for model is of the form $N = 1000\mathrm{e}^{\ t}$

M1: Uses the model to set up a correct equation in k. Award for substituting $N = 2000, t = 5$ following through on their value for A.

M1: Uses correct ln work to solve an equation of the form $a\mathrm{e}^{5k} = b$ and obtain a value for k

A1: Correct equation of model. Condone an ambiguous $N = 1000\mathrm{e}^{\frac{1}{5}\ln 2t}$ unless followed by something incorrect. Watch for $N = 1000 \times 2^{\frac{1}{5}t}$ which is also correct

(b)

M1: Differentiates $\alpha\mathrm{e}^{kt}$ to $\beta\mathrm{e}^{kt}$ and substitutes $t = 8$ (Condone $\alpha = \beta$ so long as you can see an attempt to differentiate)

A1: For awrt 420 (2sf).

(c)

M1: Uses both models to set up an equation in T using their value for k, but also allow in terms of k

M1: Uses correct processing using lns to obtain a linear equation in T (or t)

A1: Awrt 12.5

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Answers to (b) and (c) appearing without working (i.e. from a calculator).

It is important that candidates show sufficient working to make their methods clear.

(b) If candidate has for example $N = 1000\mathrm{e}^{0\ 139t}$, and then writes at $t = 8$ $\dfrac{\mathrm{d}N}{\mathrm{d}t} = $ awrt 420 award both marks. Just the answer from a correct model equation score SC 1,0.

(c) The first M1 should be seen E.g $500\mathrm{e}^{1\ 4\times\text{"}0\ 139\text{"}t} = 1000\mathrm{e}^{\text{"}0\ 139\text{"}t}$

If the answer $T = 12.5$ appears without any further working score SC M1 M1 A0

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Question	Scheme	Marks	AOs
6	$\dfrac{9^{x-1}}{3^{y+2}} = 81 \Rightarrow \dfrac{3^{2x-2}}{3^{y+2}} = 3^4$ or $\dfrac{9^{x-1}}{3^{y+2}} = 81 \Rightarrow \dfrac{9^{x-1}}{9^{\frac{1}{2}(y+2)}} = 9^2$	M1	1.1b
	$\Rightarrow 2x-2-y-2=4 \Rightarrow y=$ or $\Rightarrow x-1-\dfrac{1}{2}y-1=2 \Rightarrow y=$	dM1	1.1b
	$\Rightarrow y = 2x-8$	A1	1.1b
		(3)	
Alt	Eg. $\log_3\left(\dfrac{9^{x-1}}{3^{y+2}}\right) = \log_3 81$	M1	1.1b
	$\Rightarrow (x-1)\log_3\left(9^{x-1}\right) - (y+2)\log_3\left(3^{y+2}\right) = 4$ $\Rightarrow 2(x-1) - y - 2 = 4 \Rightarrow y =$	dM1	1.1b
	$\Rightarrow y = 2x-8$	A1	1.1b
			(3 marks)

Notes

M1: Attempts to set 9^{x-1} and 81 as powers of 3. Condone $9^{x-1} = 3^{2x-1}$ and $9^{x-1} = 3^{3x-3}$.
Alternatively attempts to write each term as a logarithm of base 3 or 9. You must see the base written to award this mark.

dM1: Attempts to use the addition (or subtraction) index law, or laws or logarithms, correctly and rearranges the equation to reach y in terms of x. Condone slips in their rearrangement.

A1: $y = 2x-8$

Question	Scheme	Marks	AOs
7 (a)	35 (km^2)	B1	3.4
		(1)	
(b)	Sets their $60 = 80 - 45e^{14c} \Rightarrow 45e^{14c} = 20$	M1 A1	1.1b 1.1b
	$\Rightarrow c = \frac{1}{14}\ln\left(\frac{20}{45}\right) = \cdots - 0.0579$	dM1	3.1b
	$A = 80 - 45e^{-0\,0579t}$	A1	3.3
		(4)	
(c)	Gives a suitable answer • The maximum area covered by trees is only 80km^2 • The "80" would need to be "100" • Substitutes 100 into the equation of the model and shows that the formula fails with a reason eg. you cannot take a log of a negative number	B1	3.5b
		(1)	
		(6 marks)	

Notes

(a)

B1: Uses the equation of the model to find that 35 (km^2) of the reserve was covered on 1st January 2005. Do not accept eg. 35 m^2

(b)

M1: Sets their $60 = 80 - 45e^{14c} \Rightarrow Ae^{14c} = B$

A1: $45e^{14c} = 20$ or equivalent.

dM1: A full and careful method using precise algebra, correct log laws and a knowledge that e^x and ln*x* are inverse functions and proceeds to a value for *c*.

A1: Gives a complete equation for the model $A = 80 - 45e^{-0\,0579t}$

(c)

B1: Gives a suitable interpretation (See scheme)

Question	Scheme		Marks	AOs
8 (a)	$\log_{10} h = 2.25 - 0.235\log_{10} m$ $\Rightarrow h = 10^{2\ 25-0\ 235\log_{10} m}$ $\Rightarrow h = 10^{2\ 25} \times m^{-0\ 235}$	$h = pm^{q}$ $\Rightarrow \log_{10} h = \log_{10} p + \log_{10} m^{q}$ $\Rightarrow \log_{10} h = \log_{10} p + q\log_{10} m$	M1	1.1b
	Either one of $p = 10^{2\ 25}$ $q = -0.235$	Or either one of $\log_{10} p = 2.25$ $q = -0.235$	A1	1.1b
	$\Rightarrow p = 178$ and $q = -0.235$		A1	2.2a
			(3)	
(b)	$h = "178" \times 5^{"-0\ 235"}$	$\log_{10} h = "2.25" - "0.235"\log_{10} 5$	M1	3.1b
	$h = 122$	$h = 122$	A1	1.1b
	Reasonably accurate (to 2 sf) so suitable		A1ft	3.2b
			(3)	
(c)	"p" would be the (resting) heart rate (in bpm) of a mammal with a mass of 1 kg		B1	3.4
			(1)	
				(7 marks)

Notes

(a)

M1: Establishes a link between $h = pm^{q}$ and $\log_{10} h = 2.25 - 0.235\log_{10} m$.
May be implied by a correct equation in p or q

A1: For a correct equation in p or q

A1: $p = 178$ and $q = -0.235$

(b)

M1: Uses either model to set up an equation in h (or m)

A1: h = awrt 122. Condone h = awrt 122 bpm

A1ft: Comments on the suitability of the model. Follow through on their answer.

Requires a comment consistent with their answer from using the model.

E.g. It is a suitable model as it is only “3” bpm away from the real value ✓
Do not allow an argument stating that it should be the same.
It is an unsuitable model as “122” bpm is not equal to 119 bpm ×

(c)

B1: "p" would be the (resting) heart rate of a mammal with a mass of 1 kg

Question	Scheme	Marks	AOs
9	$4^{3p-1}=5^{210}\Rightarrow(3p-1)\log 4=210\log 5$	M1	1.1b
	$\Rightarrow 3p=\dfrac{210\log 5}{\log 4}+1\Rightarrow p=...$	dM1	2.1
	$p=$ awrt 81.6	A1	1.1b
		(3)	
			(3 marks)
Notes:			

M1: Takes logs of both sides and uses the power law on **each** side.
Condone a missing bracket on lhs and slips.
Award for any base including ln but the logs must be the same base.

dM1: A full method leading to a value for *p*.
It is dependent upon the previous M mark and there must be an attempt to change the subject of the equation in the correct order.

Look for $(3p-1)\log 4=210\log 5\Rightarrow 3p=\dfrac{210\log 5}{\log 4}\pm1\Rightarrow p=...$ condoning slips.

You may see numerical versions E.g. $(3p-1)\times 0.60=210\times 0.7\Rightarrow 1.8p-0.6=147\Rightarrow p=82$

Use of incorrect log laws would be dM0. E.g $(3p-1)\log 4=210\log 5\Rightarrow 3p=210\log\dfrac{5}{4}\pm1$

A1: awrt 81.6 following a correct method. Bracketing errors can be recovered for full marks
A correct answer with no working scores 0 marks. The demand in the question is clear.

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There are alternatives:

E.g. A starting point could be $4^{3p-1}=5^{210}\Rightarrow\dfrac{4^{3p}}{4}=5^{210}$

but the first M mark is still for using the power law correctly on each side

In such a method the dM1 mark is for using **all** log rules correctly and proceeding to a value for *p*.

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Using base 4 or 5

E.g. $4^{3p-1}=5^{210}\Rightarrow(3p-1)=\log_4 5^{210}$

The M mark is not scored until $(3p-1)=210\log_4 5$

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Question	Scheme		Marks	AOs
10 (a)	$\log_{10} V = 0.072t + 2.379$ $\Rightarrow V = 10^{0.072t+2.379}$ $\Rightarrow V = 10^{0.072t} \times 10^{2.379}$	$V = ab^t$ $\Rightarrow \log_{10} V = \log_{10} a + \log_{10} b^t$ $\Rightarrow \log_{10} V = \log_{10} a + t\log_{10} b$	B1	2.1
	States either $a = 10^{2.379}$ or $b = 10^{0.072}$	States either $\log_{10} a = 2.379$ or $\log_{10} b = 0.072$	M1	1.1b
	$a = 239$ or $b = 1.18$	$a = 239$ or $b = 1.18$	A1	1.1b
	Either $V = 239 \times 1.18^t$ or imply by $a = 239, b = 1.18$		A1	1.1b
			(4)	
(b)	The value of *ab* is the (total) number of views of the advert **1 day after** it went live.		B1	3.4
			(1)	
(c)	Substitutes $t = 20$ in either equation and finds V Eg $V = 239 \times 1.18^{20}$		M1	3.4
	Awrt 6500 or 6600		A1	1.1b
			(2)	
				(7 marks)

(a) **Condone** $\log_{10}$ **written** log **or** lg **written throughout the question**

B1: Scored for showing that $\log_{10} V = 0.072t + 2.379$ can be written in the form $V = ab^t$ or vice versa

Either starts with $\log_{10} V = 0.072t + 2.379$ (may be implied) and **shows lines**

$$V = 10^{0.072t+2.379} \text{ and } V = 10^{0.072t} \times 10^{2.379}$$

Or starts with $V = ab^t$ (implied) and **shows the lines**

$$\log_{10} V = \log_{10} a + \log_{10} b^t \text{ and } \log_{10} V = \log_{10} a + t\log_{10} b$$

M1: For a correct equation in *a* or a correct equation in *b*

A1: Finds either constant. Allow $a =$ awrt 240 or $b =$ awrt 1.2 following a correct method

A1: Correct solution: Look for $V = 239 \times 1.18^t$ or $a = 239, b = 1.18$
Note that this is NOT awrt

(b)

B1: See scheme. Condone not seeing total. Do not allow number of views at the start or similar.

(c)

M1: Substitutes $t = 20$ in either their $V = 239 \times 1.18^t$ or $\log_{10} V = 0.072t + 2.379$ and uses a correct method to find V

A1: Awrt 6500 or 6600

Question	Scheme	Marks	AOs
11 (a)	Uses a model $V = Ae^{\pm kt}$ oe (See next page for other suitable models)	M1	3.3
	Eg. Substitutes $t = 0, V = 20\,000 \Rightarrow A = 20000$	M1	1.1b
	Eg. Substitutes $t = 1, V = 16000 \Rightarrow 16000 = 20000e^{-1k} \Rightarrow k = ..$	dM1	3.1b
	$V = 20000e^{-0.223t}$	A1	1.1b
		(4)	
(b)	Substitutes $t = 10$ in their $V = 20000e^{-0.223t} \Rightarrow V = (£\,2150)$	M1	3.4
	Eg. The model is reliable as £2150 ≈ £2000	A1	3.5a
		(2)	
(c)	Make the "-0.223" less negative. Alt: Adapt model to for example $V = 18000e^{-0.223t} + 2000$	B1ft	3.3
		(1)	
			(7 marks)

(**a**) **Option 1**

M1: For $V = Ae^{\pm kt}$ Do not allow if *k* is fixed, eg $k = -0.5$

Condone different variables $V \leftrightarrow y$ $t \leftrightarrow x$ for this mark, but for A1 *V* and *t* must be used.

M1: Substitutes $t = 0 \Rightarrow A = 20000$ into their exponential model

Candidates may start by simply writing $V = 20000e^{kt}$ which would be M1 M1

dM1: Substitutes $t = 1 \Rightarrow 16000 = 20000e^{-1k} \Rightarrow k = ..$via the correct use of logs.

It is dependent upon both previous M's.

A1: $V = 20000e^{-0.223t}$ (with accuracy to at least 3sf) or $V = 20000e^{t\ln 0.8}$

A correct linking formula with correct constants must be seen somewhere in the question

(b)

M1: Uses a model of the form $V = Ae^{\pm kt}$ to find the value of *V* when *t* = 10.

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants with accuracy to at least 2sf .

Compares *V* = (£) 2150 with (£) 2 000 and states "reliable as $2150 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

In the alternative it is for comparing their value of *t* with 10 and making a suitable comment as to the reliability of their model with a reason.

$V = 20000e^{-0.223t} \Rightarrow 2000 = 20000e^{-0.223t} \Rightarrow t = 10.3$ years.

Deduction Reliable model as the time is approximately the same as 10 years. A candidate can argue that the model is unreliable if they can give a suitable reason.

(c)

B1ft: For a correct statement. Eg states that the value of their '-0.223' should become less negative.

Alt states that the value of their '0.223' should become smaller. If they refer to *k* then refer to the model and apply the same principles.

Condone the fact that they don't state their -0.223 doesn't lie in the range $(-0.223, 0)$

(a) Option 2

M1: For $V = Ar^t$ or equivalent such as $V = kr^{t-1}$

Condone different variables $V \leftrightarrow y$ $t \leftrightarrow x$ for this mark, but for A1 V and t must be used.

M1: Uses $t = 0 \Rightarrow A = 20\,000$ in their model. Alternatively uses $(0,20000)$ and $(1,16000)$ to give $r = \frac{4}{5}$ oe

You may award if one of the number pair $(0,20000)$ or $(1,16000)$ works in an allowable model

dM1: $t = 1 \Rightarrow 16\,000 = 20\,000r^1 \Rightarrow r = ..$ Dependent upon both previous M's

In the alternative it would be for using $r = \frac{4}{5}$ with one of the points to find $A = 20\,000$

You may award if both number pairs $(0,20000)$ or $(1,16000)$ work in an allowable model

A1: $V = 20000 \times 0.8^t$ Note that $V = 20000 \times 1.25^{-t}$ $V = 16000 \times 0.8^{t-1}$ and is also correct

(b)

M1: Uses a model of the form $V = Ar^t$ oe to find the value of V when t = 10. Eg. 20000×0.8^{10}

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants also allowing an accuracy to 2sf.

Compares (£) 2147 with (£) 2 000 and states "reliable as $2147 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

(c)

B1ft: States a value of r in the range $(0.8,1)$ or states would increase the value of "0.8"

They do not need to state that "0.8" must lie in the range $(0.8,1)$

Condone increase the 0.8. Also allow decrease the "1.25" for $V = 20000 \times 1.25^{-t}$

....

(a) Option 3

M1: They may suggest an exponential model with a lower bound. For example, for $V = Ae^{\pm kt} + 2000$ The bound must be stated but do not allow k to be fixed . Allow as long as the bound < 10 000

M1: $t = 0, V = 20\,000 \Rightarrow A = 18\,000$

dM1: $t = 1, V = 16\,000 \Rightarrow 16\,000 = 2\,000 + 18\,000e^k \Rightarrow k = ..$ Dependent upon both previous M's

A1: $V = 18\,000 \times e^{-0\,251t} + 2\,000$

(b)

M1: Uses their model to find the value of V when t = 10.

Alternatively substitutes $V = 2000$ into their model and finds t

A1: For $V = 18\,000 \times e^{-0\,251 \times 10} + 2\,000 = £3462.83$ Deduction: Unreliable model as £3462.83 is not close to £2 000 This can only be scored from an acceptable model with correct constants

(c)

B1: States make the value of k or the -0.251 greater (or less negative) so that it lies in the range $(-0.251, 0)$

Condone 'make the value of k or the -0.251 greater (or less negative)'

....

It is entirely possible that they start part (a) from a differential equation.

M1: $\frac{dV}{dt} = kV \Rightarrow \int \frac{dV}{V} = \int k\,dt \Rightarrow \ln V = kt + c$ M1: $\ln 20000 = c$

dM1: Using $t = 1, V = 16\,000 \Rightarrow k = ..$ A1: $\ln V = -\ln\left(\frac{5}{4}\right)t + \ln 20000$

Question	Scheme	Marks	AOs
12 (a)	States $\log a - \log b = \log \frac{a}{b}$	B1	1.2
	Proceeds from $\frac{a}{b} = a - b \to ...... \to ab - a = b^2$	M1	1.1b
	$ab - a = b^2 \to a(b-1) = b^2 \Rightarrow a = \frac{b^2}{b-1}$ *	A1*	2.1
		(3)	
(b)	States either $b > 1$ or $b \neq 1$ with reason $\frac{b^2}{b-1}$ is not defined at $b = 1$ oe	B1	2.2a
	States $b > 1$ and explains that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$	B1	2.4
		(2)	
			(5 marks)

(**a)**

B1: States or uses $\log a - \log b = \log \frac{a}{b}$. This may be awarded anywhere in the question and may be implied by a **starting line** of $\frac{a}{b} = a - b$ oe. Alternatively takes $\log b$ to the rhs and uses the addition law $\log(a-b) + \log b = \log(a-b)b$. Watch out for $\log a - \log b = \frac{\log a}{\log b} = \log\left(\frac{a}{b}\right)$ which could score 010

M1: Attempts to make 'a' the subject. Awarded for proceeding from $\frac{a}{b} = a - b$ to a point where the two terms in a are on the same side of the equation and the term in b is on the other.

A1*: CSO. Shows clear reasoning and correct mathematics leading to $a = \frac{b^2}{b-1}$. Bracketing must be correct.

Allow a candidate to proceed from $ab - a = b^2$ to $a = \frac{b^2}{b-1}$ without the intermediate line.

(b)

B1: For deducing $b \neq 1$ as $a \to \infty$ oe such as "you cannot divide by 0" or correctly deducing that $b > 1$. They may state that b cannot be less than 1.

B1: For $b > 1$ and explaining that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$ (as b^2 is positive)

As a minimum accept that $b > 1$ as a cannot be negative.

Note that $a > b > 1$ is a correct statement but not sufficient on its own without an explanation.

....

Alt (a)

Note that it is possible to attempt part (a) by substituting $a = \frac{b^2}{b-1}$ into both sides of the given identity.

$$\log a - \log b = \log(a-b) \Rightarrow \log\left(\frac{b^2}{b-1}\right) - \log b = \log\left(\frac{b^2}{b-1} - b\right)$$

B1: Score for $\log\left(\frac{b^2}{b-1}\right) - \log b = \log\left(\frac{b}{b-1}\right)$

M1: Attempts to write $\frac{b^2}{b-1} - b$ as a single fraction $\frac{b^2}{b-1} - b = \frac{b^2 - b(b-1)}{b-1}$

Allow as two separate fractions with the same common denominator

A1*: Achieves lhs and rhs as $\log\left(\frac{b}{b-1}\right)$ **and** makes a comment such as "hence true"

Question	Scheme	Marks	AOs
13 (a)	(£)18 000	B1	3.4
		(1)	
(b)	(i) $\frac{dV}{dt} = -3925e^{-0.25t}$	M1 A1	3.1b 1.1b
	Sets $-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ * cso	A1*	3.4
	(ii) $e^{-0.25T} = 0.127... \Rightarrow -0.25T = \ln 0.127...$	M1	1.1b
	$T = 8.24$ (awrt)	A1	1.1b
	8 years 3 months	A1	3.2a
		(6)	
(c)	2 300	B1	1.1b
		(1)	
(d)	Any suitable reason such as • Other factors affect price such as condition/mileage • If the car has had an accident it will be worth less than the model predicts • The price may go up in the long term as it becomes rare • £2300 is too large a value for a car's scrap price. Most cars scrap for around £400	B1	3.5b
		(1)	
		(9 marks)	

Notes

(a)

B1: £18 000 There is no requirement to have the units

(b)(i)

M1: Award for making the link between gradient and rate of change.

Score for attempting to differentiate V to $\frac{dV}{dt} = ke^{-0.25t}$ An attempt at both sides are required.

For the left hand side you may condone attempts such as $\frac{dy}{dx}$

A1: Achieves $\frac{dV}{dt} = -3925e^{-0.25t}$ or $\frac{dV}{dt} = 15700 \times -0.25e^{-0.25t}$ with both sides correct

A1*: Sets $-3925e^{-0.25T} = -500$ oe and proceeds to $3925e^{-0.25T} = 500$

This is a given answer and to achieve this mark, all aspects must be seen and be correct.

t must be changed to T at some point even if just at the end of their solution/proof

SC: Award SC 110 candidates who simply write

$-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ without any mention or reference to $\frac{dV}{dt}$

Or $15700 \times -0.25e^{-0.25t} = -500 \Rightarrow 3925e^{-0.25T} = 500$ without any mention or reference to $\frac{dV}{dt}$

(b)(ii)

M1: Proceeds from $e^{-0.25T} = A, A > 0$ using ln's to $\pm 0.25T = ..$

Alternatively takes lns first $3925e^{-0.25T} = 500 \Rightarrow \ln 3925 - 0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$

but $3925e^{-0.25T} = 500 \Rightarrow \ln 3925 \times -0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$ is M0

A1: $T =$ awrt 8.24 or $-\frac{1}{0.25}\ln\left(\frac{20}{157}\right)$ Allow $t =$ awrt 8.24

Notes on Question 13 continue
A1: 8 years 3 months. Correct answer and solution only Answers obtained numerically score 0 marks. The M mark must be scored. **(c)** **B1:** 2 300 but condone £2 300 **(d)** **B1:** Any suitable reason. See scheme Accept "Scrappage" schemes may pay more (or less) than £ 2 300. Do not accept "does not take into account inflation" It asks for a limitation of the model so candidates cannot score marks by suggesting other suitable models " the value may fall by the same amount each year"

Question	Scheme	Marks	AOs
14 (a)	(i) Method to find p Eg. Divides $32000 = Ap^4$ by $50000 = Ap^{11}$ $$p^7 = \frac{50000}{32000} \Rightarrow p = \sqrt[7]{\frac{50000}{32000}} = ...$$	M1	3.1a
	$p = 1.0658$	A1	1.1b
	(ii) Substitutes their $p = 1.0658$ into either equation and finds A $$A = \frac{32000}{'1.0658'^4} \text{ or } A = \frac{50000}{'1.0658'^{11}}$$	M1	1.1b
	$A = 24795 \rightarrow 24805 \simeq 24\,800$ *	A1*	1.1b
		(4)	
(b)	A / $(£)24\,800$ is the value of the car on 1st January 2001	B1	3.4
	p /1.0658 is the factor by which the value rises each year. Accept that the value rises by 6.6 % a year (ft on their p)	B1	3.4
		(2)	
(c)	Attempts $100000 = '24800' \times '1.0658'^t$		
	$'1.0658'^t = \frac{100000}{24800}$	M1	3.4
	$t = \log_{1.0658}\left(\frac{100000}{24800}\right)$	dM1	1.1b
	$t = 21.8$ or 21.9	A1	1.1b
	cso 2022	A1	3.2a
		(4)	
			(10 marks)

(a) (i)

M1: Attempts to use both pieces of information within $V = Ap^t$, eliminates A correctly and solves an equation of the form $p^n = k$ to reach a value for p.

Allow for slips on the 32 000 and 50 000 and the values of t.

A1: $p =$ awrt 1.0658

Both marks can be awarded from incorrect but consistent interpretations of t. Eg.

$32000 = Ap^5, 50000 = Ap^{12}$

(a)(ii)

M1: Substitutes their $p = 1.0658$ into either of their equations and finds A

Eg $A = \frac{32000}{1.0658^4}$ or $A = \frac{50000}{1.0658^7}$ but you may follow through on incorrect equations from part (i)

A1*: Shows that A is between 24 795 and 24 805 before you see ' =24 800' or '≈ 24800'. Accept with or without units.

An alternative to (ii) is to start with the given answer.

