Maths Questions By Topic:

Integration
Mark Scheme

A-Level Edexcel

0207 060 4494
www.expert-tuition.co.uk
online.expert-tuition.co.uk
enquiries@expert-tuition.co.uk
The Foundry, 77 Fulham Palace Road, W6 8JA

Table Of Contents

New Spec

Paper 1 .... Page 1

Paper 2 .... Page 36

Old Spec

Core 1 .... Page 70

Core 2 .... Page 105

Core 4 .... Page 133

Question	Scheme	Marks	AOs
1 (a)	$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x = \int_{2.1}^{6.3} \frac{2}{x}\,dx$	B1	1.2
		(1)	
(b)	$= [2\ln x]_{2.1}^{6.3} = 2\ln 6.3 - 2\ln 2.1$	M1	1.1b
	$= \ln 9$ CSO	A1	1.1b
		(2)	
			(3 marks)
Notes:			

Mark (a) and (b) as one

(a)

B1: States that $\int_{2.1}^{6.3} \frac{2}{x}\,dx$ or equivalent such as $2\int_{2.1}^{6.3} x^{-1}\,dx$ but must include the limits and the dx.

Condone $dx \leftrightarrow \delta x$ as it is very difficult to tell one from another sometimes

(b)

M1: Know that $\int \frac{1}{x}\,dx = \ln x$ and attempts to apply the limits (either way around)

Condone $\int \frac{2}{x}\,dx = p\ln x$ (including $p = 1$) or $\int \frac{2}{x}\,dx = p\ln qx$ as long as the limits are applied.

Also be aware that $\int \frac{2}{x}\,dx = \ln x^2$, $\int \frac{2}{x}\,dx = 2\ln|x| + c$ and $\int \frac{2}{x}\,dx = 2\ln cx$ o.e. are also correct

$[p\ln x]_{2.1}^{6.3} = p\ln 6.3 - p\ln 2.1$ is sufficient evidence to award this mark

A1: CSO ln 9 . Also answer = $\ln 3^2$ so $k = 9$ is fine. Condone $\ln|9|$

The method mark must have been awarded. Do not accept answers such as $\ln\frac{39.69}{4.41}$

Note that solutions appearing from "rounded" decimal work when taking lns should not score the final mark. It is a "show that" question

E.g. $[2\ln x]_{2.1}^{6.3} = 2\ln 6.3 - 2\ln 2.1 = 2.197 = \ln k \Rightarrow k = e^{2.197} = 8.998 = 9$

Question	Scheme	Marks	AOs
2	$\int x^3 \ln x \, dx = \frac{x^4}{4}\ln x - \int \frac{x^4}{4} \times \frac{1}{x} \, dx$	M1	1.1b
	$= \frac{x^4}{4}\ln x - \frac{x^4}{16}(+c)$	M1 A1	1.1b 1.1b
	$\int_1^{e^2} x^3 \ln x \, dx = \left[\frac{x^4}{4}\ln x - \frac{x^4}{16}\right]_1^{e^2} = \left(\frac{e^8}{4}\ln e^2 - \frac{e^8}{16}\right) - \left(-\frac{1^4}{16}\right)$	M1	2.1
	$= \frac{7}{16}e^8 + \frac{1}{16}$	A1	1.1b
		(5)	
(5 marks)			
Notes:			

M1: Integrates by parts the right way round.

Look for $kx^4 \ln x - \int kx^4 \times \frac{1}{x} \, dx$ o.e. with $k > 0$. Condone a missing dx

M1: Uses a correct method to integrate an expression of the form $\int kx^4 \times \frac{1}{x} \, dx \to c\,x^4$

A1: $\int x^3 \ln x \, dx = \frac{x^4}{4}\ln x - \frac{x^4}{16}$ $(+\,c)$ which may be left unsimplified

M1: Attempts to substitute 1 and e^2 into an expression of the form $\pm px^4 \ln x \pm qx^4$, subtracts and uses $\ln e^2 = 2$ (which may be implied).

A1: $\frac{7}{16}e^8 + \frac{1}{16}$ o.e. Allow $0.4375e^8 + 0.0625$ or uncancelled fractions. NOT ISW: $7e^8 + 1$ is A0

....

You may see attempts where substitution has been attempted.

E.g. $u = \ln x \Rightarrow x = e^u$ and $\frac{dx}{du} = e^u$

M1: Attempts to integrate the correct way around condoning slips on the coefficients

$$\int x^3 \ln x \, dx = \int e^{4u} u \, du = \frac{e^{4u}}{4}u - \int \frac{e^{4u}}{4} \, du$$

M1 A1: $\int x^3 \ln x \, dx = \frac{e^{4u}}{4}u - \frac{e^{4u}}{16}(+c)$

M1 A1: Substitutes 0 and 2 into an expression of the form $\pm pue^{4u} \pm qe^{4u}$ and subtracts

....

It is possible to use integration by parts "the other way around"

To do this, candidates need to know or use $\int \ln x \, dx = x\ln x - x$

FYI $I = \int x^3 \ln x \, dx = x^3(x\ln x - x) - \int (x\ln x - x) \times 3x^2 \, dx = x^3(x\ln x - x) - 3I + \frac{3}{4}x^4$

Hence $4I = x^4 \ln x - \frac{1}{4}x^4 \Rightarrow I = \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4$

Score M1 for a full attempt at line 1 (condoning bracketing and coefficient slips) followed by M 1 for line 2 where terms in I o.e. to form the answer.

Question	Scheme	Marks	AOs
3 (a)	Attempts $y.\dfrac{dx}{dt}=(2\sin 2t+3\sin t)\times 16\sin t\cos t$ and uses $\sin 2t = 2\sin t\cos t$	M1	2.1
	Correct expanded integrand. **Usually** for one of $(R)=\int \underline{\underline{48\sin^2 t\cos t+16\sin^2 2t}}\ dt$ $(R)=\int \underline{\underline{48\sin^2 t\cos t+64\sin^2 t\cos^2 t}}\ dt$ $(R)=\int \underline{\underline{24\sin 2t\sin t+16\sin^2 2t}}\ dt$	A1	1.1b
	Attempts to use $\cos 4t = 1-2\sin^2 2t = (1-8\sin^2 t\cos^2 t)$	M1	1.1b
	$R=\int_0^a 8-8\cos 4t+48\sin^2 t\cos t\ dt$ *	A1*	2.1
	Deduces $a=\dfrac{\pi}{4}$	B1	2.2a
		(5)	
(b)	$\int 8-8\cos 4t+48\sin^2 t\cos t\ dt = 8t-2\sin 4t+16\sin^3 t$	M1 A1	2.1 1.1b
	$\left[8t-2\sin 4t+16\sin^3 t\right]_0^{\frac{\pi}{4}}=2\pi+4\sqrt{2}$	M1 A1	2.1 1.1b
		(4)	
			(9 marks)
Notes:			

(a) Condone work in another variable, say $\theta \leftrightarrow t$ if used consistently for the first 3 marks

M1: For the key step in attempting $y.\dfrac{dx}{dt}=(2\sin 2t+3\sin t)\times 16\sin t\cos t$ with an attempt to use $\sin 2t = 2\sin t\cos t$ Condone slips in finding $\dfrac{dx}{dt}$ but it must be of the form $k\sin t\cos t$

E.g. I $\quad y.\dfrac{dx}{dt}=(2\sin 2t+3\sin t)\times k\sin t\cos t=(4\sin t\cos t+3\sin t)\times k\sin t\cos t$

E.g. II $\quad y.\dfrac{dx}{dt}=(2\sin 2t+3\sin t)\times k\sin t\cos t=(2\sin 2t+3\sin t)\times\dfrac{k}{2}\sin 2t$

A1: A correct (expanded) integrand in t . Don't be concerned by the absence of $\int$ or dt or limits

$(R)=\int \underline{\underline{48\sin^2 t\cos t+16\sin^2 2t}}\ dt$ or $(R)=\int \underline{\underline{48\sin^2 t\cos t+64\sin^2 t\cos^2 t}}\ dt$

but watch for other correct versions such as $(R)=\int \underline{\underline{24\sin 2t\sin t+16\sin^2 2t}}\ dt$

M1: Attempts to use $\cos 4t = \pm 1 \pm 2\sin^2 2t$ to get the integrand in the correct form.

If they have the form $P\sin^2 2t$ it is acceptable to write $P\sin^2 2t = \frac{P}{2}(\pm 1 \pm \cos 4t)$

If they have the form $Q\sin^2 t\cos^2 t$ sight and use of $\sin 2t$ and/or $\cos 2t$ will usually be seen first.

There are many ways to do this, below is such an example

$$Q\sin^2 t\cos^2 t = Q\left(\frac{1-\cos 2t}{2}\right)\left(\frac{1+\cos 2t}{2}\right) = Q\left(\frac{1-\cos^2 2t}{4}\right) = Q\left(\frac{1}{4} - \frac{\cos^2 2t}{4}\right) = Q\left(\frac{1}{4} - \frac{1+\cos 4t}{8}\right)$$

Allow candidates to start with the given answer and work backwards using the same rules. So expect to see $\cos 4t = \pm 1 \pm 2\times\sin^2 2t$ or $\cos 4t = \pm 2\times\cos^2 2t \pm 1$ before double angle identities for $\sin 2t$ or $\cos 2t$ are used.

A1*: Proceeds to the given answer with correct working. The order of the terms is not important.
Ignore limits for this mark. The integration sign and the dt must be seen on their final answer.
If they have worked backwards there must be a concluding statement to the effect that they know that they have shown it. The integration sign and the dt must also be seen

E.g. Reaches $\int 48\sin^2 t\cos t + 64\sin^2 t\cos^2 t \, dt$

Answer is $\int 8 - 8\cos 4t + 48\sin^2 t\cos t \, dt$

$$= \int 8 - 8\left(1 - 2\sin^2 2t\right) + 48\sin^2 t\cos t \, dt$$

$$= \int 16\sin^2 2t + 48\sin^2 t\cos t \, dt$$

$$= \int 64\sin^2 t\cos^2 t + 48\sin^2 t\cos t \, dt$$

which is the same, ✓

B1: Deduces $a = \frac{\pi}{4}$. It may be awarded from the upper limit and can be awarded from (b)

(b)

M1: For the key process in using a correct approach to integrating the trigonometric terms.
May be done separately.
There may be lots of intermediate steps (e.g. let $u = \sin t$) .
There are other more complicated methods so look carefully at what they are doing.

$\int 8 - 8\cos 4t + 48\sin^2 t\cos t \, dt = ... \pm P\sin 4t \pm Q\sin^3 t$ where P and Q are constants

A1: $\int 8 - 8\cos 4t + 48\sin^2 t\cos t \, dt = 8t - 2\sin 4t + 16\sin^3 t(+c)$

If they have written $16\sin^3 t$ as $16\sin t^3$ only award if further work implies a correct answer.
Similarly, 8t may be written as 8x. Award if further work implies 8t, e.g. substituting in their limits.
Do not penalise this sort of slip at all, these are intermediate answers.

M1: Uses the limits their a and 0 where $a = \frac{\pi}{6}, \frac{\pi}{4}$ or $\frac{\pi}{3}$ in an expression of the form $kt \pm P\sin 4t \pm Q\sin^3 t$ leading to an exact answer. Ignore evidence at lower limit as terms are 0

A1: CSO $2\pi + 4\sqrt{2}$ or exact **simplified** equivalent such as $2\pi + \frac{8}{\sqrt{2}}$ or $2\pi + \sqrt{32}$.

Be aware that $\int_0^{\frac{\pi}{4}} 8 - 8\cos 4t + 48\sin^2 t\cos t \, dt = 8t \underset{=}{+\lambda} \sin 4t + 16\sin^3 t(+c)$ would lead to the correct answer but would only score M1 A0 M1 A0

Question	Scheme	Marks	AOs
4	$x^n \to x^{n+1}$	M1	1.1b
	$\int\left(8x^3 - \dfrac{3}{2\sqrt{x}} + 5\right)dx = \dfrac{8x^4}{4} ... + 5x$	A1	1.1b
	$= ... - 2\times\dfrac{3}{2}x^{\frac{1}{2}} + ...$	A1	1.1b
	$= 2x^4 - 3x^{\frac{1}{2}} + 5x + c$	A1	1.1b
		(4)	
			(4 marks)

Notes

M1: For raising any correct power of x by 1 including $5 \to 5x$ (not for + c) Also allow eg $x^3 \to x^{3+1}$

A1: For 2 correct non-fractional power terms (allow unsimplified coefficients) and may appear on separate lines. The indices must be processed. The + c does not count as a correct term here. Condone the 1 appearing as a power or denominator such as $\dfrac{5x^1}{1}$ for this mark.

A1: For the correct fractional power term (allow unsimplified) Allow eg $+-2\times 1.5\sqrt{x^1}$.

Also allow fractions within fractions for this mark such as $\dfrac{\frac{3}{2}}{\frac{1}{2}}x^{\frac{1}{2}}$

A1: All correct and simplified and on one line including + c. Allow $-3\sqrt{x}$ or $-\sqrt{9x}$ for $-3x^{\frac{1}{2}}$. Do not accept $+-3x^{\frac{1}{2}}$ for this mark.

Award once a correct expression is seen and isw but if there is any additional/incorrect notation and no correct expression has been seen on its own, withhold the final mark.

Eg. $\int 2x^4 - 3x^{\frac{1}{2}} + 5x + c \text{ d}x$ or $2x^4 - 3x^{\frac{1}{2}} + 5x + c = 0$ with no correct expression seen earlier are both A0.

EXPERT TUITION

Question	Scheme	Marks	AOs
5(a)	$y=\frac{1}{3}x^2-2\sqrt{x}+3\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{3}x-x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$x=4\Rightarrow y=\frac{13}{3}$	B1	1.1b
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=4}=\frac{2}{3}\times4-4^{-\frac{1}{2}}\left(=\frac{13}{6}\right)\quad\therefore y-\frac{13}{3}=\frac{13}{6}(x-4)$	M1	2.1
	$13x-6y-26=0$*	A1*	1.1b
		(5)	
(b)	$\int\left(\frac{x^2}{3}-2\sqrt{x}+3\right)\mathrm{d}x=\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x(+c)$	M1 A1	1.1b 1.1b
	$y=0\Rightarrow x=2$	B1	2.2a
	Area of R is $\left[\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x\right]_0^4-\frac{1}{2}\times(4-"2")\times"\frac{13}{3}"=\frac{76}{9}-\frac{13}{3}$	M1	3.1a
	$=\frac{37}{9}$	A1	1.1b
		(5)	
		(10 marks)	

Notes

(a) **Calculators: If no algebraic differentiation seen then maximum in a) is M0A0B1M1A0***

M1: $x^n\to x^{n-1}$ seen at least once $...x^2\to...x^1,\ ...x^{\frac{1}{2}}\to...x^{-\frac{1}{2}},\ 3\to0$.

Also accept on sight of eg $...x^{\frac{1}{2}}\to...x^{\frac{1}{2}-1}$

A1: $\frac{2}{3}x-x^{-\frac{1}{2}}$ or any unsimplified equivalent (indices must be processed) accept the use of $0.\dot{6}x$ but not rounded or ambiguous values eg $0.6x$ or eg $0.66\ldots x$

B1: Correct y coordinate of P. May be seen embedded in an attempt of the equation of l

M1: Fully correct strategy for an equation for l. Look for $y-"\frac{13}{3}"="\frac{13}{6}"(x-4)$ where their $\frac{13}{6}$ is from differentiating the equation of the curve and substituting in $x=4$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and the y coordinate is from substituting $x=4$ into the given equation.
If they use $y=mx+c$ they must proceed as far as $c=...$ to score this mark.
Do not allow this mark if they use a perpendicular gradient.

A1*: Obtains the printed answer with no errors.

(b) **Calculators: If no algebraic integration seen then maximum in b) is M0A0B1M1A0**

M1: $x^n\to x^{n+1}$ seen at least once. Eg $...x^2\to...x^3,\ ...x^{\frac{1}{2}}\to...x^{\frac{3}{2}},\ 3\to3x^1$. Allow eg $...x^2\to...x^{2+1}$ The $+c$ is not a valid term for this mark.

A1: $\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x$ or any unsimplified equivalent (indices must be processed) accept the use of exact decimals for $\frac{1}{9}$ $(0.\dot{1})$ and $-\frac{4}{3}$ $(-1.\dot{3})$ but not rounded or ambiguous values.

B1: Deduces the correct value for x for the intersection of l with the x-axis. May be seen indicated on Figure 2.

M1: Fully correct strategy for the area. This needs to include

- a correct attempt at the area of the triangle using their values (**could use integration**)
- a correct attempt at the area under the curve using 0 and 4 in their integrated expression
- the two values subtracted.

Be aware of those who mix up using the y-coordinate of P and the gradient at P which is M0. The values embedded in an expression is sufficient to score this mark.

A1: $\frac{37}{9}$ or exact equivalent eg $4\frac{1}{9}$ or $4.\dot{1}$ but not 4.111…. isw after a correct answer

---

Be aware of other strategies to find the area *R*

eg Finding the area under the curve between 0 and 2 and then the difference between the curve and the straight line between 2 and 4:

$$\int_0^2 \frac{x^2}{3}-2\sqrt{x}+3\ \mathrm{d}x+\int_2^4 \frac{x^2}{3}-2\sqrt{x}-\frac{13}{6}x+\frac{22}{3}\ \mathrm{d}x$$

M1 $x^n \to x^{n+1}$ seen at least once on either integral (or on the equation of the line $y=\frac{1}{3}x+3$)

A1 for correct integration of **either** integral $\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x$ or $\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}-\frac{13}{12}x^2+\frac{22}{3}x$ (may be unsimplified/uncollected terms but the indices must be processed with/without the $+C$)

B1 Correct value for x can be seen from the top of the first integral (or bottom value of the second integral)

M1 Correct strategy for the area eg.

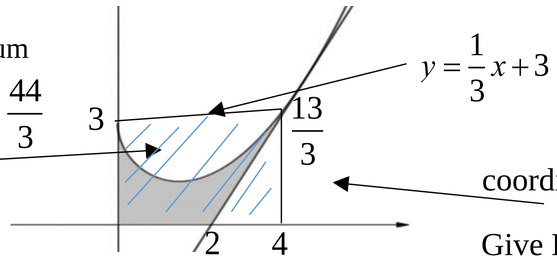
$$\left[\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x\right]_0^2+\left[\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}-\frac{13}{12}x^2+\frac{22}{3}x\right]_2^4=\frac{62}{9}-\frac{4}{3}(2)^{\frac{3}{2}}+\frac{76}{9}-\frac{101}{9}+\frac{4}{3}(2)^{\frac{3}{2}}$$

A1: $\frac{37}{9}$ or exact equivalent eg $4\frac{1}{9}$ or $4.\dot{1}$ but not 4.1 or 4.111….

You could also see use of the area of a trapezium and/or the use of the line $y=\frac{1}{3}x+3$ to find other areas which could be combined or used as part of the strategy to find *R*. Ignore areas which are not used. The marks should still be able to be applied as per the scheme

Area of large trapezium

$=\frac{1}{2}\times\left(3+\frac{13}{3}\right)\times 4=\frac{44}{3}$



Area of triangle and the y coordinate of point P are both $\frac{13}{3}$

Give BOD if in doubt of intention

Area of trapezium – (Area between $y=\frac{1}{3}x+3$ and curve C + area of triangle)

$$=\frac{44}{3}-\frac{56}{9}-\frac{13}{3}=\frac{37}{9}$$

Question	Scheme	Marks	AOs
6(a)	$h = 0.5$	B1	1.1b
	$A \approx \frac{1}{2}\times\frac{1}{2}\{0.4805+1.9218+2(0.8396+1.2069+1.5694)\}$	M1	1.1b
	$= 2.41$	A1	1.1b
		(3)	
(b)	$\int(\ln x)^2\ \mathrm{d}x = x(\ln x)^2 - \int x \times \frac{2\ln x}{x}\ \mathrm{d}x$	M1 A1	3.1a 1.1b
	$= x(\ln x)^2 - 2\int \ln x\ \mathrm{d}x = x(\ln x)^2 - 2\left(x\ln x - \int \mathrm{d}x\right)$ $= x(\ln x)^2 - 2\int \ln x\ \mathrm{d}x = x(\ln x)^2 - 2x\ln x + 2x$	dM1	2.1
	$\int_2^4 (\ln x)^2\ \mathrm{d}x = \left[x(\ln x)^2 - 2x\ln x + 2x\right]_2^4$ $= 4(\ln 4)^2 - 2\times 4\ln 4 + 2\times 4 - \left(2(\ln 2)^2 - 2\times 2\ln 2 + 2\times 2\right)$ $= 4(2\ln 2)^2 - 16\ln 2 + 8 - 2(\ln 2)^2 + 4\ln 2 - 4$	ddM1	2.1
	$= 14(\ln 2)^2 - 12\ln 2 + 4$	A1	1.1b
		(5)	
		(8 marks)	

Notes

(a)

B1: Correct strip width. May be implied by $\frac{1}{2}\times\frac{1}{2}\{....\}$ or $\frac{1}{4}\times\{....\}$

M1: Correct application of the trapezium rule.

Look for $\frac{1}{2}\times"h"\{0.4805+1.9218+2(0.8396+1.2069+1.5694)\}$ condoning slips in the digits.

The bracketing must be correct but it is implied by awrt 2.41

A1: 2.41 only. This is not awrt

(b)

M1: Attempts parts the correct way round to achieve $\alpha x(\ln x)^2 - \beta\int \ln x\ \mathrm{d}x$ o.e.

May be unsimplified (see scheme). Watch for candidates who know or learn $\int \ln x\ \mathrm{d}x = x\ln x - x$

who may write $\int(\ln x)^2\ \mathrm{d}x = \int(\ln x)(\ln x)\ \mathrm{d}x = \ln x(x\ln x - x) - \int \frac{x\ln x - x}{x}\ \mathrm{d}x$

A1: Correct expression which may be unsimplified

dM1: Attempts parts again to (only condone coefficient errors) to achieve $\alpha x(\ln x)^2 - \beta x\ln x \pm \gamma x$ o.e.

ddM1: Applies the limits 4 and 2 to an expression of the form $\pm\alpha x(\ln x)^2 \pm \beta x\ln x \pm \gamma x$, subtracts and applies $\ln 4 = 2\ln 2$ at least once. Both M's must have been awarded

A1: Correct answer

....

It is possible to do $\int(\ln x)^2\ \mathrm{d}x$ via a substitution $u = \ln x$ but it is very similar.

M1 A1, dM1: $\int u^2 \mathrm{e}^u\ \mathrm{d}u = u^2 \mathrm{e}^u - \int 2u\,\mathrm{e}^u\ \mathrm{d}u, = u^2 \mathrm{e}^u - 2u\mathrm{e}^u \pm 2\mathrm{e}^u$

ddM1: Applies appropriate limits and uses $\ln 4 = 2\ln 2$ at least once to an expression of the form $u^2 \mathrm{e}^u - \beta u\mathrm{e}^u \pm \gamma\mathrm{e}^u$ Both M's must have been awarded

....

Question	Scheme	Marks	AOs
7	$\int \frac{3x^4-4}{2x^3}\,dx = \int \frac{3}{2}x - 2x^{-3}\,dx$	M1 A1	1.1b 1.1b
	$= \frac{3}{2}\times\frac{x^2}{2} - 2\times\frac{x^{-2}}{-2} \quad (+c)$	dM1	3.1a
	$= \frac{3}{4}x^2 + \frac{1}{x^2} + c \quad$ o.e	A1	1.1b
		(4)	
		(4 marks)	

Notes:

(i)

M1: Attempts to divide to form a sum of terms. Implied by two terms with one correct index.

$\int \frac{3x^4}{2x^3} - \frac{4}{2x^3}\,dx$ scores this mark.

A1: $\int \frac{3}{2}x - 2x^{-3}\,dx$ o.e such as $\frac{1}{2}\int\left(3x - 4x^{-3}\right)\,dx$. The indices must have been processed on both terms. Condone spurious notation or lack of the integral sign for this mark.

dM1: For the full strategy to integrate the expression. It requires two terms with one correct index. Look for $= ax^p + bx^q$ where $p = 2$ or $q = -2$

A1: Correct answer $\frac{3}{4}x^2 + \frac{1}{x^2} + c$ o.e. such as $\frac{3}{4}x^2 + x^{-2} + c$

Question	Scheme	Marks	AOs
8	$\displaystyle\int_k^9 \frac{6}{\sqrt{x}}\,\mathrm{d}x = \left[ax^{\frac{1}{2}}\right]_k^9 = 20 \Rightarrow 36-12\sqrt{k}=20$	M1 A1	1.1b 1.1b
	Correct method of solving Eg. $36-12\sqrt{k}=20 \Rightarrow k=$	dM1	3.1a
	$\Rightarrow k=\dfrac{16}{9}$ oe	A1	1.1b
		(4)	
			(4 marks)

Notes:

M1: For setting $\left[ax^{\frac{1}{2}}\right]_k^9 = 20$

A1: A correct equation involving *p* Eg. $36-12\sqrt{k}=20$

dM1: For a whole strategy to find *k*. In the scheme it is awarded for setting $\left[ax^{\frac{1}{2}}\right]_k^9 = 20$, using both limits and proceeding using correct index work to find *k*. It cannot be scored if $k^{\frac{1}{2}}<0$

A1: $k=\dfrac{16}{9}$

Question	Scheme	Marks	AOs
9(a)	$\text{f}(x)=-3x^2+12x+8=-3(x\pm 2)^2+\ldots$	M1	1.1b
	$=-3(x-2)^2+\ldots$	A1	1.1b
	$=-3(x-2)^2+20$	A1	1.1b
		(3)	
(b)	Coordinates of $M=(2,20)$	B1ft B1ft	1.1b 2.2a
		(2)	
(c)	$\int -3x^2+12x+8\,\text{d}x=-x^3+6x^2+8x$	M1 A1	1.1b 1.1b
	Method to find $R=$ their $2\times 20-\int_0^2\left(-3x^2+12x+8\right)\text{d}x$	M1	3.1a
	$R=40-\left[-2^3+24+16\right]$	dM1	1.1b
	$=8$	A1	1.1b
		(5)	
			(10 marks)
Alt(c)	$\int 3x^2-12x+12\,\text{d}x=x^3-6x^2+12x$	M1 A1	1.1b 1.1b
	Method to find $R=\int_0^2 3x^2-12x+12\,\text{d}x$	M1	3.1a
	$R=2^3-6\times 2^2+12\times 2$	dM1	1.1b
	$=8$	A1	1.1b

Notes:

(a)

M1: Attempts to take out a common factor and complete the square. Award for $-3(x\pm 2)^2+\ldots$

Alternatively attempt to compare $-3x^2+12x+8$ to $ax^2+2abx+ab^2+c$ to find values of a and b

A1: Proceeds to a form $-3(x-2)^2+\ldots$ or via comparison finds $a=-3, b=-2$

A1: $-3(x-2)^2+20$

(b)

B1ft: One correct coordinate

B1ft: Correct coordinates. Allow as $x = ..., y = ...$
Follow through on their $\left(-b, c\right)$

(c)

M1: Attempts to integrate. Award for any correct index

A1: $\int -3x^2 + 12x + 8 \, dx = -x^3 + 6x^2 + 8x \ (+c)$ (which may be unsimplified)

M1: Method to find area of R. Look for their $2 \times "20" - \int_0^{'2"} f(x) \, dx$

dM1: Correct application of limits on their integrated function. Their 2 must be used

A1: Shows that area of $R = 8$

Question	Scheme	Marks	AOs
10 (a)	$x=u^2+1 \Rightarrow \mathrm{d}x=2u\mathrm{d}u$ oe	B1	1.1b
	Full substitution $\displaystyle\int \frac{3\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)}=\int \frac{3\times 2u\ \mathrm{d}u}{\left(u^2+1-1\right)(3+2u)}$	M1	1.1b
	Finds correct limits e.g. $p=2, q=3$	B1	1.1b
	$\displaystyle=\int \frac{3\times 2\cancel{u}\ \mathrm{d}u}{u^{\cancel{2}}(3+2u)}=\int \frac{6\ \mathrm{d}u}{u(3+2u)}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{6}{u(3+2u)}=\dfrac{A}{u}+\dfrac{B}{3+2u} \Rightarrow A=\ldots, B=\ldots$	M1	1.1b
	Correct PF. $\dfrac{6}{u(3+2u)}=\dfrac{2}{u}-\dfrac{4}{3+2u}$	A1	1.1b
	$\displaystyle\int \frac{6\ \mathrm{d}u}{u(3+2u)}=2\ln u-2\ln(3+2u) \qquad (+c)$	dM1 A1ft	3.1a 1.1b
	Uses limits $u="3", u="2"$ with some correct ln work leading to $k\ln b$ E.g. $(2\ln 3-2\ln 9)-(2\ln 2-2\ln 7)=2\ln\dfrac{7}{6}$	M1	1.1b
	$\ln\dfrac{49}{36}$	A1	2.1
		(6)	
			(10 marks)
Notes: Mark (a) and (b) together as one complete question			

(a)

B1: $\mathrm{d}x=2u\mathrm{d}u$ or exact equivalent. E.g. $\dfrac{\mathrm{d}x}{\mathrm{d}u}=2u$, $\dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{1}{2}(x-1)^{-\frac{1}{2}}$

M1: Attempts a full substitution of $x=u^2+1$, including $\mathrm{d}x \to \ldots u\mathrm{d}u$ to form an integrand in terms of u. Condone slips but there should be an attempt to use the correct substitution on the denominator.

B1: Finds correct limits either states $p=2, q=3$ or sight of embedded values as limits to the integral

A1*: Clear reasoning including one fully correct intermediate line, including the integral signs, leading to the given expression ignoring limits. So B1, M1, B0, A1 is possible if the limits are incorrect, omitted or left as 5 and 10.

(b)

M1: Uses correct form of PF leading to values of A and B.

A1: Correct PF $\dfrac{6}{u(3+2u)}=\dfrac{2}{u}-\dfrac{4}{3+2u}$ (Not scored for just the correct values of A and B)

dM1: This is an overall problem solving mark. It is for using the correct PF form and integrating using lns. Look for $P\ln u+Q\ln(3+2u)$

A1ft: Correct integration for their $\dfrac{A}{u}+\dfrac{B}{3+2u} \to A\ln u+\dfrac{B}{2}\ln(3+2u)$ with or without modulus signs

M1: Uses their 2 and 3 as limits, with at least one correct application of the addition law or subtraction law leading to the form $k\ln b$ or $\ln a$. PF's must have been attempted. Condone bracketing slips. Alternatively changing the u's back to x's and use limits of 5 and 10.

A1: Proceeds to $\ln\dfrac{49}{36}$. Answers without working please send to review.

Question	Scheme	Marks	AOs
11 (a)	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(\frac{5}{2\sqrt{x}}+3\right) dx = 5\sqrt{x}+3x$	A1	1.1b
	$\left[5\sqrt{x}+3x\right]_1^k = 4 \Rightarrow 5\sqrt{k}+3k-8=4$	dM1	1.1b
	$3k+5\sqrt{k}-12=0$ *	A1*	2.1
		(4)	
(b)	$3k+5\sqrt{k}-12=0 \Rightarrow \left(3\sqrt{k}-4\right)\left(\sqrt{k}+3\right)=0$	M1	3.1a
	$\sqrt{k}=\frac{4}{3},(-3)$	A1	1.1b
	$\sqrt{k}=\ldots \Rightarrow k=\ldots$ oe	dM1	1.1b
	$k=\frac{16}{9},\cancel{9}$	A1	2.3
		(4)	
			(8 marks)

Notes

(a)

M1: For $x^n \to x^{n+1}$ on correct indices. This can be implied by the sight of either $x^{\frac{1}{2}}$ or x

A1: $5\sqrt{x}+3x$ or $5x^{\frac{1}{2}}+3x$ but may be unsimplified. Also allow with + c and condone any spurious notation.

dM1: Uses both limits, subtracts, and sets equal to 4. They cannot proceed to the given answer without a line of working showing this.

A1*: Fully correct proof with no errors (bracketing or otherwise) leading to given answer.

(b)

M1: For a correct method of solving. This could be as the scheme, treating as a quadratic in $\sqrt{k}$ and using allowable method to solve including factorisation, formula etc.

Allow values for $\sqrt{k}$ to be just written down, e.g. allow $\sqrt{k}=\pm\frac{4}{3},(\pm 3)$

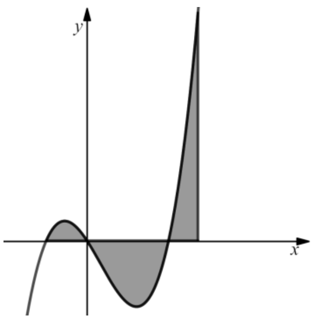
Alternatively score for rearranging to $5\sqrt{k}=12-3k$ and then squaring to get $\ldots k=(12-3k)^2$

A1: $\sqrt{k}=\frac{4}{3},(-3)$

Or in the alt method it is for reaching a correct 3TQ equation $9k^2-97k+144=0$

dM1: For solving to find at least one value for k. It is dependent upon the first M mark.
In the main method it is scored for squaring their value(s) of $\sqrt{k}$
In the alternative scored for solving their 3TQ by an appropriate method

A1: Full and rigorous method leading to $k=\frac{16}{9}$ only. The 9 must be rejected.

Question	Scheme	Marks	AOs
12 (a)	$y = x(x+2)(x-4) = x^3 - 2x^2 - 8x$	B1	1.1b
	$\int x^3 - 2x^2 - 8x \, dx \rightarrow \frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2$	M1	1.1b
	Attempts area using the correct strategy $\int_{-2}^{0} y \, dx$	dM1	2.2a
	$\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0} = (0) - \left(4 - \frac{-16}{3} - 16\right) \quad = \frac{20}{3}$ *	A1*	2.1
		(4)	
(b)	For setting 'their' $\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = \pm\frac{20}{3}$	M1	1.1b
	For correctly deducing that $3b^4 - 8b^3 - 48b^2 + 80 = 0$	A1	2.2a
	Attempts to factorise $3b^4 - 8b^3 - 48b^2 \pm 80 = (b+2)(b+2)(3b^2 ... b ... 20)$	M1	1.1b
	Achieves $(b+2)^2(3b^2 - 20b + 20) = 0$ with no errors	A1*	2.1
		(4)	
(c)	 States that between $x = -2$ and $x = 5.442$ the area above the x-axis = area below the x-axis	B1 B1	1.1b 2.4
		(2)	
			(10 marks)

(a)

B1: Expands $x(x+2)(x-4)$ to $x^3 - 2x^2 - 8x$ (They may be in a different order)

M1: Correct attempt at integration of their cubic seen in at least two terms.

Look for an expansion to a cubic and $x^n \rightarrow x^{n+1}$ seen at least twice

dM1: For a correct strategy to find the area of R_1

It is dependent upon the previous M and requires a substitution of -2 into $\pm$ their integrated function.

The limit of 0 may not be seen. Condone $\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2} = \frac{20}{3}$ oe for this mark

A1*: For a rigorous argument leading to area of $R_1 = \frac{20}{3}$ For this to be awarded the integration must be correct and the limits must be the correct way around and embedded or calculated values must be seen.

Eg. Look for $-\left(4+\frac{16}{3}-16\right)$ or $-\left(\frac{1}{4}(-2)^4-\frac{2}{3}(-2)^3-4(-2)^2\right)$ oe before you see the $\frac{20}{3}$

Note: It is possible to do this integration by parts.

(b)

M1: For setting their $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=\pm\frac{20}{3}$ or $\left[\frac{1}{4}x^4-\frac{2}{3}x^3-4x^2\right]_{-2}^{b}=0$

A1: Deduces that $3b^4-8b^3-48b^2+80=0$. Terms may be in a different order but expect integer coefficients.

It must have followed $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=-\frac{20}{3}$ oe.

Do not award this mark for $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2+\frac{20}{3}=0$ unless they attempt the second part of this question by expansion and then divide the resulting expanded expression by 12

M1: Attempts to factorise $3b^4-8b^3-48b^2\pm80=(b+2)(b+2)(3b^2...b...20)$ via repeated division or inspection. FYI $3b^4-8b^3-48b^2+80=(b+2)(3b^3-14b^2-20b+40)$ Allow an attempt via inspection

$3b^4-8b^3-48b^2\pm80=(b^2+4b+4)(3b^2...b...20)$ but do not allow candidates to just write out

$3b^4-8b^3-48b^2\pm80=(b+2)^2(3b^2-20b+20)$ which is really just copying out the given answer.

Alternatively attempts to expand $(b+2)^2(3b^2-20b+20)$ achieving terms of a quartic expression

A1*: Correctly reaches $(b+2)^2(3b^2-20b+20)=0$ with no errors and must have $=0$

In the alternative obtains both equations in the same form **and states that they are same**. Allow ✓ QED etc here.

(c) Please watch for candidates who answer this on Figure 2 which is fine

B1: Sketches the curve and a vertical line to the right of 4 ($x = 5.442$ may not be labelled.)

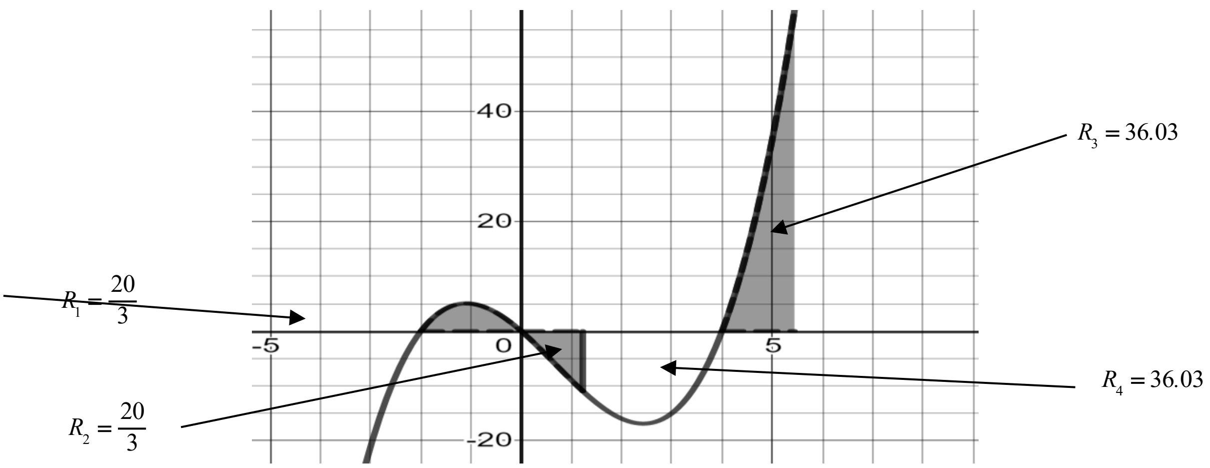
B1: Explains that (between $x=-2$ and $x=5.442$) the area above the x-axis = area below the x-axis with appropriate areas shaded or labelled**.**

Alternatively states that the area between 1.225 and 4 is the same as the area between 4 and 5.442

Another correct statement is that the net area between 0 and 5.442 is $-\frac{20}{3}$

Look carefully at what is written. There are many correct statements/ deductions.

Eg. " (area between 0 and 4) - (area between 4 and 5.442) = 20/3". Diagram below for your information.



Question	Scheme	Marks	AOs
13 (a)	(i) **Explains** $2x-q=0$ when $x=2$ oe Hence $q=4$ *	B1*	2.4
	(ii) Substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves	M1	1.1b
	$\frac{1}{2}=\frac{p-9}{(2)\times(6)}\Rightarrow p-9=6\Rightarrow p=15$*	A1*	2.1
		(3)	
(b)	Attempts to write $\frac{15-3x}{(2x-4)(x+3)}$ in PF's and integrates using lns between 3 and another value of x.	M1	3.1a
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to A and B	M1	1.1b
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$ or $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$ oe	A1	1.1b
	$\mathrm{I}=\int\frac{15-3x}{(2x-4)(x+3)}\mathrm{d}x=m\ln(2x-4)+n\ln(x+3)+(c)$	M1	1.1b
	$\mathrm{I}=\int\frac{15-3x}{(2x-4)(x+3)}\mathrm{d}x=0.9\ln(2x-4)-2.4\ln(x+3)$ oe	A1ft	1.1b
	Deduces that Area Either $\int_3^5\frac{15-3x}{(2x-4)(x+3)}\mathrm{d}x$ Or $[..............]_3^5$	B1	2.2a
	Uses correct ln work seen at least once for $\ln 6=\ln 2+\ln 3$ **or** $\ln 8=3\ln 2$ $[0.9\ln(6)-2.4\ln(8)]-[0.9\ln(2)-2.4\ln(6)]$ $=3.3\ln 6-7.2\ln 2-0.9\ln 2$	dM1	2.1
	$=3.3\ln 3-4.8\ln 2$	A1	1.1b
		(8)	
			(11marks)

(a)

B1*: Is able to link $2x-q=0$ and $x=2$ to explain why $q=4$

Eg "The asymptote $x=2$ is where $2x-q=0$ so $4-q=0 \Rightarrow q=4$ "

"The curve is not defined when $2\times 2-q=0 \Rightarrow q=4$"

There **must be some words** explaining why $q=4$ and in most cases, you should see a reference to either "the asymptote $x=2$", "the curve is not defined at $x=2$" , 'the denominator is 0 at $x=2$"

M1: Substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves

Alternatively substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{15-3x}{(2x-4)(x+3)}$ and shows $\frac{1}{2}=\frac{6}{(2)\times(6)}$ oe

A1*: Full proof showing all necessary steps $\frac{1}{2}=\frac{p-9}{(2)\times(6)} \Rightarrow p-9=6 \Rightarrow p=15$

In the alternative there would have to be some recognition that these are equal eg ✓ hence $p=15$

(b)

M1: Scored for an overall attempt at using PF's and integrating with lns seen with sight of limits 3 and another value of x.

M1: $\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to A and B

A1: $\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$, or for example $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$, $\frac{9}{(10x-20)}-\frac{12}{(5x+15)}$ oe

Must be written in PF form, not just for correct A and B

M1: Area $R=\int \frac{15-3x}{(2x-4)(x+3)}\,\mathrm{d}x = m\ln(2x-4)+n\ln(x+3)$

OR $\int \frac{15-3x}{(2x-4)(x+3)}\,\mathrm{d}x = m\ln(x-2)+n\ln(x+3)$

Note that $\int \frac{l}{(x-2)}\,\mathrm{d}x \to l\ln(kx-2k)$ and $\int \frac{m}{(x+3)}\,\mathrm{d}x \to m\ln(nx+3n)$

A1ft: $=\int \frac{15-3x}{(2x-4)(x+3)}\,\mathrm{d}x = 0.9\ln(2x-4)-2.4\ln(x+3)$ oe. FT on their A and B

B1: Deduces that the limits for the integral are 3 and 5. It cannot just be awarded from 5 being marked on Figure 4. So award for sight of $\int_3^5 \frac{15-3x}{(2x-4)(x+3)}(\mathrm{d}x)$ or $[.............]_3^5$ having performed an integral which may be incorrect

dM1: Uses correct ln work seen at least once eg $\ln 6=\ln 2+\ln 3$, $\ln 8=3\ln 2$ or $m\ln 6k-m\ln 2k=m\ln 3$

This is an attempt to get either of the above ln's in terms of ln2 and/or ln3

It is dependent upon the correct limits and having achieved $m\ln(2x-4)+n\ln(x+3)$ oe

A1: $=3.3\ln 3-4.8\ln 2$ oe

Question	Scheme	Marks	AOs
14(a)	$x^n \to x^{n+1}$	M1	1.1b
	$\int\left(\frac{4}{x^3}+kx\right)\mathrm{d}x = -\frac{2}{x^2}+\frac{1}{2}kx^2+c$	A1 A1	1.1b 1.1b
		(3)	
(b)	$\left[-\frac{2}{x^2}+\frac{1}{2}kx^2\right]_{0.5}^{2} = \left(-\frac{2}{2^2}+\frac{1}{2}k\times 4\right)-\left(-\frac{2}{(0.5)^2}+\frac{1}{2}k\times(0.5)^2\right)=8$	M1	1.1b
	$7.5+\frac{15}{8}k=8 \Rightarrow k=...$	dM1	1.1b
	$k=\frac{4}{15}$ oe	A1	1.1b
		(3)	
			(6 marks)

Notes

Mark parts (a) and (b) as one

(a)

M1: For $x^n \to x^{n+1}$ for either x^{-3} or x^1. This can be implied by the sight of either x^{-2} or x^2.

Condone "unprocessed" values here. Eg. x^{-3+1} and x^{1+1}

A1: Either term correct (un simplified).

Accept $4\times\frac{x^{-2}}{-2}$ or $k\frac{x^2}{2}$ **with** the indices processed.

A1: Correct (and simplified) with $+c$.

Ignore spurious notation e.g. answer appearing with an $\int$ sign or with $\mathrm{d}x$ on the end.

Accept $-\frac{2}{x^2}+\frac{1}{2}kx^2+c$ or exact simplified equivalent such as $-2x^{-2}+k\frac{x^2}{2}+c$

(b)

M1: For substituting both limits into their $-\frac{2}{x^2}+\frac{1}{2}kx^2$, subtracting either way around and setting equal to 8. Allow this when using a changed function. (so the M in part (a) may not have been awarded). Condone missing brackets. Take care here as substituting 2 into the original function gives the same result as the integrated function so you will have to consider both limits.

dM1: For solving a **linear** equation in k. It is dependent upon the previous M only

Don't be too concerned by the mechanics here. Allow for a linear equation in k leading to $k=$

A1: $k=\frac{4}{15}$ or exact equivalent. Allow for $\frac{m}{n}$ where m and n are integers and $\frac{m}{n}=\frac{4}{15}$

Condone the recurring decimal $0.2\dot{6}$ but not 0.266 or 0.267

Please remember to isw after a correct answer

	Scheme	Marks	AOs
15.	The overall method of finding the x coordinate of A.	M1	3.1a
	$y=2x^3-17x^2+40x \Rightarrow \frac{dy}{dx}=6x^2-34x+40$	B1	1.1b
	$\frac{dy}{dx}=0 \Rightarrow 6x^2-34x+40=0 \Rightarrow 2(3x-5)(x-4)=0 \Rightarrow x=...$	M1	1.1b
	Chooses $x=4$ $\quad \cancel{x=\frac{5}{3}}$	A1	3.2a
	$\int 2x^3-17x^2+40x \, dx=\left[\frac{1}{2}x^4-\frac{17}{3}x^3+20x^2\right]$	B1	1.1b
	Area $=\frac{1}{2}(4)^4-\frac{17}{3}(4)^3+20(4)^2$	M1	1.1b
	$=\frac{256}{3}$ *	A1*	2.1
		(7)	
		(7 marks)	

Notes

M1: An overall problem -solving method mark to find the minimum point. To score this you need to see

- an attempt to differentiate with at least **two correct terms**
- an attempt to set their $\frac{dy}{dx}=0$ and then solve to find x. Don't be overly concerned by the mechanics of this solution

B1: $\left(\frac{dy}{dx}=\right)6x^2-34x+40$ which may be unsimplified

M1: Sets their $\frac{dy}{dx}=0$, which must be a 3TQ in x, and attempts to solve via factorisation, formula or calculator. If a calculator is used to find the roots, they must be correct for their quadratic.

If $\frac{dy}{dx}$ is correct allow them to just choose the root 4 for M1 A1. Condone $(x-4)\left(x-\frac{5}{3}\right)$

A1: Chooses $x=4$ This may be awarded from the upper limit in their integral

B1: $\int 2x^3-17x^2+40x \, dx=\left[\frac{1}{2}x^4-\frac{17}{3}x^3+20x^2\right]$ which may be unsimplified

M1: Correct attempt at area. There may be slips on the integration but expect **two correct terms**

The upper limit used must be their larger solution of $\frac{dy}{dx}=0$ and the lower limit used must be 0.

So if their roots are 6 and 10, then they must use 10 and 0. If only one value is found then the limits must be 0 to that value.

Expect to see embedded or calculated values.

Don't accept $\int_0^4 2x^3-17x^2+40x \, dx=\frac{256}{3}$ without seeing the integration and the embedded or calculated values

A1*: Area $=\frac{256}{3}$ **with** correct notation and no errors. Note that this is a given answer.

Notes on Question 15 continue
For correct notation expect to see • $\frac{dy}{dx}$ or $\frac{d}{dx}$ used correctly at least once. If $f(x)$ is used accept $f'(x)$. Condone y' • $\int 2x^3 - 17x^2 + 40x \, dx$ used correctly at least once with or without the limits.

Question	Scheme	Marks	AOs
16 (a)	$\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$	M1 A1	1.1a 1.1b
	$\int_k^{3k} \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(9k-k) - \frac{2}{3}\ln(3k-k)$	dM1	1.1b
	$= \frac{2}{3}\ln\left(\frac{8\cancel{k}}{2\cancel{k}}\right) = \frac{2}{3}\ln 4$ oe	A1	2.1
		(4)	
(b)	$\int \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(2x-k)}$	M1	1.1b
	$\int_k^{2k} \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(4k-k)} + \frac{1}{(2k-k)}$	dM1	1.1b
	$= \frac{2}{3k} \quad \left(\propto \frac{1}{k}\right)$	A1	2.1
		(3)	
			(7 marks)

(a)

M1: $\int \frac{2}{(3x-k)}\,dx = A\ln(3x-k)$ Condone a missing bracket

A1: $\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$

Allow recovery from a missing bracket if in subsequent work $A\ln 9k-k \to A\ln 8k$

dM1: For substituting k and $3k$ into their $A\ln(3x-k)$ and subtracting either way around

A1: Uses correct ln work and notation to show that I $= \frac{2}{3}\ln\left(\frac{8}{2}\right)$ or $\frac{2}{3}\ln 4$ oe (ie independent of k)

(b)

M1: $\int \frac{2}{(2x-k)^2}\,dx = \frac{C}{(2x-k)}$

dM1: For substituting k and $2k$ into their $\frac{C}{(2x-k)}$ and subtracting

A1: Shows that it is inversely proportional to k Eg proceeds to the answer is of the form A/k with $A = 2/3$

There is no need to perform the whole calculation. Accept from $-\frac{1}{(3k)} + \frac{1}{(k)} = \left(-\frac{1}{3}+1\right)\times\frac{1}{k} \propto \frac{1}{k}$

If the calculation is performed it must be correct.

Do not isw here. They should know when they have an expression that is inversely proportional to k.

You may see substitution used but the mark is scored for the same result. See below

$u = 2x-k \to \left[\frac{C}{u}\right]$ for M1 with limits $3k$ and k used for dM1

Question	Scheme	Marks	AOs
17(a)	$\frac{dH}{dt}=\frac{H\cos 0.25t}{40} \Rightarrow \int \frac{1}{H}\,dH=\int \frac{\cos 0.25t}{40}\,dt$	M1	3.1a
	$\ln H=\frac{1}{10}\sin 0.25t\,(+c)$	M1 A1	1.1b 1.1b
	Substitutes $t=0, H=5 \Rightarrow c=\ln(5)$	dM1	3.4
	$\ln\left(\frac{H}{5}\right)=\frac{1}{10}\sin 0.25t \Rightarrow H=5e^{0.1\sin 0.25t}$ *	A1*	2.1
		(5)	
(b)	Max height = $5e^{0.1}=5.53$ m (Condone lack of units)	B1	3.4
		(1)	
(c)	Sets $0.25t=\frac{5\pi}{2}$	M1	3.1b
	31.4	A1	1.1b
		(2)	
			(8 marks)

(a)

M1: Separates the variables to reach $\int \frac{1}{H}\,dH=\int \frac{\cos 0.25t}{40}\,dt$ or equivalent.

The integral signs need to be present on both sides and the dH AND dt need to be in the correct positions.

M1: Integrates both sides to reach $\ln H = A\sin 0.25t$ or equivalent with or without the + c

A1: $\ln H=\frac{1}{10}\sin 0.25t+c$ or equivalent with or without the + c. Allow two constants, one either side

If the 40 was on the lhs look for $40\ln H=4\sin 0.25t+c$ or equivalent.

dM1: Substitutes $t=0, H=5 \Rightarrow c=..$ There needs to have been a single "+ c" to find.

It is dependent upon the previous M mark. You may allow even if you don't explicitly see "$t=0, H=5$" as it may be implied from their previous line

If the candidate has attempted to change the subject and made an error/ slip then condone it for this M but not the final A. Eg. $40\ln H=4\sin 0.25t+c \Rightarrow\Rightarrow H^{40}=e^{4\sin 0.25t}+e^{c} \Rightarrow 5^{40}=1+e^{c} \Rightarrow c=...$

Also many students will be attempting to get to the given answer so condone the method of finding c =...
These students will lose the A1* mark

A1*: Proceeds via $\ln H=\frac{1}{10}\sin 0.25t+\ln 5$ or equivalent to the given answer $H=5e^{0.1\sin 0.25t}$ with at least one correct intermediate line and no incorrect work.

DO NOT condone c's going to c's when they should be e^{c} or A

Accept as a minimum $\ln H = \frac{1}{10}\sin 0.25t + \ln 5 \Rightarrow H = e^{\frac{1}{10}\sin 0.25t + \ln 5}$ or $H = e^{\frac{1}{10}\sin 0.25t} \times e^{+\ln 5}$ before sight of the given answer

If the only error was to omit the integration signs on line 1, thus losing the first M1, allow the candidate to have access to this mark following a correct intermediate line (see above).

If they attempt to change the subject first then the constant of integration must have been adapted if the A1* is to be awarded. $\ln H = \frac{1}{10}\sin 0.25t + c \Rightarrow H = e^{\frac{1}{10}\sin 0.25t + c} \Rightarrow H = Ae^{\frac{1}{10}\sin 0.25t}$

The dM1 and A1* under this method are awarded at virtually the same time.

Also, for the final two marks, you may see a proof from $\int_0^H \frac{40}{H}\, dH = \int_5^t \cos 0.25t\, dt$

....

There is an alternative via the use of an integrating factor.

....

(b)

B1: States that the maximum height is 5.53 m Accept $5e^{0.1}$ Condone a lack of units here, but penalise if incorrect units are used.

(c)

M1: For identifying that it would reach the maximum height for the 2nd time when $0.25t = \frac{5\pi}{2}$ or 450

A1: Accept awrt 31.4 or 10π Allow if units are seen

Question	Scheme for Substitution		Marks	AOs
18	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x$ Award for • Using a valid substitution $u=...$, changing the terms to u's • integrating and using appropriate limts .		M1	3.1a
	Substitution $u=\sqrt{x+2} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=2u$ oe	Substitution $u=x+2 \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=1$ oe	B1	1.1b
	$\int 2x\sqrt{x+2}\,\mathrm{d}x$ $=\int A\left(u^2 \pm 2\right)u^2\mathrm{d}u$	$\int 2x\sqrt{x+2}\,\mathrm{d}x$ $=\int A\left(u \pm 2\right)\sqrt{u}\mathrm{d}u$	M1	1.1b
	$=Pu^5 \pm Qu^3$	$=Su^{\frac{5}{2}} \pm Tu^{\frac{3}{2}}$	dM1	2.1
	$=\frac{4}{5}u^5-\frac{8}{3}u^3$	$=\frac{4}{5}u^{\frac{5}{2}}-\frac{8}{3}u^{\frac{3}{2}}$	A1	1.1b
	Uses limits 2 and $\sqrt{2}$ the correct way around	Uses limits 4 and 2 the correct way around	ddM1	1.1b
	$=\frac{32}{15}\left(2+\sqrt{2}\right)$ *		A1*	2.1
			(7)	
				(7 marks)

M1: For attempting to integrate using substitution. Look for

- terms and limits changed to u's. Condone slips and errors/omissions on changing $\mathrm{d}x \rightarrow \mathrm{d}u$
- attempted multiplication of terms and raising of at least one power of u by one. Condone slips
- Use of at least the top correct limit. For instance if they go back to x's the limit is 2

B1: For substitution it is for giving the substitution and stating a correct $\frac{\mathrm{d}x}{\mathrm{d}u}$

Eg, $u=\sqrt{x+2} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=2u$ or equivalent such as $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2\sqrt{x+2}}$

M1: It is for attempting to get all aspects of the integral in terms of 'u'.
All terms must be attempted including the dx. You are only condoning slips on signs and coefficients
dM1: It is for using a correct method of expanding and integrating each term (seen at least once) . It is dependent upon the previous M
A1: Correct answer in x or u See scheme
ddM1: Dependent upon the previous M, it is for using the correct limits for their integral, **the correct way around**

A1*: Proceeds correctly to $=\frac{32}{15}\left(2+\sqrt{2}\right)$. **Note that this is a given answer**

There must be at one least correct intermediate line between $\left[\frac{4}{5}u^5-\frac{8}{3}u^3\right]_{\sqrt{2}}^{2}$ and $\frac{32}{15}\left(2+\sqrt{2}\right)$

Question Alt	Scheme for by parts	Marks	AOs
18	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x$ Award for • using by parts the correct way around • and using limits	M1	3.1a
	$\int\left(\sqrt{x+2}\right)\mathrm{d}x=\frac{2}{3}(x+2)^{\frac{3}{2}}$	B1	1.1b
	$\int 2x\sqrt{x+2}\,\mathrm{d}x = Ax(x+2)^{\frac{3}{2}}-\int B(x+2)^{\frac{3}{2}}(\mathrm{d}x)$	M1	1.1b
	$=Ax(x+2)^{\frac{3}{2}}-C(x+2)^{\frac{5}{2}}$	dM1	2.1
	$=\frac{4}{3}x(x+2)^{\frac{3}{2}}-\frac{8}{15}(x+2)^{\frac{5}{2}}$	A1	1.1b
	Uses limits 2 and 0 the correct way around	ddM1	1.1b
	$=\frac{32}{15}\left(2+\sqrt{2}\right)$	A1*	2.1
		(7)	

M1: For attempting using by parts to solve It is a problem- solving mark and all elements do not have to be correct.

- the formula applied the correct way around. You may condone incorrect attempts at integrating $\sqrt{x+2}$ for this problem solving mark
- further integration, again, this may not be correct, and the use of at least the top limit of 2

B1: For $\int\left(\sqrt{x+2}\right)\mathrm{d}x=\frac{2}{3}(x+2)^{\frac{3}{2}}$ oe May be awarded $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x \to x^2\times\frac{2(x+2)^{\frac{3}{2}}}{3}$

M1: For integration by parts the right way around. Award for $Ax(x+2)^{\frac{3}{2}}-\int B(x+2)^{\frac{3}{2}}(\mathrm{d}x)$

dM1: For integrating a second time. Award for $Ax(x+2)^{\frac{3}{2}}-C(x+2)^{\frac{5}{2}}$

A1: $\frac{4}{3}x(x+2)^{\frac{3}{2}}-\frac{8}{15}(x+2)^{\frac{5}{2}}$ which may be un simplified

ddM1: Dependent upon the previous M, it is for using the limits 2 and 0 the **correct way around**

A1*: Proceeds to $=\frac{32}{15}\left(2+\sqrt{2}\right)$. **Note that this is a given answer**.

At least one correct intermediate line must be seen. (See substitution). You would condone missing dx's

Question	Scheme	Marks	AOs
19	$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x$		
	Attempts to integrate awarded for any correct power	M1	1.1a
	$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x = \frac{2}{3}\times\frac{x^4}{4} + ... + x$	A1	1.1b
	$= ... - 6\frac{x^{\frac{3}{2}}}{3/2} + ...$	A1	1.1b
	$= \frac{1}{6}x^4 - 4x^{\frac{3}{2}} + x + c$	A1	1.1b
			(4 marks)

Notes

M1: Allow for raising power by one. $x^n \to x^{n+1}$
Award for any correct power including sight of $1x$

A1: Correct two '**non fractional power**' terms (may be un-simplified at this stage)

A1: Correct '**fractional power**' term (may be un-simplified at this stage)

A1: Completely correct, simplified and including constant of integration seen on one line.
Simplification is expected for full marks.

Accept correct exact equivalent expressions such as $\frac{x^4}{6} - 4x\sqrt{x} + 1x^1 + c$

Accept $\frac{x^4 - 24x^{\frac{3}{2}} + 6x}{6} + c$

Remember to isw after a correct answer.

Condone poor notation. Eg answer given as $\int\frac{1}{6}x^4 - 4x^{\frac{3}{2}} + x + c$

Question	Scheme	Marks	AOs
20.	For the complete strategy of finding where the normal cuts the x-axis. Key points that must be seen are • Attempt at differentiation	M1	3.1a

	• Attempt at using a changed gradient to find equation of normal • Correct attempt to find where normal cuts the x - axis		
	$y=\frac{32}{x^2}+3x-8 \Rightarrow \frac{dy}{dx}=-\frac{64}{x^3}+3$	M1 A1	1.1b 1.1b
	For a correct method of attempting to find Either the equation of the normal: this requires substituting $x=4$ in their $\frac{dy}{dx}=-\frac{64}{x^3}+3=(2)$, then using the perpendicular gradient rule to find the equation of normal $y-6="-\frac{1}{2}"(x-4)$ Or where the equation of the normal at (4,6) cuts the x - axis. As above but may not see equation of normal. Eg $0-6="-\frac{1}{2}"(x-4) \Rightarrow x=...$ or an attempt using just gradients $"-\frac{1}{2}"=\frac{6}{a-4} \Rightarrow a=...$	dM1	2.1
	Normal cuts the x-axis at $x=16$	A1	1.1b
	For the complete strategy of finding the values of the two key areas. Points that must be seen are • There must be an attempt to find the area under the curve by integrating between 2 and 4 • There must be an attempt to find the area of a triangle using $\frac{1}{2}\times('16'-4)\times 6$ or $\int_4^{"16"} "\left(-\frac{1}{2}x+8\right)" dx$	M1	3.1a
	$\int \frac{32}{x^2}+3x-8\, dx=-\frac{32}{x}+\frac{3}{2}x^2-8x$	M1 A1	1.1b 1.1b
	Area under curve = $=\left[-\frac{32}{x}+\frac{3}{2}x^2-8x\right]_2^4=(-16)-(-26)=(10)$	dM1	1.1b
	Total area =10 + 36 =46 *	A1*	2.1
		(10)	
			(10 marks)

(a)

The first 5 marks are for finding the normal to the curve cuts the x - axis

M1: For the complete strategy of finding where the normal cuts the x- axis. See scheme

M1: Differentiates with at least one index reduced by one

A1: $\frac{dy}{dx}=-\frac{64}{x^3}+3$

dM1: Method of finding

either the equation of the normal at (4, 6) .

or where the equation of the normal at (4, 6) cuts the x - axis

See scheme. It is dependent upon having gained the M mark for differentiation.

A1: Normal cuts the x-axis at $x=16$

The next 5 marks are for finding the area *R*

M1: For the complete strategy of finding the values of two key areas. See scheme

M1: Integrates $\int \frac{32}{x^2}+3x-8\,\mathrm{d}x$ raising the power of at least one index

A1: $\int \frac{32}{x^2}+3x-8\,\mathrm{d}x=-\frac{32}{x}+\frac{3}{2}x^2-8x$ which may be unsimplified

dM1: Area $=\left[-\frac{32}{x}+\frac{3}{2}x^2-8x\right]_2^4=(-16)-(-26)=(10)$

It is dependent upon having scored the M mark for integration, for substituting in both 4 and 2 and subtracting either way around. The above line shows the minimum allowed working for a correct answer.

A1*: Shows that the area under curve = 46. No errors or omissions are allowed

....

A number of candidates are equating the line and the curve (or subtracting the line from the curve)
The last 5 marks are scored as follows.

M1: For the complete strategy of finding the values of the two key areas. Points that must be seen are

- There must be an attempt to find the area BETWEEN the line and the curve either way around by integrating between 2 and 4
- There must be an attempt to find the area of a triangle using $\frac{1}{2}\times('16'-2)\times\left(-\frac{1}{2}\times 2+8\right)$ or

  via integration $\int_2^{16}\left("-\frac{1}{2}x+8"\right)\mathrm{d}x$

M1: Integrates $\int\left("-\frac{1}{2}x+8"\right)-\left(\frac{32}{x^2}+3x-8\right)\mathrm{d}x$ either way around and raises the power of at least one index by one

A1: $\pm\left(-\frac{32}{x}+\frac{7}{4}x^2-16x\right)$ must be correct

dM1: Area = $\int_2^4\left("-\frac{1}{2}x+8"\right)-\left(\frac{32}{x^2}+3x-8\right)\mathrm{d}x=$ ......either way around

A1: Area = $49-3=46$

NB: Watch for candidates who calculate the area under the curve between 2 and 4 = 10 and subtract this from the large triangle = 56. They will lose both the strategy mark and the answer mark.

Question	Scheme	Marks	AOs
21(a)	(i) $\int_1^a \sqrt{8x}\,dx = \sqrt{8}\times\int_1^a \sqrt{x}\,dx = 10\sqrt{8} = 20\sqrt{2}$	M1 A1	2.2a 1.1b
	(ii) $\int_0^a \sqrt{x}\,dx = \int_0^1 \sqrt{x}\,dx + \int_1^a \sqrt{x}\,dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^1 + 10 = \frac{32}{3}$	M1 A1	2.1 1.1b
		(4)	
(b)	$R = \int_1^a \sqrt{x}\,dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^a$	M1 A1	1.1b 1.1b
	$\frac{2}{3}a^{\frac{3}{2}} - \frac{2}{3} = 10 \Rightarrow a^{\frac{3}{2}} = 16 \Rightarrow a = 16^{\frac{2}{3}}$	dM1	3.1a
	$\Rightarrow a = 2^{4\times\frac{2}{3}} = 2^{\frac{8}{3}}$	A1	2.1
		(4)	
		(8 marks)	

Notes:

(a)(i)

M1: For deducing that $\int_1^a \sqrt{8x}\,dx = \sqrt{8}\times\int_1^a \sqrt{x}\,dx$ attempting to multiply $\int_1^a \sqrt{x}\,dx$ by $\sqrt{8}$

A1: $20\sqrt{2}$ or exact equivalent

(a)(ii)

M1: For identifying and attempting to use $\int_0^a \sqrt{x}\,dx = \int_0^1 \sqrt{x}\,dx + \int_1^a \sqrt{x}\,dx$

A1: For $\frac{32}{3}$ or exact equivalent

(b)

M1: Attempts to integrate, $x^{\frac{1}{2}} \to x^{\frac{3}{2}}$

A1: $\int_1^a \sqrt{x}\,dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^a$

dM1: For a whole strategy to find a. In the scheme it is awarded for setting $\left[...x^{\frac{3}{2}}\right]_1^a$ =10, using both limits and proceeding using correct index work to find a. Alternatively a candidate could assume $a = 2^k$. In such a case it is awarded for setting $\left[...x^{\frac{3}{2}}\right]_1^{2^k} = 10$, using both limits and proceeding using correct index work to k=..

A1: $a = 2^{4\times\frac{2}{3}} = 2^{\frac{8}{3}}$

In the alternative case, a further statement must be seen following $k = \frac{8}{3}$ Hence True

Question	Scheme	Marks	AOs
22	$y=(x-2)^2(x+3)=(x^2-4x+4)(x+3)=x^3-1x^2-8x+12$	B1	1.1b
	An attempt to find x coordinate of the maximum point. To score this you must see either • an attempt to expand $(x-2)^2(x+3)$, an attempt to differentiate the result, followed by an attempt at solving $\frac{dy}{dx}=0$ • an attempt to differentiate $(x-2)^2(x+3)$ by the product rule followed by an attempt at solving $\frac{dy}{dx}=0$	M1	3.1a
	$y=x^3-1x^2-8x+12\Rightarrow\frac{dy}{dx}=3x^2-2x-8$	M1	1.1b
	Maximum point occurs when $\frac{dy}{dx}=0\Rightarrow(x-2)(3x+4)=0$ $\Rightarrow x=-\frac{4}{3}$	M1 A1	1.1b 1.1b
	An attempt to find the area under $y=(x-2)^2(x+3)$ between two values. To score this you must see an attempt to expand $(x-2)^2(x+3)$ followed by an attempt at using two limits	M1	3.1a
	Area $=\int(x^3-1x^2-8x+12)\,dx=\left[\frac{x^4}{4}-\frac{x^3}{3}-4x^2+12x\right]$	M1	1.1b
	Uses a top limit of 2 and a bottom limit of their $x=-\frac{4}{3}$ $R=\left[\frac{x^4}{4}-\frac{x^3}{3}-4x^2+12x\right]_{-\frac{4}{3}}^{2}$	M1	2.2a
	Uses $=\frac{28}{3}--\frac{1744}{81}=\frac{2500}{81}$	A1	2.1
		(9)	
			(9 marks)

Notes:

B1: Expands $(x-2)^2(x+3)$ to $x^3-1x^2-8x+12$ seen at some point in their solution. It may appear just on their integral for example.
M1: This is a problem solving mark for knowing the method of finding the maximum point. You should expect to see the key points used (i) differentiation (ii) solution of their $\frac{dy}{dx}=0$

M1: For correctly differentiating their cubic with at least two terms correct (for their cubic).

