EXPERT TUITION

Maths Questions By Topic:

Exponentials & Logarithms

A-Level Edexcel

0207 060 4494
www.expert-tuition.co.uk
online.expert-tuition.co.uk
enquiries@expert-tuition.co.uk
The Foundry, 77 Fulham Palace Road, W6 8JA

Table Of Contents

New Spec

Old Spec

1. A scientist is studying the number of bees and the number of wasps on an island.

The number of bees, measured in thousands, N_b, is modelled by the equation

$$N_b = 45 + 220\,e^{0.05t}$$

where t is the number of years from the start of the study.

According to the model,

(a) find the number of bees at the start of the study,

(1)

(b) show that, exactly 10 years after the start of the study, the number of bees was increasing at a **rate** of approximately 18 thousand per year.

(3)

The number of wasps, measured in thousands, N_w, is modelled by the equation

$$N_w = 10 + 800\,e^{-0.05t}$$

where t is the number of years from the start of the study.

When $t = T$, according to the models, there are an equal number of bees and wasps.

(c) Find the value of T to 2 decimal places.

(4)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2. The mass, A kg, of algae in a small pond, is modelled by the equation

$$A = pq^t$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by the equation

$$\log_{10} A = 0.03t + 0.5$$

(a) Use this relationship to find a complete equation for the model in the form

$$A = pq^t$$

giving the value of p and the value of q each to 4 significant figures.

(4)

(b) With reference to the model, interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find, according to the model,

(i) the mass of algae in the pond when $t = 8$, giving your answer to the nearest 0.5 kg,

(ii) the number of weeks it takes for the mass of algae in the pond to reach 4 kg.

(3)

(d) State one reason why this may not be a realistic model in the long term.

(1)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 10 marks)

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The air pressure, P kg/cm^2, inside a car tyre, t minutes from the instant when the tyre developed a puncture is given by the equation

$$P = k + 1.4e^{-0.5t} \qquad t \in \mathbb{R} \qquad t \geqslant 0$$

where k is a constant.

Given that the initial air pressure inside the tyre was 2.2 kg/cm^2

(a) state the value of k.

(1)

From the instant when the tyre developed the puncture,

(b) find the time taken for the air pressure to fall to 1 kg/cm^2
Give your answer in minutes to one decimal place.

(3)

(c) Find the rate at which the air pressure in the tyre is decreasing exactly 2 minutes from the instant when the tyre developed the puncture.
Give your answer in kg/cm^2 per minute to 3 significant figures.

(2)

Question 3 continued

(Total for Question 3 is 6 marks)

4. (a) Given that $p = \log_3 x$, where $x > 0$, find in simplest form in terms of p,

(i) $\log_3\left(\frac{x}{9}\right)$

(ii) $\log_3\left(\sqrt{x}\right)$

(2)

(b) Hence, or otherwise, solve

$$2\log_3\left(\frac{x}{9}\right) + 3\log_3\left(\sqrt{x}\right) = -11$$

giving your answer as a simplified fraction.

Solutions relying on calculator technology are not acceptable.

(4)

Question 4 continued

(Total for Question 4 is 6 marks)

5. A scientist is studying the growth of two different populations of bacteria.

The number of bacteria, N, in the **first** population is modelled by the equation

$$N = Ae^{kt} \qquad t \geqslant 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 1000 bacteria in this population at the start of the study
- it took exactly 5 hours from the start of the study for this population to double

(a) find a complete equation for the model.

(4)

(b) Hence find the rate of increase in the number of bacteria in this population exactly 8 hours from the start of the study. Give your answer to 2 significant figures.

(2)

The number of bacteria, M, in the **second** population is modelled by the equation

$$M = 500e^{1\ 4kt} \qquad t \geqslant 0$$

where k has the value found in part (a) and t is the time in hours from the start of the study.

Given that T hours after the start of the study, the number of bacteria in the two different populations was the same,

(c) find the value of T.

(3)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

express y in terms of x, writing your answer in simplest form.

(3)

Question 6 continued

(Total for Question 6 is 3 marks)

7. The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, A km^2, is modelled by the equation

$$A = 80 - 45e^{ct}$$

where c is a constant and t is the number of years after 1st January 2005.

Using the model,

(a) find the area of the nature reserve that was covered by trees just before tree planting started.

(1)

On 1st January 2019 an area of 60 km^2 of the nature reserve was covered by trees.

(b) Use this information to find a complete equation for the model, giving your value of c to 3 significant figures.

(4)

On 1st January 2020, the owners of the nature reserve announced a long-term plan to have 100 km^2 of the nature reserve covered by trees.

(c) State a reason why the model is not appropriate for this plan.

(1)

Question 7 continued

(Total for Question 7 is 6 marks)

8.

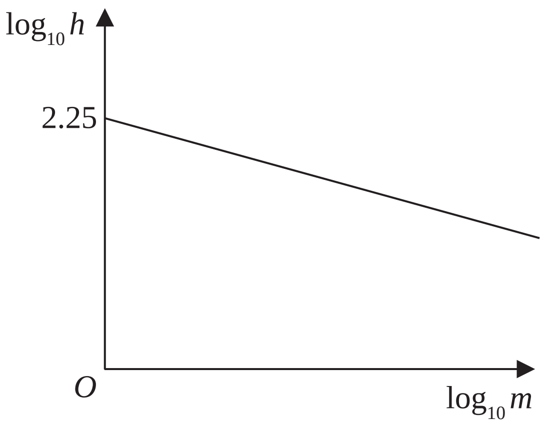


Figure 2

The resting heart rate, h, of a mammal, measured in beats per minute, is modelled by the equation

$$h = pm^q$$

where p and q are constants and m is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between $\log_{10} h$ and $\log_{10} m$

The line meets the vertical $\log_{10} h$ axis at 2.25 and has a gradient of -0.235

(a) Find, to 3 significant figures, the value of p and the value of q.

(3)

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

(b) Comment on the suitability of the model for this mammal.