M1: Attempts $24800 \times '1.0658'^4 = (32000.34)$

A1: 24800×1.0658^4, achieves a value between 31095 and 32005 followed by $\approx 32\,000$ hence A must be $\approx 24\,800$

(b)

B1: States that A is the value of the car on 1st January 2001.
The statement must reference **the car**, its **cost/value**, and **"0" time**
Allow 'it is the initial value of the car" "it is the cost of the car at $t = 0$" "it is the cars starting value"

B1: States that p is the rate at which the value of the car rises each year.
The statement must reference **a yearly rate** and **an increase in value or multiplier**.
They could reference the 1.0658 Eg "The cars value rises by 6.5 % each year."
Allow "p is the rate the cars value is rising each year" "it is the proportional increase in value of the car each year" "the factor by which the value of the car is rising each year" 'its value appreciates by 6.5% per year' Allow ' the value of the car multiplies by p each year'
Do not allow "by how much the value of the car rises each year " or "it is the rate of inflation"

(c)

M1: Uses the model $100000 = '24800' \times '1.0658'^t$ and proceeds to their $'1.0658'^t = k$
Allow use of any inequality here.

dM1: For the complete method of (i) using the information given with their equation of the model **and** (ii) translating the situation into a correct method to find 't'

A1: $(t) =$ awrt 21.8 or 21.9 or $\log_{1.0658}\left(\dfrac{100000}{24800}\right)$ oe

A1: States in the year 2022. A candidate using a GP formula can be awarded full marks
Allow different methods in part (c).
Eg Via GP a formula
M1: $24800 \times '1.0658'^{n-1} = 100000 \Rightarrow '1.0658'^{n-1} = K$
dM1: Uses a correct method to find n.
A2: 2022
Via (trial and improvement)

M1: Uses the model by substituting integer values of t into their $V = Ap^t$ so that for $t = n, V < 100\,000$ or $t = n+1, V > 100\,000$
(So for the correct A and p this would be scored for $t = 21, V \approx £95000$ or $t = 21, V \approx £101\,000$

dM1: For a complete method showing that this is the least value. So both of the above values

A1: Allow for 22 following correct and accurate results (awrt nearest £1000 is sufficient accuracy)

A1: As before

Question	Scheme		Marks	AOs
15 (a)	Identifies one of the two errors "You cannot use the subtraction law without dealing with the 2 first" " They undo the logs incorrectly. It should be $x = 2^3 = 8$ "		B1	2.3
	Identifies both errors. See above.		B1	2.3
			(2)	
(b)	$\log_2\left(\dfrac{x^2}{\sqrt{x}}\right) = 3$	$\dfrac{3}{2}\log_2(x) = 3$	M1	1.1b
	$x^{\frac{3}{2}} = 2^3$ or $\dfrac{x^2}{\sqrt{x}} = 2^3$	$x = 2^2$	M1	1.1b
	$x = \left(2^3\right)^{\frac{2}{3}} = 4$	$x = 4$	A1	1.1b
			(3)	
				(5 marks)

(a)

B1: States one of the two errors.

Error One: Either in words states 'They cannot use the subtraction law without dealing with the 2 first' or writes ' that line 2 should be $\log_2\left(\dfrac{x^2}{\sqrt{x}}\right)$ $(=3)$' If they rewrite line two it must be correct. Allow 'the coefficient of each log term is different so we cannot use the subtraction law'

Allow responses such as 'it must be $\log x^2$ before subtracting the logs'

Do not accept an incomplete response such as "the student ignored the 2". **There must be some reference to the subtraction law as well.**

Error Two: Either in words states 'They undo the log incorrectly' or writes that 'if $\log_2 x = 3$ then $x = 2^3 = 8$' If it is rewritten it must be correct. Eg $x = \log_2 9$ is B0

B1: States both of the two errors. (See above)

(b)

M1: Uses a correct method of combining the two log terms. Either uses both the power law and the subtraction law to reach a form $\log_2\left(\dfrac{x^2}{\sqrt{x}}\right) = 3$ oe. Or uses both the power law and subtraction to reach $\dfrac{3}{2}\log_2(x) = 3$

M1: Uses correct work to "undo" the log. Eg moves from $\log_2\left(Ax^n\right) = b \Rightarrow Ax^n = 2^b$

This is independent of the previous mark so allow following earlier error.

A1: cso $x = 4$ achieved with at least one intermediate step shown. Extra solutions would be A0

SC: If the "answer" rather than the "solution" is given score 1,0,0.

Question	Scheme	Marks	AOs
16(a)	For a correct equation in p or q $\quad p=10^{4.8}$ or $q=10^{0.05}$	M1	1.1b
	For $p=$ awrt 63100 or $q=$ awrt 1.122	A1	1.1b
	For correct equations in p and q $\quad p=10^{4.8}$ and $q=10^{0.05}$	dM1	3.1a
	For $p=$ awrt 63100 and $q=$ awrt 1.122	A1	1.1b
		(4)	
(b)	(i) The value of the painting on 1st January 1980	B1	3.4
	(ii) The proportional increase in value each year	B1	3.4
		(2)	
(c)	Uses $V=63100\times1.122^{30}$ or $\log V=0.05\times30+4.8$ leading to $V=$	M1	3.4
	$=$ awrt $(£)2000000$	A1	1.1b
		(2)	
			(8 marks)

Notes

(a)

M1: For a correct equation in p or q This is usually $p=10^{4.8}$ or $q=10^{0.05}$ but may be $\log q=0.05$ or $\log p=4.8$

A1: For $p=$ awrt 63100 or $q=$ awrt 1.122

M1: For linking the two equations and forming correct equations in p and q. This is usually $p=10^{4.8}$ and $q=10^{0.05}$ but may be $\log q=0.05$ and $\log p=4.8$

A1: For $p=$ awrt 63100 and $q=$ awrt 1.122 Both these values implies M1 M1

....

ALT I(a)

M1: Substitutes $t=0$ and states that $\log p=4.8$

A1: $p=$ awrt 63100

M1: Uses their found value of p and another value of t to find form an equation in q

A1: $p=$ awrt 63100 and $q=$ awrt 1.122

....

(b)(i)

B1: The value of the painting on 1st January 1980 (is £63 100)

Accept the original value/cost of the painting or the initial value/cost of the painting

(b)(ii)

B1: The proportional increase in value each year. Eg Accept an explanation that explains that the value of the painting will rise 12.2% a year. (Follow through on their value of q.)

Accept "the rate" by which the value is rising/price is changing. "1.122 is the decimal multiplier representing the year on year increase in value"

Do not accept "the amount" by which it is rising or "how much" it is rising by

If they are not labelled (b)(i) and (b)(ii) mark in the order given but accept any way around as long as clearly labelled " p is............ " and "q is ...."

(c)

M1: For substituting $t=30$ into $V=pq^t$ using their values for p and q or substituting $t=30$ into $\log_{10}V=0.05t+4.8$ and proceeds to V

A1: For awrt either £1.99 million or £2.00 million. Condone the omission of the £ sign. Remember to isw after a correct answer

Question	Scheme	Marks	AOs
17	$2\log_4(2-x)-\log_4(x+5)=1$		
	Uses the power law $\log_4(2-x)^2-\log_4(x+5)=1$	M1	1.1b
	Uses the subtraction law $\log_4\dfrac{(2-x)^2}{(x+5)}=1$	M1	1.1b
	$\dfrac{(2-x)^2}{(x+5)}=4\rightarrow 3\text{TQ in } x$	dM1	3.1a
	$x^2-8x-16=0$	A1	1.1b
	$(x-4)^2=32\Rightarrow x=$	M1	1.1b
	$x=4-4\sqrt{2}$ oe only	A1	2.3
		(6)	
			(6 marks)

Notes:

M1: Uses the power law of logs $2\log_4(2-x)=\log_4(2-x)^2$

M1: Uses the subtraction law of logs following the above $\log_4(2-x)^2-\log_4(x+5)=\log_4\dfrac{(2-x)^2}{(x+5)}$

Alternatively uses the addition law following use of $1=\log_4 4$ That is $1+\log_4(x+5)=\log_4 4(x+5)$

dM1: This can be awarded for the overall strategy leading to a 3TQ in x. It is dependent upon the correct use of both previous M's and for undoing the logs to reach a 3TQ equation in x

A1: For a correct equation in x

M1: For the correct method of solving their 3TQ =0

A1: $x=4-4\sqrt{2}$ or exact equivalent only. (For example accept $x=4-\sqrt{32}$)

Question	Scheme	Marks	AOs
18(a)	0.2 m^2	B1	3.4
		(1)	
(b)	$A=0.2e^{0.3t}$ Rate of change = gradient $=\frac{dA}{dt}=0.06e^{0.3t}$	M1	3.1b
	At $t=5 \Rightarrow$ Rate of Growth is $0.06e^{1.5}=0.269\ \text{m}^2/\text{day}$	A1	1.1b
		(2)	
(c)	$100=0.2e^{0.3t}\Rightarrow e^{0.3t}=500$	M1 A1	3.1a 1.1b
	$\Rightarrow t=\frac{\ln(500)}{0.3}=20.7$ days 20 days 17 hours	M1 A1	1.1b 3.2a
		(4)	
	At $t=5 \Rightarrow$ Rate of Growth is $0.06e^{1.5}=0.269\ \text{m}^2/\text{day}$	A1	1.1b
		(2)	
(d)	The model given suggests that the pond is fully covered after 20 days 17 hours. Observed data is inconsistent with this as the pond is only 90% covered by the end of one month (28/29/30/31 days). Hence the model is not accurate	B1	3.5a
		(1)	
			(8 marks)

Notes:

(a)

B1: 0.2 m^2 oe

(b)

M1: Links rate of change to gradient and differentiates $0.2e^{0.3t}\rightarrow ke^{0.3t}$

A1: Correct answer $0.269\ \text{m}^2/\text{day}$

(c)

M1: Substitutes $A=100$ and proceeds to $e^{0.3t}=k$

A1: $e^{0.3t}=500$

M1: Correct method when proceeding from $e^{0.3t}=k\Rightarrow t=..$

A1: 20 days 17 hours

(d)

B1: Valid conclusion following through on their answer to (c).

Question	Scheme		Marks	AOs
19(a)	$2^{2x}+2^4$ is wrong in line 2 - it should be $2^{2x}\times 2^4$		B1	2.3
	In line 4, 2^4 has been replaced by 8 instead of by 16		B1	2.3
			(2)	
(b)	**Way 1:** $2^{2x+4}-9(2^x)=0$ $2^{2x}\times 2^4-9(2^x)=0$ Let $2^x=y$ $16y^2-9y=0$	**Way 2:** $(2x+4)\log 2-\log 9-x\log 2=0$	M1	2.1
	$y=\frac{9}{16}$ or $y=0$ So $x=\log_2\left(\frac{9}{16}\right)$ or $\dfrac{\log\left(\frac{9}{16}\right)}{\log 2}$ o.e. with no second answer	$x=\dfrac{\log 9}{\log 2}-4$ o.e.	A1	1.1b
			(2)	
			(4 marks)	

Notes:

(a)

B1: Lists error in line 2 (as above)

B1: Lists error in line 4 (as above)

(b)

M1: Correct work with powers reaching this equation

A1: Correct answer here – there are many exact equivalents

Question	Scheme		Marks	AOs
20(a)	$\log_{10} P = mt + c$		M1	1.1b
	$\log_{10} P = \frac{1}{200}t + 5$		A1	1.1b
			(2)	
(b)	**Way 1:** As $P = ab^t$ then $\log_{10} P = t\log_{10} b + \log_{10} a$	**Way 2:** As $\log_{10} P = \frac{t}{200} + 5$ then $P = 10^{\left(\frac{t}{200}+5\right)} = 10^5 10^{\left(\frac{t}{200}\right)}$	M1	2.1
	$\log_{10} b = \frac{1}{200}$ or $\log_{10} a = 5$	$a = 10^5$ or $b = 10^{\left(\frac{1}{200}\right)}$	M1	1.1b
	So a = 100 000 **or** b = 1.0116		A1	1.1b
	Both a = 100 000 **and** b = 1.0116 (awrt 1.01)		A1	1.1b
			(4)	
(c)(i)	The initial population		B1	3.4
(c)(ii)	The proportional increase of population each year		B1	3.4
			(2)	
(d)(i)	300000 to nearest hundred thousand		B1	3.4
(d)(ii)	Uses $200\,000 = ab^t$ with their values of a and b or $\log_{10} 200000 = \frac{1}{200}t + 5$ and rearranges to give t =		M1	3.4
	60.2 years to 3sf		A1ft	1.1b
			(3)	
(e)	Any two valid reasons- e.g. • 100 years is a long time and population may be affected by wars and disease • Inaccuracies in measuring gradient may result in widely different estimates • Population growth may not be proportional to population size • The model predicts unlimited growth		B2	3.5b
			(2)	

Question 20 continued
Notes:
(a) **M1:** Uses a linear equation to relate log P and t **A1:** Correct use of gradient and intercept to give a correct line equation
(b) **M1:** **<u>Way 1</u>:** Uses logs correctly to give log equation; **<u>Way 2:</u>** Uses powers correctly to "undo" log equation and expresses as product of two powers **M1:** **<u>Way 1</u>:** Identifies log b or log a or both; **<u>Way 2:</u>** Identifies a or b as powers of 10 **A1:** Correct value for a or b **A1:** Correct values for both
(c)(i) **B1:** Accept equivalent answers e.g. The population at $t = 0$ **(c)(ii)** **B1:** So accept rate at which the population is increasing each year or scale factor 1.01 or increase of 1% per year
(d)(i) **B1:** cao **(d)(ii)** **M1:** As in the scheme **A1ft:** On their values of a and b with correct log work
(e) **B2:** As given in the scheme – any two valid reasons

Question	Scheme	Marks	AOs
21 (a)(i)	10750 barrels	B1	3.4
(ii)	Gives a valid limitation, for example • The model shows that the daily volume of oil extracted would become negative as t increases, which is impossible • States when $t=10, V=-1500$ which is impossible • States that the model will only work for $0 \leqslant t \leqslant \frac{64}{7}$	B1	3.5b
		(2)	
(b)(i)	Suggests a suitable exponential model, for example $V = Ae^{kt}$	M1	3.3
	Uses $(0,16000)$ and $(4,9000)$ in $\Rightarrow 9000 = 16000e^{4k}$	dM1	3.1b
	$\Rightarrow k = \frac{1}{4}\ln\left(\frac{9}{16}\right)$ awrt -0.144	M1	1.1b
	$V = 16000e^{\frac{1}{4}\ln\left(\frac{9}{16}\right)t}$ or $V = 16000e^{-0\,144t}$	A1	1.1b
(ii)	Uses their exponential model with $t=3 \Rightarrow V =$ awrt 10 400 barrels	B1ft	3.4
		(5)	
		(7 marks)	

Notes:

(a)(i)

B1: 10750 barrels

(a)(ii)

B1: See scheme

(b)(i)

M1: Suggests a suitable exponential model, for example $V = Ae^{kt}$, $V = Ar^t$ or any other suitable function such as $V = Ae^{kt} + b$ where the candidate chooses a value for b.

dM1: Uses both $(0,16000)$ and $(4,9000)$ in their model.

With $V = Ae^{kt}$ candidates need to proceed to $9000 = 16000e^{4k}$

With $V = Ar^t$ candidates need to proceed to $9000 = 16000r^4$

With $V = Ae^{kt} + b$ candidates need to proceed to $9000 = (16000 - b)e^{4k} + b$ where b is given as a positive constant and $A + b = 16000$.

M1: Uses a correct method to find all constants in the model.

A1: Gives a suitable equation for the model passing through (or approximately through in the case of decimal equivalents) both values $(0,16000)$ and $(4,9000)$. Possible equations for the model could be for example

$V = 16000e^{-0\,144t}$ $\quad V = 16000 \times (0.866)^t$ $\quad V = 15800e^{-0\,146t} + 200$

(b)(ii)

B1ft: Follow through on their exponential model

Question	Scheme	Marks	AOs
22 (a)	$N = aT^b \Rightarrow \log_{10} N = \log_{10} a + \log_{10} T^b$	M1	2.1
	$\Rightarrow \log_{10} N = \log_{10} a + b\log_{10} T$ so $m = b$ **and** $c = \log_{10} a$	A1	1.1b
		(2)	
(b)	Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** b = gradient	M1	3.1b
	Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** b = gradient	M1	1.1b
	Uses $T = 3$ in $N = aT^b$ with their a and b	M1	3.1b
	Number of microbes ≈ 800	A1	1.1b
		(4)	
(c)	$N = 1000000 \Rightarrow \log_{10} N = 6$	M1	3.4
	We cannot 'extrapolate' the graph and assume that the model still holds	A1	3.5b
		(2)	
(d)	States that 'a' is the number of microbes 1 day after the start of the experiment	B1	3.2a
		(1)	
			(9 marks)

Question 22 continued
Notes:
(a) **M1:** Takes logs of both sides and shows the addition law **M1:** Uses the power law, writes $\log_{10} N = \log_{10} a + b\log_{10} T$ and states $m = b$ **and** $c = \log_{10} a$
(b) **M1:** Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** b = gradient. This would be implied by the sight of $b = 2.3$ **or** $a = 10^{1.8} \approx 63$ **M1:** Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** b = gradient. This would be implied by the sight of $b = 2.3$ **and** $a = 10^{1.8} \approx 63$ **M1:** Uses $T = 3 \Rightarrow N = aT^b$ with their a and b. This is implied by an attempt at $63 \times 3^{2.3}$ **A1:** Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work. There is an alternative to this using a graphical approach. **M1**: Finds the value of $\log_{10} T$ from T=3. Accept as $T = 3 \Rightarrow \log_{10} T \approx 0.48$ **M1**: Then using the line of best fit finds the value of $\log_{10} N$ from their "0.48" Accept $\log_{10} N \approx 2.9$ **M1:** Finds the value of N from their value of $\log_{10} N$ $\log_{10} N \approx 2.9 \Rightarrow N = 10^{'2.9'}$ **A1:** Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work
(c) **M1** For using N = 1000000 and stating that $\log_{10} N = 6$ **A1:** Statement to the effect that "we only have information for values of $\log N$ between 1.8 and 4.5 so we cannot be certain that the relationship still holds". "We cannot extrapolate with any certainty, we could only interpolate" There is an alternative approach that uses the formula. **M1:** Use N = 1000000 in their $N = 63 \times T^{2.3} \Rightarrow \log_{10} T = \dfrac{\log_{10}\left(\dfrac{1000000}{63}\right)}{2.3} \approx 1.83$. **A1:** The reason would be similar to the main scheme as we only have $\log_{10} T$ values from 0 to 1.2. We cannot 'extrapolate' the graph and assume that the model still holds
(d) **B1:** Allow a numerical explanation $T = 1 \Rightarrow N = a1^b \Rightarrow N = a$ giving a is the value of N at T=1

Question	Scheme	Marks	AOs
23	$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \log_3 \frac{12y+5}{1-3y}=2$ or e.g. $2=\log_3 9$	B1 M1 on EPEN	1.1b
	$\log_3 \frac{12y+5}{1-3y}=2 \Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=...$ or e.g. $\log_3(12y+5)=\log_3\left(3^2(1-3y)\right) \Rightarrow (12y+5)=3^2(1-3y) \Rightarrow y=...$	M1	2.1
	$y=\frac{4}{39}$	A1	1.1b
		(3)	
			(3 marks)

Notes

B1(M1 on EPEN): Applies at least one addition or subtraction law of logs correctly.
Can also be awarded for using $2=\log_3 9$. This may be implied by e.g.
$\log_3 ... = 2 \Rightarrow ... = 9$

M1: A rigorous argument with no incorrect working to remove the log or logs correctly and obtain a correct equation in any form **and** solve for y.

A1: Correct **exact** value. Allow equivalent fractions.

Guidance on how to mark particular cases:

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{\log_3(12y+5)}{\log_3(1-3y)}=2$$
$$\Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M0A0

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{\log_3(12y+5)}{\log_3(1-3y)}=2 \Rightarrow \log_3 \frac{12y+5}{1-3y}=2$$
$$\Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M0A0

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M1A1

Question	Scheme	Marks	AOs
24(a)	$T = al^b \Rightarrow \log_{10} T = \log_{10} a + \log_{10} l^b$	M1	2.1
	$\Rightarrow \log_{10} T = \log_{10} a + b\log_{10} l$ * or $\Rightarrow \log_{10} T = b\log_{10} l + \log_{10} a$ *	A1*	1.1b
		(2)	
(b)	$b = 0.495$ or $b = \dfrac{45}{91}$	B1	2.2a
	$0 = "0.495" \times -0.7 + \log_{10} a \Rightarrow a = 10^{0\,346}$ or $0.45 = "0.495" \times 0.21 + \log_{10} a \Rightarrow a = 10^{0\,346}$	M1	3.1a
	$T = 2.22l^{0.495}$	A1	3.3
		(3)	
(c)	The time taken for one swing of a pendulum of length 1 m	B1	3.2a
		(1)	
			(6 marks)

Notes

(a)

M1: Takes logs of both sides and shows the addition law.

Implied by $T = al^b \Rightarrow \log_{10} a + \log_{10} l^b$

A1*: Uses the power law to obtain the given equation with no errors. Allow the bases to be missing in the working but they must be present in the final answer.
Also allow *t* rather than *T* and *A* rather than *a*.

Allow working backwards e.g.

$$\log_{10} T = b\log_{10} l + \log_{10} a \Rightarrow \log_{10} T = \log_{10} l^b + \log_{10} a$$

$$\Rightarrow \log_{10} T = \log_{10} al^b \Rightarrow T = al^b \ *$$

M1: Uses the given answer and uses the power law and addition law correctly

A1: Reaches the given equation with no errors as above

(b)

B1: Deduces the correct value for *b* (Allow awrt 0.495 or $\frac{45}{91}$)

M1: Correct strategy to find the value of *a*.
E.g. substitutes one of the given points and their value for *b* into $\log_{10} T = \log_{10} a + b\log_{10} l$ and uses correct log work to identify the value of *a*. Allow slips in rearranging their equation but must be correct log work to find *a*.
Alternatively finds the equation of the straight line and equates the constant to $\log_{10} a$ and uses correct log work to identify the value of *a*.
E.g. $y - 0.45 = "0.495"(x - 0.21) \Rightarrow y = "0.495"x + 0.346 \Rightarrow a = 10^{0\,346} = ...$

A1: Complete equation $T = 2.22l^{0.495}$ or $T = 2.22l^{\frac{45}{91}}$

(Allow awrt 2.22 and awrt 0.495 or $\frac{45}{91}$)

Must see the <u>equation</u> not just correct values as it is a requirement of the question.

(c)

B1: Correct interpretation

Question	Scheme	Marks	AOs
25(a)	$2\log(4-x)=\log(4-x)^2$	B1	1.2
	$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2=\log(x+8)$ $(4-x)^2=(x+8)$ **or** $2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0$ $\dfrac{(4-x)^2}{(x+8)}=1$	M1	1.1b
	$16-8x+x^2=x+8\Rightarrow x^2-9x+8=0$ *	A1*	2.1
		(3)	
	(a) Alternative - working backwards:		
	$x^2-9x+8=0\Rightarrow(4-x)^2-x-8=0$	B1	1.2
	$\Rightarrow(4-x)^2=x+8$ $\Rightarrow\log(4-x)^2=\log(x+8)$	M1	1.1b
	$\Rightarrow 2\log(4-x)=\log(x+8)$ * Hence proved.	A1	2.1
(b)	(i) (x =) 1, 8	B1	1.1b
	(ii) 8 is **not** a solution as log(4 – 8) cannot be found	B1	2.3
		(2)	
			(5 marks)

Notes:

(a)

B1: States or uses $2\log(4-x)=\log(4-x)^2$

M1: Correct attempt at eliminating the logs to form a quadratic equation in x.

Note that this may be implied by e.g. $\log\dfrac{(4-x)^2}{(x+8)}=0\Rightarrow(4-x)^2=x+8$

A1*: Proceeds to the given answer with at least one line where the $(4-x)^2$ has been multiplied out.
There must be no errors or omissions but condone invisible brackets around the arguments of the logs e.g. allow $\log 16-8x+x^2$ for $\log(16-8x+x^2)$ and $\log x+8$ for $\log(x+8)$

Note we will allow a start of $(4-x)^2=x+8$ with no previous work for full marks.

Some examples of how to mark (a) in particular cases:

$$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0\Rightarrow(4-x)^2-x-8=0$$

$$\Rightarrow 16-8x+x^2-x-8\Rightarrow x^2-9x+8=0$$

Scores B1M1A1

$$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0\Rightarrow\frac{\log(4-x)^2}{\log(x+8)}=0$$

$$\Rightarrow\frac{(4-x)^2}{(x+8)}=1\Rightarrow 16-8x+x^2=x+8\Rightarrow x^2-9x+8=0$$

Scores B1M0A0

(a) Alternative:

B1: Writes $x^2 - 9x + 8 = 0$ as $(4 - x)^2 - x - 8 = 0$ or equivalent

M1: Proceeds correctly to reach $\log(4-x)^2=\log(x+8)$

A1: Obtains $2\log(4-x)=\log(x+8)$ and makes a (minimal) conclusion e.g. hence proved, QED, #, square etc.