M1: For setting their $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and solves using a correct method (including calculator methods)

A1: $\Rightarrow x=-\frac{4}{3}$

M1: This is a problem solving mark for knowing how integration is used to find the area underneath a curve between two points.

M1: For correctly integrating their cubic with at least two correct terms (for their cubic).

M1: For deducing the top limit is 2, the bottom limit is their $x=-\frac{4}{3}$ and applying these correctly within their integration.

A1: Shows above steps clearly and proceeds to $R=\frac{2500}{81}$

Question	Scheme	Marks	AOs
23	f(x) = 2x + 3 + 12 x^{-2}	B1	1.1b
	Attempts to integrate	M1	1.1a
	$\int\left(+2x+3+\frac{12}{x^2}\right)\mathrm{d}x = x^2+3x-\frac{12}{x}$	A1	1.1b
	$\left((2\sqrt{2})^2+3(2\sqrt{2})-\frac{12(\sqrt{2})}{2\times 2}\right)-\left(-8\right)$	M1	1.1b
	$=16+3\sqrt{2}$ *	A1*	1.1b
			(5 marks)

Notes:

B1: Correct function with numerical powers

M1: Allow for raising power by one. $x^n \to x^{n+1}$

A1: Correct three terms

M1: Substitutes limits and rationalises denominator

A1*: Completely correct, no errors seen

Question	Scheme	Marks	AOs
24	Writes $\displaystyle\int \frac{t+1}{t}\,\mathrm{d}t = \int 1+\frac{1}{t}\,\mathrm{d}t$ and attempts to integrate	M1	2.1
	$= t + \ln t \ (+c)$	M1	1.1b
	$(2a + \ln 2a) - (a + \ln a) = \ln 7$	M1	1.1b
	$a = \ln \frac{7}{2}$ with $k = \frac{7}{2}$	A1	1.1b
			(4 marks)

Notes:

M1: Attempts to divide each term by t or alternatively multiply each term by t^{-1}

M1: Integrates each term and knows $\int \frac{1}{t}\,\mathrm{d}t = \ln t$. The + c is not required for this mark

M1: Substitutes in both limits, subtracts and sets equal to ln7

A1: Proceeds to $a = \ln \frac{7}{2}$ and states $k = \frac{7}{2}$ or exact equivalent such as 3.5

Question	Scheme	Marks	AOs
25	$y=\dfrac{(x-2)(x-4)}{4\sqrt{x}}=\dfrac{x^2-6x+8}{4\sqrt{x}}=\dfrac{1}{4}x^{\frac{3}{2}}-\dfrac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$\displaystyle\int \frac{1}{4}x^{\frac{3}{2}}-\frac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}\,dx=\frac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}\,(+c)$	dM1 A1	3.1a 1.1b
	Deduces limits of integral are 2 and 4 and applies to their $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}$	M1	2.2a
	$\left(\dfrac{32}{10}-8+8\right)-\left(\dfrac{2}{5}\sqrt{2}-2\sqrt{2}+4\sqrt{2}\right)=\dfrac{16}{5}-\dfrac{12}{5}\sqrt{2}$ Area $R=\dfrac{12}{5}\sqrt{2}-\dfrac{16}{5}\ \left(\text{or } \dfrac{16}{5}-\dfrac{12}{5}\sqrt{2}\right)$	A1	2.1
		(6)	
			(6 marks)
Notes:			

M1: Correct attempt to write $\dfrac{(x-2)(x-4)}{4\sqrt{x}}$ as a sum of terms with **indices**.

Look for at least two different terms with the correct index e.g. two of $x^{\frac{3}{2}}, x^{\frac{1}{2}}, x^{-\frac{1}{2}}$ which have come from the correct places.

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\dfrac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

A1: $\dfrac{1}{4}x^{\frac{3}{2}}-\dfrac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$ which can be left unsimplified e.g. $\dfrac{1}{4}x^{2-\frac{1}{2}}-\dfrac{1}{2}x^{\frac{1}{2}}-x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$

or as e.g. $\dfrac{1}{4}\left(x^{\frac{3}{2}}-6x^{\frac{1}{2}}+8x^{-\frac{1}{2}}\right)$

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\dfrac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

dM1: Integrates $x^n \to x^{n+1}$ for at least 2 correct indices

i.e. at least 2 of $x^{\frac{3}{2}}\to x^{\frac{5}{2}},\ \ x^{\frac{1}{2}}\to x^{\frac{3}{2}},\ \ x^{-\frac{1}{2}}\to x^{\frac{1}{2}}$

It is dependent upon the first M so at least two terms must have had a correct index.

A1: $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}(+c)$. Allow unsimplified e.g. $\dfrac{1}{4}\times\dfrac{2}{5}x^{\frac{3}{2}+1}-\dfrac{1}{2}\times\dfrac{2}{3}x^{\frac{1}{2}+1}-\dfrac{2}{3}x^{\frac{1}{2}+1}+2\times2x^{\frac{1}{2}}$

or as e.g. $\dfrac{1}{4}\left(\dfrac{2}{5}x^{\frac{5}{2}}-4x^{\frac{3}{2}}+16x^{\frac{1}{2}}\right)(+c)$.

M1: Substitutes the limits 4 and 2 to their $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}$ and subtracts either way round.

There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\dfrac{1}{10}\times4^{\frac{5}{2}}-4^{\frac{3}{2}}+4\times4^{\frac{1}{2}}-\dfrac{1}{10}\times2^{\frac{5}{2}}-2^{\frac{3}{2}}+4\times2^{\frac{1}{2}}$

This is an independent mark but the limits must be applied to an expression that is not y so they may even have differentiated.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2}-\frac{16}{5}$ but also allow $\frac{16}{5}-\frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw

See overleaf for integration by parts and integration by substitution.

Integration by parts:

$\int \frac{(x-2)(x-4)}{4\sqrt{x}}\mathrm{d}x = \int \frac{1}{4}(x-2)(x-4)x^{-\frac{1}{2}}\mathrm{d}x = \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \int \frac{1}{2}(2x-6)x^{\frac{1}{2}}\mathrm{d}x$	M1 A1	1.1b 1.1b
$\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \underline{\int \frac{1}{2}(2x-6)x^{\frac{1}{2}}\mathrm{d}x} = \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \underline{\int x^{\frac{3}{2}} - 3x^{\frac{1}{2}}\mathrm{d}x}$ $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} \underline{- \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}}}$ Or e.g. $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}(2x-6) + \frac{4}{15}x^{\frac{5}{2}}$	<u>dM1</u> A1	3.1a 1.1b
Deduces limits of integral are 2 and 4 and applies to their $\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}(2x-6) + \frac{4}{15}x^{\frac{5}{2}}$	M1	2.2a
$0 - \frac{16}{3} + \frac{128}{15} - \left(0 + \frac{4}{3}\sqrt{2} + \frac{16}{15}\sqrt{2}\right)$ Area $R = \frac{12}{5}\sqrt{2} - \frac{16}{5}$ $\left(\text{or } \frac{16}{5} - \frac{12}{5}\sqrt{2}\right)$	A1	2.1
	(6)	

Notes:

M1: Applies integration by parts and reaches the form $\alpha(x-2)(x-4)x^{\frac{1}{2}} \pm \int (px+q)x^{\frac{1}{2}}\mathrm{d}x$ $\alpha, p \neq 0$

oe e.g. $\alpha\left(x^2 - 6x + 8\right)x^{\frac{1}{2}} \pm \int (px+q)x^{\frac{1}{2}}\mathrm{d}x$ $\alpha, p \neq 0$

A1: Correct first application of parts in any form

dM1: Attempts their $\int (px+q)x^{\frac{1}{2}}\mathrm{d}x$ by expanding and integrating or may attempt parts again.

E.g. $\int (2x-6)x^{\frac{1}{2}}\mathrm{d}x = \int \left(2x^{\frac{3}{2}} - 6x^{\frac{1}{2}}\right)\mathrm{d}x = ...$ or e.g. $\int (2x-6)x^{\frac{1}{2}}\mathrm{d}x = \frac{2}{3}x^{\frac{3}{2}}(2x-6) - \frac{4}{3}\int x^{\frac{3}{2}}\mathrm{d}x$

If they expand then at least one term requires $x^n \to x^{n+1}$ or if parts is attempted again, the structure must be correct.

A1: Fully correct integration in any form

M1: Substitutes the limits 4 and 2 to their $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}}$ and subtracts

either way round. There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $0 - \frac{16}{3} + \frac{128}{15} - 0 + \frac{4}{3}\sqrt{2} + \frac{16}{15}\sqrt{2}$

This is an independent mark but the limits must be applied to a "changed" function.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2} - \frac{16}{5}$ but also allow $\frac{16}{5} - \frac{12}{5}\sqrt{2}$ or exact equivalents

Attempts at integration by parts "the other way round" should be sent to review.

Integration by substitution example:

$u=\sqrt{x}\left(x=u^2\right)\Rightarrow\int\frac{(x-2)(x-4)}{4\sqrt{x}}\mathrm{d}x=\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{4u}\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$ $=\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{4u}2u\mathrm{d}u$	M1 A1	1.1b 1.1b
$=\frac{1}{2}\int\left(u^4-6u^2+8\right)\mathrm{d}u=\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)(+c)$	dM1 A1	3.1a 1.1b
Deduces limits of integral are $\sqrt{2}$ and 2 and applies to their $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)$	M1	2.2a
$\frac{1}{2}\left(\frac{32}{5}-16+16-\left(\frac{4\sqrt{2}}{5}-4\sqrt{2}+8\sqrt{2}\right)\right)$ Area $R=\frac{12}{5}\sqrt{2}-\frac{16}{5}\left(\text{or }\frac{16}{5}-\frac{12}{5}\sqrt{2}\right)$	A1	2.1
	(6)	

Notes:

M1: Applies the substitution e.g. $u=\sqrt{x}$ and attempts $k\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{u}\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$

A1: Fully correct integral in terms of u in any form e.g. $\frac{1}{2}\int\left(u^2-2\right)\left(u^2-4\right)\mathrm{d}u$

dM1: Expands the bracket and integrates $u^n\to u^{n+1}$ for at least 2 correct indices

i.e. at least 2 of $u^4\to u^5,\ u^2\to u^3,\ k\to ku$

A1: $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)(+c)$. Allow unsimplified.

M1: Substitutes the limits 2 and $\sqrt{2}$ to their $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)$ and subtracts either way round.

There is no requirement to evaluate but 2 and $\sqrt{2}$ must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\frac{1}{2}\left(\frac{32}{5}-16+16-\frac{4\sqrt{2}}{5}-4\sqrt{2}+8\sqrt{2}\right)$

Alternatively reverses the substitution and applies the limits 4 and 2 with the same conditions.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2}-\frac{16}{5}$ but also allow $\frac{16}{5}-\frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw.

There may be other substitutions seen and the same marking principles apply.

Question	Scheme	Marks	AOs
26(a)	$\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow A = ..., B = ...$	M1	1.1b
	Either $A = 2$ or $B = -1$	A1	1.1b
	$\frac{3}{(2x-1)(x+1)} = \frac{2}{2x-1} - \frac{1}{x+1}$	A1	1.1b
		(3)	
(b)	$\int \frac{1}{V} \mathrm{d}V = \int \frac{3}{(2t-1)(t+1)} \mathrm{d}t$	B1	1.1a
	$\int \frac{2}{2t-1} - \frac{1}{t+1} \mathrm{d}t = ...\ln(2t-1) - ...\ln(t+1)(+c)$	M1	3.1a
	$\ln V = \ln(2t-1) - \ln(t+1)(+c)$	A1ft	1.1b
	Substitutes $t = 2, V = 3 \Rightarrow c = (\ln 3)$	M1	3.4
	$\ln V = \ln(2t-1) - \ln(t+1) + \ln 3$ $V = \frac{3(2t-1)}{(t+1)}$ *	A1*	2.1
		(5)	
	(b) Alternative separation of variables:		
	$\int \frac{1}{3V} \mathrm{d}V = \int \frac{1}{(2t-1)(t+1)} \mathrm{d}t$	B1	1.1a
	$\frac{1}{3}\int \frac{2}{2t-1} - \frac{1}{t+1} \mathrm{d}t = ...\ln(2t-1) - ...\ln(t+1)(+c)$	M1	3.1a
	$\frac{1}{3}\ln 3V = \frac{1}{3}\ln(2t-1) - \frac{1}{3}\ln(t+1)(+c)$	A1ft	1.1b
	Substitutes $t = 2, V = 3 \Rightarrow c = \left(\frac{1}{3}\ln 3\right)$	M1	3.4
	$\frac{1}{3}\ln V = \frac{1}{3}\ln(2t-1) - \frac{1}{3}\ln(t+1) + \frac{1}{3}\ln 3$ $V = \frac{3(2t-1)}{(t+1)}$ *	A1*	2.1
		(5)	
(c)	(i) 30 (minutes)	B1	3.2a
	(ii) 6 (m^3)	B1	3.4
		(2)	
			(10 marks)
Notes:			

(a)

M1: Correct method of partial fractions leading to values for their A and B

E.g. substitution: $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = A(x+1) + B(2x-1) \Rightarrow A = ..., B = ...$

Or compare coefficients $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = x(A+2B) + A - B \Rightarrow A = ..., B = ...$

Note that $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = A(2x-1) + B(x+1) \Rightarrow A = ..., B = ...$ scores M0

A1: Correct value for "A" or "B"

A1: Correct partial fractions not just values for "A" and "B". $\dfrac{2}{2x-1}-\dfrac{1}{x+1}$ or e.g. $\dfrac{2}{2x-1}+\dfrac{-1}{x+1}$

Must be seen as **fractions** but if not stated here, allow if the correct fractions appear later.

(b)

B1: Separates variables $\displaystyle\int \frac{1}{V}\,\mathrm{d}V=\int \frac{3}{(2t-1)(t+1)}\,\mathrm{d}t$. May be implied by later work.

Condone omission of the integral signs but the dV and dt must be in the correct positions if awarding this mark in isolation but they may be implied by subsequent work.

M1: Correct attempt at integration of the partial fractions.

Look for $...\ln(2t-1)+...\ln(t+1)$ where … are constants.

Condone missing brackets around the $(2t-1)$ and/or the $(t+1)$ for this mark

A1ft: Fully correct equation following through their A and B **only**.

No requirement for $+c$ here.

The brackets around the $(2t-1)$ and/or the $(t+1)$ must be seen or implied for this mark

M1: Attempts to find "c" or e.g. "ln k" using $t=2,\ V=3$ following an attempt at integration.

Condone poor algebra as long as $t=2,\ V=3$ is used to find a value of their constant.

Note that the constant may be found immediately after integrating or e.g. after the ln's have been combined.

A1*: Correct processing leading to the given answer $V=\dfrac{3(2t-1)}{(t+1)}$

Alternative:

B1: Separates variables $\displaystyle\int \frac{1}{3V}\,\mathrm{d}V=\int \frac{1}{(2t-1)(t+1)}\,\mathrm{d}t$. May be implied by later work.

Condone omission of the integral signs but the dV and dt must be in the correct positions if awarding this mark in isolation but they may be implied by subsequent work.

M1: Correct attempt at integration of the partial fractions.

Look for $...\ln(2t-1)+...\ln(t+1)$ where … are constants.

Condone missing brackets around the $(2t-1)$ and/or the $(t+1)$ for this mark

A1ft: Fully correct equation following through their A and B **only**.

No requirement for $+c$ here.

The brackets around the $(2t-1)$ and/or the $(t+1)$ must be seen or implied for this mark

M1: Attempts to find "c" or e.g. "ln k" using $t=2,\ V=3$ following an attempt at integration.

Condone poor algebra as long as $t=2,\ V=3$ is used to find a value of their constant.

Note that the constant may be found immediately after integrating or e.g. after the ln's have been combined.

A1*: Correct processing leading to the given answer $V=\dfrac{3(2t-1)}{(t+1)}$

(Note the working may look like this:

$$\frac{1}{3}\ln 3V=\frac{1}{3}\ln(2t-1)-\frac{1}{3}\ln(t+1)+c,\ \frac{1}{3}\ln 9=\frac{1}{3}\ln(3)-\frac{1}{3}\ln 3+c,\ c=\frac{1}{3}\ln 9$$

$$\ln 3V=\ln\frac{9(2t-1)}{(t+1)}\Rightarrow 3V=\frac{9(2t-1)}{(t+1)}\Rightarrow V=\frac{3(2t-1)}{(t+1)}*)$$

Note that B0M1A1M1A1 is not possible in (b) as the B1 must be implied if all the other marks have been awarded.

Note also that some candidates may use different variables in (b) e.g.

$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{3y}{(2x-1)(x+1)}\Rightarrow\displaystyle\int \frac{1}{y}\,\mathrm{d}y=\int \frac{3}{(2x-1)(x+1)}\,\mathrm{d}x$ etc. In such cases you should award marks for equivalent work but they must revert to the given variables at the end to score the final mark.

Also if e.g. a "t" becomes an "x" within their working but is recovered allow full marks.

(c)

B1: Deduces 30 minutes. Units not required so just look for 30 but allow equivalents e.g. ½ an hour.
If units are given they must be correct so do not allow e.g. 30 hours.

B1: Deduces 6 m^3. Units not required so just look for 6. Condone $V < 6$ or $V \leq 6$
If units are given they must be correct so do not allow e.g. 6 m.

Question	Scheme	Marks	AOs
27(a)	$y=x^3-10x^2+27x-23 \Rightarrow \frac{dy}{dx}=3x^2-20x+27$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=5}=3\times5^2-20\times5+27(=2)$	M1	1.1b
	$y+13=2(x-5)$	M1	2.1
	$y=2x-23$	A1	1.1b
		(4)	
(b)	Both C and l pass through (0, −23) and so C meets l again on the y-axis	B1	2.2a
		(1)	
(c)	$\pm\int\left(x^3-10x^2+27x-23-(2x-23)\right)\,dx$ $=\pm\left(\frac{x^4}{4}-\frac{10}{3}x^3+\frac{25}{2}x^2\right)$	M1 A1ft	1.1b 1.1b
	$\left[\frac{x^4}{4}-\frac{10}{3}x^3+\frac{25}{2}x^2\right]_0^5$ $=\left(\frac{625}{4}-\frac{1250}{3}+\frac{625}{2}\right)(-0)$	dM1	2.1
	$=\frac{625}{12}$	A1	1.1b
		(4)	
	(c) Alternative:		
	$\pm\int\left(x^3-10x^2+27x-23\right)\,dx$ $=\pm\left(\frac{x^4}{4}-\frac{10}{3}x^3+\frac{27}{2}x^2-23x\right)$	M1 A1	1.1b 1.1b
	$\left[\frac{x^4}{4}-\frac{10}{3}x^3+\frac{27}{2}x^2-23x\right]_0^5+\frac{1}{2}\times5(23+13)$ $=-\frac{455}{12}+90$	dM1	2.1
	$=\frac{625}{12}$	A1	1.1b
			(9 marks)

Notes
(a) B1: Correct derivative M1: Substitutes $x = 5$ into their derivative. This may be implied by their value for $\frac{dy}{dx}$ M1: Fully correct straight line method using $(5, -13)$ and their $\frac{dy}{dx}$ at $x = 5$ A1: cao. Must see the full equation in the required form. (b) B1: Makes a suitable deduction. Alternative via equating l and C and factorising e.g. $x^3 - 10x^2 + 27x - 23 = 2x - 23$ $x^3 - 10x^2 + 25x = 0$ $x\left(x^2 - 10x + 25\right) = 0 \Rightarrow x = 0$ So they meet on the y-axis (c) M1: For an attempt to integrate $x^n \to x^{n+1}$ for ±" $C - l$" A1ft: Correct integration in any form which may be simplified or unsimplified. (follow through their equation from (a)) If they attempt as 2 separate integrals e.g. $\int \left(x^3 - 10x^2 + 27x - 23\right) dx - \int \left(2x - 23\right) dx$ then award this mark for the correct integration of the curve as in the alternative. If they combine the curve with the line first then the subsequent integration must be correct or a correct ft for their line and allow for ±" $C - l$" dM1: Fully correct strategy for the area. Award for use of 5 as the limit and condone the omission of the "– 0". **Depends on the first method mark.** A1: Correct exact value **Alternative:** M1: For an attempt to integrate $x^n \to x^{n+1}$ for $\pm C$ A1: Correct integration for $\pm C$ dM1: Fully correct strategy for the area e.g. correctly attempts the area of the trapezium and subtracts the area enclosed between the curve and the x-axis. Need to see the use of 5 as the limit condoning the omission of the "– 0" **and** a correct attempt at the trapezium **and** the subtraction. May see the trapezium area attempted as $\int \left(2x - 23\right) dx$ in which case the integration and use of the limits needs to be correct or correct follow through for their straight line equation. **Depends on the first method mark.** A1: Correct exact value Note if they do $l - C$ rather than $C - l$ and the working is otherwise correct allow full marks if their final answer is given as a positive value. E.g. correct work with $l - C$ leading to $-\frac{625}{12}$ and then e.g. hence area is $\frac{625}{12}$ is acceptable for full marks. If the answer is left as $-\frac{625}{12}$ then score A0

Question	Scheme	Marks	AOs
28(a)	$u=1+\sqrt{x}\Rightarrow x=(u-1)^2\Rightarrow\dfrac{dx}{du}=2(u-1)$ or $u=1+\sqrt{x}\Rightarrow\dfrac{du}{dx}=\dfrac{1}{2}x^{-\frac{1}{2}}$	B1	1.1b
	$\displaystyle\int\frac{x}{1+\sqrt{x}}dx=\int\frac{(u-1)^2}{u}2(u-1)du$ or $\displaystyle\int\frac{x}{1+\sqrt{x}}dx=\int\frac{x}{u}\times2x^{\frac{1}{2}}du=\int\frac{2x^{\frac{3}{2}}}{u}du=\int\frac{2(u-1)^3}{u}du$	M1	2.1
	$\displaystyle\int_0^{16}\frac{x}{1+\sqrt{x}}dx=\int_1^5\frac{2(u-1)^3}{u}du$	A1	1.1b
		(3)	
(b)	$\displaystyle2\int\frac{u^3-3u^2+3u-1}{u}du=2\int\left(u^2-3u+3-\frac{1}{u}\right)du=...$	M1	3.1a
	$=(2)\left[\dfrac{u^3}{3}-\dfrac{3u^2}{2}+3u-\ln u\right]$	A1	1.1b
	$=2\left[\dfrac{5^3}{3}-\dfrac{3(5)^2}{2}+3(5)-\ln5-\left(\dfrac{1}{3}-\dfrac{3}{2}+3-\ln1\right)\right]$	dM1	2.1
	$=\dfrac{104}{3}-2\ln5$	A1	1.1b
		(4)	
		(7 marks)	

Notes

(a)

B1: Correct expression for $\dfrac{dx}{du}$ or $\dfrac{du}{dx}$ (or u') or dx in terms of du or du in terms of dx

M1: Complete method using the given substitution.

This needs to be a correct method for their $\dfrac{dx}{du}$ or $\dfrac{du}{dx}$ leading to an integral in terms of u only (ignore any limits if present) so for each case you need to see:

$$\frac{dx}{du}=f(u)\rightarrow\int\frac{x}{1+\sqrt{x}}dx=\int\frac{(u-1)^2}{u}f(u)du$$

$\displaystyle\frac{du}{dx}=g(x)\rightarrow\int\frac{x}{1+\sqrt{x}}dx=\int\frac{x}{u}\times\frac{du}{g(x)}=\int h(u)du$. In this case you can condone slips with coefficients e.g. allow $\displaystyle\frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}}\rightarrow\int\frac{x}{1+\sqrt{x}}dx=\int\frac{x}{u}\times\frac{x^{\frac{1}{2}}}{2}du=\int h(u)du$

but not $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}} \rightarrow \int \frac{x}{1+\sqrt{x}}\mathrm{d}x = \int \frac{x}{u} \times \frac{x^{-\frac{1}{2}}}{2}\mathrm{d}u = \int \mathrm{h}(u)\mathrm{d}u$

A1: All correct with correct limits and no errors. The "du" must be present but may have been omitted along the way but it must have been seen at least once before the final answer. The limits can be seen as part of the integral or stated separately.

(b)

M1: Realises the requirement to cube the bracket and divide through by u and makes progress with the integration to obtain at least 3 terms of the required form e.g. 3 from ku^3, ku^2, ku, $k\ln u$

A1: Correct integration. This mark can be scored with the "2" still outside the integral or even if it has been omitted. But if the "2" has been combined with the integrand, the integration must be correct.

dM1: Completes the process by applying their "changed" limits and subtracts the right way round **Depends on the first method mark.**

A1: Cao (Allow equivalents for $\frac{104}{3}$ e.g. $\frac{208}{6}$)

Question	Scheme	Marks	AOs
29(a)	$\frac{dV}{dt} = 0.48 - 0.1h$	B1	3.1b
	$V = 24h \Rightarrow \frac{dV}{dh} = 24$ or $\frac{dh}{dV} = \frac{1}{24}$	B1	3.1b
	$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV} = \frac{0.48 - 0.1h}{24}$ or e.g. $\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} \Rightarrow 0.48 - 0.1h = 24\frac{dh}{dt}$	M1	2.1
	$1200\frac{dh}{dt} = 24 - 5h$ *	A1*	1.1b
		(4)	
(b)	$1200\frac{dh}{dt} = 24 - 5h \Rightarrow \int \frac{1200}{24-5h}\,dh = \int dt$ $\Rightarrow$ *e.g.* $\alpha \ln(24-5h) = t(+c)$ oe or $1200\frac{dh}{dt} = 24 - 5h \Rightarrow \frac{dt}{dh} = \frac{1200}{24-5h}$ $\Rightarrow$ *e.g.* $t(+c) = \alpha \ln(24-5h)$ oe	M1	3.1a
	$t = -240\ln(24-5h)(+c)$ oe	A1	1.1b
	$t = 0, h = 2 \Rightarrow 0 = -240\ln(24-10) + c \Rightarrow c = \ldots(240\ln 14)$	M1	3.4
	$t = 240\ln(14) - 240\ln(24-5h)$	A1	1.1b
	$t = 240\ln\frac{14}{24-5h} \Rightarrow \frac{t}{240} = \ln\frac{14}{24-5h} \Rightarrow e^{\frac{t}{240}} = \frac{14}{24-5h}$ $\Rightarrow 14e^{-\frac{t}{240}} = 24 - 5h \Rightarrow h = \ldots$	ddM1	2.1
	$h = 4.8 - 2.8e^{-\frac{t}{240}}$ oe e.g. $h = \frac{24}{5} - \frac{14}{5}e^{-\frac{t}{240}}$	A1	3.3
		(6)	
(c)	**Examples:** • As $t \to \infty, e^{-\frac{t}{240}} \to 0$ • When $h > 4.8$, $\frac{dV}{dt} < 0$ • Flow in = flow out at max h so $0.1h = 4.8 \rightarrow h = 4.8$ • As $e^{-\frac{t}{240}} > 0$, $h < 4.8$ • $h = 5 \Rightarrow \frac{dV}{dt} = -0.02$ or $\frac{dh}{dt} = -\frac{1}{1200}$ • $\frac{dh}{dt} = 0 \Rightarrow h = 4.8$ • $h = 5 \Rightarrow 4.8 - 2.8e^{-\frac{t}{240}} = 5 \Rightarrow e^{-\frac{t}{240}} < 0$	M1	3.1b
	• The limit for h (according to the model) is 4.8m and the tank is 5m high so the tank will never become full • If $h = 5$ the tank would be emptying so can never be full • The equation can't be solved when $h = 5$	A1	3.2a

		(2)	
			(12 marks)

Notes

(a)

B1: Identifies the correct expression for $\frac{\mathrm{d}V}{\mathrm{d}t}$ according to the model

B1: Identifies the correct expression for $\frac{\mathrm{d}V}{\mathrm{d}h}$ according to the model

M1: Applies $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \times \frac{\mathrm{d}h}{\mathrm{d}V}$ or equivalent correct formula with their $\frac{\mathrm{d}V}{\mathrm{d}t}$ and $\frac{\mathrm{d}V}{\mathrm{d}h}$ which may be implied by their working

A1*: Correct equation obtained with no errors

Note that: $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.48 - 0.1h \Rightarrow \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{0.48 - 0.1h}{24} \Rightarrow 1200\frac{\mathrm{d}h}{\mathrm{d}t} = 24 - 5h$ * scores B1B0M0A0. There must be clear evidence where the "24" comes from and evidence of the correct chain rule being applied.

(b)

M1: Adopts a correct strategy by separating the variables correctly or rearranges to obtain $\frac{\mathrm{d}t}{\mathrm{d}h}$ correctly in terms of h **and** integrates to obtain $t = \alpha \ln(24 - 5h)(+c)$ or equivalent (condone missing brackets around the "24 – 5h") and + c not required for this mark.

A1: Correct equation in any form and + c not required. Do not condone missing brackets unless they are implied by subsequent work.

M1: Substitutes t = 0 and h = 2 to find their constant of integration (there must have been some attempt to integrate)

A1: Correct equation in any form

ddM1: Uses fully correct log work to obtain h in terms of t.

This depends on <u>both</u> previous method marks.

A1: Correct equation

Note that the marks may be earned in a different order e.g.:

$$t + c = -240\ln(24 - 5h) \Rightarrow -\frac{t}{240} + d = \ln(24 - 5h) \Rightarrow Ae^{-\frac{t}{240}} = 24 - 5h$$

$$t = 0, h = 2 \Rightarrow A = 14 \Rightarrow 14e^{-\frac{t}{240}} = 24 - 5h \Rightarrow h = 4.8 - 2.8e^{-\frac{t}{240}}$$

Score as M1 A1 as in main scheme then

M1: Correct work leading to $Ae^{\alpha t} = 24 - 5h$ (must have a constant "A")

A1: $Ae^{-\frac{t}{240}} = 24 - 5h$

ddM1: Uses t = 0, h = 2 in an expression of the form above to find A

A1: $h = 4.8 - 2.8e^{-\frac{t}{240}}$

(c)

M1: See scheme for some examples

A1: Makes a correct interpretation for their method.

There must be no incorrect working or contradictory statements.

This is not a follow through mark and if their equation in (b) is used it must be correct.

Question	Scheme	Marks	AOs
30(a)	$x^2+8x-3=(Ax+B)(x+2)+C$ or $Ax(x+2)+B(x+2)+C$ $\Rightarrow A=\ldots, B=\ldots, C=\ldots$ **or** $x+2\overline{)x^2+8x-3}$ with quotient $x+6$: x^2+2x $6x-3$ $6x+12$ -15	M1	1.1b
	Two of $A=1, B=6, C=-15$	A1	1.1b
	All three of $A=1, B=6, C=-15$	A1	1.1b
		(3)	
30(b)	$\int \frac{x^2+8x-3}{x+2}\,\mathrm{d}x = \int x+6-\frac{15}{x+2}\,\mathrm{d}x = \ldots -15\ln(x+2)$	M1	1.1b
	$=\frac{1}{2}x^2+6x-15\ln(x+2) \quad (+c)$	A1ft	1.1b
	$\int_0^6 \frac{x^2+8x-3}{x+2}\,\mathrm{d}x = \left[\frac{1}{2}x^2+6x-15\ln(x+2)\right]_0^6$ = (18 + 36 – 15ln8) – (0 + 0 – 15ln2) = 18 + 36 – (15 – 45)ln2 or e.g. $18+36+15\ln\left(\frac{2}{8}\right)$	M1	2.1
	= 54 – 30 ln2	A1	1.1b
		(4)	
			(7 marks)

Notes:

(a)

M1: Multiplies by (x + 2) and attempts to find values for A, B and C e.g. by comparing coefficients or substituting values for x. If the method is unclear, at least 2 terms must be correct on rhs.

Or attempts to divide x^2+8x-3 by x + 2 and obtains a linear quotient and a constant remainder.

This mark may be implied by 2 correct values for A, B or C

A1: Two of A = 1, B = 6, C = –15. But note that **just** performing the division correctly is insufficient and they must clearly identify their A, B, C to score any accuracy marks.

A1: All three of A = 1, B = 6, C = –15

This is implied by stating $\frac{x^2+8x-3}{x+2}=x+6-\frac{15}{x+2}$ or within the integral in (b)

(b)

M1: Integrates an expression of the form $\frac{C}{x+2}$ to obtain $k\ln(x+2)$.

Condone the omission of brackets around the "x + 2"

A1ft: Correct integration ft on their $Ax+B+\frac{C}{x+2}$, $(A,B,C\neq 0)$ The brackets should be present around the "x + 2" unless they are implied by subsequent work.

M1: Substitutes both limits 0 and 6 into an expression that contains an x or x^2 term or both and a ln term and subtracts either way round **WITH** fully correct log work to combine two log terms (but allow sign errors when removing brackets) leading to an answer of the form $a + b\ln c$ (a, b and c not necessarily integers)

e.g. if they expand to get –15ln8 – 15ln2 followed by -15ln16 and reach $a + b\ln c$ then allow the M mark

A1: 54 – 30 ln2 (Apply isw once a correct answer is seen)

Examples: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{2\sqrt{x}\,(8x+1)-\left(4x^2+x\right)x^{-\frac{1}{2}}}{\left(2\sqrt{x}\right)^2}$, $\frac{1}{2}x^{-\frac{1}{2}}(8x+1)-\frac{1}{4}\left(4x^2+x\right)x^{-\frac{3}{2}}$, $2\times\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$

A1*: Obtains $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ via $3\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{4}{x}$ or a correct application of the quotient or product rule and with sufficient working shown to reach the printed answer.

<u>**There must be no errors e.g. missing brackets.**</u>

(b)

M1: Sets $12x^2+x-16\sqrt{x}=0$ and divides by $\sqrt{x}$ or equivalent e.g. divides by x and multiplies by $\sqrt{x}$

dM1: Makes the term in $x^{\frac{3}{2}}$ the subject of the formula

A1*: A correct and rigorous argument leading to the given solution.

Alternative - working backwards:

$$x=\left(\frac{4}{3}-\frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}\Rightarrow x^{\frac{3}{2}}=\frac{4}{3}-\frac{\sqrt{x}}{12}\Rightarrow 12x^{\frac{3}{2}}=16-\sqrt{x}\Rightarrow 12x^2=16\sqrt{x}-x\Rightarrow 12x^2-16\sqrt{x}+x=0$$

M1: For raising to power of 3/2 both sides. dM1: Multiplies through by $\sqrt{x}$. A1: Achieves printed answer and makes a minimal comment e.g. tick, #, QED, true etc.

(c)

M1: Attempts to use the iterative formula with x_1 = 2. This is implied by sight of $x_2=\left(\frac{4}{3}-\frac{\sqrt{2}}{12}\right)^{\frac{2}{3}}$ or awrt 1.14

A1: x_2 = awrt 1.13894

A1: Deduces that x = 1.15650

Via firstly integrating

Question	Scheme	Marks	AOs
31	$f'(x)=6x^2+ax-23 \Rightarrow f(x)=2x^3+\frac{1}{2}ax^2-23x+c$	M1 A1	1.1b 1.1b
	$"c"=-12$	B1	2.2a
	$f(-4)=0 \Rightarrow 2\times(-4)^3+\frac{1}{2}a(-4)^2-23(-4)-12=0$	dM1	3.1a
	$a=\ldots\ (6)$	dM1	1.1b
	$(f(x)=)2x^3+3x^2-23x-12$ Or Equivalent e.g. $(f(x)=)(x+4)(2x^2-5x-3)$ $(f(x)=)(x+4)(2x+1)(x-3)$	A1cso	2.1
		(6)	
			(6 marks)

Notes:

M1: Integrates $f'(x)$ with two correct indices. There is no requirement for the + c

A1: Fully correct integration (may be unsimplified). The + c must be seen (or implied by the –12)

B1: Deduces that the constant term is –12

dM1: Dependent upon having done some integration. It is for setting up a linear equation in a by using $f(-4)=0$

May also see long division attempted for this mark. Need to see a complete method leading to a remainder in terms of a which is then set = 0.

For reference, the quotient is $2x^2+\left(\frac{a}{2}-8\right)x+9-2a$ and the remainder is $8a-48$

May also use $(x+4)(px^2+qx+r)=2x^3+\frac{1}{2}ax^2-23x-12$ and compare coefficients to find p, q and r and hence a. Allow this mark if they solve for p, q and r

Note that some candidates use 2f(x) which is acceptable and gives the same result if executed correctly.

dM1: Solves the linear equation in a or uses p, q and r to find a.

It is dependent upon having attempted some integration and used $f(\pm 4)=0$ or long division/comparing coefficients with $(x+4)$ as a factor.

A1cso: For $(f(x)=)2x^3+3x^2-23x-12$ oe. Note that "f(x) =" does not need to be seen and ignore any "= 0"

Via firstly using factor

Question	Scheme	Marks	AOs
31 Alt	$f(x)=(x+4)(Ax^2+Bx+C)$	M1 A1	1.1b 1.1b
	$f(x)=Ax^3+(4A+B)x^2+(4B+C)x+4C \Rightarrow C=-3$	B1	2.2a
	$f'(x)=3Ax^2+2(4A+B)x+(4B+C)$ and $f'(x)=6x^2+ax-23$ $\Rightarrow A=\ldots$	dM1	3.1a
	Full method to get A, B and C	dM1	1.1b
	$f(x)=(x+4)(2x^2-5x-3)$	A1cso	2.1
		(6)	
			(6 marks)

Notes:

M1: Uses the fact that $f(x)$ is a cubic expression with a factor of $(x+4)$

A1: For $f(x)=(x+4)(Ax^2+Bx+C)$

B1: Deduces that $C=-3$

dM1: Attempts to differentiate either by product rule or via multiplication and compares to $f'(x) = 6x^2 + ax - 23$ to find A.

dM1: Full method to get A, B and C

A1cso: $f(x) = (x + 4)(2x^2 - 5x - 3)$ or $f(x) = (x + 4)(2x + 1)(x - 3)$

Question	Scheme	Marks	AOs
32(a)(i)	$y\times\dfrac{dx}{dt}=5\sin 2t\times 6\cos t$ or $5\times 2\sin t\cos t\times 6\cos t$	M1	1.2
	(Area =) $\int 5\sin 2t\times 6\cos t\,dt=\int 5\times 2\sin t\cos t\times 6\cos t\,dt$ or $\int 5\sin 2t\times 6\cos t\,dt=\int 60\sin t\cos^2 t\,dt$	dM1	1.1b
	(Area =) $\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t\,dt$ *	A1*	2.1*
		(3)	
(a)(ii)	$\int 60\sin t\cos^2 t\,dt=-20\cos^3 t$	M1 A1	1.1b 1.1b
	Area $=\left[-20\cos^3 t\right]_0^{\frac{\pi}{2}}=0-(-20)=20$ *	A1*	2.1
		(3)	
(b)	$5\sin 2t=4.2\Rightarrow \sin 2t=\dfrac{4.2}{5}$	M1	3.4
	$t=0.4986...,1.072...$	A1	1.1b
	Attempts to finds the x values at both t values	dM1	3.4
	$t=0.4986...\Rightarrow x=2.869...$ $t=1.072\Rightarrow x=5.269...$	A1	1.1b
	Width of path = 2.40 metres	A1	3.2a
		(5)	
			(11 marks)

Notes:

(a)(i)

M1: Attempts to multiply y by $\dfrac{dx}{dt}$ to obtain $A\sin 2t\cos t$ but may apply $\sin 2t=2\sin t\cos t$ here

dM1: Attempts to use $\sin 2t=2\sin t\cos t$ within an integral which may be implied by

e.g. $A\int \sin 2t\times\cos t\,dt=\int k\sin t\cos^2 t\,dt$

A1*: Fully correct work leading to the given answer.

This must include $\sin 2t=2\sin t\cos t$ or e.g. $5\sin 2t=10\sin t\cos t$ seen <u>explicitly</u> in their proof and a correct intermediate line that includes an integral sign and the "dt"

Allow the limits to just "appear" in the final answer e.g. working need not be shown for the limits.

(a)(ii)

M1: Obtains $\int 60\sin t\cos^2 t\,dt=k\cos^3 t$. This may be attempted via a substitution of $u=\cos t$ to obtain $\int 60\sin t\cos^2 t\,dt=ku^3$

A1: Correct integration $-20\cos^3 t$ or equivalent e.g. $-20u^3$

A1*: Rigorous proof with all aspects correct including the correct limits and the $0-(-20)$ and

not <u>just:</u> $-20\cos^3\dfrac{\pi}{2}-\left(-20\cos^3 0\right)=20$

(b)

M1: Uses the given model and attempts to find value(s) of t when $\sin 2t=\dfrac{4.2}{5}$. Look for $2t=\sin^{-1}\dfrac{4.2}{5}\Rightarrow t=...$

A1: At least one correct value for t, correct to 2 dp. FYI $t=0.4986...,1.072...$ or in degrees $t=28.57…, 61.42…$

dM1: Attempts to find **TWO** distinct values of x when $\sin 2t = \dfrac{4.2}{5}$. Condone poor trig work and allow this mark if 2 values of x are attempted from 2 values of t.

A1: Both values correct to 2 dp. NB x = 2.869…, 5.269…

Or may take Cartesian approach

$$5\sin 2t = 4.2 \Rightarrow 10\sin t\cos t = 4.2 \Rightarrow 10\frac{x}{6}\sqrt{1-\frac{x^2}{36}} = 4.2 \Rightarrow x^4 - 36x^2 + 228.6144 = 0 \Rightarrow x = 2.869...,5.269...$$

M1: For converting to Cartesian form A1: Correct quartic M1: Solves quartic A1: Correct values

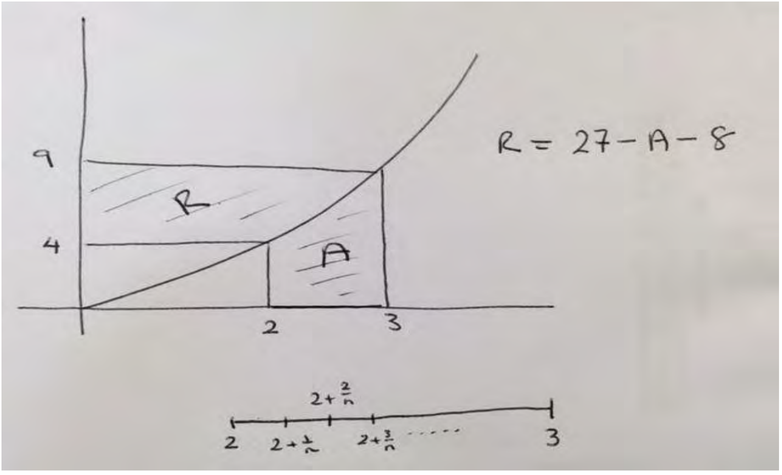
A1: 2.40 metres or 240 cm

Allow awrt 2.40 m or allow 2.4m (not awrt 2.4 m) and allow awrt 240 cm. **Units are required**.

Question	Scheme	Marks	AOs
33	States $\left\{\lim\limits_{\delta x\to 0}\sum\limits_{x=4}^{9}\sqrt{x}\,\delta x \text{ is}\right\}$ $\int_4^9 \sqrt{x}\,\mathrm{d}x$	B1	1.2
	$= \left[\frac{2}{3}x^{\frac{3}{2}}\right]_4^9$	M1	1.1b
	$= \frac{2}{3}\times 9^{\frac{3}{2}} - \frac{2}{3}\times 4^{\frac{3}{2}} = \frac{54}{3} - \frac{16}{3}$		
	$= \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7	A1	1.1b
		(3)	
			(3 marks)

Notes for Question **33**	
B1:	States $\int_4^9 \sqrt{x}\,\mathrm{d}x$ with or without the '$\mathrm{d}x$'
M1:	Integrates $\sqrt{x}$ to give $\lambda x^{\frac{3}{2}}$; $\lambda \neq 0$
A1:	See scheme
Note:	You can imply B1 for $\left[\lambda x^{\frac{3}{2}}\right]_4^9$ or for $\lambda \times 9^{\frac{3}{2}} - \lambda \times 4^{\frac{3}{2}}$
Note:	Give B0 for $\int_1^9 \sqrt{x}\,\mathrm{d}x - \int_1^3 \sqrt{x}\,\mathrm{d}x$ or for $\int_3^9 \sqrt{x}\,\mathrm{d}x$ without reference to a correct $\int_4^9 \sqrt{x}\,\mathrm{d}x$
Note:	Give B1 M1 A1 for no working leading to a correct $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\int_4^9 \sqrt{x}\,\mathrm{d}x = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\left[\frac{2}{3}x^{\frac{3}{2}} + c\right]_4^9 = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for no working followed by an answer $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give M0 A0 for use of a trapezium rule method to give an answer of awrt 12.7, but allow B1 if $\int_4^9 \sqrt{x}\,\mathrm{d}x$ is seen in a trapezium rule method
Note:	Otherwise, give B0 M0 A0 for using the trapezium rule to give an answer of awrt 12.7

Notes for Question **33** Continued	
Alt	The following method is correct:



$$\text{Area }(A) = \lim_{n\to\infty}\sum_{i=1}^{n}(x_i - x_{i-1})\mathrm{f}(x_i) = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left(2+\frac{i}{n}\right)^2$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{4i}{n}\right) + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{i^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{4}{n^2}\sum_{i=1}^{n}i + \frac{1}{n^3}\sum_{i=1}^{n}i^2\right]$$

$$= \lim_{n\to\infty}\left[\frac{4n}{n} + \frac{4}{n^2}\left(\frac{1}{2}n(n+1)\right) + \frac{1}{n^3}\left(\frac{1}{6}n(n+1)(2n+1)\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{4}{n} + \frac{4n^2+4n}{2n^2} + \frac{2n^3+3n^2+n}{6n^3}\right]$$

$$= \lim_{n\to\infty}\left[4 + 2 + \frac{2}{n} + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right]$$

$$= 4 + 2 + \frac{1}{3} = \frac{19}{3}$$

So, $\lim_{\delta x\to 0}\sum_{x=4}^{9}\sqrt{x}\;\delta x = \text{Area}(R) = (3\times 9) - (2\times 4) - \dfrac{19}{3}$

$$= \frac{38}{3} \quad \text{or} \quad 12\frac{2}{3} \quad \text{or} \quad \text{awrt } 12.7$$

Question	Scheme	Marks	AOs
34 (a)	$\{u = 4-\sqrt{h} \Rightarrow\}\ \dfrac{\mathrm{d}u}{\mathrm{d}h} = -\dfrac{1}{2}h^{-\frac{1}{2}}$ or $\dfrac{\mathrm{d}h}{\mathrm{d}u} = -2(4-u)$ or $\dfrac{\mathrm{d}h}{\mathrm{d}u} = -2\sqrt{h}$	B1	1.1b
	$\left\{\displaystyle\int \frac{\mathrm{d}h}{4-\sqrt{h}} = \right\}\ \displaystyle\int \frac{-2(4-u)}{u}\,\mathrm{d}u$	M1	2.1
	$= \displaystyle\int \left(-\frac{8}{u}+2\right)\mathrm{d}u$	M1	1.1b
	$= -8\ln u + 2u\ \{+c\}$	M1	1.1b
		A1	1.1b
	$= -8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) + c = -8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$ *	A1*	2.1
		(6)	
(b)	$\left\{\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{t^{0.25}(4-\sqrt{h})}{20} = 0 \Rightarrow\right\}\ 4-\sqrt{h} = 0$	M1	3.4
	Deduces any of $0<h<16,\ 0\le h<16,\ 0<h\le 16,\ 0\le h\le 16,$ $h<16,\ h\le 16$ or all values up to 16	A1	2.2a
		(2)	
(c) Way 1	$\displaystyle\int \frac{1}{(4-\sqrt{h})}\,\mathrm{d}h = \int \frac{1}{20}t^{0.25}\,\mathrm{d}t$	B1	1.1b
	$-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} = \dfrac{1}{25}t^{1.25}\ \{+c\}$	M1	1.1b
		A1	1.1b
	$\{t=0,\ h=1 \Rightarrow\}\ -8\ln(4-1) - 2\sqrt{(1)} = \dfrac{1}{25}(0)^{1.25} + c$	M1	3.4
	$\Rightarrow c = -8\ln(3) - 2 \Rightarrow\ -8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} = \dfrac{1}{25}t^{1.25} - 8\ln(3) - 2$ $\{h=12 \Rightarrow\}\ -8\ln\left\|4-\sqrt{12}\right\| - 2\sqrt{12} = \dfrac{1}{25}t^{1.25} - 8\ln(3) - 2$	dM1	3.1a
	$t^{1.25} = 221.2795202... \Rightarrow t = \sqrt[1.25]{221.2795...}$ or $t = (221.2795...)^{0.8}$	M1	1.1b
	$t = 75.154... \Rightarrow t = 75.2$ (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	**(7)**	
(c) Way 2	$\displaystyle\int_1^{12} \frac{20}{(4-\sqrt{h})}\,\mathrm{d}h = \int_0^T t^{0.25}\,\mathrm{d}t$	B1	1.1b
	$\left[20(-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h})\right]_1^{12} = \left[\dfrac{4}{5}t^{1.25}\right]_0^T$	M1	1.1b
		A1	1.1b
	$20(-8\ln(4-\sqrt{12}) - 2\sqrt{12}) - 20(-8\ln(4-1) - 2\sqrt{1}) = \dfrac{4}{5}T^{1.25} - 0$	M1	3.4
		dM1	3.1a
	$T^{1.25} = 221.2795202... \Rightarrow T = \sqrt[1.25]{221.2795...}$ or $T = (221.2795...)^{0.8}$	M1	1.1b
	$T = 75.154... \Rightarrow T = 75.2$ (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	**(7)**	
			(15 marks)

EXPERT TUITION

Notes for Question 34	
(a)	
B1:	See scheme. Allow $\mathrm{d}u = -\frac{1}{2}h^{-\frac{1}{2}}\mathrm{d}h$, $\mathrm{d}h = -2(4-u)\mathrm{d}u$, $\mathrm{d}h = -2\sqrt{h}\,\mathrm{d}u$ o.e.
M1: **Note:**	Complete method for applying $u = 4-\sqrt{h}$ to $\int \frac{\mathrm{d}h}{4-\sqrt{h}}$ to give an expression of the form $\int \frac{k(4-u)}{u}\mathrm{d}u$; $k \neq 0$ Condone the omission of an integral sign and/or $\mathrm{d}u$
M1:	Proceeds to obtain an integral of the form $\int\left(\frac{A}{u}+B\right)\{\mathrm{d}u\}$; $A, B \neq 0$
M1:	$\int\left(\frac{A}{u}+B\right)\{\mathrm{d}u\} \rightarrow D\ln u + Eu$; $A, B, D, E \neq 0$; with or without a constant of integration
A1:	$\int\left(-\frac{8}{u}+2\right)\{\mathrm{d}u\} \rightarrow -8\ln u + 2u$; with or without a constant of integration
A1*:	**dependent on all previous marks** Substitutes $u = 4-\sqrt{h}$ into their integrated result and completes the proof by obtaining the printed result $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$. Condone the use of brackets instead of the modulus sign.
Note:	They must combine 2(4) and their $+c$ correctly to give $+k$
Note:	Going from $-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) + c$ to $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$, with no intermediate working or with no incorrect working is required for the final A1* mark.
Note:	Allow A1* for correctly reaching $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + c + 8$ and stating $k = c+8$
Note:	Allow A1* for correctly reaching $-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) + k = -8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$
	Alternative (integration by parts) method for the 2nd M, 3rd M and 1st A mark $\left\{\int \frac{-2(4-u)}{u}\mathrm{d}u = \int \frac{2u-8}{u}\mathrm{d}u\right\} = (2u-8)\ln u - \int 2\ln u\,\mathrm{d}u = (2u-8)\ln u - 2(u\ln u - u)\ \{+c\}$
2nd M1:	Proceeds to obtain an integral of the form $(Au+B)\ln u - \int A\ln u\{\mathrm{d}u\}$; $A, B \neq 0$
3rd M1:	Integrates to give $D\ln u + Eu$; $D, E \neq 0$; which can be simplified or un-simplified with or without a constant of integration.
Note:	Give 3rd M1 for $(2u-8)\ln u - 2(u\ln u - u)$ because it is an un-simplified form of $D\ln u + Eu$
1st A1:	Integrates to give $(2u-8)\ln u - 2(u\ln u - u)$ or $-8\ln u + 2u$ o.e. with or without a constant of integration.
(b)	
M1: **Note:**	Uses the context of the model and has an understanding that the tree keeps growing until $\frac{\mathrm{d}h}{\mathrm{d}t} = 0 \Rightarrow 4-\sqrt{h} = 0$. Alternatively, they can write $\frac{\mathrm{d}h}{\mathrm{d}t} > 0 \Rightarrow 4-\sqrt{h} > 0$ Accept $h = 16$ or 16 used in their inequality statement for this mark.
A1:	See scheme
Note:	A correct answer can be given M1 A1 from any working.

Notes for Question 34	
(c)	**Way 1**
B1:	Separates the variables correctly. $\mathrm{d}h$ and $\mathrm{d}t$ should not be in the wrong positions, although this mark can be implied by later working. Condone absence of integral signs.
M1:	Integrates $t^{0.25}$ to give $\lambda t^{1.25}$; $\lambda \neq 0$
A1:	Correct integration. E.g. $-8\ln\left\|4-\sqrt{h}\right\|-2\sqrt{h} = \frac{1}{25}t^{1.25}$ or $20(-8\ln\left\|4-\sqrt{h}\right\|-2\sqrt{h}) = \frac{4}{5}t^{1.25}$ $-8\ln\left\|4-\sqrt{h}\right\|+2(4-\sqrt{h}) = \frac{1}{25}t^{1.25}$ or $20(-8\ln\left\|4-\sqrt{h}\right\|+2(4-\sqrt{h})) = \frac{4}{5}t^{1.25}$ with or without a constant of integration, e.g. *k*, *c* or *A*
Note:	There is no requirement for modulus signs.
M1:	Some evidence of ***applying*** both $t=0$ and $h=1$ to their model (which can be a changed equation) which contains a constant of integration, e.g. *k*, *c* or *A*
dM1:	**dependent on the previous M mark** Complete process of finding their constant of integration, followed by applying $h=12$ and their constant of integration to their changed equation
M1:	Rearranges their equation to make $t^{\text{their } 1.25} = ...$ followed by a correct method to give $t = ...$; $t > 0$
Note:	$t^{\text{their } 1.25} = ...$ can be negative, but their '$t = ...$' must be positive
Note:	"their 1.25" cannot be 0 or 1 for this mark
Note:	Do not give this mark if $t^{\text{their } 1.25} = ...$ (usually $t^{0.25} = ...$) is a result of substituting $t=12$ (or $t=11$) into the given $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}(4-\sqrt{h})}{20}$. **Note:** They will usually write $\frac{\mathrm{d}h}{\mathrm{d}t}$ as either 12 or 11.
A1:	awrt 75.2
(c)	**Way 2**
B1:	Separates the variables correctly. $\mathrm{d}h$ and $\mathrm{d}t$ should not be in the wrong positions, although this mark can be implied by later working.
Note:	Integral signs and limits are not required for this mark.
M1:	Same as Way 1 (ignore limits)
A1:	Same as Way 1 (ignore limits)
M1:	Applies limits of 1 and 12 to their model (i.e. to their changed expression in *h*) and subtracts
dM1	**dependent on the previous M mark** Complete process of applying limits of 1 and 12 and 0 and *T* (or '*t*') appropriately to their changed equation
M1:	Same as Way 1
A1:	Same as Way 1

Question	Scheme	Marks	AOs
35	$C: y = x\ln x$; l is a normal to C at $P(\mathrm{e}, \mathrm{e})$ Let x_A be the x-coordinate of where l cuts the x-axis		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \ln x + x\left(\frac{1}{x}\right) \quad \{= 1 + \ln x\}$	M1 A1	2.1 1.1b
	$x = \mathrm{e},\ m_T = 2 \Rightarrow m_N = -\frac{1}{2} \Rightarrow y - \mathrm{e} = -\frac{1}{2}(x - \mathrm{e})$ $y = 0 \Rightarrow -\mathrm{e} = -\frac{1}{2}(x - \mathrm{e}) \Rightarrow x = ...$	M1	3.1a
	l meets x-axis at $x = 3\mathrm{e}$ (allow $x = 2\mathrm{e} + \mathrm{e}\ln\mathrm{e}$)	A1	1.1b
	{Areas:} **either** $\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ ...\ ]_1^{\mathrm{e}} = ...$ **or** $\frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$	M1	2.1
	$\left\{\int x\ln x\,\mathrm{d}x = \right\}\ \frac{1}{2}x^2\ln x - \int \frac{1}{x}.\left(\frac{x^2}{2}\right)\{\mathrm{d}x\}$	M1	2.1
	$\left\{= \frac{1}{2}x^2\ln x - \int \frac{1}{2}x\,\{\mathrm{d}x\}\right\} = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2$	dM1 A1	1.1b 1.1b
	$\text{Area}(R_1) = \int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ ...\ ]_1^{\mathrm{e}} = ...$; $\text{Area}(R_2) = \frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$ and so, $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2) \quad \{= \frac{1}{4}\mathrm{e}^2 + \frac{1}{4} + \mathrm{e}^2\}$	M1	3.1a
	$\text{Area}(R) = \frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$	A1	1.1b
		(10)	

Notes for Question **35**	
M1:	Differentiates by using the product rule to give $\ln x + x(\text{their } \mathrm{g}'(x))$, where $\mathrm{g}(x) = \ln x$
A1:	Correct differentiation of $y = x\ln x$, which can be un-simplified or simplified
M1:	Complete strategy to find the x coordinate where their normal to C at $P(\mathrm{e}, \mathrm{e})$ meets the x-axis i.e. Sets $y = 0$ in $y - \mathrm{e} = m_N(x - \mathrm{e})$ to find $x = ...$
Note:	m_T is found by using calculus and $m_N \neq m_T$
A1:	l meets x-axis at $x = 3\mathrm{e}$, allowing un-simplified values for x such as $x = 2\mathrm{e} + \mathrm{e}\ln\mathrm{e}$
Note:	Allow $x =$ awrt 8.15
M1:	Scored for either • Area under curve $= \int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ ...\ ]_1^{\mathrm{e}} = ...$, with limits of e and 1 and some attempt to substitute these and subtract • **or** Area under line $= \frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$, with a valid attempt to find x_A
M1:	Integration by parts the correct way around to give $Ax^2\ln x - \int B\left(\frac{x^2}{x}\right)\{\mathrm{d}x\}$; $A \neq 0, B > 0$
dM1:	**dependent on the previous M mark** Integrates the second term to give $\pm\lambda x^2$; $\lambda \neq 0$
A1:	$\frac{1}{2}x^2\ln x - \frac{1}{4}x^2$
M1:	Complete strategy of finding the area of R by finding the sum of two key areas. See scheme.
A1:	$\frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$

Notes for Question **35** Continued	
Note:	$\text{Area}(R_2)$ can also be found by integrating the line l between limits of e and their x_A i.e. $\text{Area}(R_2) = \int_{\text{e}}^{\text{their } x_A} \left(-\frac{1}{2}x + \frac{3}{2}\text{e}\right) \text{d}x = \left[\ \dots\ \right]_{\text{e}}^{\text{their } x_A} = \dots$
Note:	**<u>Calculator approach with no algebra, differentiation or integration seen:</u>** • Finding l cuts through the x-axis at awrt 8.15 is 2nd M1 2nd A1 • Finding area between curve and the x-axis between $x = 1$ and $x = \text{e}$ to give awrt 2.10 is 3rd M1 • Using the above information (must be seen) to apply $\text{Area}(R) = 2.0972... + 7.3890... = 9.4862...$ is final M1 Therefore, a maximum of 4 marks out of the 10 available.

Question	Scheme	Marks	AOs
36 (a)	$x > \ln\left(\frac{4}{3}\right)$	B1	2.2a
		(1)	
(b)	Attempts to apply $\int y \frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t$	M1	3.1a
	$\left\{\int y \frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t \;=\right\} = \int \left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right)\mathrm{d}t$	A1	1.1b
	$\frac{1}{(t+1)(t+2)} \equiv \frac{A}{(t+1)} + \frac{B}{(t+2)} \Rightarrow 1 \equiv A(t+2) + B(t+1)$	M1	3.1a
	$\{A=1,\ B=-1 \Rightarrow\}$ gives $\frac{1}{(t+1)} - \frac{1}{(t+2)}$	A1	1.1b
	$\left\{\int\left(\frac{1}{(t+1)} - \frac{1}{(t+2)}\right)\mathrm{d}t \;=\right\}\ \ln(t+1) - \ln(t+2)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = \left[\ln(t+1) - \ln(t+2)\right]_0^2 = (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	M1	2.2a
	$= \ln 3 - \ln 4 + \ln 2 = \ln\left(\frac{(3)(2)}{4}\right) = \ln\left(\frac{6}{4}\right)$		
	$= \ln\left(\frac{3}{2}\right)$ *	A1*	2.1
		(8)	
(b) Alt 1	Attempts to apply $\int y\,\mathrm{d}x = \int \frac{1}{\mathrm{e}^x - 2 + 1}\,\mathrm{d}x = \int \frac{1}{\mathrm{e}^x - 1}\,\mathrm{d}x$, with a substitution of $u = \mathrm{e}^x - 1$	M1	3.1a
	$\left\{\int y\,\mathrm{d}x\right\} = \int\left(\frac{1}{u}\right)\left(\frac{1}{u+1}\right)\mathrm{d}u$	A1	1.1b
	$\frac{1}{u(u+1)} \equiv \frac{A}{u} + \frac{B}{(u+1)} \Rightarrow 1 \equiv A(u+1) + Bu$	M1	3.1a
	$\{A=1,\ B=-1 \Rightarrow\}$ gives $\frac{1}{u} - \frac{1}{(u+1)}$	A1	1.1b
	$\left\{\int\left(\frac{1}{u} - \frac{1}{(u+1)}\right)\mathrm{d}u \;=\right\}\ \ln u - \ln(u+1)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = \left[\ln u - \ln(u+1)\right]_1^3 = (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	M1	2.2a
	$= \ln 3 - \ln 4 + \ln 2 = \ln\left(\frac{(3)(2)}{4}\right) = \ln\left(\frac{6}{4}\right)$		
	$= \ln\left(\frac{3}{2}\right)$ *	A1 *	2.1
		(8)	
	(9 marks)		

Question	Scheme	Marks	AOs
36 (b) Alt 2	Attempts to apply $\int y\,\mathrm{d}x = \int \frac{1}{\mathrm{e}^x - 2 + 1}\,\mathrm{d}x = \int \frac{1}{\mathrm{e}^x - 1}\,\mathrm{d}x$, with a substitution of $v = \mathrm{e}^x$	M1	3.1a
	$\left\{\int y\,\mathrm{d}x\right\} = \int \left(\frac{1}{v-1}\right)\left(\frac{1}{v}\right)\mathrm{d}v$	A1	1.1b
	$\frac{1}{(v-1)v} \equiv \frac{A}{(v-1)} + \frac{B}{v} \Rightarrow 1 \equiv Av + B(v-1)$	M1	3.1a
	$\{A = 1,\ B = -1 \Rightarrow\}$ gives $\frac{1}{(v-1)} - \frac{1}{v}$	A1	1.1b
	$\left\{\int \left(\frac{1}{(v-1)} - \frac{1}{v}\right)\mathrm{d}v = \right\} \ln(v-1) - \ln v$	M1 A1	1.1b 1.1b
	$\text{Area}(R) = \left[\ln(v-1) - \ln v\right]_2^4 = (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	M1	2.2a
	$= \ln 3 - \ln 4 + \ln 2 = \ln\left(\frac{(3)(2)}{4}\right) = \ln\left(\frac{6}{4}\right)$		
	$= \ln\left(\frac{3}{2}\right)$ *	A1 *	2.1
		(8)	

Question **36** Notes:	
(a)	
B1:	Uses $x = \ln(t+2)$ with $t > -\dfrac{2}{3}$ to deduce the correct domain, $x > \ln\left(\dfrac{4}{3}\right)$
(b)	
M1:	Attempts to solve the problem by either • a parametric process or • a Cartesian process with a substitution of either $u = e^x - 1$ or $v = e^x$
A1:	Obtains • $\int \left(\dfrac{1}{t+1}\right)\left(\dfrac{1}{t+2}\right) dt$ from a parametric approach • $\int \left(\dfrac{1}{u}\right)\left(\dfrac{1}{u+1}\right) du$ from a Cartesian approach with $u = e^x - 1$ • $\int \left(\dfrac{1}{v-1}\right)\left(\dfrac{1}{v}\right) dv$ from a Cartesian approach with $v = e^x$
M1:	Applies a strategy of attempting to express either $\dfrac{1}{(t+1)(t+2)}$, $\dfrac{1}{u(u+1)}$ or $\dfrac{1}{(v-1)v}$ as partial fractions
A1:	Correct partial fractions for their method
M1:	Integrates to give either • $\pm\alpha \ln(t+1) \pm \beta \ln(t+2)$ • $\pm\alpha \ln u \pm \beta \ln(u+1)$; $\alpha, \beta \neq 0$, where $u = e^x - 1$ • $\pm\alpha \ln(v-1) \pm \beta \ln v$; $\alpha, \beta \neq 0$, where $v = e^x$
A1:	Correct integration for their method
M1:	Either • Parametric approach: Deduces and applies limits of 2 and 0 in t and subtracts the correct way round • Cartesian approach: Deduces and applies limits of 3 and 1 in u, where $u = e^x - 1$, and subtracts the correct way round • Cartesian approach: Deduces and applies limits of 4 and 2 in v, where $v = e^x$, and subtracts the correct way round
A1*:	Correctly shows that the area of R is $\ln\left(\dfrac{3}{2}\right)$, with no errors seen in their working

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="4">37</td><td colspan="2">$\int\left(3x^{0\,5}+A\right)\mathrm{d}x = 2x^{1\,5}+Ax(+c)$</td><td>M1
A1</td><td>3.1a
1.1b</td></tr>
<tr><td colspan="2">Uses limits and sets = $2A^2 \Rightarrow (2\times 8+4A)-(2\times 1+A)=2A^2$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>Sets up quadratic and attempts to solve</td><td>Sets up quadratic and attempts b^2-4ac</td><td>M1</td><td>1.1b</td></tr>
<tr><td>$\Rightarrow A=-2, \frac{7}{2}$ and states that there are two roots</td><td>States $b^2-4ac=121>0$ and hence there are two roots</td><td>A1</td><td>2.4</td></tr>
<tr><td colspan="5" align="right">(5 marks)</td></tr>
</table>

Notes:

M1: Integrates the given function and achieves an answer of the form $kx^{1\,5}+Ax(+c)$ where k is a non- zero constant

A1: Correct answer but may not be simplified

M1: Substitutes in limits and subtracts. This can only be scored if $\int A\,\mathrm{d}x = Ax$ and not $\frac{A^2}{2}$

M1: Sets up quadratic equation in A and either attempts to solve or attempts b^2-4ac

A1: Either $A=-2, \frac{7}{2}$ **and** states that there are two roots

Or states $b^2-4ac=121>0$ and hence there are two roots

Question	Scheme	Marks	AOs
38	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{15}{2}x^{\frac{1}{2}} - 9$	M1 A1	3.1a 1.1b
	Substitutes $x = 4 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 6$	M1	2.1
	Uses (4, 15) and gradient $\Rightarrow y - 15 = 6(x - 4)$	M1	2.1
	Equation of l is $y = 6x - 9$	A1	1.1b
	Area $R = \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) - (6x - 9)\,\mathrm{d}x$	M1	3.1a
	$= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x(+c)\right]_0^4$	A1	1.1b
	Uses both limits of 4 and 0 $\left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x\right]_0^4 = 2 \times 4^{\frac{5}{2}} - \frac{15}{2} \times 4^2 + 20 \times 4 - 0$	M1	2.1
	Area of $R = 24$ *	A1*	1.1b
	Correct notation with good explanations	A1	2.5
		(10)	
			(10 marks)

Question 38 continued

Notes:

M1: Differentiates $5x^{\frac{3}{2}} - 9x + 11$ to a form $Ax^{\frac{1}{2}} + B$

A1: $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{15}{2}x^{\frac{1}{2}} - 9$ but may not be simplified

M1: Substitutes $x = 4$ in their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to find the gradient of the tangent

M1: Uses their gradient and the point (4, 15) to find the equation of the tangent

A1: Equation of l is $y = 6x - 9$

M1: Uses Area $R = \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) - (6x - 9)\,\mathrm{d}x$ following through on their $y = 6x - 9$

Look for a form $Ax^{\frac{5}{2}} + Bx^2 + Cx$

A1: $= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x(+c)\right]_0^4$ This must be correct but may not be simplified

M1: Substitutes in both limits and subtracts

A1*: Correct area for $R = 24$

A1: Uses correct notation and produces a well explained and accurate solution. Look for

- Correct notation used consistently and accurately for both differentiation and integration
- Correct explanations in producing the equation of l. See scheme.
- Correct explanation in finding the area of R. In way 2 a diagram may be used.