(3)

(c) With reference to the model, interpret the value of the constant p.

(1)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 7 marks)

9. By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of p to one decimal place.

(3)

Question 9 continued

(Total for Question 9 is 3 marks)

10. An advertising agency is monitoring the number of views of an online advert.

The equation

$$\log_{10} V = 0.072t + 2.379 \qquad 1 \leqslant t \leqslant 30,\ t \in \mathbb{N}$$

is used to model the total number of views of the advert, V, in the first t days after the advert went live.

(a) Show that $V = ab^t$ where a and b are constants to be found.

Give the value of a to the nearest whole number and give the value of b to 3 significant figures.

(4)

(b) Interpret, with reference to the model, the value of ab.

(1)

Using this model, calculate

(c) the total number of views of the advert in the first 20 days after the advert went live. Give your answer to 2 significant figures.

(2)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 7 marks)

11. In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car A, a possible equation linking V with t.

(4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

(2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B.

(1)

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 7 marks)

12. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b - 1}$$

(3)

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2)

Question 12 continued

(Total for Question 12 is 5 marks)

13. The value of a car, £V, can be modelled by the equation

$$V = 15\,700\mathrm{e}^{-0\ 25t} + 2300 \qquad t \in \mathbb{R},\ t \geqslant 0$$

where the age of the car is t years.

Using the model,

(a) find the initial value of the car.

(1)

Given the model predicts that the value of the car is decreasing at a rate of £500 per year at the instant when $t = T$,

(b) (i) show that

$$3925\mathrm{e}^{-0\ 25T} = 500$$

(ii) Hence find the age of the car at this instant, giving your answer in years and months to the nearest month.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(6)

The model predicts that the value of the car approaches, but does not fall below, £A.

(c) State the value of A.

(1)

(d) State a limitation of this model.

(1)

Question 13 continued

Question 13 continued

Question 13 continued

(Total for Question 13 is 9 marks)

14. The value, £V, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find p to 4 decimal places,

(ii) show that A is approximately 24 800

(4)

(b) With reference to the model, interpret

(i) the value of the constant A,

(ii) the value of the constant p.

(2)

Using the model,

(c) find the year during which the value of the car first exceeds £100 000

(4)

Question 14 continued

Question 14 continued

Question 14 continued

(Total for Question 14 is 10 marks)

15. A student's attempt to solve the equation $2\log_2 x - \log_2 \sqrt{x} = 3$ is shown below.

$$2\log_2 x - \log_2 \sqrt{x} = 3$$

$$2\log_2\left(\frac{x}{\sqrt{x}}\right) = 3$$ using the subtraction law for logs

$$2\log_2\left(\sqrt{x}\right) = 3$$ simplifying

$$\log_2 x = 3$$ using the power law for logs

$$x = 3^2 = 9$$ using the definition of a log

(a) Identify two errors made by this student, giving a brief explanation of each.

(2)

(b) Write out the correct solution.

(3)

Question 15 continued

(Total for Question 15 is 5 marks)

16.

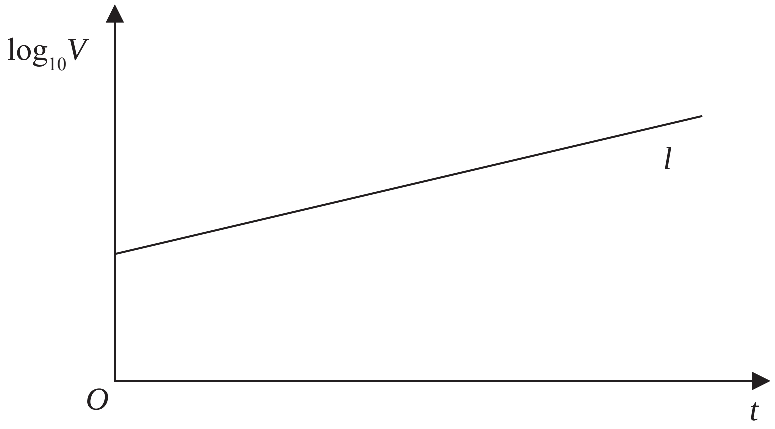


Figure 3

The value of a rare painting, £V, is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10} V$ since 1st January 1980.

The equation of line l is $\log_{10} V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q.

(4)

(b) With reference to the model interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds.

(2)

Question 16 continued

Question 16 continued

Question 16 continued

(Total for Question 16 is 8 marks)

17. Find any real values of x such that

$$2\log_4(2-x) - \log_4(x+5) = 1$$

(6)

Question 17 continued

(Total for Question 17 is 6 marks)

18. The growth of pond weed on the surface of a pond is being investigated.

The surface area of the pond covered by the weed, A m^2, can be modelled by the equation

$$A = 0.2\mathrm{e}^{0\ 3t}$$

where t is the number of days after the start of the investigation.

(a) State the surface area of the pond covered by the weed at the start of the investigation.

(1)

(b) Find the rate of increase of the surface area of the pond covered by the weed, in m^2/day, exactly 5 days after the start of the investigation.

(2)

Given that the pond has a surface area of 100 m^2,

(c) find, to the nearest hour, the time taken, according to the model, for the surface of the pond to be fully covered by the weed.

(4)

The pond is observed for one month and by the end of the month 90% of the surface area of the pond was covered by the weed.

(d) Evaluate the model in light of this information, giving a reason for your answer.

(1)

Question 18 continued

(Total for Question 18 is 8 marks)

19. A student was asked to give the exact solution to the equation

$$2^{2x+4} - 9(2^x) = 0$$

The student's attempt is shown below:

$$2^{2x+4} - 9(2^x) = 0$$

$$2^{2x} + 2^4 - 9(2^x) = 0$$

$$\text{Let} \quad 2^x = y$$

$$y^2 - 9y + 8 = 0$$

$$(y - 8)(y - 1) = 0$$

$$y = 8 \text{ or } y = 1$$

$$\text{So } x = 3 \text{ or } x = 0$$

(a) Identify the two errors made by the student.