(b)

B1: Writes down (x =) 1, 8

B1: Chooses 8 (no follow through here) and gives a reason why it should be rejected by referring to logs and which log it is.

They must refer to the 8 as the required value but allow e.g. $x \neq 8$ and there must be a reference to $\log(4 - x)$ or log of lhs or log(–4) or the 4 – 8. Some acceptable reasons are: log(–4) can't be found/worked out/is undefined, log(–4) gives math error, log(–4) = n/a, lhs is log(negative) so reject, you can't do the log of a negative number which would happen with 4 – 8

Do **not** allow "you can't have a negative log" unless this is clarified further and do **not** allow "you get a math error" in isolation

There must be no contradictory statements.

Note that this is an independent mark but must have $x = 8$ (i.e. may have solved to get $x = -1, 8$ for first B mark)

Question	Scheme	Marks	AOs
26	$15-2^{x+1}=3\times 2^x$	B1	1.1b
	$\Rightarrow 15-2\times 2^x=3\times 2^x \Rightarrow 2^x=3$ or e.g. $\Rightarrow \dfrac{15}{2^x}-2=3 \Rightarrow 2^x=3$	M1	1.1b
	$2^x=3 \Rightarrow x=\ldots$	dM1	1.1b
	$x=\log_2 3$	A1cso	1.1b
		(4)	
	Alternative		
	$y=3\times 2^x \Rightarrow 2^x=\dfrac{y}{3} \Rightarrow y=15-2\times\dfrac{y}{3}$	B1	1.1b
	$3y+2y=45 \Rightarrow y=9 \Rightarrow 3\times 2^x=9 \Rightarrow 2^x=3$	M1	1.1b
	$2^x=3 \Rightarrow x=\ldots$	dM1	1.1b
	$x=\log_2 3$	A1cso	1.1b
			(4 marks)

Notes:

B1: Combines the equations to reach $15-2^{x+1}=3\times 2^x$ or equivalent e.g. $15-2^{x+1}-3\times 2^x=0$

M1: Uses $2^{x+1}=2\times 2^x$ oe e.g. $\dfrac{2^{x+1}}{2^x}=2$ to obtain an equation in 2^x and attempts to make 2^x the subject.

See scheme but e.g. $y=2^x \Rightarrow 3\times 2^x=15-2^{x+1} \Rightarrow 3y=15-2y \Rightarrow y=\ldots$ is also possible

dM1: Uses logs correctly and proceeds to a value for x from an equation of the form $2^x=k$ where $k>1$

e.g. $2^x=k \Rightarrow x=\log_2 k$

or $2^x=k \Rightarrow \log 2^x=\log k \Rightarrow x\log 2=\log k \Rightarrow x=\ldots$

or $2^x=k \Rightarrow \ln 2^x=\ln k \Rightarrow x\ln 2=\ln k \Rightarrow x=\ldots$

Depends on the first method mark

This may be implied if they go straight to decimals e.g. 2^x = 3 so x = 1.584.. but you may need to check

A1cso: $x=\log_2 3$ or $\dfrac{\log 3}{\log 2}$ or $\dfrac{\ln 3}{\ln 2}$

Ignore any attempts to find the y-coordinate

<u>Alternative</u>

B1: Correct equation in y

M1: Solves their equation in y and attempts to make 2^x the subject.

dM1: Uses logs correctly and proceeds to a value for x from an equation of the form $2^x=k$ where $k>1$

e.g. $2^x=k \Rightarrow x=\log_2 k$

or $2^x=k \Rightarrow \log 2^x=\log k \Rightarrow x\log 2=\log k \Rightarrow x=\ldots$

or $2^x=k \Rightarrow \ln 2^x=\ln k \Rightarrow x\ln 2=\ln k \Rightarrow x=\ldots$

Depends on the first method mark

This may be implied if they go straight to decimals e.g. 2^x = 3 so x = 1.584.. but you may need to check

A1cso: $x=\log_2 3$ or $\dfrac{\log 3}{\log 2}$ or $\dfrac{\ln 3}{\ln 2}$

Ignore any attempts to find the y-coordinate

Question	Scheme	Marks	AOs
27(a)	$t = 0,\ \theta = 18 \Rightarrow 18 = A - B$ **or** $t = 10,\ \theta = 44 \Rightarrow 44 = A - B\mathrm{e}^{-0\ 7}$	M1	3.1b
	$t = 0,\ \theta = 18 \Rightarrow 18 = A - B$ **and** $t = 10,\ \theta = 44 \Rightarrow 44 = A - B\mathrm{e}^{-0\ 7}$ **and** $\Rightarrow A = \ldots, B = \ldots$	M1	3.1a
	At least one of: A = 69.6, B = 51.6 but allow awrt 70/awrt 52	A1 **M1 on EPEN**	1.1b
	$\theta = 69.6 - 51.6\mathrm{e}^{-0\ 07t}$	A1	3.3
		(4)	
(b)	The maximum temperature is "69.6"(°C) (according to the model) (The model has an) upper limit of "69.6"(°C) (The model suggests that) the boiling point is "69.6"(°C)	B1ft	3.4
	Model is not appropriate as 69.6(°C) is much lower than 78(°C)	B1ft	3.5a
		(2)	
			(6 marks)

Notes:

(a)

M1: Makes the first key step in the solution of the problem. Substitutes t = 0 and θ = 18 **or** t = 10 and θ = 44 into the equation of the model to obtain an equation connecting A and B.
Note that 18 = A – $B\mathrm{e}^0$ scores M0 unless 18 = A – B is seen or implied later.
If they do not obtain an equation in A and B using the first conditions e.g. they have 18 = A – 1 then they can score this mark if they substitute A = 19 directly into $44 = A - B\mathrm{e}^{-0\ 7}$ as an equation in A and B is implied.

M1: Substitutes t = 0 and θ = 18 **and** t = 10 and θ = 44 to obtain 2 equations connecting A and B **and** then proceeds to solves their equations in A and B simultaneously to obtain values for both constants. Do not be too concerned with the processing as long as values for A and B are obtained.

A1(M1 on EPEN): For A = awrt 70 **or** B = awrt 52

A1: For $\theta = 69.6 - 51.6\mathrm{e}^{-0\ 07t}$ **Must be a <u>fully correct equation as shown</u> but allow recovery if seen in (b).**
Note that some candidates evaluate e^0 as 0 and so obtain A = 18 and then write 44 = 18 – $B\mathrm{e}^{-0\ 7}$ and solve for B.
Such attempts can score M1M0A0A0 only.

(b)

B1ft: Identifies A as the boiling point/maximum temperature in the model. Follow through their A.

B1ft: Makes a valid conclusion (valid/not valid, good/not good etc.) that refers to the 78 and includes a reference to a significant/large difference

Alternative provided their A < 78

B1ft: $\theta = 69.6 - 51.6\mathrm{e}^{-0\ 07t} = 78 \Rightarrow 51.6\mathrm{e}^{-0\ 07t} = 69.6 - 78 = -8.4$

$\Rightarrow \mathrm{e}^{-0\ 07t} = -\dfrac{7}{43}$ and $\ln\left(-\dfrac{7}{43}\right)$ and makes a reference to the fact that the equation cannot be solved or e.g. cannot take log of a negative number. You can condone numerical slips in the calculation.

B1ft: Model is not appropriate as 69.6(°C) is much lower than 78(°C)

Minimum for both marks: The model is not appropriate as "69.6"(°C) is much lower than 78(°C)

Note that these marks are not available if their equation is solvable. Note also that B0B1 is not possible.

Question	Scheme	Marks	AOs
28(a)	$k = e^2$ **or** $x \neq e^2$	B1	2.2a
		(1)	
(b)	$g'(x) = \dfrac{(\ln x - 2)\times\dfrac{3}{x} - (3\ln x - 7)\times\dfrac{1}{x}}{(\ln x - 2)^2} = \dfrac{1}{x(\ln x - 2)^2}$ or $g'(x) = \dfrac{d}{dx}\left(3 - (\ln(x) - 2)^{-1}\right) = (\ln x - 2)^{-2}\times\dfrac{1}{x} = \dfrac{1}{x(\ln x - 2)^2}$ or $g'(x) = (\ln x - 2)^{-1}\times\dfrac{3}{x} - (3\ln x - 7)(\ln x - 2)^{-2}\times\dfrac{1}{x} = \dfrac{1}{x(\ln x - 2)^2}$	M1 A1	1.1b 2.1
	As $x > 0$ (or $1/x > 0$) **AND** $\ln x - 2$ is squared so $g'(x) > 0$	A1cso	2.4
		(3)	
(c)	Attempts to solve either $3\ln x - 7 \ldots 0$ or $\ln x - 2 \ldots 0$ or $3\ln a - 7 \ldots 0$ or $\ln a - 2 \ldots 0$ where … is "=" or ">" to reach a value for x or a but may be seen as an inequality e.g. $x > \ldots$ or $a > \ldots$	M1	3.1a
	$0 < a < e^2, \quad a > e^{\frac{7}{3}}$	A1	2.2a
		(2)	

(6 marks)

Notes:

(a)

B1: Deduces $k = \mathrm{e}^2$ **or** $x \neq \mathrm{e}^2$ Condone k = awrt 7.39 or $x \neq$ awrt 7.39

(b)

M1: Attempts to differentiate via the quotient rule and with $\ln x \to \frac{1}{x}$ so allow for:

$$\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{g}(x)) = \frac{(\ln x - 2)\times\frac{\alpha}{x} - (3\ln x - 7)\times\frac{\beta}{x}}{(\ln x - 2)^2},\ \beta > 0$$

But a correct rule may be implied by their *u*, *v*, *u′*, *v′* followed by applying $\frac{vu' - uv'}{v^2}$ etc.

Alternatively attempts to write $\mathrm{g}(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} = 3 - (\ln(x) - 2)^{-1}$ and attempts the chain rule so allow for:

$$3 - (\ln(x) - 2)^{-1} \to (\ln(x) - 2)^{-2} \times \frac{\alpha}{x}$$

Alternatively writes $\mathrm{g}(x) = (3\ln(x) - 7)(\ln(x) - 2)^{-1}$ and attempts the product rule so allow for:

$$\mathrm{g}'(x) = (\ln x - 2)^{-1} \times \frac{\alpha}{x} - (3\ln x - 7)(\ln x - 2)^{-2} \times \frac{\beta}{x}$$

In general condone missing brackets for the M mark. E.g. if they quote $u = 3\ln x - 7$ and $v = \ln x - 2$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have v rather than v^2 in the denominator.

A1: $\frac{1}{x(\ln x - 2)^2}$ **Allow** $\frac{\frac{1}{x}}{(\ln x - 2)^2}$ **i.e. we need to see the numerator simplified to $1/x$**

Note that some candidates establish the correct numerator and correct denominator independently and provided they obtain the correct expressions, this mark can be awarded.

But allow a correctly expanded denominator.

A1cso: States that as $x > 0$ **AND** $\ln x - 2$ is squared so $\mathrm{g}'(x) > 0$

(c)

M1: Attempts to solve either $3\ln x - 7 = 0$ or $\ln x - 2 = 0$ or using inequalities e.g. $3\ln x - 7 > 0$

A1: $0 < a < \mathrm{e}^2,\ a > \mathrm{e}^{\frac{7}{3}}$

Question	Scheme	Marks	AOs
29	$2^x \times 4^y = \frac{1}{2\sqrt{2}} \quad \left\{= \frac{\sqrt{2}}{4}\right\}$		
Special Case	If 0 marks are scored on application of the mark scheme then allow Special Case B1 M0 A0 (total of 1 mark) for any of • $2^x \times 4^y \rightarrow 2^{x+2y}$ • $2^x \times 4^y \rightarrow 4^{\frac{1}{2}x+y}$ • $\frac{1}{2^x 2\sqrt{2}} \rightarrow 2^{-x-\frac{3}{2}}$ • $\log 2^x + \log 4^y \rightarrow x\log 2 + y\log 4$ or $x\log 2 + 2y\log 2$ • $\ln 2^x + \ln 4^y \rightarrow x\ln 2 + y\ln 4$ or $x\ln 2 + 2y\ln 2$ • $y = \log\left(\frac{1}{2^x 2\sqrt{2}}\right)$ o.e. {base of 4 omitted}		
Way 1	$2^x \times 2^{2y} = 2^{-\frac{3}{2}}$	B1	1.1b
	$2^{x+2y} = 2^{-\frac{3}{2}} \Rightarrow x + 2y = -\frac{3}{2} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	
Way 2	$\log(2^x \times 4^y) = \log\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\frac{1}{2\sqrt{2}}\right)$ $\Rightarrow x\log 2 + y\log 4 = \log 1 - \log(2\sqrt{2}) \Rightarrow y = ...$	M1	2.1
	$y = \frac{-\log(2\sqrt{2}) - x\log 2}{\log 4} \quad \left\{\Rightarrow y = -\frac{1}{2}x - \frac{3}{4}\right\}$	A1	1.1b
		(3)	
Way 3	$\log(2^x \times 4^y) = \log\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow \log 2^x + y\log 4 = \log\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow y = ...$	M1	2.1
	$y = \frac{\log\left(\frac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4} \quad \left\{\Rightarrow y = -\frac{1}{2}x - \frac{3}{4}\right\}$	A1	1.1b
		(3)	
Way 4	$\log_2(2^x \times 4^y) = \log_2\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log_2 2^x + \log_2 4^y = \log_2\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow x + 2y = -\frac{3}{2} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	
			(3 marks)

Question	Scheme	Marks	AOs
Way 5	$4^{\frac{1}{2}x} \times 4^y = 4^{-\frac{3}{4}}$	B1	1.1b
	$4^{\frac{1}{2}x+y} = 4^{-\frac{3}{4}} \Rightarrow \frac{1}{2}x + y = -\frac{3}{4} \Rightarrow y = \ldots$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	

Notes for Question 29

	Way 1
B1:	Writes a correct equation in powers of 2 only
M1:	Complete process of writing a correct equation in powers of 2 only and using correct index laws to obtain *y* written as a function of *x*.
A1:	$y = -\frac{1}{2}x - \frac{3}{4}$ o.e.
	Way 2, Way 3 and Way 4
B1:	Writes a correct equation involving logarithms
M1:	Complete process of writing a correct equation involving logarithms and using correct log laws to obtain *y* written as a function of *x*.
A1:	$y = \dfrac{-\log(2\sqrt{2}) - x\log 2}{\log 4}$ or $y = \dfrac{-\ln(2\sqrt{2}) - x\ln 2}{\ln 4}$ or $y = \dfrac{\log\left(\frac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4}$ or $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$ o.e.
	Way 5
B1:	Writes a correct equation in powers of 4 only
M1:	Complete process of writing a correct equation in powers of 4 only and using correct index laws to obtain *y* written as a function of *x*.
A1:	$y = -\frac{1}{2}x - \frac{3}{4}$ o.e.
Note:	Allow equivalent results for A1 where *y* is written as a function of *x*
Note:	You can ignore subsequent working following on from a correct answer.
Note:	Allow B1 for $2^x \times 4^y = \frac{1}{2\sqrt{2}} \Rightarrow 4^y = \frac{1}{2^x 2\sqrt{2}} \Rightarrow \log_4(4^y) = \log_4\left(\frac{1}{2^x 2\sqrt{2}}\right)$ followed by M1 A1 for $y = \log_4\left(\frac{1}{2^x 2\sqrt{2}}\right)$ or $y = \log_4\left(\frac{2^{-x}}{2\sqrt{2}}\right)$ or $y = \log_4\left(\frac{\sqrt{2}}{4(2^x)}\right)$ or $y = -\log_4\left(2^{x+\frac{3}{2}}\right)$ or $y = -\log_4(\sqrt{2}(2^{x+1}))$

Question	Scheme	Marks	AOs
30 (a) **Way 1**	$\{d = kV^n \Rightarrow\}\ \log_{10} d = \log_{10} k + n\log_{10} V$ or $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ seen or used as part of their argument	M1	2.1
	Alludes to $d = kV^n$ and gives a full explanation by comparing their result with a linear model e.g. $Y = MX + C$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
30 (a) **Way 2**	$\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ or $\log_{10} d = \log_{10} k + n\log_{10} V$ seen or used as part of their argument	M1	2.1
	$\{d = kV^n \Rightarrow\}\ \log_{10} d = \log_{10}(kV^n)$ $\Rightarrow \log_{10} d = \log_{10} k + \log_{10} V^n \Rightarrow \log_{10} d = \log_{10} k + n\log_{10} V$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(a) **Way 3**	Starts from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$	M1	2.1
	$\log_{10} d = m\log_{10} V + c \Rightarrow d = 10^{m\log_{10} V + c} \Rightarrow d = 10^c V^m \Rightarrow d = kV^n$ **or** $\log_{10} d = m\log_{10} V - 1.77 \Rightarrow d = 10^{m\log_{10} V - 1.77}$ $\Rightarrow d = 10^{-1.77} V^m \Rightarrow d = kV^n$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(b)	$\{d = 20, V = 30 \Rightarrow\}\quad 20 = k(30)^n$ or $\log_{10} 20 = \log_{10} k + n\log_{10} 30$	M1	3.4
	$20 = k(30)^n \Rightarrow \log 20 = \log k + n\log 30 \Rightarrow n = \dfrac{\log 20 - \log k}{\log 30} \Rightarrow n = ...$ $\log_{10} 20 = \log_{10} k + n\log_{10} 30 \Rightarrow n = \dfrac{\log_{10} 20 - \log_{10} k}{\log_{10} 30} \Rightarrow n = ...$	M1	1.1b
	$\{n = \text{awrt } 2.08 \Rightarrow\}\ d = (0.017)V^{2.08}$ **or** $\log_{10} d = -1.77 + 2.08\log_{10} V$	A1	1.1b
	Note: You can recover the A1 mark for a correct model equation given in part (c)	**(3)**	
(c)	$d = (0.017)(60)^{2.08}$	M1	3.4
	• $13.333... + 84.918... = 98.251... \Rightarrow$ Sean stops in time • $100 - 13.333... = 86.666...$ & $d = 84.918 \Rightarrow$ Sean stops in time	M1 A1ft	3.1b 3.2a
		(3)	
			(9 marks)

ADVICE: Ignore labelling (a), (b), (c) when marking this question

Note: Give B0 in (a) for $10^{-1.77} = 0.01698...$ without reference to 0.017 in the same part

Notes for Question **30**	
Note:	In their solution to (a) and/or (b) condone writing $\log$ in place of $\log_{10}$
(a)	**Way 1**
M1:	See scheme
A1:	See scheme
B1*:	See scheme
(a)	**Way 2**
M1:	See scheme
A1:	Starts from $d = kV^n$ (which they do not have to state) and progresses to $\log_{10} d = \log_{10} k + n\log_{10} V$ with an intermediate step in their working.
B1*:	See scheme
(a)	**Way 3**
M1:	Starts their argument from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$
A1:	Mathematical explanation is seen by showing any of either • $\log_{10} d = m\log_{10} V + c \rightarrow d = 10^c V^m$ or $d = kV^n$ • $\log_{10} d = m\log_{10} V - 1.77 \rightarrow d = 10^{-1.77} V^m$ or $d = kV^n$ with no errors seen in their working
B1*:	See scheme
Note:	Allow B1 for $\log_{10} 0.017 = -1.77$ or $\log 0.017 = -1.77$
(b)	
M1:	Applies $V = 30$ and $d = 20$ to their model **(correct way round)**
M1:	Applies $(V, d) = (30, 20)$ or $(20, 30)$ and applies logarithms correctly leading to $n = ...$
A1:	$d = (0.017)V^{2.08}$ or $\log_{10} d = -1.77 + 2.08\log_{10} V$ or $\log_{10} d = \log_{10}(0.017) + 2.08\log_{10} V$
Note:	Allow $k =$ awrt 0.017 and/or $n =$ awrt 2.08 in their final model equation
Note:	M0 M1 A0 is a possible score for (b)
(c)	
M1:	Applies $V = 60$ to their exponential model or their logarithmic model
M1:	Uses their model in a correct problem-solving process of either • adding a "thinking distance" to their value of their *d* to find an overall stopping distance • applying 100 – "thinking distance" and finds their value of *d*
Note:	$\frac{1}{75}$ or 48 are examples of acceptable thinking distances
A1ft:	**Either** adds 13.3… to their *d* to find a total stopping distance and gives a correct ft conclusion **or** finds their *d* and a comparative 86.666…(m) or awrt 87 (m) and gives a correct ft conclusion
Note:	The thinking distance must be dimensionally correct for the M1 mark. i.e. $0.8\times$ their velocity
Note:	A thinking distance of awrt 13 and a value of *d* in the range [81.5, 88.5] are required for A1ft
Note:	Allow "Sean stops in time" or "Yes he stops in time" or "he misses the puddle" as relevant conclusions.
Note:	A mark of M0 M1 A0 is possible in (c)

Question	Scheme	Marks	AOs
31 (a)	$\{t=0,\ \theta=75 \Rightarrow 75 = 25 + A \Rightarrow A = 50\} \Rightarrow \theta = 25 + 50\mathrm{e}^{-0.03t}$	B1	3.3
		(1)	
(b)	$\{\theta = 60 \Rightarrow\} \Rightarrow 60 = 25 + \text{"50"}\mathrm{e}^{-0.03t} \Rightarrow \mathrm{e}^{-0.03t} = \dfrac{60-25}{\text{"50"}}$	M1	3.4
	$t = \dfrac{\ln(0.7)}{-0.03} = 11.8891648 = 11.9$ minutes (1 dp)	A1	1.1b
		(2)	
(c)	A valid evaluation of the model, which relates to the large values of t. E.g. • As $20.3 < 25$ then the model is not true for large values of t • $\mathrm{e}^{-0.03t} = \dfrac{20.3-25}{\text{"50"}} = -0.094$ does not have any solutions and so the model predicts that tea in the room will never be 20.3 °C. So the model does not work for large values of t • $t = 120 \Rightarrow \theta = 25 + 50e^{-0.03(120)} = 26.36\ldots$ which is not approximately equal to 20.3, so the model is not true for large values of t	B1	3.5a
		(1)	
		(4 marks)	

Question 31 Notes:

(a)	
B1:	Applies $t = 0, \theta = 75$ to give the complete model $\theta = 25 + 50\mathrm{e}^{-0.03t}$
(b)	
M1:	Applies $\theta = 60$ and their value of A to the model and rearranges to make $\mathrm{e}^{-0.03t}$ the subject.
	Note: Later working can imply this mark.
A1	Obtains 11.9 (minutes) with no errors in manipulation seen.
(c)	
B1	See scheme

Question	Scheme	Marks	AOs
32(a)	$\frac{\mathrm{d}p}{\mathrm{d}t} \propto p \Rightarrow \frac{\mathrm{d}p}{\mathrm{d}t} = kp$	B1	3.3
	$\int \frac{1}{p}\,\mathrm{d}p = \int k\,\mathrm{d}t$	M1	1.1b
	$\ln p = kt\ \{+c\}$	A1	1.1b
	$\ln p = kt + c \Rightarrow \quad p = \mathrm{e}^{kt+c} = \mathrm{e}^{kt}\mathrm{e}^{c} \quad \Rightarrow \quad p = a\mathrm{e}^{kt}$ *	A1 *	2.1
		(4)	
(b)	$p = a\mathrm{e}^{kt} \Rightarrow \ln p = \ln a + kt$ and evidence of understanding that either • gradient $= k$ or $"M" = k$ • vertical intercept $= \ln a$ or $"C" = \ln a$	M1	2.1
	gradient $= k = 0.14$	A1	1.1b
	vertical intercept $= \ln a = 3.95 \Rightarrow a = \mathrm{e}^{3.95} = 51.935 = 52$ (2 sf)	A1	1.1b
		(3)	
(c)	e.g. • $p = a\mathrm{e}^{kt} \Rightarrow p = a(\mathrm{e}^{k})^{t} = ab^{t}$, • $p = 52\mathrm{e}^{0.14t} \Rightarrow p = 52(\mathrm{e}^{0.14})^{t}$	B1	2.2a
	$b = 1.15$ which can be implied by $p = 52(1.15)^{t}$	B1	1.1b
		(2)	
(d)(i)	Initial area (i.e. $"52"\ \mathrm{mm}^2$) of bacterial culture that was first placed onto the circular dish.	B1	3.4
(d)(ii)	E.g. • Rate of increase per hour of the area of bacterial culture • The area of bacterial culture increases by "15%" each hour	B1	3.4
		(2)	
(e)	The model predicts that the area of the bacteria culture will increase indefinitely, but the size of the circular dish will be a constraint on this area.	B1	3.5b
		(1)	
			(12 marks)

Question **32** Notes:	
(a)	
B1:	Translates the scientist's statement regarding proportionality into a differential equation, which involves a constant of proportionality. e.g. $\frac{\mathrm{d}p}{\mathrm{d}t} \propto p \Rightarrow \frac{\mathrm{d}p}{\mathrm{d}t} = kp$
M1:	Correct method of separating the variables p and t in their differential equation
A1:	$\ln p = kt$, with or without a constant of integration
A1*:	Correct proof with no errors seen in working.
(b)	
M1:	See scheme
A1:	Correctly finds $k = 0.14$
A1:	Correctly finds $a = 52$
(c)	
B1:	Uses algebra to correctly deduce either • $p = ab^t$ from $p = a\mathrm{e}^{kt}$ • $p = "52"(\mathrm{e}^{"0.14"})^t$ from $p = "52"\mathrm{e}^{"0.14"t}$
B1:	See scheme
(d)(i)	
B1:	See scheme
(d)(ii)	
B1:	See scheme
(e)	
B1:	Gives a correct long-term limitation of the model for p. (See scheme).