Alternative method for the area using area under curve and triangles. (Way 2)

M1: Area under curve $= \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) = \left[Ax^{\frac{5}{2}} + Bx^2 + Cx\right]_0^4$

A1: $= \left[2x^{\frac{5}{2}} - \frac{9}{2}x^2 + 11x\right]_0^4 = 36$

M1: This requires a full method with all triangles found using a correct method

Look for Area $R = \text{their } 36 - \frac{1}{2} \times 15 \times \left(4 - \text{their } \frac{3}{2}\right) + \frac{1}{2} \times \text{their } 9 \times \text{their } \frac{3}{2}$

Question	Scheme	Marks	AOs
39(a)	Sets $\dfrac{1}{P(11-2P)}=\dfrac{A}{P}+\dfrac{B}{(11-2P)}$	B1	1.1a
	Substitutes either $P=0$ or $P=\dfrac{11}{2}$ into $1=A(11-2P)+BP\Rightarrow A\ or\ B$	M1	1.1b
	$\dfrac{1}{P(11-2P)}=\dfrac{1/11}{P}+\dfrac{2/11}{(11-2P)}$	A1	1.1b
		(3)	
(b)	Separates the variables $\displaystyle\int\frac{22}{P(11-2P)}\mathrm{d}P=\int 1\,\mathrm{d}t$	B1	3.1a
	Uses (a) and attempts to integrate $\displaystyle\int\frac{2}{P}+\frac{4}{(11-2P)}\mathrm{d}P=t+c$	M1	1.1b
	$2\ln P-2\ln(11-2P)=t+c$	A1	1.1b
	Substitutes $t=0,P=1\Rightarrow t=0,P=1\Rightarrow\ c=(-2\ln 9)$	M1	3.1a
	Substitutes $P=2\Rightarrow t=2\ln 2+2\ln 9-2\ln 7$	M1	3.1a
	Time $=1.89$ years	A1	3.2a
		(6)	
(c)	Uses ln laws $2\ln P-2\ln(11-2P)=t-2\ln 9$ $\Rightarrow\ln\left(\dfrac{9P}{11-2P}\right)=\dfrac{1}{2}t$	M1	2.1
	Makes 'P' the subject $\Rightarrow\left(\dfrac{9P}{11-2P}\right)=\mathrm{e}^{\frac{1}{2}t}$ $\Rightarrow 9P=(11-2P)\mathrm{e}^{\frac{1}{2}t}$ $\Rightarrow P=\mathrm{f}\left(\mathrm{e}^{\frac{1}{2}t}\right)$ or $\Rightarrow P=\mathrm{f}\left(\mathrm{e}^{-\frac{1}{2}t}\right)$	M1	2.1
	$\Rightarrow P=\dfrac{11}{2+9\mathrm{e}^{-\frac{1}{2}t}}\Rightarrow A=11,B=2,C=9$	A1	1.1b
		(3)	
		(12 marks)	

Question 39 continued

Notes:

(a)

B1: Sets $\dfrac{1}{P(11-2P)}=\dfrac{A}{P}+\dfrac{B}{(11-2P)}$

M1: Substitutes $P=0$ or $P=\dfrac{11}{2}$ into $1=A(11-2P)+BP \Rightarrow A \text{ or } B$

Alternatively compares terms to set up and solve two simultaneous equations in A and B

A1: $\dfrac{1}{P(11-2P)}=\dfrac{1/11}{P}+\dfrac{2/11}{(11-2P)}$ or equivalent $\dfrac{1}{P(11-2P)}=\dfrac{1}{11P}+\dfrac{2}{11(11-2P)}$

Note: The correct answer with no working scores all three marks.

(b)

B1: Separates the variables to reach $\displaystyle\int \frac{22}{P(11-2P)}\,\mathrm{d}P=\int 1\,\mathrm{d}t$ or equivalent

M1: Uses part (a) and $\displaystyle\int \frac{A}{P}+\frac{B}{(11-2P)}\mathrm{d}P = A\ln P \pm C\ln(11-2P)$

A1: Integrates both sides to form a correct equation including a 'c' Eg $2\ln P-2\ln\left(11-2P\right)=t+c$

M1: Substitutes $t=0$ and $P=1$ to find c

M1: Substitutes $P=2$ to find t. This is dependent upon having scored both previous M's

A1: Time = 1.89 years

(c)

M1: Uses correct log laws to move from $2\ln P-2\ln\left(11-2P\right)=t+c$ to $\ln\left(\dfrac{P}{11-2P}\right)=\dfrac{1}{2}t+d$ for their numerical 'c'

M1: Uses a correct method to get P in terms of $\mathrm{e}^{\frac{1}{2}t}$

This can be achieved from $\ln\left(\dfrac{P}{11-2P}\right)=\dfrac{1}{2}t+d \Rightarrow \dfrac{P}{11-2P}=\mathrm{e}^{\frac{1}{2}t+d}$ followed by cross multiplication and collection of terms in P (See scheme)

Alternatively uses a correct method to get P in terms of $\mathrm{e}^{-\frac{1}{2}t}$ For example $\dfrac{P}{11-2P}=\mathrm{e}^{\frac{1}{2}t+d} \Rightarrow \dfrac{11-2P}{P}=\mathrm{e}^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \dfrac{11}{P}-2=\mathrm{e}^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \dfrac{11}{P}=2+\mathrm{e}^{-\left(\frac{1}{2}t+d\right)}$ followed by division

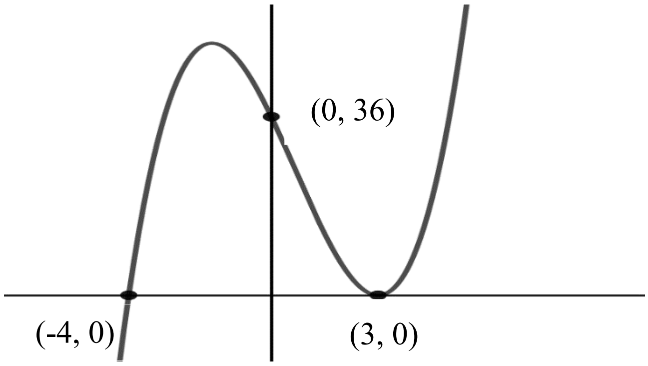
A1: Achieves the correct answer in the form required. $P=\dfrac{11}{2+9\mathrm{e}^{-\frac{1}{2}t}} \Rightarrow A=11, B=2, C=9$ oe

Question Number	Scheme		Marks
40.	$2x^{1.5}-3x^{2}+4x+c$	M1: For $x^n \to x^{n+1}$ i.e. $x^{1.5}$ or x^2or x seen (not for "+ c")	M1A1A1
		A1: For two out of three terms correct un-simplified or simplified (Ignore + c for this mark)	
		A1: cao $2x^{1.5}-3x^{2}+4x+c$. All correct and simplified and on one line including "+ c". Allow $\sqrt{x^3}$ for $x^{1.5}$ but not x^1 for x.	
	Ignore any spurious integral signs.		
			(3)
			(3 marks)

EXPERT TUITION

Question Number	Scheme		Marks
41.(a)	$(x-3)(3x+5)=3x^2-4x-15$ Allow $3x^2+5x-9x-15$	Correct expansion simplified or un-simplified.	B1
	$\mathrm{f}(x)=x^3-2x^2-15x+c$	M1: $x^n \to x^{n+1}$ for any term. Follow through on incorrect indices but not for "+ c" A1: All terms correct. Need not be simplified. No need for + c here.	M1A1
	$x=1, y=20 \Rightarrow 20=1-2-15+c$ $\Rightarrow c=36$	Substitutes x = 1 and y = 20 into their f(x) to find c. Must have + c at this stage. **Dependent on the first method mark.**	**d**M1
	$(\mathrm{f}(x)=)x^3-2x^2-15x+36$	Cao $(\mathrm{f}(x)=)x^3-2x^2-15x+36$ (All together and on one line)	A1
			(5)
(b) **Way 1**	$A=4$	Correct value (may be implied)	B1
	$\mathrm{f}(x)=(x-3)^2(x+A)=(x^2-6x+9)(x+A)$ $\mathrm{f}(x)=x^3+(A-6)x^2+(9-6A)x+9A$ $A-6=-2\Rightarrow A=4 \quad 9-6A=-15\Rightarrow A=4 \quad 9A=36\Rightarrow A=4$ M1: Expands $(x-3)^2(x+A)$ and compares coefficients with their f(x) from part (a) to **form 3 equations** and attempts to solve **at least two of them** in an attempt to show that A is the same in each case or substitutes their A to show that the coefficients are the same. A1: Fully correct proof – must use all 3 coefficients		M1A1
			(3)
Way 2	$A=4$	Correct value (may be implied)	B1
	$\mathrm{f}(x)=(x-3)^2(x+4)=(x^2-6x+9)(x+4)$ $=x^3-6x^2+4x^2+9x-24x+36=x^3-2x^2-15x+36$ M1: Expands $(x-3)^2(x+"4")$ fully in an attempt to show that the expansion gives the same expression found as found in part (a) A1: Fully correct proof (Condone invisible brackets here e.g. around x + 4 provided sufficient working is shown)		M1A1
			(3)
Way 3	$A=4$	Correct value (may be implied)	B1
	$(x^3-2x^2-15x+36)\div(x-3)=x^2+x-12$ $(x^2+x-12)\div(x-3)=x+4$ or $(x^2+x-12)=(x+4)(x-3)$ M1: Divides their f(x) from part (a) by (x – 3) and divides their quotient by (x – 3) in an attempt to establish the value of A. Alternatively divides their f(x) from part (a) by $(x-3)^2$ (Allow $x^2\pm6x\pm9$) in an attempt to establish the value of A. A1: Fully correct proof		M1A1
			(3)
	Note that this is an acceptable proof: A = 4 (may be implied) $x^3-2x^2-15x+36=(x-3)(x^2+x-12)$ $=(x-3)(x-3)(x+4)$ $=(x-3)^2(x+4)$		

<u>Remember to check the last page for their sketch</u>

41(c)		
	A positive cubic shape. Its position is not important but must be a curve and not straight lines and the "ends" must not clearly turn back in on themselves.	B1
	Touches at the point (3, 0) (could be a maximum). Accept 3 marked on the x-axis and accept (0, 3) as long as it is in the correct place. Allow (3, 0) in the body of the script but it must correspond with the sketch. If ambiguous, the sketch takes precedence.	B1
	Crosses or reaches the x-axis at $(-4, 0)$. Accept -4 marked on the x-axis and accept (0, -4) as long as it is in the correct place. FT on their $-A$ from part (b) and allow "$-A$" and allow a "made up" A. Allow $(-4, 0)$ in the body of the script but it must correspond with the sketch. If ambiguous, the sketch takes precedence.	B1ft
	Crosses the y-axis at (0, 36) **and with a maximum in the second quadrant**. Accept 36 marked on the y – axis and accept (36, 0) as long as it is in the correct place. FT on their numerical 'c' **<u>from part (a) only</u>**. Allow (0, 36) in the body of the script but it must correspond with the sketch. If ambiguous, the sketch takes precedence.	B1ft
		(4)
		(12 marks)

Question Number	Scheme		Marks
42.	$\int\left(2x^5-\frac{1}{4}x^{-3}-5\right)dx$		
	Ignore any spurious integral signs throughout		
	$x^n \to x^{n+1}$	Raises any of their powers by 1. E.g. $x^5 \to x^6$ or $x^{-3} \to x^{-2}$ or $k \to kx$ or $x^{\text{their } n} \to x^{\text{their } n+1}$. Allow the powers to be un-simplified e.g. $x^5 \to x^{5+1}$ or $x^{-3} \to x^{-3+1}$ or $kx^0 \to kx^{0+1}$.	M1
	$2\times\frac{x^{5+1}}{6}$ or $-\frac{1}{4}\times\frac{x^{-3+1}}{-2}$	Any one of the first two terms correct **<u>simplified or un-simplified</u>**.	A1
	Two of: $\frac{1}{3}x^6,\ \frac{1}{8}x^{-2},\ -5x$	Any two correct **<u>simplified</u>** terms. Accept $+\frac{1}{8x^2}$ for $+\frac{1}{8}x^{-2}$ but not x^1 for x. Accept 0.125 for $\frac{1}{8}$ but $\frac{1}{3}$ would clearly need to be identified as 0.3 recurring.	A1
	$\frac{1}{3}x^6+\frac{1}{8}x^{-2}-5x+c$	All correct and simplified and including + c all on one line. Accept $+\frac{1}{8x^2}$ for $+\frac{1}{8}x^{-2}$ but not x^1 for x. Apply isw here.	A1
			(4 marks)

<table>
<tr><td>Question Number</td><td colspan="2">Scheme</td><td>Marks</td></tr>
<tr><td rowspan="7">43</td><td colspan="2">Allow the marks in (b) to score in (a) i.e. <u>mark (a) and (b) together</u></td><td></td></tr>
<tr><td rowspan="3">$\Rightarrow \mathrm{f}(x)=30x+6\dfrac{x^{\frac{1}{2}}}{0.5}-5\dfrac{x^{\frac{5}{2}}}{2.5}(+c)$</td><td>M1: $30 \to 30x$ or $\dfrac{6}{\sqrt{x}} \to \alpha x^{\frac{1}{2}}$ or
$-\dfrac{5x^2}{\sqrt{x}} \to \beta x^{\frac{5}{2}}$ (these cases only)</td><td rowspan="3">M1A1A1</td></tr>
<tr><td>A1: Any 2 correct terms which can be simplified or un-simplified. This includes the powers – so allow $-\frac{1}{2}+1$ for $\frac{1}{2}$ and allow $\frac{3}{2}+1$ for $\frac{5}{2}$
(With or without + c)</td></tr>
<tr><td>A1: All 3 terms correct which can be simplified or un-simplified.
(With or without + c)</td></tr>
<tr><td colspan="2">Ignore any spurious integral signs</td><td></td></tr>
<tr><td>$x=4, \mathrm{f}(x)=-8 \Rightarrow$
$-8=120+24-64+c \Rightarrow c=\ldots$</td><td>Substitutes $x=4, \mathrm{f}(x)=-8$ into their f(x) (not $\mathrm{f}'(x)$) i.e. a changed $\mathrm{f}'(x)$ containing $+c$ and rearranges to obtain a value or numerical expression for c.</td><td>M1</td></tr>
<tr><td>$\Rightarrow (\mathrm{f}(x)=)\,30x+12x^{\frac{1}{2}}-2x^{\frac{5}{2}}-88$</td><td>Cao and cso (Allow $\sqrt{x}$ for $x^{\frac{1}{2}}$ and e.g. $\sqrt{x^5}$ or $x^2\sqrt{x}$ for $x^{\frac{5}{2}}$).
Isw here so as soon as you see the correct answer, award this mark.
Note that the “f(x) =” is not needed.</td><td>A1</td></tr>
<tr><td colspan="2"></td><td>(5)</td></tr>
<tr><td></td><td colspan="2"></td><td>(5 marks)</td></tr>
</table>

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td rowspan="6">44</td><td colspan="2">$\int (2x^4 - \frac{4}{\sqrt{x}} + 3)\,\mathrm{d}x$</td><td></td></tr>
<tr><td rowspan="3">$\frac{2}{5}x^5 - \frac{4}{\frac{1}{2}}x^{\frac{1}{2}} + 3x$</td><td>M1: $x^n \to x^{n+1}$. One power increased by 1 but not for just + c.
This could be for 3 → $3x$ or for $x^n \to x^{n+1}$ on what they think $\frac{1}{\sqrt{x}}$ is as a power of x.</td><td rowspan="3">M1A1A1</td></tr>
<tr><td>A1: One of these 3 terms correct.
Allow un-simplified e.g. $\frac{2x^{4+1}}{4+1}$, $-\frac{4x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$, $3x^1$</td></tr>
<tr><td>A1: Two of these 3 terms correct.
Allow un-simplified e.g. $\frac{2x^{4+1}}{4+1}$, $-\frac{4x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$, $3x^1$</td></tr>
<tr><td>$= \frac{2}{5}x^5 - 8x^{\frac{1}{2}} + 3x + c$</td><td><u>Complete fully correct simplified expression appearing all on one line with constant</u>. Allow 0.4 for $\frac{2}{5}$.
Do not allow $3x^1$ for $3x$
Allow $\sqrt{x}$ or $x^{0.5}$ for $x^{\frac{1}{2}}$</td><td>A1</td></tr>
<tr><td colspan="2">Ignore any spurious integral signs and ignore subsequent working following a fully correct answer.</td><td></td></tr>
<tr><td></td><td></td><td></td><td>[4]</td></tr>
<tr><td></td><td></td><td></td><td>4 marks</td></tr>
</table>

Question Number	Scheme		Marks
45.	$y = 4x^3 - \frac{5}{x^2}$		
	$x^4 + \frac{5}{x} + c$ or $x^4 + 5x^{-1} + c$	M1: $x^n \to x^{n+1}$. e.g. Sight of x^4 or x^{-1} or $\frac{1}{x^1}$ **Do <u>not</u> award for integrating their answer to part (a)**	M1A1A1
		A1: $4\frac{x^4}{4}$ or $-5\times\frac{x^{-1}}{-1}$	
		A1: For fully correct and simplified answer with + c <u>all on one line</u>. Allow $x^4 + 5\times\frac{1}{x} + c$ Allow $1x^4$ for x^4	
	Apply ISW here and award marks when first seen. Ignore spurious integral signs for all marks.		
			(3)
			(3 marks)

Question Number	Scheme		Marks
46	$\mathrm{f}(x)=x^{\frac{3}{2}}-\frac{9}{2}x^{\frac{1}{2}}+2x(+c)$	M1: $x^n \to x^{n+1}$ A1: Two terms in x correct, simplification is not required in coefficients or powers A1: All terms in x correct. Simplification not required in coefficients or powers and + c is not required	M1A1A1
	Sub $x=4, y=9$ into $\mathrm{f}(x) \Rightarrow c=...$	M1: Sub $x = 4$, $y = 9$ into f (x) to obtain a value for c. If no + c then M0. Use of $x = 9$, $y = 4$ is M0.	M1
	$(\mathrm{f}(x)=)x^{\frac{3}{2}}-\frac{9}{2}x^{\frac{1}{2}}+2x+2$	Accept equivalents **but must be simplified** e.g. $\mathrm{f}(x)=x^{\frac{3}{2}}-4.5\sqrt{x}+2x+2$ Must be all 'on one line' **and simplified**. Allow $x\sqrt{x}$ for $x^{\frac{3}{2}}$	A1
			(5)
			(5 marks)

Question Number	Scheme	Marks
47.	$\int(8x^3+4)\,dx \;=\; \dfrac{8x^4}{4}+4x$	M1, A1
	$= 2x^4+4x+c$	A1
		(3 marks)

Notes

M1 $x^n \to x^{n+1}$ so $x^3 \to x^4$ or $4 \to 4x$ or $4x^1$

A1 This is for either term with coefficient unsimplified (power must be simplified)– so $\dfrac{8}{4}x^4$ or $4x$ (accept $4x^1$)

A1 Fully correct simplified solution with c i.e. $2\,x^4 + 4x + c$ [allow $2x^4+4x+cx^0$]

If the answer is given as $\int\; 2x^4+4x+c$, with an integral sign – having never been seen as the fully correct simplified answer without an integral sign – then give M1A1A0 but allow anything before the = sign e.g. $y = 2\,x^4 + 4x + c$, $f(x) = 2\,x^4 + 4x + c$, $\int = \; 2x^4+4x+c$, etc….

If this answer is followed by (for example) x^4+2x+k then treat this as **isw** (ignore subsequent work)
If they follow it by finding a value for c, also **isw**, provided correct answer with c has been seen and credited

Question Number	Scheme	Marks
48.	$\mathrm{f}(x)=\int\left(\frac{3}{8}x^2-10x^{-\frac{1}{2}}+1\right)\mathrm{d}x$	
	$x^n \to x^{n+1} \Rightarrow \mathrm{f}(x)=\frac{3}{8}\times\frac{x^3}{3}-10\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+x(+c)$	M1, A1, A1
	Substitute $x=4, y=25 \Rightarrow 25 = 8 - 40 + 4 + c \Rightarrow c =$	M1
	$(\mathrm{f}(x))=\frac{x^3}{8}-20x^{\frac{1}{2}}+x+53$	A1
		(5)
		(5 Marks)

Notes

M1 Attempt to integrate $x^n \to x^{n+1}$

A1 Term in x^3 **or** term in $x^{\frac{1}{2}}$ correct, coefficient need not be simplified, no need for $+x$ nor +c

A1 ALL three terms correct, coefficients need not be simplified, no need for +c

M1 For using $x = 4$, $y = 25$ in their f(x) to form a linear equation in c and attempt to find c

A1 $=\frac{x^3}{8}-20x^{\frac{1}{2}}+x+53$ cao (all coefficients and powers must be simplified to give this answer- do not need a left hand side and if there is one it may be f(x) or y). Need full expression with 53

These marks need to be scored in part (a)

Question Number	Scheme	Marks
49	$$\int 2x^5 + \frac{6}{\sqrt{x}}\,\mathrm{d}x$$	
	$x^n \to x^{n+1}$	M1
	$$= \frac{x^6}{3} + 12x^{\frac{1}{2}} + c$$	A1 A1
		(3) (3 marks)

M1 For $x^n \to x^{n+1}$. ie. x^6 or $x^{\frac{1}{2}}$ or $\left(\sqrt{x}\right)$ seen

Do not award for integrating their answer to part (a)

A1 For either $2\frac{x^6}{6}$ **or** $6\times\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$ or simplified or unsimplified equivalents

A1 For fully correct and simplified answer with $+c$.

EXPERT TUITION

Question Number	Scheme	Marks
50.	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^{-\frac{1}{2}} + x\sqrt{x}$	
	$x\sqrt{x} = x^{\frac{3}{2}}$	B1
	$x^n \to x^{n+1}$	M1
	$y = \frac{6}{\frac{1}{2}}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}(+c)$	A1, A1
	Use $x=4, y=37$ to give equation in c, $37 = 12\sqrt{4} + \frac{2}{5}(\sqrt{4})^5 + c$	M1
	$\Rightarrow c = \frac{1}{5}$ or equivalent eg. 0.2	A1
	$(y) = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$	A1
		(7 marks)

B1 $x\sqrt{x} = x^{\frac{3}{2}}$. This may be implied by $+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ oe in the subsequent work.

M1 $x^n \to x^{n+1}$ in at least one case so see either $x^{\frac{1}{2}}$ or $x^{\frac{5}{2}}$ or both

A1 One term integrated correctly. It does not have to be simplified Eg. $\frac{6}{\frac{1}{2}}x^{\frac{1}{2}}$ or $+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}$.

No need for $+c$

A1 Other term integrated correctly. See above. No need to simplify nor for $+c$. Need to see

$\frac{6}{\frac{1}{2}}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ or a simplified correct version

M1 Substitute $x = 4, y = 37$ to produce an equation in c.

A1 Correctly calculates $c = \frac{1}{5}$ or equivalent e.g. 0.2

A1 cso $y = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$. Allow $5y = 60x^{\frac{1}{2}} + 2x^{\frac{5}{2}} + 1$ and accept fully simplified equivalents.

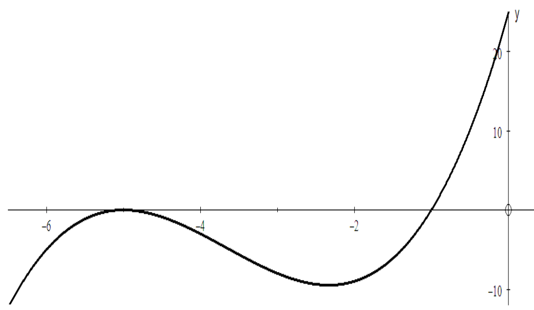
e.g. $y = \frac{1}{5}(60x^{\frac{1}{2}} + 2x^{\frac{5}{2}} + 1)$, $y = 12\sqrt{x} + \frac{2}{5}\sqrt{x^5} + \frac{1}{5}$

Question Number	Scheme		Marks
51	$(\int =)\dfrac{10x^5}{5}-\dfrac{4x^2}{2},-\dfrac{3x^{\frac{1}{2}}}{\frac{1}{2}}$	M1: Some attempt to integrate: $x^n \to x^{n+1}$ on at least one term. (not for + c) (If they think $\dfrac{3}{\sqrt{x}}$ is $3x^{\frac{1}{2}}$ you can still award the method mark for $x^{\frac{1}{2}} \to x^{\frac{3}{2}}$	M1A1, A1
		A1: $\dfrac{10x^5}{5}$ **and** $\dfrac{-4x^2}{2}$ or better	
		A1: $-\dfrac{3x^{\frac{1}{2}}}{\frac{1}{2}}$ or better	
	$= \underline{2x^5 - 2x^2 - 6x^{\frac{1}{2}} + c}$	Each term correct and simplified and the + c all appearing together on the same line. Allow $\sqrt{x}$ for $x^{\frac{1}{2}}$. Ignore any spurious integral or signs and/or dy/dx's.	A1
	Do **not** apply isw. If they obtain the correct answer and then e.g. divide by 2 they lose the last mark.		
			[4]

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td>52 (a)</td><td>$(3-x^2)^2 = 9-6x^2+x^4$</td><td>An attempt to expand the numerator obtaining an expression of the form $9 \pm px^2 \pm qx^4, \quad p,q \neq 0$</td><td>M1</td></tr>
<tr><td></td><td>$9x^{-2}+x^2$</td><td>Must come from $\dfrac{9+x^4}{x^2}$</td><td>A1</td></tr>
<tr><td></td><td>-6</td><td>Must come from $\dfrac{-6x^2}{x^2}$</td><td>A1</td></tr>
<tr><td></td><td colspan="2">Alternative 1: Writes $\dfrac{(3-x^2)^2}{x^2}$ as $(3x^{-1}-x)^2$ and attempts to expand = M1
then A1A1 as in the scheme.</td><td></td></tr>
<tr><td></td><td colspan="2">Alternative 2: Sets $(3-x^2)^2 = 9+Ax^2+Bx^4$, expands $(3-x^2)^2$ and compares coefficients = M1 then A1A1 as in the scheme.</td><td></td></tr>
<tr><td></td><td></td><td></td><td>(3)</td></tr>
<tr><td></td><td colspan="2">$(f'(x) = 9x^{-2}-6+x^2)$</td><td></td></tr>
<tr><td rowspan="2">(b)</td><td rowspan="2">$-18x^{-3}+2x$</td><td>M1: $x^n \to x^{n-1}$ on separate terms at least once. Do not award for $A \to 0$ (Integrating is M0)</td><td rowspan="2">M1 A1ft</td></tr>
<tr><td>A1ft: $-18x^{-3}+2"B"x$ with a numerical B and no extra terms. (A may have been incorrect or even zero)</td></tr>
<tr><td></td><td></td><td></td><td>(2)</td></tr>
<tr><td rowspan="2">(c)</td><td rowspan="2">$f(x) = -9x^{-1}-6x+\dfrac{x^3}{3}(+c)$</td><td>M1: $x^n \to x^{n+1}$ on separate terms at least once. (Differentiating is M0)</td><td rowspan="2">M1A1ft</td></tr>
<tr><td>A1ft: $-9x^{-1}+Ax+\dfrac{Bx^3}{3}(+c)$ with numerical A and B, $A,B \neq 0$</td></tr>
<tr><td></td><td>$10 = \dfrac{-9}{-3}-6(-3)+\dfrac{(-3)^3}{3}+c$ so c = ...</td><td>Uses x = -3 and y =10 in what they think is f(x) (They may have differentiated here) but it must be a changed function i.e. not the original $f'(x)$, to form a linear equation in c and attempts to find c. No + c gets M0 and A0 unless their method implies that they are correctly finding a constant.</td><td>M1</td></tr>
<tr><td></td><td>c = -2</td><td>cso</td><td>A1</td></tr>
<tr><td></td><td>$(f(x) =) -9x^{-1}-6x+\dfrac{x^3}{3}$ + their c</td><td>Follow through their c in an otherwise (possibly un-simplified) correct expression.
Allow $-\dfrac{9}{x}$ for $-9x^{-1}$ or even $\dfrac{9x^{-1}}{-1}$.</td><td>A1ft</td></tr>
<tr><td></td><td colspan="2">Note that if they integrate in (b), no marks there but if they then go on to use their integration in (c), the marks for integration are available.</td><td></td></tr>
<tr><td></td><td></td><td></td><td>(5)</td></tr>
<tr><td></td><td></td><td></td><td>[10]</td></tr>
</table>

Question Number	Scheme	Notes	Marks
53.	$\int 3x^2 - \frac{4}{x^2}\mathrm{d}x = 3\frac{x^3}{3} - 4\frac{x^{-1}}{-1}$	M1: $x^n \to x^{n+1}$ for either term. If they write $\frac{4}{x^2}$ *as* $4x^2$ allow $x^2 \to x^3$ here.	M1,A1,A1
		A1: $3\frac{x^3}{3}$ **or** $-4\frac{x^{-1}}{-1}$ (one correct term which may be un-simplified)	
		A1: $3\frac{x^3}{3}$ **and** $-4\frac{x^{-1}}{-1}$ (both terms correct which may be un-simplified)	
	Note that M1A0A1 is not possible		
	$= x^3 + \frac{4}{x} + c$ or $x^3 + 4x^{-1} + c$	Fully correct simplified answer with + c all appearing on the same line.	A1
			[4]

Question Number	Scheme	Notes	Marks
54	$\text{f}'(x)=\dfrac{x+9}{\sqrt{x}}=\dfrac{x}{\sqrt{x}}+\dfrac{9}{\sqrt{x}}=x^{\frac{1}{2}}+9x^{-\frac{1}{2}}$	M1: Correct attempt to split into 2 separate terms or fractions. May be implied by one correct term. Divides by $x^{\frac{1}{2}}$ or multiplies by $x^{-\frac{1}{2}}$. A1: $x^{\frac{1}{2}}+9x^{-\frac{1}{2}}$ or equivalent	M1A1
	$\text{f}(x)=\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}+9\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}}(+c)$	M1: Independent method mark for $x^n \rightarrow x^{n+1}$ **on separate terms** A1: Allow un-simplified answers. No requirement for + c here	M1A1
	$\dfrac{(9)^{\frac{3}{2}}}{\frac{3}{2}}+9\dfrac{(9)^{\frac{1}{2}}}{\frac{1}{2}}+c=0\Rightarrow c=\ldots$	Substitutes x = 9 and y = 0 into their integrated expression leading to a value for c. If no c at this stage M0A0 follows unless their method implies that they are correctly finding a constant of integration.	M1
	$\text{f}(x)=\dfrac{2}{3}x^{\frac{3}{2}}+18x^{\frac{1}{2}}-72$	There is no requirement to simplify their f(x) so accept any correct un-simplified form.	A1
			(6)

Question Number	Scheme		Marks
55 **(a)**		Horizontal translation – does **not** have to cross the y-axis on the right but must at least reach the x-axis.	B1
		Touching at (-5, 0). This could be stated anywhere or -5 could be marked on the x-axis. Or (0, -5) **marked in the correct place.** Be fairly generous with 'touching' if the intention is clear.	B1
		The right hand tail of their cubic shape crossing at (-1, 0). This could be stated anywhere or -1 could be marked on the x-axis. Or (0, -1) **marked in the correct place.** The curve must **cross** the x-axis and not stop at -1.	B1
			(3)
(b)	$(x+5)^2(x+1)$	Allow $(x+3+2)^2(x-1+2)$	B1
			(1)
(c)	When x = 0, y = 25	M1: Substitutes x = 0 into their expression in **part (b)** which is not f(x). This may be implied by their answer. Note that the question asks them to use part (b) but allow independent methods.	M1 A1
		A1: y = 25 (Coordinates not needed)	
	If they expand <u>incorrectly</u> prior to substituting x = 0, score M1 A0 **NB f(x + 2) =** $x^3+11x^2+35x+25$		
			(2)
			[6]

Question Number	Scheme	Marks
56.	$\left\{\int\left(6x^2+\frac{2}{x^2}+5\right)\mathrm{d}x\right\} = \frac{6x^3}{3}+\frac{2x^{-1}}{-1}+5x\,(+c)$	M1 A1
	$= 2x^3-2x^{-1}\,;+5x+c$	A1; A1
		4
	Notes	

M1: for some attempt to integrate a term in x: $x^n \to x^{n+1}$

So seeing either $6x^2 \to \pm\lambda x^3$ or $\frac{2}{x^2} \to \pm\mu x^{-1}$ or $5 \to 5x$ is M1.

1st A1: for a correct un-simplified x^3 or x^{-1} $\left(\text{or } \frac{1}{x}\right)$ term.

2nd A1: for both x^3 and x^{-1} terms correct and simplified on the same line. Ie. $2x^3-2x^{-1}$ or $2x^3-\frac{2}{x}$.

3rd A1: for $+5x+c$. Also allow $+5x^1+c$. This needs to be written on the same line.

Ignore the incorrect use of the integral sign in candidates' responses.

Note: If a candidate scores M1A1A1A1 and their answer is NOT ON THE SAME LINE then withhold the final accuracy mark.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>57</td><td>$\mathrm{f}(x) = \dfrac{x^{1+1}}{2(2)} - \dfrac{6x^{-\frac{1}{2}+1}}{(\frac{1}{2})} + 3x\,(+c)$ or equivalent.
$\{\mathrm{f}(4) = -1 \Rightarrow\}\ \dfrac{16}{4} - 12(2) + 3(4) + c = -1$
$\{4 - 24 + 12 + c = -1 \quad \Rightarrow \quad c = 7\}$
So, $\{\mathrm{f}(x) =\}\ \dfrac{x^2}{2(2)} - \dfrac{6x^{\frac{1}{2}}}{(\frac{1}{2})} + 3x + 7$
$\left\{\text{NB: } \mathrm{f}(x) = \dfrac{x^2}{4} - 12\sqrt{x} + 3x + 7\right\}$</td><td>M1 A1
dM1

A1 cso
[4]
4</td></tr>
<tr><td></td><td>Notes</td><td></td></tr>
<tr><td>57</td><td colspan="2">1st M1: for a clear attempt to integrate $\mathrm{f}'(\ )$ with at least one correct application of $x^n \to x^{n+1}$ on $\mathrm{f}'(x) = \dfrac{1}{2}x - \dfrac{6}{\sqrt{x}} + 3$.
So seeing either $\dfrac{1}{2}x \to \pm\lambda x^{1+1}$ or $-\dfrac{6}{\sqrt{x}} \to \pm\mu x^{-\frac{1}{2}+1}$ or $3 \to 3x^{0+1}$ is M1.
1st A1: for correct un-simplified coefficients and powers (or equivalent) with or without $+c$.
2nd dM1: for use of $x = 4$ and $y = -1$ in an integrated equation to form a linear equation in c equal to -1. ie: applying $\mathrm{f}(4) = -1$. This mark is dependent on the first method mark being awarded.
A1: For $\{\mathrm{f}(x) =\}\ \dfrac{x^2}{2(2)} - \dfrac{6x^{\frac{1}{2}}}{(\frac{1}{2})} + 3x + 7$ stated on one line where coefficients can be un-simplified or simplified, but must contain one term powers. Note this mark is for correct solution only.
<u>Note: For a candidate attempting to find f(x) in part (a)</u>
If it is clear that they understand that they are finding $\mathrm{f}(x)$ in part (a); ie. by writing $\mathrm{f}(x) = \ldots$ or $y = \ldots$ then you can give credit for this working in part (b).</td></tr>
</table>

Question	Scheme	Marks
58.	$\dfrac{x^5}{5}+4x^{\frac{3}{2}}+C$	M1A1A1 (3) **3 marks**
	Notes	
	M1 for $x^n \to x^{n+1}$ applied to y only so x^5 or $x^{\frac{3}{2}}$ seen. Do not award for integrating their answer to part (a) 1st A1 for $\dfrac{x^5}{5}$ or $\dfrac{6x^{\frac{3}{2}}}{\frac{3}{2}}$ (or better). Allow $1/5x^5$ here but not for 2nd A1 2nd A1 for fully correct and simplified answer with $+C$. Allow $(1/5)x^5$ If + C appears earlier but not on a line where 2nd A1 could be scored then A0	

Question	Scheme	Marks
59.	$\left[\text{f}(x)=\right]\dfrac{3x^3}{3}-\dfrac{3x^2}{2}+5x\left[+c\right]$ <u>or</u> $\left\{x^3-\dfrac{3}{2}x^2+5x(+c)\right\}$	M1A1
	$10 = 8 - 6 + 10 + c$	M1
	$c = -2$	A1
	$\text{f}(1) = 1-\dfrac{3}{2}+5 \;\;\text{"}-2\text{"} = \underline{\dfrac{5}{2}}$ (o.e.)	A1ft (5)
		5 marks
	Notes	
	1^{st} M1 for attempt to integrate $x^n \rightarrow x^{n+1}$ 1^{st} A1 all correct, possibly unsimplified. Ignore $+c$ here. 2^{nd} M1 for using $x = 2$ <u>and</u> f(2) = 10 to form a linear equation in c. Allow sign errors. They should be substituting into a <u>changed</u> expression 2^{nd} A1 for $c = -2$ 3^{rd} A1ft for $\dfrac{9}{2}+c$ Follow through their <u>numerical</u> c ($\neq 0$) This mark is dependent on 1^{st} M1 and 1^{st} A1 only.	

Question Number	Scheme	Marks
60.	$\left(\int =\right) \dfrac{2x^6}{6} + 7x + \dfrac{x^{-2}}{-2} = \dfrac{x^6}{3} + 7x - \dfrac{x^{-2}}{2}$ $+ C$	M1 A1 A1 B1 (4) **4**
	<u>Notes</u> M1: Attempt to integrate $x^n \to x^{n+1}$ (i.e. ax^6 or ax or ax^{-2}, where a is any non-zero constant). 1st A1: Two correct terms, possibly unsimplified. 2nd A1: All three terms correct and **simplified**. Allow correct equivalents to printed answer , e.g. $\dfrac{x^6}{3} + 7x - \dfrac{1}{2x^2}$ or $\dfrac{1}{3}x^6 + 7x - \dfrac{1}{2}x^{-2}$ Allow $\dfrac{1x^6}{3}$ or $7x^1$ B1: + C appearing at any stage in part (b) (independent of previous work)	

Question Number	Scheme	Marks
61. **(a)**	$p=\frac{1}{2},\ q=2$ or $6x^{\frac{1}{2}},\ 3x^2$	B1, B1 (2)
(b)	$\frac{6x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+\frac{3x^3}{3}\quad\left(=4x^{\frac{3}{2}}+x^3\right)$	M1 A1ft
	$x=4,\ y=90$: $32+64+C=90\ \Rightarrow\ C=-6$	M1 A1
	$y=4x^{\frac{3}{2}}+x^3+\text{"}their-6\text{"}$	A1 (5) **7**
	Notes	
	(a) Accept any equivalent answers, e.g. $p=0.5$, $q=4/2$ (b) 1st M: Attempt to integrate $x^n\to x^{n+1}$ (for either term) 1st A: ft their p and q, but terms need not be simplified ($+C$ not required for this mark) 2nd M: Using $x=4$ and $y=90$ to form an equation in C. 2nd A: cao 3rd A: answer as shown with simplified correct coefficients and powers – but follow through their value for C If there is a 'restart' in part (b) it can be marked independently of part (a), but marks for part (a) cannot be scored for work seen in (b). Numerator and denominator integrated separately: First M mark **cannot** be awarded so only mark available is second M mark. So 1 out of 5 marks.	

Question Number	Scheme	Marks
62.	$\left(\int =\right)\dfrac{12x^6}{6}, -\dfrac{3x^3}{3}, +\dfrac{4x^{\frac{4}{3}}}{\frac{4}{3}}, (+c)$ $= \underline{2x^6 - x^3 + 3x^{\frac{4}{3}} + c}$	M1A1,A1,A1 A1 **5**
	Notes	
	M1 for some attempt to integrate: $x^n \to x^{n+1}$ i.e ax^6 or ax^3 or $ax^{\frac{4}{3}}$ or $ax^{1\frac{1}{3}}$, where a is a non zero constant 1st A1 for $\dfrac{12x^6}{6}$ or better 2nd A1 for $-\dfrac{3x^3}{3}$ or better 3rd A1 for $\dfrac{4x^{\frac{4}{3}}}{\frac{4}{3}}$ or better 4th A1 for each term correct and simplified and the $+c$ occurring in the final answer	

Question Number	Scheme	Marks
63.	$\left(\mathrm{f}(x)=\right)\dfrac{12x^3}{3}-\dfrac{8x^2}{2}+x(+c)$	M1 A1 A1
	$\left(\mathrm{f}(-1)=0\Rightarrow\right)\ 0=4\times(-1)-4\times1-1+c$	M1
	$c=\underline{9}$	A1
	$\left[\mathrm{f}(x)=4x^3-4x^2+x+9\right]$	
		5
	Notes	
	1st M1 for an attempt to integrate $x^n\to x^{n+1}$ 1st A1 for at least 2 terms in x correct - needn't be simplified, ignore $+c$ 2nd A1 for all the terms in x correct but they need not be simplified. No need for $+c$ 2nd M1 for using x = -1 and y =0 to form a linear equation in c. No $+c$ gets M0A0 3rd A1 for c = 9. Final form of f(x) is not required.	

Question Number	Scheme	Marks
64.	$\frac{8x^4}{4} + \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - 5x + c$	M1 A1
	$= 2x^4 + 4x^{\frac{3}{2}}, -5x + c$	A1 A1
		4
	Notes	

M1 for some attempt to integrate a term in x: $x^n \to x^{n+1}$

1st A1 for correct, possibly un-simplified x^4 or $x^{\frac{3}{2}}$ term. e.g. $\frac{8x^4}{4}$ or $\frac{6x^{\frac{3}{2}}}{\frac{3}{2}}$

2nd A1 for both $2x^4$ and $4x^{\frac{3}{2}}$ terms correct and simplified on the same line

N.B. some candidates write $4\sqrt{x^3}$ or $4x^{1\frac{1}{2}}$ which are, of course, fine for A1

3rd A1 for $-5x + c$. Accept $-5x^1 + c$.
The $+c$ must appear on the same line as the $-5x$
N.B. We do not need to see one line with a fully correct integral

Ignore ISW (ignore incorrect subsequent working) if a correct answer is followed by an incorrect version.

Condone poor use of notation e.g. $\int 2x^4 + 4x^{\frac{3}{2}} - 5x + c$ will score full marks.

Question Number	Scheme	Marks
65. **(a)**	$(y=)\dfrac{3x^2}{2}-\dfrac{5x^{\frac{1}{2}}}{\frac{1}{2}}-2x\quad(+c)$	M1A1A1
	$f(4)=5\quad\Rightarrow 5=\frac{3}{2}\times16-10\times2-8+c$	M1
	$\underline{c=9}$	A1 (5)
	$\left[f(x)=\frac{3}{2}x^2-10x^{\frac{1}{2}}-2x+9\right]$	
(b)	$m=3\times4-\dfrac{5}{2}-2\quad\left(=7.5\text{ or }\dfrac{15}{2}\right)$	M1
	Equation is: $y-5=\dfrac{15}{2}(x-4)$	M1A1
	$\underline{2y-15x+50=0}$ o.e.	A1 (4) **(9marks)**

(a)

1st M1 for an attempt to integrate $x^n\to x^{n+1}$
1st A1 for at least 2 correct terms in x (unsimplified)
2nd A1 for all 3 terms in x correct (condone missing $+c$ at this point). Needn't be simplified
2nd M1 for using the point (4, 5) to form a linear equation for c. Must use x = 4 and y = 5 and have no x term and the function must have "changed".
3rd A1 for c = 9. The final expression is not required.

(b)

1st M1 for an attempt to evaluate $f'(4)$. Some correct use of x = 4 in $f'(x)$ but condone slips. They must therefore have at least 3×4 or $-\frac{5}{2}$ and clearly be using $f'(x)$ with x = 4. Award this mark wherever it is seen.

2nd M1 for using their value of m [or their $-\dfrac{1}{m}$] (provided it clearly comes from using x = 4 in f'(x)) to form an equation of the line through (4,5)).

Allow this mark for an attempt at a normal or tangent. Their m must be numerical. Use of $y = mx + c$ scores this mark when c is found.
1st A1 for any correct expression for the equation of the line
2nd A1 for any correct equation with integer coefficients. An "=" is required. e.g. $2y = 15x - 50$ etc as long as the equation is correct and has integer coefficients.

Normal

Attempt at normal can score both M marks in (b) but A0A0

Question number	Scheme	Marks
66	$x\sqrt{x} = x^{\frac{3}{2}}$ (Seen, or implied by correct integration)	B1
	$x^{-1/2} \to kx^{1/2}$ or $x^{3/2} \to kx^{5/2}$ (k a non-zero constant)	M1
	$(y=)\ \dfrac{5x^{1/2}}{1/2}\ldots + \dfrac{x^{5/2}}{5/2}\ (+C)$ ("$y =$" and " $+C$" are not required for these marks)	A1… A1
	$35 = \dfrac{5\times 4^{1/2}}{1/2} + \dfrac{4^{5/2}}{5/2} + C$ An equation in C is required (see conditions below). (With their terms simplified or unsimplified).	M1
	$C = \dfrac{11}{5}$ or equivalent $2\dfrac{1}{5}$, 2.2	A1
	$y = 10x^{1/2} + \dfrac{2x^{5/2}}{5} + \dfrac{11}{5}$ (Or equivalent <u>simplified</u>)	A1 ft
	I.s.w. if necessary, e.g. $y = 10x^{1/2} + \dfrac{2x^{5/2}}{5} + \dfrac{11}{5} = 50x^{1/2} + 2x^{5/2} + 11$ The final A mark requires an <u>equation</u> "$y =$..." with correct x terms (see below).	**[7]**
	B mark: $x^{\frac{3}{2}}$ often appears from integration of $\sqrt{x}$, which is B0. 1st A: Any unsimplified or simplified correct form, e.g. $\dfrac{5\sqrt{x}}{0.5}$. 2nd A: Any unsimplified or simplified correct form, e.g. $\dfrac{x^2\sqrt{x}}{2.5}$, $\dfrac{2(\sqrt{x})^5}{5}$. 2nd M: Attempting to use $x = 4$ <u>and</u> $y = 35$ in a changed function (even if differentiated) to form an equation in C. 3rd A: Obtaining $C = \dfrac{11}{5}$ with no earlier incorrect work. 4th A: Follow-through <u>only</u> the value of C (i.e. the other terms must be correct). Accept equivalent <u>simplified</u> terms such as $10\sqrt{x} + 0.4x^2\sqrt{x}$ ...	

Question Number	Scheme	Marks
67	$\dfrac{2x^4}{4}+\dfrac{3x^{-1}}{-1}(+C)$	M1 A1
	$\dfrac{x^4}{2}-3x^{-1}+C$	A1
		(3) **[3]**
	M1 for some attempt to integrate an x term of the given y. $x^n \rightarrow x^{n+1}$ 1st A1 for **both** x terms correct but unsimplified- as printed or better. Ignore $+c$ here 2nd A1 for both x terms correct and simplified and $+c$. Accept $-\dfrac{3}{x}$ but <u>NOT</u> $+-3x^{-1}$ Condone the $+c$ appearing on the first (unsimplified) line but missing on the final (simplified) line Apply ISW if a correct answer is seen If part (b) is attempted first and this is clearly labelled then apply the scheme and allow the marks. Otherwise assume the first solution is for part (a).	

Question Number	Scheme	Marks
68	$(I=)\frac{12}{6}x^6-\frac{8}{4}x^4+3x+c$ $=2x^6-2x^4+3x+c$	M1 A1A1A1 **[4]**
	M1 for an attempt to integrate $x^n \to x^{n+1}$ (i.e. ax^6 or ax^4 or ax, where a is any non-zero constant). Also, this M mark can be scored for just the + c (seen at some stage), even if no other terms are correct. 1st A1 for $2x^6$ 2nd A1 for $-2x^4$ 3rd A1 for $3x+c$ (or $3x+k$, etc., any appropriate letter can be used as the constant) Allow $3x^1+c$, but <u>not</u> $\frac{3x^1}{1}+c$. Note that the A marks can be awarded at separate stages, e.g. $\frac{12}{6}x^6-2x^4+3x$ scores 2nd A1 $\frac{12}{6}x^6-2x^4+3x+c$ scores 3rd A1 $2x^6-2x^4+3x$ scores 1st A1 (even though the c has now been lost). Remember that all the A marks are dependent on the M mark. If applicable, isw (ignore subsequent working) after a correct answer is seen. Ignore wrong notation if the intention is clear, e.g. Answer $\int 2x^6-2x^4+3x+c\,dx$.	

Question Number	Scheme	Marks
69	$\left(\mathrm{f}(x)=\right)\dfrac{3x^3}{3}-\dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}}-7x\left(+c\right)$	M1
	$= \ x^3-2x^{\frac{3}{2}}-7x \quad (+c)$	A1A1
	f(4) = 22 $\Rightarrow$ $22=64-16-28+c$	M1
	$\boldsymbol{c=2}$	A1cso (5)
		[5]
	1st M1 for an attempt to integrate (x^3 or $x^{\frac{3}{2}}$ seen). The x term is insufficient for this mark and similarly the + c is insufficient. 1st A1 for $\frac{3}{3}x^3$ or $-\dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}}$ (An unsimplified or simplified correct form) 2nd A1 for all three x terms correct and simplified… (the simplification may be seen later). The + c is not required for this mark. Allow $-7x^1$, but <u>not</u> $-\dfrac{7x^1}{1}$. 2nd M1 for an attempt to use x = 4 <u>and</u> y = 22 in a changed function (even if differentiated) to form an equation in c. 3rd A1 for c = 2 with no earlier incorrect work (a final expression for f(x) is not required).	

Question number	Scheme	Marks
70.	$2x+\frac{5}{3}x^3+c$	M1A1A1 (3) **3**
	M1 for an attempt to integrate $x^n \to x^{n+1}$. Can be given if $+c$ is only correct term. 1st A1 for $\frac{5}{3}x^3$ or $2x+c$. Accept $1\frac{2}{3}$ for $\frac{5}{3}$. Do <u>not</u> accept $\frac{2x}{1}$ or $2x^1$ as final answer 2nd A1 for as printed (no extra or omitted terms). Accept $1\frac{2}{3}$ or $1.\dot{6}$ for $\frac{5}{3}$ but not 1.6 or 1.67 etc Give marks for the first time correct answers are seen e.g. $\frac{5}{3}$ that later becomes 1.67, the 1.67 is treated as ISW NB M1A0A1 is not possible	

Question number	Scheme	Marks	
71. (a)	$\left(x^2+3\right)^2 = x^4+3x^2+3x^2+3^2$	M1	
	$\dfrac{\left(x^2+3\right)^2}{x^2} = \dfrac{x^4+6x^2+9}{x^2} = x^2+6+9x^{-2}$ **(*)**	A1cso	(2)
(b)	$y = \dfrac{x^3}{3}+6x+\dfrac{9}{-1}x^{-1}\left(+c\right)$	M1A1A1	
	$20 = \dfrac{27}{3}+6\times3-\dfrac{9}{3}+c$	M1	
	$c = -4$	A1	
	$[y=]\dfrac{x^3}{3}+6x-9x^{-1}-4$	A1ft	(6)
			8
(a)	M1 for attempting to expand $\left(x^2+3\right)^2$ and having at least 3(out of the 4) correct terms. A1 at least this should be seen and no incorrect working seen. If they never write $\dfrac{9}{x^2}$ as $9x^{-2}$ they score A0.		
(b)	1st M1 for some correct integration, one correct x term as printed or better Trying $\dfrac{\int u}{\int v}$ loses the first M mark but could pick up the second. 1st A1 for two correct x terms, un-simplified, as printed or better 2nd A1 for a fully correct expression. Terms need not be simplified and $+c$ is not required. No + c loses the next 3 marks 2nd M1 for using x = 3 and y = 20 in their expression for f(x) $\left[\neq \dfrac{\mathrm{d}y}{\mathrm{d}x}\right]$ to form a linear equation for c 3rd A1 for c = −4 4th A1ft for an expression for y with simplified x terms: $\dfrac{9}{x}$ for $9x^{-1}$ is OK . Condone missing "y = " Follow through their numerical value of c only.		

Question number	Scheme	Marks	
72.	$3x^2 \to kx^3$ or $4x^5 \to kx^6$ or $-7 \to kx$ (k a non-zero constant)	M1	
	$\frac{3x^3}{3}$ or $\frac{4x^6}{6}$ (Either of these, simplified or unsimplified)	A1	
	$x^3 + \frac{2x^6}{3} - 7x$ or equivalent unsimplified, such as $\frac{3x^3}{3} + \frac{4x^6}{6} - 7x^1$	A1	
	$+ C$ (or any other constant, e.g. $+ K$)	B1	(4)
			4
	M: Given for increasing by one the power of x in one of the three terms. A marks: 'Ignore subsequent working' after a correct unsimplified version of a term is seen. B: Allow the mark (independently) for an integration constant appearing at any stage (even if it appears, then disappears from the final answer). This B mark can be allowed even when no other marks are scored.		

Question number	Scheme	Marks
73.	(a) $4x \to kx^2$ or $6\sqrt{x} \to kx^{3/2}$ or $\dfrac{8}{x^2} \to kx^{-1}$ (k a non-zero constant)	M1
	$\text{f}(x) = 2x^2,\ -4x^{3/2},\ -8x^{-1}$ (+ C) (+ C not required)	A1, A1, A1
	At $x = 4$, $y = 1$: $1 = (2\times 16) - \left(4\times 4^{3/2}\right) - \left(8\times 4^{-1}\right) + C$ <u>Must be in part (a)</u>	M1
	$C = 3$	A1 (6)
	(b) $\text{f}'(4) = 16 - (6\times 2) + \dfrac{8}{16} = \dfrac{9}{2}\ (= m)$ M: Attempt $\text{f}'(4)$ with the <u>given</u> f'. <u>Must be in part (b)</u>	M1
	Gradient of normal is $-\dfrac{2}{9}\left(= -\dfrac{1}{m}\right)$ M: Attempt perp. grad. rule. Dependent on the use of their $\text{f}'(x)$	M1
	Eqn. of normal: $y - 1 = -\dfrac{2}{9}(x-4)$ (or any equiv. form, e.g. $\dfrac{y-1}{x-4} = -\dfrac{2}{9}$)	M1 A1 (4)
	Typical answers for A1: $\left(y = -\dfrac{2}{9}x + \dfrac{17}{9}\right)$ $(2x + 9y - 17 = 0)$ $(y = -0.\dot{2}x + 1.\dot{8})$	
	Final answer: gradient $-\dfrac{1}{\left(9/2\right)}$ or $-\dfrac{1}{4.5}$ is A0 (but all M marks are available).	
		10
	(a) The first 3 A marks are awarded in the order shown, and the terms must be simplified. 'Simplified' coefficient means $\dfrac{a}{b}$ where a and b are integers with no common factors. Only a single + or – sign is allowed (e.g. + – must be replaced by –). 2nd M: Using $x = 4$ <u>and</u> $y = 1$ (<u>not</u> $y = 0$) to form an eqn in C. (No C is M0) (b) 2nd M: Dependent upon use of their $\text{f}'(x)$. 3rd M: eqn. of a straight line through (4, 1) with any gradient except 0 or ∞. <u>Alternative for 3rd M:</u> Using (4, 1) in $y = mx + c$ to <u>find a value</u> of c, but an equation (general or specific) must be seen. Having coords the <u>wrong way round</u>, e.g. $y - 4 = -\dfrac{2}{9}(x-1)$, loses the 3rd M mark <u>unless</u> a correct general formula is seen, e.g. $y - y_1 = m(x - x_1)$. N.B. The A mark is scored for <u>any</u> form of the correct equation… be prepared to apply isw if necessary.	

Question Number	Scheme	Marks
74. (a)	$\frac{dy}{dx} = 70x \ -35x^{\frac{3}{2}}$	M1A1
	Put $\frac{dy}{dx} = 0$ to give $70x \ -35x^{\frac{3}{2}} = 0$ *so* $x^{\frac{1}{2}} = 2$	M1
	$x = 4$	A1
	$y = 112$	A1 **(5)**
(b) (Way 1)	When $y = 0$, $\quad 35x^2 = 14x^{\frac{5}{2}}$ *and* $x^{\frac{1}{2}} = \frac{35}{14}$ *or* $5 = 2\sqrt{x}$ *so* $\sqrt{x} = \frac{5}{2}$	M1
	$x = \frac{25}{4}$	A1 **(2)**
(b) (Way 2)	When $y = 0$, $\quad 35x^2 = 14x^{\frac{5}{2}}$ *so* $1225x^4 = 196x^5$ *or* $5 = 2\sqrt{x}$ *so* $25 = 4x$	M1
	$x = \frac{25}{4}$ or $x = \frac{1225}{196}$	A1 **(2)**
(c) Way 1	$\int 35x^2 - 14x^{\frac{"5"}{2}} dx = \frac{35}{3}x^3 - \frac{14x^{\frac{"7"}{2}}}{\frac{"7"}{2}} \ (+c)$	M1A1ft
	$\left[\frac{35}{3}x^3 - 4x^{\frac{7}{2}}\right]_4^{\frac{25}{4}} = 406.901.. - 234.667 = 172.23$	dM1
	Hence Area $=$ "*their* $112 \times (6\frac{1}{4} - 4)$" $-$ "172.23" or "252" – " 172.23"	ddM1
	79.77	A1 **(5)**
(c) Way 2	$\int "112" - \{35x^2 - 14x^{\frac{5}{2}}\} dx = (112x) - \frac{35}{3}x^3 + \frac{14x^{\frac{7}{2}}}{\frac{7}{2}} \ (+c)$	M1A1ft
	$\left[(112x) - (\frac{35}{3}x^3 - 4x^{\frac{7}{2}})\right]_{"4"}^{"\frac{25}{4}"}$ with correct use of limits	dM1
	Integrated their 112 to give $112x$ with correct use of limits	ddM1
	79.77	A1 **(5)** **[12]**

Notes

(a)
M1: Attempt at differentiation after multiplying out - may be awarded for $70x$ term correct
(If product rule is used it must be of correct form i.e. $\frac{dy}{dx} = 7x^2\left(-2kx^{k-1}\right) + 14x(5-2x^k)$)
A1: the derivative must be completely correct but may be unsimplified
For product rule this is $\frac{dy}{dx} = 7x^2\left(-x^{-\frac{1}{2}}\right) + 14x(5-2\sqrt{x})$
M1: uses derivative = 0 to find $x^k =$ or $x =$ with correct work for their equation (even without fractional powers)
A1: obtains $x = 4$ then
A1: for $y = 112$ (may be credited if seen in part (a) or in part(c))
(b)
Way 1 (Dividing first)
M1: Puts $y = 0$ and obtains expression of the form $x^k = A$ (where k is not equal to 1) after correct algebra for their equation (may be a sign slip)
A1: Obtains $x = 6.25$ or equivalent correct answer
(b)
Way 2 (dealing with fractional power first i.e. Squaring)
M1: Puts $y = 0$ and squares each term correctly for their equation obtaining expression of the form $A^2 x^m = B^2 x^n$ after correct algebra
A1: Obtains $x = 6.25$ or equivalent correct answer
(c)
Way 1
M1: Correct integration of one of their terms – e.g. see x^2 term integrated correctly (not just raised power)
A1ft: completely correct integral for their power which must have been a fraction **(may be unsimplified)**
dM1: (dependent on previous M) substituting their 25/4 and their 4 and subtracting
ddM1 (depends on both method marks) **Correct** method **to obtain shaded area** so their rectangle minus their area under curve
A1: Accept answers which round to 79.77
(c)
Way 2
M1: Attempt at integration – x^2 term integrated correctly
A1ft: completely correct integral for second and their third terms (provided one has a fractional power) (ignore sign errors) **(may be unsimplified)**
dM1: (dependent on previous M) substituting their 25/4 and their 4 and subtracting (either way)
ddM1 (depends on both method marks) **Correct** method **to obtain shaded area** so their 112 integrated correctly and correct signs for the other two terms in the integrand
A1: Accept answers which round to 79.77
Answer with no working – send to review
If they have the wrong fractional power on their second term after expansion in part (a) (usually 3/2), all the method marks are available throughout the question and the A1ft is available in (c). The A mark in part (b) may also be accessible. Maximum score is likely to be 8/12
If they have the trivial power 1 on their second term, then two method marks are available in (a) and three method marks are available in part (c) Maximum score is likely to be 5/12

Question Number	Scheme	Marks
75 Way 1	When $x = 1$, $y = 4 + 9 - 30 - 8 = -25$	B1
	Area of triangle ABP = $\frac{1}{2}\times 1\times 25 = 12.5$ (Where P is at (1, 0))	B1
	Way 1: $\int(4x^3+9x^2-30x-8)\,\mathrm{d}x = x^4+\frac{9}{3}x^3-\frac{30x^2}{2}-8x\ \{+c\}$ *or* $x^4+3x^3-15x^2-8x\ \{+c\}$	M1A1
	$\left[x^4+3x^3-15x^2-8x\right]_{-\frac{1}{4}}^{1} = (1+3-15-8)-\left(\left(-\frac{1}{4}\right)^4+3\left(-\frac{1}{4}\right)^3-15\left(-\frac{1}{4}\right)^2-8\left(-\frac{1}{4}\right)\right)$	
	$= (-19)-\frac{261}{256}$ or $-19-1.02$	dM1
	So Area = "*their* 12.5" + "*their* $20\frac{5}{256}$" or "12.5" + " 20.02" or "12.5" + "*their* $\frac{5125}{256}$"	ddM1
	$= 32.52$ (NOT -32.52)	A1 (7)
	Less efficient alternative methods for first two marks in part (b) with Way 1 or 2	
	For first mark: Finding equation of the line AB as $y = 25x - 50$ as this implies the -25	B1
	For second mark: Integrating to find triangle area	
	$\int_1^2(25x-50)\,\mathrm{d}x = \left[\frac{25}{2}x^2-50x\right]_1^2 = -50+37.5 = -12.5$ so area is 12.5	B1
	Then mark as before if they use Method in original scheme	
(75) Way 2	**Way 2:** Those who use area for **original curve between -1/4 and 2** and subtract area **between line and curve between 1 and 2** have a correct (long) method .	
	The first B1 (if y=−25 is not seen) is for equation of straight line $y = 25x - 50$	B1
	The second B1 may be implied by final answer correct, or 4.5 seen for area of "segment shaped" region between line and curve, or by area between line and axis/triangle found as 12.5	B1
	$\int(4x^3+9x^2-55x+42)\,\mathrm{d}x = x^4+\frac{9}{3}x^3-\frac{55x^2}{2}+42x\ \{+c\}$ (or integration as in Way 1)	M1A1
	The dM1 is for correct use of the different correct limits for each of the two areas: i.e.	
	$\left[x^4+3x^3-15x^2-8x\right]_{-\frac{1}{4}}^{2} = (16+24-60-16)-\left(\left(-\frac{1}{4}\right)^4+3\left(-\frac{1}{4}\right)^3-15\left(-\frac{1}{4}\right)^2-8\left(-\frac{1}{4}\right)\right)$	
	And $\left[x^4+\frac{9}{3}x^3-\frac{55x^2}{2}+42x\right]_1^2 = 16+24-110+84-(1+3-27.5+42)$	dM1
	So Area =their $\left[x^4+3x^3-15x^2-8x\right]_{-\frac{1}{4}}^{2}$ **minus their** $\left[x^4+\frac{9}{3}x^3-\frac{55x^2}{2}+42x\right]_1^2$	ddM1
	i.e. "*their* 37.0195" − "*their* 4.5" (with both sets of limits correct for the integral)	
	Reaching $= 32.52$ (NOT -32.52)	A1
	See over for special case with wrong limits	

	NB: Those who attempt curve – line wrongly with limits −1/4 to 2 may earn M1A1 for correct integration of their cubic. Usually e.g. $\int(4x^3+9x^2-55x+42)\,\mathrm{d}x = x^4+\frac{9}{3}x^3-\frac{55x^2}{2}+42x\ \{+c\}$ (They will not earn any of the last 3 marks) They may also get first B1 mark for the correct equation of the straight line (usually seen but may be implied by correct line −curve equation) and second B1 if they also use limits 1 and 2 to obtain 4.5 (or find the triangle area 12.5).	M1A1

	Notes
75	**Way 1:** B1: Obtains y = - 25 when x = 1 (may be seen anywhere – even in (a)) or finds correct equation of line is $y = 25x - 50$ B1: Obtains area of triangle = 12.5 (may be seen anywhere). Allow −12.5. Accept $\frac{1}{2}\times1\times25$ M1: Attempt at integration of cubic; two correct terms for their integration. No limits needed A1: completely correct integral for the cubic **(may be unsimplified)** dM1: We are looking for the start of a correct method here (dependent on previous M). It is for substituting 1 and -1/4 and subtracting. May use 2 and -1/4 and also 2 and 1 **AND subtract** (which is equivalent) ddM1 (depends on both method marks) **Correct** method **to obtain shaded area** so **adds** two positive numbers (areas) together – one is area of triangle, the other is area of region obtained from integration of correct function with correct limits (may add two negatives then makes positive) **Way 2:** This is a long method and needs to be a correct method B1: Finds y=−25 at x=1 ,or correct equation of line is $y = 25x - 50$ B1: May be implied where WAY 2 is used and final correct answer obtained so award of final A1 results in the award of this B1. It may also be implied by correct integration of line equation or of curve minus line expression between limits 1 and 2. So if only slip is final subtraction (giving final A0, this mark may still be awarded) So may be implied by 4.5 seen for area of "segment shaped" region between line and curve. M1: Attempt at integration of given cubic or after attempt at subtracting their line equation (no limits needed). Two correct terms needed A1: Completely correct integral for their cubic **(may be unsimplified) – may have wrong coefficients of *x* and wrong constant term through errors in subtraction** dM1: **Use limits for original curve between -1/4 and 2 and use limits of 1 and 2 for area between line and curve–** needs completely correct limits– see scheme- this is dependent on two integrations ddM1: (depends on both method marks) Subtracts $"\textit{their } 37.0195" - "\textit{their } 4.5"$ Needs consistency of signs. A1: 32.52 or awrt 32.52 e.g. $32\frac{133}{256}$ **NB: This correct answer implies the second B mark** (Trapezium rule **gets no marks after first two B marks) The first two B marks may be given wherever seen. The integration of a cubic gives the following M1 and correct integration of their cubic** $\int(4x^3+9x^2+Ax+B)\,\mathrm{d}x = x^4+\frac{9}{3}x^3+\frac{Ax^2}{2}+Bx\ \{+c\}$ gives the A1

Question Number	Scheme		Marks
76. (a)	$\left\{\int\left(3x - x^{\frac{3}{2}}\right)\mathrm{d}x\right\} = \dfrac{3x^2}{2} - \dfrac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}\{+c\}$	Either $3x \to \pm\lambda x^2$ or $x^{\frac{3}{2}} \to \pm\mu x^{\frac{5}{2}}$, $\lambda, \mu \neq 0$	M1
		At least one term correctly integrated	A1
		Both terms correctly integrated	A1
			[3]
(b)	$0 = 3x - x^{\frac{3}{2}} \Rightarrow 0 = 3 - x^{\frac{1}{2}}$ or $0 = x\left(3 - x^{\frac{1}{2}}\right) \Rightarrow x = ...$	Sets $y = 0$, in order to find the correct $x^{\frac{1}{2}} = 3$ or $x = 9$	M1
	$\left\{\text{Area}(S) = \left[\dfrac{3x^2}{2} - \dfrac{2}{5}x^{\frac{5}{2}}\right]_0^9\right\}$		
	$= \left(\dfrac{3(9)^2}{2} - \left(\dfrac{2}{5}\right)(9)^{\frac{5}{2}}\right) - \{0\}$	Applies the limit 9 on an integrated function with **no wrong lower limit** .	ddM1
	$\left\{= \left(\dfrac{243}{2} - \dfrac{486}{5}\right) - \{0\}\right\} = \dfrac{243}{10}$ or 24.3	$\dfrac{243}{10}$ or 24.3	A1 oe
			[3]
			6

		Question 76 Notes
(a)	**M1**	Either $3x \to \pm\lambda x^2$ or $x^{\frac{3}{2}} \to \pm\mu x^{\frac{5}{2}}$, $\lambda, \mu \neq 0$
	1st A1	At least one term correctly integrated. Can be simplified or un-simplified but power must be simplified. Then isw.
	2nd A1	Both terms correctly integrated. Can be un-simplified (as in the scheme) but the n+1 in each denominator and power should be a single number. (e.g. 2 – not 1+1) Ignore subsequent work if there are errors simplifying. Ignore the omission of "$+ c$". Ignore integral signs in their answer.
(b)	**1st M1**	Sets $y = 0$, and reaches the **correct** $x^{\frac{1}{2}} = 3$ or $x = 9$ (isw if $x^{\frac{1}{2}} = 3$ is followed by $x = \sqrt{3}$) Just seeing $x = \sqrt{3}$ without the correct $x^{\frac{1}{2}} = 3$ gains M0. May just see $x = 9$. Use of trapezium rule to find area is M0A0 as hence implies integration needed.
	ddM1	This mark is dependent on the two previous method marks and needs both to have been awarded. Sees the limit **9** substituted in an integrated function. (Do not follow through their value of x) Do not need to see MINUS 0 but if another value is used as lower limit – this is M0. This mark may be implied by 9 in the limit and a correct answer.
	A1	$\dfrac{243}{10}$ or 24.3
	Common Error	**Common Error** $0 = 3x - x^{\frac{3}{2}} \Rightarrow x^{\frac{1}{2}} = 3$ *so* $x = \sqrt{3}$ **Then** uses limit $\sqrt{3}$ etc gains M1 M0 A0 so 1/3

Question Number	Scheme	Marks
	May mark (a) and (b) together	
77. (a)	Expands to give $10x^{\frac{3}{2}} - 20x$	B1
	Integrates to give $\frac{10}{"\frac{5}{2}"}x^{"\frac{5}{2}"} + \frac{-"20"x^2}{2}$ $(+c)$	M1 A1ft
	Simplifies to $4x^{\frac{5}{2}} - 10x^2 (+c)$	A1cao **(4)**
(b)	Use limits 0 and 4 either way round on their integrated function (may only see 4 substituted)	M1
	Use limits 4 and 9 either way round on their integrated function	dM1
	Obtains either ± -32 or $\pm$ 194 needs at least one of the previous M marks for this to be awarded	A1
	(So area = $\left\lvert\int_0^4 ydx\right\rvert + \int_4^9 ydx$) i.e. 32 + 194, = 226	ddM1,A1 **(5)** **[9]**

Notes

(a) **B1**: Expands the bracket correctly

M1: Correct integration process **on at least one term** after attempt at multiplication. (Follow correct expansion or one slip resulting in $10x^k - 20x$ where k may be $\frac{1}{2}$ or $\frac{5}{2}$ or resulting in $10x^{\frac{3}{2}} - Bx$, where B may be 2 or 5)

So $x^{\frac{3}{2}} \to \frac{x^{\frac{5}{2}}}{5/2}$ or $x^{\frac{1}{2}} \to \frac{x^{\frac{3}{2}}}{3/2}$ or $x^{\frac{5}{2}} \to \frac{x^{\frac{7}{2}}}{7/2}$ and/or $x \to \frac{x^2}{2}$.