(2)

(b) Find the exact solution to the equation.

(2)

Question 19 continued

(Total for Question 19 is 4 marks)

20.

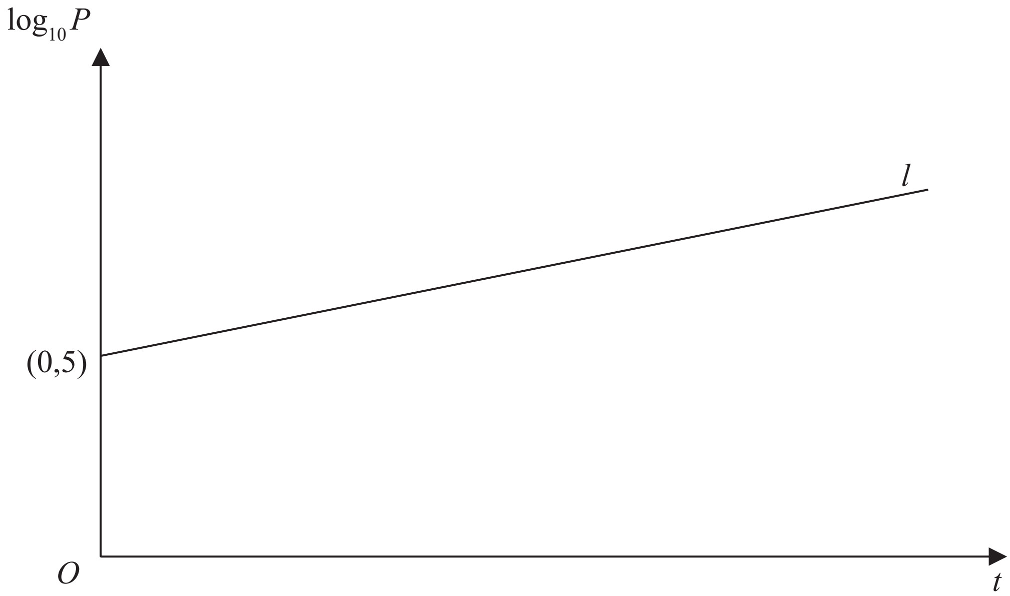


Figure 2

A town's population, P, is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded.

The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10}P$ for the population over a period of 100 years.

The line l meets the vertical axis at (0, 5) as shown. The gradient of l is $\frac{1}{200}$.

(a) Write down an equation for l.

(2)

(b) Find the value of a and the value of b.

(4)

(c) With reference to the model interpret

(i) the value of the constant a,

(ii) the value of the constant b.

(2)

(d) Find

(i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

(3)

(e) State two reasons why this may not be a realistic population model.

(2)

Question 20 continued

(Total for Question 20 is 13 marks)

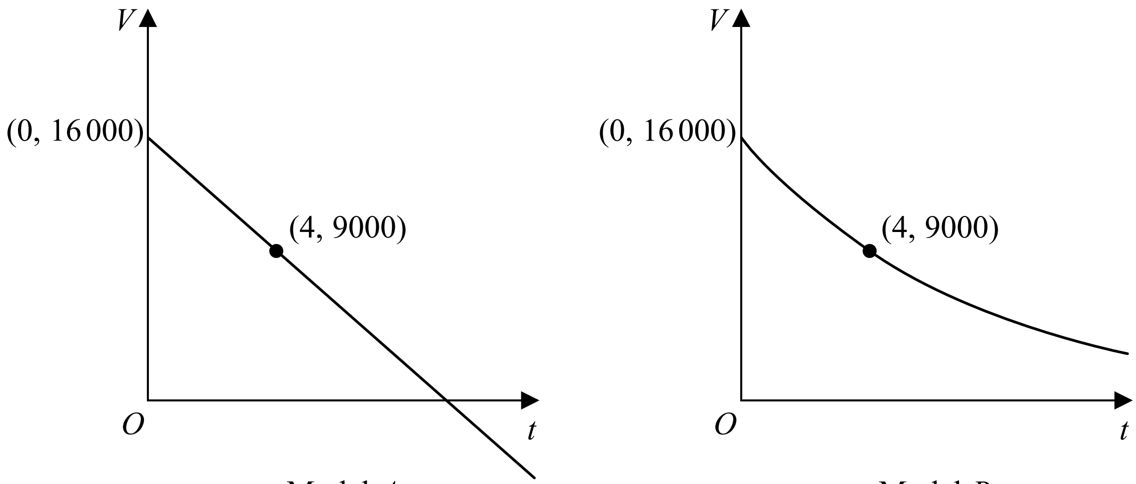
21. A company plans to extract oil from an oil field.

The daily volume of oil V, measured in barrels that the company will extract from this oil field depends upon the time, t years, after the start of drilling.

The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16 000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.



Model A Model B

(a) (i) Use model A to estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(ii) Write down a limitation of using model A.

(2)

(b) (i) Using an exponential model and the information given in the question, find a possible equation for model B.

(ii) Using your answer to (b)(i) estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(5)

Question 21 continued

(Total for Question 21 is 7 marks)

22. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b, \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b.