Question	Scheme	Marks	AOs
33(a)	Substitutes $t=0.5$ into $m=25\mathrm{e}^{-0.05t} \Rightarrow m=25\mathrm{e}^{-0.05\times0.5}$	M1	3.4
	$\Rightarrow m=24.4\text{g}$	A1	1.1b
		(2)	
(b)	States or uses $\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{-0.05t}\right)=\pm C\,\mathrm{e}^{-0.05t}$	M1	2.1
	$\frac{\mathrm{d}m}{\mathrm{d}t}=-0.05\times25\mathrm{e}^{-0.05t}=-0.05m \Rightarrow k=-0.05$	A1	1.1b
		(2)	
			(4 marks)

Notes:

(a)

M1: Substitutes $t=0.5$ into $m=25\mathrm{e}^{-0.05t} \Rightarrow m=25\mathrm{e}^{-0.05\times0.5}$

A1: $m=24.4\text{g}$ An answer of $m=24.4\text{g}$ with no working would score both marks

(b)

M1: Applies the rule $\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{kx}\right)=k\,\mathrm{e}^{kx}$ in this context by stating or using $\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{-0.05t}\right)=\pm C\,\mathrm{e}^{-0.05t}$

A1: $\frac{\mathrm{d}m}{\mathrm{d}t}=-0.05\times25\mathrm{e}^{-0.05t}=-0.05m \Rightarrow k=-0.05$

Question Number	Scheme		Marks
34(a)	Replaces 2^{2x+1} with $2^{2x}\times 2$ or states $2^{2x+1}=2^{2x}\times 2$ or states $\left(2^x\right)^2=2^{2x}$	Uses the addition **or** power law of indices on 2^{2x} or 2^{2x+1}. E.g. $2^x\times 2^x=2^{2x}$ or $\left(2^x\right)^2=2^{2x}$ or $2^{2x+1}=2\times 2^{2x}$ or $2^{x+0.5}=2^x\times\sqrt{2}$ or $2^{2x+1}=\left(2^{x+0.5}\right)^2$.	M1
	$2^{2x+1}-17\times 2^x+8=0$ $\Rightarrow 2y^2-17y+8=0$*	Cso. Complete proof that includes explicit statements for the addition **and** power law of indices on 2^{2x+1} with no errors. The equation needs to be as printed including the "= 0". If they work backwards, they do not need to write down the printed answer first but must end with the version in 2^x including '= 0'.	A1*
	The following are examples of acceptable proofs.		
	$2^{2x+1}=\left(2^{x+0.5}\right)^2=\left(2^x\sqrt{2}\right)^2=\left(y\sqrt{2}\right)^2=2y^2$ $\Rightarrow 2^{2x+1}-17\left(2^x\right)+8=2y^2-17y+8=0$		
	$2y^2=2\times 2^x\times 2^x=2^{2x+1}$ $\Rightarrow 2^{2x+1}-17\left(2^x\right)+8=2y^2-17y+8=0$		
	$2y^2-17y+8=0\Rightarrow 2\left(2^x\right)^2-17\left(2^x\right)+8=0$ $\Rightarrow 2\times 2^{2x}-17\left(2^x\right)+8=0\Rightarrow 2^{2x+1}-17\left(2^x\right)+8=0$		
	$2^{2x+1}=2\times 2^{2x}\Rightarrow 2\times 2^{2x}-17\left(2^x\right)+8=0$ $\Rightarrow 2y^2-17y+8=0$ Scores **M1A0** as $2^{2x}=\left(2^x\right)^2$ has not been shown explicitly		
	Special Case: $2^{2x+1}=2^1\times\left(2^x\right)^2$ or $2^{2x+1}=\left(2^x\right)^2\times 2^1$ **With or without the multiplication signs and with no subsequent explicit evidence of the power law scores M1A0**		
	Example of insufficient working: $2^{2x+1}=2\left(2^x\right)^2=2y^2$ **scores no marks as neither rule has been shown explicitly.**		
			(2)

(b)	$2y^2-17y+8=0\Rightarrow(2y-1)(y-8)(=0)\Rightarrow y=...$ or $2\left(2^x\right)^2-17\left(2^x\right)+8=0\Rightarrow\left(2\left(2^x\right)-1\right)\left(\left(2^x\right)-8\right)(=0)\Rightarrow 2^x=...$ Solves the **given** quadratic either in terms of y or in terms of 2^x See General Principles for solving a 3 term quadratic Note that completing the square on e.g. $y^2-\frac{17}{2}y+4=0$ requires $\left(y\pm\frac{17}{4}\right)^2\pm q\pm4=0\Rightarrow y=...$		M1
	$(y=)\frac{1}{2},8$ or $\left(2^x=\right)\frac{1}{2},8$	Correct values	A1
	$\Rightarrow 2^x=\frac{1}{2},8\Rightarrow x=-1,3$	M1: Either finds one correct value of x for their 2^x or obtains a correct numerical expression in terms of logs e.g. for $k>0$ $2^x=k\Rightarrow x=\log_2 k$ or $\frac{\log k}{\log 2}$ A1: $x=-1,3$ only. Must be values of x.	M1 A1
			(4)
			(6 marks)

Question Number	Scheme	Notes	Marks
35	9^{3x+1} = for example $3^{2(3x+1)}$ or $\left(3^2\right)^{3x+1}$ or $\left(3^{(3x+1)}\right)^2$ or $3^{3x+1}\times 3^{3x+1}$ or $(3\times 3)^{3x+1}$ or $3^2\times\left(3^2\right)^{3x}$ or $\left(9^{\frac{1}{2}}\right)^y$ or $9^{\frac{1}{2}y}$ or $y=2(3x+1)$	Expresses 9^{3x+1} correctly as a power of 3 or expresses 3^y correctly as a power of 9 or expresses y correctly in terms of x (**This mark is <u>not</u> for just** $3^2=9$)	M1
	$=3^{6x+2}$ or $y = 6x + 2$ or $a = 6$, $b = 2$	Cao (isw if necessary)	A1
	Providing there is no incorrect work, allow sight of $6x + 2$ to score both marks Correct answer only implies both marks **Special case:** 3^{6x+1} **only** scores M1A0		
			[2]
	Alternative using logs		
	$9^{3x+1}=3^y\Rightarrow\log 9^{3x+1}=\log 3^y$		
	$(3x+1)\log 9=y\log 3$	Use power law correctly on both sides	M1
	$y=\dfrac{\log 9}{\log 3}(3x+1)$		
	$y=6x+2$	cao	A1
			2 marks

Question Number	Scheme		Marks
36(a)	$\left(4^x =\right) y^2$	Allow y^2 or $y \times y$ or "y squared" "$4^x =$" not required	B1
	Must be seen in part (a)		
			(1)
(b)	$8y^2 - 9y + 1 = (8y-1)(y-1) = 0 \Rightarrow y = ...$ or $(8(2^x)-1)((2^x)-1) = 0 \Rightarrow 2^x = ...$	For attempting to solve the given equation as a **3 term quadratic** in y or as a **3 term quadratic** in 2^x leading to a value of y or 2^x (Apply usual rules for solving the quadratic – see general guidance) Allow x (or any other letter) instead of y for this mark e.g. an attempt to solve $8x^2 - 9x + 1 = 0$	M1
	$2^x (\text{or } y) = \frac{1}{8},\ 1$	Both correct answers of $\frac{1}{8}$ (oe) and 1 for 2^x or y or their letter but **not x** unless $2^x (\text{or } y)$ is implied later	A1
	$x = -3 \quad x = 0$	M1: A correct attempt to find one **numerical value** of x from their $2^x (\text{or } y)$ **which must have come from a 3 term quadratic equation**. If logs are used then they must be evaluated. A1: Both $x = -3$ and/or $x = 0$ May be implied by e.g. $2^{-3} = \frac{1}{8}$ and $2^0 = 1$ **and no extra values.**	M1A1
			(4)
			(5 marks)

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Question Number	Scheme	Marks
37.	(a) $32^{\frac{1}{5}} = 2$	B1 (1)
	(b) For 2^{-2} or $\frac{1}{4}$ or $\left(\frac{1}{2}\right)^2$ or 0.25 as coefficient of x^k, for any value of k including $k = 0$	M1
	Correct index for x so Ax^{-2} or $\frac{A}{x^2}$ o.e. for any value of A	B1
	$= \frac{1}{4x^2}$ or $0.25\,x^{-2}$	A1 cao (3)
		4 Marks

Notes

(a) B1 Answer 2 must be in part (a) for this mark

(b) Look at their final answer

M1 For 2^{-2} or $\frac{1}{4}$ or $\left(\frac{1}{2}\right)^2$ or 0.25 in their answer as coefficient of x^k for numerical value of k (including $k = 0$) so final answer $\frac{1}{4}$ is M1 B0 A0

B1 Ax^{-2} or $\frac{A}{x^2}$ or equivalent e.g. $Ax^{-\frac{10}{5}}$ or $Ax^{-\frac{50}{25}}$ i.e. correct power of x seen in final answer

May have a bracket provided it is $(Ax)^{-2}$ or $\left(\frac{A}{x}\right)^2$

A1 $\frac{1}{4x^2}$ or $\frac{1}{4}x^{-2}$ or $0.25\,x^{-2}$ oe but must be correct power **and** coefficient combined correctly and must not be followed by a different wrong answer.

Poor bracketing: $2x^{-2}$ earns M0 B1 A0 as correct power of x is seen in this solution (They can recover if they follow this with $\frac{1}{4x^2}$ etc)

Special case $(2x)^{-2}$ as a **final** answer and $\left(\frac{1}{2x}\right)^2$ can have M0 B1 A0 if the correct expanded answer is not seen

The correct answer $\frac{1}{4x^2}$ etc. followed by $\left(\frac{1}{2x}\right)^2$ or $(2x)^{-2}$, treat $\frac{1}{4x^2}$ as final answer so M1 B1 A1 isw

But the correct answer $\frac{1}{4x^2}$ etc clearly followed by the wrong $2x^{-2}$ or $4x^{-2}$, gets M1 B1 A0 do not ignore subsequent wrong work here

Question Number	Scheme	Marks
38.(a)	$81^{\frac{3}{2}} = (81^{\frac{1}{2}})^3 = 9^3$ or $81^{\frac{3}{2}} = (81^3)^{\frac{1}{2}} = (531441)^{\frac{1}{2}}$	M1
	$=729$	A1
		(2)
(b)	$(4x^{-\frac{1}{2}})^2 = 16x^{-\frac{2}{2}}$ or $\frac{16}{x}$ or equivalent	M1
	$x^2(4x^{-\frac{1}{2}})^2 = 16x$	A1
		(2)
		(4 marks)

(a) M1 Dealing with either the 'cube' or the 'square root' first. A correct answer will imply this mark.

Also accept a law of indices approach $81^{\frac{3}{2}} = 81^1 \times 81^{\frac{1}{2}} = 81 \times 9$

A1 Cao 729. Accept $(\pm)729$

(b) M1 For correct use of power 2 on both 4 and the $x^{-\frac{1}{2}}$ term.

A1 Cao $=16x$

Question Number	Scheme		Marks
39(a)	$8^{\frac{1}{3}} = 2$ or $8^5 = 32768$	A correct attempt to deal with the $\frac{1}{3}$ or the 5. $8^{\frac{1}{3}} = \sqrt[3]{8}$ or $8^5 = 8\times8\times8\times8\times8$	M1
	$\left(8^{\frac{5}{3}} =\right)$ 32	Cao	A1
	A correct answer with no working scores full marks		
	Alternative $8^{\frac{5}{3}} = 8\times 8^{\frac{2}{3}} = 8\times 2^2 =$ M1 (Deals with the 1/3) = 32 A1		
			(2)
(b)	$\left(2x^{\frac{1}{2}}\right)^3 = 2^3 x^{\frac{3}{2}}$	One correct power either 2^3 or $x^{\frac{3}{2}}$. $\left(2x^{\frac{1}{2}}\right)\times\left(2x^{\frac{1}{2}}\right)\times\left(2x^{\frac{1}{2}}\right)$ on its own is not sufficient for this mark.	M1
	$\dfrac{8x^{\frac{3}{2}}}{4x^2} = 2x^{-\frac{1}{2}}$ or $\dfrac{2}{\sqrt{x}}$	M1: Divides coefficients of x **and** subtracts their powers of x. **Dependent on the previous M1**	dM1A1
		A1: Correct answer	
	Note that unless the power of x implies that they have subtracted their powers you would need to see evidence of subtraction. E.g. $\dfrac{8x^{\frac{3}{2}}}{4x^2} = 2x^{\frac{1}{2}}$ would score dM0 unless you see some evidence that 3/2 – 2 was intended for the power of x.		
	Note that there is a misconception that $\dfrac{\left(2x^{\frac{1}{2}}\right)^3}{4x^2} = \left(\dfrac{2x^{\frac{1}{2}}}{4x^2}\right)^3$ - this scores 0/3		
			(3)
			[5]

Question Number	Scheme	Notes	Marks
40(a)	$2^y = 8 \Rightarrow y = 3$	Cao (Can be implied i.e. by 2^3)	B1
	(Alternative: Takes logs base 2: $\log_2 2^y = \log_2 8 \Rightarrow y\log_2 2 = 3\log_2 2 \Rightarrow y = 3$)		
			(1)
(b)	$8 = 2^3$	Replaces 8 by 2^3 (May be implied)	M1
	$4^{x+1} = (2^2)^{x+1}$ or $(2^{x+1})^2$	Replaces 4 by 2^2 **correctly.**	M1
	$2^{3x+2} = 2^3 \Rightarrow 3x+2=3 \Rightarrow x = \frac{1}{3}$	M1: Adds their powers of 2 on the lhs and puts this equal to 3 leading to a solution for *x*. A1: $x = \frac{1}{3}$ or $x = 0.\dot{3}$ or awrt 0.333	M1A1
			(4)
(b) Way 2	$4^{x+1} = 4 \times 4^x$	Obtains 4^{x+1} in terms of 4^x **correctly**	M1
	$2^x \times 4^x = 8^x$	Combines their 2^x and 4^x **correctly**	M1
	$4 \times 8^x = 8 \Rightarrow 8^x = 2 \Rightarrow x = \frac{1}{3}$	M1: Solves $8^x = k$ leading to a solution for *x*. A1: $x = \frac{1}{3}$ or $x = 0.\dot{3}$ or awrt 0.333	M1A1
			[5]

Question Number	Scheme	Marks
41.	$(8^{2x+3} = \left(2^3\right)^{2x+3}) = 2^{3(2x+3)}$ or 2^{ax+b} with a = 6 **or** b = 9	M1
	$= 2^{6x+9}$ *or* $= 2^{3(2x+3)}$ as **final** answer with no errors or $(y =) 6x + 9$ or $3(2x + 3)$	A1 **[2]**
		2 marks
	Notes	
	M1: Uses $8 = 2^3$, and **multiplies** powers $3(2x + 3)$. Does **not** add powers. (Just $8 = 2^3$ or $8^{\frac{1}{3}} = 2$ is M0) **A1:** Either 2^{6x+9} *or* $= 2^{3(2x+3)}$ or $(y =) 6x + 9$ or $3(2x + 3)$	
	Note: Examples: 2^{6x+3} scores M1A0 **:** $8^{2x+3} = \left(2^3\right)^{2x+3} = 2^{3+2x+3}$ gets M0A0 **Special case:** **:** $= 2^{6x}\,2^9$ without seeing as single power M1A0 **Alternative method using logs:** $8^{2x+3} = 2^y \Rightarrow (2x+3)\log 8 = y\log 2 \Rightarrow y = \dfrac{(2x+3)\log 8}{\log 2}$ **So** $(y =) 6x + 9$ or $3(2x + 3)$	**M1** **A1 [2]**

Question Number	Scheme	Marks
42. (a)	$\left\{(32)^{\frac{3}{5}}\right\} = \left(\sqrt[5]{32}\right)^3$ or $\sqrt[5]{(32)^3}$ or 2^3 or $\sqrt[5]{32768}$	M1
	$= 8$	A1
		[2]
(b)	$\left\{\left(\dfrac{25x^4}{4}\right)^{-\frac{1}{2}}\right\} = \left(\dfrac{4}{25x^4}\right)^{\frac{1}{2}}$ or $\left(\dfrac{5x^2}{2}\right)^{-1}$ or $\dfrac{1}{\left(\dfrac{25x^4}{4}\right)^{\frac{1}{2}}}$ ***See notes below***	M1
	$= \dfrac{2}{5x^2}$ or $\dfrac{2}{5}x^{-2}$ ***See notes for other alternatives***	A1
		[2]
		4
	Notes	
(a)	**M1**: for a full correct interpretation of the fractional power. **Note:** $5\times(32)^3$ is M0.	
	A1: for 8 only.	
	Note: Award M1A1 for writing down 8.	
(b)	**M1:** For use of $\dfrac{1}{2}$ OR use of -1	
	Use of $\dfrac{1}{2}$: Candidate needs to demonstrate the they have rooted all three elements in their bracket.	
	Use of -1: Either Candidate has $\dfrac{1}{\text{Bracket}}$ or $\left(\dfrac{Ax^C}{B}\right)$ becomes $\left(\dfrac{B}{Ax^C}\right)$.	
	Allow M1 for...	
	• $\left(\dfrac{4}{25x^4}\right)^{\frac{1}{2}}$ or $\left(\dfrac{5x^2}{2}\right)^{-1}$ or $\dfrac{1}{\left(\dfrac{25x^4}{4}\right)^{\frac{1}{2}}}$ or $\sqrt{\left(\dfrac{4}{25x^4}\right)}$ or $\dfrac{1}{\sqrt{\left(\dfrac{25x^4}{4}\right)}}$ or $\left(\dfrac{\frac{1}{25x^4}}{\frac{1}{4}}\right)^{\frac{1}{2}}$ or $\dfrac{\frac{1}{5x^2}}{\frac{1}{2}}$ or $\dfrac{\frac{1}{5}x^{-2}}{\frac{1}{2}}$	
	or $-\left(\dfrac{5x^2}{2}\right)$ or $\left(\dfrac{-5x^{-2}}{-2}\right)$ or $-\left(\dfrac{5x^{-2}}{2}\right)$ or $\dfrac{5x^{-2}}{2}$	
	• $\left(\dfrac{4}{25x^4}\right)^K$ or $\left(\dfrac{5x^2}{2}\right)^C$ where K, C are any powers including 1.	
	A1: for either $\dfrac{2}{5x^2}$ or $\dfrac{2}{5}x^{-2}$ or $0.4x^{-2}$ or $\dfrac{0.4}{x^2}$.	
	Note: $\left(\sqrt{\left(\dfrac{25x^4}{4}\right)}\right)^{-1}$ is not enough work by itself for the method mark.	
	Note: A final answer of $\dfrac{1}{\frac{5}{2}x^2}$ or $\dfrac{1}{2\frac{1}{2}x^2}$ or $\dfrac{1}{2.5x^2}$ is A0.	
	Note: Also allow $\pm\dfrac{2}{5x^2}$ or $\pm\dfrac{2}{5}x^{-2}$ or $\pm 0.4x^{-2}$ or $\pm\dfrac{0.4}{x^2}$ for A1.	

Question Number	Scheme	Marks
43. **(a)**	5 (or ± 5)	B1 (1)
(b)	$25^{-\frac{3}{2}} = \frac{1}{25^{\frac{3}{2}}}$ **or** $25^{\frac{3}{2}}=125$ or better	M1
	$\frac{1}{125}$ or 0.008 (or $\pm\frac{1}{125}$)	A1
		(2) **3**
	Notes (a) Give B1 for 5 or ± 5 Anything else is B0 (including just –5) (b) M: Requires reciprocal OR $25^{\frac{3}{2}} = 125$ Accept $\frac{1}{5^3}, \frac{1}{\sqrt{15625}}, \frac{1}{25\times 5}, \frac{1}{25\sqrt{25}}, \frac{1}{\sqrt{25}^3}$ for M1 Correct answer with no working (or notation errors in working) scores both marks i.e. M1 A1 M1A0 for - $\frac{1}{125}$ without $+\frac{1}{125}$	

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Question Number	Scheme	Marks
44.		
(a)	$16^{\frac{1}{4}}=2$ or $\dfrac{1}{16^{\frac{1}{4}}}$ or better	M1
	$\left(16^{-\frac{1}{4}}=\right)\ \dfrac{1}{2}$ or 0.5 (ignore $\pm$)	A1
		(2)
(b)	$\left(2x^{-\frac{1}{4}}\right)^4=2^4x^{-\frac{4}{4}}$ or $\dfrac{2^4}{x^{\frac{4}{4}}}$ or equivalent	M1
	$x\left(2x^{-\frac{1}{4}}\right)^4=2^4$ or 16	A1 cao
		(2) **4**
	<u>Notes</u>	
(a)	M1 for a correct statement dealing with the $\frac{1}{4}$ or the $-$ power This may be awarded if 2 is seen or for reciprocal of their $16^{\frac{1}{4}}$ s.c ¼ is M1 A0 , also 2^{-1} is M1 A0 $\pm\frac{1}{2}$ is not penalised so M1 A1	
(b)	M1 for **correct** use of the power 4 on both the 2 and the *x* terms A1 for cancelling the *x* and simplifying to one of these two forms. Correct answers with no working get full marks	

Question Number	Scheme	Marks
45	$32=2^5$ or $2048=2^{11}$, $\sqrt{2}=2^{1/2}$ or $\sqrt{2048}=(2048)^{\frac{1}{2}}$ $a=\frac{11}{2}$ $\left(\text{or } 5\frac{1}{2} \text{ or } 5.5\right)$	B1, B1 B1 **[3]**
	1st B1 for $32=2^5$ or $2048=2^{11}$ This should be explicitly seen: $32\sqrt{2}=2^a$ followed by $2^5\sqrt{2}=2^a$ is OK Even writing $32\times 2=2^5\times 2\left(=2^6\right)$ is OK but simply writing $32\times 2=2^6$ is NOT 2nd B1 for $2^{\frac{1}{2}}$ or $(2048)^{\frac{1}{2}}$ seen. This mark may be implied 3rd B1 for answer as written. **Need *a* =** … so $2^{\frac{11}{2}}$ is B0 $a=\frac{11}{2}$ $\left(\text{or } 5\frac{1}{2} \text{ or } 5.5\right)$ with no working scores full marks. If a = 5.5 seen then award 3/3 unless it is clear that the value follows from totally incorrect work. Part solutions: e.g. $2^5\sqrt{2}$ scores the first B1. Special case: If $\sqrt{2}=2^{1/2}$ is not explicitly seen, but the final answer includes $\frac{1}{2}$, e.g. $a=2\frac{1}{2}$, $a=4\frac{1}{2}$, the second B1 is given by implication.	

Question Number	Scheme	Marks
46 (a)	**5** $\qquad (\pm 5$ is B0)	B1 (1)
(b)	$\dfrac{1}{(\text{their } 5)^2}$ or $\left(\dfrac{1}{\text{their } 5}\right)^2$	M1
	$= \dfrac{1}{25}$ **or 0.04** $\qquad (\pm\dfrac{1}{25}$ is A0)	A1 (2) **[3]**
(b)	M1 follow through their value of 5. Must have reciprocal and square. 5^{-2} is <u>not</u> sufficient to score this mark, unless $\dfrac{1}{5^2}$ follows this. A negative introduced at any stage can score the M1 but not the A1, e.g. $125^{-2/3} = \left(-\dfrac{1}{5}\right)^2 = \dfrac{1}{25}$ scores M1 A0 $125^{-2/3} = -\left(\dfrac{1}{5}\right)^2 = -\dfrac{1}{25}$ scores M1 A0. Correct answer with no working scores both marks. <u>Alternative</u>: $\dfrac{1}{\sqrt[3]{125^2}}$ or $\dfrac{1}{(125^2)^{1/3}}$ M1 (reciprocal and the correct number squared) $\left(= \dfrac{1}{\sqrt[3]{15625}}\right)$ $= \dfrac{1}{25}$ A1	

Question number	Scheme	Marks	
47.	(a) 2	B1	(1)
	(b) x^9 seen, or $(\text{answer to (a)})^3$ seen, or $(2x^3)^3$ seen.	M1	
	$8x^9$	A1	(2)
			3
	(b) M: Look for x^9 first… if seen, this is M1. If not seen, look for $(\text{answer to (a)})^3$, e.g. 2^3… this would score M1 even if it does not subsequently become 8. (Similarly for other answers to (a)). In $(2x^3)^3$, the 2^3 is implied, so this scores the M mark. <u>Negative answers</u>: (a) Allow -2. Allow ± 2. Allow '2 or -2'. (b) Allow $\pm 8x^9$. Allow '$8x^9$ or $-8x^9$'. N.B. If part (a) is wrong, it is possible to 'restart' in part (b) and to score full marks in part (b).		