A1: Correct unsimplified follow through for both terms of their integration. Does not need (+ c)

A1: Must be simplified and correct– allow answer in scheme or $4x^{2\frac{1}{2}} - 10x^2$. Does not need (+ c)

(b) **M1**: (does not depend on first method mark) Attempt to substitute 4 into their integral (however obtained but must not be differentiated) or seeing their evaluated number (usually 32) is enough – do not need to see minus zero.

d**M1:** (depends on first method mark in (a)) Attempt to subtract either way round using the limits 4 and 9 $A \times 9^{\frac{5}{2}} - B \times 9^2$ with $A \times 4^{\frac{5}{2}} - B \times 4^2$ is enough – or seeing 162 –(-32) {but not 162 – 32 }

A1: At least one of the values (32 and 194) correct (needs just one of the two previous M marks in (b)) or may see 162 + 32 + 32 or 162 + 64 or may be implied by correct final answer if not evaluated until last line of working

ddM1: Adds 32 and 194 (may see 162 + 32 + 32 or may be implied by correct final answer if not evaluated until last line of working). This depends on everything being correct to this point.

A1cao: Final answer of 226 not (- 226)

Common errors: $4 \times 4^{\frac{5}{2}} - 10 \times 4^2 + 4 \times 9^{\frac{5}{2}} - 10 \times 9^2 - 4 \times 4^{\frac{5}{2}} - 10 \times 4^2 = \pm$ 162 obtains M1 M1 A0 (neither 32 nor 194 seen and final answer incorrect) then M0 A0 so 2/5

Uses correct limits to obtain -32 +162 +32 = +/-162 is M1 M1 A1 (32 seen) M0 A0 so 3/5

Special case: In part (b) Uses limits 9 and 0 = 972 – 810 - 0 = 162 M0 M1 A0 M0A0 scores 1/5
This also applies if 4 never seen.

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="5">78.</td><td>$\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \frac{x^4}{6(4)}+\frac{x^{-1}}{(3)(-1)}$</td><td>M1: $x^n \to x^{n+1}$
A1: At least one of either $\frac{x^4}{6(4)}$ or $\frac{x^{-1}}{(3)(-1)}$.
A1: $\frac{x^4}{6(4)}+\frac{x^{-1}}{(3)(-1)}$ or equivalent.
e.g. $\frac{\frac{x^4}{6}}{4}+\frac{\frac{x^{-1}}{3}}{-1}$ (they will lose the final mark if they cannot deal with this correctly)</td><td>M1A1A1</td></tr>
<tr><td colspan="2">Note that some candidates may change the function prior to integrating e.g.
$\int\frac{x^3}{6}+\frac{1}{3x^2}\mathrm{d}x=\int 3x^5+6\,\mathrm{d}x$ in which case allow the M1 if $x^n \to x^{n+1}$ for their changed function and allow the M1 for limits if scored</td><td></td></tr>
<tr><td colspan="2">$\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \left(\frac{(\sqrt{3})^4}{24}+\frac{(\sqrt{3})^{-1}}{-1(3)}\right)-\left(\frac{(1)^4}{24}+\frac{(1)^{-1}}{-1(3)}\right)$</td><td>dM1</td></tr>
<tr><td colspan="2">2nd dM1: For using limits of $\sqrt{3}$ and 1 on an integrated expression and subtracting the correct way round. The 2nd M1 is dependent on the 1st M1 being awarded.</td><td></td></tr>
<tr><td>$=\left(\frac{9}{24}-\frac{1}{3\sqrt{3}}\right)-\left(\frac{1}{24}-\frac{1}{3}\right)=\frac{2}{3}-\frac{1}{9}\sqrt{3}$</td><td>$\frac{2}{3}-\frac{1}{9}\sqrt{3}$ or $a=\frac{2}{3}$ and $b=-\frac{1}{9}$.
Allow equivalent fractions for a and/or b and 0.6 recurring and/or 0.1 recurring but do not allow $\frac{6-\sqrt{3}}{9}$</td><td>A1cso</td></tr>
<tr><td></td><td colspan="2">This final mark is cao and cso – there must have been no previous errors</td><td></td></tr>
<tr><td></td><td colspan="2"></td><td>Total 5</td></tr>
<tr><td></td><td colspan="2">Common Errors (Usually 3 out of 5)</td><td></td></tr>
<tr><td></td><td colspan="2">$\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\}=\int\left(\frac{x^3}{6}+3x^{-2}\right)\mathrm{d}x=\frac{x^4}{6(4)}+\frac{3x^{-1}}{(-1)}$ M1A1A0
$\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\}=\left(\frac{(\sqrt{3})^4}{24}+\frac{3(\sqrt{3})^{-1}}{-1}\right)-\left(\frac{(1)^4}{24}+\frac{3(1)^{-1}}{-1}\right)$ dM1
$=\left(\frac{9}{24}-\frac{3}{\sqrt{3}}\right)-\left(\frac{1}{24}+\frac{3}{-1}\right)=\frac{10}{3}-\sqrt{3}$ A0</td><td></td></tr>
<tr><td></td><td colspan="2">$\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\}=\int\left(\frac{x^3}{6}+(3x)^{-2}\right)\mathrm{d}x=\frac{x^4}{6(4)}+\frac{(3x)^{-1}}{(-1)}$ M1A1A0
$\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\}=\left(\frac{(\sqrt{3})^4}{24}+\frac{(3\sqrt{3})^{-1}}{-1}\right)-\left(\frac{(1)^4}{24}+\frac{(3\times 1)^{-1}}{-1}\right)$ dM1
$=\left(\frac{9}{24}-\frac{1}{3\sqrt{3}}\right)-\left(\frac{1}{24}-\frac{1}{3}\right)=\frac{2}{3}-\frac{\sqrt{3}}{9}$ A0
Note this is the correct answer but follows incorrect work.</td><td></td></tr>
</table>

Question Number	Scheme		Marks
	$\int\left(\frac{1}{8}x^3+\frac{3}{4}x^2\right)\,\mathrm{d}x = \frac{x^4}{32}+\frac{x^3}{4}\{+c\}$	M1: $x^n \to x^{n+1}$ on either term A1: $\frac{x^4}{32}+\frac{x^3}{4}$. Any correct simplified or un-simplified form. (+ c not required)	M1A1
	$\left[\frac{x^4}{32}+\frac{x^3}{4}\right]_{-4}^{2} = \left(\frac{16}{32}+\frac{8}{4}\right)-\left(\frac{256}{32}+\frac{(-64)}{4}\right)$ or $\left[\frac{x^4}{32}+\frac{x^3}{4}\right]_{-4}^{0} = (0)-\left(\frac{(-4)^4}{32}+\frac{(-4)^3}{4}\right)$ added to $\left[\frac{x^4}{32}+\frac{x^3}{4}\right]_{0}^{2} = \left(\frac{(2)^4}{32}+\frac{(2)^3}{4}\right)-(0)$		dM1
	Substitutes limits of 2 and -4 into an "integrated function" and subtracts either way round. Or substitutes limits of 0 and -4 **and** 2 and 0 into an "integrated function" and subtracts either way round and adds the two results.		
79.	$= \frac{21}{2}$	$\frac{21}{2}$ or 10.5	A1
	$\{\text{At } x=-4,\ y=-8+12=4 \text{ or at } x=2,\ y=1+3=4\}$		
	Area of Rectangle $=6\times4=24$ or Area of Rectangles $=4\times4=16$ and $2\times4=8$		M1
	Evidence of $(4--2)\times$ their y_{-4} **or** $(4--2)\times$ their y_2 or Evidence of $4\times$ their y_{-4} **and** $2\times$ their y_2		
	So, $\text{area}(R)=24-\frac{21}{2}=\frac{27}{2}$	dddM1: Area rectangle – integrated answer. Dependent on all previous method marks **and requires:** **Rectangle > integration > 0** A1: $\frac{27}{2}$ or 13.5	dddM1A1
			[7]
			Total 7

	<u>Alternative:</u>		
	$\pm\int \text{"their4"} - \left(\frac{1}{8}x^3 + \frac{3}{4}x^2\right)\,\mathrm{d}x$	Line – curve. **Condone missing brackets and allow either way round.**	4^{th} M1
	$= 4x - \frac{x^4}{32} - \frac{x^3}{4}\{+c\}$	M1: $x^n \to x^{n+1}$ on either curve term A1ft: $\text{"}-\frac{x^4}{32} - \frac{x^3}{4}.\text{"}$ Any correct simplified or un-simplified form of their curve terms, **follow through sign errors.** (+ c not required)	1^{st} M1,1^{st} A1ft
	$[\ ]_{-4}^{2} = \underline{\underline{\left(8 - \frac{16}{32} - \frac{8}{4}\right) - \left(-16 - \frac{256}{32} - \frac{(-64)}{4}\right)}}$	2^{nd} M1 Substitutes limits of 2 and -4 into an "integrated curve" and subtracts either way round. 3^{rd} M1 for $\pm(\text{"}8\text{"} - \text{"}-16\text{"})$ **Substitutes limits into the 'line part' and subtracts either way round.** 2^{nd} A1 for correct $\pm$ (underlined expression). **Now needs to be correct but allow $\pm$ the correct expression.**	2^{nd} M1, 3^{rd} M1 2^{nd} A1
	$= \frac{27}{2}$	A1: $\frac{27}{2}$ or 13.5	3^{rd} A1
	If the final answer is -13.5 you can withhold the final A1 **If -13.5 then "becomes" +13.5 allow the A1**		

Question Number	Scheme	Marks
80. (a)	Seeing -4 and 2.	B1 **(1)**
(b)	$x(x+4)(x-2) = \underline{x^3+2x^2-8x}$ or $\underline{x^3-2x^2+4x^2-8x}$ (without simplifying)	<u>B1</u>
	$\int(x^3+2x^2-8x)\mathrm{d}x = \frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\{+c\}$ or $\frac{x^4}{4}-\frac{2x^3}{3}+\frac{4x^3}{3}-\frac{8x^2}{2}\{+c\}$	M1A1ft
	$\left[\frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\right]_{-4}^{0} = (0)-\left(64-\frac{128}{3}-64\right)$ **or** $\left[\frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\right]_{0}^{2} = \left(4+\frac{16}{3}-16\right)-(0)$	dM1
	One integral $=\pm\,42\frac{2}{3}$ (42.6 or awrt 42.7) **or** other integral $=\pm\,6\frac{2}{3}$ (6.6 or awrt 6.7)	A1
	Hence Area $=$ "$their\,42\frac{2}{3}$" $+$ "$their\,6\frac{2}{3}$" or Area $=$ "$their\,42\frac{2}{3}$" $-$ "$-their\,6\frac{2}{3}$"	dM1
	$= 49\frac{1}{3}$ or 49.3 or $\frac{148}{3}$ (NOT $-\frac{148}{3}$)	A1
	(An answer of $= 49\frac{1}{3}$ may not get the final two marks – check solution carefully)	**(7)** **[8]**
	Notes for Question 80	
(a)	B1: Need both –4 and 2. May see (-4,0) and (2,0) (correct) but allow (0,-4) and (0, 2) or A = -4, B = 2 or indeed any indication of -4 and 2 – check graph also	
(b)	B1: Multiplies out cubic correctly (terms may not be collected, but if they are, mark collected terms here) M1: Tries to integrate their expansion with $x^n \to x^{n+1}$ for at least one of the terms A1ft: completely correct integral **following through** from their CUBIC expansion (if only quadratic or quartic this is A0) dM1: (dependent on previous M) substituting EITHER -a and 0 and subtracting either way round OR similarly for 0 and b. **If their limits –*a* and *b* are used in ONE integral, apply the Special Case below**. A1: Obtain **either** $\pm 42\frac{2}{3}$ (or 42.6 or awrt 42.7)*from the integral from -4 to 0* **or** $\pm 6\frac{2}{3}$ (6.6 or awrt 6.7) *from the integral from 0 to 2;* NO follow through on their cubic (allow decimal or improper equivalents $\frac{128}{3}$ **or** $\frac{20}{3}$) isw such as subtracting from rectangles. This will be penalized in the next two marks, which will be M0A0. dM1 (depends on first method mark) **Correct** method **to obtain shaded area** so adds two positive numbers (areas) together or uses their **positive** value minus **their negative** value, **obtained from** two separate **definite integrals**. A1: Allow 49.3, 49.33, 49.333 etc. Must follow correct logical work with **no errors** seen. For full marks on this question there must be **two** definite integrals, from -4 to 0 and from 0 to 2, though the evaluations for 0 may not be seen. (Trapezium rule **gets no marks after first two B marks)**	
(b)	**Special Case: one integral only from –*a* to *b*:** B1M1A1 available as before, then $\left[\frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\right]_{-4}^{2} = (4+\frac{16}{3}-16)-\left(64-\frac{128}{3}-64\right) = -6\tfrac{2}{3}+42\frac{2}{3}$ **=….** **dM1** for correct use of their limits –a and b and **subtracting** either way round**.** A1 for 36: NO follow through. Final M and A marks not available. Max 5/7 for part (b)	

81.	$y = 27 - 2x - 9\sqrt{x} - \dfrac{16}{x^2}$		
	$\int y\,\mathrm{d}x = 27x - x^2 - 6x^{\frac{3}{2}} + 16x^{-1}\ (+c)$	M1: $x^n \to x^{n+1}$ on any term A1: $27x - x^2$ A1: $-6x^{\frac{3}{2}}$ A1: $+16x^{-1}$	M1A1A1A1
	$\left(27(4) - (4)^2 - 6(4)^{\frac{3}{2}} + 16(4)^{-1}\right)$ $-\left(27(1) - (1)^2 - 6(1)^{\frac{3}{2}} + 16(1)^{-1}\right)$	**Attempt to subtract either way round using the limits 4 and 1. Dependent on the previous M1**	dM1
	= (48 – 36)		
	12	Cao	A1
			(6)
			[6]

Question number	Scheme		Marks
Method 1 82 (a)	Puts $10 - x = 10x - x^2 - 8$ and rearranges to give three term quadratic	Or puts $y = 10(10-y) - (10-y)^2 - 8$ and rearranges to give three term quadratic	M1
	Solves their "$x^2 - 11x + 18 = 0$" using acceptable method as in general principles to give x =	Solves their "$y^2 - 9y + 8 = 0$" using acceptable method as in general principles to give y =	M1
	Obtains x = 2 , x = 9 (may be on diagram or in part (b) in limits)	Obtains y = 8, y = 1 (may be on diagram)	A1
	Substitutes their x into a given equation to give y = (may be on diagram)	Substitutes their y into a given equation to give x= (may be on diagram or in part (b))	M1
	y = 8, y = 1	x = 2, x = 9	A1 (5)
(b)	$\int (10x - x^2 - 8)\,dx = \frac{10x^2}{2} - \frac{x^3}{3} - 8x\ \{+c\}$		M1 A1 A1
	$\left[\frac{10x^2}{2} - \frac{x^3}{3} - 8x\right]_2^9 = (......) - (......)$		dM1
	$= 90 - \frac{4}{3} = 88\frac{2}{3}$ or $\frac{266}{3}$		
	Area of trapezium $= \frac{1}{2}(8+1)(9-2) = 31.5$		B1
	So area of R is $88\frac{2}{3} - 31.5 = 57\frac{1}{6}$ or $\frac{343}{6}$		M1A1 cao (7)
			12 marks
Notes (a)	First **M1:** See scheme Second **M1**: See notes relating to solving quadratics Third **M1**: This may be awarded if one substitution is made Two correct Answers following tables of values, or from Graphical calculator are 5/5 **Just one pair of correct coordinates – no working or from table is M0M0A0M1A0**		
(b)	**M1**: $x^n \to x^{n+1}$ for any one term. **1st A1**: at least two out of three terms correct **2nd A1**: All three correct **dM1**: Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round (NB: If candidate **changes all signs** to get $\int(-10x + x^2 + 8)\,dx = -\frac{10x^2}{2} + \frac{x^3}{3} + 8x\ \{+c\}$ This is M1 A1 A1 Then uses limits dM1 and trapezium is B1 Needs to *change sign of value obtained* from integration for final M1A1 so $-88\frac{2}{3} - 31.5$ is M0A0) **B1**: Obtains 31.5 for area under line using any correct method (could be integration) or triangle minus triangle $\frac{1}{2}\times 8\times 8 - \frac{1}{2}$ or rectangle plus triangle [may be implied by correct 57 1/6] **M1**: Their Area under curve – Their Area under line (if integrate both need same limits) **A1**: Accept 57.16recurring but not 57.16 **PTO for Alternative method**		

Method 2 for (b)	Area of R $= \int_2^9 (10x - x^2 - 8) - (10 - x)\,dx$ $\int_2^9 -x^2 + 11x - 18\,dx$ $= -\frac{x^3}{3} + \frac{11x^2}{2} - 18x\ \{+c\}$ $\left[-\frac{x^3}{3} + \frac{11x^2}{2} - 18x\right]_2^9 = (......) - (......)$ This mark is implied by final answer which rounds to 57.2 ***See above working(allow bracketing errors) to decide to award 3rd M1 mark for (b) here:*** $40.5 - (-16\tfrac{2}{3}) = 57\tfrac{1}{6}$ cao	3rd M1 (in (b)): Uses difference between two functions in integral. M: $x^n \to x^{n+1}$ for any one term. A1 at least two out of these three simplified terms Correct integration. (Ignore + c). Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	M1 A1 A1 dM1 B1 M1 A1 (7)
Special case of above method	$\int_2^9 x^2 - 11x + 18\,dx = \frac{x^3}{3} - \frac{11x^2}{2} + 18x\ \{+c\}$ $\left[\frac{x^3}{3} - \frac{11x^2}{2} + 18x\right]_2^9 = (......) - (......)$ This mark is implied by final answer which rounds to 57.2 (not -57.2) Difference of functions implied (see above expression) $40.5 - (-16\tfrac{2}{3}) = 57\tfrac{1}{6}$ cao		M1A1A1 DM1 B1 M1 A1 (7)
Special Case 2	Integrates expression in y e.g. "$y^2 - 9y + 8 = 0$": This can have first M1 in part (b) and no other marks. (It is not a method for finding this area)		
Notes	Take away trapezium again having used Method 2 loses last two marks Common Error: Integrates $-x^2 + 9x - 18$ is likely to be M1A1A0dM1B0M1A0 Integrates $2 - 11x - x^2$ is likely to e M1A0A0dM1B0M1A0 Writing $\int_2^9 (10x - x^2 - 8) - (10 - x)\,dx$ only earns final M mark		

EXPERT TUITION

Question number	Scheme	Marks
83 (a)	<table><tr><td>x</td><td>1</td><td>1.5</td><td>2</td><td>2.5</td><td>3</td><td>3.5</td><td>4</td></tr><tr><td>y</td><td>16.5</td><td>7.361</td><td>**4**</td><td>**2.31**</td><td>1.278</td><td>0.556</td><td>0</td></tr></table>	B1, B1 (2)
(b)	$\frac{1}{2}\times 0.5, \quad \{(16.5+0)+2(7.361+4+2.31+1.278+0.556)\}$ = 11.88 (or answers listed below in note)	B1, M1A1ft A1 (4)
(c)	$\int_1^4 \frac{16}{x^2}-\frac{x}{2}+1\,dx = \left[-\frac{16}{x}-\frac{x^2}{4}+x\right]_1^4$ $=[-4-4+4]-[-16-\frac{1}{4}+1]$ $= 11\frac{1}{4}$ or equivalent	M1 A1 A1 dM1 A1 (5) **11**
Notes	(a) **B1** for 4 or any correct equivalent e.g. 4.000 **B1** for 2.31 or 2.310 (b) **B1:** Need 0.25 or ½ of 0.5 **M1:** requires first bracket to contain first y value plus last y value (0 may be omitted or be at end) **and** second bracket to include no additional y values from those in the scheme. They may however omit one value as a slip. N.B. Special Case - Bracketing mistake $\frac{1}{2}\times 0.5(16.5+0)+2(7.361+4+2.31+1.278+0.556)$ scores **B1 M1 A0 A0** unless the final answer implies that the calculation has been done correctly (then full marks) A1ft: This should be correct but ft their 4 and 2.31 **A1**: Accept 11.8775 or 11.878 or 11.88 only (c) **M1** Attempt to integrate ie power increased by 1 or 1 becomes x , **A1** two correct terms, next **A1** all three correct unsimplified (ignore +c) (Allow $-16x^{-1}-0.25x^2+1x$ or equivalent) d**M1** **(This cannot be earned if previous M mark has not been awarded)** Uses limits 4 and 1 in their integrated expression and subtracts (either way round) **A1** 11.25 or 11 ¼ or 45/4 or equivalent (penalise negative final answer here)	
Alternative Method for (b)	Separate trapezia may be used : B1 for 0.25, M1 for $\frac{1}{2}h(a+b)$ used 5 or 6 times (and A1ft all correct for their "4" and "2.31") final A1 for 11.88 etc. as before	
	In part (b) Need to use trapezium rule – answer only (with no working) is 0/4 -any doubts send to review In part (c) need to see integration	

Question Number	Scheme		Marks
84. (a)	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$		
	$\{\text{Curve} = \text{Line}\} \Rightarrow -x^2 + 2x + 24 = x + 4$	Eliminating y correctly.	B1
	$x^2 - x - 20 \{= 0\} \Rightarrow (x-5)(x+4) \{= 0\} \Rightarrow x = ...$	Attempt to solve a ***resulting*** quadratic to give x = their values.	M1
	So, $x = 5, -4$	Both $x = 5$ and $x = -4$.	A1
	So corresponding y-values are $y = 9$ and $y = 0$.	See notes below.	B1ft **[4]**
(b)	$\left\{\int(-x^2 + 2x + 24)\,\mathrm{d}x\right\} = \underline{-\frac{x^3}{3} + \frac{2x^2}{2} + 24x} \{+c$	M1: $x^n \to x^{n+1}$ for any one term. 1st A1 at least two out of three terms. 2nd A1 for <u>correct answer</u>.	M1A1A1
	$\left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{-4}^{5} = (......) - (......)$	Substitutes 5 and -4 (or their limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{\left(-\frac{125}{3} + 25 + 120\right) - \left(\frac{64}{3} + 16 - 96\right) = \left(103\frac{1}{3}\right) - \left(-58\frac{2}{3}\right) = 162\right\}$		
	Area of $\Delta = \frac{1}{2}(9)(9) = 40.5$	Uses correct method for finding area of triangle.	M1
	So area of R is $162 - 40.5 = 121.5$	Area under curve – Area of triangle.	M1
		121.5	A1 oe **cao**
			[7]
			11

Question Number	Scheme	Marks
84(a)	1st B1: For correctly eliminating either x or y. Candidates will usually write $-x^2 + 2x + 24 = x + 4$. This mark can be implied by the resulting quadratic. M1: For solving their quadratic (which must be different to $-x^2 + 2x + 24$) to give $x = ...$ See introduction for Method mark for solving a 3TQ. It must result from some attempt to eliminate one of the variables. A1: For both $x = 5$ and $x = -4$. 2nd B1ft: For correctly substituting their values of x in equation of line or parabola to give ***both correct ft*** y-values. (You may have to get your calculators out if they substitute their x into $y = -x^2 + 2x + 24$). **Note:** For $x = 5, -4 \Rightarrow y = 9$ and $y = 0 \Rightarrow$ eg. $(-4, 9)$ and $(5, 0)$, award B1 isw. If the candidate gives additional answers to $(-4, 0)$ and $(5, 9)$, then withhold the final B1 mark. **Special Case**: Award SC: B0M0A0B1 for $\{A\}(-4, 0)$. You may see this point marked on the diagram. **Note:** SC: B0M0A0B1 for solving $0 = -x^2 + 2x + 24$ to give $\{A\}(-4, 0)$ and/or $(6, 10)$. Note: Do not give marks for working in part (b) which would be creditable in part (a).	
84(b)	1st M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms. Note that $24 \to 24x$ is sufficient for M1. 1st A1 at least two out of three terms correctly integrated. 2nd A1 for correct integration only and no follow through. Ignore the use of a $'+c'$. 2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -4 (and not 4 if the candidate has stated $x = -4$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. Allow one slip! 3rd M1: Area of triangle = $\frac{1}{2}$(their x_2 – their x_1)(their y_2) or Area of triangle = $\int_{x_1}^{x_2} x + 4 \{dx\}$. Where $x_1 =$ their -4, $x_2 =$ their 5 and $y_2 =$ their y usually found in part (a). 4th M1: Area under curve – Area under triangle, where both Area under curve > 0 and Area under triangle > 0 and Area under curve > Area under triangle. 3rd A1: 121.5 or $\frac{243}{2}$ oe **cao.**	

EXPERT TUITION

Question Number	Scheme	Marks
Aliter **84.**(b) **Way 2**	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$	
	Area of $R = \int_{-4}^{5} (-x^2 + 2x + 24) - (x + 4)\,dx$ — 3[rd] M1: Uses integral of $(x + 4)$ with correct ft limits. 4[th] M1: Uses "curve" – "line" function with correct ft limits.	
	M: $x^n \to x^{n+1}$ for any one term.	M1
	$= -\frac{x^3}{3} + \frac{x^2}{2} + 20x \{+c\}$ — A1 at least two out of three terms	A1ft
	Correct answer (Ignore + c).	A1
	$\left[-\frac{x^3}{3} + \frac{x^2}{2} + 20x\right]_{-4}^{5} = (......) - (......)$ — Substitutes 5 and -4 (or ***their limits*** from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{\left(-\frac{125}{3} + \frac{25}{2} + 100\right) - \left(\frac{64}{3} + 8 - 80\right) = \left(70\frac{5}{6}\right) - \left(-50\frac{2}{3}\right)\right\}$	
	See above working to decide to award 3[rd] M1 mark here:	M1
	See above working to decide to award 4[th] M1 mark here:	M1
	So area of R is $= 121.5$ — 121.5	A1 oe **cao**
		[7] **11**
84(b)	1[st] M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms. Note that $20 \to 20x$ is sufficient for M1. 1[st] A1 at least two out of three terms correctly ft. Note this accuracy mark is ft in Way 2. 2[nd] A1 for correct integration only and no follow through. Ignore the use of a $'+c'$. Allow 2[nd] A1 also for $-\frac{x^3}{3} + \frac{2x^2}{2} + 24x - \left(\frac{x^2}{2} + 4x\right)$. Note that $\frac{2x^2}{2} - \frac{x^2}{2}$ or $24x - 4x$ only counts as one integrated term for the 1[st] A1 mark. Do not allow any extra terms for the 2[nd] A1 mark. 2[nd] M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -4 (and not 4 if the candidate has stated $x = -4$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. Allow one slip! 3[rd] M1: Uses the integral of $(x + 4)$ with correct ft limits of their x_1 and their x_2 (usually found in part (a)) {where $(x_1, y_1) = (-4, 0)$ and $(x_2, y_2) = (5, 9)$.} This mark is usually found in the first line of the candidate's working in part (b). 4[th] M1: Uses "curve" – "line" function with correct ft (usually found in part (a)) limits. Subtraction must be correct way round. This mark is usually found in the first line of the candidate's working in part (b). Allow $\int_{-4}^{5} (-x^2 + 2x + 24) - x + 4 \{dx\}$ for this method mark. 3[rd] A1: 121.5 oe **cao.** **<u>Note: SPECIAL CASE for this alternative method</u>** Area of $R = \int_{-4}^{5} (x^2 - x - 20)\,dx = \left[\frac{x^3}{3} - \frac{x^2}{2} - 20x\right]_{-4}^{5} = \left(\frac{125}{3} - \frac{25}{2} - 100\right) - \left(-\frac{64}{3} - 8 + 80\right)$ The working so far would score SPEICAL CASE M1A1A1M1M1M0A0. The candidate may then go on to state that $= \left(-70\frac{5}{6}\right) - \left(50\frac{2}{3}\right) = -\frac{243}{2}$ If the candidate then multiplies their answer by -1 then they would gain the 4[th] M1 and 121.5 would gain the final A1 mark.	

Question Number	Scheme	Marks
Aliter	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$	
84. (a)	$\{\text{Curve = Line}\} \Rightarrow y = -(y-4)^2 + 2(y-4) + 24$ — Eliminating x correctly.	B1
Way 2	$y^2 - 9y\ \{=0\} \Rightarrow y(y-9)\ \{=0\} \Rightarrow y = ...$ — Attempt to solve a resulting quadratic to give y = their values.	M1
	So, $y = 0, 9$ — Both $y = 0$ and $y = 9$.	A1
	So corresponding y-values are $x = -4$ and $x = 5$. — See notes below.	B1ft
		[4]
	2[nd] B1ft: For correctly substituting their values of y in equation of line or parabola to give ***both correct ft*** x-values.	
84. (b)	**<u>Alternative Methods for obtaining the M1 mark for use of limits:</u>** There are two alternative methods can candidates can apply for finding "162". <u>Alternative 1:</u> $\int_{-4}^{0}(-x^2+2x+24)\,dx + \int_{0}^{5}(-x^2+2x+24)\,dx$ $= \left[-\frac{x^3}{3}+\frac{2x^2}{2}+24x\right]_{-4}^{0} + \left[-\frac{x^3}{3}+\frac{2x^2}{2}+24x\right]_{0}^{5}$ $= (0) - \left(\frac{64}{3}+16-96\right) + \left(-\frac{125}{3}+25+120\right) - (0)$ $= \left(103\frac{1}{3}\right) - \left(-58\frac{2}{3}\right) = 162$ <u>Alternative 2:</u> $\int_{-4}^{6}(-x^2+2x+24)\,dx - \int_{5}^{6}(-x^2+2x+24)\,dx$ $= \left[-\frac{x^3}{3}+\frac{2x^2}{2}+24x\right]_{-4}^{6} - \left[-\frac{x^3}{3}+\frac{2x^2}{2}+24x\right]_{5}^{6}$ $= \left\{\left(-\frac{216}{3}+36+144\right) - \left(\frac{64}{3}+16-96\right)\right\} - \left\{\left(-\frac{216}{3}+36+144\right) - \left(-\frac{125}{3}+25+120\right)\right\}$ $= \left\{(108) - \left(-58\frac{2}{3}\right)\right\} - \left\{(108) - \left(103\frac{1}{3}\right)\right\}$ $= \left(166\frac{2}{3}\right) - \left(4\frac{2}{3}\right) = 162$	

Question Number	Scheme	Marks
85. (a)	Seeing -1 and 5. (See note below.)	B1 (1)
(b)	$(x+1)(x-5) = \underline{x^2-4x-5}$ or $\underline{x^2-5x+x-5}$	B1
	$\int (x^2-4x-5)\,dx = \frac{x^3}{3} - \frac{4x^2}{2} - 5x \{+c\}$ — M: $x^n \to x^{n+1}$ for any one term. 1st A1 at least two out of three terms correctly ft.	M1A1ft A1
	$\left[\frac{x^3}{3} - \frac{4x^2}{2} - 5x\right]_{-1}^{5} = (......) - (......)$ — Substitutes 5 and -1 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{ \left(\frac{125}{3} - \frac{100}{2} - 25\right) - \left(-\frac{1}{3} - 2 + 5\right) = \left(-\frac{100}{3}\right) - \left(\frac{8}{3}\right) = -36 \right\}$	
	Hence, Area $= 36$ — Final answer must be 36, not -36	A1 (6) **[7]**
	Notes	
(a)	B1: for -1 and 5. Note that $(-1, 0)$ and $(5, 0)$ are acceptable for B1. Also allow $(0, -1)$ and $(0, 5)$ generously for B1. Note that if a candidate writes down that $A:(5,0)$, $B:(-1,0)$, (ie A and B interchanged,) then B0. Also allow values inserted in the correct position on the x-axis of the graph.	
(b)	B1 for x^2-4x-5 or $x^2-5x+x-5$. If you believe that the candidate is applying the Way 2 method then $-x^2+4x+5$ or $-x^2+5x-x+5$ would then be fine for B1. 1st M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms. Note that $-5 \to 5x$ is sufficient for M1. 1st A1 at least two out of three terms correctly ft from their multiplied out brackets. 2nd A1 for correct integration only and no follow through. Ignore the use of a '$+c$'. Allow 2nd A1 also for $\frac{x^3}{3} - \frac{5x^2}{2} + \frac{x^2}{2} - 5x$. Note that $-\frac{5x^2}{2} + \frac{x^2}{2}$ only counts as one integrated term for the 1st A1 mark. Do not allow any extra terms for the 2nd A1 mark. 2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -1 (and not 1 if the candidate has stated $x=-1$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. 3rd A1: For a final answer of 36, not -36. **Note:** An alternative method exists where the candidate states from the outset that Area $(R) = -\int_{-1}^{5} (x^2-4x+5)\,dx$ is detailed in the Appendix.	

Question Number	Scheme		Marks
86.			
(a)	x: 2, 2.25, 2.5, 2.75, 3 y: 0.5, 0.38, 0.298507…, 0.241691…, 0.2		
	At $\{x = 2.5,\}$ $y = 0.30$ (only)	At least one y-ordinate correct.	B1
	At $\{x = 2.75,\}$ $y = 0.24$ (only)	Both y-ordinates correct.	B1
			(2)
(b)		Outside brackets $\frac{1}{2}\times 0.25$ or $\frac{1}{8}$	B1 aef
	$\frac{1}{2}\times 0.25$; $\times\underline{\{0.5+0.2+2(0.38+\text{ their }0.30+\text{ their }0.24)\}}$	<u>For structure of</u> $\{\ldots\ldots\ldots\ldots\}$;	M1
		Correct expression <u>inside brackets</u> which all must be multiplied by their "outside constant".	<u>A1</u> $\sqrt{\ }$
	$\left\{=\frac{1}{8}(2.54)\right\}=$ awrt 0.32	awrt 0.32	A1
			(4)
(c)	Area of triangle $=\frac{1}{2}\times 1\times 0.2 = 0.1$		B1
	$\text{Area}(S) = \text{"}0.3175\text{"} - 0.1$		M1
	$= 0.2175$		A1 ft
			(3)
			[9]

Question Number	Scheme	Marks
	Notes	
(b)	B1 for using $\frac{1}{2}\times 0.25$ or $\frac{1}{8}$ or equivalent. M1 requires the correct $\{......\}$ bracket structure. This is for the first bracket to contain first y-ordinate plus last y-ordinate and the second bracket to be the summation of the remaining y-ordinates in the table. No errors (eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y-ordinate) are allowed in the second bracket and the second bracket must be multiplied by 2. Only one copying error is allowed here in the $2(0.38+\text{ their }0.30+\text{ their }0.24)$ bracket. A1ft for the correct bracket $\{......\}$ following through candidate's y-ordinates found in part (a). A1 for answer of awrt 0.32. <u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly** then award M1A0A0 for either $\frac{1}{2}\times 0.25\times 0.5+2(0.38+\text{ their }0.30+\text{ their }0.24)+0.2$ (nb: yielding final answer of 2.1025) so that the 0.5 is only multiplied by $\frac{1}{2}\times 0.25$ or $\frac{1}{2}\times 0.25\times(0.5+0.2)+2(0.38+\text{ their }0.30+\text{ their }0.24)$ (nb: yielding final answer of 1.9275) so that the $(0.5+0.2)$ is multiplied by $\frac{1}{2}\times 0.25$. **Need to see trapezium rule – answer only (with no working) gains no marks.** **<u>Alternative:</u>** Separate trapezia may be used, and this can be marked equivalently. (See appendix.)	
(c)	B1 for the area of the triangle identified as either $\frac{1}{2}\times 1\times 0.2$ or 0.1. May be identified on the diagram. M1 for "part (b) answer" – "0.1 only" or "part (b) answer – their attempt at 0.1 only". (Strict attempt!) A1ft for correctly following through "part (b) answer" – 0.1. This is also dependent on the answer to (b) being greater than 0.1. Note: candidates may round answers here, so allow A1ft if they round their answer correct to 2 dp.	

Question Number	Scheme	Marks
87	(a) $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2-20x+k$ (Differentiation is required) At x = 2, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$, so $12-40+k=0$ k = 28 (*) <u>N.B. The ‘= 0’ must be seen at some stage to score the final mark.</u> <u>Alternatively</u>: (using k = 28) $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2-20x+28$ (M1 A1) ‘Assuming’ k = 28 only scores the final cso mark if there is justification that $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ at x = 2 represents the <u>maximum</u> turning point.	M1 A1 A1 cso (3)
	(b) $\int(x^3-10x^2+28x)\,\mathrm{d}x=\frac{x^4}{4}-\frac{10x^3}{3}+\frac{28x^2}{2}$ Allow $\frac{kx^2}{2}$ for $\frac{28x^2}{2}$ $\left[\frac{x^4}{4}-\frac{10x^3}{3}+14x^2\right]_0^2=...$ $\left(=4-\frac{80}{3}+56=\frac{100}{3}\right)$ (With limits 0 to 2, substitute the limit 2 into a 'changed function') y-coordinate of P = $8-40+56=24$ <u>Allow if seen in part (a)</u> (The B1 for 24 may be scored by implication from later working) Area of rectangle = $2\times$(their y-coordinate of P) Area of R = (their 48) $-\left(\text{their } \frac{100}{3}\right)$ $=\frac{44}{3}$ $\left(14\frac{2}{3} \text{ or } 14.\dot{6}\right)$ If the subtraction is the ‘wrong way round’, the final A mark is lost.	M1 A1 M1 B1 M1 A1 (6) **9**
	(a) M: $x^n\to cx^{n-1}$ (c constant, $c\neq 0$) for one term, seen <u>in part (a)</u>. (b) 1st M: $x^n\to cx^{n+1}$ (c constant, $c\neq 0$) for one term. Integrating the <u>gradient function</u> loses this M mark. 2ndM: Requires use of limits 0 and 2, with 2 substituted into a ‘changed function’. (It may, for example, have been differentiated). Final M: Subtract their values either way round. This mark is dependent on the use of calculus and a correct method attempt for the area of the rectangle. A1: Must be <u>exact</u>, not 14.67 or similar, but isw after seeing, say, $\frac{44}{3}$. <u>Alternative</u>: (effectively finding area of rectangle by integration) $\int\{24-(x^3-10x^2+28x)\}\mathrm{d}x=24x-\left(\frac{x^4}{4}-\frac{10x^3}{3}+\frac{28x^2}{2}\right)$, etc. This can be marked equivalently, with the 1st A being for integrating the same 3 terms correctly. The 3rd M (for subtraction) will be scored at the same stage as the 2nd M. If the subtraction is the ‘wrong way round’, the final A mark is lost.	

Question Number	Scheme	Marks
88 (a)	**Puts** $y = 0$ and attempts to solve quadratic e.g. $(x-4)(x-1)=0$ Points are (1,0) and (4, 0)	M1 A1 (2)
(b)	$x = 5$ gives $y = 25 - 25 + 4$ and so (5, 4) lies on the curve	B1cso (1)
(c)	$\int(x^2-5x+4)\mathrm{d}x = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x \quad (+c)$	M1A1 (2)
(d)	Area of triangle = $\frac{1}{2}\times4\times4=8$ or $\int(x-1)\mathrm{d}x = \frac{1}{2}x^2 - x$ with limits 1 and 5 to give 8	B1
	Area under the curve = $\int_4^5$ $\quad \frac{1}{3}\times5^3 - \frac{5}{2}\times5^2 + 4\times5 \quad \left[=-\frac{5}{6}\right]$	M1
	$\frac{1}{3}\times4^3 - \frac{5}{2}\times4^2 + 4\times4 \quad \left[=-\frac{8}{3}\right]$	M1
	$\int_4^5 \; = -\frac{5}{6} - -\frac{8}{3} = \frac{11}{6}$ or equivalent (allow 1.83 or 1.8 here)	A1 cao
	Area of R = $8 - \frac{11}{6} = 6\frac{1}{6}$ or $\frac{37}{6}$ or $6.1\dot{6}$ (not 6.17)	A1 cao (5) **[10]**
(a)	M1 for attempt to find L and M A1 Accept $x = 1$ and $x = 4$, then isw or accept L =(1,0) , M = (4,0) Do not accept L = 1, M = 4 nor (0 , 1), (0 , 4) (unless subsequent work) Do not need to distinguish L and M. Answers imply M1A1.	
(b)	See substitution, working should be shown, need conclusion which could be just y = 4 or a tick. Allow y = 25 - 25 + 4 = 4 But not 25 - 25 + 4 = 4. ($y=4$ may appear at start) Usually 0 = 0 or 4 = 4 is B0	
(c)	M1 for attempt to integrate $x^2 \to kx^3, x \to kx^2$ or $4 \to 4x$ A1 for correct integration of all three terms (do not need constant) isw. Mark correct work when seen. So e.g. $\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x$ is A1 then $2x^3 - 15x^2 + 24x$ would be ignored as subsequent work.	
(d)	B1 for this triangle only (not triangle LMN) 1st M1 for substituting 5 into their changed function 2nd M1 for substituting 4 into their changed function	
(d)	Alternative method: $\int_1^5 (x-1)-(x^2-5x+4)\mathrm{d}x + \int_1^4 x^2 - 5x + 4\mathrm{d}x$ can lead to correct answer Constructs $\int_1^5 (x-1)-(x^2-5x+4)\mathrm{d}x$ is B1 M1 for substituting 5 and 1 and subtracting in first integral M1 for substituting 4 and 1 and subtracting in second integral A1 for answer to first integral i.e. $\frac{32}{3}$ (allow 10.7) and A1 for final answer as before..	

(d)	Another alternative $\int_4^5 (x-1)-(x^2-5x+4)\mathrm{d}x +$ *area of triangle LMP* Constructs $\int_4^5 (x-1)-(x^2-5x+4)\mathrm{d}x$ is B1 M1 for substituting 5 and 4 and subtracting in first integral M1 for complete method to find area of triangle (4.5) A1 for answer to first integral i.e. $\frac{5}{3}$ and A1 for final answer as before.
(d)	Could also use $\int_4^5 (4x-16)-(x^2-5x+4)\mathrm{d}x +$ *area of triangle LMN* Similar scheme to previous one. Triangle has area 6 A1 for finding Integral has value $\frac{1}{6}$ and A1 for final answer as before.

Question Number	Scheme	Marks
89	$\int \left(2x+3x^{\frac{1}{2}}\right) \mathrm{d}x = \frac{2x^2}{2} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$	M1 A1A1
	$\int_1^4 \left(2x+3x^{\frac{1}{2}}\right) \mathrm{d}x = \left[x^2+2x^{\frac{3}{2}}\right]_1^4 = (16+2\times 8)-(1+2)$	M1
	$= 29$ (29 + C scores A0)	A1 (5) **[5]**
	1st M1 for attempt to integrate $x \to kx^2$ or $x^{\frac{1}{2}} \to kx^{\frac{3}{2}}$. 1st A1 for $\frac{2x^2}{2}$ or a simplified version. 2nd A1 for $\frac{3x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$ or $\frac{3x\sqrt{x}}{\left(\frac{3}{2}\right)}$ or a simplified version. Ignore + C, if seen, but two correct terms and an <u>extra non-constant</u> term scores M1A1A0. 2nd M1 for correct use of correct limits ('top' – 'bottom'). Must be used in a 'changed function', not just the original. (The changed function may have been found by differentiation). Ignore 'poor notation' (e.g. missing integral signs) if the intention is clear. <u>No working</u>: The answer 29 with no working scores M0A0A0M1A0 (1 mark).	

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>90</td><td>$y=(1+x)(4-x)=4+3x-x^2$ M: Expand, giving 3 (or 4) terms

$\int\left(4+3x-x^2\right)\mathrm{d}x=4x+\frac{3x^2}{2}-\frac{x^3}{3}$ M: Attempt to integrate

$=[..............]_{-1}^{4}=\left(16+24-\frac{64}{3}\right)-\left(-4+\frac{3}{2}+\frac{1}{3}\right)=\frac{125}{6}$ $\left(=20\frac{5}{6}\right)$</td><td>M1

M1 A1

M1 A1 (5)
[5]</td></tr>
<tr><td>Notes</td><td colspan="2">M1 needs expansion, there may be a slip involving a sign or simple arithmetical error e.g. $1\times 4=5$, but there needs to be a ‘constant’ an ‘x term’ and an ‘ x^2 term’. The x terms do not need to be collected. (Need not be seen if next line correct)

Attempt to integrate means that $x^n\to x^{n+1}$ for at least one of the terms, then M1 is awarded (even 4 becoming 4x is sufficient) – one correct power sufficient.

A1 is for correct answer only, not follow through. But allow $2x^2-\frac{1}{2}x^2$ or any correct equivalent. Allow + c, and even allow an evaluated extra constant term.

M1: Substitute limit 4 and limit –1 into a changed function (must be –1) and indicate subtraction (either way round).

A1 must be exact, not 20.83 or similar. If recurring indicated can have the mark. Negative area, even if subsequently positive loses the A mark.</td></tr>
<tr><td>Special cases</td><td colspan="2">(i) Uses calculator method: M1 for expansion (if seen) M1 for limits if answer correct, so 0 , 1 or 2 marks out of 5 is possible (Most likely M0 M0 A0 M1 A0)
(ii) Uses trapezium rule : not exact, no calculus – 0/5 unless expansion mark M1 gained.
(iii) Using original method, but then change all signs after expansion is likely to lead to: M1 M1 A0, M1 A0 i.e. 3/5</td></tr>
</table>

Question number	Scheme	Marks
91	Area of triangle = $\frac{1}{2}\times 2\times 22$ (M: Correct method to find area of triangle)	M1 A1
	(Area = 22 with no working is acceptable)	
	$\int 10+8x+x^2-x^3\,dx = 10x+\frac{8x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}$ (M: $x^n \to x^{n+1}$ for one of the terms)	M1 A1 A1
	Only one term correct: M1 A0 A0 2 or 3 terms correct: M1 A1 A0 Integrating the <u>gradient function</u> loses this M mark.	
	$\left[10x+\frac{8x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^2 = \ldots\ldots$ (Substitute limit 2 into a 'changed function')	M1
	$\left(=20+16+\frac{8}{3}-4\right)$ (This M can be awarded even if the other limit is wrong)	
	Area of $R = 34\frac{2}{3}-22=\frac{38}{3}\left(=12\frac{2}{3}\right)$ (Or $12.\dot{6}$)	M1 A1 (8)
	M: <u>Dependent on use of calculus in (b) and correct overall 'strategy'</u>: subtract either way round. A: Must be <u>exact</u>, not 12.67 or similar. A negative area at the end, even if subsequently made positive, loses the A mark.	
		8
	<u>Alternative</u>:	
	Eqn. of line $y = 11x$. (Marks dependent on subsequent use in integration)	M1 A1
	(M1: Correct method to find equation of line. A1: Simplified form $y = 11x$)	
	$\int 10+kx+x^2-x^3\,dx = 10x+\frac{kx^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}$ (k perhaps –3)	M1 A1 A1
	$\left[10x+\frac{kx^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^2 = \ldots\ldots$ (Substitute limit 2 into a 'changed function')	M1
	Area of $R = \left[10x-\frac{3x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^2 = 20-6+\frac{8}{3}-4=\frac{38}{3}$ $\left(=12\frac{2}{3}\right)$	M1 A1 (8)
	Final M1 for $\int(\text{curve})-\int(\text{line})$ or $\int(\text{line})-\int(\text{curve})$.	

92	(a)	Either solving $0 = x(6 - x)$ and showing $x = 6$ (and $x = 0$) or showing (6,0) (and $x = 0$) satisfies $y = 6x - x^2$ [allow for showing x = 6]	B1 (1)
	(b)	Solving $2x = 6x - x^2$ $(x^2 = 4x)$ to x = .. $x = 4$ (and $x = 0$) Conclusion: when $x = 4$, $y = 8$ and when $x = 0$, $y = 0$,	M1 A1 A1 (3)
	(c)	(Area =) $\int_{(0)}^{(4)}(6x - x^2)\,dx$ Limits not required Correct integration $3x^2 - \frac{x^3}{3}$ (+ c) Correct use of correct limits on their result above (see notes on limits) [" $3x^2 - \frac{x^3}{3}$ "]^4 – [" $3x^2 - \frac{x^3}{3}$ "]$_0$ with limits substituted $[= 48 - 21\frac{1}{3} = 26\frac{2}{3}]$ Area of triangle = $2 \times 8 = 16$ (Can be awarded even if no M scored, i.e. B1) Shaded area = $\pm$(area under curve – area of triangle) applied correctly $(= 26\frac{2}{3} - 16)$ $= 10\frac{2}{3}$ (awrt 10.7)	M1 A1 M1 A1 M1 A1 (6)**[10]**
Notes		(b) In scheme first A1: need only give $x = 4$ If *verifying approach* used: Verifying (4,8) satisfies both the line and the curve M1(attempt at both), Both shown successfully A1 For final A1, (0,0) needs to be mentioned ; accept " clear from diagram" (c) **Alternative** Using Area = $\pm\int_{(0)}^{(4)}\{(6x - x^2); \quad -2x\}\,dx$ approach (i) If candidate integrates separately can be marked as main scheme If combine to work with = $\pm\int_{(0)}^{(4)}(4x - x^2)\,dx$, **first** M mark and **third** M mark $= (\pm)\,[\,2x^2 - \frac{x^3}{3}$ (+ c)] A1, Correct use of correct limits on their result **second** M1, Totally correct, unsimplified $\pm$ expression (may be implied by correct ans.) A1 10⅔ A1 [Allow this if, having given - 10⅔, they correct it] M1 *for correct use of correct limits*: Must substitute correct limits for their strategy into a changed expression and subtract, either way round, e.g $\pm\{[\ ]^4 - [\ ]_0\}$ If a long method is used, e,g, finding three areas, this mark only gained for correct strategy and all limits need to be correct for this strategy. Use of trapezium rule: M0A0MA0,possibleA1for triangle M1(if correct application of trap. rule from $x = 0$ to $x = 4$) A0	

Question Number	Scheme	Notes	Marks
93. (i)	$\dfrac{13-4x}{(2x+1)^2(x+3)} \equiv \dfrac{A}{(2x+1)} + \dfrac{B}{(2x+1)^2} + \dfrac{C}{(x+3)}$		
(a)	$B=6,\ C=1$	At least one of $B=6$ or $C=1$	B1
		Both $B=6$ and $C=1$	B1
	$13-4x \equiv A(2x+1)(x+3) + B(x+3) + C(2x+1)^2$ $x=-3 \Rightarrow 25 = 25C \Rightarrow C=1$ $x=-\dfrac{1}{2} \Rightarrow 13--2 = \dfrac{5}{2}B \Rightarrow 15 = 2.5B \Rightarrow B=6$	Writes down a correct identity and attempts to find the value of either one of A or B or C	M1
	Either $x^2: 0 = 2A+4C$, constant : $13 = 3A+3B+C$, x: - $4 = 7A+B+4C$ or $x=0 \Rightarrow 13 = 3A+3B+C$ leading to $A=$ - 2	Using a correct identity to find $A=$ - 2	A1
			[4]
(b)	$\displaystyle\int \frac{13-4x}{(2x+1)^2(x+3)}\,dx = \int \frac{-2}{(2x+1)} + \frac{6}{(2x+1)^2} + \frac{1}{(x+3)}\,dx$		
	$= \dfrac{(-2)}{2}\ln(2x+1) + \dfrac{6(2x+1)^{-1}}{(-1)(2)} + \ln(x+3)\ \{+c\}$ o.e. $\{= -\ln(2x+1) - 3(2x+1)^{-1} + \ln(x+3)\ \{+c\}\}$	See notes	M1
		At least two terms correctly integrated	A1ft
		Correct answer, o.e. Simplified or un-simplified. The correct answer must be stated on one line Ignore the absence of '$+c$'	A1
			[3]
(ii)	$\{(e^x+1)^3 =\}\ e^{3x} + 3e^{2x} + 3e^x + 1$	$e^{3x} + 3e^{2x} + 3e^x + 1$, simplified or un-simplified	B1
	$\left\{\displaystyle\int (e^x+1)^3\,dx\right\} = \dfrac{1}{3}e^{3x} + \dfrac{3}{2}e^{2x} + 3e^x + x\ \{+c\}$	At least 3 examples (see notes) of correct ft integration	M1
		$\dfrac{1}{3}e^{3x} + \dfrac{3}{2}e^{2x} + 3e^x + x$, simplified or un-simplified with or without $+c$	A1
			[3]
(iii)	$\displaystyle\int \frac{1}{4x+5x^{\frac{1}{3}}}\,dx,\ x>0;\ u^3 = x$		
	$3u^2\dfrac{du}{dx} = 1$	$3u^2\dfrac{du}{dx} = 1$ or $\dfrac{dx}{du} = 3u^2$ or $\dfrac{du}{dx} = \dfrac{1}{3}x^{-\frac{2}{3}}$ or $3u^2du = dx$ o.e.	B1
	$= \displaystyle\int \frac{1}{4u^3+5u}.3u^2\,du\ \left\{= \int \frac{3u}{4u^2+5}\,du\right\}$	Expression of the form $\displaystyle\int \frac{\pm ku^2}{4u^3 \pm 5u}\{du\}$, $k \neq 0$ Does not have to include integral sign or Can be implied by later working	M1
	$\equiv \dfrac{3}{8}\ln(4u^2+5)\ \{+c\}$ $\dfrac{3}{8}\ln\left(4x^{\frac{2}{3}}+5\right)\ \{+c\}$	**dependent on the previous M mark** $\pm\lambda\ln(4u^2+5)$; λ is a constant; $\lambda \neq 0$	dM1
		Correct answer in x with or without $+c$	A1
			[4]
			14

EXPERT TUITION

	Question 93 Notes		
93. (iii) **Alt 1**	**Alternative method 1 for part (iii)**		
	$\left\{\int \frac{1}{4x+5x^{\frac{1}{3}}}\,\mathrm{d}x\right\} = \int \frac{x^{-\frac{1}{3}}}{4x^{\frac{2}{3}}+5}\,\mathrm{d}x$	Attempts to multiply numerator and denominator by $x^{-\frac{1}{3}}$	M1
		Expression of the form $\int \frac{\pm kx^{-\frac{1}{3}}}{4x^{\frac{2}{3}}\pm 5}\,\mathrm{d}x,\ k\neq 0$ Does not have to include integral sign or $\mathrm{d}u$ Can be implied by later working	M1
	$= \frac{3}{8}\ln\left(4x^{\frac{2}{3}}+5\right)\ \{+c\}$	$\pm\lambda\ln(4x^{\frac{2}{3}}+5)$; λ is a constant; $\lambda\neq 0$	dM1
		Correct answer in x with or without $+c$	A1
			[4]

93. (i) (a)	**M1**	Writes down ***a correct identity*** (although this can be implied) and attempts ***to find the value of at least one*** of either A or B or C. This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients.
	Note	The correct partial fraction from no working scores B1B1M1A1
(i) (b)	**M1**	At least 2 of either $\pm\frac{P}{(2x+1)} \to \pm D\ln(2x+1)$ or $\pm D\ln(x+\frac{1}{2})$ or $\pm\frac{Q}{(2x+1)^2} \to \pm E(2x+1)^{-1}$ or $\pm\frac{R}{(x+3)} \to \pm F\ln(x+3)$ for their constants P, Q, R.
	A1ft	At least two terms from any of $\pm\frac{P}{(2x+1)}$ or $\pm\frac{Q}{(2x+1)^2}$ or $\pm\frac{R}{(x+3)}$ correctly integrated.
	Note	Can be un-simplified for the A1ft mark.
	A1	Correct answer of $\frac{(-2)}{2}\ln(2x+1)+\frac{6(2x+1)^{-1}}{(-1)(2)}+\ln(x+3)\ \{+c\}$ simplified or un-simplified. with or without '+ c'.
	Note	Allow final A1 for equivalent answers, e.g. $\ln\left(\frac{x+3}{2x+1}\right)-\frac{3}{2x+1}\ \{+c\}$ or $\ln\left(\frac{2x+6}{2x+1}\right)-\frac{3}{2x+1}\ \{+c\}$
	Note	**Beware that** $\int\frac{-2}{(2x+1)}\,\mathrm{d}x = \int\frac{-1}{(x+\frac{1}{2})}\,\mathrm{d}x = -\ln(x+\frac{1}{2})\ \{+c\}$ is correct integration
	Note	E.g. Allow M1 A1ft A1 for a correct un-simplified $\ln(x+3)-\ln(x+\frac{1}{2})-\frac{3}{2}(x+\frac{1}{2})^{-1}\ \{+c\}$
	Note	Condone 1st A1ft for poor bracketing, but do not allow poor bracketing for the final A1 E.g. Give final A0 for $-\ln 2x+1-3(2x+1)^{-1}+\ln x+3\ \{+c\}$ unless recovered
(ii)	**Note**	Give B1 for an un-simplified $\mathrm{e}^{3x}+2\mathrm{e}^{2x}+\mathrm{e}^{2x}+2\mathrm{e}^{x}+\mathrm{e}^{x}+1$
	M1	At least 3 of either $\alpha\mathrm{e}^{3x}\to\frac{\alpha}{3}\mathrm{e}^{3x}$ **or** $\beta\mathrm{e}^{2x}\to\frac{\beta}{2}\mathrm{e}^{2x}$ **or** $\delta\mathrm{e}^{x}\to\delta\mathrm{e}^{x}$ **or** $\mu\to\mu x$; $\alpha, \beta, \delta, \mu\neq 0$
	Note	Give A1 for an un-simplified $\frac{1}{3}\mathrm{e}^{3x}+\mathrm{e}^{2x}+\frac{1}{2}\mathrm{e}^{2x}+2\mathrm{e}^{x}+\mathrm{e}^{x}+x$, with or without $+c$
(iii)	**Note**	1st M1 can be implied by $\int\frac{\pm ku}{4u^2\pm 5}\{\mathrm{d}u\},\ k\neq 0$. Does not have to include integral sign or $\mathrm{d}u$
	Note	Condone 1st M1 for expressions of the form $\int\left(\frac{\pm 1}{4u^3\pm 5u}.\frac{\pm k}{u^{-2}}\right)\{\mathrm{d}u\},\ k\neq 0$
	Note	Give 2nd M0 for $\frac{3u}{8u}\ln(4u^2+5)\ \{+c\}$ (u's not cancelled) unless recovered in later working
	Note	E.g. Give 2nd M0 for integration leading to $\frac{3}{4}u\ln(4u^2+5)$ as this is not in the form $\pm\lambda\ln(4u^2+5)$

	Note	Condone 2nd M1 for poor bracketing, but do not allow poor bracketing for the final A1 E.g. Give final A0 for $\frac{3}{8}\ln 4x^{\frac{2}{3}} + 5\ \{+c\}$ unless recovered

Question Number	Scheme	Notes	Marks
93. (ii) **Alt 1**	$\int (e^x+1)^3\,dx;\quad u = e^x + 1 \Rightarrow \frac{du}{dx} = e^x$		
	$\left\{= \int \frac{u^3}{(u-1)}\,du = \right\} \int \left(u^2 + u + 1 + \frac{1}{u-1}\right) du$	$\int \left(u^2 + u + 1 + \frac{1}{u-1}\right) \{du\}$ where $u = e^x + 1$	B1
	$= \frac{1}{3}u^3 + \frac{1}{2}u^2 + u + \ln(u-1)\ \{+c\}$	At least 3 of either $\alpha u^2 \to \frac{\alpha}{3}u^3$ **or** $\beta u \to \frac{\beta}{2}u^2$ **or** $\delta \to \delta u$ **or** $\frac{\lambda}{u-1} \to \lambda\ln(u-1);\ \alpha, \beta, \delta, \lambda \neq 0$	M1
	$= \frac{1}{3}(e^x+1)^3 + \frac{1}{2}(e^x+1)^2 + (e^x+1) + \ln(e^x+1-1)\ \{+c\}$		
	$= \frac{1}{3}(e^x+1)^3 + \frac{1}{2}(e^x+1)^2 + (e^x+1) + x\ \{+c\}$	$\frac{1}{3}(e^x+1)^3 + \frac{1}{2}(e^x+1)^2 + (e^x+1) + x$ or $\frac{1}{3}(e^x+1)^3 + \frac{1}{2}(e^x+1)^2 + e^x + x$ simplified or un-simplified with or without $+c$ **Note:** $\ln(e^x+1-1)$ needs to be simplified to x for this mark	A1
			[3]
93. (ii) **Alt 2**	$\int (e^x+1)^3\,dx;\quad u = e^x \Rightarrow \frac{du}{dx} = e^x$		
	$\left\{= \int \frac{(u+1)^3}{u}\,du = \right\} \int \left(u^2 + 3u + 3 + \frac{1}{u}\right) du$	$\int \left(u^2 + 3u + 3 + \frac{1}{u}\right) \{du\}$ where $u = e^x$	B1
	$= \frac{1}{3}u^3 + \frac{3}{2}u^2 + 3u + \ln u\ \{+c\}$	At least 3 of either $\alpha u^2 \to \frac{\alpha}{3}u^3$ **or** $\beta u \to \frac{\beta}{2}u^2$ **or** $\delta \to \delta u$ **or** $\frac{\lambda}{u} \to \lambda\ln u;\ \alpha, \beta, \delta, \lambda \neq 0$	M1
	$= \frac{1}{3}e^{3x} + \frac{3}{2}e^{2x} + 3e^x + x\ \{+c\}$	$\frac{1}{3}e^{3x} + \frac{3}{2}e^{2x} + 3e^x + x$, simplified or un-simplified with or without $+c$ **Note:** $\ln(e^x)$ needs to be simplified to x for this mark	A1
			[3]

Question Number	Scheme	Notes	Marks
94.	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2}{3\cos^2 2x}; \ -\frac{1}{2} < x < \frac{1}{2}; \ y = 2$ at $x = -\frac{\pi}{8}$		
	$\int \frac{1}{y^2}\,\mathrm{d}y = \int \frac{1}{3\cos^2 2x}\,\mathrm{d}x$	Separates variables as shown. Can be implied by a correct attempt at integration. Ignore the integral signs	B1
	$\int \frac{1}{y^2}\,\mathrm{d}y = \int \frac{1}{3}\sec^2 2x\,\mathrm{d}x$		
	$-\frac{1}{y} = \frac{1}{3}\left(\frac{\tan 2x}{2}\right)\{+c\}$	$\pm\frac{A}{y^2} \to \pm\frac{B}{y}; \ A, B \neq 0$	M1
			M1
		$-\frac{1}{y} = \frac{1}{3}\left(\frac{\tan 2x}{2}\right)$	A1
	$-\frac{1}{2} = \frac{1}{6}\tan\left(2\left(-\frac{\pi}{8}\right)\right) + c$	Use of $x = -\frac{\pi}{8}$ and $y = 2$ in an integrated equation ***containing a constant of integration***, e.g. c	M1
	$-\frac{1}{2} = -\frac{1}{6} + c \Rightarrow c = -\frac{1}{3}$		
	$-\frac{1}{y} = \frac{1}{6}\tan 2x - \frac{1}{3} = \frac{\tan(2x) - 2}{6}$		
	$y = \frac{-1}{\frac{1}{6}\tan 2x - \frac{1}{3}}$ or $y = \frac{6}{2 - \tan 2x}$ or $y = \frac{6\cot 2x}{-1 + 2\cot 2x}$ $\left\{-\frac{1}{2} < x < \frac{1}{2}\right\}$		A1 o.e.
			[6]
			6

		Question 94 Notes
94.	**B1**	Separates variables as shown. $\mathrm{d}y$ and $\mathrm{d}x$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. The number "3" may appear on either side. E.g. $\int \frac{1}{y^2}\,\mathrm{d}y = \int \frac{1}{3}\sec^2 2x\,\mathrm{d}x$ or $\int \frac{3}{y^2}\,\mathrm{d}y = \int \frac{1}{\cos^2 2x}\,\mathrm{d}x$ are fine for B1
	Note	Allow e.g. $\int \frac{1}{y^2}\frac{\mathrm{d}y}{\mathrm{d}x}\,\mathrm{d}x = \int \frac{1}{3}\sec^2 2x\,\mathrm{d}x$ for B1 or condone $\int \frac{1}{y^2} = \int \frac{1}{3}\sec^2 2x$ for B1
	Note	B1 can be implied by correct integration of both sides
	M1	$\pm\frac{A}{y^2} \to \pm\frac{B}{y}; \ A, B \neq 0$
	M1	$\frac{1}{\cos^2 2x}$ or $\sec^2 2x \to \pm\lambda\tan 2x; \ \lambda \neq 0$
	A1	$-\frac{1}{y} = \frac{1}{3}\left(\frac{\tan 2x}{2}\right)$ with or without '$+c$'. E.g. $-\frac{6}{y} = \tan 2x$
	M1	Evidence of using both $x = -\frac{\pi}{8}$ and $y = 2$ in an integrated or changed equation containing c
	Note	This mark can be implied by the correct value of c
	Note	You may need to use your calculator to check that they have satisfied the final M mark
	Note	Condone using $x = \frac{\pi}{8}$ instead of $x = -\frac{\pi}{8}$
	A1	$y = \frac{-1}{\frac{1}{6}\tan 2x - \frac{1}{3}}$ or $y = \frac{6}{2 - \tan 2x}$ **or any equivalent correct answer in the form** $y = \mathrm{f}(x)$
	Note	You can ignore subsequent working, which follows from a correct answer