(2)

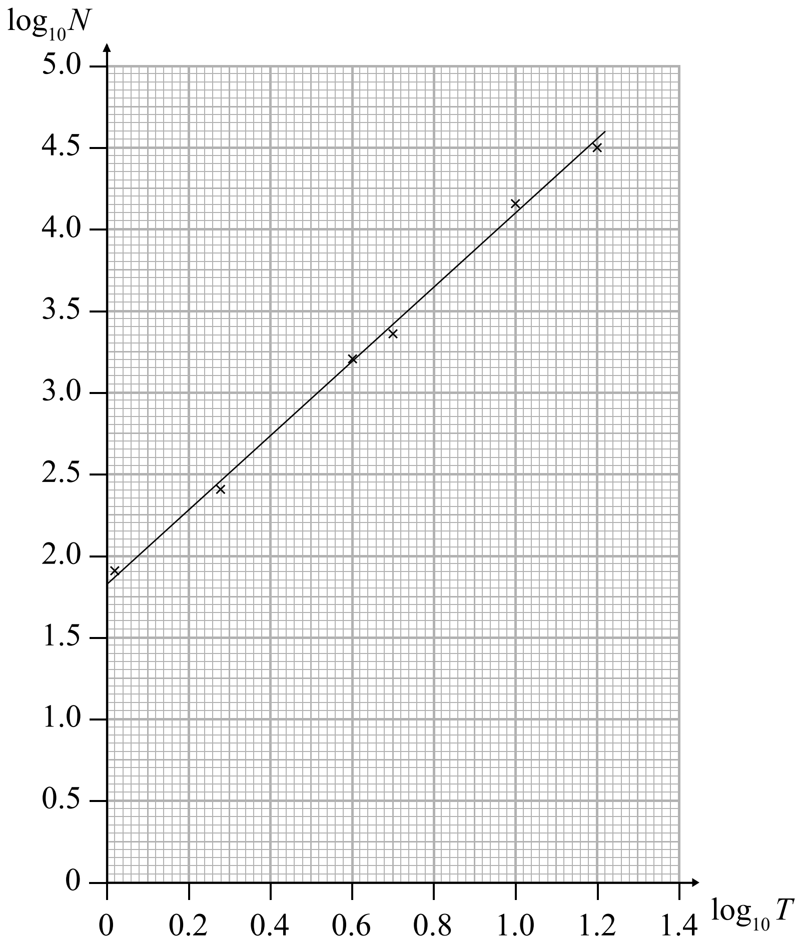


Figure 3

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a.

(1)

Question 22 continued

Question 22 continued

Question 22 continued

(Total for Question 22 is 9 marks)

23. Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

(3)

Question 23 continued

(Total for Question 23 is 3 marks)

24. The time, T seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where l metres is the length of the pendulum and a and b are constants.

(a) Show that this relationship can be written in the form

$$\log_{10} T = b\log_{10} l + \log_{10} a$$

(2)

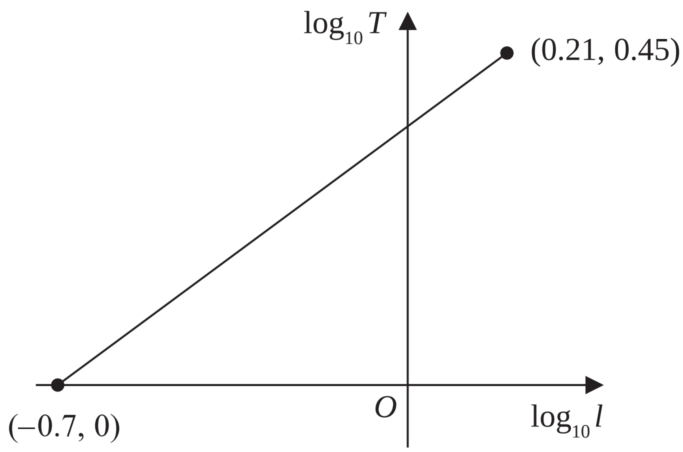


Figure 3

A student carried out an experiment to find the values of the constants a and b.

The student recorded the value of T for different values of l.

Figure 3 shows the linear relationship between $\log_{10} l$ and $\log_{10} T$ for the student's data.
The straight line passes through the points (–0.7, 0) and (0.21, 0.45)

Using this information,

(b) find a complete equation for the model in the form

$$T = al^b$$

giving the value of a and the value of b, each to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant a.

(1)

Question 24 continued

Question 24 continued

Question 24 continued

(Total for Question 24 is 6 marks)

25. (a) Given that

$$2\log(4 - x) = \log(x + 8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2\log(4 - x) = \log(x + 8)$$

giving a reason for your answer.

(2)

Question 25 continued

(Total for Question 25 is 5 marks)

26. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P.

Find, using algebra, the exact x coordinate of P.

(4)

Question 26 continued

(Total for Question 26 is 4 marks)

27. A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol, θ°C, at time t seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0\ 07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18 °C
- after 10 seconds the temperature of the ethanol was 44 °C

(a) find a complete equation for the model, giving the values of A and B to 3 significant figures.

(4)

Ethanol has a boiling point of approximately 78 °C

(b) Use this information to evaluate the model.

(2)

Question 27 continued

(Total for Question 27 is 6 marks)

28. The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \qquad x > 0 \qquad x \neq k$$

where k is a constant.

(a) Deduce the value of k.

(1)

(b) Prove that

$$g'(x) > 0$$

for all values of x in the domain of g.

(3)

(c) Find the range of values of a for which

$$g(a) > 0$$

(2)

Question 28 continued

Question 28 continued

Question 28 continued

(Total for Question 28 is 6 marks)

29. Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x.

(3)

Question 29 continued

(Total for Question 29 is 3 marks)

30. A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of $V \text{ km h}^{-1}$.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.