Question Number	Scheme	Marks
48. (i)	Use of power rule so $(y-1)\log 1.01 = \log 500$ or $(y-1) = \log_{1.01} 500$	M1
	625.56	A1 **(2)**
(ii) (a)	Ignore labels (a) and (b) in part ii and mark work as seen	
	$\log_4 (3x+5)^2 =$ Applies power law of logarithms	M1
	Uses $\log_4 4 = 1$ or $4^1 = 4$	M1
	Uses quotient or product rule so e.g. $\log(3x+5)^2 = \log 4(3x+8)$ or $\log \frac{(3x+5)^2}{(3x+8)} = 1$	M1
	Obtains with no errors $9x^2 + 18x - 7 = 0$ *	A1* cso **(4)**
(b)	Solves given or "their" quadratic equation by any of the standard methods	M1
	Obtains $x = \frac{1}{3}$ and $-\frac{7}{3}$ and rejects $-\frac{7}{3}$ to give just $\frac{1}{3}$	A1 **(2)** **[8]**

Notes

(i)
M1: Applies power law of logarithms correctly or changes base (Allow missing brackets)
A1: Accept answers which round to 625.56 (This may follow 624.56 + 1 = or may follow $y = \log_{1.01} 505$ or $\frac{\log 505}{\log 1.01}$ or may appear with no working)

(ii) (a)
M1: Applies power law of logarithms $2\log_4(3x+5) = \log_4(3x+5)^2$
M1: Uses $\log_4 4 = 1$ or $4^1 = 4$
M1: Applying the subtraction or addition law of logarithms **correctly** to make **two** log **terms into one** log term in x (*see note below)
A1cso: This is a given answer and needs a correct algebraic statement such as $9x^2 + 30x + 25 = 4(3x+8)$ followed by a conclusion, such as $9x^2 + 18x - 7 = 0$
(ii) (b)
M1: Solves by factorisation or by completion of the square or by correct use of formula (see general principles)
A1: Needs to find two answers and reject one to give the correct $\frac{1}{3}$ (This may be indicated by underlining just the 1/3 for example).
Special case: States $\frac{\log(3x+5)^2}{\log(3x+8)} = \log\frac{(3x+5)^2}{(3x+8)} = 1$, loses the third M mark in part ii(a) and the A1 cso

Question Number	Scheme	Marks
49. (i)	Use of power rule so $\log(x+a)^2 = \log 16a^6$ or $2\log(x+a) = 2\log 4a^3$ or $\log(x+a) = \log(16a^6)^{\frac{1}{2}}$	M1
	Removes logs and square roots, **or** halves then removes logs to give $(x+a) = 4a^3$ Or $x^2 + 2ax + a^2 - 16a^6 = 0$ followed by factorisation or formula to give $x = \sqrt{16a^6} - a$	M1
	$(x =)\ 4a^3 - a$ (depends on previous M's and must be this expression or equivalent)	A1cao **(3)**
(ii) Way 1	$\log_3 \dfrac{(9y+b)}{(2y-b)} = 2$ Applies quotient law of logarithms	M1
	$\dfrac{(9y+b)}{(2y-b)} = 3^2$ Uses $\log_3 3^2 = 2$	M1
	$(9y+b) = 9(2y-b) \Rightarrow y =$ Multiplies across and makes y the subject	M1
	$y = \frac{10}{9}b$	A1cso **(4)**
Way 2	Or : $\log_3(9y+b) = \log_3 9 + \log_3(2y-b)$ 2nd M mark	M1
	$\log_3(9y+b) = \log_3 9(2y-b)$ 1st M mark	M1
	$(9y+b) = 9(2y-b) \Rightarrow y = \frac{10}{9}b$ Multiplies across and makes y the subject	M1 A1cso **(4)** **[7]**

	Notes
(i)	1st M1: Applies power law of logarithms correctly to one side of the equation M1: Correct log work in correct order. If they square and obtain a quadratic the algebra should be correct. The marks is for $x+a=\sqrt{16a^6}$ isw so allow $x+a=\pm\ 4a^3$ for Method mark. Also allow $x+a = 4a^4$ or $x+a=\pm\ 4a^{5.5}$ or even $x+a=16a^3$ as there is evidence of attempted square root. May see the correct $x+a=10^{(\log 4+3\log a)}$ *so* $x=-a+10^{(\log 4+3\log a)}$ which gains M1A0 unless followed by the answer in the scheme. A1: Do not allow $x=\pm\ 4a^3-a$ for accuracy mark. You may see the factorised $a(2a+1)(2a-1)$ o.e.
(ii)	M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms into one** log term in y M1: Uses $\log_3 3^2 = 2$ 3rd M1: Obtains **correct** linear equation in y usually the one in the scheme and attempts $y =$ A1cso: $y = \frac{10}{9}b$ or correct equivalent after **completely correct** work. **Special case:** $\dfrac{\log_3(9y+b)}{\log_3(2y-b)} = 2$ is M0 unless clearly crossed out and replaced by the correct $\log_3 \dfrac{(9y+b)}{(2y-b)} = 2$ Candidates may then write $\dfrac{(9y+b)}{(2y-b)} = 3^2$ and proceed to the *correct* answer – allow M0M1M1A0 as the answer requires a completely correct solution.

Question Number	Scheme	Marks
50(i)	Two Ways of answering the question are given in part (i)	
Way 1	$\log_3\left(\dfrac{3b+1}{a-2}\right) = -1$ or $\log_3\left(\dfrac{a-2}{3b+1}\right) = 1$ — Applying the subtraction law of logarithms	M1
	$\dfrac{3b+1}{a-2} = 3^{-1}\left\{= \dfrac{1}{3}\right\}$ or $\left(\dfrac{a-2}{3b+1}\right) = 3$ — Making a correct connection between log base 3 and 3 to a power.	M1
	$\{9b+3 = a-2 \Rightarrow\}\ b = \dfrac{1}{9}a - \dfrac{5}{9}$ — $b = \dfrac{1}{9}a - \dfrac{5}{9}$ or $b = \dfrac{a-5}{9}$	A1 oe
		[3]
	In **Way 2** a correct connection between log base 3 and "3 to a power" is used before applying the subtraction or addition law of logs	
(i) **Way 2**	Either $\log_3(3b+1) - \log_3(a-2) = -\log_3 3$ or $\log_3(3b+1) + \log_3 3 = \log_3(a-2)$	2nd M1
	$\log_3(3b+1) = \log_3(a-2) - \log_3 3 = \log_3\left(\dfrac{a-2}{3}\right)$ or $\log_3 3(3b+1) = \log_3(a-2)$	1st M1
	$\{3b+1 = \dfrac{a-2}{3}\}\ b = \dfrac{1}{9}a - \dfrac{5}{9}$	A1
		[3]
	Five Ways of answering the question are given in part (ii)	
(ii) **Way 1** **See also common approach below in notes**	$32(2^{2x}) - 7(2^x) = 0$ — Deals with power 5 correctly giving $\times 32$	M1
	So, $2^x = \dfrac{7}{32}$ — $2^x = \dfrac{7}{32}$ or $y = \dfrac{7}{32}$ or awrt 0.219	A1 oe
	$x\log 2 = \log\left(\dfrac{7}{32}\right)$ or $x = \dfrac{\log\left(\frac{7}{32}\right)}{\log 2}$ or $x = \log_2\left(\dfrac{7}{32}\right)$ — A valid method for solving $2^x = \frac{7}{32}$ Or $2^x = k$ to achieve $x = \ldots$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]
	Begins with $2^{2x+5} = 7(2^x)$ (for Way 2 and Way 3) (see notes below)	
(ii) **Way 2**	$(2x+5)\log 2 = \log 7 + x\log 2$ — Correct application of **either** the power law **or** addition law of logarithms	M1
	Correct result after applying the power **and** addition laws of logarithms.	A1
	$2x\log 2 + 5\log 2 = \log 7 + x\log 2$ $\Rightarrow x = \dfrac{\log 7 - 5\log 2}{\log 2}$ — Multiplies out, collects x terms to achieve $x = \ldots$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]
(ii) **Way 3**	$2x+5 = \log_2 7 + x$ — Evidence of $\log_2$ and either $2^{2x+5} \to 2x+5$ or $7(2^x) \to \log_2 7 + \log_2(2^x)$	M1
	$2x+5 = \log_2 7 + x$ oe.	A1
	$2x - x = \log_2 7 - 5$ $\Rightarrow x = \log_2 7 - 5$ — Collects x terms to achieve $x = \ldots$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]

(ii) **Way 4**	$2^{2x+5} = 7(2^x) \Rightarrow 2^{x+5} = 7$	
	$x+5 = \log_2 7$ or $\dfrac{\log 7}{\log 2}$ — Evidence of $\log_2$ and either $2^{x+5} \to x+5$ or $7 \to \log_2 7$	M1
	$x+5 = \log_2 7$ oe.	A1
	$x = \log_2 7 - 5$ — Rearranges to achieve $x = ...$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]
Way 5 (similar to Way 3)	$2^{2x+5} = 2^{\log_2 7}(2^x)$ — 7 is replaced by $2^{\log_2 7}$	M1
	$2x+5 = \log_2 7 + x$ — $2x+5 = \log_2 7 + x$ oe.	A1
	$2x - x = \log_2 7 - 5$ $\Rightarrow x = \log_2 7 - 5$ — Collects x terms to achieve $x = ...$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]
		7

	Question 50 Notes	
(i)	**1st M1**	Applying either the addition or subtraction law of logarithms correctly to combine any **two** log terms into **one** log term.
	2nd M1	For making a correct connection between log base 3 and 3 to a power.
	A1	$b = \frac{1}{9}a - \frac{5}{9}$ or $b = \frac{a-5}{9}$ o.e. e.g. Accept $b = \frac{1}{3}\left(\frac{a}{3} - \frac{5}{3}\right)$ but not $b = \frac{a-2}{9} - \frac{3}{9}$ nor $b = \frac{\left(\frac{a}{3} - \frac{5}{3}\right)}{3}$
(ii)	**1st M1**	First step towards solution – an equation with one side or other correct or one term dealt with correctly (see five* possible methods above)
	1st A1	Completely correct first step – giving a correct equation as shown above
	dM1	Correct complete method (all log work correct) and working to reach x = in terms of logs reaching a correct expression or one where the only errors are slips solving linear equations
	2nd A1	Accept answers which round to -2.19 If a second answer is also given this becomes A0
	Special Case in (i)	Writes $\frac{\log_3(3b+1)}{\log_3(a-2)} = -1$ and proceeds to $\frac{3b+1}{a-2} = 3^{-1}\left\{= \frac{1}{3}\right\}$ and to correct answer- Give M0M1A1 (special case)
	Common approach to part (ii)	Let $2^x = y$ Treat this as **Way 1** They get $32y^2 - 7y = 0$ for M1 and need to reach $y = \frac{7}{32}$ for A1 Then back to **Way 1** as before. Any letter may be used for the new variable which I have called y. If they use x and obtain $x = \frac{7}{32}$, this may be awarded M1A0M0A0 Those who get $y^2 - 7y + 32 = 0$ or $y^7 - 7y = 0$ will be awarded M0,A0,M0,A0
	Common Presentation of Work in ii	**Many begin with** $\log\left(2^{2x+5}\right) - \log\left(7(2^x)\right) = 0$. It is possible to reach this in two stages correctly so do not penalise this and award the full marks if they continue correctly as in **Way 2**. If however the solution continues with $(2x+5)\log 2 - x\log 14 = 0$ or with $(2x+5)\log 2 - 7x\log 2 = 0$ (both incorrect) then they are awarded M1A0M0A0 just getting credit for the $(2x + 5)$ log2 term.
	Note	N.B. The answer (+)2.19 results from “algebraic errors solving linear equations” leading to $2^x = \frac{32}{7}$ and gets M1A0M1A0

Question Number	Scheme	Marks
51. (i)	$8^{2x+1}=24$ $(2x+1)\log 8 \;=\log 24$ or $(2x+1)=\log_8 24$ | or $8^{2x}=3$ and so $(2x)\log 8 \;=\log 3$ or $(2x)=\log_8 3$ $x=\dfrac{1}{2}\left(\dfrac{\log 24}{\log 8}-1\right)$ or $x=\dfrac{1}{2}\left(\log_8 24-1\right)$ | $x=\dfrac{1}{2}\left(\dfrac{\log 3}{\log 8}\right)$ or $x=\dfrac{1}{2}\left(\log_8 3\right)$ o.e. =0.264	M1 dM1 A1 **(3)**
(ii)	$\log_2(11y-3)-\log_2 3-2\log_2 y=1$ $\log_2(11y-3)-\log_2 3-\log_2 y^2=1$. $\log_2\dfrac{(11y-3)}{3y^2}=1$ or $\log_2\dfrac{(11y-3)}{y^2}=1+\log_2 3=2.58496501$ $\log_2\dfrac{(11y-3)}{3y^2}=\log_2 2$ or $\log_2\dfrac{(11y-3)}{y^2}=\log_2 6$ (allow awrt 6 if replaced by 6 later) Obtains $6y^2-11y+3=0$ o.e. i.e. $6y^2=11y-3$ for example Solves quadratic to give $y=$ $y=\frac{1}{3}$ and $\frac{3}{2}$ (need both- one should not be rejected)	 M1 dM1 B1 A1 ddM1 A1 **(6)** **[9]**
Notes (i) (ii)	**M1**: Takes logs and uses law of powers correctly. (Any log base may be used) Allow lack of brackets. **dM1**: Make x subject of their formula correctly (may evaluate the log before subtracting 1 and calculate e.g. (1.528 -1)/2) **A1**: Allow answers which round to 0.264 **M1**: Applies power law of logarithms replacing $2\log_2 y$ by $\log_2 y^2$ **dM1**: Applies quotient or product law of logarithms correctly to the three log terms including term in y^2. (dependent on first M mark) or applies quotient rule to two terms and collects constants (allow "triple" fractions) $1+\log_2 3$ on RHS is not sufficient – need $\log_2 6$ or 2.58… e.g. $\log_2(11y-3)=\log_2 3+\log_2 y^2+\log_2 2$ becoming $\log_2(11y-3)=\log_2 6y^2$ **B1**: States or uses $\log_2 2=1$ or $2^1=2$ at any point in the answer so may be given for $\log_2(11y-3)-\log_2 3-2\log_2 y=\log_2 2$ or for $\dfrac{(11y-3)}{3y^2}=2$, for example (Sometimes this mark will be awarded before the second M mark, and it is possible to score M1M0B1in some cases) Or may be given for $\log_2 6=2.584962501..$ or $2^{2.584962501..}=6$ **A1**: This or equivalent quadratic equation (does not need to be in this form but should be equation) **ddM1:** (dependent on the two previous M marks) Solves their quadratic equation following reasonable log work using factorising, completion of square, formula or implied by both answers correct. **A1:** Any equivalent correct form – need both answers- allow awrt 0.333 for the answer 1/3 *NB: If "=0" is missing from the equation but candidate continues correctly and obtains correct answers then allow the penultimate A1 to be implied (Allow use of x or other varable instead of y throughout)	

Question Number	Scheme		Marks
	$5^y = 8$		
52. (i)	$y\log 5 = \log 8$	$y\log 5 = \log 8$ or $y = \log_5 8$	M1
	$\left\{y = \dfrac{\log 8}{\log 5}\right\} = 1.2920...$	awrt 1.29	A1
	Allow correct answer only		
			[2]
	$\log_2(x+15) - 4 = \frac{1}{2}\log_2 x$		
(ii)	$\log_2(x+15) - 4 = \log_2 x^{\frac{1}{2}}$	Applies the power law of logarithms seen **at any point in their working**	M1
	$\log_2\left(\dfrac{x+15}{x^{\frac{1}{2}}}\right) = 4$	Applies the subtraction or addition law of logarithms **at any point in their working**	M1
	$\left(\dfrac{x+15}{x^{\frac{1}{2}}}\right) = 2^4$	Obtains a correct expression with logs removed and no errors	M1
	$x - 16x^{\frac{1}{2}} + 15 = 0$ or e.g. $x^2 + 225 = 226x$	Correct **three** term quadratic in any form	A1
	$(\sqrt{x}-1)(\sqrt{x}-15) = 0 \Rightarrow \sqrt{x} = ...$	A **valid** attempt to factorise or solve their **three term quadratic** to obtain $\sqrt{x} = ...$ or $x = ...$ Dependent on all previous method marks.	**ddd**M1
	$\{\sqrt{x} = 1, 15\}$		
	$x = 1, 225$	**Both $x = 1$ and $x = 225$ (If both are seen, ignore any other values of $x \le 0$ from an otherwise correct solution)**	A1
			[6]
			Total 8
	<u>Alternative:</u>		
	$2\log_2(x+15) - 8 = \log_2 x$		
	$\log_2(x+15)^2 - 8 = \log_2 x$	Applies the power law of logarithms	M1
	$\log_2\left(\dfrac{(x+15)^2}{x}\right) = 8$	Applies the subtraction law of logarithms	M1
	$\dfrac{(x+15)^2}{x} = 2^8$	Obtains a correct expression with logs removed	M1
	$x^2 + 30x + 225 = 256x$		
	$x^2 - 226x + 225 = 0$	Correct three term quadratic in any form	A1
	$(x-1)(x-225) = 0 \Rightarrow x = ...$	A valid attempt to factorise or solve their 3TQ to obtain $x = ...$ Dependent on all previous method marks.	dddM1
	$x = 1, 225$	**Both $x = 1$ and $x = 225$ (If both are seen, ignore any other values of $x \le 0$ from an otherwise correct solution)**	A1

Question Number	Scheme	Marks
53 (i) Method 1	$\log_2\left(\frac{2x}{5x+4}\right) = -3$ or $\log_2\left(\frac{5x+4}{2x}\right) = 3$, or $\log_2\left(\frac{5x+4}{x}\right) = 4$ (see special case 2) $\left(\frac{2x}{5x+4}\right) = 2^{-3}$ or $\left(\frac{5x+4}{2x}\right) = 2^3$ or $\left(\frac{5x+4}{x}\right) = 2^4$ or $\left(\log_2\left(\frac{2x}{5x+4}\right)\right) = \log_2\left(\frac{1}{8}\right)$ $16x = 5x+4 \Rightarrow x =$ (depends on previous Ms and must be this equation or equivalent) $x = \frac{4}{11}$ or exact recurring decimal $0.\dot{3}\dot{6}$ after correct work	M1 M1 dM1 A1 **cso** **(4)**
53(i) Method 2	$\log_2(2x) + 3 = \log_2(5x+4)$ So $\log_2(2x) + \log_2(8) = \log_2(5x+4)$ (3 replaced by $\log_2 8$) Then $\log_2(16x) = \log_2(5x+4)$ (addition law of logs) Then final M1 A1 as before	 2nd M1 1st M1 dM1A1
(ii)	$\log_a y + \log_a 2^3 = 5$ $\log_a 8y = 5$ Applies product law of logarithms. $y = \frac{1}{8}a^5$ $y = \frac{1}{8}a^5$	M1 dM1 A1cao **(3)** **[7]**
	Notes for Question 53	
(i)	1st M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms in** ***x*** **into one** log term in x 2nd M1: For RHS of either 2^{-3}, 2^3, 2^4 or $\log_2\left(\frac{1}{8}\right)$, $\log_2 8$ or $\log_2 16$ i.e. using connection between log base 2 and 2 to a power. This may follow an earlier error. **Use of** 3^2 **is M0** 3rd dM1: Obtains **correct** linear equation in x. usually the one in the scheme and attempts $x =$ A1: cso Answer of 4/11 with **no** suspect log work preceding this.	
(ii)	M1: Applies power law of logarithms to replace $3\log_a 2$ by $\log_a 2^3$ *or* $\log_a 8$ dM1: (should not be following M0) Uses addition law of logs to give $\log_a 2^3 y = 5$ or $\log_a 8y = 5$	
(i)	**Special case 1:** $\log_2(2x) = \log_2(5x+4) - 3 \Rightarrow \frac{\log_2(2x)}{\log_2(5x+4)} = -3 \Rightarrow \frac{2x}{5x+4} = 2^{-3} \Rightarrow x = \frac{4}{11}$ or $\log_2(2x) = \log_2(5x+4) - 3 \Rightarrow \frac{\log_2(2x)}{\log_2(5x+4)} = -3 \Rightarrow \log_2\frac{2x}{5x+4} = -3 \Rightarrow \frac{2x}{5x+4} = 2^{-3} \Rightarrow x = \frac{4}{11}$ each attempt scores M0M1M1A0 – special case	
	Special case 2: $\log_2(2x) = \log_2(5x+4) - 3 \Rightarrow \log_2 2 + \log_2 x = \log_2(5x+4) - 3$, is M0 until the two log terms are combined to give $\log_2\left(\frac{5x+4}{x}\right) = 3 + \log_2 2$. This earns M1 Then $\left(\frac{5x+4}{x}\right) = 2^4$ or $\log_2\left(\frac{5x+4}{x}\right) = \log_2 2^4$ gets second M1. Then scheme as before.	

Question Number	Scheme	Marks
54(a)	Way 1: $\log_3(9x) = \log_3 9 + \log_3 x$ or Way 2: $\log_3(9x) = \log_3 3^{a+2}$	M1
	$= 2 + a$ $= 2 + a$	A1 (2)
(b)	Way 1: $\log_3\left(\frac{x^5}{81}\right) = \log_3 x^5 - \log_3 81$ or Way 2 $= \log_3 \frac{3^{5a}}{3^4}$	M1
	$\log x^5 = 5\log x$ or log81=4log3 or log81=4 $= \log_3 3^{5a-4}$	M1
	$= 5a - 4$	A1 cso (3)
(c)	$\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$	
	Method 1 Method 2	
	$\Rightarrow 2 + a + 5a - 4 = 3$ $\log_3\left(9x.\frac{x^5}{81}\right)$ = (3 or log27)	M1
	$\Rightarrow a = \frac{5}{6}$ $\log_3\left(\frac{x^6}{9}\right) = 3$ or log 27	A1
	$\Rightarrow x = 3^{\frac{5}{6}}$ or $\log_{10} x = a\log_{10} 3$ so $x =$ $\Rightarrow \frac{x^6}{9} = 3^3 \Rightarrow x^6 = 3^5 \Rightarrow x =$	M1
	$x = 2.498$ **or awrt** $x = 2.498$ **or awrt**	A1
	If x = -2.498 appears as well or instead this is A0	(4) **Total 9**

	Notes for Question 54
(a)	Way 1: M1: Use of log(ab) = log(a) + log(b) A1: must be a + 2 or 2 + a **Way 2: Uses** $x = 3^a$ to give $\log_3(9x) = \log_3 3^{a+2}$, A1 for a + 2 or 2 + a
(b)	Way 1: M1: Use of log(a/b) = log(a) – log(b) M1: Use of nlog(a) = $\log(a)^n$ Way 2: M1 Use of correct powers of 3 in numerator and denominator M1: Subtracts powers A1: No errors seen
(c)	**Method 1**: M1: Uses (a) and (b) results to form an equation in a (may not be linear) A1: a = awrt 0.833 M1: Finds x by use of 3 to a power, or change of base performed correctly A1: x = 2.498 (accept answer which round to this value from 2.498049533…) **Method 2:** M1: Use of log(ab) = log(a) + log(b) in an equation (RHS may be wrong) A1: Equation correct and simplified M1: Tries to undo log by 3 to power correctly, and uses root to obtain x A1: x = 2.498 (accept answer which round to this value from 2.498049533…) Lose this mark if negative answer is given as well as or instead of positive answer.