	Question 94 Notes Continued	
94.	**Note**	Writing $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2}{3\cos^2 2x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}y^2\sec^2 2x$ leading to e.g. • $y = \frac{1}{9}y^3\left(\frac{1}{2}\tan 2x\right)$ gets 2nd M0 for $\pm\lambda\tan 2x$ • $u = \frac{1}{3}y^2,\ \frac{\mathrm{d}v}{\mathrm{d}x} = \sec^2 2x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2}{3}y,\ v = \frac{1}{2}\tan 2x$ gets 2nd M0 for $\pm\lambda\tan 2x$ because the variables have not been separated

Question Number	Scheme	Notes	Marks
95. (a)	$\left\{\int x\cos 4x\,\mathrm{d}x\right\}$ $= \frac{1}{4}x\sin 4x - \int \frac{1}{4}\sin 4x\ \{\mathrm{d}x\}$	$\pm\alpha x\sin 4x \pm \beta\int \sin 4x\ \{\mathrm{d}x\}$, with or without $\mathrm{d}x$; $\alpha,\ \beta \neq 0$	M1
		$\frac{1}{4}x\sin 4x - \int \frac{1}{4}\sin 4x\ \{\mathrm{d}x\}$, with or without $\mathrm{d}x$ **Can be simplified or un-simplified**	A1
	$= \frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x\ \{+c\}$	$\frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x$ o.e. with or without $+c$ **Can be simplified or un-simplified**	A1
	Note: You can ignore subsequent working following on from a correct solution		**[3]**
(b) **Way 1**	$\{V =\}\ \pi\int_0^{\frac{\pi}{4}} \left(\sqrt{x}\sin 2x\right)^2 \{\mathrm{d}x\}$	$\pi\int \left(\sqrt{x}\sin 2x\right)^2 \{\mathrm{d}x\}$ Ignore limits and $\mathrm{d}x$. Can be implied	B1
	$\left\{\int x\sin^2 2x\,\mathrm{d}x = \right\}$ $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$	For writing down a correct equation linking $\sin^2 2x$ and $\cos 4x$ (e.g. $\cos 4x = 1 - 2\sin^2 2x$) **and** some attempt at applying this equation (or a manipulation of this equation which can be incorrect) to their integral Can be implied.	M1
		Simplifies $\int x\sin^2 2x\ \{\mathrm{d}x\}$ to $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$	A1
	$\left\{\int\left(\frac{1}{2}x - \frac{1}{2}x\cos 4x\right)\mathrm{d}x\right\}$ $= \frac{1}{4}x^2 - \frac{1}{2}\left(\frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x\right)\{+c\}$	Integrates to give $\pm Ax^2 \pm Bx\sin 4x \pm C\cos 4x;\ A, B,\ C \neq 0$ which can be simplified or un-simplified. **Note:** Allow one transcription error (on $\sin 4x$ or $\cos 4x$) in the copying of their answer from part (a) to part (b)	M1
	$\left\{\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\mathrm{d}x = \left[\frac{1}{4}x^2 - \frac{1}{8}x\sin 4x - \frac{1}{32}\cos 4x\right]_0^{\frac{\pi}{4}}\right\}$		
	$= \left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - \left(0 - 0 - \frac{1}{32}\cos 0\right)$	**dependent on the previous M mark** see notes	dM1
	$= \left(\frac{\pi^2}{64} + \frac{1}{32}\right) - \left(-\frac{1}{32}\right) = \frac{\pi^2}{64} + \frac{1}{16}$		
	So, $V = \pi\left(\frac{\pi^2}{64} + \frac{1}{16}\right)$ or $\frac{1}{64}\pi^3 + \frac{1}{16}\pi$ or $\frac{\pi}{2}\left(\frac{\pi^2}{32} + \frac{1}{8}\right)$ o.e.	two term exact answer	A1 o.e.
			[6]
			9

Question 95 Notes

	SC	**Special Case for the 2nd M and 3rd M mark for those who use their answer from part (a)** You can apply the 2nd M and 3rd M marks for integration of the form $\pm Ax^2 \pm$ (their answer to part (a)) where their answer to part (a) is in the form • $\pm Bx\sin kx \pm C\cos px$ to give $\pm Ax^2 \pm Bx\sin kx \pm C\cos px$ • $\pm Bx\sin kx \pm C\sin px$ to give $\pm Ax^2 \pm Bx\sin kx \pm C\sin px$ • $\pm Bx\cos kx \pm C\sin px$ to give $\pm Ax^2 \pm Bx\cos kx \pm C\sin px$ • $\pm Bx\cos kx \pm C\cos px$ to give $\pm Ax^2 \pm Bx\cos kx \pm C\cos px$ $k, p \neq 0$, k, p can be 1

Question Number	Scheme	Notes	Marks
95. (b) **Way 2**	$\{V=\}\ \pi\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\{\mathrm{d}x\}$	$\pi\int\left(\sqrt{x}\sin 2x\right)^2\{\mathrm{d}x\}$ Ignore limits and $\mathrm{d}x$. Can be implied	B1
	$\left\{\int x\sin^2 2x\,\mathrm{d}x=\right\}$ $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$	For writing down a correct equation linking $\sin^2 2x$ and $\cos 4x$ (e.g. $\cos 4x = 1-2\sin^2 2x$) **and** some attempt at applying this equation (or a manipulation of this equation which can be incorrect) to their integral. Can be implied	M1
		Simplifies $\int x\sin^2 2x\ \{\mathrm{d}x\}$ to $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$ **Note:** This mark can be implied for stating $u=x$ **and** $\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{1-\cos 4x}{2}$ **or** $u=\frac{1}{2}x$ **and** $\frac{\mathrm{d}v}{\mathrm{d}x}=1-\cos 4x$	A1
	$=x\left(\frac{1}{2}x-\frac{1}{8}\sin 4x\right)-\int\left(\frac{1}{2}x-\frac{1}{8}\sin 4x\right)\mathrm{d}x$		
	$=x\left(\frac{1}{2}x-\frac{1}{8}\sin 4x\right)-\left(\frac{1}{4}x^2+\frac{1}{32}\cos 4x\right)\{+c\}$	Integrates to give $\pm Ax^2\pm Bx\sin 4x\pm C\cos 4x;\ A,B,C\neq 0$ or an expression that can be simplified to this form	M1 **(B1 on ePEN)**
	$\left\{\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\mathrm{d}x=\left[\frac{1}{4}x^2-\frac{1}{8}x\sin 4x-\frac{1}{32}\cos 4x\right]_0^{\frac{\pi}{4}}\right\}$		
	$=\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2-\frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right)-\frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right)-\left(0-0-\frac{1}{32}\cos 0\right)$	**dependent on the previous M mark** see notes	dM1
	$=\left(\frac{\pi^2}{64}+\frac{1}{32}\right)-\left(-\frac{1}{32}\right)=\frac{\pi^2}{64}+\frac{1}{16}$		
	So, $V=\pi\left(\frac{\pi^2}{64}+\frac{1}{16}\right)$ or $\frac{1}{64}\pi^3+\frac{1}{16}\pi$ or $\frac{\pi}{2}\left(\frac{\pi^2}{32}+\frac{1}{8}\right)$ o.e.		A1 o.e.
			[6]

	Question 95 Notes Continued	
95. (a)	**SC**	Give ***Special Case*** M1A0A0 for writing down the correct "by parts" formula and using $u=x,\ \frac{\mathrm{d}v}{\mathrm{d}x}=\cos 4x$, but making only one error in the application of the correct formula
(b)	**Note**	You can imply B1 for seeing $\pi\int y^2\{\mathrm{d}x\}$, followed by $y^2=\left(\sqrt{x}\sin 2x\right)^2$ or $y^2=x\sin^2 2x$
	Note	If the form $\cos 4x=\cos^2 2x-\sin^2 2x$ or $\cos 4x=2\cos^2 2x-1$ is used, the 1st M cannot be gained until $\cos^2 2x$ has been replaced by $\cos^2 2x=1-\sin^2 2x$ and the result is applied to their integral
	Note	Mixing $x's$ and e.g. $\theta's$: Condone $\cos 4\theta=1-2\sin^2 2\theta,\ \sin^2 2\theta=\frac{1-\cos 4\theta}{2}$ or $\lambda\sin^2 2\theta=\lambda\left(\frac{1-\cos 4\theta}{2}\right)$ if recovered in their integration
	Final M1	**Complete** method of applying limits of $\frac{\pi}{4}$ and 0 to all terms of an expression of the form $\pm Ax^2\pm Bx\sin 4x\pm C\cos 4x;\ A,B,C\neq 0$ and subtracting the correct way round.
	Note	For the final M1 mark in Way 1, allow one transcription error (on $\sin 4x$ or $\cos 4x$) in the copying of their answer from part (a) to part (b)

	Question 95 Notes Continued	
95. (b)	**Note**	Evidence of a proper consideration of the limit of 0 on $\cos 4x$ **where applicable** is needed for the final M mark E.g. $\left[\frac{1}{4}x^2 - \frac{1}{8}x\sin 4x - \frac{1}{32}\cos 4x\right]_0^{\frac{\pi}{4}} =$ • $=\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) + \frac{1}{32}$ is final M1 • $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - 0$ is final M0 • $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - \frac{1}{32}$ is final M0 (adding) • $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - \left(\frac{1}{32}\right)$ is final M1 (condone) • $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - (0+0+0)$ is final M0
95. (b)	**Note**	**<u>Alternative Method</u>**: $\left\{\begin{matrix} u = \sin^2 2x & \frac{dv}{dx} = x \\ \frac{du}{dx} = 2\sin 4x & v = \frac{1}{2}x^2 \end{matrix}\right\}, \left\{\begin{matrix} u = x^2 & \frac{dv}{dx} = \sin 4x \\ \frac{du}{dx} = 2x & v = -\frac{1}{4}\cos 4x \end{matrix}\right\}$ $\int x\sin^2 2x\,dx$ $= \frac{1}{2}x^2\sin^2 2x - \int \frac{1}{2}x^2(2\sin 4x)dx$ $= \frac{1}{2}x^2\sin^2 2x - \int x^2\sin 4x\,dx$ $= \frac{1}{2}x^2\sin^2 2x - \left(-\frac{1}{4}x^2\cos 4x - \int 2x.\left(-\frac{1}{4}\cos 4x\right)dx\right)$ $= \frac{1}{2}x^2\sin^2 2x - \left(-\frac{1}{4}x^2\cos 4x + \frac{1}{2}\int x\cos 4x\,dx\right)$ $= \frac{1}{2}x^2\sin^2 2x + \frac{1}{4}x^2\cos 4x - \frac{1}{2}\int x\cos 4x\,dx$ $= \frac{1}{2}x^2\sin^2 2x + \frac{1}{4}x^2\cos 4x - \frac{1}{2}\left(\frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x\right)\ \{+c\}$ $= \frac{1}{2}x^2\sin^2 2x + \frac{1}{4}x^2\cos 4x - \frac{1}{8}x\sin 4x - \frac{1}{32}\cos 4x\ \{+c\}$ $V = \pi\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2 dx = \pi\left(\frac{\pi^2}{64} + \frac{1}{16}\right)$ or $\frac{1}{64}\pi^3 + \frac{1}{16}\pi$ or $\frac{\pi}{2}\left(\frac{\pi^2}{32} + \frac{1}{8}\right)$ o.e.

Question Number	Scheme		Marks
96 **Way 1**	$\left\{ \mathrm{I} = \int x^2 \ln x \, \mathrm{d}x \right\}$, $\left\{ \begin{matrix} u = \ln x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = x^2 \Rightarrow v = \frac{1}{3}x^3 \end{matrix} \right\}$		
	$= \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)\{\mathrm{d}x\}$	Either $x^2 \ln x \to \pm \lambda x^3 \ln x - \int \mu x^3 \left(\frac{1}{x}\right)\{\mathrm{d}x\}$ or $\pm \lambda x^3 \ln x - \int \mu x^2 \{\mathrm{d}x\}$, where $\lambda, \mu > 0$	M1
		$x^2 \ln x \to \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)\{\mathrm{d}x\}$, simplified or un-simplified	A1
	$= \frac{x^3}{3}\ln x - \frac{x^3}{9}$	$\frac{x^3}{3}\ln x - \frac{x^3}{9}$, simplified or un-simplified	A1
	$\text{Area}(R) = \left\{ \left[\frac{x^3}{3}\ln x - \frac{x^3}{9} \right]_1^2 \right\} = \left(\frac{8}{3}\ln 2 - \frac{8}{9}\right) - \left(0 - \frac{1}{9}\right)$	**dependent on the previous M mark.** Applies limits of 2 and 1 and subtracts the correct way round	dM1
	$= \frac{8}{3}\ln 2 - \frac{7}{9}$	$\frac{8}{3}\ln 2 - \frac{7}{9}$ or $\frac{1}{9}(24\ln 2 - 7)$	A1 oe **cso**
			[5]
96 **Way 2**	$\mathrm{I} = x^2(x\ln x - x) - \int 2x(x\ln x - x)\mathrm{d}x$	$\left\{ \begin{matrix} u = x^2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x \\ \frac{\mathrm{d}v}{\mathrm{d}x} = \ln x \Rightarrow v = x\ln x - x \end{matrix} \right\}$	
	So, $3\mathrm{I} = x^2(x\ln x - x) + \int 2x^2 \{\mathrm{d}x\}$		
	and $\mathrm{I} = \frac{1}{3}x^2(x\ln x - x) + \frac{1}{3}\int 2x^2 \{\mathrm{d}x\}$	A full method of applying $u = x^2, v' = \ln x$ to give $\pm \lambda x^2(x\ln x - x) \pm \mu \int x^2 \{\mathrm{d}x\}$	M1
		$\frac{1}{3}x^2(x\ln x - x) + \frac{1}{3}\int 2x^2 \{\mathrm{d}x\}$ simplified or un-simplified	A1
	$= \frac{1}{3}x^2(x\ln x - x) + \frac{2}{9}x^3$	$\frac{x^3}{3}\ln x - \frac{x^3}{9}$, simplified or un-simplified	A1
		Then award dM1A1 in the same way as above	M1 A1
			[5]

96	A1	Exact answer needs to be a two term expression in the form $a\ln b + c$
	Note	Give A1 e.g. $\frac{8}{3}\ln 2 - \frac{7}{9}$ or $\frac{1}{9}(24\ln 2 - 7)$ or $\frac{4}{3}\ln 4 - \frac{7}{9}$ or $\frac{1}{3}\ln 256 - \frac{7}{9}$ or $-\frac{7}{9} + \frac{8}{3}\ln 2$ or $\ln 2^{\frac{8}{3}} - \frac{7}{9}$ or equivalent.
	Note	Give final A0 for a final answer of $\frac{8\ln 2 - \ln 1}{3} - \frac{7}{9}$ or $\frac{8\ln 2}{3} - \frac{1}{3}\ln 1 - \frac{7}{9}$ or $\frac{8\ln 2}{3} - \frac{8}{9} + \frac{1}{9}$ or $\frac{8}{3}\ln 2 - \frac{7}{9} + c$
	Note	$\left[\frac{x^3}{3}\ln x - \frac{x^3}{9}\right]_1^2$ followed by awrt 1.07 with no correct answer seen is dM1A0
	Note	Give dM0A0 for $\left[\frac{x^3}{3}\ln x - \frac{x^3}{9}\right]_1^2 \to \left(\frac{8}{3}\ln 2 - \frac{8}{9}\right) - \frac{1}{9}$ (adding rather than subtracting)
	Note	Allow dM1A0 for $\left[\frac{x^3}{3}\ln x - \frac{x^3}{9}\right]_1^2 \to \left(\frac{8}{3}\ln 2 - \frac{8}{9}\right) - \left(0 + \frac{1}{9}\right)$
	SC	A candidate who uses $u = \ln x$ and $\frac{dv}{dx} = x^2$, $\frac{du}{dx} = \frac{\alpha}{x}$, $v = \beta x^3$, writes down the correct "by parts" formula but makes only one error when applying it can be awarded Special Case 1st M1.

Question Number	Scheme	Notes	Marks
97.	$\dfrac{dx}{dt} = -\dfrac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) **Way 1**	$\int \dfrac{1}{x}\,dx = \int -\dfrac{5}{2}\,dt$	Separates variables as shown. dx and dt should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\ln x = -\dfrac{5}{2}t + c$	Integrates both sides to give **either** $\pm\dfrac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$\ln x = -\dfrac{5}{2}t + c$, including "$+c$"	A1
	$\{t=0, x=60 \Rightarrow\}\ \ln 60 = c$ $\ln x = -\dfrac{5}{2}t + \ln 60 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \dfrac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \dfrac{60}{e^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 **cso**
			[4]
(a) **Way 2**	$\dfrac{dt}{dx} = -\dfrac{2}{5x}$ or $t = \int -\dfrac{2}{5x}\,dx$	**Either** $\dfrac{dt}{dx} = -\dfrac{2}{5x}$ **or** $t = \int -\dfrac{2}{5x}\,dx$	B1
	$t = -\dfrac{2}{5}\ln x + c$	Integrates both sides to give **either** $t = \ldots$ **or** $\pm\alpha\ln px;\ \alpha \neq 0,\ p > 0$	M1
		$t = -\dfrac{2}{5}\ln x + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}\ c = \dfrac{2}{5}\ln 60 \Rightarrow t = -\dfrac{2}{5}\ln x + \dfrac{2}{5}\ln 60$ $\Rightarrow -\dfrac{5}{2}t = \ln x - \ln 60 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \dfrac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \dfrac{60}{e^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 **cso**
			[4]
(a) **Way 3**	$\int_{60}^{x} \dfrac{1}{x}\,dx = \int_{0}^{t} -\dfrac{5}{2}\,dt$	Ignore limits	B1
	$[\ln x]_{60}^{x} = \left[-\dfrac{5}{2}t\right]_{0}^{t}$	Integrates both sides to give **either** $\pm\dfrac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$[\ln x]_{60}^{x} = \left[-\dfrac{5}{2}t\right]_{0}^{t}$ including the correct limits	A1
	$\ln x - \ln 60 = -\dfrac{5}{2}t \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \dfrac{60}{e^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 **cso**
			[4]
(b)	$20 = 60e^{-\frac{5}{2}t}$ or $\ln 20 = -\dfrac{5}{2}t + \ln 60$	Substitutes $x = 20$ into an equation in the form of **either** $x = \pm\lambda e^{\pm\mu t} \pm \beta$ **or** $x = \pm\lambda e^{\pm\mu t \pm \alpha\ln\delta x}$ **or** $\pm\alpha\ln\delta x = \pm\mu t \pm \beta$ **or** $t = \pm\lambda\ln\delta x \pm \beta$; $\alpha, \lambda, \mu, \delta \neq 0$ and β can be 0	M1
	$t = -\dfrac{2}{5}\ln\left(\dfrac{20}{60}\right)$ $\{= 0.4394449\ldots \text{ (days)}\}$ **Note: t must be greater than 0**	**dependent on the previous M mark** Uses correct algebra to achieve an equation of the form of either $t = A\ln\left(\frac{60}{20}\right)$ or $A\ln\left(\frac{20}{60}\right)$ or $A\ln 3$ or $A\ln\left(\frac{1}{3}\right)$ o.e. or $t = A(\ln 20 - \ln 60)$ or $A(\ln 60 - \ln 20)$ o.e. ($A \in$ □ , $t > 0$)	dM1
	$\Rightarrow t = 632.8006\ldots = 633$ (to the nearest minute)	awrt 633 **or** 10 hours and awrt 33 minutes	A1 **cso**
	Note: dM1 can be implied by $t =$ awrt 0.44 from no incorrect working.		
			7

Question Number	Scheme	Notes	Marks
97.	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = -\dfrac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) **Way 4**	$\displaystyle\int \frac{2}{5x}\,\mathrm{d}x = -\int \mathrm{d}t$	Separates variables as shown. $\mathrm{d}x$ and $\mathrm{d}t$ should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\dfrac{2}{5}\ln(5x) = -t + c$	Integrates both sides to give **either** $\pm\alpha\ln(px)$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$; $p > 0$	M1
		$\dfrac{2}{5}\ln(5x) = -t + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}\ \dfrac{2}{5}\ln 300 = c$ $\dfrac{2}{5}\ln(5x) = -t + \dfrac{2}{5}\ln 300 \Rightarrow \underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \dfrac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60\mathrm{e}^{-\frac{5}{2}t}$ or $x = \dfrac{60}{\mathrm{e}^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 **cso**
			[4]
(a) **Way 5**	$\left\{\dfrac{\mathrm{d}t}{\mathrm{d}x} = -\dfrac{2}{5x} \Rightarrow\right\}\quad t = \displaystyle\int_{60}^{x} -\frac{2}{5x}\,\mathrm{d}x$	Ignore limits	B1
	$t = \left[-\dfrac{2}{5}\ln x\right]_{60}^{x}$	Integrates both sides to give **either** $\pm k \to \pm kt$ (with respect to t) **or** $\pm\dfrac{\alpha}{x} \to \pm\alpha\ln x$; $k, \alpha \neq 0$	M1
		$t = \left[-\dfrac{2}{5}\ln x\right]_{60}^{x}$ including the correct limits	A1
	$t = -\dfrac{2}{5}\ln x + \dfrac{2}{5}\ln 60 \Rightarrow -\dfrac{5}{2}t = \ln x - \ln 60$ $\Rightarrow \underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \dfrac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 **cso**
			[4]

Question 97 Notes

97. (a)	**B1**	For the correct separation of variables. E.g. $\displaystyle\int \frac{1}{5x}\,\mathrm{d}x = \int -\frac{1}{2}\,\mathrm{d}t$
	Note	B1 can be implied by seeing **either** $\ln x = -\dfrac{5}{2}t + c$ **or** $t = -\dfrac{2}{5}\ln x + c$ with or without $+c$
	Note	B1 can also be implied by seeing $[\ln x]_{60}^{x} = \left[-\dfrac{5}{2}t\right]_{0}^{t}$
	Note	Allow A1 for $x = 60\sqrt{\mathrm{e}^{-5t}}$ or $x = \dfrac{60}{\sqrt{\mathrm{e}^{5t}}}$ with no incorrect working seen
	Note	Give final A0 for $x = \mathrm{e}^{-\frac{5}{2}t} + 60 \to x = 60\mathrm{e}^{-\frac{5}{2}t}$
	Note	Give final A0 for writing $x = \mathrm{e}^{-\frac{5}{2}t + \ln 60}$ as their final answer (without seeing $x = 60\mathrm{e}^{-\frac{5}{2}t}$)
	Note	Way 1 to Way 5 do not exhaust all the different methods that candidates can give.
	Note	Give B0M0A0A0 for writing down $x = 60\mathrm{e}^{-\frac{5}{2}t}$ or $x = \dfrac{60}{\mathrm{e}^{\frac{5}{2}t}}$ with no evidence of working or integration seen.
(b)	**A1**	You can apply **cso** for the work only seen in part (b).
	Note	Give dM1(Implied) A1 for $\dfrac{5}{2}t = \ln 3$ followed by $t =$ awrt 633 from no incorrect working.
	Note	Substitutes $x = 40$ into their equation from part (a) is M0dM0A0

Question Number	Scheme	Notes	Marks
98.	(i) $\displaystyle\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y,\ y>0$, (ii) $\displaystyle\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x,\ x=4\sin^2\theta$		
(i) **Way 1**	$\dfrac{3y-4}{y(3y+2)} \equiv \dfrac{A}{y}+\dfrac{B}{(3y+2)} \Rightarrow 3y-4 = A(3y+2)+By$	See notes	M1
	$y=0 \Rightarrow -4=2A \Rightarrow A=-2$	At least one of their $A=-2$ **or** their $B=9$	A1
	$y=-\frac{2}{3} \Rightarrow -6=-\frac{2}{3}B \Rightarrow B=9$	Both their $A=-2$ **and** their $B=9$	A1
	$\displaystyle\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{-2}{y}+\frac{9}{(3y+2)}\,\mathrm{d}y$	Integrates to give at least one of **either** $\dfrac{A}{y}\to\pm\lambda\ln y$ **or** $\dfrac{B}{(3y+2)}\to\pm\mu\ln(3y+2)$; $A\neq0,\ B\neq0$	M1
		At least one term correctly followed through from their A or from their B	A1 ft
	$= -2\ln y + 3\ln(3y+2)\ \{+c\}$	$-2\ln y+3\ln(3y+2)$ **or** $-2\ln y+3\ln(y+\frac{2}{3})$ with correct bracketing, simplified or un-simplified. Can apply isw.	A1 **cao**
			[6]
(ii) (a) **Way 1**	$\{x=4\sin^2\theta \Rightarrow\}\ \dfrac{\mathrm{d}x}{\mathrm{d}\theta}=8\sin\theta\cos\theta$ or $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=4\sin2\theta$ or $\mathrm{d}x=8\sin\theta\cos\theta\,\mathrm{d}\theta$		B1
	$\displaystyle\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}.8\sin\theta\cos\theta\,\{\mathrm{d}\theta\}$ or $\displaystyle\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}.4\sin2\theta\,\{\mathrm{d}\theta\}$		M1
	$=\displaystyle\int \underline{\underline{\tan\theta}}.8\sin\theta\cos\theta\,\{\mathrm{d}\theta\}$ or $\displaystyle\int \underline{\underline{\tan\theta}}.4\sin2\theta\,\{\mathrm{d}\theta\}$	$\sqrt{\left(\dfrac{x}{4-x}\right)}\to\pm K\tan\theta$ or $\pm K\left(\dfrac{\sin\theta}{\cos\theta}\right)$	$\underline{\underline{\text{M1}}}$
	$=\displaystyle\int 8\sin^2\theta\,\mathrm{d}\theta$	$\displaystyle\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$	A1
	$3=4\sin^2\theta$ **or** $\dfrac{3}{4}=\sin^2\theta$ **or** $\sin\theta=\dfrac{\sqrt{3}}{2} \Rightarrow \theta=\dfrac{\pi}{3}$ $\{x=0\to\theta=0\}$	Writes down a correct equation involving $x=3$ leading to $\theta=\dfrac{\pi}{3}$ and no incorrect work seen regarding limits	B1
			[5]
(ii) (b)	$=\{8\}\displaystyle\int\left(\frac{1-\cos2\theta}{2}\right)\mathrm{d}\theta\ \left\{=\int(4-4\cos2\theta)\,\mathrm{d}\theta\right\}$	Applies $\cos2\theta=1-2\sin^2\theta$ to their integral. (See notes)	M1
	$=\{8\}\left(\dfrac{1}{2}\theta-\dfrac{1}{4}\sin2\theta\right)\ \{=4\theta-2\sin2\theta\}$	For $\pm\alpha\theta\pm\beta\sin2\theta,\ \alpha,\beta\neq0$	M1
		$\sin^2\theta\to\left(\dfrac{1}{2}\theta-\dfrac{1}{4}\sin2\theta\right)$	A1
	$\left\{\displaystyle\int_0^{\frac{\pi}{3}}8\sin^2\theta\,\mathrm{d}\theta=8\left[\frac{1}{2}\theta-\frac{1}{4}\sin2\theta\right]_0^{\frac{\pi}{3}}\right\}=8\left(\left(\dfrac{\pi}{6}-\dfrac{1}{4}\left(\dfrac{\sqrt{3}}{2}\right)\right)-(0+0)\right)$		
	$=\dfrac{4}{3}\pi-\sqrt{3}$	"two term" exact answer of e.g. $\dfrac{4}{3}\pi-\sqrt{3}$ or $\dfrac{1}{3}(4\pi-3\sqrt{3})$	A1 o.e.
			[4]
			15

98. (i)	**1st M1**	Writing $\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)}$ and a complete method for finding the value of at least one of their A or their B.
	Note	M1A1 can be implied ***for writing down*** either $\frac{3y-4}{y(3y+2)} \equiv \frac{-2}{y} + \frac{\text{their } B}{(3y+2)}$ or $\frac{3y-4}{y(3y+2)} \equiv \frac{\text{their } A}{y} + \frac{9}{(3y+2)}$ with no working.
	Note	Correct bracketing is not necessary for the penultimate A1ft, but is required for the final A1 in (i)
	Note	**Give** 2nd M0 for $\frac{3y-4}{y(3y+2)}$ going directly to $\pm\alpha\ln(3y^2+2y)$
	Note	**…but allow** 2nd M1 for either $\frac{M(6y+2)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ or $\frac{M(3y+1)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$
98. (ii)(a)	**1st M1**	Substitutes $x = 4\sin^2\theta$ and their $\mathrm{d}x$ $\left(\text{from their correctly rearranged } \frac{\mathrm{d}x}{\mathrm{d}\theta}\right)$ into $\sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x$
	Note	$\mathrm{d}x \neq \lambda\mathrm{d}\theta$. For example $\mathrm{d}x \neq \mathrm{d}\theta$
	Note	Allow substituting $\mathrm{d}x = 4\sin 2\theta$ for the 1st M1 after a correct $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sin 2\theta$ or $\mathrm{d}x = 4\sin 2\theta\,\mathrm{d}\theta$
	2nd M1	Applying $x = 4\sin^2\theta$ to $\sqrt{\left(\frac{x}{4-x}\right)}$ to give $\pm K\tan\theta$ or $\pm K\left(\frac{\sin\theta}{\cos\theta}\right)$
	Note	Integral sign is not needed for this mark.
	1st A1	Simplifies to give $\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$
	2nd B1	Writes down a correct equation involving $x = 3$ leading to $\theta = \frac{\pi}{3}$ and no incorrect work seen regarding limits
	Note	Allow 2nd B1 for $x = 4\sin^2\left(\frac{\pi}{3}\right) = 3$ **and** $x = 4\sin^2 0 = 0$
	Note	Allow 2nd B1 for $\theta = \sin^{-1}\left(\sqrt{\frac{x}{4}}\right)$ followed by $x = 3,\ \theta = \frac{\pi}{3};\ x = 0,\ \theta = 0$
(ii)(b)	**M1**	Writes down a correct equation involving $\cos 2\theta$ and $\sin^2\theta$ **E.g.:** $\cos 2\theta = 1 - 2\sin^2\theta$ or $\sin^2\theta = \frac{1-\cos 2\theta}{2}$ or $K\sin^2\theta = K\left(\frac{1-\cos 2\theta}{2}\right)$ and ***applies*** it to their integral. **Note:** Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm\alpha\theta \pm \beta\sin 2\theta$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$, $\alpha \neq 0,\ \beta \neq 0$ (can be simplified or un-simplified).
	1st A1	Integrating $\sin^2\theta$ to give $\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta$, un-simplified or simplified. Correct solution only. Can be implied by $k\sin^2\theta$ giving $\frac{k}{2}\theta - \frac{k}{4}\sin 2\theta$ or $\frac{k}{4}(2\theta - \sin 2\theta)$ un-simplified or simplified.
	2nd A1	A ***correct solution in part (ii)*** leading to a "two term" exact answer of e.g. $\frac{4}{3}\pi - \sqrt{3}$ or $\frac{8}{6}\pi - \sqrt{3}$ or $\frac{4}{3}\pi - \frac{2\sqrt{3}}{2}$ or $\frac{1}{3}(4\pi - 3\sqrt{3})$
	Note	A decimal answer of 2.456739397... (without a correct **exact** answer) is A0.
	Note	Candidates can work in terms of λ (note that λ is not given in (ii)) and gain the 1st three marks (i.e. M1M1A1) in part (b).
	Note	If they incorrectly obtain $\int_0^{\frac{\pi}{3}} 8\sin^2\theta\,\mathrm{d}\theta$ in part (i)(a) (or correctly guess that $\lambda = 8$) then the final A1 is available for a correct solution in part (ii)(b).

	Scheme	Notes	Marks
98. (i) **Way 2**	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{6y+2}{3y^2+2y}\,\mathrm{d}y - \int \frac{3y+6}{y(3y+2)}\,\mathrm{d}y$		
	$\frac{3y+6}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y+6 = A(3y+2) + By$	See notes	M1
	$y = 0 \Rightarrow 6 = 2A \Rightarrow A = 3$	At least one of their $A = 3$ **or** their $B = -6$	A1
	$y = -\frac{2}{3} \Rightarrow 4 = -\frac{2}{3}B \Rightarrow B = -6$	Both their $A = 3$ **and** their $B = -6$	A1
	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y$ $= \int \frac{6y+2}{3y^2+2y}\,\mathrm{d}y - \int \frac{3}{y}\,\mathrm{d}y + \int \frac{6}{(3y+2)}\,\mathrm{d}y$	Integrates to give at least one of **either** $\frac{M(6y+2)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$ $M \neq 0, A \neq 0, B \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)\ \{+c\}$	$\ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 **cao**
			[6]
98. (i) **Way 3**	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{3y+1}{3y^2+2y}\,\mathrm{d}y - \int \frac{5}{y(3y+2)}\,\mathrm{d}y$		
	$\frac{5}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 5 = A(3y+2) + By$	See notes	M1
	$y = 0 \Rightarrow 5 = 2A \Rightarrow A = \frac{5}{2}$	At least one of their $A = \frac{5}{2}$ **or** their $B = -\frac{15}{2}$	A1
	$y = -\frac{2}{3} \Rightarrow 5 = -\frac{2}{3}B \Rightarrow B = -\frac{15}{2}$	Both their $A = \frac{5}{2}$ **and** their $B = -\frac{15}{2}$	A1
	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y$ $= \int \frac{3y+1}{3y^2+2y}\,\mathrm{d}y - \int \frac{\frac{5}{2}}{y}\,\mathrm{d}y + \int \frac{\frac{15}{2}}{(3y+2)}\,\mathrm{d}y$	Integrates to give at least one of **either** $\frac{M(3y+1)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$ $M \neq 0, A \neq 0, B \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)\ \{+c\}$	$\frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 **cao**
			[6]

	Scheme	Notes	
98. (i) **Way 4**	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{3y}{y(3y+2)}\,\mathrm{d}y - \int \frac{4}{y(3y+2)}\,\mathrm{d}y$		
	$= \int \frac{3}{(3y+2)}\,\mathrm{d}y - \int \frac{4}{y(3y+2)}\,\mathrm{d}y$		
	$\frac{4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 4 = A(3y+2) + By$	See notes	M1
	$y = 0 \Rightarrow 4 = 2A \Rightarrow A = 2$	At least one of their $A = 2$ **or** their $B = -6$	A1
	$y = -\frac{2}{3} \Rightarrow 4 = -\frac{2}{3}B \Rightarrow B = -6$	Both their $A = 2$ **and** their $B = -6$	A1
	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y$ $= \int \frac{3}{3y+2}\,\mathrm{d}y - \int \frac{2}{y}\,\mathrm{d}y + \int \frac{6}{(3y+2)}\,\mathrm{d}y$	Integrates to give at least one of **either** $\frac{C}{(3y+2)} \to \pm\alpha\ln(3y+2)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$, $A \neq 0,\ B \neq 0,\ C \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \ln(3y+2) - 2\ln y + 2\ln(3y+2)\ \{+c\}$	$\ln(3y+2) - 2\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 **cao**
			[6]
	Alternative methods for B1M1M1A1 in (ii)(a)		
(ii)(a) **Way 2**	$\{x = 4\sin^2\theta \Rightarrow\}\ \frac{\mathrm{d}x}{\mathrm{d}\theta} = 8\sin\theta\cos\theta$	As in Way 1	B1
	$\int \sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,8\sin\theta\cos\theta\ \{\mathrm{d}\theta\}$	As before	M1
	$= \int \sqrt{\frac{\sin^2\theta}{(1-\sin^2\theta)}}\,.\,8\cos\theta\sin\theta\ \{\mathrm{d}\theta\}$		
	$= \int \frac{\sin\theta}{\sqrt{(1-\sin^2\theta)}}\,.\,8\sqrt{(1-\sin^2\theta)}\sin\theta\ \{\mathrm{d}\theta\}$		
	$= \int \sin\theta\,.\,8\sin\theta\ \{\mathrm{d}\theta\}$	Correct method leading to $\sqrt{(1-\sin^2\theta)}$ being cancelled out	M1
	$= \int 8\sin^2\theta\,\mathrm{d}\theta$	$\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$	A1 **cso**
(ii)(a) **Way 3**	$\{x = 4\sin^2\theta \Rightarrow\}\ \frac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sin 2\theta$	As in Way 1	B1
	$x = 4\sin^2\theta = 2 - 2\cos 2\theta,\ \ 4 - x = 2 + 2\cos 2\theta$		
	$\int \sqrt{\frac{2-2\cos 2\theta}{2+2\cos 2\theta}}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\}$		M1
	$= \int \frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2+2\cos 2\theta}}\,.\,\frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2-2\cos 2\theta}}\,4\sin 2\theta\ \{\mathrm{d}\theta\} = \int \frac{2-2\cos 2\theta}{\sqrt{4-4\cos^2 2\theta}}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\}$		
	$= \int \frac{2-2\cos 2\theta}{2\sin 2\theta}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\} = \int 2(2-2\cos 2\theta).\ \{\mathrm{d}\theta\}$	Correct method leading to $\sin 2\theta$ being cancelled out	M1
	$= \int 8\sin^2\theta\,\mathrm{d}\theta$	$\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$	A1 **cso**

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>99.</td><td>$y=(2x-1)^{\frac{3}{4}}, \quad x \geqslant \frac{1}{2}$ passes though $P(k, 8)$</td><td></td><td></td></tr>
<tr><td rowspan="2">(a)</td><td rowspan="2">$\left\{\int (2x-1)^{\frac{3}{2}}\,dx\right\} = \frac{1}{5}(2x-1)^{\frac{5}{2}}\ \{+c\}$</td><td>$(2x \pm 1)^{\frac{3}{2}} \to \pm\lambda(2x \pm 1)^{\frac{5}{2}}$ or $\pm\lambda u^{\frac{5}{2}}$ where $u = 2x \pm 1$; $\lambda \neq 0$</td><td>M1</td></tr>
<tr><td>$\frac{1}{5}(2x-1)^{\frac{5}{2}}$ with or without $+c$. Must be simplified.</td><td>A1</td></tr>
<tr><td></td><td></td><td></td><td>[2]</td></tr>
<tr><td rowspan="2">(b)</td><td>$\{P(k, 8) \Rightarrow\}\ 8 = (2k-1)^{\frac{3}{4}} \Rightarrow k = \frac{8^{\frac{4}{3}}+1}{2}$</td><td>Sets $8 = (2k-1)^{\frac{3}{4}}$ or $8 = (2x-1)^{\frac{3}{4}}$ and rearranges to give $k=$ (or $x=$) a numerical value.</td><td>M1</td></tr>
<tr><td>So, $k = \frac{17}{2}$</td><td>k (or x) $= \frac{17}{2}$ or 8.5</td><td>A1</td></tr>
<tr><td></td><td></td><td></td><td>[2]</td></tr>
<tr><td rowspan="4">(c)</td><td>$\pi\int\left((2x-1)^{\frac{3}{4}}\right)^2 dx$</td><td>For $\pi\int\left((2x-1)^{\frac{3}{4}}\right)^2$ or $\pi\int(2x-1)^{\frac{3}{2}}$
Ignore limits and dx. Can be implied.</td><td>B1</td></tr>
<tr><td>$\left\{\int_{\frac{1}{2}}^{\frac{17}{2}} y^2\,dx\right\} = \left[\frac{(2x-1)^{\frac{5}{2}}}{5}\right]_{\frac{1}{2}}^{\frac{17}{2}} = \left(\left(\frac{16^{\frac{5}{2}}}{5}\right)-(0)\right)\ \left\{=\frac{1024}{5}\right\}$
Note: It is not necessary to write the "-0"</td><td>Applies x-limits of "8.5" (their answer to part (b)) and 0.5 to an expression of the form $\pm\beta(2x-1)^{\frac{5}{2}}$; $\beta \neq 0$ and subtracts the correct way round.</td><td>M1</td></tr>
<tr><td>$\{V_{\text{cylinder}}\} = \pi(8)^2\left(\frac{17}{2}\right)\ \{=544\pi\}$</td><td>$\pi(8)^2$(their answer to part (b))
$V_{\text{cylinder}} = 544\pi$ implies this mark</td><td>B1 ft</td></tr>
<tr><td>$\left\{\text{Vol}(S) = 544\pi - \frac{1024\pi}{5}\right\} \Rightarrow \text{Vol}(S) = \frac{1696}{5}\pi$</td><td>An exact correct answer in the form $k\pi$
E.g. $\frac{1696}{5}\pi$, $\frac{3392}{10}\pi$ or 339.2π</td><td>A1</td></tr>
<tr><td></td><td></td><td></td><td>[4]</td></tr>
<tr><td rowspan="5">Alt. (c)</td><td>$\text{Vol}(S) = \pi(8)^2\left(\frac{1}{2}\right) + \underline{\underline{\pi}}\int_{0.5}^{8.5}\left(8^2 - \underline{\underline{(2x-1)^{\frac{3}{2}}}}\right)dx$</td><td>For $\underline{\underline{\pi}}\int \ldots\ldots \underline{\underline{(2x-1)^{\frac{3}{2}}}}$
Ignore limits and dx.</td><td>B1</td></tr>
<tr><td>$= \pi(8)^2\left(\frac{1}{2}\right) + \pi\left[64x - \frac{1}{5}(2x-1)^{\frac{5}{2}}\right]_{0.5}^{8.5}$</td><td></td><td></td></tr>
<tr><td rowspan="2">$= \underline{\underline{\pi(8)^2\left(\frac{1}{2}\right)}} + \underline{\underline{\pi}}\left(\left(\underline{\underline{64(\text{"}8.5\text{"})}} - \frac{1}{5}(2(8.5)-1)^{\frac{5}{2}}\right) - \left(\underline{\underline{64(0.5)}} - \frac{1}{5}(2(0.5)-1)^{\frac{5}{2}}\right)\right)$</td><td>as above</td><td>M1</td></tr>
<tr><td></td><td>$\underline{\underline{\text{B1}}}$</td></tr>
<tr><td>$\left\{= 32\pi + \pi\left(\left(544 - \frac{1024}{5}\right) - (32-0)\right)\right\} \Rightarrow \text{Vol}(S) = \frac{1696}{5}\pi$</td><td></td><td>A1</td></tr>
<tr><td></td><td></td><td></td><td>[4]</td></tr>
<tr><td></td><td></td><td></td><td>8</td></tr>
</table>

99. (b)	**SC**	Allow Special Case SC M1 for a candidate who sets $8=(2k-1)^{\frac{3}{2}}$ or $8=(2x-1)^{\frac{3}{2}}$ and rearranges to give $k=$ (or $x=$) a numerical value.
99. (c)	**M1**	Can also be given for applying u-limits of "16" $(2(\text{"part }(b)\text{"})-1)$ and 0 to an expression of the form $\pm\beta u^{\frac{5}{2}}$; $\beta\neq 0$ and subtracts the correct way round.
	Note	You can give M1 for $\left[\dfrac{(2x-1)^{\frac{5}{2}}}{5}\right]_{\frac{1}{2}}^{\frac{17}{2}}=\dfrac{1024}{5}$
	Note	Give M0 for $\left[\dfrac{(2x-1)^{\frac{5}{2}}}{5}\right]_{0}^{\frac{17}{2}}=\left(\left(\dfrac{16^{\frac{5}{2}}}{5}\right)-(0)\right)$
	B1ft	Correct expression for the volume of a cylinder with radius 8 and their (part (b)) height k.
	Note	If a candidate uses integration to find the volume of this cylinder they need to apply their limits to give a correct expression for its volume. So $\pi\int_0^{8.5}8^2\,dx=\pi\left[64x\right]_0^{8.5}$ is **not sufficient** for B1 but $\pi(64(8.5)-0)$ **is sufficient** for B1.

99.	**MISREADING IN BOTH PARTS (B) AND (C)**		
	Apply the misread rule (MR) for candidates who apply $y=(2x-1)^{\frac{3}{2}}$ to **both** parts (b) and (c)		
(b)	$\{P(k,8)\Rightarrow\}\ 8=(2k-1)^{\frac{3}{2}}\Rightarrow k=\dfrac{8^{\frac{2}{3}}+1}{2}$	Sets $8=(2k-1)^{\frac{3}{2}}$ or $8=(2x-1)^{\frac{3}{2}}$ and rearranges to give $k=$ (or $x=$) a numerical value.	M1
	So, $k=\dfrac{5}{2}$	k (or x) $=\dfrac{5}{2}$ or 2.5	A1
			[2]
(c)	$\pi\int\left((2x-1)^{\frac{3}{2}}\right)^2 dx$	For $\pi\int\left((2x-1)^{\frac{3}{2}}\right)^2$ or $\pi\int(2x-1)^3$ Ignore limits and dx. Can be implied.	B1
	$\left\{\int_{\frac{1}{2}}^{\frac{17}{2}}y^2\,dx\right\}=\left[\dfrac{(2x-1)^4}{8}\right]_{\frac{1}{2}}^{\frac{5}{2}}=\left(\left(\dfrac{4^4}{8}\right)-(0)\right)\ \{=32\}$	Applies x-limits of "2.5" (their answer to part (b)) and 0.5 to an expression of the form $\pm\beta(2x-1)^4$; $\beta\neq 0$ and subtracts the correct way round.	M1
	$V_{\text{cylinder}}=\pi(8)^2\left(\dfrac{5}{2}\right)\ \{=160\pi\}$	$\pi(8)^2(\text{their answer to part }(b))$ Sight of 160π implies this mark	B1 ft
	$\{\text{Vol}(S)=160\pi-32\pi\}\Rightarrow\text{Vol}(S)=128\pi$	An exact correct answer in the form $k\pi$ E.g. 128π	A1
			[4]
	Note	Mark parts (b) and (c) using the mark scheme above and then working forwards from part (b) deduct two from any A or B marks gained. E.g. (b) M1A1 (c) B1M1B1A1 would score (b) M1A0 (c) B0M1B1A1 E.g. (b) M1A1 (c) B1M1B0A0 would score (b) M1A0 (c) B0M1B0A0	
	Note	If a candidate uses $y=(2x-1)^{\frac{3}{4}}$ in part (b) and then uses $y=(2x-1)^{\frac{3}{2}}$ in part (c) do not apply a misread in part (c).	

Question Number	Scheme		Marks
100.	$y = 4x - x\mathrm{e}^{\frac{1}{2}x},\ x \geqslant 0$		
(a)	$\left\{y = 0 \Rightarrow 4x - x\mathrm{e}^{\frac{1}{2}x} = 0 \Rightarrow x(4 - \mathrm{e}^{\frac{1}{2}x}) = 0 \Rightarrow\right\}$		
	$\mathrm{e}^{\frac{1}{2}x} = 4 \Rightarrow x_A = 4\ln 2$	Attempts to solve $\mathrm{e}^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda\ln\mu$ where $\mu > 0$	M1
		$4\ln 2$ **cao** (Ignore $x = 0$)	A1
			[2]
(b)	$\left\{\int x\mathrm{e}^{\frac{1}{2}x}\,\mathrm{d}x\right\} = 2x\mathrm{e}^{\frac{1}{2}x} - \int 2\mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$	$\alpha x\mathrm{e}^{\frac{1}{2}x} - \beta\int \mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$, $\alpha > 0, \beta > 0$	M1
		$2x\mathrm{e}^{\frac{1}{2}x} - \int 2\mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$, with or without $\mathrm{d}x$	A1 **(M1 on ePEN)**
	$= 2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}\ \{+c\}$	$2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}$ o.e. with or without $+c$	A1
			[3]
(c)	$\left\{\int 4x\,\mathrm{d}x\right\} = 2x^2$	$4x \to 2x^2$ or $\dfrac{4x^2}{2}$ o.e.	B1
	$\left\{\int_0^{4\ln 2}(4x - x\mathrm{e}^{\frac{1}{2}x})\,\mathrm{d}x\right\} = \left[2x^2 - \left(2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}\right)\right]_0^{4\ln 2 \text{ or } \ln 16 \text{ or their limits}}$		
	$= \left(2(4\ln 2)^2 - 2(4\ln 2)\mathrm{e}^{\frac{1}{2}(4\ln 2)} + 4\mathrm{e}^{\frac{1}{2}(4\ln 2)}\right) - \left(2(0)^2 - 2(0)\mathrm{e}^{\frac{1}{2}(0)} + 4\mathrm{e}^{\frac{1}{2}(0)}\right)$	**See notes**	M1
	$= \left(32(\ln 2)^2 - 32(\ln 2) + 16\right) - (4)$ $= 32(\ln 2)^2 - 32(\ln 2) + 12$	$32(\ln 2)^2 - 32(\ln 2) + 12$, **see notes**	A1
			[3] **8**

Question 100 Notes

100. (a)	**M1**	Attempts to solve $\mathrm{e}^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda\ln\mu$ where $\mu > 0$
	A1	$4\ln 2$ **cao** stated in part (a) only (Ignore $x = 0$)
(b)	**NOTE**	**Part (b) appears as M1M1A1 on ePEN, but is now marked as M1A1A1.**
	M1	Integration by parts is applied in the form $\alpha x\mathrm{e}^{\frac{1}{2}x} - \beta\int \mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$, where $\alpha > 0, \beta > 0$. (must be in this form) with or without $\mathrm{d}x$
	A1	$2x\mathrm{e}^{\frac{1}{2}x} - \int 2\mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$ or equivalent, with or without $\mathrm{d}x$. **Can be un-simplified.**
	A1	$2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}$ or equivalent with or without + c. **Can be un-simplified.**
	Note	You can also allow $2\mathrm{e}^{\frac{1}{2}x}(x-2)$ or $\mathrm{e}^{\frac{1}{2}x}(2x-4)$ for the final A1.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	**<u>SPECIAL CASE:</u>** A candidate who uses $u = x$, $\dfrac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^{\frac{1}{2}x}$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1. (Applying their v counts for one consistent error.)

100. (c)	**B1**	$4x \to 2x^2$ or $\dfrac{4x^2}{2}$ oe
	M1	**Complete** method of applying limits of their x_A and 0 to all terms of an expression of the form $\pm Ax^2 \pm Bx\mathrm{e}^{\frac{1}{2}x} \pm C\mathrm{e}^{\frac{1}{2}x}$ (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) and subtracting the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.
	Note	$\ln 16$ or $2\ln 4$ or equivalent is fine as an upper limit.
	A1	A correct three term exact quadratic expression in $\ln 2$. For example allow for A1 • $32(\ln 2)^2 - 32(\ln 2) + 12$ • $8(2\ln 2)^2 - 8(4\ln 2) + 12$ • $2(4\ln 2)^2 - 32(\ln 2) + 12$ • $2(4\ln 2)^2 - 2(4\ln 2)\mathrm{e}^{\frac{1}{2}(4\ln 2)} + 12$
	Note	Note that the constant term of 12 needs to be combined from $4\mathrm{e}^{\frac{1}{2}(4\ln 2)} - 4\mathrm{e}^{\frac{1}{2}(0)}$ o.e.
	Note	Also allow $32\ln 2(\ln 2 - 1) + 12$ or $32\ln 2\left(\ln 2 - 1 + \dfrac{12}{32\ln 2}\right)$ for A1.
	Note	***Do not apply "ignore subsequent working" for incorrect simplification.*** Eg: $32(\ln 2)^2 - 32(\ln 2) + 12 \to 64(\ln 2) - 32(\ln 2) + 12$ or $32(\ln 4) - 32(\ln 2) + 12$
	Note	**Bracketing error:** $32\ln 2^2 - 32(\ln 2) + 12$, unless recovered is final A0.
	Note	**Notation:** Allow $32(\ln^2 2) - 32(\ln 2) + 12$ for the final A1.
	Note	5.19378… without seeing $32(\ln 2)^2 - 32(\ln 2) + 12$ is A0.
	Note	5.19378… following from a correct $2x^2 - \left(2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}\right)$ is M1A0.
	Note	5.19378… from no working is M0A0.

Question Number	Scheme		Marks
101. (a)	$A = \int_0^3 \sqrt{(3-x)(x+1)}\ \mathrm{d}x\ ,\ \ x = 1 + 2\sin\theta$		
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$ or $2\cos\theta$ used correctly in their working. Can be implied.	B1
	$\left\{\int \sqrt{(3-x)(x+1)}\ \mathrm{d}x \text{ or } \int \sqrt{(3+2x-x^2)}\ \mathrm{d}x\right\}$		
	$= \int \sqrt{\big(3-(1+2\sin\theta)\big)\big((1+2\sin\theta)+1\big)}\ 2\cos\theta\ \{\mathrm{d}\theta\}$	Substitutes for both x and $\mathrm{d}x$, where $\mathrm{d}x \neq \lambda \mathrm{d}\theta$. Ignore $\mathrm{d}\theta$	M1
	$= \int \sqrt{(2-2\sin\theta)(2+2\sin\theta)}\ 2\cos\theta\ \{\mathrm{d}\theta\}$ $= \int \sqrt{\left(4-4\sin^2\theta\right)}\ 2\cos\theta\ \{\mathrm{d}\theta\}$		
	$= \int \sqrt{\left(4-4(1-\cos^2\theta)\right)}\ 2\cos\theta\ \{\mathrm{d}\theta\}$ or $\int \sqrt{4\cos^2\theta}\ 2\cos\theta\ \{\mathrm{d}\theta\}$	Applies $\cos^2\theta = 1 - \sin^2\theta$ **see notes**	M1
	$= 4\int \cos^2\theta\,\mathrm{d}\theta,\ \{k = 4\}$	$4\int \cos^2\theta\mathrm{d}\theta$ or $\int 4\cos^2\theta\mathrm{d}\theta$ **Note:** $\mathrm{d}\theta$ is required here.	A1
	$0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2} \Rightarrow \underline{\theta = -\frac{\pi}{6}}$ **and** $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1 \Rightarrow \underline{\theta = \frac{\pi}{2}}$	See notes	B1
			[5]
(b)	$\left\{k\int \cos^2\theta\,\{\mathrm{d}\theta\}\right\} = \{k\}\int \left(\frac{1+\cos 2\theta}{2}\right)\{\mathrm{d}\theta\}$	Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral	M1
	$= \{k\}\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)$	Integrates to give $\pm\alpha\theta \pm \beta\sin 2\theta,\ \alpha \neq 0,\ \beta \neq 0$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$	M1 **(A1 on ePEN)**
	$\left\{\text{So } 4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\mathrm{d}\theta = \left[2\theta + \sin 2\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{2}}\right\}$ $= \left(2\left(\frac{\pi}{2}\right) + \sin\left(\frac{2\pi}{2}\right)\right) - \left(2\left(-\frac{\pi}{6}\right) + \sin\left(-\frac{2\pi}{6}\right)\right)$		
	$\left\{= (\pi) - \left(-\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\right\} = \frac{4\pi}{3} + \frac{\sqrt{3}}{2}$	$\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ **or** $\frac{1}{6}\left(8\pi + 3\sqrt{3}\right)$	A1 **cao cso**
			[3] **8**

	Question 101 Notes	
101. (a)	**B1**	$\dfrac{dx}{d\theta} = 2\cos\theta$. Also allow $dx = 2\cos\theta\, d\theta$. This mark can be implied by later working.
	Note	You can give B1 for $2\cos\theta$ used correctly in their working.
	M1	Substitutes $x = 1 + 2\sin\theta$ and their dx $\left(\text{from their rearranged } \dfrac{dx}{d\theta}\right)$ into $\sqrt{(3-x)(x+1)}\, dx$.
	Note	Condone bracketing errors here.
	Note	$dx \neq \lambda d\theta$. For example $dx \neq d\theta$.
	Note	Condone substituting $dx = \cos\theta$ for the 1st M1 after a correct $\dfrac{dx}{d\theta} = 2\cos\theta$ or $dx = 2\cos\theta d\theta$
	M1	Applies either • $1 - \sin^2\theta = \cos^2\theta$ • $\lambda - \lambda\sin^2\theta$ or $\lambda(1 - \sin^2\theta) = \lambda\cos^2\theta$ • $4 - 4\sin^2\theta = 4 + 2\cos 2\theta - 2 = 2 + 2\cos 2\theta = 4\cos^2\theta$ to their expression where λ is a numerical value.
	A1	Correctly proves that $\int \sqrt{(3-x)(x+1)}\, dx$ is equal to $4\int \cos^2\theta\, d\theta$ or $\int 4\cos^2\theta\, d\theta$
	Note	All three previous marks must have been awarded before A1 can be awarded.
	Note	Their final answer must include $d\theta$.
	Note	You can ignore limits for the final A1 mark.
	B1	Evidence of a correct equation in $\sin\theta$ or $\sin^{-1}\theta$ for both x-values leading to both θ values. Eg: • $0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\dfrac{1}{2}$ which then leads to $\theta = -\dfrac{\pi}{6}$, **and** • $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1$ which then leads to $\theta = \dfrac{\pi}{2}$
	Note	Allow B1 for $x = 1 + 2\sin\left(-\dfrac{\pi}{6}\right) = 0$ **and** $x = 1 + 2\sin\left(\dfrac{\pi}{2}\right) = 3$
	Note	Allow B1 for $\sin\theta = \left(\dfrac{x-1}{2}\right)$ or $\theta = \sin^{-1}\left(\dfrac{x-1}{2}\right)$ followed by $x = 0,\ \theta = -\dfrac{\pi}{6};\ x = 3,\ \theta = \dfrac{\pi}{2}$
(b)	**NOTE**	**Part (b) appears as M1A1A1 on ePEN, but is now marked as M1M1A1.**
	M1	Writes down a correct equation involving $\cos 2\theta$ and $\cos^2\theta$ **Eg:** $\cos 2\theta = 2\cos^2\theta - 1$ or $\cos^2\theta = \dfrac{1 + \cos 2\theta}{2}$ or $\lambda\cos^2\theta = \lambda\left(\dfrac{1 + \cos 2\theta}{2}\right)$ and ***applies*** it to their integral. **Note:** Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm\alpha\theta \pm \beta\sin 2\theta$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$, $\alpha \neq 0, \beta \neq 0$ (can be simplified or un-simplified).
	A1	A ***correct solution in part (b)*** leading to a "two term" exact answer. Eg: $\dfrac{4\pi}{3} + \dfrac{\sqrt{3}}{2}$ **or** $\dfrac{8\pi}{6} + \dfrac{\sqrt{3}}{2}$ **or** $\dfrac{1}{6}\left(8\pi + 3\sqrt{3}\right)$
	Note	5.054815… from no working is M0M0A0.
	Note	Candidates can work in terms of k (note that k is not given in (a)) for the M1M1 marks in part (b).
	Note	If they incorrectly obtain $4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$ in part (a) (or guess $k = 4$) then the final A1 is available for a correct solution in part (b) only.

Question Number	Scheme		Marks
102. (a)	$\dfrac{2}{P(P-2)} = \dfrac{A}{P} + \dfrac{B}{(P-2)}$		
	$2 \equiv A(P-2) + BP$	Can be implied.	M1
	$A = -1,\ B = 1$	Either one.	A1
	giving $\dfrac{1}{(P-2)} - \dfrac{1}{P}$	See notes. **cao, aef**	A1
			[3]
(b)	$\dfrac{dP}{dt} = \dfrac{1}{2}P(P-2)\cos 2t$		
	$\displaystyle\int \frac{2}{P(P-2)}\,dP = \int \cos 2t\,dt$	can be implied by later working	B1 oe
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t\ \ (+c)$	$\pm\lambda\ln(P-2) \pm \mu\ln P$, $\lambda \neq 0,\ \mu \neq 0$	M1
		$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t$	A1
	$\{t = 0,\ P = 3 \Rightarrow\}\ \ \ln 1 - \ln 3 = 0 + c\ \ \{\Rightarrow\ c = -\ln 3\ \text{or}\ \ln(\tfrac{1}{3})\}$	See notes	M1
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t - \ln 3$ $\ln\left(\dfrac{3(P-2)}{P}\right) = \dfrac{1}{2}\sin 2t$		
	$\dfrac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda,\mu,\beta,K,\delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	M1
	$3(P-2) = Pe^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = Pe^{\frac{1}{2}\sin 2t}$ gives $3P - Pe^{\frac{1}{2}\sin 2t} = 6 \Rightarrow P(3 - e^{\frac{1}{2}\sin 2t}) = 6$	A complete method of rearranging to make P the subject. **Must have a constant of integration that need not be evaluated (see note)**	dM1
	$P = \dfrac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	A1 * cso
			[7]
(c)	$\{\text{population} = 4000 \Rightarrow\}\ P = 4$	States $P = 4$ or applies $P = 4$	M1
	$\dfrac{1}{2}\sin 2t = \ln\left(\dfrac{3(4-2)}{4}\right)\ \left\{= \ln\left(\dfrac{3}{2}\right)\right\}$	Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, $\lambda \neq 0,\ k > 0$ where λ and k are numerical values and λ can be 1	M1
	$t = 0.4728700467...$	anything that rounds to 0.473 Do not apply isw here	A1
			[3] **13**

Question Number	Scheme		Marks
	Method 2 for Q7(b)		
102. (b)	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t \ \ (+c)$	As before for…	**B1M1A1**
	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$		
	$\frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t + c}$ or $\frac{(P-2)}{P} = Ae^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda, \mu, \beta, K, \delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	**3rd M1**
	$(P-2) = APe^{\frac{1}{2}\sin 2t} \Rightarrow P - APe^{\frac{1}{2}\sin 2t} = 2$ $\Rightarrow P(1 - Ae^{\frac{1}{2}\sin 2t}) = 2 \Rightarrow P = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2t})}$	A complete method of rearranging to make P the subject. Condone sign slips or constant errors. **Must have a constant of integration that need not be evaluated (see note)**	**4th dM1**
	$\{t = 0, P = 3 \Rightarrow\}\quad 3 = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2(0)})}$	See notes **(Allocate this mark as the 2nd M1 mark on ePEN).**	**2nd M1**
	$\left\{\Rightarrow\ 3 = \frac{2}{(1-A)} \Rightarrow A = \frac{1}{3}\right\}$		
	$\Rightarrow P = \frac{2}{\left(1 - \frac{1}{3}e^{\frac{1}{2}\sin 2t}\right)} \Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	**A1 * cso**

Question 102 Notes

102. (a)	**M1**	Forming a correct identity. For example, $2 \equiv A(P-2) + BP$ from $\frac{2}{P(P-2)} = \frac{A}{P} + \frac{B}{(P-2)}$
	Note	A and B are not referred to in question.
	A1	Either one of $A = -1$ or $B = 1$.
	A1	$\frac{1}{(P-2)} - \frac{1}{P}$ or any equivalent form. This answer ***cannot*** be recovered from part (b).
	Note	M1A1A1 can also be given for a candidate who finds both $A = -1$ and $B = 1$ and $\frac{A}{P} + \frac{B}{(P-2)}$ is seen in their working.
	Note	Candidates can use 'cover-up' rule to write down $\frac{1}{(P-2)} - \frac{1}{P}$, so as to gain all three marks.
	Note	Equating coefficients from $2 \equiv A(P-2) + BP$ gives $A + B = 2, -2A = 2 \Rightarrow A = -1,\ B = 1$

102. (b)	**B1**	Separates variables as shown on the Mark Scheme. $\mathrm{d}P$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	**Eg:** $\int \frac{2}{P^2-2P}\,\mathrm{d}P = \int \cos 2t\,\mathrm{d}t$ or $\int \frac{1}{P(P-2)}\,\mathrm{d}P = \frac{1}{2}\int \cos 2t\,\mathrm{d}t$ **o.e.** are also fine for B1.
	1st M1	$\pm\lambda\ln(P-2)\pm\mu\ln P,\ \lambda\neq 0,\ \mu\neq 0$. Also allow $\pm\lambda\ln(M(P-2))\pm\mu\ln NP;\ M,N$ can be 1.
	Note	Condone $2\ln(P-2)+2\ln P$ or $2\ln(P(P-2))$ or $2\ln(P^2-2P)$ or $\ln(P^2-2P)$
	1st A1	Correct result of $\ln(P-2)-\ln P=\frac{1}{2}\sin 2t$ or $2\ln(P-2)-2\ln P=\sin 2t$ o.e. with or without $+c$
	2nd M1	Some evidence of using both $t=0$ and $P=3$ in an integrated equation containing a constant of integration. Eg: c or A, etc.
	3rd M1	Starting from an equation of the form $\pm\lambda\ln(P-\beta)\pm\mu\ln P=\pm K\sin\delta t+c,\ \lambda,\mu,\beta,K,\delta\neq 0$, applies a fully correct method to eliminate their logarithms.
	4th M1	**dependent on the third method mark being awarded.** A complete method of rearranging to make P the subject. Condone sign slips or constant errors.
	Note	For the 3rd M1 and 4th M1 marks, a candidate needs to have included a constant of integration, in their working. eg. c, A, $\ln A$ or an evaluated constant of integration.
	2nd A1	Correct proof of $P=\frac{6}{(3-\mathrm{e}^{\frac{1}{2}\sin 2t})}$. **Note:** This answer is given in the question.
	Note	$\ln\left(\frac{(P-2)}{P}\right)=\frac{1}{2}\sin 2t+c$ followed by $\frac{(P-2)}{P}=\mathrm{e}^{\frac{1}{2}\sin 2t}+\mathrm{e}^c$ is 3rd M0, 4th M0, 2nd A0.
	Note	$\ln\left(\frac{(P-2)}{P}\right)=\frac{1}{2}\sin 2t+c\ \rightarrow\ \frac{(P-2)}{P}=\mathrm{e}^{\frac{1}{2}\sin 2t+c}\ \rightarrow\ \frac{(P-2)}{P}=\mathrm{e}^{\frac{1}{2}\sin 2t}+\mathrm{e}^c$ is final M1M0A0
		<u>**4th M1 for making *P* the subject**</u> **Note there are three type of manipulations here which are considered acceptable for making *P* the subject.** **(1)** M1 for $\frac{3(P-2)}{P}=\mathrm{e}^{\frac{1}{2}\sin 2t}\Rightarrow 3(P-2)=P\mathrm{e}^{\frac{1}{2}\sin 2t}\Rightarrow 3P-6=P\mathrm{e}^{\frac{1}{2}\sin 2t}\Rightarrow P(3-\mathrm{e}^{\frac{1}{2}\sin 2t})=6$ $\Rightarrow P=\frac{6}{(3-\mathrm{e}^{\frac{1}{2}\sin 2t})}$ **(2)** M1 for $\frac{3(P-2)}{P}=\mathrm{e}^{\frac{1}{2}\sin 2t}\Rightarrow 3-\frac{6}{P}=\mathrm{e}^{\frac{1}{2}\sin 2t}\Rightarrow 3-\mathrm{e}^{\frac{1}{2}\sin 2t}=\frac{6}{P}\Rightarrow\Rightarrow P=\frac{6}{(3-\mathrm{e}^{\frac{1}{2}\sin 2t})}$ **(3)** M1 for $\left\{\ln(P-2)+\ln P=\frac{1}{2}\sin 2t+\ln 3\Rightarrow\right\}P(P-2)=3\mathrm{e}^{\frac{1}{2}\sin 2t}\Rightarrow P^2-2P=3\mathrm{e}^{\frac{1}{2}\sin 2t}$ $\Rightarrow(P-1)^2-1=3\mathrm{e}^{\frac{1}{2}\sin 2t}$ leading to $P=..$
(c)	**M1**	States $P=4$ or applies $P=4$
	M1	Obtains $\pm\lambda\sin 2t=\ln k$ or $\pm\lambda\sin t=\ln k$, where λ and k are numerical values and λ can be 1
	A1	anything that rounds to 0.473. ***(Do not apply isw here)***
	Note	***Do not apply ignore subsequent working for A1.*** (Eg: 0.473 followed by 473 years is A0.)
	Note	<u>Use of $P=4000$</u>: Without the mention of $P=4$, $\frac{1}{2}\sin 2t=\ln 2.9985$ or $\sin 2t=2\ln 2.9985$ or $\sin 2t=2.1912...$ will usually imply M0M1A0
	Note	<u>Use of Degrees:</u> $t=$ awrt 27.1 will usually imply M1M1A0

Question Number	Scheme		Marks
103. (a)	$\left\{y = 3^x \Rightarrow\right\} \dfrac{dy}{dx} = 3^x \ln 3$	$\dfrac{dy}{dx} = 3^x \ln 3$ or $\ln 3\left(e^{x\ln 3}\right)$ or $y \ln 3$	B1
	Either T: $y - 9 = 3^2 \ln 3(x-2)$ **or T:** $y = (3^2\ln 3)x + 9 - 18\ln 3$, where $9 = (3^2 \ln 3)(2) + c$	See notes	M1
	{Cuts x-axis $\Rightarrow y = 0 \Rightarrow$}		
	$-9 = 9\ln 3(x-2)$ or $0 = (3^2 \ln 3)x + 9 - 18\ln 3$,	Sets $y = 0$ in their tangent equation and progresses to $x = ...$	M1
	So, $x = 2 - \dfrac{1}{\ln 3}$	$2 - \dfrac{1}{\ln 3}$ or $\dfrac{2\ln 3 - 1}{\ln 3}$ o.e.	A1 **cso**
			[4]
(b)	$V = \pi\int \left(3^x\right)^2 \{dx\}$ or $\pi\int 3^{2x}\{dx\}$ or $\pi\int 9^x\{dx\}$	$V = \pi\int\left(3^x\right)^2$ with or without dx, which can be implied	B1 o.e.
	$= \{\pi\}\left(\dfrac{3^{2x}}{2\ln 3}\right)$ or $= \{\pi\}\left(\dfrac{9^x}{\ln 9}\right)$	Eg: either $3^{2x} \to \dfrac{3^{2x}}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)3^{2x}$ or $9^x \to \dfrac{9^x}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)9^x$, $\alpha \in$	M1
		$3^{2x} \to \dfrac{3^{2x}}{2\ln 3}$ or $9^x \to \dfrac{9^x}{\ln 9}$ or $e^{2x\ln 3} \to \dfrac{1}{2\ln 3}\left(e^{2x\ln 3}\right)$	A1 o.e.
	$\left\{V = \pi\int_0^2 3^{2x}\,dx = \{\pi\}\left[\dfrac{3^{2x}}{2\ln 3}\right]_0^2\right\} = \{\pi\}\left(\dfrac{3^4}{2\ln 3} - \dfrac{1}{2\ln 3}\right) \left\{= \dfrac{40\pi}{\ln 3}\right\}$	**Dependent on the previous method mark.** Substitutes $x = 2$ and $x = 0$ and subtracts the correct way round.	dM1
	$V_{\text{cone}} = \dfrac{1}{3}\pi(9)^2\left(\dfrac{1}{\ln 3}\right) \left\{= \dfrac{27\pi}{\ln 3}\right\}$	$V_{\text{cone}} = \dfrac{1}{3}\pi(9)^2(2 - \text{their } (a))$. See notes.	B1ft
	$\left\{\text{Vol}(S) = \dfrac{40\pi}{\ln 3} - \dfrac{27\pi}{\ln 3}\right\} = \underline{\dfrac{13\pi}{\ln 3}}$	$\dfrac{13\pi}{\ln 3}$ or $\dfrac{26\pi}{\ln 9}$ or $\dfrac{26\pi}{2\ln 3}$ etc., isw	A1 o.e.
		{Eg: $p = 13\pi$, $q = \ln 3$}	**[6]** **10**
(b)	**<u>Alternative Method 1: Use of a substitution</u>**		
	$V = \pi\int\left(3^x\right)^2\{dx\}$		B1 o.e.
	$\left\{u = 3^x \Rightarrow \dfrac{du}{dx} = 3^x\ln 3 = u\ln 3\right\}$ $V = \{\pi\}\int \dfrac{u^2}{u\ln 3}\{du\} = \{\pi\}\int\dfrac{u}{\ln 3}\{du\}$		
	$= \{\pi\}\left(\dfrac{u^2}{2\ln 3}\right)$	$\left(3^x\right)^2 \to \dfrac{u^2}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)u^2$, where $u = 3^x$	M1
		$\left(3^x\right)^2 \to \dfrac{u^2}{2(\ln 3)}$, where $u = 3^x$	A1
	$\left\{V = \pi\int_0^2\left(3^x\right)^2 dx = \{\pi\}\left[\dfrac{u^2}{2\ln 3}\right]_1^9\right\} = \{\pi\}\left(\dfrac{9^2}{2\ln 3} - \dfrac{1}{2\ln 3}\right) \left\{= \dfrac{40\pi}{\ln 3}\right\}$	Substitutes limits of 9 and 1 in u (or 2 and 0 in x) and subtracts the correct way round.	dM1
	then apply the main scheme.		