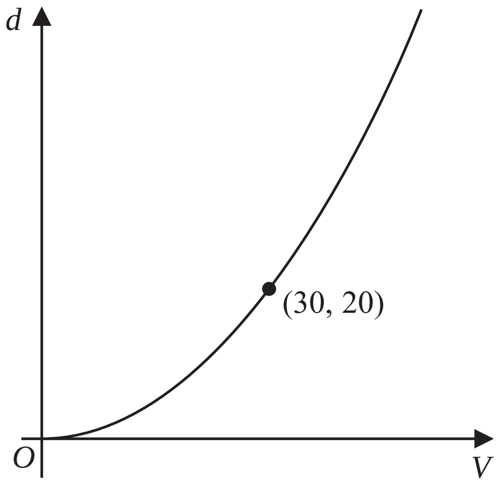


Figure 5

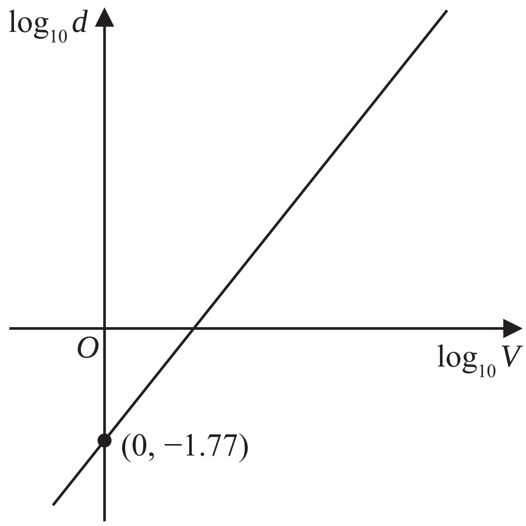


Figure 6

(a) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \qquad \text{where } k \text{ and } n \text{ are constants}$$

with $k \approx 0.017$

(3)

Using the information given in Figure 5, with $k = 0.017$

(b) find a complete equation for the model giving the value of n to 3 significant figures.

(3)

Sean is driving this car at 60 km h^{-1} in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

(3)

Question 30 continued

Question 30 continued

Question 30 continued

(Total for Question 30 is 9 marks)

31. A cup of hot tea was placed on a table. At time t minutes after the cup was placed on the table, the temperature of the tea in the cup, θ °C, is modelled by the equation

$$\theta = 25 + Ae^{-0\,03t}$$

where A is a constant.

The temperature of the tea was 75 °C when the cup was placed on the table.

(a) Find a complete equation for the model.

(1)

(b) Use the model to find the time taken for the tea to cool from 75 °C to 60 °C, giving your answer in minutes to one decimal place.

(2)

Two hours after the cup was placed on the table, the temperature of the tea was measured as 20.3 °C.

Using this information,

(c) evaluate the model, explaining your reasoning.

(1)

Question 31 continued

(Total for Question 31 is 4 marks)

32. A bacterial culture has area p mm^2 at time t hours after the culture was placed onto a circular dish.

A scientist states that at time t hours, the rate of increase of the area of the culture can be modelled as being proportional to the area of the culture.

(a) Show that the scientist's model for p leads to the equation

$$p = ae^{kt}$$

where a and k are constants.

(4)

The scientist measures the values for p at regular intervals during the first 24 hours after the culture was placed onto the dish.

She plots a graph of $\ln p$ against t and finds that the points on the graph lie close to a straight line with gradient 0.14 and vertical intercept 3.95

(b) Estimate, to 2 significant figures, the value of a and the value of k.

(3)

(c) Hence show that the model for p can be rewritten as

$$p = ab^t$$

stating, to 3 significant figures, the value of the constant b.

(2)

With reference to this model,

(d) (i) interpret the value of the constant a,

(ii) interpret the value of the constant b.

(2)

(e) State a long term limitation of the model for p.

(1)

Question 32 continued

Question 32 continued

Question 32 continued

(Total for Question 32 is 12 marks)

33. The mass, m grams, of a radioactive substance, t years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

(2)

(b) show that $\frac{dm}{dt} = km$, where k is a constant to be found.

(2)

(Total for Question 33 is 4 marks)

Leave blank

34. (a) Given $y = 2^x$, show that

$$2^{2x+1} - 17(2^x) + 8 = 0$$

can be written in the form

$$2y^2 - 17y + 8 = 0$$

(2)

(b) Hence solve

$$2^{2x+1} - 17(2^x) + 8 = 0$$

(4)

Leave blank

Question 34 continued

(Total 6 marks)

Leave blank

35. Express 9^{3x+1} in the form 3^y, giving y in the form $ax + b$, where a and b are constants.

(2)

(Total 2 marks)

Leave blank

36. Given that $y = 2^x$,

(a) express 4^x in terms of y.

(1)

(b) Hence, or otherwise, solve

$$8(4^x) - 9(2^x) + 1 = 0$$

(4)

(Total 5 marks)

Leave blank

37. (a) Write down the value of $32^{\frac{1}{5}}$

(1)

(b) Simplify fully $(32x^5)^{-\frac{2}{5}}$

(3)

(Total 4 marks)

EXPERT TUITION

Leave blank

38. (a) Evaluate $81^{\frac{3}{2}}$

(2)

(b) Simplify fully $x^2\left(4x^{-\frac{1}{2}}\right)^2$

(2)

(Total 4 marks)

Leave blank

39. (a) Find the value of $8^{\frac{5}{3}}$

(2)

(b) Simplify fully $\dfrac{\left(2x^{\frac{1}{2}}\right)^3}{4x^2}$

(3)

(Total 5 marks)

Leave blank

40. Solve

(a) $2^y = 8$

(1)

(b) $2^x \times 4^{x+1} = 8$

(4)

(Total 5 marks)

Leave blank

41. Express 8^{2x+3} in the form 2^y, stating y in terms of x.

(2)

(Total 2 marks)

Leave blank

42. (a) Evaluate $(32)^{\frac{3}{5}}$, giving your answer as an integer.

(2)

(b) Simplify fully $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$

(2)

(Total 4 marks)

Leave blank

43. Find the value of

(a) $25^{\frac{1}{2}}$

(1)

(b) $25^{-\frac{3}{2}}$

(2)

(Total 3 marks)

Leave blank

44. (a) Find the value of $16^{-\frac{1}{4}}$

(2)

(b) Simplify $x\left(2x^{-\frac{1}{4}}\right)^{4}$

(2)

(Total 4 marks)

Leave blank

45. Given that $32\sqrt{2} = 2^{a}$, find the value of a.

(3)

(Total 3 marks)

Leave blank

46. (a) Write down the value of $125^{\frac{1}{3}}$.

(1)

(b) Find the value of $125^{-\frac{2}{3}}$.

(2)

(Total 3 marks)

Leave blank

47. (a) Write down the value of $16^{\frac{1}{4}}$.