55.			
(a)	$2\log(x+15)=\log(x+15)^2$		B1
	$\log(x+15)^2-\log x=\log\dfrac{(x+15)^2}{x}$	Correct use of $\log a-\log b=\log\dfrac{a}{b}$	M1
	$2^6=64$ or $\log_2 64=6$	64 used in the correct context	B1
	$\log_2\dfrac{(x+15)^2}{x}=6\Rightarrow\dfrac{(x+15)^2}{x}=64$	Removes logs correctly	M1
	$\Rightarrow x^2+30x+225=64x$ $or\ x+30+225x^{-1}=64$	Must see expansion of $(x+15)^2$ to score the final mark.	
	$\therefore x^2-34x+225=0$ *		A1
			(5)
(b)	$(x-25)(x-9)=0\Rightarrow x=25$ or $x=9$	M1: Correct attempt to solve the **given** quadratic as far as x =... A1: Both 25 and 9	M1 A1
			(2)
			[7]

Question number	Scheme	Marks
56	$2\log x = \log x^2$	B1
	$\log_3 x^2 - \log_3(x-2) = \log_3 \dfrac{x^2}{x-2}$	M1
	$\dfrac{x^2}{x-2} = 9$	A1 o.e.
	Solves $x^2 - 9x + 18 = 0$ to give $x = ...$	M1
	$x = 3$, $x = 6$	A1
		Total 5
Notes	**B1** for **this** correct use of power rule (may be implied) **M1:** for **correct** use of subtraction rule (or addition rule) for logs N.B. $2\log_3 x - \log_3(x-2) = 2\log_3 \dfrac{x}{x-2}$ is **M0** **A1**. for correct equation without logs (Allow any correct equivalent including 3^2 instead of 9.) **M1** for attempting to solve $x^2 - 9x + 18 = 0$ to give $x =$ (see notes on marking quadratics) **A1** for these t**wo** correct answers	
Alternative Method	$\log_3 x^2 = 2 + \log_3(x-2)$ is B1, *so* $x^2 = 3^{2+\log_3(x-2)}$ needs to be followed by $(x^2) = 9(x-2)$ for M1 A1 Here M1 is for **complete** method i.e.correct use of powers after logs are used correctly	
Common Slips	$2\log x - \log x + \log 2 = 2$ may obtain B1 if $\log x^2$ appears but **the statement is M0** and so leads to no further marks $2\log_3 x - \log_3(x-2) = 2$ *so* $\log_3 x - \log_3(x-2) = 1$ and $\log_3 \dfrac{x}{x-2} = 1$ can earn M1 for *correct* subtraction rule following error, but no other marks	
Special Case	$\dfrac{\log x^2}{\log(x-2)} = 2$ leading to $\dfrac{x^2}{x-2} = 9$ and then to x =3, x =6, usually earns B1M0A0, but may then earn M1A1 (special case) so 3/5 [This *recovery* after uncorrected error is very common] Trial and error, Use of a table or just stating answer **with both** x=3 and x=6 should be awarded B0M0A0 then final M1A1 i.e. 2/5	

Question number	Scheme	Marks
57. (a)	$\log_3 3x^2 = \log_3 3 + \log_3 x^2$ or $\log y - \log x^2 = \log 3$ or $\log y - \log 3 = \log x^2$ $\log_3 x^2 = 2\log_3 x$ **Using** $\log_3 3 = 1$	B1 B1 B1 (3)
(b)	$3x^2 = 28x - 9$ Solves $3x^2 - 28x + 9 = 0$ to give $x = \frac{1}{3}$ or $x = 9$	M1 M1 A1 (3) **6**
Notes (a)	**B1** for correct use of addition rule (or correct use of subtraction rule) **B1:** replacing $\log x^2$ by $2\log x$ – **not** $\log 3x^2$ by $2\log 3\, x$ this is **B0** **These first two B marks are often earned in the first line of working** **B1**. for replacing $\log 3$ by 1 (or use of $3^1 = 3$) If candidate has been awarded 3 marks and their proof includes an error or omission of reference to log*y* withhold the last mark. So just B1 B1 B0 These marks must be awarded for **work in part (a)** only	
(b)	**M1** for removing logs to get an equation in *x*– statement in scheme is sufficient. This needs to be accurate without any errors seen in part (b). **M1** for attempting to solve three term quadratic to give *x* = (see notes on marking quadratics) **A1** for the two correct answers – this depends on second M mark only. Candidates often begin again in part (b) and do not use part (a). If such candidates make errors in log work in part (b) they score first **M0**. The second **M** and the **A** are earned as before. It is possible to get M0M1A1 or M0M1A0.	
Alternative to (b) using *y*	Eliminates *x* to give $3y^2 - 730y + 243 = 0$ with no errors is M1 Solves quadratic to find *y*, then uses values to find *x* M1 A1 as before **See extra sheet with examples illustrating the scheme.**	

Question Number	Scheme		Marks
58.	(a) $5^x = 10$ and (b) $\log_3(x-2) = -1$		
(a)	$x = \dfrac{\log 10}{\log 5}$ or $x = \log_5 10$		M1
	$x\left\{= 1.430676558...\right\} = 1.43$ (3 sf)	1.43	A1 **cao**
			[2]
(b)	$(x-2) = 3^{-1}$	$(x-2) = 3^{-1}$ or $\frac{1}{3}$	M1 oe
	$x\left\{= \frac{1}{3} + 2\right\} = 2\frac{1}{3}$	$2\frac{1}{3}$ or $\frac{7}{3}$ or $2.\dot{3}$ or awrt 2.33	A1
			[2] **4**
(a)	M1: for $x = \dfrac{\log 10}{\log 5}$ or $x = \log_5 10$. Also allow M1 for $x = \dfrac{1}{\log 5}$ 1.43 with no working (or any working) scores M1A1 (even if left as $5^{1.43}$). Other answers which round to 1.4 with no working score M1A0. **<u>Trial & Improvement Method:</u>** M1: For a method of trial and improvement by trialing f(value between 1.4 and 1.43) = Value below 10 and f(value between 1.431 and 1.5) = Value over 10. A1 for 1.43 cao. **<u>Note</u>**: $x = \log_{10} 5$ by itself is M0; but $x = \log_{10} 5$ followed by $x = 1.430676558...$ is M1.		
(b)	M1: Is for correctly eliminating log out of the equation. **Eg 1:** $\log_3(x-2) = \log_3(\frac{1}{3}) \Rightarrow x - 2 = \frac{1}{3}$ only gets M1 when the logs are correctly removed. **Eg 2:** $\log_3(x-2) = -\log_3(3) \Rightarrow \log_3(x-2) + \log_3(3) = 0 \Rightarrow \log_3(3(x-2)) = 0$ $\Rightarrow 3(x-2) = 3^0$ only gets M1 when the logs are correctly removed, but $3(x-2) = 0$ would score M0. **Note**: $\log_3(x-2) = -1 \Rightarrow \log_3\left(\dfrac{x}{2}\right) = -1 \Rightarrow \dfrac{x}{2} = 3^{-1}$ would score M0 for incorrect use of logs. **<u>Alternative: changing base</u>** $\dfrac{\log_{10}(x-2)}{\log_{10} 3} = -1 \Rightarrow \log_{10}(x-2) = -\log_{10} 3 \Rightarrow \log_{10}(x-2) + \log_{10} 3 = 0$ $\Rightarrow \log_{10} 3(x-2) = 0 \Rightarrow 3(x-2) = 10^0$. At this point M1 is scored. A correct answer in (b) without any working scores M1A1.		

Question Number	Scheme	Marks
59	(a) $2\log_3(x-5)=\log_3(x-5)^2$	B1
	$\log_3(x-5)^2-\log_3(2x-13)=\log_3\dfrac{(x-5)^2}{2x-13}$	M1
	$\log_3 3=1$ seen or used correctly	B1
	$\log_3\left(\dfrac{P}{Q}\right)=1 \;\Rightarrow\; P=3Q \qquad \left\{\dfrac{(x-5)^2}{2x-13}=3 \;\Rightarrow\; (x-5)^2=3(2x-13)\right\}$	M1
	$x^2-16x+64=0$ (*)	A1 cso
		(5)
	(b) $(x-8)(x-8)=0 \;\Rightarrow\; x=8$ <u>Must</u> be seen in part (b). Or: Substitute $x=8$ into original equation and verify. Having additional solution(s) such as $x=-8$ loses the A mark. $x=8$ with no working scores both marks.	M1 A1 (2) **7**

(a) Marks may be awarded if equivalent work is seen in part (b).

1st M: $\log_3(x-5)^2-\log_3(2x-13)=\dfrac{\log_3(x-5)^2}{\log_3(2x-13)}$ is M0

$2\log_3(x-5)-\log_3(2x-13)=2\log\dfrac{x-5}{2x-13}$ is M0

2$^{\text{nd}}$ M: <u>After the first mistake above</u>, this mark is available only if there is 'recovery' to the required $\log_3\left(\dfrac{P}{Q}\right)=1 \;\Rightarrow\; P=3Q$. Even then the final mark (cso) is lost.

<u>'Cancelling logs'</u>, e.g. $\dfrac{\log_3(x-5)^2}{\log_3(2x-13)}=\dfrac{(x-5)^2}{2x-13}$ will also lose the 2$^{\text{nd}}$ M.

<u>A typical wrong solution</u>:

$$\log_3\frac{(x-5)^2}{2x-13}=1 \;\Rightarrow\; \log_3\frac{(x-5)^2}{2x-13}=3 \;\Rightarrow\; \frac{(x-5)^2}{2x-13}=3 \;\Rightarrow\; (x-5)^2=3(2x-13)$$

(Wrong step here)

This, with no evidence elsewhere of $\log_3 3=1$, scores B1 M1 B0 M0 A0

However, $\log_3\dfrac{(x-5)^2}{2x-13}=1 \;\Rightarrow\; \dfrac{(x-5)^2}{2x-13}=3$ is correct and could lead to full marks.

(Here $\log_3 3=1$ is implied).

<u>No log methods shown</u>:

It is <u>not</u> acceptable to jump immediately to $\dfrac{(x-5)^2}{2x-13}=3$. The only mark this scores is the 1$^{\text{st}}$ B1 (by generous implication).

(b) M1: Attempt to solve the <u>given</u> quadratic equation (usual rules), so the factors $(x-8)(x-8)$ with no solution is M0.

Question Number	Scheme	Marks
60	(a) $25\,000 \times 1.03 = 25750$ $\left\{25000+750=25750,\ \text{or}\ \ 25000\dfrac{(1-0.03^2)}{1-0.03}=25750\right\}$ (*)	B1 (1)
	(b) r = 1.03 Allow $\dfrac{103}{100}$ or $1\dfrac{3}{100}$ but no other alternatives	B1 (1)
	(c) $25000r^{N-1} > 40000$ (Either letter r or their r value) Allow ' = ' or '<' $r^M > 1.6 \Rightarrow \log r^M > \log 1.6$ Allow ' = ' or '<' (<u>See below</u>) OR (by change of base), $\log_{1.03} 1.6 < M \Rightarrow \dfrac{\log 1.6}{\log 1.03} < M$ $(N-1)\log 1.03 > \log 1.6$ (Correct bracketing required) (*) Accept work for part (c) seen in part (d)	M1 M1 A1 cso (3)
	(d) Attempt to evaluate $\dfrac{\log 1.6}{\log 1.03}+1$ {or $25000(1.03)^{15}$ <u>and</u> $25000(1.03)^{16}$ } N = 17 (<u>not</u> 16.9 and not e.g. $N \geq 17$) Allow '17th year' Accept work for part (d) seen in part (c)	M1 A1 (2)
	(e) Using formula $\dfrac{a(1-r^n)}{1-r}$ with values of a and r, and n = 9, 10 or 11 $\dfrac{25000(1-1.03^{10})}{1-1.03}$ 287 000 (<u>must</u> be rounded to the nearest 1 000) Allow 287000.00	M1 A1 A1 (3) **10**

(c) 2nd M: Requires $\dfrac{40000}{25000}$ to be dealt with, and 'two' logs introduced.

With, say, N instead of $N-1$, this mark is still available.

<u>Jumping</u> straight from $1.03^{N-1} > 1.6$ to $(N-1)\log 1.03 > \log 1.6$ can score only M1 M0 A0.
(The intermediate step $\log 1.03^{N-1} > \log 1.6$ must be seen).

<u>Longer methods</u> require correct log work throughout for 2nd M, e.g.:

$\log(25000r^{N-1}) > \log 40000 \Rightarrow \log 25000 + \log r^{N-1} > \log 40000 \Rightarrow$

$\log r^{N-1} > \log 40000 - \log 25000 \Rightarrow \log r^{N-1} > \log 1.6$

(d) Correct answer with no working scores both marks.

Evaluating $\log\left(\dfrac{1.6}{1.03}\right)+1$ does <u>not</u> score the M mark.

(e) M1 can also be scored by a "year by year" method, <u>with terms added</u>.
(Allow the M mark if there is evidence of adding 9, 10 or 11 terms).
1st A1 is scored if the 10 correct terms have been added (allow terms to be to the nearest 100).
To the nearest 100, these terms are:
25000, 25800, 26500, 27300, 28100, 29000, 29900, 30700, 31700, 32600

<u>No</u> working shown: Special case: 287 000 scores 1 mark, scored on ePEN as 1, 0, 0.
(Other answers with no working score no marks).

Question Number	Scheme	Marks
61 (a)	$\log_x 64 = 2 \quad \Rightarrow \quad 64 = x^2$	M1
	So $x = 8$	A1 (2)
(b)	$\log_2(11-6x) = \log_2(x-1)^2 + 3$	M1
	$\log_2\left[\dfrac{11-6x}{(x-1)^2}\right] = 3$	M1
	$\dfrac{11-6x}{(x-1)^2} = 2^3$	M1
	$\{11-6x = 8(x^2-2x+1)\}$ and so $0 = 8x^2 - 10x - 3$	A1
	$0 = (4x+1)(2x-3) \quad \Rightarrow x = \ldots$	dM1
	$x = \dfrac{3}{2}, \left[-\dfrac{1}{4}\right]$	A1 (6)
		[8]
(a)	M1 for getting out of logs A1 Do not need to see x = -8 appear and get rejected. Ignore x = -8 as extra solution. x= 8 with no working is M1 A1	
(b)	1st M1 for using the $n$log$x$ rule 2nd M1 for using the log$x$ - log$y$ rule or the log$x$ + log$y$ rule as appropriate 3rd M1 for using 2 to the power– need to see 2^3 or 8 (May see $3 = \log_2 8$ used) **If all three M marks have been earned and logs are still present** in equation **do not give** final M1. So solution stopping at $\log_2\left[\dfrac{11-6x}{(x-1)^2}\right] = \log_2 8$ would earn M1M1M0 1st A1 for a correct 3TQ 4th dependent M1 for attempt to solve or factorize their 3TQ to obtain x =… (mark depends on three previous M marks) 2nd A1 for 1.5 (ignore -0.25) s.c 1.5 only – no working – is 0 marks	
(a)	<u>Alternatives</u> Change base : (i) $\dfrac{\log_2 64}{\log_2 x} = 2$, so $\log_2 x = 3$ and $x = 2^3$, is M1 or (ii) $\dfrac{\log_{10} 64}{\log_{10} x} = 2$, $\log x = \frac{1}{2}\log 64$ so $x = 64^{\frac{1}{2}}$ is M1 then x = 8 is A1 **BUT** $\log x = 0.903$ so x = 8 is M1A0 (loses accuracy mark) (iii) $\log_{64} x = \frac{1}{2}$ so $x = 64^{\frac{1}{2}}$ is M1 then x = 8 is A1	

Question Number	Scheme	Marks
62 (a)	$\log_2 y = -3 \quad \Rightarrow \quad y = 2^{-3}$	M1
	$y = \frac{1}{8}$ or 0.125	A1 (2)
(b)	$32 = 2^5$ or $16 = 2^4$ or $512 = 2^9$	M1
	[or $\log_2 32 = 5\log_2 2$ or $\log_2 16 = 4\log_2 2$ or $\log_2 512 = 9\log_2 2$]	
	[or $\log_2 32 = \dfrac{\log_{10} 32}{\log_{10} 2}$ or $\log_2 16 = \dfrac{\log_{10} 16}{\log_{10} 2}$ or $\log_2 512 = \dfrac{\log_{10} 512}{\log_{10} 2}$]	
	$\log_2 32 + \log_2 16 = 9$	A1
	$(\log x)^2 = \ldots$ or $(\log x)(\log x) = \ldots$ (May not be seen explicitly, so M1 may be implied by later work, and the base may be 10 rather than 2)	M1
	$\log_2 x = 3 \quad \Rightarrow \quad x = 2^3 = 8$	A1
	$\log_2 x = -3 \quad \Rightarrow \quad x = 2^{-3} = \dfrac{1}{8}$	A1ft (5) **[7]**

(a) M1 for <u>getting out of logs</u> correctly.

If done by change of base, $\log_{10} y = -0.903\ldots$ is insufficient for the M1, but $y = 10^{-0.903}$ scores M1.

A1 for the <u>exact</u> answer, e.g. $\log_{10} y = -0.903 \Rightarrow y = 0.12502..$ scores M1 (implied) A0.

<u>Correct answer</u> with no working scores both marks.

<u>Allow</u> both marks for implicit statements such as $\log_2 0.125 = -3$.

(b) 1st M1 for expressing 32 or 16 or 512 as a power of 2, or for a change of base enabling evaluation of $\log_2 32$, $\log_2 16$ or $\log_2 512$ by calculator. (Can be implied by 5, 4 or 9 respectively).

1st A1 for 9 (exact).

2nd M1 for getting $(\log_2 x)^2$ = constant. The constant can be a log or a sum of logs.

If written as $\log_2 x^2$ instead of $(\log_2 x)^2$, allow the M mark <u>only</u> if subsequent work implies correct interpretation.

2nd A1 for 8 (exact). Change of base methods leading to a non-exact answer score A0.

3rd A1ft for an answer of $\dfrac{1}{\text{their } 8}$. An ft answer may be non-exact.

<u>Possible mistakes</u>:

$\log_2(2^9) = \log_2(x^2) \Rightarrow x^2 = 2^9 \Rightarrow x = \ldots$ scores M1A1(implied by 9)M0A0A0

$\log_2 512 = \log_2 x \times \log_2 x \Rightarrow x^2 = 512 \Rightarrow x = \ldots$ scores M0A0(9 never seen)M1A0A0

$\log_2 48 = (\log_2 x)^2 \Rightarrow (\log_2 x)^2 = 5.585 \Rightarrow x = 5.145,\ x = 0.194$ scores M0A0M1A0A1ft

<u>No working</u> (or 'trial and improvement'):

$x = 8$ scores M0 A0 M1 A1 A0

Question Number	Scheme	Marks
63	$2\log_5 x = \log_5\left(x^2\right)$, $\quad \log_5(4-x) - \log_5(x^2) = \log_5 \dfrac{4-x}{x^2}$ $\log\left(\dfrac{4-x}{x^2}\right) = \log 5 \quad 5x^2 + x - 4 = 0$ **or** $5x^2 + x = 4$ **o.e.** $(5x-4)(x+1) = 0 \quad x = \dfrac{4}{5} \quad (x=-1)$	B1, M1 M1 A1 dM1 A1 (6) **[6]**
Notes	**B1** is awarded for $2\log x = \log x^2$ anywhere. **M1** for correct use of log A – log B = log $\frac{A}{B}$ **M1** for replacing 1 by $\log_k k$. **A1** for correct quadratic ($\log(4-x) - \log x^2 = \log 5 \Rightarrow 4 - x - x^2 = 5$ is **B1M0M1A0 M0A0**) **dM1** for attempt to solve quadratic with usual conventions. (Only award if previous two **M** marks have been awarded) **A1** for 4/5 or 0.8 or equivalent (Ignore extra answer).	
Alternative 1	$\log_5(4-x) - 1 = 2\log_5 x$ so $\log_5(4-x) - \log_5 5 = 2\log_5 x$ $\log_5 \dfrac{4-x}{5} = 2\log_5 x$ then could complete solution with $2\log_5 x = \log_5\left(x^2\right)$ $\left(\dfrac{4-x}{5}\right) = x^2 \quad 5x^2 + x - 4 = 0$ Then as in first method $(5x-4)(x+1) = 0 \quad x = \dfrac{4}{5} \quad (x=-1)$	M1 M1 B1 A1 dM1 A1 (6) **[6]**
Special cases	Complete trial and error yielding 0.8 is **M3** and **B1** for 0.8 **A1, A1** awarded for each of two tries evaluated. i.e. 6/6 Incomplete trial and error with wrong or no solution is 0/6 Just answer 0.8 with no working is **B1**	
	If log base 10 or base e used throughout - can score **B1M1M1A0M1A0**	

Question number	Scheme	Marks
64.	(a) $x = \frac{\log 7}{\log 5}$ or $x = \log_5 7$ (i.e. correct method up to x =…)	M1
	1.21 Must be this answer (3 s.f.)	A1 (2)
	(b) $(5^x - 7)(5^x - 5)$ Or another variable, e.g. $(y-7)(y-5)$, even $(x-7)(x-5)$	M1 A1
	$(5^x = 7$ or $5^x = 5)$ x = 1.2 (awrt) ft from the answer to (a), if used	A1ft
	x = 1 (allow 1.0 or 1.00 or 1.000)	B1 (4)
		6
	(a) 1.21 with no working: M1 A1 (even if it left as $5^{1.21}$). Other answers which round to 1.2 with no working: M1 A0. (b) M: Using the correct quadratic equation, attempt to factorise $(5^x \pm 7)(5^x \pm 5)$, or attempt quadratic formula. Allow $\log_5 7$ or $\frac{\log 7}{\log 5}$ instead of 1.2 for A1ft. No marks for simply substituting a decimal answer from (a) into the given equation (perhaps showing that it gives approximately zero). However, note the following special case: Showing that $5^x = 7$ satisfies the given equation, therefore 1.21 is a solution scores 0, 0, 1, 0 (and could score full marks if the x = 1 were also found). e.g. If $5^x = 7$, then $5^{2x} = 49$, and $5^{2x} - 12(5^x) + 35 = 49 - 84 + 35 = 0$, so one solution is x = 1.21 (‘conclusion’ must be seen). To score this special case mark, values substituted into the equation must be exact. Also, the mark would not be scored in the following case: e.g. If $5^x = 7$, $5^{2x} - 84 + 35 = 0 \Rightarrow 5^{2x} = 49 \Rightarrow x = 1.21$ (Showing no appreciation that $5^{2x} = (5^x)^2$) B1: Do not award this mark if x = 1 clearly follows from wrong working.	

65.	<u>Method 1</u> (Substituting a = 3b into second equation at some stage)	
	Using a law of logs correctly (anywhere) e.g. $\log_3 ab = 2$	M1
	Substitution of 3*b* for *a* (or a/3 for b) e.g. $\log_3 3b^2 = 2$	M1
	Using base correctly on correctly derived $\log_3 p = q$ e.g. $3b^2 = 3^2$	M1
	First correct value $b = \sqrt{3}$ (allow $3^{½}$)	A1
	Correct method to find other value (dep. on at least first M mark)	M1
	Second answer $a = 3b = 3\sqrt{3}$ or $\sqrt{27}$	A1
	<u>Method 2</u> (Working with two equations in $\log_3 a$ and $\log_3 b$)	
	" Taking logs" of first equation and " separating" $\log_3 a = \log_3 3 + \log_3 b$ $(= 1 + \log_3 b)$	M1
	Solving simultaneous equations to find $\log_3 a$ or $\log_3 b$ [$\log_3 a = 1½$, $\log_3 b = ½$]	M1
	Using base correctly to find a or b	M1
	Correct value for *a* or *b* $a = 3\sqrt{3}$ or $b = \sqrt{3}$	A1
	Correct method for second answer, dep. on first M; correct second answer [Ignore negative values]	M1;A1**[6]**
Notes:	Answers must be exact; decimal answers lose both A marks There are several variations on Method 1, depending on the stage at which *a* = 3*b* is used, but they should all mark as in scheme. In this method, the first three method marks on Epen are for (i) First M1: correct use of log law, (ii) Second M1: substitution of *a* = 3*b*, (iii) Third M1: requires using base correctly on correctly derived $\log_3 p = q$	

Question Number	Scheme	Marks
66(a)	$A = 1500$	B1 **(1)**
(b)	Sub $t = 2, V = 13500 \Rightarrow 16000\mathrm{e}^{-2k} = 12000$	M1
	$\Rightarrow e^{-2k} = \dfrac{3}{4} \quad 0.75 \quad \text{oe}$	A1
	$\Rightarrow k = -\dfrac{1}{2}\ln\dfrac{3}{4}, = \ln\sqrt{\dfrac{4}{3}} = \ln\left(\dfrac{2}{\sqrt{3}}\right)$	dM1, A1*
		(4)
(c)	Sub $6000 = 16000\mathrm{e}^{-\ln\left(\frac{2}{\sqrt{3}}\right)T} + '1500' \Rightarrow \mathrm{e}^{-\ln\left(\frac{2}{\sqrt{3}}\right)T} = C$	M1
	$\Rightarrow \mathrm{e}^{-\ln\left(\frac{2}{\sqrt{3}}\right)T} = \dfrac{45}{160} = 0.28125$	A1
	$\Rightarrow T = -\dfrac{\ln\left(\frac{45}{160}\right)}{\ln\left(\frac{2}{\sqrt{3}}\right)} = 8.82$	M1 A1
		(4)
		(9 marks)
Alt (b)	Sub $t = 2, V = 13500 \Rightarrow 13500 = 16000\mathrm{e}^{-2k} + '1500' \Rightarrow 1600e^{-2k} = 1200$	M1
	$\Rightarrow \ln 1600 - 2k = \ln 1200$	A1
	$\Rightarrow k = -\dfrac{1}{2}\ln\dfrac{1200}{1600}, = \ln\sqrt{\dfrac{4}{3}} = \ln\left(\dfrac{2}{\sqrt{3}}\right)$	dM A1*
		(4)

You may mark parts (a) and (b) together

(a)

B1: Sight of $A = 1500$

(b)

M1: Substitutes $t = 2, V = 13500 \Rightarrow 13500 = 16000\mathrm{e}^{-2k} + '\textit{their}\ 1500'$ and proceeds to $P\mathrm{e}^{-2k} = \ldots$ or $Q\mathrm{e}^{2k} = \ldots$ Condone slips, for example, V may be 1350. It is for an **attempt** to make $\mathrm{e}^{\pm 2k}$ the subject.