		Question 103 Notes
103. (a)	**B1**	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3^x \ln 3$ or $\ln 3\left(\mathrm{e}^{x\ln 3}\right)$ or $y\ln 3$. Can be implied by later working.
	M1	Substitutes either $x = 2$ **or** $y = 9$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ which is a function of x or y to find m_T **and** • **either** applies $y - 9 = \left(\text{their } m_T\right)\left(x - 2\right)$, where m_T is a numerical value. • **or** applies $y = \left(\text{their } m_T\right)x + \text{their } c$, where m_T is a numerical value and c is found by solving $9 = \left(\text{their } m_T\right)(2) + c$
	Note	The first M1 mark can be implied from later working.
	M1	Sets $y = 0$ in their ***tangent*** equation, where m_T is a numerical value, (seen or implied) and progresses to $x = ...$
	A1	An exact value of $2 - \frac{1}{\ln 3}$ or $\frac{2\ln 3 - 1}{\ln 3}$ or $\frac{\ln 9 - 1}{\ln 3}$ by a correct solution only.
	Note	Allow A1 for $2 - \frac{\lambda}{\lambda \ln 3}$ or $\frac{\lambda(2\ln 3 - 1)}{\lambda \ln 3}$ or $\frac{\lambda(\ln 9 - 1)}{\lambda \ln 3}$ or $2 - \frac{\lambda}{\lambda \ln 3}$, where λ is an integer, and ignore subsequent working.
	Note	Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{\mathrm{d}y}{\mathrm{d}x}}$ or $\frac{1}{\text{their } \frac{\mathrm{d}y}{\mathrm{d}x}}$) is M0 M0 in part (a).
	Note	Candidates who invent a value for m_T (which bears no resemblance to their gradient function) **cannot** gain the 1st M1 and 2nd M1 mark in part (a).
	Note	A decimal answer of 1.089760773… (without a correct **exact** answer) is A0.
103. (b)	**B1**	A correct expression for the volume with or without $\mathrm{d}x$
	Note	**Eg:** Allow B1 for $\pi\int\left(3^x\right)^2\left\{\mathrm{d}x\right\}$ or $\pi\int 3^{2x}\left\{\mathrm{d}x\right\}$ or $\pi\int 9^x\left\{\mathrm{d}x\right\}$ or $\pi\int\left(\mathrm{e}^{x\ln 3}\right)^2\left\{\mathrm{d}x\right\}$ or $\pi\int\left(\mathrm{e}^{2x\ln 3}\right)\left\{\mathrm{d}x\right\}$ or $\pi\int \mathrm{e}^{x\ln 9}\left\{\mathrm{d}x\right\}$ with or without $\mathrm{d}x$
	M1	Either $3^{2x} \to \frac{3^{2x}}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)3^{2x}$ or $9^x \to \frac{9^x}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)9^x$ $\mathrm{e}^{2x\ln 3} \to \frac{\mathrm{e}^{2x\ln 3}}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)\mathrm{e}^{2x\ln 3}$ or $\mathrm{e}^{x\ln 9} \to \frac{\mathrm{e}^{x\ln 9}}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)\mathrm{e}^{x\ln 9}$, etc where $\alpha \in$
	Note	$3^{2x} \to \frac{3^{2x+1}}{\pm\alpha(\ln 3)}$ or $9^x \to \frac{9^{x+1}}{\pm\alpha(\ln 3)}$ are allowed for M1
	Note	$3^{2x} \to \frac{3^{2x+1}}{2x+1}$ or $9^x \to \frac{9^{x+1}}{x+1}$ are both M0
	Note	M1 can be given for $9^{2x} \to \frac{9^{2x}}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)9^{2x}$
	A1	Correct integration of 3^{2x}. Eg: $3^{2x} \to \frac{3^{2x}}{2\ln 3}$ or $\frac{3^{2x}}{\ln 9}$ or $9^x \to \frac{9^x}{\ln 9}$ or $\mathrm{e}^{2x\ln 3} \to \frac{1}{2\ln 3}\left(\mathrm{e}^{2x\ln 3}\right)$
	dM1	**dependent on the previous method mark being awarded.** Attempts to apply $x = 2$ and $x = 0$ to integrated expression and subtracts the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.

	dM1	**dependent on the previous method mark being awarded.** Attempts to apply $x = 2$ and $x = 0$ to integrated expression and subtracts the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.
	B1ft	$V_{\text{cone}} = \frac{1}{3}\pi(9)^2\left(2 - \text{their answer to part }(a)\right)$. Sight of $\frac{27\pi}{\ln 3}$ implies the B1 mark.
	Note	Alternatively they can apply the volume formula to the line segment. They need to achieve the result highlighted by **** on either page 29 or page 30 in order to obtain the B1ft mark.
	A1	$\frac{13\pi}{\ln 3}$ or $\frac{26\pi}{\ln 9}$ or $\frac{26\pi}{2\ln 3}$, etc. , where their answer is in the form $\frac{p}{q}$
	Note	The π in the volume formula is only needed for the 1st B1 mark and the final A1 mark.
	Note	A decimal answer of 37.17481128… (without a correct **exact** answer) is A0.
	Note	A candidate who applies $\int 3^x \mathrm{d}x$ will either get B0 M0 A0 M0 **B0** A0 **or** B0 M0 A0 M0 **B1** A0
	Note	$\pi\int 3^{x^2}\,\mathrm{d}x$ unless recovered is B0.
	Note	**<u>Be careful! A correct answer may follow from incorrect working</u>** $V = \pi\int_0^2 3^{x^2}\,\mathrm{d}x - \frac{1}{3}\pi(9)^2\left(\frac{1}{\ln 3}\right) = \pi\left[\frac{3^{x^2}}{2\ln 3}\right]_0^2 - \frac{27\pi}{\ln 3} = \frac{\pi 3^4}{2\ln 3} - \frac{\pi}{2\ln 3} - \frac{27\pi}{\ln 3} = \frac{13\pi}{\ln 3}$ would score B0 M0 A0 dM0 M1 A0.
103. (b)	**<u>2nd B1ft mark for finding the Volume of a Cone</u>**	

$$V_{\text{cone}} = \pi\int_{2-\frac{1}{\ln 3}}^{2}\left(9x\ln 3 - 18\ln 3 + 9\right)^2\,\mathrm{d}x$$

$$= \pi\left[\frac{\left(9x\ln 3 - 18\ln 3 + 9\right)^3}{27\ln 3}\right]_{2-\frac{1}{\ln 3}\text{ or their part (a) answer}}^{2} \qquad ****$$

Award B1ft here where their lower limit is $2 - \frac{1}{\ln 3}$ or their part (a) answer.

$$= \pi\left(\left(\frac{\left(18\ln 3 - 18\ln 3 + 9\right)^3}{27\ln 3}\right) - \left(\frac{\left(9\left(2 - \frac{1}{\ln 3}\right)\ln 3 - 18\ln 3 + 9\right)^3}{27\ln 3}\right)\right)$$

$$= \pi\left(\left(\frac{729}{27\ln 3}\right) - \left(\frac{\left(18\ln 3 - 9 - 18\ln 3 + 9\right)^3}{27\ln 3}\right)\right)$$

$$= \frac{27\pi}{\ln 3}$$

103. (b)	**2nd B1ft mark for finding the Volume of a Cone** **Alternative method 2:**

$$V_{\text{cone}} = \pi\int_{2-\frac{1}{\ln 3}}^{2} \left(9x\ln 3 - 18\ln 3 + 9\right)^2 \, \mathrm{d}x$$

$$= \pi\int_{2-\frac{1}{\ln 3}}^{2} \left(81x^2(\ln 3)^2 - 324x(\ln 3)^2 + 162x\ln 3 - 324\ln 3 + 324(\ln 3)^2 + 81\right) \mathrm{d}x$$

$$= \pi\left[27x^3(\ln 3)^2 - 162x^2(\ln 3)^2 + 81x^2\ln 3 - 324x\ln 3 + 324x(\ln 3)^2 + 81x\right]_{2-\frac{1}{\ln 3}}^{2}$$

Award B1ft here where their lower limit is $2 - \dfrac{1}{\ln 3}$ or their part (a) answer.

****

$$= \pi\left(\begin{array}{l}\left(216(\ln 3)^2 - 648(\ln 3)^2 + 324\ln 3 - 648\ln 3 + 648(\ln 3)^2 + 162\right) \\ \quad - \left(\begin{array}{l} 27\left(2-\dfrac{1}{\ln 3}\right)^3(\ln 3)^2 - 162\left(2-\dfrac{1}{\ln 3}\right)^2(\ln 3)^2 + 81\left(2-\dfrac{1}{\ln 3}\right)^2\ln 3 \\ -324\left(2-\dfrac{1}{\ln 3}\right)\ln 3 + 324\left(2-\dfrac{1}{\ln 3}\right)(\ln 3)^2 + 81\left(2-\dfrac{1}{\ln 3}\right)\end{array}\right)\end{array}\right)$$

$$= \pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(\begin{array}{l} 27\left(8-\dfrac{12}{\ln 3}+\dfrac{6}{(\ln 3)^2}-\dfrac{1}{(\ln 3)^3}\right)(\ln 3)^2 - 162\left(4-\dfrac{4}{\ln 3}+\dfrac{1}{(\ln 3)^2}\right)(\ln 3)^2 \\ +81\left(4-\dfrac{4}{\ln 3}+\dfrac{1}{(\ln 3)^2}\right)\ln 3 - 324\left(2-\dfrac{1}{\ln 3}\right)\ln 3 \\ +324\left(2-\dfrac{1}{\ln 3}\right)(\ln 3)^2 + 81\left(2-\dfrac{1}{\ln 3}\right)\end{array}\right)\right)$$

$$= \pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(\begin{array}{l} 216(\ln 3)^2 - 324\ln 3 + 162 - \dfrac{27}{\ln 3} - 648(\ln 3)^2 + 648\ln 3 - 162 \\ +324\ln 3 - 324 + \dfrac{81}{\ln 3} - 648\ln 3 + 324 \\ +648(\ln 3)^2 - 324\ln 3 + 162 - \dfrac{81}{\ln 3}\end{array}\right)\right)$$

$$= \pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(216(\ln 3)^2 - 324\ln 3 + 162 - \frac{27}{\ln 3}\right)\right)$$

$$= \frac{27\pi}{\ln 3}$$

EXPERT TUITION

Question Number	Scheme		Marks
104	$\left\{u=\sqrt{x} \Rightarrow\right\} \dfrac{du}{dx}=\dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{dx}{du}=2u$		B1
	$\displaystyle\int \frac{10}{2u^2+5u}.\, 2u\, du$	Either $\left\{\int\right\}\dfrac{\pm ku}{\alpha u^2 \pm \beta u}\{du\}$ or $\left\{\int\right\}\dfrac{\pm k}{u(\alpha u^2 \pm \beta u)}\{du\}$	M1
	$\left\{= \displaystyle\int \frac{20}{2u+5}\, du\right\} = \dfrac{20}{2}\ln(2u+5)$	$\pm\lambda\ln(2u+5)$ or $\pm\lambda\ln\left(u+\dfrac{5}{2}\right)$, $\lambda \neq 0$ **with no other terms.**	M1
		$\dfrac{20}{2u+5} \to \dfrac{20}{2}\ln(2u+5)$ or $10\ln\left(u+\dfrac{5}{2}\right)$	A1 **cso**
	$\left\{\left[\dfrac{20}{2}\ln(2u+5)\right]_1^2\right\} = 10\ln(2(2)+5) - 10\ln(2(1)+5)$	Substitutes limits of 2 and 1 in u (or 4 and 1 in x) and subtracts the correct way round.	M1
	$10\ln 9 - 10\ln 7$ or $10\ln\left(\dfrac{9}{7}\right)$ or $20\ln 3 - 10\ln 7$		A1 oe **cso**
			[6]

104	**B1**	$\dfrac{du}{dx}=\dfrac{1}{2}x^{-\frac{1}{2}}$ or $du=\dfrac{1}{2\sqrt{x}}dx$ or $2\sqrt{x}\,du = dx$ or $dx = 2u\,du$ or $\dfrac{dx}{du}=2u$ o.e.
	M1	Applying the substitution and achieving $\left\{\int\right\}\dfrac{\pm ku}{\alpha u^2 \pm \beta u}\{du\}$ or $\left\{\int\right\}\dfrac{\pm k}{u(\alpha u^2 \pm \beta u)}\{du\}$, $k, \alpha, \beta \neq 0$. Integral sign and du not required for this mark.
	M1	Cancelling u and integrates to achieve $\pm\lambda\ln(2u+5)$ or $\pm\lambda\ln\left(u+\dfrac{5}{2}\right)$, $\lambda \neq 0$ **with no other terms.**
	A1	**cso**. Integrates $\dfrac{20}{2u+5}$ to give $\dfrac{20}{2}\ln(2u+5)$ or $10\ln\left(u+\dfrac{5}{2}\right)$, un-simplified or simplified.
	Note	BE CAREFUL! Candidates must be integrating $\dfrac{20}{2u+5}$ or equivalent.
		So $\displaystyle\int \frac{10}{2u+5}\, du = 10\ln(2u+5)$ WOULD BE A0 and final A0.
	M1	Applies limits of 2 and 1 in u or 4 and 1 in x in their (i.e. any) changed function and subtracts the correct way round.
	A1	Exact answers of either $10\ln 9 - 10\ln 7$ or $10\ln\left(\dfrac{9}{7}\right)$ or $20\ln 3 - 10\ln 7$ or $20\ln\left(\dfrac{3}{\sqrt{7}}\right)$ or $\ln\left(\dfrac{9^{10}}{7^{10}}\right)$ or equivalent. **Correct solution only.**
	Note	You can ignore subsequent working which follows from a correct answer.
	Note	A decimal answer of 2.513144283... (without a correct **exact** answer) is A0.

Question Number	Scheme		Marks
105. (i)	$\int xe^{4x}\,dx = \frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,\{dx\}$	$\pm\alpha xe^{4x} - \int \beta e^{4x}\{dx\},\quad \alpha \neq 0,\ \beta > 0$	M1
		$\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,\{dx\}$	A1
	$= \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}\ \{+c\}$	$\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$	A1
			[3]
(ii)	$\int \frac{8}{(2x-1)^3}\,dx = \frac{8(2x-1)^{-2}}{(2)(-2)}\ \{+c\}$	$\pm\lambda(2x-1)^{-2}$	M1
		$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or equivalent.	A1
	$\{= -2(2x-1)^{-2}\ \{+c\}\}$	*{Ignore subsequent working}*.	**[2]**
(iii)	$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y \qquad y = \frac{\pi}{6}$ at $x = 0$		
	Main Scheme		
	$\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y}\,dy = \int e^x\,dx$ **or** $\int \sin 2y \sin y\,dy = \int e^x\,dx$		B1 oe
	$\int 2\sin y\cos y \sin y\,dy = \int e^x\,dx$	Applying $\frac{1}{\operatorname{cosec}2y}$ or $\sin 2y \to 2\sin y\cos y$	M1
		Integrates to give $\pm\mu\sin^3 y$	M1
	$\frac{2}{3}\sin^3 y = e^x\ \{+c\}$	$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$	A1
		$e^x \to e^x$	B1
	$\frac{2}{3}\sin^3\left(\frac{\pi}{6}\right) = e^0 + c$ **or** $\frac{2}{3}\left(\frac{1}{8}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$	$\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$	A1
			[7]
	Alternative Method 1		
	$\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y}\,dy = \int e^x\,dx$ **or** $\int \sin 2y \sin y\,dy = \int e^x\,dx$		B1 oe
	$\int -\frac{1}{2}(\cos 3y - \cos y)\,dy = \int e^x\,dx$	$\sin 2y \sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$	M1
		Integrates to give $\pm\alpha\sin 3y \pm \beta\sin y$	M1
	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right) = e^x\ \{+c\}$	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$	A1
		$e^x \to e^x$ ***as part of solving their DE.***	B1
	$-\frac{1}{2}\left(\frac{1}{3}\sin\left(\frac{3\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right)\right) = e^0 + c$ **or** $-\frac{1}{2}\left(\frac{1}{3} - \frac{1}{2}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	A1
			[7]
			12

	Question 105 Notes	
105. (i)	**M1**	Integration by parts is applied in the form $\pm\alpha x\mathrm{e}^{4x} - \int \beta\mathrm{e}^{4x}\{\mathrm{d}x\}$, where $\alpha \neq 0,\ \beta > 0$. (must be in this form).
	A1	$\frac{1}{4}x\mathrm{e}^{4x} - \int \frac{1}{4}\mathrm{e}^{4x}\{\mathrm{d}x\}$ or equivalent.
	A1	$\frac{1}{4}x\mathrm{e}^{4x} - \frac{1}{16}\mathrm{e}^{4x}$ with/without + c. Can be un-simplified.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	**SPECIAL CASE:** A candidate who uses $u = x$, $\frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^{4x}$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1.
(ii)	**M1**	$\pm\lambda(2x-1)^{-2}$, $\lambda \neq 0$. **Note** that λ can be 1.
	A1	$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or $-2(2x-1)^{-2}$ or $\frac{-2}{(2x-1)^2}$ with/without + c. Can be un-simplified.
	Note	You can ignore subsequent working which follows from a correct answer.
(iii)	**B1**	Separates variables as shown. $\mathrm{d}y$ and $\mathrm{d}x$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	**Allow B1 for** $\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y} = \int \mathrm{e}^x$ **or** $\int \sin 2y \sin y = \int \mathrm{e}^x$
	M1	$\frac{1}{\operatorname{cosec} 2y} \to 2\sin y\cos y$ **or** $\sin 2y \to 2\sin y\cos y$ **or** $\sin 2y\sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$ seen anywhere in the candidate's working to (iii).
	M1	Integrates to give $\pm\mu\sin^3 y$, $\mu \neq 0$ **or** $\pm\alpha\sin 3y \pm \beta\sin y$, $\alpha \neq 0, \beta \neq 0$
	A1	$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$ (with no extra terms) **or** integrates to give $-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$
	B1	Evidence that e^x has been integrated to give e^x ***as part of solving their DE.***
	M1	Some evidence of using both $y = \frac{\pi}{6}$ and $x = 0$ in an integrated or changed equation containing c.
	Note	that is mark can be implied by the correct value of c.
	A1	$\frac{2}{3}\sin^3 y = \mathrm{e}^x - \frac{11}{12}$ **or** $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = \mathrm{e}^x - \frac{11}{12}$ **or any equivalent correct answer.**
	Note	You can ignore subsequent working which follows from a correct answer.

Alternative Method 2 (Using integration by parts twice)

$\int \sin 2y \sin y\ \mathrm{d}y = \int \mathrm{e}^x\ \mathrm{d}x$		B1 oe
	Applies integration by parts **twice** to give $\pm\alpha\cos y\sin 2y \pm \beta\sin y\cos 2y$	**M2**
$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y = \mathrm{e}^x\ \{+c\}$	$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y$ (simplified or un-simplified)	A1
	$\mathrm{e}^x \to \mathrm{e}^x$ ***as part of solving their DE.***	B1
	as in the main scheme	M1
$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y = \mathrm{e}^x - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = \mathrm{e}^x - \frac{11}{12}$	A1
		[7]

Question Number	Scheme		Marks
106.	$x = 3\tan\theta$, $y = 4\cos^2\theta$ or $y = 2 + 2\cos 2\theta$, $0 \leqslant \theta < \frac{\pi}{2}$.		
(a)	$\frac{dx}{d\theta} = 3\sec^2\theta$, $\frac{dy}{d\theta} = -8\cos\theta\sin\theta$ **or** $\frac{dy}{d\theta} = -4\sin 2\theta$		
	$\frac{dy}{dx} = \frac{-8\cos\theta\sin\theta}{3\sec^2\theta}$ $\left\{= -\frac{8}{3}\cos^3\theta\sin\theta = -\frac{4}{3}\sin 2\theta\cos^2\theta\right\}$	their $\frac{dy}{d\theta}$ divided by their $\frac{dx}{d\theta}$ Correct $\frac{dy}{dx}$	M1 A1 oe
	At $P(3, 2)$, $\theta = \frac{\pi}{4}$, $\frac{dy}{dx} = -\frac{8}{3}\cos^3\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)$ $\left\{= -\frac{2}{3}\right\}$ So, $m(\mathbf{N}) = \frac{3}{2}$	Some evidence of substituting $\theta = \frac{\pi}{4}$ into their $\frac{dy}{dx}$ applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1 M1
	Either **N:** $y - 2 = \frac{"3"}{2}(x - 3)$ **or** $2 = \left(\frac{"3"}{2}\right)(3) + c$	**see notes**	M1
	{At Q, $y = 0$, so, $-2 = \frac{3}{2}(x - 3)$ } **giving** $x = \frac{5}{3}$	$x = \frac{5}{3}$ or $1\frac{2}{3}$ or awrt 1.67	A1 **cso** **[6]**
(b)	$\left\{\int y^2 dx = \int y^2 \frac{dx}{d\theta} d\theta\right\} = \left\{\int\right\}(4\cos^2\theta)^2 3\sec^2\theta\ \{d\theta\}$	**see notes**	M1
	So, $\pi\int y^2 dx = \pi\int(4\cos^2\theta)^2 3\sec^2\theta\,\{d\theta\}$	**see notes**	A1
	$\int y^2 dx = \int 48\cos^2\theta\, d\theta$	$\int 48\cos^2\theta\{d\theta\}$	A1
	$= \{48\}\int\left(\frac{1+\cos 2\theta}{2}\right)d\theta$ $\left\{= \int(24 + 24\cos 2\theta)d\theta\right\}$	Applies $\cos 2\theta = 2\cos^2\theta - 1$	M1
	$= \{48\}\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)$ $\{= 24\theta + 12\sin 2\theta\}$	**Dependent on the first method mark.** For $\pm\alpha\theta \pm \beta\sin 2\theta$ $\cos^2\theta \to \left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)$	dM1 A1
	$\int_0^{\frac{\pi}{4}} y^2 dx \left\{= 48\left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{4}}\right\} = \{48\}\left(\left(\frac{\pi}{8} + \frac{1}{4}\right) - (0 + 0)\right)$ $\{= 6\pi + 12\}$	**Dependent on the third method mark.**	dM1
	{So $V = \pi\int_0^{\frac{\pi}{4}} y^2 dx = 6\pi^2 + 12\pi$ }		
	$V_{cone} = \frac{1}{3}\pi(2)^2\left(3 - \frac{5}{3}\right)$ $\left\{= \frac{16\pi}{9}\right\}$	$V_{cone} = \frac{1}{3}\pi(2)^2(3 - \text{their }(a))$	M1
	$\left\{\text{Vol}(S) = 6\pi^2 + 12\pi - \frac{16\pi}{9}\right\} \Rightarrow \text{Vol}(S) = \frac{92}{9}\pi + 6\pi^2$	$\frac{92}{9}\pi + 6\pi^2$	A1
		$\left\{p = \frac{92}{9},\ q = 6\right\}$	**[9]**
			15

		Question 106 Notes
106. (a)	**1st M1**	Applies their $\frac{dy}{d\theta}$ divided by their $\frac{dx}{d\theta}$ **or** applies $\frac{dy}{d\theta}$ multiplied by their $\frac{d\theta}{dx}$
	SC	Award **Special Case 1st M1** if **both** $\frac{dx}{d\theta}$ **and** $\frac{dy}{d\theta}$ are both correct.
	1st A1	Correct $\frac{dy}{dx}$ i.e. $\frac{-8\cos\theta\sin\theta}{3\sec^2\theta}$ or $-\frac{8}{3}\cos^3\theta\sin\theta$ or $-\frac{4}{3}\sin 2\theta\cos^2\theta$ or any equivalent form.
	2nd M1	***Some evidence*** of substituting $\theta = \frac{\pi}{4}$ or $\theta = 45^\circ$ into their $\frac{dy}{dx}$
	Note	For 3rd M1 and 4th M1, $m(\mathbf{T})$ must be found by using $\frac{dy}{dx}$.
	3rd M1	applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$. Numerical value for $m(\mathbf{N})$ is required here.
	4th M1	• Applies $y - 2 = (\text{their } m_N)(x-3)$, where $\text{m}(\mathbf{N})$ is a numerical value, • or ***finds c*** by solving $2 = (\text{their } m_N)3 + c$, where $\text{m}(\mathbf{N})$ is a numerical value, and $m_N = -\frac{1}{\text{their m}(\mathbf{T})}$ or $m_N = \frac{1}{\text{their m}(\mathbf{T})}$ or $m_N = -\text{their m}(\mathbf{T})$.
	Note	This mark can be implied by subsequent working.
	2nd A1	$x = \frac{5}{3}$ or $1\frac{2}{3}$ or awrt 1.67 **from a correct solution only.**
(b)	**1st M1**	Applying $\int y^2 dx$ as $y^2 \frac{dx}{d\theta}$ with their $\frac{dx}{d\theta}$. Ignore π or $\frac{1}{3}\pi$ outside integral.
	Note	You can ignore the omission of an integral sign and/or $d\theta$ for the 1st M1.
	Note	Allow 1st M1 for $\int(\cos^2\theta)^2 \times \text{"their } 3\sec^2\theta\text{"}\, d\theta$ or $\int 4(\cos^2\theta)^2 \times \text{"their } 3\sec^2\theta\text{"}\, d\theta$
	1st A1	Correct expression $\left\{\pi\int y^2 dx\right\} = \pi\int(4\cos^2\theta)^2 3\sec^2\theta\,\{d\theta\}$ (Allow the omission of $d\theta$)
	Note	**IMPORTANT:** The π can be recovered later, **but as a correct statement only**.
	2nd A1	$\left\{\int y^2 dx\right\} = \int 48\cos^2\theta\{d\theta\}$. (Ignore $d\theta$). **Note**: 48 can be written as 24(2) for example.
	2nd M1	Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral. (Seen or **implied**.)
	3rd dM1*	**which is dependent on the 1st M1 mark.** Integrating $\cos^2\theta$ to give $\pm\alpha\theta \pm \beta\sin 2\theta$, $\alpha \neq 0$, $\beta \neq 0$, un-simplified or simplified.
	3rd A1	**which is dependent on the 3rd M1 mark and the 1st M1 mark.** Integrating $\cos^2\theta$ to give $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$, un-simplified or simplified. This can be implied by $k\cos^2\theta$ giving $\frac{k}{2}\theta + \frac{k}{4}\sin 2\theta$, un-simplified or simplified.
	4th dM1	**which is dependent on the 3rd M1 mark and the 1st M1 mark.** Some evidence of applying limits of $\frac{\pi}{4}$ and 0 (0 can be implied) to an integrated function in θ
	5th M1	Applies $V_{\text{cone}} = \frac{1}{3}\pi(2)^2(3 - \text{their part}(a)\text{ answer})$.
	Note	Also allow the 5th M1 for $V_{\text{cone}} = \pi\int_{\text{their }\frac{5}{3}}^{3}\left(\frac{3}{2}x - \frac{5}{2}\right)^2\{dx\}$, which includes the correct limits.
	4th A1	$\frac{92}{9}\pi + 6\pi^2$ or $10\frac{2}{9}\pi + 6\pi^2$
	Note	A decimal answer of 91.33168464... (without a correct **exact** answer) is A0.
	Note	The π in the volume formula is only needed for the 1st A1 mark and the final accuracy mark.

106.		**Working with a Cartesian Equation** A cartesian equation for C is $y = \dfrac{36}{x^2 + 9}$
(a)	**1st M1** **1st A1** **2nd dM1**	$\dfrac{dy}{dx} = \pm\lambda x\left(\pm\alpha x^2 \pm \beta\right)^{-2}$ **or** $\dfrac{dy}{dx} = \dfrac{\pm\lambda x}{\left(\pm\alpha x^2 \pm \beta\right)^2}$ $\dfrac{dy}{dx} = -36(x^2+9)^{-2}(2x)$ **or** $\dfrac{dy}{dx} = \dfrac{-72x}{(x^2+9)^2}$ un-simplified or simplified. **Dependent on the 1st M1 mark if a candidate uses this method** For substituting $x = 3$ into their $\dfrac{dy}{dx}$ i.e. at $P(3, 2)$, $\dfrac{dy}{dx} = \dfrac{-72(3)}{(3^2+9)^2}\left\{= -\dfrac{2}{3}\right\}$ From this point onwards the original scheme can be applied.
(b)	**1st M1** **A1**	For $\displaystyle\int\left(\frac{\pm\lambda}{\pm\alpha x^2 \pm \beta}\right)^2 \{dx\}$ (π not required for this mark) For $\displaystyle\pi\int\left(\frac{36}{x^2+9}\right)^2 \{dx\}$ (π required for this mark) To integrate, a substitution of $x = 3\tan\theta$ is required which will lead to $\displaystyle\int 48\cos^2\theta\, d\theta$ and so from this point onwards the original scheme can be applied.
(a)	**1st M1** **1st A1** **2nd dM1**	Another cartesian equation for C is $x^2 = \dfrac{36}{y} - 9$ $\pm\alpha x = \pm\dfrac{\beta}{y^2}\dfrac{dy}{dx}$ or $\pm\alpha x\dfrac{dx}{dy} = \pm\dfrac{\beta}{y^2}$ $2x = -\dfrac{36}{y^2}\dfrac{dy}{dx}$ or $2x\dfrac{dx}{dy} = -\dfrac{36}{y^2}$ **Dependent on the 1st M1 mark if a candidate uses this method** For substituting $x = 3$ to find $\dfrac{dy}{dx}$ i.e. at $P(3, 2)$, $2(3) = -\dfrac{36}{4}\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = ...$ From this point onwards the original scheme can be applied.

Question Number	Scheme		Marks
107	$\left\{\int (2-x)\mathrm{e}^{2x}\,\mathrm{d}x\right\}$, $\left\{\begin{array}{ll} u = 2-x & \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = -1 \\ \dfrac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^{2x} & \Rightarrow v = \dfrac{1}{2}\mathrm{e}^{2x} \end{array}\right\}$		
	$= \dfrac{1}{2}(2-x)\mathrm{e}^{2x} - \int -\dfrac{1}{2}\mathrm{e}^{2x}\{\mathrm{d}x\}$	Either $(2-x)\mathrm{e}^{2x} \to \pm\lambda(2-x)\mathrm{e}^{2x} \pm \int \mu\mathrm{e}^{2x}\{\mathrm{d}x\}$ or $\pm x\mathrm{e}^{2x} \to \pm\lambda x\mathrm{e}^{2x} \pm \int \mu\mathrm{e}^{2x}\{\mathrm{d}x\}$	M1
		$(2-x)\mathrm{e}^{2x} \to \dfrac{1}{2}(2-x)\mathrm{e}^{2x} - \int -\dfrac{1}{2}\mathrm{e}^{2x}\{\mathrm{d}x\}$	A1
	$= \dfrac{1}{2}(2-x)\mathrm{e}^{2x} + \dfrac{1}{4}\mathrm{e}^{2x}$	$\dfrac{1}{2}(2-x)\mathrm{e}^{2x} + \dfrac{1}{4}\mathrm{e}^{2x}$	A1 oe
	$\text{Area} = \left\{\left[\dfrac{1}{2}(2-x)\mathrm{e}^{2x} + \dfrac{1}{4}\mathrm{e}^{2x}\right]_0^2\right\}$		
	$= \left(0 + \dfrac{1}{4}\mathrm{e}^4\right) - \left(\dfrac{1}{2}(2)\mathrm{e}^0 + \dfrac{1}{4}\mathrm{e}^0\right)$	Applies limits of 2 and 0 ***to all terms*** and subtracts the correct way round.	dM1
	$= \dfrac{1}{4}\mathrm{e}^4 - \dfrac{5}{4}$	$\dfrac{1}{4}\mathrm{e}^4 - \dfrac{5}{4}$ or $\dfrac{\mathrm{e}^4 - 5}{4}$ **cao**	A1 oe
			[5]

		Question 107 Notes
107	**M1**	Either $(2-x)\mathrm{e}^{2x} \to \pm\lambda(2-x)\mathrm{e}^{2x} \pm \int \mu\mathrm{e}^{2x}\{\mathrm{d}x\}$ or $\pm x\mathrm{e}^{2x} \to \pm\lambda x\mathrm{e}^{2x} \pm \int \mu\mathrm{e}^{2x}\{\mathrm{d}x\}$
	A1	$(2-x)\mathrm{e}^{2x} \to \dfrac{1}{2}(2-x)\mathrm{e}^{2x} - \int -\dfrac{1}{2}\mathrm{e}^{2x}\{\mathrm{d}x\}$ either un-simplified or simplified.
	A1	Correct expression, i.e. $\dfrac{1}{2}(2-x)\mathrm{e}^{2x} + \dfrac{1}{4}\mathrm{e}^{2x}$ or $\dfrac{5}{4}\mathrm{e}^{2x} - x\mathrm{e}^{2x}$ (or equivalent)
	dM1	**which is dependent on the 1st M1 mark being awarded.** **Complete method** of applying limits of 2 and 0 **to all terms** and subtracting the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So, just subtracting zero is M0.
	A1	$\dfrac{1}{4}\mathrm{e}^4 - \dfrac{5}{4}$ or $\dfrac{\mathrm{e}^4 - 5}{4}$. Do not allow $\dfrac{1}{4}\mathrm{e}^4 - \dfrac{5}{4}\mathrm{e}^0$ unless simplified to give $\dfrac{1}{4}\mathrm{e}^4 - \dfrac{5}{4}$
	Note	12.39953751... **without seeing** $\dfrac{1}{4}\mathrm{e}^4 - \dfrac{5}{4}$ is A0.
	Note	12.39953751... from **NO** working is M0A0A0M0A0.

Question Number	Scheme		Marks
108. (a)	$\dfrac{25}{x^2(2x+1)} \equiv \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{(2x+1)}$		
		At least one of "B" or "C" correct.	B1
	$B = 25,\ \ C = 100$	Breaks up their partial fraction correctly into ***three terms* and** both "B" $= 25$ and "C" $= 100$. **See notes.**	B1 **cso**
	$25 \equiv Ax(2x+1) + B(2x+1) + Cx^2$ $x = 0, \quad 25 = B$ $x = -\dfrac{1}{2}, \quad 25 = \dfrac{1}{4}C \Rightarrow C = 100$ x^2 terms: $\quad 0 = 2A + C$ $0 = 2A + 100 \quad \Rightarrow \quad A = -50$ $x^2:\ 0 = 2A + C,\quad x:\ 0 = A + 2B,$ constant: $25 = B$	Writes down **a correct identity** and attempts to **find the value** of either one of "A", "B" or "C".	M1
	leading to $A = -50$	Correct value for "A" which is found using a correct identity and follows from their partial fraction decomposition.	A1
	$\left\{\dfrac{25}{x^2(2x+1)} \equiv -\dfrac{50}{x} + \dfrac{25}{x^2} + \dfrac{100}{(2x+1)}\right\}$		**[4]**
(b)	$V = \pi \displaystyle\int_1^4 \left(\dfrac{5}{x\sqrt{(2x+1)}}\right)^2 \mathrm{d}x$	For $\pi \displaystyle\int \left(\dfrac{5}{x\sqrt{(2x+1)}}\right)^2$ Ignore limits and $\mathrm{d}x$. Can be implied.	B1
	For their partial fraction		
	$\left\{\displaystyle\int \dfrac{25}{x^2(2x+1)}\mathrm{d}x = \displaystyle\int -\dfrac{50}{x} + \dfrac{25}{x^2} + \dfrac{100}{(2x+1)}\mathrm{d}x\right\}$ $= -50\ln x + \dfrac{25x^{-1}}{(-1)} + \dfrac{100}{2}\ln(2x+1)\ \{+c\}$	Either $\pm\dfrac{A}{x} \to \pm a\ln x$ or $\pm a\ln kx$ or $\pm\dfrac{B}{x^2} \to \pm bx^{-1}$ or $\dfrac{C}{(2x+1)} \to \pm c\ln(2x+1)$ ***At least*** two terms correctly integrated ***All three*** terms correctly integrated.	M1 A1ft A1ft
	$\left\{\displaystyle\int_1^4 \dfrac{25}{x^2(2x+1)}\mathrm{d}x\right\} = \left[-50\ln x - \dfrac{25}{x} + 50\ln(2x+1)\right]_1^4$ $= \left(-50\ln 4 - \dfrac{25}{4} + 50\ln 9\right) - (0 - 25 + 50\ln 3)$ $= 50\ln 9 - 50\ln 4 - 50\ln 3 - \dfrac{25}{4} + 25$ $= 50\ln\left(\dfrac{3}{4}\right) + \dfrac{75}{4}$ So, $V = \dfrac{75}{4}\pi + 50\pi\ln\left(\dfrac{3}{4}\right)$ or allow $\pi\left(\dfrac{75}{4} + 50\ln\left(\dfrac{3}{4}\right)\right)$	Applies limits of 4 and 1 and subtracts the correct way round.	dM1 A1 oe
			[6] **10**

EXPERT TUITION

		Question 108 Notes
108. (a)		**BE CAREFUL!** Candidates will assign ***their own*** "A, B and C" for this question.
	B1	At least one of "B" or "C" are correct.
	B1	Breaks up their partial fraction correctly into three terms **and** both $"B" = 25$ and $"C" = 100$.
	Note	If a candidate does not give partial fraction decomposition then: • the 2nd B1 mark can follow from a correct identity.
	M1	Writes down ***a correct identity*** (although this can be implied) and attempts to find the value of either one of "A" or "B" or "C". This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously.
	A1	Correct value for "A" which is found using a correct identity and follows from their partial fraction decomposition.
	Note	If a candidate does not give partial fraction decomposition then the final A1 mark can be awarded for a correct "A" if a candidate writes out their partial fractions at the end.
	Note	The correct partial fraction from no working scores B1B1M1A1.
	Note	A number of candidates will start this problem by writing out the correct identity and then attempt to find "A" or "B" or "C". Therefore the B1 marks can be awarded from this method.
	Note	Award SC B1B0M0A0 for $\dfrac{25}{x^2(2x+1)} \equiv \dfrac{B}{x^2} + \dfrac{C}{(2x+1)}$ leading to $"B" = 25$ **or** $"C" = 100$
(b)	**B1**	For a correct statement of $\pi\displaystyle\int\left(\frac{5}{x\sqrt{(2x+1)}}\right)^2$ or $\pi\displaystyle\int\frac{25}{x^2(2x+1)}$. Ignore limits and d$x$. Can be implied.
	Note	**The π can only be recovered later from a correct expression.**
		For their partial fraction, (not $\sqrt{\text{their partial fraction}}$), where A, B, C are "their" part (a) constants
	M1	Either $\pm\dfrac{A}{x} \to \pm a\ln x$ or $\pm\dfrac{B}{x^2} \to \pm bx^{-1}$ or $\dfrac{C}{(2x+1)} \to \pm c\ln(2x+1)$.
	Note	$\sqrt{\dfrac{B}{x^2}} \to \dfrac{\sqrt{B}}{x}$ which integrates to $\sqrt{B}\ln x$ is **not** worthy of M1.
	A1ft	At least two terms from any of $\pm\dfrac{A}{x}$ or $\pm\dfrac{B}{x^2}$ or $\dfrac{C}{(2x+1)}$ correctly integrated. Can be un-simplified.
	A1ft	All 3 terms from $\pm\dfrac{A}{x}$, $\pm\dfrac{B}{x^2}$ and $\dfrac{C}{(2x+1)}$ correctly integrated. Can be un-simplified.
	Note	The 1st A1 and 2nd A1 marks in part (b) are both follow through accuracy marks.
	dM1	**Dependent on the previous M mark.** Applies limits of 4 and 1 and subtracts the correct way round.
	A1	Final correct **exact** answer in the form $a + b\ln c$. i.e. either $\dfrac{75}{4}\pi + 50\pi\ln\left(\dfrac{3}{4}\right)$ or $50\pi\ln\left(\dfrac{3}{4}\right) + \dfrac{75}{4}\pi$ or $50\pi\ln\left(\dfrac{9}{12}\right) + \dfrac{75}{4}\pi$ or $\dfrac{75}{4}\pi - 50\pi\ln\left(\dfrac{4}{3}\right)$ or $\dfrac{75}{4}\pi + 25\pi\ln\left(\dfrac{9}{16}\right)$ etc. Also allow $\pi\left(\dfrac{75}{4} + 50\ln\left(\dfrac{3}{4}\right)\right)$ or equivalent.
	Note	A candidate who achieves full marks in (a), but then mixes up the correct constants when writing their partial fraction can only achieve a maximum of B1M1A1A0M1A0 in part (b).
	Note	The π in the volume formula is only required for the B1 mark and the final A1 mark.

108. (b)	**Alternative method of integration**		
	$V = \pi \int_1^4 \left(\dfrac{5}{x\sqrt{(2x+1)}}\right)^2 \mathrm{d}x$	**B1**	For $\pi \int \left(\dfrac{5}{x\sqrt{(2x+1)}}\right)^2$ Ignore limits and $\mathrm{d}x$. Can be implied.
	$\int \dfrac{25}{x^2(2x+1)}\,\mathrm{d}x$; $u = \dfrac{1}{x} \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = -\dfrac{1}{x^2}$ $= \int \dfrac{-25}{\left(\frac{2}{u}+1\right)}\,\mathrm{d}u = \int \dfrac{-25}{\left(\frac{2+u}{u}\right)}\,\mathrm{d}u = \int \dfrac{-25u}{(2+u)}\,\mathrm{d}u = -25\int \dfrac{2+u-2}{(2+u)}\,\mathrm{d}u$		
	$= -25\int 1 - \dfrac{2}{(2+u)}\,\mathrm{d}u = -25\,(u - 2\ln(2+u))$	**M1**	Achieves $\pm\,\alpha \pm \dfrac{\beta}{(k+u)}$ **and** integrates to give either $\pm\,\alpha u$ or $\pm\,\beta\ln(k+u)$
		A1	**Dependent on the M mark.** Either $-25u$ or $50\ln(2+u)$
		A1	$-25\,(u - 2\ln(2+u))$
	$\left\{\int_1^4 \dfrac{25}{x^2(2x+1)}\,\mathrm{d}x = \left[-25u + 50\ln(2+u)\right]_1^{\frac{1}{4}}\right\}$ $= \left(-\dfrac{25}{4} + 50\ln\left(\dfrac{9}{4}\right)\right) - \left(-25 + 50\ln 3\right)$ $= 50\ln\left(\dfrac{9}{4}\right) - 50\ln 3 - \dfrac{25}{4} + 25$ $= 50\ln\left(\dfrac{3}{4}\right) + \dfrac{75}{4}$	**dM1**	Applies limits of $\frac{1}{4}$ and 1 in u or 4 and 1 in x in their integrated function and subtracts the correct way round.
	So, $V = \dfrac{75}{4}\pi + 50\pi\ln\left(\dfrac{3}{4}\right)$	**A1**	$\dfrac{75}{4}\pi + 50\pi\ln\left(\dfrac{3}{4}\right)$ or allow $\pi\left(\dfrac{75}{4} + 50\ln\left(\dfrac{3}{4}\right)\right)$

Question Number	Scheme		Marks
109.	$\dfrac{dN}{dt} = \dfrac{(kt-1)(5000-N)}{t}, \quad t > 0,\ 0 < N < 5000$		
(a)	$\displaystyle\int \frac{1}{5000-N}\,dN = \int \frac{(kt-1)}{t}\,dt \quad \left\{\text{or } = \int\left(k - \frac{1}{t}\right)dt\right\}$	See notes	B1
	$-\ln(5000-N) = kt - \ln t;\ +c$	See notes	M1 A1; A1
	then eg either... $-kt + c = \ln(5000-N) - \ln t$ $-kt + c = \ln\left(\dfrac{5000-N}{t}\right)$ $e^{-kt+c} = \dfrac{5000-N}{t}$ ***or...*** $kt + c = \ln t - \ln(5000-N)$ $kt + c = \ln\left(\dfrac{t}{5000-N}\right)$ $e^{kt+c} = \dfrac{t}{5000-N}$ ***or...*** $\ln(5000-N) = -kt + \ln t + c$ $5000 - N = e^{-kt+\ln t + c}$ $5000 - N = t\,e^{-kt+c}$		
	leading to $N = 5000 - Ate^{-kt}$ with **no incorrect working/statements. See notes**		A1 * **cso**
			[5]
(b)	$\{t = 1,\ N = 1200 \Rightarrow\}\quad 1200 = 5000 - Ae^{-k}$ $\{t = 2,\ N = 1800 \Rightarrow\}\quad 1800 = 5000 - 2Ae^{-2k}$	At least one correct statement written down using the boundary conditions	B1
	So $Ae^{-k} = 3800$ and $2Ae^{-2k} = 3200$ or $Ae^{-2k} = 1600$		
	Eg. $\dfrac{e^{-k}}{2e^{-2k}} = \dfrac{3800}{3200}$ or $\dfrac{2e^{-2k}}{e^{-k}} = \dfrac{3200}{3800}$ So $\dfrac{1}{2}e^{k} = \dfrac{3800}{3200}$ or $2e^{-k} = \dfrac{3200}{3800}$	An attempt to eliminate A by producing an equation in only k.	M1
	$k = \ln\left(\dfrac{7600}{3200}\right)$ or equivalent $\left\{\text{eg } k = \ln\left(\dfrac{19}{8}\right)\right\}$	At least one of $A = 9025$ cao or $k = \ln\left(\dfrac{7600}{3200}\right)$ or exact equivalent	A1
	$\left\{A = 3800(e^{k}) = 3800\left(\dfrac{19}{8}\right) \Rightarrow\right\}\ A = 9025$	Both $A = 9025$ cao or $k = \ln\left(\dfrac{7600}{3200}\right)$ or exact equivalent	A1
			[4]
	<u>**Alternative Method for the M1 mark in (b)**</u> $e^{-k} = \dfrac{3800}{A}$ $2A\left(\dfrac{3800}{A}\right)^2 = 3200$	An attempt to eliminate k by producing an equation in only A	M1
(c)	$\left\{t = 5,\ N = 5000 - 9025(5)e^{-5\ln\left(\frac{19}{8}\right)}\right\}$		
	$N = 4402.828401... = 4400$ (fish) (nearest 100)	anything that rounds to 4400	B1
			[1] **10**

		Question 109 Notes
109. (a)		
	B1	Separates variables as shown. dN and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	M1	**Either** $\pm\lambda\ln(5000-N)$ or $\pm\lambda\ln(N-5000)$ **or** $kt-\ln t$ where $\lambda\neq 0$ is a constant.
	A1	For $-\ln(5000-N)=kt-\ln t$ or $\ln(5000-N)=-kt+\ln t$ or $-\frac{1}{k}\ln(5000-N)=t-\frac{1}{k}\ln t$ oe
	A1	**which is dependent on the 1st M1 mark being awarded.** For applying a constant of integration, eg. $+c$ or $+\ln e^c$ or $+\ln c$ or A to their integrated equation
	Note	$+c$ can be on either side of their equation for the 2nd A1 mark.
	A1	Uses a constant of integration eg. "c" or "$\ln e^c$" "$\ln c$" or and applies a fully correct method to prove the result $N=5000-Ate^{-kt}$ with no incorrect working seen. (Correct solution only.)
	NOTE	**IMPORTANT** There needs to be an intermediate stage of justifying the A **and** the e^{-kt} in Ate^{-kt} by for example • **either** $5000-N=e^{\ln t-kt+c}$ • **or** $5000-N=te^{-kt+c}$ • **or** $5000-N=te^{-kt}e^c$ or equivalent needs to be stated before achieving $N=5000-Ate^{-kt}$
(b)	**B1**	At least one of **either** $1200=5000-Ae^{-k}$ (or equivalent) **or** $1800=5000-2Ae^{-2k}$ (or equivalent)
	M1	• **Either** an attempt to eliminate A by producing an equation in only k. • **or** an attempt to eliminate k by producing an equation in only A
	A1	At least one of $A=9025$ cao or $k=\ln\left(\frac{7600}{3200}\right)$ or equivalent
	A1	Both $A=9025$ cao or $k=\ln\left(\frac{7600}{3200}\right)$ or equivalent
	Note	Alternative correct values for k are $k=\ln\left(\frac{19}{8}\right)$ or $k=-\ln\left(\frac{8}{19}\right)$ or $k=\ln 7600-\ln 3200$ or $k=-\ln\left(\frac{3800}{9025}\right)$ or equivalent.
	Note	$k=0.8649...$ without a correct exact equivalent is A0.
(c)	**B1**	anything that rounds to 4400

Question Number	Scheme		Marks
110. (a)	$\int x^2 e^x \, dx$, 1st Application: $\left\{\begin{array}{ll} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array}\right\}$, 2nd Application: $\left\{\begin{array}{ll} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array}\right\}$		
	$= x^2 e^x - \int 2x e^x dx$	$x^2 e^x - \int \lambda x e^x \{dx\}$, $\lambda > 0$	M1
		$x^2 e^x - \int 2x e^x \{dx\}$	A1 oe
	$= x^2 e^x - 2\left(x e^x - \int e^x dx\right)$	**Either** $\pm Ax^2 e^x \pm Bxe^x \pm C\int e^x \{dx\}$ **or** for $\pm K\int xe^x \{dx\} \to \pm K\left(xe^x - \int e^x \{dx\}\right)$	M1
	$= x^2 e^x - 2(xe^x - e^x)$ $\{+c\}$	$\pm Ax^2 e^x \pm Bxe^x \pm C e^x$	M1
		Correct answer, with/without $+c$	A1
			[5]
(b)	$\left\{\left[x^2 e^x - 2(xe^x - e^x)\right]_0^1\right\}$ $= \left(1^2 e^1 - 2(1e^1 - e^1)\right) - \left(0^2 e^0 - 2(0e^0 - e^0)\right)$	Applies limits of 1 and 0 to an expression of the form $\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$, $A \neq 0$, $B \neq 0$ and $C \neq 0$ and subtracts the correct way round.	M1
	$= e - 2$	$e - 2$ **cso**	A1 oe
			[2] **7**

Notes for Question 110

(a)	**M1:** Integration by parts is applied in the form $x^2 e^x - \int \lambda x e^x \{dx\}$, where $\lambda > 0$. (must be in this form). **A1:** $x^2 e^x - \int 2xe^x \{dx\}$ or equivalent. **M1:** **Either** achieving a result in the form $\pm Ax^2 e^x \pm Bxe^x \pm C\int e^x \{dx\}$ (can be implied) (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) **or** for $\pm K\int xe^x \{dx\} \to \pm K\left(xe^x - \int e^x \{dx\}\right)$ **M1:** $\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$ (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) **A1:** $x^2 e^x - 2(xe^x - e^x)$ or $x^2 e^x - 2xe^x + 2e^x$ or $(x^2 - 2x + 2)e^x$ or equivalent with/without $+c$.
(b)	**M1:** **Complete method** of applying limits of 1 and 0 to their part (a) answer in the form $\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$, (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) and subtracting the correct way round. Evidence of a proper consideration of the limit of 0 (as detailed above) is needed for M1. So, just subtracting zero is M0. **A1:** $e - 2$ or $e^1 - 2$ or $-2 + e$. Do not allow $e - 2e^0$ unless simplified to give $e - 2$. **Note:** that 0.718... without seeing $e - 2$ or equivalent is A0. **WARNING:** Please note that this A1 mark is for correct solution only. So incorrect $[.......]_0^1$ leading to $e - 2$ is A0. **Note:** If their part (a) is correct candidates can get M1A1 in part (b) for $e - 2$ from no working. **Note:** 0.718... from no working is M0A0

Question Number	Scheme		Marks
111	$V = \pi \int_0^{\frac{\pi}{2}} \left(\sec\left(\frac{x}{2}\right)\right)^2 dx$	For $\pi \int \left(\sec\left(\frac{x}{2}\right)\right)^2$. Ignore limits and dx. Can be implied.	B1
		$\pm\lambda \tan\left(\frac{x}{2}\right)$	M1
	$= \{\pi\}\left[2\tan\left(\frac{x}{2}\right)\right]_0^{\frac{\pi}{2}}$	$2\tan\left(\frac{x}{2}\right)$ or equivalent	A1
	$= 2\pi$	2π	A1 **cao cso**
			[4]

Notes for Question 111

111

B1: For a correct statement of $\pi \int \left(\sec\left(\frac{x}{2}\right)\right)^2$ or $\pi \int \sec^2\left(\frac{x}{2}\right)$ or $\pi \int \frac{1}{\left(\cos\left(\frac{x}{2}\right)\right)^2} \{dx\}$.

Ignore limits and dx. Can be implied.

Note: Unless a correct expression stated $\pi \int \sec\left(\frac{x^2}{4}\right)$ would be B0.

M1: $\pm\lambda \tan\left(\frac{x}{2}\right)$ from any working.

A1: $2\tan\left(\frac{x}{2}\right)$ or $\frac{1}{\left(\frac{1}{2}\right)}\tan\left(\frac{x}{2}\right)$ from any working.

A1: 2π **from a correct solution only.**

Note: The π in the volume formula is only required for the B1 mark and the final A1 mark.
Note: Decimal answer of 6.283... without correct exact answer is A0.
Note: The B1 mark can be implied by later working – as long as it is clear that the candidate has applied $\pi \int y^2$ in their working.
Note: Writing the correct formula of $V = \pi \int y^2 \{dx\}$, but incorrectly applying it is B0.

Question Number	Scheme	Marks
112. (a)	$\{x = u^2 \Rightarrow\}\ \dfrac{dx}{du} = 2u$ or $\dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$	B1
	$\left\{\displaystyle\int \frac{1}{x(2\sqrt{x}-1)}\,dx\right\} = \displaystyle\int \frac{1}{u^2(2u-1)}\,2u\,du$	M1
	$= \displaystyle\int \frac{2}{u(2u-1)}\,du$	A1 * **cso**
		[3]
(b)	$\dfrac{2}{u(2u-1)} \equiv \dfrac{A}{u} + \dfrac{B}{(2u-1)} \Rightarrow 2 \equiv A(2u-1) + Bu$	
	$u = 0 \Rightarrow 2 = -A \Rightarrow A = -2$ See notes	M1 A1
	$u = \frac{1}{2} \Rightarrow 2 = \frac{1}{2}B \Rightarrow B = 4$	
	So $\displaystyle\int \frac{2}{u(2u-1)}\,du = \int \frac{-2}{u} + \frac{4}{(2u-1)}\,du$ Integrates $\dfrac{M}{u} + \dfrac{N}{(2u-1)}$, $M \neq 0$, $N \neq 0$ to obtain any one of $\pm\lambda \ln u$ or $\pm\mu\ln(2u-1)$	M1
	$= -2\ln u + 2\ln(2u-1)$ At least one term correctly followed through	A1 ft
	$-2\ln u + 2\ln(2u-1)$.	A1 **cao**
	So, $\left[-2\ln u + 2\ln(2u-1)\right]_1^3$	
	$= \left(-2\ln 3 + 2\ln(2(3)-1)\right) - \left(-2\ln 1 + 2\ln(2(1)-1)\right)$ Applies limits of 3 and 1 in u or 9 and 1 in x in their integrated function and subtracts the correct way round.	M1
	$= -2\ln 3 + 2\ln 5 - (0)$	
	$= 2\ln\left(\dfrac{5}{3}\right)$ $2\ln\left(\dfrac{5}{3}\right)$	A1 **cso cao**
		[7] **10**

Notes for Question 112

(a)	**B1:** $\dfrac{dx}{du} = 2u$ or $dx = 2u\,du$ or $\dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$ or $du = \dfrac{dx}{2\sqrt{x}}$
	M1: A full substitution producing an integral in u only (including the du) (Integral sign not necessary). The candidate needs to deal with the "x", the "$(2\sqrt{x}-1)$" and the "dx" and converts from an integral term in x to an integral in u. (Remember the integral sign is not necessary for M1).
	A1*: leading to the result printed on the question paper (including the du). (Integral sign is needed).
(b)	**M1:** Writing $\dfrac{2}{u(2u-1)} \equiv \dfrac{A}{u} + \dfrac{B}{(2u-1)}$ or writing $\dfrac{1}{u(2u-1)} \equiv \dfrac{P}{u} + \dfrac{Q}{(2u-1)}$ and a complete method for finding the value of at least one of their A or their B (or their P or their Q).
	A1: Both their $A = -2$ **and** their $B = 4$. (Or their $P = -1$ **and** their $Q = 2$ with the multiplying factor of 2 in front of the integral sign).
	M1: Integrates $\dfrac{M}{u} + \dfrac{N}{(2u-1)}$, $M \neq 0$, $N \neq 0$ (i.e. ***a two term partial fraction***) to obtain any one of $\pm\lambda\ln u$ or $\pm\mu\ln(2u-1)$ or $\pm\mu\ln\left(u - \frac{1}{2}\right)$
	A1ft: At least one term correctly followed through from their A or from their B (or their P and their Q).
	A1: $-2\ln u + 2\ln(2u-1)$

Notes for Question 112 Continued

112. (b) ctd	**M1:** Applies limits of 3 and 1 in u or 9 and 1 in x in their (i.e. any) changed function and subtracts the

	correct way round. **Note:** If a candidate just writes $\left(-2\ln 3 + 2\ln(2(3)-1)\right)$ oe , this is ok for M1. **A1:** $2\ln\left(\frac{5}{3}\right)$ **correct answer only.** (Note: $a = 5,\, b = 3$). **Important note: Award M0A0M1A1A0** for a candidate who writes $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = \int \frac{2}{u} + \frac{2}{(2u-1)}\,\mathrm{d}u = 2\ln u + \ln(2u-1)$ **AS EVIDENCE OF WRITING** $\frac{2}{u(2u-1)}$ **AS PARTIAL FRACTIONS IS GIVEN.** **Important note: Award M0A0M0A0A0** for a candidate who writes down either $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = 2\ln u + 2\ln(2u-1)$ **or** $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = 2\ln u + \ln(2u-1)$ **WITHOUT ANY EVIDENCE OF WRITING** $\frac{2}{u(2u-1)}$ as partial fractions. **Important note: Award M1A1M1A1A1** for a candidate who writes down $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = -2\ln u + 2\ln(2u-1)$ **WITHOUT ANY EVIDENCE OF WRITING** $\frac{2}{u(2u-1)}$ as partial fractions. **Note:** In part (b) if they lose the "2" and find $\int \frac{1}{u(2u-1)}\,\mathrm{d}u$ we can allow a maximum of M1A0 M1A1ftA0 M1A0.

Question Number	Scheme		Marks
113.	$\frac{d\theta}{dt} = \lambda(120-\theta), \quad \theta \leqslant 100$		
(a)	$\int \frac{1}{120-\theta}\, d\theta = \int \lambda\, dt$ or $\int \frac{1}{\lambda(120-\theta)}\, d\theta = \int dt$		B1
	$-\ln(120-\theta); = \lambda t + c$ or $-\frac{1}{\lambda}\ln(120-\theta); = t + c$	See notes	M1 A1; M1 A1
	$\{t=0,\ \theta=20 \Rightarrow\}\ -\ln(120-20) = \lambda(0) + c$	See notes	M1
	$c = -\ln 100 \Rightarrow -\ln(120-\theta) = \lambda t - \ln 100$		
	then either... $-\lambda t = \ln(120-\theta) - \ln 100$; $-\lambda t = \ln\left(\frac{120-\theta}{100}\right)$; $e^{-\lambda t} = \frac{120-\theta}{100}$ ***or...*** $\lambda t = \ln 100 - \ln(120-\theta)$; $\lambda t = \ln\left(\frac{100}{120-\theta}\right)$; $e^{\lambda t} = \frac{100}{120-\theta}$		dddM1
	either: $100e^{-\lambda t} = 120-\theta$ ***or:*** $(120-\theta)e^{\lambda t} = 100 \Rightarrow 120-\theta = 100e^{-\lambda t}$ leading to $\theta = 120 - 100e^{-\lambda t}$		A1 *
			[8]
(b)	$\{\lambda = 0.01,\ \theta = 100 \Rightarrow\}\quad 100 = 120 - 100e^{-0.01t}$		M1
	$\Rightarrow 100e^{-0.01t} = 120 - 100 \Rightarrow -0.01t = \ln\left(\frac{120-100}{100}\right)$ $t = \frac{1}{-0.01}\ln\left(\frac{120-100}{100}\right)$ $\left\{t = \frac{1}{-0.01}\ln\left(\frac{1}{5}\right) = 100\ln 5\right\}$	Uses correct order of operations by moving from $100 = 120 - 100e^{-0.01t}$ to give $t = ...$ and $t = A\ln B$, where $B > 0$	dM1
	$t = 160.94379... = 161$ (s) (nearest second)	awrt 161	A1
			[3]
			11

Notes for Question 113

<table>
<tr><td>(a)</td><td>

B1: Separates variables as shown. $\mathrm{d}\theta$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.

Either / ***or***

M1: $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \to \pm A\ln(120-\theta)$ | or $\int \frac{1}{\lambda(120-\theta)}\,\mathrm{d}\theta \to \pm A\ln(120-\theta)$, A is a constant.

A1: $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \to -\ln(120-\theta)$ | or $\int \frac{1}{\lambda(120-\theta)}\,\mathrm{d}\theta \to -\frac{1}{\lambda}\ln(120-\theta)$ or $-\frac{1}{\lambda}\ln(120\lambda-\lambda\theta)$,

M1: $\int \lambda\,\mathrm{d}t \to \lambda t$ | or $\int 1\,\mathrm{d}t \to t$

A1: $\int \lambda\,\mathrm{d}t \to \lambda t + c$ | or $\int 1\,\mathrm{d}t \to t + c$ The $+c$ can appear on either side of the equation.

IMPORTANT: $+c$ can be on either side of their equation for the 2nd A1 mark.

M1: Substitutes $t = 0$ AND $\theta = 20$ in an integrated or changed equation containing c (or A or $\ln A$).

Note that this mark can be implied by the correct value of c. { Note that $-\ln 100 = -4.60517...$ }.

dddM1: Uses their value of c which must be a $\ln$ term, and uses fully correct method to eliminate their logarithms. **Note:** This mark is dependent on all three previous method marks being awarded.

A1*: This is a given answer. All previous marks must have been scored and there must not be any errors in the candidate's working. Do not accept huge leaps in working at the end. So a minimum of either:

(1): $\mathrm{e}^{-\lambda t} = \frac{120-\theta}{100} \Rightarrow 100\mathrm{e}^{-\lambda t} = 120-\theta \Rightarrow \theta = 120 - 100\mathrm{e}^{-\lambda t}$

or (2): $\mathrm{e}^{\lambda t} = \frac{100}{120-\theta} \Rightarrow (120-\theta)\mathrm{e}^{\lambda t} = 100 \Rightarrow 120-\theta = 100\mathrm{e}^{-\lambda t} \Rightarrow \theta = 120 - 100\mathrm{e}^{-\lambda t}$

is required for A1.

Note: $\int \frac{1}{(120\lambda-\lambda\theta)}\,\mathrm{d}\theta \to -\frac{1}{\lambda}\ln(120\lambda-\lambda\theta)$ is ok for the first M1A1 in part (a).

</td></tr>
<tr><td>(b)</td><td>

M1: Substitutes $\lambda = 0.01$ and $\theta = 100$ into the printed equation or one of their earlier equations connecting θ and t. This mark can be implied by subsequent working.

dM1: Candidate uses correct order of operations by moving from $100 = 120 - 100\mathrm{e}^{-0.01t}$ to $t = ...$

Note: that the 2nd Method mark is dependent on the 1st Method mark being awarded in part (b).

A1: awrt 161 or "awrt" 2 minutes 41 seconds. (Ignore incorrect units).