(1)

(b) Simplify $(16x^{12})^{\frac{3}{4}}$.

(2)

(Total 3 marks)

Leave blank

48. (i) Find the value of y for which

$$1.01^{y-1} = 500$$

Give your answer to 2 decimal places.

(2)

(ii) Given that

$$2\log_4(3x+5) = \log_4(3x+8) + 1, \qquad x > -\frac{5}{3}$$

(a) show that

$$9x^2 + 18x - 7 = 0$$

(4)

(b) Hence solve the equation

$$2\log_4(3x+5) = \log_4(3x+8) + 1, \qquad x > -\frac{5}{3}$$

(2)

Leave blank

Question 48 continued

Leave blank

Question 48 continued

Leave blank

Question 48 continued

(Total 8 marks)

EXPERT TUITION

Leave blank

49. (i) $$2\log(x + a) = \log(16a^6), \text{ where } a \text{ is a positive constant}$$

Find x in terms of a, giving your answer in its simplest form.

(3)

(ii) $$\log_3(9y + b) - \log_3(2y - b) = 2, \text{ where } b \text{ is a positive constant}$$

Find y in terms of b, giving your answer in its simplest form.

(4)

Leave blank

Question 49 continued

(Total 7 marks)

Leave blank

50. (i) Given that

$$\log_3(3b + 1) - \log_3(a - 2) = -1, \qquad a > 2$$

express b in terms of a.

(3)

(ii) Solve the equation

$$2^{2x+5} - 7(2^x) = 0$$

giving your answer to 2 decimal places.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total 7 marks)

Leave blank

51. (i) Use logarithms to solve the equation $8^{2x+1} = 24$, giving your answer to 3 decimal places.

(3)

(ii) Find the values of y such that

$$\log_2(11y - 3) - \log_2 3 - 2\log_2 y = 1, \qquad y > \frac{3}{11}$$

(6)

Leave blank

Question 51 continued

(Total 9 marks)

EXPERT TUITION

Leave blank

52. (i) Solve

$$5^y = 8$$

giving your answer to 3 significant figures.

(2)

(ii) Use algebra to find the values of x for which

$$\log_2(x+15) - 4 = \frac{1}{2}\log_2 x$$

(6)

(Total 8 marks)

Leave blank

53. (i) Find the exact value of x for which

$$\log_2(2x) = \log_2(5x + 4) - 3$$

(4)

(ii) Given that

$$\log_a y + 3\log_a 2 = 5$$

express y in terms of a.
Give your answer in its simplest form.

(3)

(Total 7 marks)

Leave blank

54. Given that $\log_3 x = a$, find in terms of a,

(a) $\log_3 (9x)$

(2)

(b) $\log_3 \left(\frac{x^5}{81}\right)$

(3)

giving each answer in its simplest form.

(c) Solve, for x,

$$\log_3 (9x) + \log_3 \left(\frac{x^5}{81}\right) = 3$$

giving your answer to 4 significant figures.

(4)

Leave blank

Question 54 continued

(Total 9 marks)

Leave blank

55. Given that

$$2\log_2(x+15) - \log_2 x = 6$$

(a) Show that

$$x^2 - 34x + 225 = 0$$

(5)

(b) Hence, or otherwise, solve the equation

$$2\log_2(x+15) - \log_2 x = 6$$

(2)

(Total 7 marks)

56. Find the values of x such that

$$2\log_3 x - \log_3(x - 2) = 2$$

(5)

(Total 5 marks)

Leave blank

57. Given that $y = 3x^2$,

(a) show that $\log_3 y = 1 + 2\log_3 x$

(3)

(b) Hence, or otherwise, solve the equation

$$1 + 2\log_3 x = \log_3(28x - 9)$$

(3)

(Total 6 marks)

Leave blank

58. Find, giving your answer to 3 significant figures where appropriate, the value of x for which

(a) $5^x = 10$,

(2)

(b) $\log_3(x-2) = -1$.

(2)

(Total 4 marks)

Leave blank

59. (a) Given that

$$2\log_3(x-5)-\log_3(2x-13)=1,$$

show that $x^2-16x+64=0$.

(5)

(b) Hence, or otherwise, solve $2\log_3(x-5)-\log_3(2x-13)=1$.

(2)

(Total 7 marks)

Leave blank

60. The adult population of a town is 25 000 at the end of Year 1.

A model predicts that the adult population of the town will increase by 3% each year, forming a geometric sequence.

(a) Show that the predicted adult population at the end of Year 2 is 25 750.

(1)

(b) Write down the common ratio of the geometric sequence.

(1)

The model predicts that Year N will be the first year in which the adult population of the town exceeds 40 000.

(c) Show that

$$(N-1)\log 1.03 > \log 1.6$$

(3)

(d) Find the value of N.

(2)

At the end of each year, each member of the adult population of the town will give £1 to a charity fund.

Assuming the population model,

(e) find the total amount that will be given to the charity fund for the 10 years from the end of Year 1 to the end of Year 10, giving your answer to the nearest £1000.

(3)

Leave blank

Question 60 continued

(Total 10 marks)

Leave blank

61. (a) Find the positive value of x such that

$$\log_x 64 = 2$$

(2)

(b) Solve for x

$$\log_2(11 - 6x) = 2\log_2(x - 1) + 3$$

(6)

(Total 8 marks)

Leave blank

62. (a) Find the value of y such that

$$\log_2 y = -3$$

(2)

(b) Find the values of x such that

$$\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$$

(5)

(Total 7 marks)

Leave blank

63. Given that $0 < x < 4$ and

$$\log_5(4-x) - 2\log_5 x = 1,$$

find the value of x.

(6)

(Total 6 marks)

Leave blank

64. (a) Find, to 3 significant figures, the value of x for which $5^x = 7$.

(2)

(b) Solve the equation $5^{2x} - 12(5^x) + 35 = 0$.