A1: $e^{-2k} = \dfrac{3}{4} \quad 0.75 \quad \text{or } e^{2k} = \dfrac{4}{3} \quad \left(1.\dot{3}\right) \quad \text{oe}$

dM1: For taking ln's and proceeding to $k = \ldots$ For example $k = -\dfrac{1}{2}\ln\dfrac{3}{4}$ oe

May be implied by the correct decimal answer awrt 0.144 . This mark cannot be awarded from impossible to solve equations, that is ones of the type $\Rightarrow e^{\pm 2k} = c, \quad c \leqslant 0$

A1*: cso $k=\ln\left(\frac{2}{\sqrt{3}}\right)$ (brackets not required) **with a correct intermediate line** of either

$$\frac{1}{2}\ln\frac{4}{3},\ \frac{1}{2}\ln 4-\frac{1}{2}\ln 3,\ \ln\sqrt{\frac{4}{3}}\ \text{ or }\ \ln\left(\frac{3}{4}\right)^{-\frac{1}{2}}$$

Note: $\mathrm{e}^{-2k}=\frac{3}{4}\Rightarrow \mathrm{e}^{2k}=\frac{4}{3}\Rightarrow \mathrm{e}^{k}=\frac{2}{\sqrt{3}}$ are perfectly acceptable steps

See scheme for alternative method when ln's are taken before e^{-2k} is made the subject.

It is also possible to substitute $k=\ln\left(\frac{2}{\sqrt{3}}\right)$ into $13500=16000\mathrm{e}^{-k\times 2}+1500$ and show that 12000 =12000 or similar. This is fine as long as a minimal conclusion (eg ✓)is given for the A1*.

(c)

M1: Sub $V=6000\Rightarrow 6000=16000\mathrm{e}^{\pm kT}+\text{'their 1500'}$ and proceeds to $\mathrm{e}^{\pm kT}=c,\quad c>0$

Allow candidates to write $k=$ awrt 0.144 or leave as 'k'. Condone slips on k. Eg $k=2\ln\left(\frac{2}{\sqrt{3}}\right)$

Allow this when the = sign is replaced by any inequality.

If the candidate attempts to simplify the exponential function score for $\left(\frac{2}{\sqrt{3}}\right)^{\pm T}=c,\quad c>0$

A1: $\mathrm{e}^{-\ln\left(\frac{2}{\sqrt{3}}\right)T}=\frac{45}{160}=0.28125$, $\mathrm{e}^{-kT}=\frac{45}{160}$ or $\left(\frac{2}{\sqrt{3}}\right)^{-T}=\frac{45}{160}$ Condone inequalities for =

Allow solutions from rounded values (3sf). Eg. $\mathrm{e}^{-0.144T}=0.281$

M1: Correct order of operations using ln's and division leading to a value of T. It is implied by awrt 8.8

$\left(\frac{2}{\sqrt{3}}\right)^{-T}=\frac{45}{160}\Rightarrow -T=\log_{\frac{2}{\sqrt{3}}}\frac{45}{160}$ is equivalent work for this M mark.

A1: cso 8.82 only following correct work. Note that this is not awrt

Allow a solution using an inequality as long as it arrives at the solution 8.82.

There may be solutions using trial and improvement. Score (in this order) as follows

M1: Trial at value of $V=16000\mathrm{e}^{-0.144t}+1500$ (oe) at either $t=8$ or $t=9$ and shows evidence $V_{t=8}=awrt\ 6500$ $V_{t=9}=awrt\ 5900$ This may be implied by the subsequent M1

M1: Trial at value of $V=16000\mathrm{e}^{-0.144t}+1500$ (oe) at either $t=8.81$ or $t=8.82$ and shows evidence. (See below for answers. Allow to 2sf)

A1: Correct answers for V at **both** $t=8.81$ **and** $t=8.82$ $V_{t=8.81}=awrt\ 6006$ $V_{t=8.82}=awrt\ 5999$

A1: Correctly deduces 8.82 with all evidence.

Hence candidates who **just** write down 8.82 will score 1, 1, 0, 0

Question Number	Scheme	Marks
67(a)	$e^{3x-9}=8 \Rightarrow 3x-9=\ln 8$	M1
	$\Rightarrow x=\dfrac{\ln 8+9}{3}, =\ln 2+3$	A1, A1
		(3)
(b)	$\ln(2y+5)=2+\ln(4-y)$	
	$\ln\left(\dfrac{2y+5}{4-y}\right)=2$	M1
	$\left(\dfrac{2y+5}{4-y}\right)=e^2$	M1
	$2y+5=e^2(4-y)\Rightarrow 2y+e^2y=4e^2-5\Rightarrow y=\dfrac{4e^2-5}{2+e^2}$	dM1, A1
		(4)
		7 marks

(a)

M1 Takes ln's of both sides and uses the power law. You may even accept candidates taking logs of both sides

A1 A correct unsimplified answer $\dfrac{\ln 8+9}{3}$ or equivalent such as $\dfrac{\ln 8e^9}{3}$, $3+\ln\left(\sqrt[3]{8}\right)$, $\dfrac{\log 8}{3\log e}+3$ or even 3.69

A1 cso $\ln 2+3$. Accept $\ln 2e^3$

....

Alt I (a)

$e^{3x-9}=8\Rightarrow \dfrac{e^{3x}}{e^9}=8\Rightarrow e^{3x}=8e^9\Rightarrow 3x=\ln\left(8e^9\right)$ for M1 (Condone slips on index work and lack of bracket)

Alt II (a)

$e^{x-3}=\sqrt[3]{8}\Rightarrow x-3=\ln\left(\sqrt[3]{8}\right)$ for M1 (Condone slips on the 9. Eg $e^{x-9}=2\Rightarrow x-9=\ln 2$)

....

(b)

M1 Uses a correct method to combine two terms to create a single ln term.

Eg. Score for $2+\ln(4-y)=\ln\left(e^2(4-y)\right)$ or $\ln(2y+5)-\ln(4-y)=\ln\left(\dfrac{2y+5}{4-y}\right)$

Condone slips on the signs and coefficients of the terms, but not on the e^2

M1 Scored for an attempt to undo the ln's to get an equation in y This must be awarded after an attempt to combine the ln terms. Award for $\ln(g(y))=2\Rightarrow g(y)=e^2$ and can be scored eg where $g(y)=2y+5-(4-y)$

It cannot be awarded for just $2y+5=e^2+4-y$ where the candidate attempts to undo term by term

dM1 Dependent upon **both** previous M's. It is for making y the subject. Expect to see both terms in y collected and factorised (may be implied) before reaching y =. Condone slips, for eg, on signs. y =2.615 scores this.

A1 $y=\dfrac{4e^2-5}{2+e^2}$ or equivalent such as $y=4-\dfrac{13}{2+e^2}$ ISW after you see the correct answer.

....

Special Case: $\ln(2y+5)-\ln(4-y)=2\Rightarrow \dfrac{\ln(2y+5)}{\ln(4-y)}=2\Rightarrow \dfrac{2y+5}{4-y}=e^2\Rightarrow$ Correct answer score M0 M1 M1 A0

Question	Scheme	Marks
68(a)	Subs $D=15$ and $t=4$ $\quad x=15e^{-0.2\times 4}=6.740\ (mg)$	M1A1
		(2)
(b)	$15e^{-0.2\times 7}+15e^{-0.2\times 2}=13.754(mg)$	M1A1*
		(2)
(c)	$15e^{-0.2\times T}+15e^{-0.2\times (T+5)}=7.5$	M1
	$15e^{-0.2\times T}+15e^{-0.2\times T}e^{-1}=7.5$	
	$15e^{-0.2\times T}(1+e^{-1})=7.5\Rightarrow e^{-0.2\times T}=\dfrac{7.5}{15(1+e^{-1})}$	dM1
	$T=-5\ln\left(\dfrac{7.5}{15\left(1+e^{-1}\right)}\right)=5\ln\left(2+\dfrac{2}{e}\right)$	A1, A1
		(4)
		(8 marks)

(a)

M1 Attempts to substitute both $D=15$ and $t=4$ in $x=De^{-0.2t}$

It can be implied by sight of $15e^{-0.8}$, $15e^{-0.2\times 4}$ or awrt 6.7

Condone slips on the power. Eg you may see -0.02

A1 CAO 6.740 (mg) Note that 6.74 (mg) is A0

(b)

M1 Attempt to find the sum of two expressions with $D=15$ in both terms with t values of 2 and 7

Evidence would be $15e^{-0.2\times 7}+15e^{-0.2\times 2}$ or similar expressions such as $\left(15e^{-1}+15\right)e^{-0.2\times 2}$

Award for the sight of the two numbers awrt **3.70** and awrt **10.05**, followed by their total awrt **13.75**

Alternatively finds the amount after 5 hours, $15e^{-1}=$ awrt **5.52** adds the second dose = **15** to get a total of awrt **20.52** then multiplies this by $e^{-0.4}$ to get awrt **13.75**.

Sight of 5.52+15=20.52 $\rightarrow$ 13.75 is fine.

A1* cso so both the expression $15e^{-0.2\times 7}+15e^{-0.2\times 2}$ and $13.754(mg)$ are required

Alternatively both the expression $\left(15e^{-0.2\times 5}+15\right)\times e^{-0.2\times 2}$ and $13.754(mg)$ are required.

Sight of just the numbers is not enough for the A1*

(c)

M1 Attempts to write down a correct equation involving T or t. Accept with or without correct bracketing

Eg. accept $15e^{-0.2\times T}+15e^{-0.2\times (T\pm 5)}=7.5$ or similar equations $\left(15e^{-1}+15\right)e^{-0.2\times T}=7.5$

dM1 Attempts to solve their equation, dependent upon the previous mark, by proceeding to $e^{-0.2\times T}=\ldots$

An attempt should involve an attempt at the index law $x^{m+n}=x^m\times x^n$ and taking out a factor of $e^{-0.2\times T}$ Also score for candidates who make $e^{+0.2\times T}$ the subject using the same criteria

A1 Any correct form of the answer, for example, $-5\ln\left(\dfrac{7.5}{15\left(1+e^{-1}\right)}\right)$

A1 CSO $T=5\ln\left(2+\dfrac{2}{e}\right)$ Condone t appearing for T throughout this question.

Alt (c) using lns

(c)	$15e^{-0.2\times T}+15e^{-0.2\times(T+5)}=7.5$ $15e^{-0.2\times T}+15e^{-0.2\times T}e^{-1}=7.5$ $e^{-0.2\times T}(1+e^{-1})=0.5\Rightarrow -0.2\times T+\ln(1+e^{-1})=\ln 0.5$ $\Rightarrow T=\dfrac{\ln 0.5-\ln(1+e^{-1})}{-0.2},\Rightarrow T=5\ln\left(2+\dfrac{2}{e}\right)$	M1 dM1 A1, A1 **(4)** **(8 marks)**

You may see numerical attempts at part (c).

Such an attempt can score a maximum of two marks.

This can be achieved either by

Method One

1st Mark (Method): $15e^{-0.2\times T}+\text{awrt }5.52e^{-0.2\times T}=7.5\Rightarrow e^{-0.2\times T}=\text{awrt }0.37$

2nd Mark (Accuracy): $T=-5\ln(\text{awrt }0.37)$ or awrt 5.03 or $T=-5\ln\left(\dfrac{7.5}{\text{awrt }20.52}\right)$

Method Two

1st Mark (Method): $13.754e^{-0.2\times T}=7.5\Rightarrow T=-5\ln\left(\dfrac{7.5}{13.754}\right)$ or equivalent such as 3.03

2nd Mark (Accuracy): 3.03+2=5.03 Allow $-5\ln\left(\dfrac{7.5}{13.754}\right)+2$

Method Three (by trial and improvement)

1st Mark (Method): $15e^{-0.2\times 5}+15e^{-0.2\times 10}=7.55$ or $15e^{-0.2\times 5.1}+15e^{-0.2\times 10.1}=7.40$ or any value between
2nd Mark (Accuracy): Answer T =5.03.

Question Number	Scheme	Marks
69(a)	$(\theta =)20$	B1 **(1)**
(b)	Sub $t = 40, \theta = 70 \Rightarrow 70 = 120 - 100e^{-40\lambda}$	
	$\Rightarrow e^{-40\lambda} = 0.5$	M1A1
	$\Rightarrow \lambda = \dfrac{\ln 2}{40}$	M1A1
		(4)
(c)	$\theta = 100 \Rightarrow T = \dfrac{\ln 0.2}{-\text{their}'\lambda'}$	M1
	$T =$ awrt 93	A1
		(2)
		(7 marks)
Alt (b)	Sub $t = 40, \theta = 70 \Rightarrow 100e^{-40\lambda} = 50$	
	$\Rightarrow \ln 100 - 40\lambda = \ln 50$	M1A1
	$\Rightarrow \lambda = \dfrac{\ln 100 - \ln 50}{40} = \dfrac{\ln 2}{40}$	M1A1
		(4)

(a)

B1 Sight of $(\theta =)20$

(b)

M1 Sub $t = 40, \theta = 70 \Rightarrow 70 = 120 - 100e^{-40\lambda}$ and proceed to $e^{\pm 40\lambda} = A$ where A is a constant. Allow sign slips and copying errors.

A1 $e^{-40\lambda} = 0.5$, or $e^{40\lambda} = 2$ or exact equivalent

M1 For undoing the e's by taking ln's and proceeding to $\lambda = ..$

May be implied by the correct decimal answer awrt 0.017 or $\lambda = \frac{\ln 0.5}{-40}$

A1 cso $\lambda = \frac{\ln 2}{40}$

Accept equivalents in the form $\frac{\ln a}{b}$, $a, b \in \mathbb{Z}$ such as $\lambda = \frac{\ln 4}{80}$

(c)

M1 Substitutes $\theta = 100$ and their numerical value of λ into $\theta = 120 - 100e^{-\lambda t}$ and proceed to $T = \pm \frac{\ln 0.2}{\text{their}'\lambda'}$ or $T = \pm \frac{\ln 5}{\text{their}'\lambda'}$ Allow inequalities here.

A1 awrt $T = 93$

Watch for candidates who lose the minus sign in (b) and use $\lambda = \frac{\ln \frac{1}{2}}{40}$ in (c). Many then reach $T = -93$ and ignore the minus. This is M1 A0

Question Number	Scheme	Marks
70(a)	$2^{x+1} - 3 = 17 - x \Rightarrow 2^{x+1} = 20 - x$ $(x+1)\ln 2 = \ln(20 - x) \Rightarrow x = \ldots$ $x = \dfrac{\ln(20-x)}{\ln 2} - 1$	M1 dM1 A1* **(3)** **(3 marks)**
70(a)Alt	$2^{x+1} - 3 = 17 - x \Rightarrow 2^{x} = \dfrac{20 - x}{2}$ $x\ln 2 = \ln \dfrac{20 - x}{2} \Rightarrow x = \ldots$ $x = \dfrac{\ln(20-x)}{\ln 2} - 1$	M1 dM1 A1* **(3)**
70(a) **backwards**	$x = \dfrac{\ln(20-x)}{\ln 2} - 1 \Rightarrow (x+1)\ln 2 = \ln(20 - x)$ $\Rightarrow 2^{x+1} = 20 - x$ Hence $y = 2^{x+1} - 3$ meets $y = 17 - x$	M1 dM1 A1* **(3)**

M1 Setting equations in x equal to each other and proceeding to make 2^{x+1} the subject

dM1 Take ln's or logs of both sides, use the power law and proceed to x =..

A1* This is a given answer and all aspects must be correct including ln or $\log_e$ rather than $\log_{10}$

Bracketing on both $(x+1)$ and $\ln(20-x)$ must be correct.

Eg $x+1\ln 2=\ln(20-x) \Rightarrow x=\dfrac{\ln(20-x)}{\ln 2}-1$ is A0*

Special case: Students who start from the point $2^{x+1}=20-x$ can score M1 dM1A0*

EXPERT TUITION

Question Number	Scheme	Marks
71(a)	$2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \quad \Rightarrow 2x+1=e^5 \Rightarrow x=..$	M1
	$\Rightarrow x=\dfrac{e^5-1}{2}$	A1
		(2)
(b)	$3^x e^{4x}=e^7 \Rightarrow \ln(3^x e^{4x})=\ln e^7$	
	$\ln 3^x+\ln e^{4x}=\ln e^7 \Rightarrow x\ln 3+4x\ln e=7\ln e$	M1,M1
	$x(\ln 3+4)=7 \Rightarrow x=...$	dM1
	$x=\dfrac{7}{(\ln 3+4)}$ oe	A1
		(4)
		6 marks
Alt 1 71(b)	$3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}}$	
	$3^x=e^{7-4x} \Rightarrow x\ln 3=(7-4x)\ln e$	M1,M1
	$x(\ln 3+4)=7 \Rightarrow x=...$	dM1
	$x=\dfrac{7}{(\ln 3+4)}$	A1
		(4)
Alt 2 71(b) Using logs	$3^x e^{4x}=e^7 \Rightarrow \log(3^x e^{4x})=\log e^7$	
	$\log 3^x+\log e^{4x}=\log e^7 \Rightarrow x\log 3+4x\log e=7\log e$	M1, M1
	$x(\log 3+4\log e)=7\log e \Rightarrow x=...$	dM1
	$x=\dfrac{7\log e}{(\log 3+4\log e)}$	A1
		(4)
Alt 3 71(b) Using $\log_3$	$3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}}$	
	$3^x=e^{7-4x} \Rightarrow x=(7-4x)\log_3 e$	M1,M1
	$x(1+4\log_3 e)=7\log_3 e \Rightarrow x=...$	dM1
	$x=\dfrac{7\log_3 e}{(1+4\log_3 e)}$	A1
		(4)
Alt 4 71(b) Using $3^x=e^{x\ln 3}$	$3^x e^{4x}=e^7 \Rightarrow e^{x\ln 3}e^{4x}=e^7$	
	$\Rightarrow e^{x\ln 3+4x}=e^7, \Rightarrow x\ln 3+4x=7$	M1,M1
	$x(\ln 3+4)=7 \Rightarrow x=... \qquad x=\dfrac{7}{(\ln 3+4)}$	dM1 A1
		(4)

(a)

M1 Proceeds from $2\ln(2x+1)-10=0$ to $\ln(2x+1)=5$ before taking exp's to achieve x in terms of e^5

Accept for M1 $2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \Rightarrow x=\mathrm{f}(e^5)$

Alternatively they could use the power law before taking exp's to achieve x in terms of $\sqrt{e^{10}}$

$2\ln(2x+1)=10 \Rightarrow \ln(2x+1)^2=10 \Rightarrow (2x+1)^2=e^{10} \Rightarrow x=\mathrm{g}\left(\sqrt{e^{10}}\right)$

A1 cso. Accept $x=\dfrac{e^5-1}{2}$ or other exact simplified alternatives such as $x=\dfrac{e^5}{2}-\dfrac{1}{2}$. Remember to isw.

The decimal answer of 73.7 will score M1A0 unless the exact answer has also been given.

The answer $\dfrac{\sqrt{e^{10}}-1}{2}$ does not score this mark unless simplified. $x=\dfrac{\pm e^5-1}{2}$ is M1A0

(b)

M1 Takes ln's or logs of both sides and applies the addition law.

$\ln(3^x e^{4x})=\ln 3^x+\ln e^{4x}$ or $\ln(3^x e^{4x})=\ln 3^x+4x$ is evidence for the addition law

If the e^{4x} was 'moved' over to the right hand side score for either e^{7-4x} or the subtraction law.

$\ln\dfrac{e^7}{e^{4x}}=\ln e^7-\ln e^{4x}$ or $3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}} \Rightarrow 3^x=e^{7-4x}$ is evidence of the subtraction law

M1 Uses the power law of logs (seen at least once in a term with x as the index Eg $3^x, e^{4x}$ or e^{7-4x}).

$\ln 3^x+\ln e^{4x}=\ln e^7 \Rightarrow x\ln 3+4x\ln e=7\ln e$ is an example after the addition law

$3^x=e^{7-4x} \Rightarrow x\log 3=(7-4x)\log e$ is an example after the subtraction law.

It is possible to score M0M1 by applying the power law after an incorrect addition/subtraction law

For example $3^x e^{4x}=e^7 \Rightarrow \ln(3^x)\times\ln(e^{4x})=\ln e^7 \Rightarrow x\ln 3\times 4x\ln e=7\ln e$

dM1 This is dependent upon **both** previous M's. Collects/factorises out term in x and proceeds to $x=$.
Condone sign slips for this mark. An unsimplified answer can score this mark.

A1 If the candidate has taken ln's then they must use $\ln e=1$ and achieve $x=\dfrac{7}{(\ln 3+4)}$ or equivalent.

If the candidate has taken log's they must be writing log as oppose to ln and achieve

$x=\dfrac{7\log e}{(\log 3+4\log e)}$ or other exact equivalents such as $x=\dfrac{7\log e}{\log 3e^4}$.

Question Number	Scheme	Marks
72(a)	$P=\frac{800e^0}{1+3e^0}, =\frac{800}{1+3}=200$	M1,A1
		(2)
(b)	$250=\frac{800e^{0.1t}}{1+3e^{0.1t}}$	
	$250(1+3e^{0.1t})=800e^{0.1t} \Rightarrow 50e^{0.1t}=250, \Rightarrow e^{0.1t}=5$	M1,A1
	$t=\frac{1}{0.1}\ln(5)$	M1
	$t=10\ln(5)$	A1
		(4)
(c)	$P=\frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow \frac{dP}{dt}=\frac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t}-800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^2}$	M1,A1
	At t=10	
	$\frac{dP}{dt}=\frac{(1+3e)\times 80e-240e^2}{(1+3e)^2}=\frac{80e}{(1+3e)^2}$	M1,A1
		(4)
(d)	$P=\frac{800e^{0.1t}}{1+3e^{0.1t}}=\frac{800}{e^{-0.1t}+3} \Rightarrow P_{max}=\frac{800}{3}=266$. Hence P cannot be 270	B1
		(1)
		(11 marks)

(a)

M1 Sub t = 0 into P **and** use $e^0=1$ in at least one of the two cases. Accept $P=\frac{800}{1+3}$ as evidence

A1 200. Accept this for both marks as long as no incorrect working is seen.

(b)

M1 Sub P=250 into $P=\frac{800e^{0.1t}}{1+3e^{0.1t}}$, cross multiply, collect terms in $e^{0.1t}$ **and** proceed to $Ae^{0.1t}=B$

Condone bracketing issues and slips in arithmetic.

If they divide terms by $e^{0.1t}$ you should expect to see $Ce^{-0.1t}=D$

A1 $e^{0.1t}=5$ or $e^{-0.1t}=0.2$

M1 Dependent upon gaining $e^{0.1t} = E$, for taking ln's of both sides and proceeding to t=…

Accept $e^{0.1t} = E \Rightarrow 0.1t = \ln E \Rightarrow t = ...$ It could be implied by $t = \text{awrt } 16.1$

A1 $t = 10\ln(5)$

Accept exact equivalents of this as long as a and b are integers. Eg. $t = 5\ln(25)$ is fine.

(c)

M1 Scored for a full application of the quotient rule and knowing that $\frac{\mathrm{d}}{\mathrm{d}t}e^{0.1t} = ke^{0.1t}$ and NOT $kte^{0.1t}$

If the rule is quoted it must be correct.