</td></tr>
</table>

***Aliter* 113. (a) Way 2**	$\int \frac{1}{120-\theta}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t$		B1
	$-\ln(120-\theta) = \lambda t + c$	See notes	M1 A1; M1 A1
	$-\ln(120-\theta) = \lambda t + c$		
	$\ln(120-\theta) = -\lambda t + c$		
	$120-\theta = A\mathrm{e}^{-\lambda t}$		
	$\theta = 120 - A\mathrm{e}^{-\lambda t}$		
	$\{t = 0, \theta = 20 \Rightarrow\}\ 20 = 120 - A\mathrm{e}^{0}$		M1
	$A = 120 - 20 = 100$		
	So, $\theta = 120 - 100\mathrm{e}^{-\lambda t}$		dddM1 A1 *
			[8]

Notes for Question 113 Continued		
(a)	**B1M1A1M1A1:** Mark as in the original scheme. **M1:** Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be c or A. **Note** that this mark can be implied by the correct value of c or A. **dddM1:** Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration. **Note:** This mark is dependent on all three previous method marks being awarded. **Note:** $\ln(120-\theta) = -\lambda t + c$ leading to $120-\theta = e^{-\lambda t} + e^{c}$ or $120-\theta = e^{-\lambda t} + A$, would be dddM0. **A1*:** Same as the original scheme. **Note:** The jump from $\ln(120-\theta) = -\lambda t + c$ to $120-\theta = Ae^{-\lambda t}$ ***with no incorrect working*** is condoned in part (a).	
Aliter 113. (a) Way 3	$\int \frac{1}{120-\theta}\,d\theta = \int \lambda\,dt \quad \left\{\Rightarrow \int \frac{-1}{\theta-120}\,d\theta = \int \lambda\,dt\right\}$	B1
	$-\ln\lvert\theta-120\rvert = \lambda t + c$ *Modulus required for 1st A1.*	M1 **A1**
	$\{t=0,\ \theta=20 \Rightarrow\}\ -\ln\lvert 20-120\rvert = \lambda(0) + c$ *Modulus not required here!*	M1 A1 M1
	$\Rightarrow c = -\ln 100 \Rightarrow -\ln\lvert\theta-120\rvert = \lambda t - \ln 100$	
	then either... $-\lambda t = \ln\lvert\theta-120\rvert - \ln 100$ $-\lambda t = \ln\left\lvert\frac{\theta-120}{100}\right\rvert$ ***or...*** $\lambda t = \ln 100 - \ln\lvert\theta-120\rvert$ $\lambda t = \ln\left\lvert\frac{100}{\theta-120}\right\rvert$ As $\theta \leqslant 100$ *either:* $-\lambda t = \ln\left(\frac{120-\theta}{100}\right)$, $e^{-\lambda t} = \frac{120-\theta}{100}$ *or:* $\lambda t = \ln\left(\frac{100}{120-\theta}\right)$, $e^{\lambda t} = \frac{100}{120-\theta}$ *Understanding of modulus is required here!*	dddM1
	either: $100e^{-\lambda t} = 120-\theta$ *or:* $(120-\theta)e^{\lambda t} = 100 \Rightarrow 120-\theta = 100e^{-\lambda t}$ leading to $\theta = 120 - 100e^{-\lambda t}$	**A1 ***
		[8]
	B1: Mark as in the original scheme. **M1:** Mark as in the original scheme ignoring the modulus. **A1:** $\int \frac{1}{120-\theta}\,d\theta \to -\ln\lvert\theta-120\rvert$. ***(The modulus is required here).*** **M1A1:** Mark as in the original scheme. **M1:** Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be c or A. Mark as in the original scheme ignoring the modulus. **dddM1:** Mark as in the original scheme **AND** the candidate must demonstrate that they have converted $\ln\lvert\theta-120\rvert$ to $\ln(120-\theta)$ in their working. **Note:** This mark is dependent on all three previous method marks being awarded. **A1:** Mark as in the original scheme.	

Notes for Question 113 Continued		
Aliter 113. (a) Way 4	***Use of an integrating factor***	
	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \lambda(120 - \theta) \Rightarrow \frac{\mathrm{d}\theta}{\mathrm{d}t} + \lambda\theta = 120\lambda$	
	$\text{IF} = \mathrm{e}^{\lambda t}$	B1
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{\lambda t}\theta\right) = 120\lambda\mathrm{e}^{\lambda t}$,	M1A1
	$\mathrm{e}^{\lambda t}\theta = 120\lambda\mathrm{e}^{\lambda t} + k$	M1A1
	$\theta = 120 + K\mathrm{e}^{-\lambda t}$	M1
	$\{t = 0, \theta = 20 \Rightarrow\} -100 = K$	
	$\theta = 120 - 100\mathrm{e}^{-\lambda t}$	M1A1

Question Number	Scheme		Marks
114.	$\int_0^4 \frac{1}{2+\sqrt{(2x+1)}}\,\mathrm{d}x\ ,\ \ u = 2+\sqrt{(2x+1)}$		
	$\frac{\mathrm{d}u}{\mathrm{d}x} = (2x+1)^{-\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = u-2$	Either $\frac{\mathrm{d}u}{\mathrm{d}x} = \pm K(2x+1)^{-\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = \pm\lambda(u-2)$	M1
		Either $\frac{\mathrm{d}u}{\mathrm{d}x} = (2x+1)^{-\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = (u-2)$	A1
	$\left\{\int \frac{1}{2+\sqrt{(2x+1)}}\,\mathrm{d}x\right\} = \int \frac{1}{u}(u-2)\,\mathrm{d}u$	Correct substitution (Ignore integral sign and $\mathrm{d}u$).	A1
	$= \int\left(1-\frac{2}{u}\right)\mathrm{d}u$	An attempt to divide each term by u.	dM1
		$\pm Au \pm B\ln u$	ddM1
	$= u - 2\ln u$	$u - 2\ln u$	A1 ft
	$\left\{\text{So } \left[u-2\ln u\right]_3^5\right\} = (5-2\ln 5)-(3-2\ln 3)$	Applies limits of 5 and 3 in u or 4 and 0 in x in their integrated function and subtracts the correct way round.	M1
	$= 2+2\ln\left(\frac{3}{5}\right)$	$2+2\ln\left(\frac{3}{5}\right)$	A1 **cao cso**
			[8] **8**

Notes for Question 114

M1: Also allow $\mathrm{d}u = \pm\lambda\frac{1}{(u-2)}\mathrm{d}x$ or $(u-2)\mathrm{d}u = \pm\lambda\,\mathrm{d}x$

Note: The expressions must contain $\mathrm{d}u$ and $\mathrm{d}x$. They can be simplified or un-simplified.

A1: Also allow $\mathrm{d}u = \frac{1}{(u-2)}\mathrm{d}x$ or $(u-2)\mathrm{d}u = \pm\lambda\,\mathrm{d}x$

Note: The expressions must contain $\mathrm{d}u$ and $\mathrm{d}x$. They can be simplified or un-simplified.

A1: $\int \frac{1}{u}(u-2)\,\mathrm{d}u$. (Ignore integral sign and $\mathrm{d}u$).

dM1: An attempt to divide each term by u.
Note that this mark is dependent on the previous M1 mark being awarded.
Note that this mark can be implied by later working.

ddM1: $\pm Au \pm B\ln u$, $A \neq 0$, $B \neq 0$
Note that this mark is dependent on the two previous M1 marks being awarded.

A1ft: $u - 2\ln u$ or $\pm Au \pm B\ln u$ being correctly followed through, $A \neq 0$, $B \neq 0$

M1: Applies limits of 5 and 3 in u or 4 and 0 in x in their integrated function and subtracts the correct way round.

A1: **cso** and **cao**. $2+2\ln\left(\frac{3}{5}\right)$ or $2+2\ln(0.6)$, $\left(= A+2\ln B\text{, so } A=2,\ B=\frac{3}{5}\right)$

Note: $2-2\ln\left(\frac{3}{5}\right)$ is A0.

Notes for Question 114 Continued	
114. ctd	**Note:** $\int \frac{1}{u}(u-2)\,\mathrm{d}u = u - 2\ln u$ with no working is 2nd M1, 3rd M1, 3rd A1.
	but **Note:** $\int \frac{1}{u}(u-2)\,\mathrm{d}u = (u-2)\ln u$ with no working is 2nd M0, 3rd M0, 3rd A0.

Question Number	Scheme		Marks
115. (a)	6.248046798... = 6.248 (3dp)	6.248 or awrt 6.248	B1
			[1]
(b)	Area $\approx \frac{1}{2}\times 2\,;\times\underline{\left[3+2\left(7.107+7.218+\text{their } 6.248\right)+5.223\right]}$		B1; <u>M1</u>
	$= 49.369 = 49.37\,(2\text{ dp})$	49.37 or awrt 49.37	A1
			[3]
(c)	$\left\{\int (4t\mathrm{e}^{-\frac{1}{3}t}+3)\,\mathrm{d}t\right\} = -12t\mathrm{e}^{-\frac{1}{3}t} - \int -12\mathrm{e}^{-\frac{1}{3}t}\,\{\mathrm{d}t\}$	$\pm At\mathrm{e}^{-\frac{1}{3}t} \pm B\int \mathrm{e}^{-\frac{1}{3}t}\{\mathrm{d}t\}$, $A\neq 0,\ B\neq 0$	M1
		See notes.	A1
	$+\,3t$	$3 \to 3t$	B1
	$= -12t\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t}\ \{+\,3t\}$	$-12t\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t}$	A1
	$\left[-12t\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t} + 3t\right]_0^8 =$		
	$= \left(-12(8)\mathrm{e}^{-\frac{1}{3}(8)} - 36\mathrm{e}^{-\frac{1}{3}(8)} + 3(8)\right) - \left(-12(0)\mathrm{e}^{-\frac{1}{3}(0)} - 36\mathrm{e}^{-\frac{1}{3}(0)} + 3(0)\right)$	Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm\lambda t\mathrm{e}^{-\frac{1}{3}t} \pm \mu\mathrm{e}^{-\frac{1}{3}t}$ or $\pm\lambda t\mathrm{e}^{-\frac{1}{3}t} \pm \mu\mathrm{e}^{-\frac{1}{3}t} + Bt$ and subtracts the correct way round.	dM1
	$= \left(-96\mathrm{e}^{-\frac{8}{3}} - 36\mathrm{e}^{-\frac{8}{3}} + 24\right) - \left(0 - 36 + 0\right)$		
	$= 60 - 132\mathrm{e}^{-\frac{8}{3}}$	$60 - 132\mathrm{e}^{-\frac{8}{3}}$	A1
			[6]
(d)	Difference $= \left\lvert 60 - 132\mathrm{e}^{-\frac{8}{3}} - 49.37\right\rvert = 1.458184439... = 1.46\ (2\text{ dp})$	1.46 or awrt 1.46	B1
			[1]
			11

Notes for Question 115

(a)	**B1:** 6.248 or awrt 6.248. Look for this on the table or in the candidate's working.
(b)	**B1**: Outside brackets $\frac{1}{2}\times 2$ or 1
	M1: <u>For structure of trapezium rule</u>$[\ \underline{............}\ ]$. Allow one miscopy of their values.
	A1: 49.37 or anything that rounds to 49.37
	Note: It can be possible to award : (a) B0 (b) B1M1A1 (awrt 49.37)
	<u>**Note:**</u> Working must be seen to demonstrate the use of the trapezium rule. <u>**Note**</u>: actual area is 50.828…
	<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly,** Award B1M0A0 for $1+3+2\left(7.107+7.218+\text{their } 6.248\right)+5.223$ (nb: answer of 50.369).

Notes for Question 115 Continued	
115. (b) ctd	<u>*Alternative method for part (b): Adding individual trapezia*</u> $\text{Area} \approx 2\times\left[\frac{3+7.107}{2}+\frac{7.107+7.218}{2}+\frac{7.218+6.248}{2}+\frac{6.248+5.223}{2}\right] = 49.369$ **B1:** 2 and a divisor of 2 on all terms inside brackets. **M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2. **A1:** anything that rounds to 49.37
(c)	**M1:** For $4te^{-\frac{1}{3}t} \to \pm Ate^{-\frac{1}{3}t} \pm B\int e^{-\frac{1}{3}t}\{\mathrm{d}t\}$, $A\neq 0,\ B\neq 0$ **A1:** For $te^{-\frac{1}{3}t} \to \left(-3te^{-\frac{1}{3}t} - \int -3e^{-\frac{1}{3}t}\right)$ (some candidates lose the 4 and this is fine for the first A1 mark). or $4te^{-\frac{1}{3}t} \to 4\left(-3te^{-\frac{1}{3}t} - \int -3e^{-\frac{1}{3}t}\right)$ or $-12te^{-\frac{1}{3}t} - \int -12e^{-\frac{1}{3}t}$ or $12\left(-te^{-\frac{1}{3}t} - \int -e^{-\frac{1}{3}t}\right)$ These results can be implied. They can be simplified or un-simplified. **B1:** $3 \to 3t$ or $3 \to 3x$ (bod) . **Note:** Award B0 for 3 integrating to $12t$ (implied), which is a common error when taking out a factor of 4. **Be careful** some candidates will factorise out 4 and have $4\left(....+\frac{3}{4}\right) \to 4\left(....+\frac{3}{4}t\right)$ which would then be fine for B1. **Note:** Allow B1 for $\int_0^8 3\mathrm{d}t = 24$ **A1:** For correct integration of $4te^{-\frac{1}{3}t}$ to give $-12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t}$ or $4\left(-3te^{-\frac{1}{3}t} - 9e^{-\frac{1}{3}t}\right)$ or equivalent. This can be simplified or un-simplified. **dM1:** Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm\lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t}$ or $\pm\lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t} + Bt$ and subtracts the correct way round. **Note:** Evidence of a proper consideration of the limit of 0 (as detailed in the scheme) is needed for dM1. So, just subtracting zero is M0. **A1:** An exact answer of $60 - 132e^{-\frac{8}{3}}$. A decimal answer of 50.82818444... without a correct answer is A0. **Note:** A decimal answer of 50.82818444... without a correct exact answer is A0. **Note:** If a candidate gains M1A1B1A1 and then writes down 50.8 or awrt 50.8 with no method for substituting limits of 8 and 0, then award the final M1A0. **IMPORTANT:** that is fine for candidates to work in terms of x rather than t in part (c). **Note:** The "$3t$" is needed for B1 and the final A1 mark.
(d)	**B1:** 1.46 or awrt 1.46 or -1.46 or awrt -1.46. Candidates may give correct decimal answers of 1.458184439... or 1.459184439... **Note:** You can award this mark whether or not the candidate has answered part (c) correctly.

Question Number	Scheme		Marks
116.	$x = 27\sec^3 t, \quad y = 3\tan t, \quad 0 \leqslant t \leqslant \frac{\pi}{3}$		
(a)	$\frac{dx}{dt} = 81\sec^2 t \sec t \tan t, \quad \frac{dy}{dt} = 3\sec^2 t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	$\frac{dy}{dx} = \frac{3\sec^2 t}{81\sec^3 t \tan t} \left\{ = \frac{1}{27\sec t \tan t} = \frac{\cos t}{27\tan t} = \frac{\cos^2 t}{27\sin t} \right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	M1;
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{3\sec^2\left(\frac{\pi}{6}\right)}{81\sec^3\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{6}\right)} = \frac{4}{72} \left\{ = \frac{3}{54} = \frac{1}{18} \right\}$	$\frac{4}{72}$	A1 **cao cso**
			[4]
(b)	$\left\{1 + \tan^2 t = \sec^2 t\right\} \Rightarrow 1 + \left(\frac{y}{3}\right)^2 = \left(\sqrt[3]{\left(\frac{x}{27}\right)}\right)^2 = \left(\frac{x}{27}\right)^{\frac{2}{3}}$		M1
	$\Rightarrow 1 + \frac{y^2}{9} = \frac{x^{\frac{2}{3}}}{9} \Rightarrow 9 + y^2 = x^{\frac{2}{3}} \Rightarrow y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ *		A1 * **cso**
	$a = 27$ and $b = 216$ or $27 \leqslant x \leqslant 216$	$a = 27$ and $b = 216$	B1
			[3]
(c)	$V = \pi \int_{27}^{125} \left(\left(x^{\frac{2}{3}} - 9 \right)^{\frac{1}{2}} \right)^2 dx$ or $\pi \int_{27}^{125} \left(x^{\frac{2}{3}} - 9 \right) dx$	For $\pi \int \left(\left(x^{\frac{2}{3}} - 9 \right)^{\frac{1}{2}} \right)^2$ or $\pi \int \left(x^{\frac{2}{3}} - 9 \right)$ Ignore limits and dx. Can be implied.	B1
	$= \{\pi\} \left[\frac{3}{5}x^{\frac{5}{3}} - 9x \right]_{27}^{125}$	Either $\pm Ax^{\frac{5}{3}} \pm Bx$ or $\frac{3}{5}x^{\frac{5}{3}}$ oe	M1
		$\frac{3}{5}x^{\frac{5}{3}} - 9x$ oe	A1
	$= \{\pi\} \left(\left(\frac{3}{5}(125)^{\frac{5}{3}} - 9(125) \right) - \left(\frac{3}{5}(27)^{\frac{5}{3}} - 9(27) \right) \right)$	Substitutes limits of 125 and 27 into an integrated function and subtracts the correct way round.	dM1
	$= \{\pi\}\left((1875 - 1125) - (145.8 - 243)\right)$		
	$= \frac{4236\pi}{5}$ or 847.2π	$\frac{4236\pi}{5}$ or 847.2π	A1
			[5] **12**

	Notes for Question 116
(a)	**B1:** At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. **Note:** that this mark can be implied from their working.
	B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. **Note:** that this mark can be implied from their working.
	M1: Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$, where both $\frac{dy}{dt}$ and $\frac{dx}{dt}$ are trigonometric functions of t.
	A1: $\frac{4}{72}$ or any equivalent correct rational answer not involving surds.
	Allow $0.0\dot{5}$ with the recurring symbol.

	Notes for Question 116 Continued
	Note: Please check that their $\frac{dx}{dt}$ is differentiated correctly. Eg. Note that $x = 27\sec^3 t = 27(\cos t)^{-3} \Rightarrow \frac{dx}{dt} = -81(\cos t)^{-2}(-\sin t)$ is correct.
(b)	**M1:** Either: • Applying a correct trigonometric identity (usually $1 + \tan^2 t = \sec^2 t$) to give a Cartesian equation in x and y only. • Starting from the RHS and goes on to achieve $\sqrt{9\tan^2 t}$ by using a correct trigonometric identity. • Starts from the LHS and goes on to achieve $\sqrt{9\sec^2 t - 9}$ by using a correct trigonometric identity. **A1*:** For a correct proof of $y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$. **Note** this result is printed on the Question Paper, so no incorrect working is allowed. **B1:** Both $a = 27$ and $b = 216$. **Note** that $27 \leqslant x \leqslant 216$ is also fine for B1.
(c)	**B1:** For a correct statement of $\pi\int\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2$ or $\pi\int\left(x^{\frac{2}{3}} - 9\right)$. Ignore limits and dx. Can be implied. **M1:** Either integrates to give $\pm Ax^{\frac{5}{3}} \pm Bx$, $A \neq 0$, $B \neq 0$ or integrates $x^{\frac{2}{3}}$ correctly to give $\frac{3}{5}x^{\frac{5}{3}}$ oe **A1:** $\frac{3}{5}x^{\frac{5}{3}} - 9x$ or. $\frac{x^{\frac{5}{3}}}{\left(\frac{5}{3}\right)} - 9x$ oe. **dM1:** Substitutes limits of 125 and 27 into an integrated function and subtracts the correct way round. **Note:** that this mark is dependent upon the previous method mark being awarded. **A1:** A correct exact answer of $\frac{4236\pi}{5}$ or 847.2π. **Note:** The π in the volume formula is only required for the B1 mark and the final A1 mark. **Note:** A decimal answer of 2661.557... without a correct exact answer is A0. **Note:** If a candidate gains the first B1M1A1 and then writes down 2661 or awrt 2662 with no method for substituting limits of 125 and 27, then award the final M1A0.
(a) **Way 2**	***<u>Alternative response using the Cartesian equation in part (a)</u>***

Working	Notes	Marks
$\left\{y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} \Rightarrow\right\} \frac{dy}{dx} = \frac{1}{2}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}x^{-\frac{1}{3}}\right)$	$\frac{dy}{dx} = \pm K x^{-\frac{1}{3}}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}$	M1
	$\frac{dy}{dx} = \frac{1}{2}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}x^{-\frac{1}{3}}\right)$ oe	A1
At $t = \frac{\pi}{6}$, $x = 27\sec^3\left(\frac{\pi}{6}\right) = 24\sqrt{3}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\left(24\sqrt{3}\right)^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}\left(24\sqrt{3}\right)^{-\frac{1}{3}}\right)$	Uses $t = \frac{\pi}{6}$ to find x and substitutes their x into an expression for $\frac{dy}{dx}$.	dM1
So, $\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{3\sqrt{3}}\right) = \frac{1}{18}$	$\frac{1}{18}$	A1 **cao cso**

Note: Way 2 is marked as M1 A1 dM1 A1
Note: For way 2 the second M1 mark is dependent on the first M1 being gained.

Notes for Question 116 Continued			
116. (b) **Way 2**	***Alternative responses for M1A1 in part (b)*: STARTING FROM THE RHS**		
	$\{\text{RHS} =\}\left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}} = \sqrt{\left(27\sec^3 t\right)^{\frac{2}{3}}-9} = \sqrt{9\sec^2 t-9} = \sqrt{9\tan^2 t}$	For applying $1+\tan^2 t = \sec^2 t$ oe to achieve $\sqrt{9\tan^2 t}$	M1
	$=3\tan t = y\ \{= \text{LHS}\}$ **cso**	Correct proof from $\left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}}$ to y.	A1*
	M1: Starts from the RHS and goes on to achieve $\sqrt{9\tan^2 t}$ by using a correct trigonometric identity.		
116. (b) **Way 3**	***Alternative responses for M1A1 in part (b)*: STARTING FROM THE LHS**		
	$\{\text{LHS} =\}\ y = 3\tan t = \sqrt{\left(9\tan^2 t\right)} = \sqrt{9\sec^2 t-9}$	For applying $1+\tan^2 t = \sec^2 t$ oe to achieve $\sqrt{9\sec^2 t-9}$	M1
	$= \sqrt{9\left(\dfrac{x}{27}\right)^{\frac{2}{3}}-9} = \sqrt{9\left(\dfrac{x^{\frac{2}{3}}}{9}\right)-9} = \left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}}$ **cso**	Correct proof from y to $\left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}}$.	A1*
	M1: Starts from the LHS and goes on to achieve $\sqrt{9\sec^2 t-9}$ by using a correct trigonometric identity.		
116. (c) **Way 2**	***Alternative response for part (c) using parametric integration***		
	$V = \pi\int 9\tan^2 t\left(81\sec^2 t\sec t\tan t\right)\,dt$	$\pi\int 3\tan t\left(81\sec^2 t\sec t\tan t\right)\,dt$ Ignore limits and dx. Can be implied.	B1
	$= \{\pi\}\int 729\sec^2 t\tan^2 t\sec t\tan t\ dt$		
	$= \{\pi\}\int 729\sec^2 t\left(\sec^2 t-1\right)\sec t\tan t\ dt$		
	$= \{\pi\}\int 729\left(\sec^4 t-\sec^2 t\right)\sec t\tan t\ dt$		
	$= \{\pi\}\int 729\left(\sec^4 t-\sec^2 t\right)\sec t\tan t\ dt$		
	$= \{\pi\}\left[729\left(\frac{1}{5}\sec^5 t-\frac{1}{3}\sec^3 t\right)\right]$	$\pm A\sec^5 t \pm B\sec^3 t$	M1
		$729\left(\frac{1}{5}\sec^5 t-\frac{1}{3}\sec^3 t\right)$	A1
	$V = \{\pi\}\left[729\left(\frac{1}{5}\left(\frac{5}{3}\right)^5-\frac{1}{3}\left(\frac{5}{3}\right)^3\right) - 729\left(\frac{1}{5}1^5-\frac{1}{3}1^3\right)\right]$	Substitutes $\sec t = \frac{5}{3}$ and $\sec t = 1$ into an integrated function and subtracts the correct way round.	dM1
	$= 729\pi\left[\left(\frac{250}{243}\right)-\left(-\frac{2}{15}\right)\right]$		
	$= \dfrac{4236\pi}{5}$ or 847.2π	$\dfrac{4236\pi}{5}$ or 847.2π	A1
			[5]

EXPERT TUITION

Question Number	Scheme	Marks
117.	$\dfrac{dx}{dt} = k(M-x)$, where M is a constant	
(a)	$\dfrac{dx}{dt}$ is the rate of increase of the mass of waste products. Any one correct explanation.	B1
	M is the total mass of unburned fuel and waste fuel (or the initial mass of unburned fuel) Both explanations are correct.	B1
		[2]
(b)	$\displaystyle\int \frac{1}{M-x}\,dx = \int k\,dt$ or $\displaystyle\int \frac{1}{k(M-x)}\,dx = \int dt$	B1
	$-\ln(M-x) = kt \; \{+c\}$ or $-\frac{1}{k}\ln(M-x) = t \; \{+c\}$ See notes	M1 A1
	$\{t=0, x=0 \Rightarrow\}\; -\ln(M-0) = k(0)+c$ See notes	M1
	$c = -\ln M \Rightarrow -\ln(M-x) = kt - \ln M$	
	then either... $-kt = \ln(M-x) - \ln M$; $-kt = \ln\left(\dfrac{M-x}{M}\right)$; $e^{-kt} = \dfrac{M-x}{M}$ ***or...*** $kt = \ln M - \ln(M-x)$; $kt = \ln\left(\dfrac{M}{M-x}\right)$; $e^{kt} = \dfrac{M}{M-x}$	ddM1
	$Me^{-kt} = M-x$ ***or*** $(M-x)e^{kt} = M$; $M-x = Me^{-kt}$	
	leading to $x = M - Me^{-kt}$ or $x = M(1-e^{-kt})$ oe	A1 * **cso**
		[6]
(c)	$\left\{x = \frac{1}{2}M,\; t = \ln 4 \Rightarrow\right\}\; \frac{1}{2}M = M(1-e^{-k\ln 4})$	M1
	$\Rightarrow \frac{1}{2} = 1 - e^{-k\ln 4} \Rightarrow e^{-k\ln 4} = \frac{1}{2} \Rightarrow -k\ln 4 = -\ln 2$	
	So $k = \frac{1}{2}$	A1
	$x = M\left(1 - e^{-\frac{1}{2}\ln 9}\right)$	dM1
	$x = \frac{2}{3}M$ $\quad x = \frac{2}{3}M$	A1 **cso**
		[4]
		12

Notes for Question 117 Continued	
117. (a)	**B1:** At least one explanation correct. **B1:** Both explanations are correct. $\frac{dx}{dt}$ is the rate of increase of the mass of waste products. or the rate of change of the mass of waste products. M is the total mass of unburned fuel and waste fuel or the initial mass of unburned fuel or the total mass of rocket fuel and waste fuel or the initial mass of rocket fuel or the initial mass of fuel or the total mass of waste and unburned products.
(b)	**B1:** Separates variables as shown. dx and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. **M1: Both** $\pm\lambda\ln(M-x)$ or $\pm\lambda\ln(x-M)$ **and** $\pm\mu t$ where λ and μ are any constants. **A1:** For $-\ln(M-x)=kt$ or $-\ln(x-M)=kt$ or $-\frac{1}{k}\ln(M-x)=t$ or $-\frac{1}{k}\ln(x-M)=t$ or $-\frac{1}{k}\ln(kM-kx)=t$ or $-\frac{1}{k}\ln(kx-kM)=t$ **Note:** $+c$ is not needed for this mark. **IMPORTANT:** $+c$ can be on either side of their equation for the 1st A1 mark. **M1:** Substitutes $t=0$ AND $x=0$ in an integrated or changed equation containing c (or A or $\ln A$, etc.) **Note** that this mark can be implied by the correct value of c. **ddM1:** Uses their value of c which must be a $\ln$ term, and uses fully correct method to eliminate their logarithms. **Note:** This mark is dependent on both previous method marks being awarded. **A1:** $x=M-Me^{-kt}$ or $x=M(1-e^{-kt})$ or $x=\frac{M(e^{kt}-1)}{e^{kt}}$ or equivalent where x is the subject. **Note:** Please check their working as incorrect working can lead to a correct answer. **Note:** $\left\{\frac{dx}{dt}=k(M-x)\Rightarrow\frac{dx}{dt}=\frac{1}{kM-kx}\Rightarrow\right\}$ $x=-\frac{1}{k}\ln(kM-kx)\{+c\}$ is B1(Implied) M1A1.
(c)	**M1:** Substitutes $x=\frac{1}{2}M$ and $t=\ln 4$ into one of their earlier equations connecting x and t. **A1:** $k=\frac{1}{2}$, which can be an un-simplified equivalent numerical value. i.e. $k=\frac{\ln 2}{\ln 4}$ is fine for A1. **dM1:** Substitutes $t=\ln 4$ and their evaluated k (which must be a numerical value) into one of their earlier equations connecting x and t. **Note:** that the 2nd Method mark is dependent on the 1st Method mark being awarded in part (c). **A1:** $x=\frac{2}{3}M$ **cso**. **Note:** Please check their working as incorrect working can lead to a correct answer.

<table>
<tr><th colspan="3">Notes for Question 117 Continued</th></tr>
<tr><td>Aliter
117. (b)
Way 2</td><td>$\int \frac{1}{M-x}\,\mathrm{d}x = \int k\,\mathrm{d}t$
$-\ln(M-x) = kt\ \{+c\}$ See notes
$\ln(M-x) = -kt + c$
$M - x = A\mathrm{e}^{-kt}$
$\{t=0,\ x=0 \Rightarrow\}\ M - 0 = A\mathrm{e}^{-k(0)}$
$\Rightarrow M = A$
$M - x = M\mathrm{e}^{-kt}$
So, $x = M - M\mathrm{e}^{-kt}$</td><td>B1
M1 A1

M1

ddM1
A1
[6]</td></tr>
<tr><td>(b)</td><td colspan="2">B1M1A1: Mark as in the original scheme.
M1: Substitutes $t=0$ AND $x=0$ in an integrated equation containing their constant of integration which could be c or A. Note that this mark can be implied by the correct value of c or A.
ddM1: Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration.
Note: This mark is dependent on both previous method marks being awarded.
Note: $\ln(M-x) = -kt + c$ leading to $\ln(M-x) = \mathrm{e}^{-kt} + \mathrm{e}^{c}$ or $\ln(M-x) = \mathrm{e}^{-kt} + A$ would be dddM0.
A1: Same as the original scheme.</td></tr>
<tr><td>Aliter
117. (b)
Way 3</td><td>$\int_0^x \frac{1}{M-x}\,\mathrm{d}x = \int_0^t k\,\mathrm{d}t$
$\left[-\ln(M-x)\right]_0^x = \left[kt\right]_0^t$
$-\ln(M-x) - (-\ln M) = kt$ Applies limits of
$-\ln(M-x) + \ln M = kt$
and then follows the original scheme.</td><td>B1
M1 A1
M1</td></tr>
<tr><td>(a)</td><td colspan="2">B1M1A1: Mark as in the original scheme (ignoring the limits).
ddM1: Applies limits 0 and x on their integrated LHS and limits of 0 and t.
M1A1: Same as the original scheme.</td></tr>
</table>

Notes for Question 117 Continued		
Aliter 117. (b) Way 4	$\int \frac{1}{M-x}\,dx = \int k\,dt \quad \left\{\Rightarrow \int \frac{-1}{x-M}\,dx = \int k\,dt\right\}$	B1
	$-\ln\lvert x-M\rvert = kt + c$ *Modulus not required for 1st A1.*	M1 A1
	$\{t=0,\ x=0 \Rightarrow\}\ -\ln\lvert 0-M\rvert = k(0)+c$ *Modulus not required here!*	M1
	$\Rightarrow c = -\ln M \Rightarrow -\ln\lvert x-M\rvert = kt - \ln M$	
	then either... $-kt = \ln\lvert x-M\rvert - \ln M$; $-kt = \ln\left\lvert\frac{x-M}{M}\right\rvert$ ***or...*** $kt = \ln M - \ln\lvert x-M\rvert$; $kt = \ln\left\lvert\frac{M}{x-M}\right\rvert$	
	As $x < M$	
	$-kt = \ln\left(\frac{M-x}{M}\right)$; $e^{-kt} = \frac{M-x}{M}$ **or** $kt = \ln\left(\frac{M}{M-x}\right)$; $e^{kt} = \frac{M}{M-x}$ *Understanding of modulus is required here!*	**ddM1**
	$Me^{-kt} = M - x$ **or** $(M-x)e^{kt} = M$; $M - x = Me^{-kt}$	
	leading to $x = M - Me^{-kt}$ or $x = M(1-e^{-kt})$ oe	**A1 * cso**
		[6]
	B1: Mark as in the original scheme. **M1A1M1:** Mark as in the original scheme ignoring the modulus. **ddM1:** Mark as in the original scheme **AND** the candidate must demonstrate that they have converted $\ln\lvert x-M\rvert$ to $\ln(M-x)$ in their working. **Note:** This mark is dependent on both the previous method marks being awarded. **A1:** Mark as in the original scheme.	
Aliter 117. (b) Way 5	***Use of an integrating factor*** **(I.F.)**	
	$\frac{dx}{dt} = k(M-x) \Rightarrow \frac{dx}{dt} + kx = kM$	
	I.F. $= e^{kt}$	B1
	$\frac{d}{dt}\left(e^{kt}x\right) = kMe^{kt}$,	
	$e^{kt}x = Me^{kt} + c$	M1A1
	$x = M + ce^{-kt}$	
	$\{t=0,\ x=0 \Rightarrow\}\ 0 = M + ce^{-k(0)}$	M1
	$\Rightarrow c = -M$	
	$x = M - Me^{-kt}$	ddM1A1

Question Number	Scheme		Marks
118(i)	$\int x\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x \Rightarrow \left\{ \begin{array}{ll} u = x & \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = 1 \\ \dfrac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^{-\frac{1}{2}x} & \Rightarrow v = -2\mathrm{e}^{-\frac{1}{2}x} \end{array} \right\}$		
	$\int x\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x = -2x\mathrm{e}^{-\frac{1}{2}x} - \int -2\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$	Use of 'integration by parts' formula in the correct direction.	M1*
		Correct expression.	A1 aef
	$= -2x\mathrm{e}^{-\frac{1}{2}x} - 4\mathrm{e}^{-\frac{1}{2}x} + c$	$\pm\lambda x\mathrm{e}^{-\frac{1}{2}x} \pm \mu\mathrm{e}^{-\frac{1}{2}x}\ (+c)$	M1
		Correct answer with/without $+c$	A1
(ii)	$\int_0^4 x\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x = \left[-2x\mathrm{e}^{-\frac{1}{2}x} - 4\mathrm{e}^{-\frac{1}{2}x}\right]_0^4$		
	$= \left(-2(4)\mathrm{e}^{-\frac{1}{2}(4)} - 4\mathrm{e}^{-\frac{1}{2}(4)}\right) - \left(-2(0)\mathrm{e}^{-\frac{1}{2}(0)} - 4\mathrm{e}^{-\frac{1}{2}(0)}\right)$	Substitutes limits of 4 and 0 and subtracts the correct way round.	dM1*
	$= \left(-8\mathrm{e}^{-2} - 4\mathrm{e}^{-2}\right) - \left(0 - 4\right)$		
	$= 4 - 12\mathrm{e}^{-2}$	$\underline{a = 4}$, $\underline{b = -12}$ or $4 - 12\mathrm{e}^{-2}$	A1
			[6]

	Notes on Question 118
118(ii)	**dM1: Complete method** of applying limits of 4 and 0 and subtracting the correct way round. Evidence of a proper consideration of the limit of 0 is needed for M1. So, just subtracting zero is M0.

Question Number	Scheme		Marks
119. (i)(a)	$\dfrac{7x}{(x+3)(2x-1)} = \dfrac{A}{(x+3)} + \dfrac{B}{(2x-1)}$		
	$7x \equiv A(2x-1) + B(x+3)$	Forms the correct identity.	B1
	When $x = -3,\ A = 3.$	Substitutes either $x = -3$ or $x = \frac{1}{2}$	
	When $x = \dfrac{1}{2},\ B = 1.$	into their identity and correctly finds one of either A or B.	M1
	Hence, $\left\{\dfrac{7x}{(x+3)(2x-1)}\right\} = \dfrac{3}{(x+3)} + \dfrac{1}{(2x-1)}$	Correct partial fraction.	A1
			[3]
(b)	$\displaystyle\int \dfrac{7x}{(x+3)(2x-1)}\,dx = \int \dfrac{3}{(x+3)} + \dfrac{1}{(2x-1)}\,dx$		
		Either $\pm a\ln(x+3)$ or $\pm b\ln(2x-1)$	M1
	$= 3\ln(x+3) + \dfrac{1}{2}\ln(2x-1) + c$	At least one ln term correct	A1 ft
		Correct integration with $+c$	A1
			[3]
(ii)	$\displaystyle\int \dfrac{1}{x + x^{\frac{1}{3}}}\,dx,\quad u^3 = x$		
	$3u^2\dfrac{du}{dx} = 1$	$3u^2\dfrac{du}{dx} = 1$ or $\dfrac{dx}{du} = 3u^2$ or $\dfrac{du}{dx} = \dfrac{1}{3}x^{-\frac{2}{3}}$	B1 oe
	$\displaystyle = \int \dfrac{1}{u^3 + u}\,.\,3u^2\,du$	Attempt to substitute $u^3 = x$ and $3u^2\dfrac{du}{dx} = 1$ or $\dfrac{du}{dx} = \dfrac{1}{3}x^{-\frac{2}{3}}$ to give an expression to be integrated which is in terms of u only.	M1
	$\displaystyle = \int \dfrac{3u}{u^2+1}\,du$	$\displaystyle\int \dfrac{3u}{u^2+1}\,du$	A1
	$= \dfrac{3}{2}\ln\left(u^2+1\right) + c$	$\pm\lambda\ln\left(u^2+1\right)$	M1
	$= \dfrac{3}{2}\ln\left(x^{\frac{2}{3}}+1\right) + c$	Correct answer in x with or without $+c$.	A1
			[5] **11**
	Notes on Question 119		
(ii)	**Note:** 1st M1 can be implied by $\displaystyle\int \dfrac{1}{u^3+u}\,.\,3u^2$ if the du is missing.		

Question Number	Scheme		Marks
120. (a)	$x = \tan\theta, \quad y = 1 + 2\cos 2\theta\,, \quad , \quad 0 \leqslant \theta < \frac{\pi}{2}$		
		attempt at $V = \underline{\pi}\int \underline{y^2}\,\mathrm{d}x$	M1
	$V = \underline{\pi}\int \underline{(1+2\cos 2\theta)^2}.\sec^2\theta\,\{\mathrm{d}\theta\}$	Correct expression ignoring limits and π.	B1
	$V = (\pi)\int (1+2(2\cos^2\theta - 1))^2\sec^2\theta\,\{\mathrm{d}\theta\}$	Using either $\cos 2\theta = 2\cos^2\theta - 1$ or $\cos 2\theta = 1 - 2\sin^2\theta$ or $\cos 2\theta = \cos^2\theta - \sin^2\theta$	M1
	$V = (\pi)\int (4\cos^2\theta - 1)^2\sec^2\theta\,\{\mathrm{d}\theta\}$		
	$V = (\pi)\int (16\cos^4\theta - 8\cos^2\theta + 1)\sec^2\theta\,\{\mathrm{d}\theta\}$		
	$V = \pi\int (16\cos^2\theta - 8 + \sec^2\theta)\{\mathrm{d}\theta\}$	Manipulates to give the final answer where $k = \pi$	A1 *
	change limits: when $x = 1 \Rightarrow 1 = \tan\theta \Rightarrow \theta = \frac{\pi}{4}$ and when $x = \sqrt{3} \Rightarrow \sqrt{3} = \tan\theta \Rightarrow \theta = \frac{\pi}{3}$	Evidence of changing both limits.	B1
			[5]
(b)	$(\pi)\int 16\left(\dfrac{1+\cos 2\theta}{2}\right) - 8 + \sec^2\theta\,\mathrm{d}\theta$	Using the identity $\cos 2\theta = 2\cos^2\theta - 1$ to substitute for $\cos^2\theta$.	M1
	$= (\pi)\int 8 + 8\cos 2\theta - 8 + \sec^2\theta\,\mathrm{d}\theta$		
	$= (\pi)\int 8\cos 2\theta + \sec^2\theta\,\mathrm{d}\theta$		
	$= (\pi)\left(\dfrac{8\sin 2\theta}{2} + \tan\theta\right)$	Either $\pm 4\sin 2\theta$ or $\tan\theta$	M1
		$\dfrac{8\sin 2\theta}{2} + \tan\theta$	A1
	So, $V = (\pi)\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(8\cos 2\theta + \sec^2\theta\right)\mathrm{d}\theta = (\pi)\left[\dfrac{8\sin 2\theta}{2} + \tan\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$		
	$= (\pi)\left[\left(\dfrac{4\sqrt{3}}{2} + \sqrt{3}\right) - (4+1)\right]$	Substitutes limits of $\dfrac{\pi}{3}$ and $\dfrac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \left(3\sqrt{3} - 5\right)\pi$	$\left(3\sqrt{3} - 5\right)\pi$	A1
			[5] **10**
	Notes on Question 120		
(a)	**Note:** The use of $\int y\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\{\mathrm{d}\theta\}$ (i.e. an expression for area and not volume) is the 1st M0, 1st B0. **Note:** For the 1st B1, the correct expression of $\int \underline{(1+2\cos 2\theta)^2}.\sec^2\theta$ must be stated on one line. **Note:** Award 2nd M0 for applying $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$ to give an expression in terms of $\cos 2\theta$. **Note:** The π in the volume formula is only required for the 1st M1 mark and the A1 mark.		

Question Number	Scheme		Marks
121. (a)	$\dfrac{dV}{dt} = -32\pi\sqrt{h}$		
	$V = \pi(40)^2 h \ \{= 1600\pi h\}$	$V = \pi(40)^2 h$	B1
	$\dfrac{dV}{dh} = 1600\pi$	$\dfrac{dV}{dh} = 1600\pi$	B1ft
	$\dfrac{dh}{dt} = \dfrac{dh}{dV} \times \dfrac{dV}{dt}$		
	$\dfrac{dh}{dt} = \dfrac{1}{1600\pi} \times -32\pi\sqrt{h}$	$\dfrac{dh}{dt} = \left(\pm 32\pi\sqrt{h}\right) \div \left(\text{their } \dfrac{dV}{dh}\right)$	M1
	So, $\dfrac{dh}{dt} = -0.02\sqrt{h}$	Correct proof.	A1 * cso
			[4]
(b)	$\displaystyle\int \frac{dh}{\sqrt{h}} = \int -0.02\,dt$	Attempt to separate variables. Integral signs not necessary.	B1
	$\Rightarrow \displaystyle\int h^{-\frac{1}{2}}\,dh = \int -0.02\,dt$		
	$\Rightarrow \dfrac{h^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = -0.02t\ (+c)$	Separates variables and integrates to give $\pm\alpha h^{\frac{1}{2}} = \pm\beta t\ (+c)$	M1
		Correct integration with/without $+c$	A1
	$t = 0,\ h = 100 \Rightarrow 2\sqrt{100} = -0.02(0) + c \Rightarrow c = 20$ $h = 50 \Rightarrow 2\sqrt{50} = -0.02t + 20$	Uses boundary conditions for t and h to find c. Then uses h with found c to form an equation in order to find t.	M1
	So, $0.02t = 20 - 2\sqrt{50}$ $\Rightarrow\ t = 1000 - 500\sqrt{2} = 292.8932188...$ $\Rightarrow\ t = 293$ (minutes) (nearest minute)	awrt 293	A1 cso
			[5] **9**
	Notes on Question 121		
(a) (b)	**Note:** Use of $V = \pi r^2 h$ is 1st B0 until $r = 40$ is substituted. **Note:** Award final A0 for dividing by 60 after achieving $t = 292.8932188...$ **Note:** The final A1 mark is for correct solution only. If the candidate makes a sign error then award final A0.		

	Notes on Question 121 continued	
(a)	**Alternative Method for part (a)**	
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\pi 40^2 h\right) = -32\pi\sqrt{h}$	**B1B1:** $\frac{\mathrm{d}}{\mathrm{d}t}\left(\pi 40^2 h\right) = -32\pi\sqrt{h}$
	$\Rightarrow \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{-32\pi\sqrt{h}}{\pi 40^2}$	**M1:** Simplifies to give an expression for $\frac{\mathrm{d}h}{\mathrm{d}t}$.
	So, $\frac{\mathrm{d}h}{\mathrm{d}t} = -0.02\sqrt{h}$ *	**A1:** Correct proof.
(b)	**Alternative Method for part (b)**	
	$\int_{100}^{50} \frac{\mathrm{d}h}{\sqrt{h}} = \int_0^T -0.02\,\mathrm{d}t$	**B1:** Attempt to separate variables. Integral signs and limits not necessary.
	$\Rightarrow \int_{100}^{50} h^{-\frac{1}{2}}\,\mathrm{d}h = \int_0^T -0.02\,\mathrm{d}t$	
	$\Rightarrow \left[\frac{h^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_{100}^{50} = \left[-0.02t\right]_0^T$	**M1:** $\pm\alpha h^{\frac{1}{2}} = \pm\beta t\ (+c)$ **A1:** Correct integration with/without limits
	$2\sqrt{50} - 2\sqrt{100} = -0.02T$	**M1:** Attempts to use limits in order to find T.
	So, $0.02T = 20 - 2\sqrt{50}$	
	$\Rightarrow T = 1000 - 500\sqrt{2} = 292.8932188...$	
	$\Rightarrow T = 293$ (minutes) (nearest minute)	**A1:** A correct solution (with a correct application of limits) leading to awrt 293.

Question Number	Scheme	
122. (a)	$\int \frac{1}{x^3}\ln x \, dx$, $\left\{\begin{array}{l} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^{-3} \Rightarrow v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{array}\right\}$	
	In the form $\frac{\pm\lambda}{x^2}\ln x \pm \int \mu \frac{1}{x^2}.\frac{1}{x}$	M1
	$= \underline{\frac{-1}{2x^2}\ln x} - \underline{\underline{\int \frac{-1}{2x^2}.\frac{1}{x}\, dx}}$ $\underline{\frac{-1}{2x^2}\ln x}$ simplified or un-simplified.	<u>A1</u>
	$\underline{\underline{-\int \frac{-1}{2x^2}.\frac{1}{x}}}$ simplified or un-simplified.	$\underline{\underline{\text{A1}}}$
	$\left\{= \frac{-1}{2x^2}\ln x + \frac{1}{2}\int \frac{1}{x^3}\, dx\right\}$	
	$= -\frac{1}{2x^2}\ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)\{+c\}$ $\pm\int \mu \frac{1}{x^2}.\frac{1}{x} \rightarrow \pm\beta x^{-2}$.	dM1
	Correct answer, with/without $+c$	A1
		[5]
(b)	$\left\{\left[-\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\right]_1^2\right\} = \left(-\frac{1}{2(2)^2}\ln 2 - \frac{1}{4(2)^2}\right) - \left(-\frac{1}{2(1)^2}\ln 1 - \frac{1}{4(1)^2}\right)$ Applies limits of 2 and 1 to their part (a) answer and subtracts the correct way round.	M1
	$= \frac{3}{16} - \frac{1}{8}\ln 2$ or $\frac{3}{16} - \ln 2^{\frac{1}{8}}$ or $\frac{1}{16}(3 - 2\ln 2)$, etc, or awrt 0.1 or equivalent.	A1
		[2] **7**

(a)

M1: Integration by parts is applied in the form $\frac{\pm\lambda}{x^2}\ln x \pm \int \mu \frac{1}{x^2}.\frac{1}{x}$ or equivalent.

<u>**A1**</u>: $\underline{\frac{-1}{2x^2}\ln x}$ simplified or un-simplified.

$\underline{\underline{\textbf{A1}}}$: $\underline{\underline{-\int \frac{-1}{2x^2}.\frac{1}{x}}}$ or equivalent. You can ignore the dx.

dM1: Depends on the previous M1. $\pm\int \mu \frac{1}{x^2}.\frac{1}{x} \rightarrow \pm\beta x^{-2}$.

A1: $-\frac{1}{2x^2}\ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)\{+c\}$ or $= -\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\{+c\}$ or $\frac{x^{-2}}{-2}\ln x - \frac{x^{-2}}{4}\{+c\}$

or $\frac{-1-2\ln x}{4x^2}\{+c\}$ or equivalent.

You can ignore subsequent working after a correct stated answer.

(b)

M1: Some evidence of applying limits of 2 and 1 to their part (a) answer and subtracts the correct way round.

A1: ***Two term exact answer*** of either $\frac{3}{16} - \frac{1}{8}\ln 2$ or $\frac{3}{16} - \ln 2^{\frac{1}{8}}$ or $\frac{1}{16}(3 - 2\ln 2)$ or $\frac{\ln\left(\frac{1}{4}\right) + 3}{16}$

or $0.1875 - 0.125\ln 2$. Also allow awrt 0.1. Also note the fraction terms must be combined.

Note: Award the final A0 in part (b) for a candidate who achieves awrt 0.1 in part (b), when their answer to part (a) is incorrect.

122. (b) ctd	**Note:** Decimal answer is 0.100856... in part (b).	
	Alternative Solution	
	$\int \frac{1}{x^3}\ln x\ \mathrm{d}x$, $\left\{\begin{array}{lll} u = x^{-3} & \Rightarrow & \frac{\mathrm{d}u}{\mathrm{d}x} = -3x^{-4} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = \ln x & \Rightarrow & v = x\ln x - x \end{array}\right\}$	
	$\int \frac{1}{x^3}\ln x\ \mathrm{d}x = \frac{1}{x^3}(x\ln x - x) - \int (x\ln x - x)\frac{-3}{x^4}\mathrm{d}x$	
	$k\int \frac{1}{x^3}\ln x\ \mathrm{d}x = \frac{1}{x^3}(x\ln x - x) \pm \int \frac{\lambda}{x^3}\mathrm{d}x$ where $k \neq 1$	M1
	$-2\int \frac{1}{x^3}\ln x\ \mathrm{d}x = \frac{1}{x^3}(x\ln x - x) - \int \frac{3}{x^3}\mathrm{d}x$ Any one of $\frac{1}{x^3}(x\ln x - x)$ or $-\int \frac{3}{x^3}\mathrm{d}x$	A1
	$\frac{1}{x^3}(x\ln x - x) - \int \frac{3}{x^3}\mathrm{d}x$ and $k = -2$	A1
	$-2\int \frac{1}{x^3}\ln x\ \mathrm{d}x = \frac{1}{x^3}(x\ln x - x) + \frac{3}{2x^2}\ \{+c\}$ $\pm\int \mu\frac{1}{x^3} \rightarrow \pm\beta x^{-2}$.	dM1
	$\int \frac{1}{x^3}\ln x\ \mathrm{d}x = -\frac{1}{2x^3}(x\ln x - x) - \frac{3}{4x^2}\ \{+c\}$ $-\frac{1}{2x^3}(x\ln x - x) - \frac{3}{4x^2}$ or equivalent with/without $+c$.	A1
	$= -\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\ \{+c\}$	

Question Number	Scheme		Marks
123. (a)	1.0981		B1 **cao** **[1]**
(b)	Area $\approx \frac{1}{2}\times 1\,;\times\underline{\left[0.5+2\left(0.8284+\text{their }1.0981\right)+1.3333\right]}$		B1; $\underline{\text{M1}}$
	$=\frac{1}{2}\times 5.6863 = 2.84315 = 2.843$ (3 dp)	2.843 or awrt 2.843	A1 **[3]**
(c)	$\left\{u=1+\sqrt{x}\right\} \Rightarrow \underline{\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}x^{-\frac{1}{2}}}$ or $\underline{\frac{\mathrm{d}x}{\mathrm{d}u}=2(u-1)}$		$\underline{\text{B1}}$
	$\left\{\int \frac{x}{1+\sqrt{x}}\,\mathrm{d}x=\right\}\int \frac{(u-1)^2}{u}.\,2(u-1)\,\mathrm{d}u$	$\int \frac{(u-1)^2}{u}......$	M1
		$\int \frac{(u-1)^2}{u}.\,2(u-1)$	A1
	$=2\int \frac{(u-1)^3}{u}\,\mathrm{d}u \;=\{2\}\int \frac{(u^3-3u^2+3u-1)}{u}\,\mathrm{d}u$	Expands to give a "four term" cubic in u. Eg: $\pm Au^3 \pm Bu^2 \pm Cu \pm D$	M1
	$=\{2\}\int \left(u^2-3u+3-\frac{1}{u}\right)\mathrm{d}u$	An attempt to divide at least three terms in ***their cubic*** by u. See notes.	M1
	$=\{2\}\left(\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right)$	$\int \frac{(u-1)^3}{u} \rightarrow \left(\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right)$	A1
	$\text{Area}(R) = \left[\frac{2u^3}{3}-3u^2+6u-2\ln u\right]_2^3$		
	$=\left(\frac{2(3)^3}{3}-3(3)^2+6(3)-2\ln 3\right)-\left(\frac{2(2)^3}{3}-3(2)^2+6(2)-2\ln 2\right)$	Applies limits of 3 and 2 in u or 4 and 1 in x and subtracts either way round.	M1
	$=\frac{11}{3}+2\ln 2-2\ln 3$ or $\frac{11}{3}+2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3}-\ln\left(\frac{9}{4}\right)$, etc	Correct exact answer or equivalent.	A1
			[8] **12**

(a)	**B1:** 1.0981 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1**: Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$
	M1: <u>For structure of trapezium rule</u>$\left[\;\underline{............}\;\right]$
	A1: anything that rounds to 2.843
	<u>**Note:**</u> Working must be seen to demonstrate the use of the trapezium rule. <u>**Note**</u>: actual area is 2.85573645…
	<u>Note:</u> Award B1M1 A1 for $\frac{1}{2}(0.5+1.3333)+\left(0.8284+\text{their }1.0981\right)=2.84315$
	<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly**
	Award B1M0A0 for $\frac{1}{2}\times 1\;+0.5+2\left(0.8284+\text{their }1.0981\right)+1.3333$ (nb: answer of 6.1863).
	Award B1M0A0 for $\frac{1}{2}\times 1\;(0.5+1.3333)+2\left(0.8284+\text{their }1.0981\right)$ (nb: answer of 4.76965).

123. (b) ctd

<u>*Alternative method for part (b): Adding individual trapezia*</u>

$$\text{Area} \approx 1\times\left[\frac{0.5+0.8284}{2}+\frac{0.8284+1.0981}{2}+\frac{1.0981+1.3333}{2}\right] = 2.84315$$

B1: 1 and a divisor of 2 on all terms inside brackets.
M1: First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
A1: anything that rounds to 2.843

(c)

B1: $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}x^{-\frac{1}{2}}$ or $\mathrm{d}u=\frac{1}{2\sqrt{x}}\,\mathrm{d}x$ or $2\sqrt{x}\,\mathrm{d}u=\mathrm{d}x$ or $\mathrm{d}x=2(u-1)\mathrm{d}u$ or $\frac{\mathrm{d}x}{\mathrm{d}u}=2(u-1)$ oe.

1st M1: $\frac{x}{1+\sqrt{x}}$ becoming $\frac{(u-1)^2}{u}$ (Ignore integral sign).

1st A1 (B1 on epen): $\frac{x}{1+\sqrt{x}}\,\mathrm{d}x$ becoming $\frac{(u-1)^2}{u}.\,2(u-1)\{\mathrm{d}u\}$ or $\frac{(u-1)^2}{u}.\frac{2}{(u-1)^{-1}}\{\mathrm{d}u\}$.

You can ignore the integral sign and the $\mathrm{d}u$.

2nd M1: Expands to give a "four term" cubic in u, $\pm Au^3 \pm Bu^2 \pm Cu \pm D$
where $A\neq 0$, $B\neq 0$, $C\neq 0$ and $D\neq 0$ The cubic does not need to be simplified for this mark.

3rd M1: An attempt to divide at least three terms in ***their cubic*** by u.

Ie. $\frac{(u^3-3u^2+3u-1)}{u}\to u^2-3u+3-\frac{1}{u}$

2nd A1: $\int\frac{(u-1)^3}{u}\,\mathrm{d}u \to \left(\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right)$

4th M1: Some evidence of limits of 3 and 2 in u and subtracting either way round.

3rd A1: Exact answer of $\frac{11}{3}+2\ln 2-2\ln 3$ or $\frac{11}{3}+2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3}-\ln\left(\frac{9}{4}\right)$ or $2\left(\frac{11}{6}+\ln 2-\ln 3\right)$

or $\frac{22}{6}+2\ln\left(\frac{2}{3}\right)$, etc. **Note**: that fractions must be combined to give either $\frac{11}{3}$ or $\frac{22}{6}$ or $3\frac{2}{3}$

<u>***Alternative method for 2nd M1 and 3rd M1 mark***</u>

$$\{2\}\int\frac{(u-1)^2}{u}.\,(u-1)\,\mathrm{d}u = \{2\}\int\frac{(u^2-2u+1)}{u}.\,(u-1)\,\mathrm{d}u$$

$$= \{2\}\int\left(u-2+\frac{1}{u}\right).\,(u-1)\,\mathrm{d}u = \{2\}\int\left(u^2-\ldots\right)\mathrm{d}u$$

An attempt to expand $(u-1)^2$, then divide the result by u and then go on to multiply by $(u-1)$. **2nd M1**

$$= \{2\}\int\left(u^2-2u+1-u+2-\frac{1}{u}\right)\mathrm{d}u$$

to give three out of four of $\pm Au^2$, $\pm Bu$, $\pm C$ or $\pm\frac{D}{u}$ **3rd M1**

$$= \{2\}\int\left(u^2-3u+3-\frac{1}{u}\right)\mathrm{d}u$$

123. (c) ctd

Final two marks in part (c): $u = 1 + \sqrt{x}$

$$\text{Area}(R) = \left[\frac{2\left(1+\sqrt{x}\right)^3}{3} - 3\left(1+\sqrt{x}\right)^2 + 6\left(1+\sqrt{x}\right) - 2\ln\left(1+\sqrt{x}\right)\right]_1^4$$

$$= \left(\frac{2\left(1+\sqrt{4}\right)^3}{3} - 3\left(1+\sqrt{4}\right)^2 + 6\left(1+\sqrt{4}\right) - 2\ln\left(1+\sqrt{4}\right)\right)$$

$$- \left(\frac{2\left(1+\sqrt{1}\right)^3}{3} - 3\left(1+\sqrt{1}\right)^2 + 6\left(1+\sqrt{1}\right) - 2\ln\left(1+\sqrt{1}\right)\right)$$

M1: Applies limits of 4 and 1 in x and subtracts either way round.

$$= \left(18 - 27 + 18 - 2\ln 3\right) - \left(\frac{16}{3} - 12 + 12 - 2\ln 2\right)$$

$$= \frac{11}{3} + 2\ln 2 - 2\ln 3 \quad \text{or} \quad \frac{11}{3} + 2\ln\left(\frac{2}{3}\right) \quad \text{or} \quad \frac{11}{3} - \ln\left(\frac{9}{4}\right), \text{ etc}$$

A1: Correct exact answer or equivalent.

Alternative method for the final 5 marks in part (b)

$$\int \frac{(u-1)^3}{u}\,\mathrm{d}u\,, \qquad \left\{\begin{array}{lll} \text{"}u\text{"} = u^{-1} & \Rightarrow & \dfrac{\mathrm{d}\text{"}u\text{"}}{\mathrm{d}x} = -u^{-2} \\ \dfrac{\mathrm{d}v}{\mathrm{d}x} = (u-1)^3 & \Rightarrow & v = \dfrac{(u-1)^4}{4} \end{array}\right\}$$

$$= \frac{(u-1)^4}{4u} - -\frac{1}{4}\int \frac{(u-1)^4}{u^2}\,\mathrm{d}u$$

$$= \frac{(u-1)^4}{4u} + \frac{1}{4}\int \frac{u^4 - 4u^3 + 6u^2 - 4u + 1}{u^2}\,\mathrm{d}u$$

M1: Applies integration by parts and expands to give a five term quartic.

$$= \frac{(u-1)^4}{4u} + \frac{1}{4}\int u^2 - 4u + 6 - \frac{4}{u} + \frac{1}{u^2}\,\mathrm{d}u$$

M1: Dividing at least 4 terms.

$$= \frac{(u-1)^4}{4u} + \frac{1}{4}\left(\frac{u^3}{3} - 2u^2 + 6u - 4\ln u - \frac{1}{u}\right)$$

A1: Correct Integration.

$$\int_2^3 \frac{(u-1)^3}{u}\,\mathrm{d}u = \left[\frac{(u-1)^4}{4u} + \frac{u^3}{12} - \frac{u^2}{2} + \frac{3u}{2} - \ln u - \frac{1}{4u}\right]_2^3$$

$$= \left(\frac{16}{12} + \frac{27}{12} - \frac{9}{2} + \frac{9}{2} - \ln 3 - \frac{1}{12}\right) - \left(\frac{1}{8} + \frac{8}{12} - \frac{4}{2} + \frac{6}{2} - \ln 2 - \frac{1}{8}\right)$$

M1

$$= \left(7 - \ln 3\right) - \left(\frac{5}{3} - \ln 2\right)$$

$$= \frac{11}{6} + \ln\frac{2}{3}$$

$$\text{Area}(R) = 2\int_2^3 \frac{(u-1)^3}{u}\,\mathrm{d}u = 2\left(\frac{11}{6} + \ln\frac{2}{3}\right)$$

A1

Question Number	Scheme		Marks
124.	***Working parametrically:***		
	$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$ or $y = e^{t\ln 2} - 1$		
(a)	$\{x = 0 \Rightarrow\}\ 0 = 1 - \frac{1}{2}t \Rightarrow t = 2$	Applies $x = 0$ to obtain a value for t.	M1
	When $t = 2$, $y = 2^2 - 1 = 3$	Correct value for y.	A1
			[2]
(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^t - 1 \Rightarrow t = 0$	Applies $y = 0$ to obtain a value for t. (Must be seen in part (b)).	M1
	When $t = 0$, $x = 1 - \frac{1}{2}(0) = 1$	$x = 1$	A1
			[2]
(c)	$\frac{dx}{dt} = -\frac{1}{2}$ and either $\frac{dy}{dt} = 2^t \ln 2$ or $\frac{dy}{dt} = e^{t\ln 2}\ln 2$		B1
	$\frac{dy}{dx} = \frac{2^t \ln 2}{-\frac{1}{2}}$	Attempts their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$.	M1
	At A, $t = "2"$, so $m(\mathbf{T}) = -8\ln 2 \Rightarrow m(\mathbf{N}) = \frac{1}{8\ln 2}$	Applies $t = "2"$ and $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.	See notes.	M1 A1 oe cso
			[5]
(d)	$\text{Area}(R) = \int (2^t - 1).\left(-\frac{1}{2}\right)dt$	Complete substitution for both y and dx	M1
	$x = -1 \to t = 4$ and $x = 1 \to t = 0$		B1
	$= \left\{-\frac{1}{2}\right\}\left(\frac{2^t}{\ln 2} - t\right)$	Either $2^t \to \frac{2^t}{\ln 2}$ or $(2^t - 1) \to \frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $(2^t - 1) \to \pm\alpha(\ln 2)(2^t) - t$	M1*
		$(2^t - 1) \to \frac{2^t}{\ln 2} - t$	A1
	$\left\{-\frac{1}{2}\left[\frac{2^t}{\ln 2} - t\right]_4^0\right\} = -\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right) - \left(\frac{16}{\ln 2} - 4\right)\right)$	**Depends on the previous method mark.** Substitutes their changed limits in t and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6]
			15

124. (a)	**M1:** Applies $x = 0$ and obtains a value of t. **A1:** For $y = 2^2 - 1 = 3$ or $y = 4 - 1 = 3$ ***Alternative Solution 1:*** **M1:** For substituting $t = 2$ into either x or y. **A1:** $x = 1 - \frac{1}{2}(2) = 0$ **and** $y = 2^2 - 1 = 3$ ***Alternative Solution 2:*** **M1:** Applies $y = 3$ and obtains a value of t. **A1:** For $x = 1 - \frac{1}{2}(2) = 0$ or $x = 1 - 1 = 0$. ***Alternative Solution 3:*** **M1:** Applies $y = 3$ or $x = 0$ and obtains a value of t. **A1:** Shows that $t = 2$ for both $y = 3$ and $x = 0$.
(b)	**M1:** Applies $y = 0$ and obtains a value of t. Working must be seen in part (b). **A1:** For finding $x = 1$. **Note:** Award M1A1 for $x = 1$.
(c)	**B1:** Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. This mark can be implied by later working. **M1:** Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. **Note:** their $\frac{dy}{dt}$ must be a function of t. **M1:** Uses their value of t found in part (a) and applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$. **M1:** $y - 3 = (\text{their normal gradient})x$ **or** $y = (\text{their normal gradient})x + 3$ or equivalent. **A1:** $y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or $y - 3 = \frac{1}{\ln 256}(x - 0)$ or $(8\ln 2)y - 24\ln 2 = x$ or $\frac{y-3}{(x-0)} = \frac{1}{8\ln 2}$. You can apply isw here. **Working in decimals** is ok for the three method marks. B1, A1 require exact values.
(d)	**M1:** Complete substitution for both y and dx. So candidate should write down $\int (2^t - 1).\left(\text{their } \frac{dx}{dt}\right)$ **B1:** Changes limits from $x \to t$. $x = -1 \to t = 4$ and $x = 1 \to t = 0$. Note $t = 4$ and $t = 0$ seen is B1. **M1*:** Integrates 2^t correctly to give $\frac{2^t}{\ln 2}$ ... or integrates $(2^t - 1)$ to give either $\frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $\pm\alpha(\ln 2)(2^t) - t$. **A1:** Correct integration of $(2^t - 1)$ with respect to t to give $\frac{2^t}{\ln 2} - t$. **dM1*: Depends upon the previous method mark.** Substitutes their limits in t and subtracts either way round. **A1:** Exact answer of $\frac{15}{2\ln 2} - 2$ or $\frac{15}{\ln 4} - 2$ or $\frac{15 - 4\ln 2}{2\ln 2}$ or $\frac{7.5}{\ln 2} - 2$ or $\frac{15}{2}\log_2 e - 2$ or equivalent.

Question Number	Scheme		Marks
124.	***Alternative: Converting to a Cartesian equation:***		
	$t = 2-2x \Rightarrow \quad y = 2^{2-2x} - 1$		
(a)	$\{x = 0 \Rightarrow\}\ y = 2^2 - 1$	Applies $x = 0$ in their Cartesian equation...	M1
	$y = 3$	... to arrive at a correct answer of 3.	A1
			[2]
(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^{2-2x} - 1 \Rightarrow 0 = 2-2x \Rightarrow x = ...$	Applies $y = 0$ to obtain a value for x. (Must be seen in part (b)).	M1
	$x = 1$	$x = 1$	A1
			[2]
(c)	$\frac{dy}{dx} = -2\left(2^{2-2x}\right)\ln 2$	$\pm\lambda 2^{2-2x},\ \lambda \neq 1$	M1
		$-2\left(2^{2-2x}\right)\ln 2$ or equivalent	A1
	At A, $x = 0$, so $m(\mathbf{T}) = -8\ln 2 \Rightarrow m(\mathbf{N}) = \frac{1}{8\ln 2}$	Applies $x = 0$ and $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.	***As in the original scheme.***	M1 A1 oe
			[5]
(d)	$\text{Area}(R) = \int \left(2^{2-2x} - 1\right)dx$	Form the integral of their Cartesian equation of C.	M1
	$= \int_{-1}^{1}\left(2^{2-2x} - 1\right)dx$	For $2^{2-2x} - 1$ with limits of $x = -1$ and $x = 1$. Ie. $\int_{-1}^{1}\left(2^{2-2x} - 1\right)$	B1
	$= \left(\frac{2^{2-2x}}{-2\ln 2} - x\right)$	Either $2^{2-2x} \rightarrow \frac{2^{2-2x}}{-2\ln 2}$ or $\left(2^{2-2x} - 1\right) \rightarrow \frac{2^{2-2x}}{\pm\alpha(\ln 2)} - x$ or $\left(2^{2-2x} - 1\right) \rightarrow \pm\alpha(\ln 2)(2^{2-2x}) - x$	M1*
		$\left(2^{2-2x} - 1\right) \rightarrow \frac{2^{2-2x}}{-2\ln 2} - x$	A1
	$\left\{\left[\frac{2^{2-2x}}{-2\ln 2} - x\right]_{-1}^{1}\right\} = \left(\left(\frac{1}{-2\ln 2} - 1\right) - \left(\frac{16}{-2\ln 2} + 1\right)\right)$	**Depends on the previous method mark.** Substitutes limits of -1 and their x_B and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6] **15**
(d)	**Alternative method: In Cartesian and applying <u>$u = 2 - 2x$</u>**		

	$\text{Area}(R) = \int \left(2^u - 1\right)\{\mathrm{d}x\}$, where $u = 2 - 2x$ $= \int_4^0 \left(2^u - 1\right)\left(-\tfrac{1}{2}\right)\{\mathrm{d}u\}$	**M0**: Unless a candidate ***writes*** $\int \left(2^{2-2x} - 1\right)\{\mathrm{d}x\}$ Then apply the "working parametrically" mark scheme.