(4)

(Total 6 marks)

Leave blank

65. Given that a and b are positive constants, solve the simultaneous equations

$$a = 3b,$$

$$\log_3 a + \log_3 b = 2.$$

Give your answers as exact numbers.

(6)

(Total 6 marks)

Leave blank

66. The value of a car is modelled by the formula

$$V = 16\,000e^{-kt} + A, \qquad t \geqslant 0,\ t \in \mathbb{R}$$

where V is the value of the car in pounds, t is the age of the car in years, and k and A are positive constants.

Given that the value of the car is £17 500 when new and £13 500 two years later,

(a) find the value of A,

(1)

(b) show that $k = \ln\left(\frac{2}{\sqrt{3}}\right)$

(4)

(c) Find the age of the car, in years, when the value of the car is £6000

Give your answer to 2 decimal places.

(4)

Leave blank

Question 66 continued

Leave blank

Question 66 continued

Leave blank

Question 66 continued

(Total 9 marks)

Leave blank

67. Find the exact solutions, in their simplest form, to the equations

(a) $e^{3x-9} = 8$

(3)

(b) $\ln(2y + 5) = 2 + \ln(4 - y)$

(4)

Leave blank

Question 67 continued

(Total 7 marks)

Leave blank

68. The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

$$x = De^{-0.2t}$$

where x is the amount of the antibiotic in the bloodstream in milligrams, D is the dose given in milligrams and t is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

(a) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 15 mg is given 5 hours after the first dose has been given. Using the same model for the second dose,

(b) show that the **total** amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754 mg to 3 decimal places.

(2)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the total amount of the antibiotic in the bloodstream is 7.5 mg.

(c) Show that $T = a\ln\left(b + \frac{b}{e}\right)$, where a and b are integers to be determined.

(4)

Leave blank

Question 68 continued

(Total 8 marks)

Leave blank

69. Water is being heated in an electric kettle. The temperature, θ °C, of the water t seconds after the kettle is switched on, is modelled by the equation

$$\theta = 120 - 100e^{-\lambda t}, \qquad 0 \leqslant t \leqslant T$$

(a) State the value of θ when $t = 0$

(1)

Given that the temperature of the water in the kettle is 70 °C when $t = 40$,

(b) find the exact value of λ, giving your answer in the form $\dfrac{\ln a}{b}$, where a and b are integers.

(4)

When $t = T$, the temperature of the water reaches 100 °C and the kettle switches off.

(c) Calculate the value of T to the nearest whole number.

(2)

(Total 7 marks)

Leave blank

70.

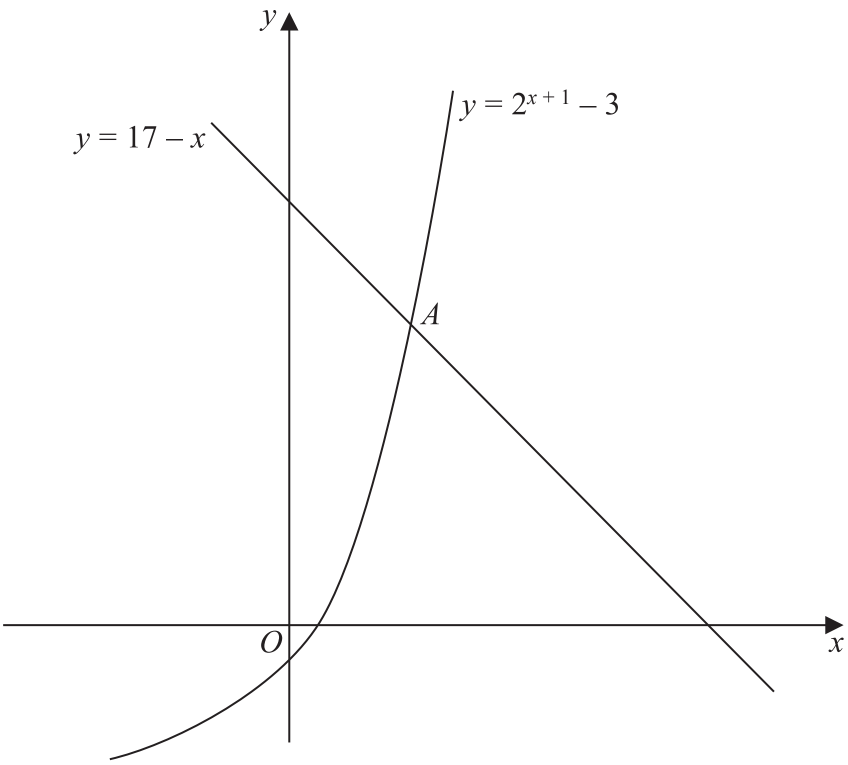


Figure 1

Figure 1 is a sketch showing part of the curve with equation $y = 2^{x+1} - 3$ and part of the line with equation $y = 17 - x$.

The curve and the line intersect at the point A.

(a) Show that the x coordinate of A satisfies the equation

$$x = \frac{\ln(20 - x)}{\ln 2} - 1$$

(3)

Leave blank

Question 70 continued

(Total 3 marks)

Leave blank

71. Find the exact solutions, in their simplest form, to the equations

(a) $2 \ln(2x + 1) - 10 = 0$

(2)

(b) $3^x e^{4x} = e^7$

(4)

(Total 6 marks)

Leave blank

72. A rare species of primrose is being studied. The population, P, of primroses at time t years after the study started is modelled by the equation

$$P = \frac{800e^{0.1t}}{1 + 3e^{0.1t}}, \qquad t \geqslant 0, \quad t \in \mathbb{R}$$

(a) Calculate the number of primroses at the start of the study.

(2)

(b) Find the exact value of t when $P = 250$, giving your answer in the form $a\ln(b)$ where a and b are integers.

(4)

(c) Find the exact value of $\frac{dP}{dt}$ when $t = 10$. Give your answer in its simplest form.