It may be implied by their $u = 800e^{0.1t}, v = 1+3e^{0.1t}, u' = pe^{0.1t}, v' = qe^{0.1t}$ followed by $\frac{vu'-uv'}{v^2}$.

If it is neither quoted nor implied only accept expressions of the form

$$\frac{(1+3e^{0.1t})\times pe^{0.1t} - 800e^{0.1t}\times qe^{0.1t}}{(1+3e^{0.1t})^2}$$

Condone missing brackets.

You may see the chain or product rule applied to

For applying the product rule see question 1 but still insist on $\frac{\mathrm{d}}{\mathrm{d}t}e^{0.1t} = ke^{0.1t}$

For the chain rule look for

$$P = \frac{800e^{0.1t}}{1+3e^{0.1t}} = \frac{800}{e^{-0.1t}+3} \Rightarrow \frac{\mathrm{d}P}{\mathrm{d}t} = 800\times\left(e^{-0.1t}+3\right)^{-2}\times -0.1e^{-0.1t}$$

A1 A correct unsimplified answer to

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t} - 800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^2}$$

M1 For substituting $t = 10$ into their $\frac{\mathrm{d}P}{\mathrm{d}t}$, NOT P

Accept numerical answers for this. 2.59 is the numerical value if $\frac{\mathrm{d}P}{\mathrm{d}t}$ was correct

A1 $\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{80e}{(1+3e)^2}$ or equivalent such as $\frac{\mathrm{d}P}{\mathrm{d}t} = 80e(1+3e)^{-2}, \frac{80e}{1+6e+9e^2}$

Note that candidates who substitute $t = 10$ before differentiation will score 0 marks

(d)

B1 Accept solutions from substituting P=270 and showing that you get an unsolvable equation

Eg. $270 = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow -27 = e^{0.1t} \Rightarrow 0.1t = \ln(-27)$ which has no answers.

Eg. $270 = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow -27 = e^{0.1t} \Rightarrow e^{0.1t} / e^x$ is never negative

Accept solutions where it implies the max value is 266.6 or 267. For example accept sight of $\frac{800}{3}$, with a comment ‘so it cannot reach 270’, or a large value of t ($t > 99$) being substituted in to get 266.6 or 267 with a similar statement, or a graph drawn with an asymptote marked at 266.6 or 267

Do not accept exp's cannot be negative or you cannot ln a negative number without numerical evidence.

Look for both a statement and a comment

Question Number	Scheme	Marks
73(a)	$\ln(4-2x)(9-3x), = \ln(x+1)^2$	M1, M1
	So $36-30x+6x^2 = x^2+2x+1$ and $5x^2-32x+35=0$	A1
	Solve $5x^2-32x+35=0$ to give $x=\frac{7}{5}$ oe (Ignore the solution $x=5$)	M1A1
		(5)
(b)	Take log$_e$'s to give $\ln 2^x + \ln e^{3x+1} = \ln 10$	M1
	$x\ln 2 + (3x+1)\ln e = \ln 10$	M1
	$x(\ln 2 + 3\ln e) = \ln 10 - \ln e \Rightarrow x = ..$	dM1
	and uses lne = 1	M1
	$x = \dfrac{-1+\ln 10}{3+\ln 2}$	A1
		(5)
	Note that the 4th M mark may occur on line 2	**(10 marks)**

Notes for Question 73

(a)

M1 Uses addition law on lhs of equation. Accept slips on the signs. If one of the terms is taken over to the rhs it would be for the subtraction law.

M1 Uses power rule for logs write the $2\ln(x+1)$ term as $\ln(x+1)^2$. Condone invisible brackets

A1 Undoes the logs to obtain the 3TQ =0. $5x^2-32x+35=0$. Accept equivalences. The equals zero may be implied by a subsequent solution of the equation.

M1 Solves a quadratic by any allowable method.
The quadratic cannot be a version of $(4-2x)(9-3x)=0$ however.

A1 Deduces x = 1.4 or equivalent. Accept both x=1.4 and x=5. Candidates do not have to eliminate $x=5$.
You may ignore any other solution as long as it is not in the range $-1<x<2$. Extra solutions in the range scores A0.

Notes for Question 73 Continued

(b)

M1 Takes logs of both sides **and** splits LHS using addition law. If one of the terms is taken to the other side it can be awarded for taking logs of both sides **and** using the subtraction law.

M1 Taking both powers down using power rule. It is not wholly dependent upon the first M1 but logs of both sides must have been taken. Below is an example of M0M1

$$\ln 2^x \times \ln e^{3x+1} = \ln 10 \Rightarrow x\ln 2 \times (3x+1)\ln e = \ln 10$$

dM1 This is dependent upon both previous two M's being scored. It can be awarded for a full method to solve their linear equation in x. The terms in x must be collected on one side of the equation and factorised. You may condone slips in signs for this mark but the process must be correct and leading to $x = \ldots$

M1 Uses ln e = 1. This could appear in line 2, but it must be part of their equation and not just a statement.

Another example where it could be awarded is $e^{3x+1} = \frac{10}{2^x} \Rightarrow 3x+1 = \ldots$

A1 Obtains answer $x = \frac{-1+\ln 10}{3+\ln 2} = \left(\frac{\ln 10 - 1}{3+\ln 2}\right) = \left(\frac{\log_e 10 - 1}{3+\log_e 2}\right) oe$. **DO NOT ISW HERE**

Note 1: If the candidate takes $\log_{10}$'s of both sides can score M1M1dM1M0A0 for 3 out of 5.

Answer = $x = \frac{-\log e + \log 10}{3\log e + \log 2} = \left(\frac{-\log e + 1}{3\log e + \log 2}\right)$

Note 2: If the candidate writes $x = \frac{-1+\log 10}{3+\log 2}$ without reference to natural logs then award M4 but with hold the last A1 mark, scoring 4 out of 5.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>Alt 1 to 73(b)</td><td>Writes lhs in e’s $\quad 2^x e^{3x+1} = 10 \Rightarrow e^{x\ln 2} e^{3x+1} = 10$
$\Rightarrow e^{x\ln 2 + 3x+1} = 10, \quad x\ln 2 + 3x + 1 = \ln 10$
$x(\ln 2 + 3) = \ln 10 - 1 \Rightarrow x = ..$
$x = \dfrac{-1 + \ln 10}{3 + \ln 2}$</td><td>1st M1
2nd M1, 4th M1
dM1
A1 (5)</td></tr>
<tr><td colspan="3">Notes for Question 73 Alt 1</td></tr>
<tr><td colspan="3">M1 Writes the lhs of the expression in e’s. Seeing $2^x = e^{x\ln 2}$ in their equation is sufficient

M1 Uses the addition law on the lhs to produce a single exponential

dM1 Takes ln’s of both sides to produce and attempt to solve a linear equation in x
You may condone slips in signs for this mark but the process must be correct leading to x= ..

M1 Uses ln e = 1. This could appear in line 2</td></tr>
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Question Number	Scheme	Marks
74.(a)	$t=0 \Rightarrow P=\dfrac{8000}{1+7}=1000$ cao	M1A1
		(2)
(b)	$t \to \infty \quad P \to \dfrac{8000}{1}=8000$	B1
		(1)
(c)	$t=3, P=2500 \Rightarrow 2500=\dfrac{8000}{1+7e^{-3k}}$	B1
	$e^{-3k}=\dfrac{2.2}{7}=(0.31..)$ oe	M1,A1
	$k=-\dfrac{1}{3}\ln\left(\dfrac{2.2}{7}\right)=$ awrt 0.386	M1A1
		(5)
(d)	Sub t=10 into $P=\dfrac{8000}{1+7e^{-0.386t}} \Rightarrow P=6970$ cao	M1A1
		(2)
(e)	$\dfrac{dP}{dt}=-\dfrac{8000}{(1+7e^{-kt})^2}\times -7ke^{-kt}$	M1,A1
	Sub t=10 $\left.\dfrac{dP}{dt}\right\|_{t=10}=346$	A1
		(3)
		(13 marks)

Notes for Question 74	
(a)	
M1	Sets t=0, giving $e^{-k\times 0}=1$. Award if candidate attempts $\frac{8000}{1+7\times 1},\frac{8000}{8}$
A1	Correct answer only 1000. Accept 1000 for both marks as long as no incorrect working is seen.
(b)	
B1	8000. Accept $P<8000$. Condone $P\leqslant 8000$ but not $P>8000$
(c)	
B1	Sets both $t=3$, and $P=2500\Rightarrow 2500=\frac{8000}{1+7e^{-3k}}$ This may be implied by a subsequent correct line.
M1	Rearranges the equation to make $e^{\pm 3k}$ the subject. They need to multiply by the $1+7e^{-3k}$ term, and proceed to $e^{\pm 3k}=A,\quad A>0$
A1	The correct intermediate answer of $e^{-3k}=\frac{2.2}{7},\frac{11}{35}$ or equivalent. Accept awrt 0.31.. Alternatively accept $e^{3k}=\frac{35}{11}, 3.18..$ or equivalent.
M1	Proceeds from $e^{\pm 3k}=A,\quad A>0$ by correctly taking *ln*'s and then making *k* the subject of the formula.
Award for $e^{-3k}=A\Rightarrow -3k=\ln(A)\Rightarrow k=\frac{\ln(A)}{-3}$	
	If e^{3k} was found accept $e^{3k}=C\Rightarrow 3k=\ln C\Rightarrow k=\frac{\ln C}{3}$ As with method 1, C >0
A1	Awrt $k=0.386$ 3dp
(d)	
M1	Substitutes t=10 into $P=\frac{8000}{1+7e^{-kt}}$ with their numerical value of *k* to find *P*
A1	$(P=)\,6970$ or other exact equivalents like 6.97×10^3
(e)	
M1	Differentiates using the chain rule to a form $\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{C}{(1+7e^{-kt})^2}\times e^{-kt}$ Accept an application of the quotient rule to achieve $\frac{(1+7e^{-kt})\times 0-C\times -e^{-kt}}{(1+7e^{-kt})^2}$
A1	A correct un simplified $\frac{\mathrm{d}P}{\mathrm{d}t}=-\frac{8000}{(1+7e^{-kt})^2}\times -7ke^{-kt}$. The derivative can be given in terms of *k*. If a numerical value is used you may follow through on incorrect values.
A1	Awrt 346. Note that M1 must have been achieved. Just the answer scores 0

Question Number	Scheme	Marks
75.	(a) (£) 19500	B1 (1)
	(b) $9500 = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$	
	$17e^{-0.25t} + 2e^{-0.5t} = 9$	
	$(\times e^{0.5t}) \Rightarrow 17e^{0.25t} + 2 = 9e^{0.5t}$	
	$0 = 9e^{0.5t} - 17e^{0.25t} - 2$	M1
	$0 = (9e^{0.25t} + 1)(e^{0.25t} - 2)$	M1
	$e^{0.25t} = 2$	A1
	$t = 4\ln(2)$ *oe*	A1 (4)
	(c)	
	$(\frac{\mathrm{d}V}{\mathrm{d}t}) = -4250e^{-0.25t} - 1000e^{-0.5t}$	M1A1
	When t=8 Decrease = 593 (£/year)	M1A1 (4)
		(9 marks)

(a) B1 19500. The £ sign is not important for this mark

(b) M1 Substitute V=9500, collect terms and set on 1 side of an equation =0. Indices must be correct
Accept $17000e^{-0.25t} + 2000e^{-0.5t} - 9000 = 0$ and $17000x + 2000x^2 - 9000 = 0$ where $x = e^{-0.25t}$

M1 Factorise the quadratic in $e^{0.25t}$ or $e^{-0.25t}$
For your information the factorised quadratic in $e^{-0.25t}$ is $(2e^{-0.25t} - 1)(e^{-0.25t} + 9) = 0$
Alternatively let $'x' = e^{0.25t}$ or otherwise and factorise a quadratic equation in x

A1 Correct solution of the quadratic. Either $e^{0.25t} = 2$ or $e^{-0.25t} = \frac{1}{2}$ oe.

A1 Correct exact value of t. Accept variations of $4\ln(2)$, such as $\ln(16)$, $\frac{\ln(\frac{1}{2})}{-0.25}, \frac{\ln(2)}{0.25}, -4\ln(\frac{1}{2})$

.(c) M1 Differentiates $V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$ by the chain rule.
Accept answers of the form $(\frac{\mathrm{d}V}{\mathrm{d}t}) = \pm Ae^{-0.25t} \pm Be^{-0.5t}$ A, B are constants $\neq 0$

A1 Correct derivative $(\frac{\mathrm{d}V}{\mathrm{d}t}) = -4250e^{-0.25t} - 1000e^{-0.5t}$.
There is no need for it to be simplified so accept

$$(\frac{\mathrm{d}V}{\mathrm{d}t}) = 17000 \times -0.25e^{-0.25t} + 2000 \times -0.5e^{-0.5t} \quad oe$$

M1 Substitute t=8 into their $\frac{\mathrm{d}V}{\mathrm{d}t}$.
This is not dependent upon the first M1 but there must have been some attempt to differentiate.
Do not accept t=8 in V

A1 ±593. Ignore the sign and the units. If the candidate then divides by 8, withhold this mark. **This would not be isw. Be aware that sub t=8 into V first and then differentiating can achieve 593. This is M0A0M0A0.**

Question No	Scheme	Marks
76	(a) 20 (mm^2)	**B1** **(1)**
	(b) $'40' = 20\ e^{1.5t} \quad \rightarrow \quad e^{1.5t} = c$ $e^{1.5t} = \frac{40}{20} = \ (2)$ Correct order $1.5t = ln'2' \rightarrow \quad t = \frac{lnc}{1.5}$ $t = \frac{ln2}{1.5} = (\ awrt\ 0.46)$ 12.28 or 28 (minutes}	**M1** **A1** **M1** **A1** **A1** **(5)** **(6 marks)**

(a)

B1 Sight of 20 relating to the value of A at t=0. Do not worry about (incorrect) units. Accept its sight in (b)

(b)

M1 Substitutes A=40 or twice their answer to (a) **and** proceeds to $e^{1.5t} = constant$. Accept non numerical answers.

A1 $e^{1.5t} = \frac{40}{20}\ or\ 2$

M1 Correct ln work to find t. Eg $e^{1.5t} = constant \rightarrow 1.5t = \ln(constant) \rightarrow t =$
The order must be correct. Accept non numerical answers**. See below for correct alternatives**

A1 Achieves either $\frac{\ln(2)}{1.5}$ or awrt 0.46 2sf

A1 Either 12:28 or 28 (minutes). Cao

Alternatives in (b)

Alt 1- taking ln's of both sides on line 1

M1 Substitutes A=40, or twice (a) takes ln's of both sides **and** proceeds to $\ln('40') = ln20 + lne^{1.5t}$

A1 $\ln(40) = ln20 + 1.5t$

M1 Make t the subject with correct *ln* work.

$$\ln('40') - ln20 = 1.5t\ \ or\ \ \ln\left(\frac{'40'}{20}\right) = 1.5t \rightarrow \ t =$$

A1,A1 are the same

Alt 2- trial and improvement-hopefully seen rarely

M1 Substitutes t= 0.46 and t=0.47 into $20e^{1.5t}$ to obtain A at both values. Must be to at least 2dp but you may accept tighter interval but the interval must span the correct value of 0.46209812

A1 Obtains A(0.46)=39.87 AND A(0.47)=40.47 or equivalent

M1 Substitutes t=0.462 and t=0.4625 into $40e^{1.5t}$

A1 Obtains A(0.462)=39.99 AND A(0.4625)=40.02 or equivalent and states t=0.462 (3sf)

A1 AS ABOVE

No working leading to fully correct accurate answer (3sf or better) send/escalate to team leader

Question Number	Scheme	Marks
77 (a)	$p=7.5$	B1 (1)
(b)	$2.5 = 7.5e^{-4k}$	M1
	$e^{-4k} = \frac{1}{3}$	M1
	$-4k = \ln(\frac{1}{3})$	dM1
	$-4k = -\ln(3)$	
	$k = \frac{1}{4}\ln(3)$	A1*
	See notes for additional correct solutions and the last A1	(4)
(c)	$\frac{dm}{dt} = -kpe^{-kt}$ ft on their p and k	M1A1ft
	$-\frac{1}{4}ln3 \times 7.5e^{-\frac{1}{4}(ln3)t} = -0.6ln3$	
	$e^{-\frac{1}{4}(ln3)t} = \frac{2.4}{7.5} = (0.32)$	M1A1
	$-\frac{1}{4}(ln3)t = \ln(0.32)$	dM1
	t=4.1486…. 4.15 or awrt 4.1	A1
		(6)
		11Marks

Question Number	Scheme		Marks
78.			
(a)	$\theta = 20 + Ae^{-kt}$ (eqn *)		
	$\{t = 0, \theta = 90 \Rightarrow\}$ $90 = 20 + Ae^{-k(0)}$	Substitutes $t = 0$ and $\theta = 90$ into eqn *	M1
	$90 = 20 + A \Rightarrow \underline{A = 70}$	$\underline{A = 70}$	A1
			(2)
(b)	$\theta = 20 + 70e^{-kt}$		
	$\{t = 5, \theta = 55 \Rightarrow\}$ $55 = 20 + 70e^{-k(5)}$ $\frac{35}{70} = e^{-5k}$	Substitutes $t = 5$ and $\theta = 55$ into eqn * and rearranges eqn * to make $e^{\pm 5k}$ the subject.	M1
	$\ln\left(\frac{35}{70}\right) = -5k$	Takes 'lns' and proceeds to make '$\pm 5k$' the subject.	dM1
	$-5k = \ln\left(\frac{1}{2}\right)$		
	$-5k = \ln 1 - \ln 2 \Rightarrow -5k = -\ln 2 \Rightarrow \underline{k = \frac{1}{5}\ln 2}$	Convincing proof that $k = \frac{1}{5}\ln 2$	A1 *
			(3)
(c)	$\theta = 20 + 70e^{-\frac{1}{5}t\ln 2}$		
	$\frac{d\theta}{dt} = -\frac{1}{5}\ln 2.(70)e^{-\frac{1}{5}t\ln 2}$	$\pm\alpha e^{-kt}$ where $k = \frac{1}{5}\ln 2$ $-14\ln 2\, e^{-\frac{1}{5}t\ln 2}$	M1 A1 oe
	When $t = 10$, $\frac{d\theta}{dt} = -14\ln 2\, e^{-2\ln 2}$		
	$\frac{d\theta}{dt} = -\frac{7}{2}\ln 2 = -2.426015132...$		
	Rate of decrease of $\theta = 2.426\ ^{\circ}C/\text{min}$ (3 dp.)	awrt ± 2.426	A1
			(3) **[8]**

Question Number	Scheme	Marks
79. (a)	$\dfrac{(x+5)(2x-1)}{(x+5)(x-3)} = \dfrac{(2x-1)}{(x-3)}$	M1 B1 A1 **aef** (3)
(b)	$\ln\left(\dfrac{2x^2+9x-5}{x^2+2x-15}\right)=1$	M1
	$\dfrac{2x^2+9x-5}{x^2+2x-15}=e$	dM1
	$\dfrac{2x-1}{x-3}=e \Rightarrow \quad 3e-1 = x(e-2)$	M1
	$\Rightarrow \; x = \dfrac{3e-1}{e-2}$	A1 **aef cso** (4) **[7]**
	(a) M1: An attempt to factorise the numerator. B1: Correct factorisation of denominator to give $(x+5)(x-3)$. Can be seen anywhere. (b) M1: Uses a correct law of logarithms to combine at least two terms. This usually is achieved by the subtraction law of logarithms to give $\ln\left(\dfrac{2x^2+9x-5}{x^2+2x-15}\right)=1.$ The product law of logarithms can be used to achieve $\ln\left(2x^2+9x-5\right)=\ln\left(e\left(x^2+2x-15\right)\right).$ The product and quotient law could also be used to achieve $\ln\left(\dfrac{2x^2+9x-5}{e(x^2+2x-15)}\right)=0.$ dM1: Removing ln's correctly by the realisation that the anti-ln of 1 is e. Note that this mark is dependent on the previous method mark being awarded. M1: Collect x terms together and factorise. Note that this is not a dependent method mark. A1: $\dfrac{3e-1}{e-2}$ or $\dfrac{3e^1-1}{e^1-2}$ or $\dfrac{1-3e}{2-e}$. aef Note that the answer needs to be in terms of e. The decimal answer is 9.9610559… Note that the solution must be correct in order for you to award this final accuracy mark. **Note: See Appendix for an alternative method of long division.**	

Question Number	Scheme		Marks
80 (i)(a)	$\ln(3x-7)=5$		
	$e^{\ln(3x-7)}=e^5$	Takes e of both sides of the equation. This can be implied by $3x-7=e^5$.	M1
		Then rearranges to make x the subject.	dM1
	$3x-7=e^5 \Rightarrow x=\dfrac{e^5+7}{3}$ $\{=51.804...\}$	***Exact answer*** of $\dfrac{e^5+7}{3}$.	A1
			(3)
(b)	$3^x e^{7x+2}=15$		
	$\ln\left(3^x e^{7x+2}\right)=\ln 15$	Takes ln (or logs) of both sides of the equation.	M1
	$\ln 3^x+\ln e^{7x+2}=\ln 15$	Applies the addition law of logarithms.	M1
	$x\ln 3+7x+2=\ln 15$	$x\ln 3+7x+2=\ln 15$	A1 oe
	$x(\ln 3+7)=-2+\ln 15$	Factorising out at least two x terms on one side and collecting number terms on the other side.	ddM1
	$x=\dfrac{-2+\ln 15}{7+\ln 3}$ $\{=0.0874...\}$	***Exact answer*** of $\dfrac{-2+\ln 15}{7+\ln 3}$	A1 oe
			(5)
			[8]

Question Number	Scheme		Marks
81	$P = 80\mathrm{e}^{\frac{t}{5}}$		
(a)	$t = 0 \Rightarrow P = 80\mathrm{e}^{\frac{0}{5}} = 80(1) = \underline{80}$	$\underline{80}$	B1
			(1)
(b)	$P = 1000 \Rightarrow 1000 = 80\mathrm{e}^{\frac{t}{5}} \Rightarrow \dfrac{1000}{80} = \mathrm{e}^{\frac{t}{5}}$	Substitutes $P = 1000$ and rearranges equation to make $\mathrm{e}^{\frac{t}{5}}$ the subject.	M1
	$\therefore\ t = 5\ln\left(\dfrac{1000}{80}\right)$		
	$t = 12.6286...$	awrt 12.6 or 13 years	A1
		Note $t = 12$ or $t = \text{awrt } 12.6 \Rightarrow t = 12$ will score A0	(2)
(c)	$\dfrac{\mathrm{d}P}{\mathrm{d}t} = 16\mathrm{e}^{\frac{t}{5}}$	$k\mathrm{e}^{\frac{1}{5}t}$ and $k \neq 80$.	M1
		$16\mathrm{e}^{\frac{1}{5}t}$	A1
			(2)
(d)	$50 = 16\mathrm{e}^{\frac{t}{5}}$		
	$\therefore\ t = 5\ln\left(\dfrac{50}{16}\right) \qquad \{= 5.69717...\}$	Using $50 = \frac{dP}{dt}$ and an attempt to solve to find the value of t or $\frac{t}{5}$.	M1
	$P = 80\mathrm{e}^{\frac{1}{5}\left(5\ln\left(\frac{50}{16}\right)\right)}$ or $P = 80\mathrm{e}^{\frac{1}{5}(5.69717...)}$	Substitutes their value of t back into the equation for P.	dM1
	$P = \dfrac{80(50)}{16} = \underline{250}$	$\underline{250}$ or awrt 250	A1
			(3)
			[8]

Question Number	Scheme	Marks
82.	(a) $e^{2x+1} = 2$ $2x + 1 = \ln 2$ $x = \frac{1}{2}(\ln 2 - 1)$	 M1 A1 (2)

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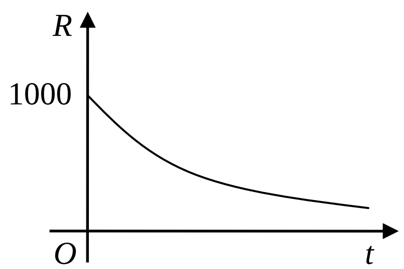
Question Number	Scheme	Marks
83.	(a) 1000	B1 (1)
	(b) $1000\,\mathrm{e}^{-5730c} = 500$	M1
	$\mathrm{e}^{-5730c} = \frac{1}{2}$	A1
	$-5730c = \ln\frac{1}{2}$	M1
	$c = 0.000121$ cao	A1 (4)
	(c) $R = 1000\,\mathrm{e}^{-22920c} = 62.5$ Accept 62-63	M1 A1 (2)
	(d)  Shape 1000	B1 B1 (2) **[9]**

EXPERT TUITION