(4)

(d) Explain why the population of primroses can never be 270

(1)

Leave blank

Question 72 continued

(Total 11 marks)

Leave blank

73. Find algebraically the exact solutions to the equations

(a) $\ln(4 - 2x) + \ln(9 - 3x) = 2\ln(x + 1), \qquad -1 < x < 2$

(5)

(b) $2^x e^{3x+1} = 10$

Give your answer to (b) in the form $\dfrac{a + \ln b}{c + \ln d}$ where a, b, c and d are integers.

(5)

(Total 10 marks)

Leave blank

74.

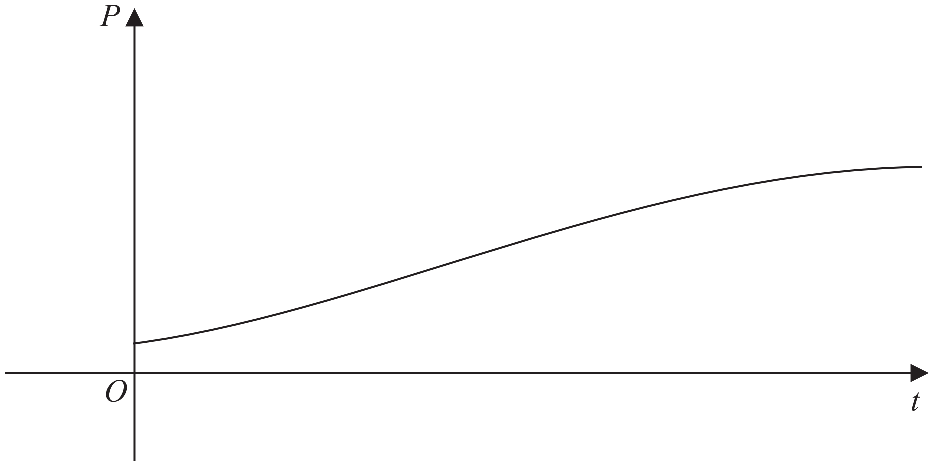


Figure 3

The population of a town is being studied. The population P, at time t years from the start of the study, is assumed to be

$$P = \frac{8000}{1+7e^{-kt}}, \qquad t \geqslant 0,$$

where k is a positive constant.

The graph of P against t is shown in Figure 3.

Use the given equation to

(a) find the population at the start of the study,

(2)

(b) find a value for the expected upper limit of the population.

(1)

Given also that the population reaches 2500 at 3 years from the start of the study,

(c) calculate the value of k to 3 decimal places.

(5)

Using this value for k,

(d) find the population at 10 years from the start of the study, giving your answer to 3 significant figures.

(2)

(e) Find, using $\frac{dP}{dt}$, the rate at which the population is growing at 10 years from the start of the study.

(3)

Leave blank

Question 74 continued

(Total 13 marks)

Leave blank

75. The value of Bob's car can be calculated from the formula

$$V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$$

where V is the value of the car in pounds (£) and t is the age in years.

(a) Find the value of the car when $t = 0$

(1)

(b) Calculate the exact value of t when $V = 9500$

(4)

(c) Find the rate at which the value of the car is decreasing at the instant when $t = 8$. Give your answer in pounds per year to the nearest pound.

(4)

Leave blank

Question 75 continued

(Total 9 marks)

Leave blank

76. The area, A mm^2, of a bacterial culture growing in milk, t hours after midday, is given by

$$A = 20\,e^{1.5t}, \quad t \geqslant 0$$

(a) Write down the area of the culture at midday.

(1)

(b) Find the time at which the area of the culture is twice its area at midday. Give your answer to the nearest minute.

(5)

(Total 6 marks)

Leave blank

77. The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = p\,e^{-kt}$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Write down the value of p.

(1)

(b) Show that $k = \frac{1}{4}\ln 3$.

(4)

(c) Find the value of t when $\frac{dm}{dt} = -0.6\ln 3$.

(6)

(Total 11 marks)

Leave blank

78. Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ°C, of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt},$$

where A and k are positive constants.

Given that the initial temperature of the tea was 90°C,

(a) find the value of A.

(2)

The tea takes 5 minutes to decrease in temperature from 90°C to 55°C.

(b) Show that $k = \frac{1}{5}\ln 2$.

(3)

(c) Find the rate at which the temperature of the tea is decreasing at the instant when $t = 10$. Give your answer, in °C per minute, to 3 decimal places.

(3)

Leave blank

Question 78 continued

(Total 8 marks)

Leave blank

79. (a) Simplify fully

$$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}$$

(3)

Given that

$$\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15), \quad x \neq -5,$$

(b) find x in terms of e.

(4)

(Total 7 marks)

Leave blank

80. (i) Find the exact solutions to the equations

(a) $\ln(3x - 7) = 5$ **(3)**

(b) $3^x e^{7x+2} = 15$ **(5)**

(Total 8 marks)

Leave blank

81. Rabbits were introduced onto an island. The number of rabbits, P, t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \qquad t \in \mathbb{R}, t \geqslant 0$$

(a) Write down the number of rabbits that were introduced to the island.

(1)

(b) Find the number of years it would take for the number of rabbits to first exceed 1000.

(2)

(c) Find $\frac{dP}{dt}$.

(2)

(d) Find P when $\frac{dP}{dt} = 50$.

(3)

(Total 8 marks)

Leave blank

82. The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y-coordinate of P is 8.

(a) Find, in terms of ln 2, the x-coordinate of P.

(2)

(Total 2 marks)

Leave blank

83. The radioactive decay of a substance is given by

$$R = 1000e^{-ct}, \qquad t \geqslant 0.$$

where R is the number of atoms at time t years and c is a positive constant.

(a) Find the number of atoms when the substance started to decay.

(1)

It takes 5730 years for half of the substance to decay.

(b) Find the value of c to 3 significant figures.

(4)

(c) Calculate the number of atoms that will be left when $t = 22\ 920$.

(2)

(d) In the space provided on page 13, sketch the graph of R against t .

(2)

(Total 9 marks)