Question Number	Scheme		Marks
124. (d)	***Alternative method: For substitution* u = 2^t**		
	$\text{Area}(R) = \int (2^t - 1).\left(-\frac{1}{2}\right)\mathrm{d}t$	Complete substitution for both y and $\mathrm{d}x$	M1
	where $u = 2^t \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = 2^t \ln 2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = u\ln 2$		
	$x = -1 \to t = 4 \to u = 16$ and $x = 1 \to t = 0 \to u = 1$	Both correct limits in t or both correct limits in u.	B1
	So $\text{area}(R) = -\frac{1}{2}\int \frac{u-1}{u\ln 2}\mathrm{d}u$ $= -\frac{1}{2}\int \frac{1}{\ln 2} - \frac{1}{u\ln 2}\mathrm{d}u$	If not awarded above, you can award M1 for this integral	
	$= \left\{-\frac{1}{2}\right\}\left(\frac{u}{\ln 2} - \frac{\ln u}{\ln 2}\right)$	Either $2^t \to \frac{u}{\ln 2}$ or $(2^t - 1) \to \frac{u}{\pm\alpha(\ln 2)} - \frac{\ln u}{\ln 2}$ or $(2^t - 1) \to \pm\alpha(\ln 2)(u) - \frac{\ln u}{\ln 2}$	M1*
		$(2^t - 1) \to \frac{u}{\ln 2} - \frac{\ln u}{\ln 2}$	A1
	$\left\{-\frac{1}{2}\left[\frac{u}{\ln 2} - \frac{\ln u}{\ln 2}\right]_{16}^{1}\right\} = -\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right) - \left(\frac{16}{\ln 2} - \frac{\ln 16}{\ln 2}\right)\right)$	**Depends on the previous method mark.** Substitutes their changed limits ***in u*** and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - \frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - \frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6]

EXPERT TUITION

Question Number	Scheme		Marks
125. (a)	$\{y=0\Rightarrow\}\ 1-2\cos x=0$	$1-2\cos x=0$, seen or implied.	M1
	$\Rightarrow x=\frac{\pi}{3},\frac{5\pi}{3}$	At least one correct value of x. (See notes).	A1
		Both $\frac{\pi}{3}$ and $\frac{5\pi}{3}$	A1 **cso**
			[3]
(b)	$V=\pi\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}}(1-2\cos x)^2\ dx$	For $\pi\int(1-2\cos x)^2$. Ignore limits and dx	B1
	$\left\{\int(1-2\cos x)^2\ dx\right\}=\int(1-4\cos x+4\cos^2 x)dx$		
	$=\int 1-4\cos x+4\left(\frac{1+\cos 2x}{2}\right)dx$	$\cos 2x=2\cos^2 x-1$ See notes.	M1
	$=\int(3-4\cos x+2\cos 2x)\ dx$		
	$=3x-4\sin x+\frac{2\sin 2x}{2}$	Attempts $\int y^2$ to give any two of $\pm A\to\pm Ax$, $\pm B\cos x\to\pm B\sin x$ or $\pm\lambda\cos 2x\to\pm\mu\sin 2x$.	M1
		Correct integration.	A1
	$V=\{\pi\}\left(\left(3\left(\frac{5\pi}{3}\right)-4\sin\left(\frac{5\pi}{3}\right)+\frac{2\sin\left(\frac{10\pi}{3}\right)}{2}\right)-\left(3\left(\frac{\pi}{3}\right)-4\sin\left(\frac{\pi}{3}\right)+\frac{2\sin\left(\frac{2\pi}{3}\right)}{2}\right)\right)$	Applying limits the correct way round. Ignore π.	ddM1
	$=\pi\left(\left(5\pi+2\sqrt{3}-\frac{\sqrt{3}}{2}\right)-\left(\pi-2\sqrt{3}+\frac{\sqrt{3}}{2}\right)\right)$		
	$=\pi\left((18.3060...)-(0.5435...)\right)=17.7625\pi=55.80$		
	$=\pi\left(4\pi+3\sqrt{3}\right)$ or $4\pi^2+3\pi\sqrt{3}$	Two term exact answer.	A1
			[6] **9**

125. (a)	**M1:** $1-2\cos x=0$. This can be implied by either $\cos x=\frac{1}{2}$ or any one of the correct values for x in radians or in degrees. **1st A1:** Any one of either $\frac{\pi}{3}$ or $\frac{5\pi}{3}$ or 60 or 300 or awrt 1.05 or 5.23 or awrt 5.24 . **2nd A1:** Both $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.
(b)	**B1:** (M1 on epen) For $\pi\int(1-2\cos x)^2$. Ignore limits and $\mathrm{d}x$. **1st M1:** Any correct form of $\cos 2x=2\cos^2 x-1$ used or written down in the same variable. This can be implied by $\cos^2 x=\frac{1+\cos 2x}{2}$ or $4\cos^2 x \to 2+2\cos 2x$ or $\cos 2A=2\cos^2 A-1$. **2nd M1:** Attempts $\int y^2$ to give any two of $\pm A\to\pm Ax$, $\pm B\cos x\to\pm B\sin x$ or $\pm\lambda\cos 2x\to\pm\mu\sin 2x$. Do not worry about the signs when integrating $\cos x$ or $\cos 2x$ for this mark. Note: $\int(1-2\cos x)^2=\int 1+4\cos^2 x$ is ok for an attempt at $\int y^2$. **1st A1:** Correct integration. Eg. $3x-4\sin x+\frac{2\sin 2x}{2}$ or $x-4\sin x+\frac{2\sin 2x}{2}+2x$ oe. **3rd ddM1:** Depends on both of the two previous method marks. (Ignore π). Some evidence of substituting their $x=\frac{5\pi}{3}$ and their $x=\frac{\pi}{3}$ and subtracting the correct way round. You will need to use your calculator to check for correct substitution of their limits into their integrand if a candidate does not explicitly give **some evidence.** **Note:** For correct integral and limits decimals gives: $\pi\left((18.3060...)-(0.5435...)\right)=17.7625\pi=55.80$ **2nd A1:** ***Two term*** exact answer of either $\pi\left(4\pi+3\sqrt{3}\right)$ or $4\pi^2+3\pi\sqrt{3}$ or equivalent. **Note:** The π in the volume formula is only required for the B1 mark and the final A1 mark. **Note:** Decimal answer of 58.802... without correct exact answer is A0. **Note:** Applying $\int(1-2\cos x)\,\mathrm{d}x$ will usually be given no marks in this part.

Question Number	Scheme	Marks
126. (a)	$\left\{\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{(3-\theta)}{125}\right\} \Rightarrow \int \frac{1}{3-\theta}\,\mathrm{d}\theta = \int \frac{1}{125}\,\mathrm{d}t$ or $\int \frac{125}{3-\theta}\,\mathrm{d}\theta = \int \mathrm{d}t$	B1
	$-\ln(\theta-3) = \frac{1}{125}t\ \{+c\}$ or $-\ln(3-\theta) = \frac{1}{125}t\ \{+c\}$ See notes.	M1 A1
	$\ln(\theta-3) = -\frac{1}{125}t + c$	
	$\theta - 3 = \mathrm{e}^{-\frac{1}{125}t + c}$ or $\mathrm{e}^{-\frac{1}{125}t}\mathrm{e}^{c}$ $\theta = A\mathrm{e}^{-0.008t} + 3$ * ***Correct*** completion to $\theta = A\mathrm{e}^{-0.008t} + 3$.	A1
		[4]
(b)	$\{t=0,\ \theta=16 \Rightarrow\}$ $16 = A\mathrm{e}^{-0.008(0)} + 3$; $\Rightarrow \underline{A=13}$ ***See notes.***	M1; A1
	$10 = 13\mathrm{e}^{-0.008t} + 3$ Substitutes $\theta = 10$ into an equation **of the form** $\theta = A\mathrm{e}^{-0.008t} + 3$, or equivalent. ***See notes.***	M1
	$\mathrm{e}^{-0.008t} = \frac{7}{13} \Rightarrow -0.008t = \ln\left(\frac{7}{13}\right)$ Correct algebra to $-0.008t = \ln k$, where k is a positive value. ***See notes.***	M1
	$\left\{t = \frac{\ln\left(\frac{7}{13}\right)}{(-0.008)}\right\} = 77.3799... = 77$ (nearest minute) awrt 77	A1
		[5] **9**

126. (a)

B1: (M1 on epen) Separates variables as shown. $\mathrm{d}\theta$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.

M1: Both $\pm\lambda\ln(3-\theta)$ or $\pm\lambda\ln(\theta-3)$ **and** $\pm\mu t$ where λ and μ are constants.

A1: For $-\ln(\theta-3) = \frac{1}{125}t$ or $-\ln(3-\theta) = \frac{1}{125}t$ or $-125\ln(\theta-3) = t$ or $-125\ln(3-\theta) = t$

Note: $+c$ is not needed for this mark.

A1: Correct completion to $\theta = A\mathrm{e}^{-0.008t} + 3$. **Note:** $+c$ is needed for this mark.

Note: $\ln(\theta-3) = -\frac{1}{125}t + c$ leading to $\theta - 3 = \mathrm{e}^{-\frac{1}{125}t} + \mathrm{e}^{c}$ or $\theta - 3 = e^{-\frac{1}{125}t} + A$, would be final A0.

Note: From $-\ln(\theta-3) = \frac{1}{125}t + c$, then $\ln(\theta-3) = -\frac{1}{125}t + c$

$\Rightarrow \theta - 3 = \mathrm{e}^{-\frac{1}{125}t + c}$ **or** $\theta - 3 = \mathrm{e}^{-\frac{1}{125}t}\mathrm{e}^{c} \Rightarrow \theta = A\mathrm{e}^{-0.008t} + 3$ is required for A1.

Note: From $-\ln(3-\theta) = \frac{1}{125}t + c$, then $\ln(3-\theta) = -\frac{1}{125}t + c$

$\Rightarrow 3 - \theta = \mathrm{e}^{-\frac{1}{125}t + c}$ **or** $3 - \theta = \mathrm{e}^{-\frac{1}{125}t}\mathrm{e}^{c} \Rightarrow \theta = A\mathrm{e}^{-0.008t} + 3$ is sufficient for A1.

Note: The jump from $3 - \theta = A\mathrm{e}^{-\frac{1}{125}t}$ to $\theta = A\mathrm{e}^{-0.008t} + 3$ is fine.

	Note: $\ln(\theta - 3) = -\frac{1}{125}t + c \Rightarrow \theta - 3 = Ae^{-\frac{1}{125}t}$, where candidate writes $A = e^c$ is also acceptable.

126. (b)

M1: (B1 on epen) Substitutes $\theta = 16,\ t = 0$, into either their equation containing an unknown constant or the printed equation. **Note:** You can imply this method mark.

A1: (M1 on epen) $A = 13$. **Note:** $\theta = 13\mathrm{e}^{-0.008t} + 3$ without any working implies the first two marks, M1A1.

M1: Substitutes $\theta = 10$ into an equation **of the form** $\theta = A\mathrm{e}^{-0.008t} + 3$, or equivalent. where A is a positive or negative numerical value and A can be equal to 1 or -1.

M1: Uses correct algebra to rearrange **their equation** into the form $-0.008t = \ln k$, where k is ***a positive numerical value***.

A1: awrt 77 or awrt 1 hour 17 minutes.

<u>Alternative Method 1 for part (b)</u>

$$\int \frac{1}{3-\theta}\,\mathrm{d}\theta = \int \frac{1}{125}\,\mathrm{d}t \Rightarrow -\ln(\theta - 3) = \frac{1}{125}t + c$$

$\{t = 0\ ,\ \theta = 16 \Rightarrow\}$ $-\ln(16-3) = \frac{1}{125}(0) + c$

$\Rightarrow c = -\ln 13$

M1: Substitutes $t = 0, \theta = 16$, into $-\ln(\theta - 3) = \frac{1}{125}t + c$

A1: $c = -\ln 13$

$-\ln(\theta - 3) = \frac{1}{125}t - \ln 13$ or $\ln(\theta - 3) = -\frac{1}{125}t + \ln 13$

$-\ln(10-3) = \frac{1}{125}t - \ln 13$

M1: Substitutes $\theta = 10$ into an equation **of the form** $\pm\lambda\ln(\theta - 3) = \pm\frac{1}{125}t \pm \mu$ where λ, μ are numerical values.

$\ln 13 - \ln 7 = \frac{1}{125}t$

M1: Uses correct algebra to rearrange **their equation** into the form $\pm 0.008t = \ln C - \ln D$, where C, D are ***positive numerical values***.

$t = 77.3799... = 77$ (nearest minute)

A1: awrt 77.

<u>Alternative Method 2 for part (b)</u>

$$\int \frac{1}{3-\theta}\,\mathrm{d}\theta = \int \frac{1}{125}\,\mathrm{d}t \Rightarrow -\ln|3 - \theta| = \frac{1}{125}t + c$$

$\{t = 0\ ,\ \theta = 16 \Rightarrow\}$ $-\ln|3-16| = \frac{1}{125}(0) + c$

$\Rightarrow c = -\ln 13$

M1: Substitutes $t = 0, \theta = 16$, into $-\ln(3 - \theta) = \frac{1}{125}t + c$

A1: $c = -\ln 13$

$-\ln|3 - \theta| = \frac{1}{125}t - \ln 13$ or $\ln|3 - \theta| = -\frac{1}{125}t + \ln 13$

$-\ln(3-10) = \frac{1}{125}t - \ln 13$

M1: Substitutes $\theta = 10$ into an equation **of the form** $\pm\lambda\ln(3 - \theta) = \pm\frac{1}{125}t \pm \mu$ where λ, μ are numerical values.

$\ln 13 - \ln 7 = \frac{1}{125}t$

M1: Uses correct algebra to rearrange **their equation** into the form $\pm 0.008t = \ln C - \ln D$,

	$t = 77.3799... = 77$ (nearest minute)	where C, D are ***positive numerical values***. **A1:** awrt 77.

126. (b)	***Alternative Method 3 for part (b)*** $\int_{16}^{10} \frac{1}{3-\theta}\,\mathrm{d}\theta = \int_0^t \frac{1}{125}\,\mathrm{d}t$ $= \left[-\ln\lvert 3-\theta\rvert\right]_{16}^{10} = \left[\frac{1}{125}t\right]_0^t$	
	$-\ln 7 - -\ln 13 = \frac{1}{125}t$	**M1A1:** $\ln 13$ **M1:** Substitutes limit of $\theta = 10$ correctly. **M1:** Uses correct algebra to rearrange **their own equation** into the form $\pm 0.008t = \ln C - \ln D$, where C, D are ***positive numerical values***.
	$t = 77.3799... = 77$ (nearest minute)	**A1:** awrt 77.
	Alternative Method 4 for part (b)	
	$\{\theta = 16 \Rightarrow\}\quad 16 = Ae^{-0.008t} + 3$ $\{\theta = 10 \Rightarrow\}\quad 10 = Ae^{-0.008t} + 3$	**M1*:** Writes down a pair of equations in A and t, for $\theta = 16$ and $\theta = 10$ with either A unknown or A being a positive or negative value. **A1:** Two equations with an unknown A.
	$-0.008t = \ln\left(\frac{13}{A}\right)$ or $-0.008t = \ln\left(\frac{7}{A}\right)$	**M1:** Uses ***correct algebra*** to solve both of **their equations** leading to answers of the form $-0.008t = \ln k$, where k is ***a positive numerical value***.
	$t_{(1)} = \frac{\ln\left(\frac{13}{A}\right)}{-0.008}$ and $t_{(2)} = \frac{\ln\left(\frac{7}{A}\right)}{-0.008}$	
	$t = t_{(1)} - t_{(2)} = \frac{\ln\left(\frac{13}{A}\right)}{-0.008} - \frac{\ln\left(\frac{7}{A}\right)}{-0.008}$	**M1:** Finds difference between the two times. (either way round).
	$\left\{t = \frac{\ln\left(\frac{7}{13}\right)}{(-0.008)}\right\} = 77.3799... = 77$ (nearest minute)	**A1:** awrt 77. Correct solution only.

Question Number	Scheme	Marks
127.	(a) $1 = A(3x-1)^2 + Bx(3x-1) + Cx$	B1
	$x \to 0$ $\quad (1 = A)$	M1
	$x \to \frac{1}{3}$ $\quad 1 = \frac{1}{3}C \;\Rightarrow\; C = 3$ any two constants correct	A1
	Coefficients of x^2	
	$0 = 9A + 3B \;\Rightarrow\; B = -3$ all three constants correct	A1 **(4)**
	(b)(i) $\displaystyle\int\left(\frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2}\right)\mathrm{d}x$	
	$\displaystyle = \ln x - \frac{3}{3}\ln(3x-1) + \frac{3}{(-1)3}(3x-1)^{-1} \quad (+C)$	M1 A1ft A1ft
	$\displaystyle\left(= \ln x - \ln(3x-1) - \frac{1}{3x-1} \quad (+C)\right)$	
	(ii) $\displaystyle\int_1^2 \mathrm{f}(x)\,\mathrm{d}x = \left[\ln x - \ln(3x-1) - \frac{1}{3x-1}\right]_1^2$	
	$\displaystyle = \left(\ln 2 - \ln 5 - \frac{1}{5}\right) - \left(\ln 1 - \ln 2 - \frac{1}{2}\right)$	M1
	$\displaystyle = \ln\frac{2\times 2}{5} + \ \ldots$	M1
	$\displaystyle = \frac{3}{10} + \ln\left(\frac{4}{5}\right)$	A1 **(6)**
		[10]

EXPERT TUITION

Question Number	Scheme		Marks	
128.	$\int y\,\mathrm{d}y = \int \frac{3}{\cos^2 x}\mathrm{d}x$ $= \int 3\sec^2 x\,\mathrm{d}x$	Can be implied. Ignore integral signs	B1	
	$\frac{1}{2}y^2 = 3\tan x \quad (+C)$		M1 A1	
	$y = 2,\ x = \frac{\pi}{4}$ $\frac{1}{2}2^2 = 3\tan\frac{\pi}{4} + C$		M1	
	Leading to $C = -1$ $\frac{1}{2}y^2 = 3\tan x - 1$	or equivalent	A1	**(5)**
				[5]

EXPERT TUITION

Question Number	Scheme	Marks
129.	(a) $\int x^{\frac{1}{2}} \ln 2x \, dx = \frac{2}{3}x^{\frac{3}{2}} \ln 2x - \int \frac{2}{3}x^{\frac{3}{2}} \times \frac{1}{x} dx$	M1 A1
	$= \frac{2}{3}x^{\frac{3}{2}} \ln 2x - \int \frac{2}{3}x^{\frac{1}{2}} \, dx$	
	$= \frac{2}{3}x^{\frac{3}{2}} \ln 2x - \frac{4}{9}x^{\frac{3}{2}} \quad (+C)$	M1 A1 **(4)**
	(b) $\left[\frac{2}{3}x^{\frac{3}{2}} \ln 2x - \frac{4}{9}x^{\frac{3}{2}}\right]_1^4 = \left(\frac{2}{3}4^{\frac{3}{2}} \ln 8 - \frac{4}{9}4^{\frac{3}{2}}\right) - \left(\frac{2}{3}\ln 2 - \frac{4}{9}\right)$	M1
	$= (16\ln 2 - \ \dots) - \ \dots$ Using or implying $\ln 2^n = n \ln 2$	M1
	$= \frac{46}{3}\ln 2 - \frac{28}{9}$	A1 **(3)**
		[7]

EXPERT TUITION

Question Number	Scheme	Marks
130 (a)	$\int x\sin 3x \,\mathrm{d}x = -\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x \,\{\mathrm{d}x\}$	M1 A1
	$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x \,\{+c\}$	A1
		[3]
(b)	$\int x^2\cos 3x \,\mathrm{d}x = \frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x \,\{\mathrm{d}x\}$	M1 A1
	$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right) \,\{+c\}$	A1 **isw**
	$\left\{= \frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x \quad \{+c\}\right\}$ *Ignore subsequent working*	**[3]**
		6

(a)

M1: Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x \to u' = 1$ and $v' = \sin 3x \to v = k\cos 3x$ (seen or implied), where k is a positive or negative constant. (Allow $k = 1$).

This means that the candidate must achieve $x(k\cos 3x) - \int (k\cos 3x)$, where k is a consistent constant.

If x^2 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0.

A1: $-\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x \,\{\mathrm{d}x\}$. Can be un-simplified. Ignore the $\{\mathrm{d}x\}$.

A1: $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x$ with/without + c. Can be un-simplified.

(b)

M1: Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x^2 \to u' = 2x$ or x and $v' = \cos 3x \to v = \lambda\sin 3x$ (seen or implied), where λ is a positive or negative constant. (Allow $\lambda = 1$).

This means that the candidate must achieve $x^2(\lambda\sin 3x) - \int 2x(\lambda\sin 3x)$, where $u' = 2x$

or $x^2(\lambda\sin 3x) - \int x(\lambda\sin 3x)$, where $u' = x$.

If x^3 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0.

A1: $\frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x \,\{\mathrm{d}x\}$. Can be un-simplified. Ignore the $\{\mathrm{d}x\}$.

A1: $\frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right)$ with/without + c, can be un-simplified.

You can ignore subsequent working here.

Special Case: If the candidate scores the first two marks of M1A1 in part (b), then you can award the final A1 as a follow through for $\frac{1}{3}x^2\sin 3x - \frac{2}{3}(\text{their follow through part(a) answer})$.

Question Number	Scheme		Marks
131.	Volume $= \pi \underline{\int_0^2 \left(\sqrt{\left(\frac{2x}{3x^2+4}\right)}\right)^2}$ dx	Use of $V = \pi \underline{\int y^2}$ dx.	$\underline{\text{B1}}$
	$= (\pi)\left[\frac{1}{3}\ln\left(3x^2+4\right)\right]_0^2$	$\pm k\ln\left(3x^2+4\right)$	M1
		$\frac{1}{3}\ln\left(3x^2+4\right)$	A1
	$= (\pi)\left[\left(\frac{1}{3}\ln 16\right) - \left(\frac{1}{3}\ln 4\right)\right]$	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1
	So Volume $= \underline{\frac{1}{3}\pi\ln 4}$	$\underline{\frac{1}{3}\pi\ln 4}$ or $\underline{\frac{2}{3}\pi\ln 2}$	A1 oe isw
			[5] 5

NOTE: π is required for the B1 mark and the final A1 mark. It is not required for the 3 intermediate marks.

B1: For applying $\pi\int y^2$. Ignore limits and dx. This can be implied by later working,

but the pi and $\int \frac{2x}{3x^2+4}$ must appear on one line somewhere in the candidate's working.

B1 can also be implied by a correct final answer. **Note**: $\pi\left(\int y\right)^2$ would be B0.

Working in x

M1: For $\pm k\ln\left(3x^2+4\right)$ or $\pm k\ln\left(x^2+\frac{4}{3}\right)$ where k is a constant and k can be 1.

Note: M0 for $\pm k\,x\ln\left(3x^2+4\right)$.

Note: M1 can also be given for $\pm k\ln\left(p\left(3x^2+4\right)\right)$, where k and p are constants and k can be 1.

A1: For $\frac{1}{3}\ln\left(3x^2+4\right)$ or $\frac{1}{3}\ln\left(\frac{1}{3}\left(3x^2+4\right)\right)$ or $\frac{1}{3}\ln\left(x^2+\frac{4}{3}\right)$ or $\frac{1}{3}\ln\left(p(3x^2+4)\right)$.

You may allow M1 A1 for $\frac{1}{3}\left(\frac{x}{x}\right)\ln\left(3x^2+4\right)$ or $\frac{1}{3}\left(\frac{2x}{6x}\right)\ln\left(3x^2+4\right)$

dM1: Substitutes limits of 2 and 0 and subtracts the correct way round. Working in decimals is fine for dM1.

A1: For either $\frac{1}{3}\pi\ln 4$, $\frac{1}{3}\ln 4^{\pi}$, $\frac{2}{3}\pi\ln 2$, $\pi\ln 4^{\frac{1}{3}}$, $\pi\ln 2^{\frac{2}{3}}$, $\frac{1}{3}\pi\ln\left(\frac{16}{4}\right)$, $2\pi\ln\left(\frac{16^{\frac{1}{6}}}{4^{\frac{1}{6}}}\right)$, etc.

Note: $\frac{1}{3}\pi(\ln 16 - \ln 4)$ would be A0.

Working in u: where $u = 3x^2+4$,

M1: For $\pm k\ln u$ where k is a constant and k can be 1.

Note: M1 can also be given for $\pm k\ln\left(pu\right)$, where k and p are constants and k can be 1.

A1: For $\frac{1}{3}\ln u$ or $\frac{1}{3}\ln 3u$ or $\frac{1}{3}\ln pu$.

dM1: Substitutes limits of 16 and 4 in u or limits of 2 and 0 in x and subtracts the correct way round.

A1: As above!

Question Number	Scheme		Marks
132. (a)	0.73508		B1 **cao**
			[1]
(b)	Area $\approx \dfrac{1}{2}\times\dfrac{\pi}{8}\,;\times\underline{\left[0+2\left(\text{their } 0.73508+1.17157+1.02280\right)+0\right]}$		B1 <u>M1</u>
	$=\dfrac{\pi}{16}\times 5.8589... \;=\; 1.150392325... \;= 1.1504 \;\; (4 \text{ dp})$	awrt 1.1504	A1 **[3]**
(c)	$\underline{\{u=1+\cos x\} \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x}=-\sin x}$		<u>B1</u>
	$\left\{\displaystyle\int \frac{2\sin 2x}{(1+\cos x)}\,\mathrm{d}x =\right\} \displaystyle\int \frac{2(2\sin x\cos x)}{(1+\cos x)}\,\mathrm{d}x$	$\sin 2x = 2\sin x\cos x$	B1
	$= \displaystyle\int \frac{4(u-1)}{u}.(-1)\,\mathrm{d}u \quad \left\{= \; 4\displaystyle\int \frac{(1-u)}{u}\,\mathrm{d}u\right\}$		M1
	$= 4\displaystyle\int \left(\frac{1}{u}-1\right)\mathrm{d}u \;=\; 4\left(\ln u - u\right)+c$		dM1
	$= 4\ln(1+\cos x) - 4(1+\cos x) + c \quad = 4\ln(1+\cos x) - 4\cos x + k$	**AG**	A1 **cso [5]**
(d)	$= \left[4\ln\left(1+\cos\dfrac{\pi}{2}\right) - 4\cos\dfrac{\pi}{2}\right] - \left[4\ln(1+\cos 0) - 4\cos 0\right]$	Applying limits $x=\dfrac{\pi}{2}$ and $x=0$ either way round.	M1
	$= [4\ln 1 - 0] - [4\ln 2 - 4]$		
	$= 4 - 4\ln 2 \;\{= 1.227411278...\}$	$\pm 4(1-\ln 2)$ or $\pm(4-4\ln 2)$ or awrt ± 1.2, ***however*** found.	A1
	Error $= \left\lvert(4-4\ln 2) - 1.1504...\right\rvert$ $= 0.0770112776... \;=\; 0.077 \;(2\text{sf})$	awrt ± 0.077 or awrt $\pm 6.3(\%)$	A1 **cso** **[3]**
			12
(a)	**B1:** 0.73508 correct answer only. Look for this on the table or in the candidate's working.		
(b)	**B1**: Outside brackets $\dfrac{1}{2}\times\dfrac{\pi}{8}$ or $\dfrac{\pi}{16}$ or awrt 0.196 **M1:** <u>For structure of trapezium rule</u>[<u>............</u>]; (0 can be implied). **A1:** anything that rounds to 1.1504 <u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly** Award B1M0A0 for $\dfrac{1}{2}\times\dfrac{\pi}{8} + 2\left(\text{their } 0.73508+1.17157+1.02280\right)$ (nb: answer of 6.0552). Award B1M0A0 for $\dfrac{1}{2}\times\dfrac{\pi}{8}\,(0+0) + 2\left(\text{their } 0.73508+1.17157+1.02280\right)$ (nb: answer of 5.8589). *<u>Alternative method for part (b): Adding individual trapezia</u>* Area $\approx \dfrac{\pi}{8}\times\left[\dfrac{0+0.73508}{2}+\dfrac{0.73508+1.17157}{2}+\dfrac{1.17157+1.02280}{2}+\dfrac{1.02280+0}{2}\right] = 1.150392325...$ B1: $\dfrac{\pi}{8}$ and a divisor of 2 on all terms inside brackets. M1: One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2. A1: anything that rounds to 1.1504		

132. (c)	**B1:** $\frac{du}{dx} = -\sin x$ or $du = -\sin x\,dx$ or $\frac{dx}{du} = \frac{1}{-\sin x}$ oe. **B1:** For seeing, applying or implying $\sin 2x = 2\sin x\cos x$. **M1:** After applying substitution candidate achieves $\pm k\int\frac{(u-1)}{u}(du)$ or $\pm k\int\frac{(1-u)}{u}(du)$. Allow M1 for "invisible" brackets here, eg: $\pm\int\frac{(\lambda u-1)}{u}(du)$ or $\pm\int\frac{(-\lambda+u)}{u}(du)$, where λ is a positive constant. **dM1:** An attempt to divide through each term by u and $\pm k\int\left(\frac{1}{u}-1\right)du \rightarrow \pm k(\ln u - u)$ with/without $+c$. Note that this mark is dependent on the previous M1 mark being awarded. **<u>Alternative method:</u>** Candidate can also gain this mark for applying integration by parts followed by a correct method for integrating $\ln u$. (See below). **A1:** Correctly combines their $+c$ and "-4" together to give $\underline{4\ln(1+\cos x) - 4\cos x + k}$ As a minimum candidate must write either $4\ln(1+\cos x) - 4(1+\cos x) + c \rightarrow 4\ln(1+\cos x) - 4\cos x + k$ or $4\ln(1+\cos x) - 4(1+\cos x) + k \rightarrow 4\ln(1+\cos x) - 4\cos x + k$ **Note**: that this mark is also for a correct solution only. **Note**: those candidates who attempt to find the value of k will usually achieve A0.
(d)	**M1:** Substitutes limits of $x = \frac{\pi}{2}$ and $x = 0$ into $\{4\ln(1+\cos x) - 4\cos x\}$ or their answer from part (c) and subtracts the either way round. **Note** that: $\left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2}\right] - [0]$ is M0. **A1:** $4(1-\ln 2)$ or $4 - 4\ln 2$ or awrt 1.2, however found. This mark can be implied by the final answer of either awrt ± 0.077 or awrt ± 6.3 **A1:** For either awrt ± 0.077 or awrt ± 6.3 (for percentage error). Note this mark is for a **correct solution only**. Therefore if there if a candidate substitutes limits the incorrect way round and final achieves (usually fudges) the final correct answer then this mark can be withheld. Note that awrt 6.7 (for percentage error) is A0. *<u>Alternative method for dM1 in part (c)</u>* $\int\frac{(1-u)}{u}\,du = \left((1-u)\ln u - \int -\ln u\,du\right) = \left((1-u)\ln u + u\ln u - \int\frac{u}{u}\,du\right) = ((1-u)\ln u + u\ln u - u)$ or $\int\frac{(u-1)}{u}\,du = \left((u-1)\ln u - \int\ln u\,du\right) = \left((u-1)\ln u - \left(u\ln u - \int\frac{u}{u}\,du\right)\right) = ((u-1)\ln u - u\ln u + u)$ So **dM1** is for $\int\frac{(1-u)}{u}\,du$ going to $((1-u)\ln u + u\ln u - u)$ or $((u-1)\ln u - u\ln u + u)$ oe. *<u>Alternative method for part (d)</u>* **M1A1** for $\left\{4\int_2^1\left(\frac{1}{u}-1\right)du = \right\} 4[\ln u - u]_2^1 = 4[(\ln 1 - 1) - (\ln 2 - 2)] = 4(1-\ln 2)$ *<u>Alternative method for part (d): Using an extra constant λ from their integration.</u>* $\left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2} + \lambda\right] - [4\ln(1+\cos 0) - 4\cos 0 + \lambda]$ λ is usually -4, but can be a value of k that the candidate has found in part (d). **Note:** The extra constant λ should cancel out and so the candidate can gain all three marks using this method, even the final A1 cso.

<table>
<tr><th>Question Number</th><th>Scheme</th><th></th><th>Marks</th></tr>
<tr><td>133. (a)</td><td>$1 = A(5-P) + BP$</td><td>Can be implied.</td><td>M1</td></tr>
<tr><td></td><td>$A = \frac{1}{5},\ B = \frac{1}{5}$</td><td>Either one.</td><td>A1</td></tr>
<tr><td></td><td>giving $\frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}$</td><td>See notes.</td><td>A1 cao, aef</td></tr>
<tr><td></td><td></td><td></td><td>[3]</td></tr>
<tr><td>(b)</td><td>$\int \frac{1}{P(5-P)}\,dP = \int \frac{1}{15}\,dt$</td><td></td><td>B1</td></tr>
<tr><td></td><td>$\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t \ \ (+c)$</td><td></td><td>M1*
A1ft</td></tr>
<tr><td></td><td>$\{t = 0,\ P = 1 \Rightarrow\}\ \ \frac{1}{5}\ln 1 - \frac{1}{5}\ln(4) = 0 + c \quad \left\{\Rightarrow\ \ c = -\frac{1}{5}\ln 4\right\}$</td><td></td><td>dM1*</td></tr>
<tr><td></td><td>eg: $\frac{1}{5}\ln\left(\frac{P}{5-P}\right) = \frac{1}{15}t - \frac{1}{5}\ln 4$</td><td>Using any of the subtraction (or addition) laws for logarithms CORRECTLY</td><td>dM1*</td></tr>
<tr><td></td><td>$\ln\left(\frac{4P}{5-P}\right) = \frac{1}{3}t$</td><td></td><td></td></tr>
<tr><td></td><td>eg: $\frac{4P}{5-P} = e^{\frac{1}{3}t}$ or eg: $\frac{5-P}{4P} = e^{-\frac{1}{3}t}$</td><td>Eliminate ln's correctly.</td><td>dM1*</td></tr>
<tr><td></td><td>gives $4P = 5e^{\frac{1}{3}t} - Pe^{\frac{1}{3}t} \Rightarrow P(4 + e^{\frac{1}{3}t}) = 5e^{\frac{1}{3}t}$</td><td></td><td></td></tr>
<tr><td></td><td>$P = \frac{5e^{\frac{1}{3}t}}{(4 + e^{\frac{1}{3}t})} \quad \left\{\frac{(\div e^{\frac{1}{3}t})}{(\div e^{\frac{1}{3}t})}\right\}$</td><td>Make P the subject.</td><td>dM1*</td></tr>
<tr><td></td><td>$P = \frac{5}{(1 + 4e^{-\frac{1}{3}t})}$ or $P = \frac{25}{(5 + 20e^{-\frac{1}{3}t})}$ etc.</td><td></td><td>A1</td></tr>
<tr><td></td><td></td><td></td><td>[8]</td></tr>
<tr><td>(c)</td><td>$1 + 4e^{-\frac{1}{3}t} > 1 \Rightarrow P < 5$. So population cannot exceed 5000.</td><td></td><td>B1</td></tr>
<tr><td></td><td></td><td></td><td>[1]
12</td></tr>
<tr><td>(a)</td><td colspan="3">M1: Forming a correct identity. For example, $1 = A(5-P) + BP$. Note A and B not referred to in question.
A1: Either one of $A = \frac{1}{5}$ or $B = \frac{1}{5}$.
A1: $\frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}$ or any equivalent form, eg: $\frac{1}{5P} + \frac{1}{25-5P}$, etc. Ignore subsequent working.
This answer must be stated in part (a) only.
A1 can also be given for a candidate who finds both $A = \frac{1}{5}$ and $B = \frac{1}{5}$ and $\frac{A}{P} + \frac{B}{5-P}$ is seen in their working.
Candidate can use 'cover-up' rule to write down $\frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}$, as so gain all three marks.
Candidate cannot gain the marks for part (a) in part (b).</td></tr>
</table>

133. (b)	**B1:** Separates variables as shown. $\mathrm{d}P$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. **M1*:** **Both** $\pm\lambda\ln P$ and $\pm\mu\ln(\pm 5 \pm P)$, where λ and μ are constants. **Or** $\pm\lambda\ln mP$ and $\pm\mu\ln(n(\pm 5 \pm P))$, where λ, μ, m and n are constants. **A1ft:** Correct follow through integration of both sides from their $\int \frac{\lambda}{P} + \frac{\mu}{(5-P)}\,\mathrm{d}P = \int K\,\mathrm{d}t$ with or without $+c$ **dM1*:** Use of $t=0$ and $P=1$ in an integrated equation containing c **dM1*:** Using ANY of the subtraction (or addition) laws for logarithms CORRECTLY. **dM1*:** Apply logarithms (or take exponentials) to eliminate ln's CORRECTLY from their equation. **dM1*:** A full ACCEPTABLE method of rearranging to make P the subject. **(See below for examples!)** **A1:** $P = \frac{5}{(1+4\mathrm{e}^{-\frac{1}{3}t})}$ $\{$where $a=5,\ b=1,\ c=4\}$. Also allow any "integer" multiples of this expression. For example: $P = \frac{25}{(5+20\mathrm{e}^{-\frac{1}{3}t})}$ **Note: If the first method mark (M1*) is not awarded then the candidate cannot gain any of the six remaining marks for this part of the question.** **Note:** $\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int 15\,\mathrm{d}t \Rightarrow \int \frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}\,\mathrm{d}P = \int 15\,\mathrm{d}t \Rightarrow \ln P - \ln(5-P) = 15t$ is B0M1A1ft. ***<u>dM1* for making P the subject</u>*** **Note there are three type of manipulations here which are considered acceptable to make *P* the subject.** **(1)** M1 for $\frac{P}{5-P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow P = 5\mathrm{e}^{\frac{1}{3}t} - P\mathrm{e}^{\frac{1}{3}t} \Rightarrow P(1+\mathrm{e}^{\frac{1}{3}t}) = 5\mathrm{e}^{\frac{1}{3}t} \Rightarrow P = \frac{5}{(1+\mathrm{e}^{-\frac{1}{3}t})}$ **(2)** M1 for $\frac{P}{5-P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \frac{5-P}{P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \frac{5}{P} - 1 = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \frac{5}{P} = \mathrm{e}^{\frac{1}{3}t} + 1 \Rightarrow P = \frac{5}{(1+\mathrm{e}^{\frac{1}{3}t})}$ **(3)** M1 for $P(5-P) = 4\mathrm{e}^{\frac{1}{3}t} \Rightarrow P^2 - 5P = -4\mathrm{e}^{\frac{1}{3}t} \Rightarrow \left(P - \frac{5}{2}\right)^2 - \frac{25}{4} = -4\mathrm{e}^{\frac{1}{3}t}$ leading to $P = \ldots$ **Note**: The incorrect manipulation of $\frac{P}{5-P} = \frac{P}{5} - 1$ or equivalent is awarded this dM0*. **Note:** $(P) - (5-P) = \mathrm{e}^{\frac{1}{3}t} \Rightarrow 2P - 5 = \frac{1}{3}t$ leading to $P = \ldots$ or equivalent is awarded this dM0*
(c)	**B1:** $1 + 4\mathrm{e}^{-\frac{1}{3}t} > 1$ **and** $P < 5$ **and** a conclusion relating population (or even P) or meerkats to 5000. **For** $P = \frac{25}{(5+20\mathrm{e}^{-\frac{1}{3}t})}$, B1 can be awarded for $5 + 20\mathrm{e}^{-\frac{1}{3}t} > 5$ **and** $P < 5$ **and** a conclusion relating population (or even P) or meerkats to 5000. B1 can only be obtained if candidates have correct values of a and b in their $P = \frac{a}{(b+c\mathrm{e}^{-\frac{1}{3}t})}$. **Award B0 for:** As $t \to \infty$, $\mathrm{e}^{-\frac{1}{3}t} \to 0$. So $P \to \frac{5}{(1+0)} = 5$, so population cannot exceed 5000, **unless** the candidate also proves that $P = \frac{5}{(1+4\mathrm{e}^{-\frac{1}{3}t})}$ oe. is an increasing function. **If unsure here, then send to review!**

133.	<u>*Alternative method for part (b)*</u> **B1M1*A1:** as before for $\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t \;\left(+c\right)$ Award 3rd M1for $\ln\left(\frac{P}{5-P}\right) = \frac{1}{3}t + c$ Award 4th M1 for $\frac{P}{5-P} = Ae^{\frac{1}{3}t}$ Award 2nd M1 for $t=0,\ P=1 \;\Rightarrow\; \frac{1}{5-1} = Ae^{0} \;\left\{\Rightarrow\; A = \frac{1}{4}\right\}$ $\frac{P}{5-P} = \frac{1}{4}e^{\frac{1}{3}t}$ then award the final M1A1 in the same way.

Question Number	Scheme	Marks
134.	(a) 0.0333, 1.3596 awrt 0.0333, 1.3596	B1 B1 **(2)**
	(b) $\text{Area}(R) \approx \frac{1}{2}\times\frac{\sqrt{2}}{4}[\ ...]$	B1
	$\approx\ ...\ \left[0+2(0.0333+0.3240+1.3596)+3.9210\right]$	M1
	≈ 1.30 Accept 1.3	A1 **(3)**
	(c) $u = x^2+2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$	B1
	$\text{Area}(R) = \int_0^{\sqrt{2}} x^3 \ln(x^2+2)\,\mathrm{d}x$	B1
	$\int x^3 \ln(x^2+2)\,\mathrm{d}x = \int x^2 \ln(x^2+2)\,x\,\mathrm{d}x = \int (u-2)(\ln u)\tfrac{1}{2}\,\mathrm{d}u$	M1
	Hence $\text{Area}(R) = \frac{1}{2}\int_2^4 (u-2)\ln u\,\mathrm{d}u$ ✱ cso	A1 **(4)**
	(d) $\int (u-2)\ln u\,\mathrm{d}u = \left(\frac{u^2}{2}-2u\right)\ln u - \int\left(\frac{u^2}{2}-2u\right)\frac{1}{u}\,\mathrm{d}u$	M1 A1
	$= \left(\frac{u^2}{2}-2u\right)\ln u - \int\left(\frac{u}{2}-2\right)\mathrm{d}u$	
	$= \left(\frac{u^2}{2}-2u\right)\ln u - \left(\frac{u^2}{4}-2u\right) \quad (+C)$	M1 A1
	$\text{Area}(R) = \frac{1}{2}\left[\left(\frac{u^2}{2}-2u\right)\ln u - \left(\frac{u^2}{4}-2u\right)\right]_2^4$	
	$= \tfrac{1}{2}\left[(8-8)\ln 4 - 4 + 8 - \left((2-4)\ln 2 - 1 + 4\right)\right]$	M1
	$= \tfrac{1}{2}(2\ln 2 + 1)$ $\quad \ln 2 + \tfrac{1}{2}$	A1 **(6)**
		[15]

EXPERT TUITION

Question Number	Scheme	Marks
135.	(a) $\tan\theta = \sqrt{3}$ *or* $\sin\theta = \dfrac{\sqrt{3}}{2}$	M1
	$\theta = \dfrac{\pi}{3}$ awrt 1.05	A1 **(2)**
	(b) $\dfrac{dx}{d\theta} = \sec^2\theta,\ \dfrac{dy}{d\theta} = \cos\theta$	
	$\dfrac{dy}{dx} = \dfrac{\cos\theta}{\sec^2\theta}\quad\left(= \cos^3\theta\right)$	M1 A1
	At P, $m = \cos^3\left(\dfrac{\pi}{3}\right) = \dfrac{1}{8}$ Can be implied	A1
	Using $mm' = -1$, $m' = -8$	M1
	For normal $y - \frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$	M1
	At Q, $y = 0$ $-\frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$	
	leading to $x = \frac{17}{16}\sqrt{3}$ $\left(k = \frac{17}{16}\right)$ 1.0625	A1 **(6)**
	(c) $\int y^2\,dx = \int y^2 \dfrac{dx}{d\theta}\,d\theta = \int \sin^2\theta\sec^2\theta\,d\theta$	M1 A1
	$= \int \tan^2\theta\,d\theta$	A1
	$= \int \left(\sec^2\theta - 1\right)d\theta$	M1
	$= \tan\theta - \theta\quad(+C)$	A1
	$V = \pi\int_0^{\frac{\pi}{3}} y^2\,dx = \left[\tan\theta - \theta\right]_0^{\frac{\pi}{3}} = \pi\left[\left(\sqrt{3} - \frac{\pi}{3}\right) - (0 - 0)\right]$	M1
	$= \sqrt{3}\pi - \frac{1}{3}\pi^2\qquad\left(p = 1, q = -\frac{1}{3}\right)$	A1 **(7)**
		[15]

Question Number	Scheme	Marks
136.	(a) $\int (4y+3)^{-\frac{1}{2}}\,\mathrm{d}x = \dfrac{(4y+3)^{\frac{1}{2}}}{(4)(\frac{1}{2})} \quad (+C)$	M1 A1 **(2)**
	$\left(= \frac{1}{2}(4y+3)^{\frac{1}{2}} + C\right)$	
	(b) $\int \dfrac{1}{\sqrt{(4y+3)}}\,\mathrm{d}y = \int \dfrac{1}{x^2}\,\mathrm{d}x$	B1
	$\int (4y+3)^{-\frac{1}{2}}\,\mathrm{d}y = \int x^{-2}\,\mathrm{d}x$	
	$\dfrac{1}{2}(4y+3)^{\frac{1}{2}} = -\dfrac{1}{x} \quad (+C)$	M1
	Using $(-2, 1.5)$ $\quad \dfrac{1}{2}(4\times 1.5+3)^{\frac{1}{2}} = -\dfrac{1}{-2} + C$	M1
	leading to $\quad C = 1$	A1
	$\dfrac{1}{2}(4y+3)^{\frac{1}{2}} = -\dfrac{1}{x} + 1$	
	$(4y+3)^{\frac{1}{2}} = 2 - \dfrac{2}{x}$	M1
	$y = \dfrac{1}{4}\left(2 - \dfrac{2}{x}\right)^2 - \dfrac{3}{4}$ or equivalent	A1 **(6)**
		[8]

Question Number	Scheme	Marks
137.	$\int x\sin 2x\,\mathrm{d}x = -\dfrac{x\cos 2x}{2} + \int \dfrac{\cos 2x}{2}\,\mathrm{d}x$	M1 A1 A1
	$= \quad \ldots \quad + \dfrac{\sin 2x}{4}$	M1
	$\left[\ \ldots\ \right]_0^{\frac{\pi}{2}} = \dfrac{\pi}{4}$	M1 A1
		[6]

Question Number	Scheme	Marks
138. (a)	$\dfrac{5}{(x-1)(3x+2)} = \dfrac{A}{x-1} + \dfrac{B}{3x+2}$	
	$5 = A(3x+2) + B(x-1)$	
	$x \to 1$ $\quad 5 = 5A \Rightarrow A = 1$	M1 A1
	$x \to -\dfrac{2}{3}$ $\quad 5 = -\dfrac{5}{3}B \Rightarrow B = -3$	A1 (3)
(b)	$\displaystyle\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x = \int \left(\frac{1}{x-1} - \frac{3}{3x+2}\right)\mathrm{d}x$	
	$= \ln(x-1) - \ln(3x+2) \quad (+C)$ ft constants	M1 A1ft A1ft
		(3)
(c)	$\displaystyle\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x = \int \left(\frac{1}{y}\right)\mathrm{d}y$	M1
	$\ln(x-1) - \ln(3x+2) = \ln y \quad (+C)$	M1 A1
	$y = \dfrac{K(x-1)}{3x+2}$ depends on first two Ms in (c)	M1 dep
	Using $(2, 8)$ $\quad 8 = \dfrac{K}{8}$ depends on first two Ms in (c)	M1 dep
	$y = \dfrac{64(x-1)}{3x+2}$	A1 (6)
		[12]

Question Number	Scheme	Marks
139.		
(a)	$\dfrac{dx}{dt}=\dfrac{1}{t}, \quad \dfrac{dy}{dt}=2t$	
	$\dfrac{dy}{dx}=2t^2$	M1 A1
	Using $mm'=-1$, at $t=3$	
	$m'=-\dfrac{1}{18}$	M1 A1
	$y-7=-\dfrac{1}{18}(x-\ln 3)$	M1 A1 (6)
(b)	$x=\ln t \Rightarrow t=e^x$	B1
	$y=e^{2x}-2$	M1 A1 (3)
(c)	$V=\pi\int\left(e^{2x}-2\right)^2 dx$	M1
	$\int\left(e^{2x}-2\right)^2 dx=\int\left(e^{4x}-4e^{2x}+4\right)dx$	M1
	$=\dfrac{e^{4x}}{4}-\dfrac{4e^{2x}}{2}+4x$	M1 A1
	$\pi\left[\dfrac{e^{4x}}{4}-\dfrac{4e^{2x}}{2}+4x\right]_{\ln 2}^{\ln 4}=\pi\left[(64-32+4\ln 4)-(4-8+4\ln 2)\right]$	M1
	$=\pi(36+4\ln 2)$	A1
		(6) **[15]**
	Alternative to (c) *using parameters*	
	$V=\pi\int\left(t^2-2\right)^2\dfrac{dx}{dt}dt$	M1
	$\int\left(\left(t^2-2\right)^2\times\dfrac{1}{t}\right)dt=\int\left(t^3-4t+\dfrac{4}{t}\right)dt$	M1
	$=\dfrac{t^4}{4}-2t^2+4\ln t$	M1 A1
	The limits are $t=2$ and $t=4$	
	$\pi\left[\dfrac{t^4}{4}-2t^2+4\ln t\right]_2^4=\pi\left[(64-32+4\ln 4)-(4-8+4\ln 2)\right]$	M1
	$=\pi(36+4\ln 2)$	A1
		(6)

Question Number	Scheme	Marks
140.		
(a)	$x=3 \Rightarrow y=0.1847$ awrt	B1
	$x=5 \Rightarrow y=0.1667$ awrt or $\frac{1}{6}$	B1
		(2)
(b)	$I \approx \underline{\underline{\frac{1}{2}}}\left[0.2+0.1667+2(0.1847+0.1745)\right]$	B1 M1 A1ft
	≈ 0.543 0.542 or 0.543	A1 (4)
(c)	$\frac{\mathrm{d}x}{\mathrm{d}u}=2(u-4)$	B1
	$\int \frac{1}{4+\sqrt{(x-1)}}\mathrm{d}x=\int \frac{1}{u}\times 2(u-4)\,\mathrm{d}u$	M1
	$=\int\left(2-\frac{8}{u}\right)\mathrm{d}u$	A1
	$=2u-8\ln u$	M1 A1
	$x=2 \Rightarrow u=5,\ x=5 \Rightarrow u=6$	B1
	$\left[2u-8\ln u\right]_5^6=(12-8\ln 6)-(10-8\ln 5)$	M1
	$=2+8\ln\left(\frac{5}{6}\right)$	A1
		(8)
		[14]

Question Number	Scheme		Marks
141.	$\dfrac{\mathrm{d}u}{\mathrm{d}x} = -\sin x$		B1
	$\int \sin x\, \mathrm{e}^{\cos x+1}\mathrm{d}x = -\int \mathrm{e}^{u}\mathrm{d}u$		M1 A1
	$= -\mathrm{e}^{u}$	ft sign error	A1ft
	$= -\mathrm{e}^{\cos x+1}$		
	$\left[-\mathrm{e}^{\cos x+1}\right]_0^{\frac{\pi}{2}} = -\mathrm{e}^{1} - \left(-\mathrm{e}^{2}\right)$	or equivalent with u	M1
	$= \mathrm{e}(\mathrm{e}-1)$ $*$	cso	A1 **(6)**

Question Number	Scheme		Marks	
142.	(a) $\quad \mathrm{f}(\theta) = 4\cos^2\theta - 3\sin^2\theta$			
	$= 4\left(\frac{1}{2}+\frac{1}{2}\cos 2\theta\right) - 3\left(\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)$		M1 M1	
	$= \frac{1}{2}+\frac{7}{2}\cos 2\theta \quad *$	cso	A1	**(3)**
	(b) $\quad \int \theta\cos 2\theta \, \mathrm{d}\theta = \frac{1}{2}\theta\sin 2\theta - \frac{1}{2}\int \sin 2\theta \, \mathrm{d}\theta$		M1 A1	
	$= \frac{1}{2}\theta\sin 2\theta + \frac{1}{4}\cos 2\theta$		A1	
	$\int \theta \, \mathrm{f}(\theta) \, \mathrm{d}\theta = \frac{1}{4}\theta^2 + \frac{7}{4}\theta\sin 2\theta + \frac{7}{8}\cos 2\theta$		M1 A1	
	$\left[\ \ldots\ \right]_0^{\frac{\pi}{2}} = \left[\frac{\pi^2}{16} + 0 - \frac{7}{8}\right] - \left[0 + 0 + \frac{7}{8}\right]$		M1	
	$= \frac{\pi^2}{16} - \frac{7}{4}$		A1	**(7)**
				[10]

Question Number	Scheme	Marks
143.	(a) $\dfrac{dV}{dt} = 0.48\pi - 0.6\pi h$	M1 A1
	$V = 9\pi h \quad \Rightarrow \dfrac{dV}{dt} = 9\pi \dfrac{dh}{dt}$	B1
	$9\pi \dfrac{dh}{dt} = 0.48\pi - 0.6\pi h$	M1
	Leading to $75\dfrac{dh}{dt} = 4 - 5h$ * cso	A1 **(5)**
	(b) $\displaystyle\int \frac{75}{4-5h}\,dh = \int 1\,dt$ separating variables	M1
	$-15\ln(4-5h) = t \ (+C)$	M1 A1
	$-15\ln(4-5h) = t + C$	
	When $t = 0$, $h = 0.2$	
	$-15\ln 3 = C$	M1
	$t = 15\ln 3 - 15\ln(4-5h)$	
	When $h = 0.5$	
	$t = 15\ln 3 - 15\ln 1.5 = 15\ln\left(\dfrac{3}{1.5}\right) = 15\ln 2$ awrt 10.4	M1 A1
	Alternative for last 3 *marks*	
	$t = \left[-15\ln(4-5h)\right]_{0.2}^{0.5}$	
	$= -15\ln 1.5 + 15\ln 3$	M1 M1
	$= 15\ln\left(\dfrac{3}{1.5}\right) = 15\ln 2$ awrt 10.4	A1 **(6)**
		[11]

Question Number	Scheme	Marks
144	(a) 1.386, 2.291 awrt 1.386, 2.291	B1 B1 (2)
	(b) $A \approx \frac{1}{2} \times 0.5(\ \dots \ )$	B1
	$= \ \dots \ \left(0+2\left(0.608+1.386+2.291+3.296+4.385\right)+5.545\right)$	M1
	$= \ 0.25\left(0+2\left(0.608+1.386+2.291+3.296+4.385\right)+5.545\right)$ ft their (a)	A1ft
	$= 0.25 \times 29.477 \ \dots \ \approx 7.37$ cao	A1 (4)
	(c)(i) $\int x \ln x \, \mathrm{d}x = \frac{x^2}{2}\ln x - \int \frac{x^2}{2} \times \frac{1}{x} \, \mathrm{d}x$	M1 A1
	$= \frac{x^2}{2}\ln x - \int \frac{x}{2} \, \mathrm{d}x$	
	$= \frac{x^2}{2}\ln x - \frac{x^2}{4} \ \ (+C)$	M1 A1
	(ii) $\left[\frac{x^2}{2}\ln x - \frac{x^2}{4}\right]_1^4 = \left(8\ln 4 - 4\right) - \left(-\frac{1}{4}\right)$	M1
	$= 8\ln 4 - \frac{15}{4}$	
	$= 8\left(2\ln 2\right) - \frac{15}{4}$ $\ln 4 = 2\ln 2$ seen or implied	M1
	$= \frac{1}{4}\left(64\ln 2 - 15\right)$ $a = 64, b = -15$	A1 (7)
		[13]

EXPERT TUITION

Question Number	Scheme		Marks	
145	(a) $\displaystyle\int \frac{9x+6}{x}\,\mathrm{d}x = \int\left(9+\frac{6}{x}\right)\mathrm{d}x$		M1	
	$= 9x + 6\ln x \;\; (+C)$		A1	(2)
	(b) $\displaystyle\int \frac{1}{y^{\frac{1}{3}}}\,\mathrm{d}y = \int \frac{9x+6}{x}\,\mathrm{d}x$	Integral signs not necessary	B1	
	$\displaystyle\int y^{-\frac{1}{3}}\,\mathrm{d}y = \int \frac{9x+6}{x}\,\mathrm{d}x$			
	$\dfrac{y^{\frac{2}{3}}}{\frac{2}{3}} = 9x + 6\ln x \;\; (+C)$	$\pm k y^{\frac{2}{3}} =$ their (a)	M1	
	$\dfrac{3}{2} y^{\frac{2}{3}} = 9x + 6\ln x \;\; (+C)$	ft their (a)	A1ft	
	$y = 8,\; x = 1$			
	$\dfrac{3}{2} 8^{\frac{2}{3}} = 9 + 6\ln 1 + C$		M1	
	$C = -3$		A1	
	$y^{\frac{2}{3}} = \dfrac{2}{3}(9x + 6\ln x - 3)$			
	$y^2 = (6x + 4\ln x - 2)^3 \quad \left(= 8(3x + 2\ln x - 1)^3\right)$		A1	(6)
				[8]

Question Number	Scheme	Marks
146	(a) $y=0 \Rightarrow t(9-t^2)=t(3-t)(3+t)=0$	
	$t=0, 3, -3$ Any one correct value	B1
	At $t=0$, $x=5(0)^2-4=-4$ Method for finding one value of x	M1
	At $t=3$, $x=5(3)^2-4=41$	
	$\left(\text{At } t=-3,\ x=5(-3)^2-4=41\right)$	
	At A, $x=-4$; at B, $x=41$ Both	A1 (3)
	(b) $\frac{\mathrm{d}x}{\mathrm{d}t}=10t$ Seen or implied	B1
	$\int y\,\mathrm{d}x=\int y\frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t=\int t(9-t^2)10t\,\mathrm{d}t$	M1 A1
	$=\int(90t^2-10t^4)\mathrm{d}t$	
	$=\frac{90t^3}{3}-\frac{10t^5}{5}\ (+C)\quad \left(=30t^3-2t^5\ (+C)\right)$	A1
	$\left[\frac{90t^3}{3}-\frac{10t^5}{5}\right]_0^3=30\times3^3-2\times3^5\quad(=324)$	M1
	$A=2\int y\,\mathrm{d}x=648\quad(\text{units}^2)$	A1 (6)
		[9]

Question Number	Scheme		Marks	
147	(a) $\dfrac{dx}{du} = -2\sin u$		B1	
	$\displaystyle\int \frac{1}{x^2\sqrt{4-x^2}}\,dx = \int \frac{1}{(2\cos u)^2\sqrt{4-(2\cos u)^2}} \times -2\sin u\,du$		M1	
	$\displaystyle= \int \frac{-2\sin u}{4\cos^2 u\sqrt{4\sin^2 u}}\,du$	Use of $1-\cos^2 u = \sin^2 u$	M1	
	$\displaystyle= -\frac{1}{4}\int \frac{1}{\cos^2 u}\,du$	$\displaystyle\pm k\int \frac{1}{\cos^2 u}\,du$	M1	
	$= -\dfrac{1}{4}\tan u \quad (+C)$	$\pm k\tan u$	M1	
	$x = \sqrt{2} \Rightarrow \sqrt{2} = 2\cos u \Rightarrow u = \dfrac{\pi}{4}$			
	$x = 1 \Rightarrow 1 = 2\cos u \Rightarrow u = \dfrac{\pi}{3}$		M1	
	$\left[-\dfrac{1}{4}\tan u\right]_{\frac{\pi}{3}}^{\frac{\pi}{4}} = -\dfrac{1}{4}\left(\tan\dfrac{\pi}{4} - \tan\dfrac{\pi}{3}\right)$			
	$= -\dfrac{1}{4}\left(1-\sqrt{3}\right) \quad \left(= \dfrac{\sqrt{3}-1}{4}\right)$		A1	(7)
	(b) $\displaystyle V = \pi\int_1^{\sqrt{2}} \left(\frac{4}{x\left(4-x^2\right)^{\frac{1}{4}}}\right)^2 dx$		M1	
	$\displaystyle= 16\pi\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4-x^2}}\,dx$	$16\pi \times$ integral in (a)	M1	
	$= 16\pi\left(\dfrac{\sqrt{3}-1}{4}\right)$	$16\pi \times$ their answer to part (a)	A1ft	(3)
				[10]

Question Number	Scheme	Marks
148 (a)	1.14805 awrt 1.14805	B1 (1)
(b)	$A \approx \frac{1}{2} \times \frac{3\pi}{8}(\ \dots\ )$	B1
	$= \dots \left(3+2\left(2.77164+2.12132+1.14805\right)+0\right)$ 0 can be implied	M1
	$= \frac{3\pi}{16}\left(3+2\left(2.77164+2.12132+1.14805\right)\right)$ ft their (a)	A1ft
	$= \frac{3\pi}{16} \times 15.08202\ \dots\ = 8.884$ cao	A1 (4)
(c)	$\int 3\cos\left(\frac{x}{3}\right)\mathrm{d}x = \frac{3\sin\left(\frac{x}{3}\right)}{\frac{1}{3}}$	M1 A1
	$= 9\sin\left(\frac{x}{3}\right)$	
	$A = \left[9\sin\left(\frac{x}{3}\right)\right]_0^{\frac{3\pi}{2}} = 9 - 0 = 9$ cao	A1 (3)
		[8]

Question Number	Scheme	Marks
149 (a)	$\mathrm{f}(x)=\dfrac{4-2x}{(2x+1)(x+1)(x+3)}=\dfrac{A}{2x+1}+\dfrac{B}{x+1}+\dfrac{C}{x+3}$	
	$4-2x=A(x+1)(x+3)+B(2x+1)(x+3)+C(2x+1)(x+1)$	M1
	A method for evaluating one constant	M1
	$x\to-\frac{1}{2},\quad 5=A\left(\frac{1}{2}\right)\left(\frac{5}{2}\right)\ \Rightarrow\ A=4$ any one correct constant	A1
	$x\to-1\ ,\quad 6=B(-1)(2)\ \Rightarrow\ B=-3$	
	$x\to-3,\ 10=C(-5)(-2)\ \Rightarrow\ C=1$ all three constants correct	A1 (4)
(b)	(i) $\displaystyle\int\left(\frac{4}{2x+1}-\frac{3}{x+1}+\frac{1}{x+3}\right)\mathrm{d}x$	
	$=\dfrac{4}{2}\ln(2x+1)-3\ln(x+1)+\ln(x+3)+C$ A1 two ln terms correct	M1 A1ft
	All three ln terms correct and "+C" ; ft constants	A1ft (3)
	(ii) $\left[2\ln(2x+1)-3\ln(x+1)+\ln(x+3)\right]_0^2$	
	$=(2\ln5-3\ln3+\ln5)-(2\ln1-3\ln1+\ln3)$	M1
	$=3\ln5-4\ln3$	
	$=\ln\left(\dfrac{5^3}{3^4}\right)$	M1
	$=\ln\left(\dfrac{125}{81}\right)$	A1 (3)
		[10]

Question Number	Scheme	Marks
150 (a)	$\int \sqrt{(5-x)}\,\mathrm{d}x = \int (5-x)^{\frac{1}{2}}\,\mathrm{d}x = \dfrac{(5-x)^{\frac{3}{2}}}{-\frac{3}{2}} \quad (+C)$ $\left(= -\dfrac{2}{3}(5-x)^{\frac{3}{2}} + C\right)$	M1 A1 (2)
(b)	(i) $\int (x-1)\sqrt{(5-x)}\,\mathrm{d}x = -\dfrac{2}{3}(x-1)(5-x)^{\frac{3}{2}} + \dfrac{2}{3}\int (5-x)^{\frac{3}{2}}\,\mathrm{d}x$	M1 A1ft
	$= \quad \dots \quad + \dfrac{2}{3} \times \dfrac{(5-x)^{\frac{5}{2}}}{-\frac{5}{2}} \quad (+C)$	M1
	$= -\dfrac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \dfrac{4}{15}(5-x)^{\frac{5}{2}} \quad (+C)$	A1 (4)
	(ii) $\left[-\dfrac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \dfrac{4}{15}(5-x)^{\frac{5}{2}}\right]_1^5 = (0-0) - \left(0 - \dfrac{4}{15} \times 4^{\frac{5}{2}}\right)$	
	$= \dfrac{128}{15}\left(= 8\dfrac{8}{15} \approx 8.53\right)$ awrt 8.53	M1 A1 (2)
		[8]
	Alternatives for (b) *and* (c)	
	(b) $u^2 = 5-x \Rightarrow 2u\dfrac{\mathrm{d}u}{\mathrm{d}x} = -1 \quad \left(\Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u} = -2u\right)$	
	$\int (x-1)\sqrt{(5-x)}\,\mathrm{d}x = \int (4-u^2)u\dfrac{\mathrm{d}x}{\mathrm{d}u}\,\mathrm{d}u = \int (4-u^2)u(-2u)\,\mathrm{d}u$	M1 A1
	$= \int (2u^4 - 8u^2)\,\mathrm{d}u = \dfrac{2}{5}u^5 - \dfrac{8}{3}u^3 \quad (+C)$	M1
	$= \dfrac{2}{5}(5-x)^{\frac{5}{2}} - \dfrac{8}{3}(5-x)^{\frac{3}{2}} \quad (+C)$	A1
	(c) $x = 1 \Rightarrow u = 2, \quad x = 5 \Rightarrow u = 0$	
	$\left[\dfrac{2}{5}u^5 - \dfrac{8}{3}u^3\right]_2^0 = (0-0) - \left(\dfrac{64}{5} - \dfrac{64}{3}\right)$	M1
	$= \dfrac{128}{15}\left(= 8\dfrac{8}{15} \approx 8.53\right)$ awrt 8.53	A1 (2)

EXPERT TUITION

Question Number	Scheme	Marks
151 (a)	$\int \sin^2\theta \, d\theta = \frac{1}{2}\int (1-\cos 2\theta)\, d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \quad (+C)$	M1 A1 (2)
(b)	$x = \tan\theta \Rightarrow \frac{dx}{d\theta} = \sec^2\theta$	
	$\pi\int y^2\, dx = \pi\int y^2 \frac{dx}{d\theta}\, d\theta = \pi\int (2\sin 2\theta)^2 \sec^2\theta\, d\theta$	M1 A1
	$= \pi\int \frac{(2\times 2\sin\theta\cos\theta)^2}{\cos^2\theta}\, d\theta$	M1
	$= 16\pi\int \sin^2\theta\, d\theta$ $\quad k = 16\pi$	A1
	$x = 0 \Rightarrow \tan\theta = 0 \Rightarrow \theta = 0, \quad x = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$	B1 (5)
	$\left(V = 16\pi\int_0^{\frac{\pi}{6}} \sin^2\theta\, d\theta\right)$	
(c)	$V = 16\pi\left[\frac{1}{2}\theta - \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{6}}$	M1
	$= 16\pi\left[\left(\frac{\pi}{12} - \frac{1}{4}\sin\frac{\pi}{3}\right) - (0-0)\right]$ $\quad$ Use of correct limits	M1
	$= 16\pi\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) = \frac{4}{3}\pi^2 - 2\pi\sqrt{3}$ $\quad p = \frac{4}{3},\ q = -2$	A1 (3)
		[10]

EXPERT TUITION

Question Number	Scheme		Marks
152. (a)	Area(R) = $\int_0^2 \frac{3}{\sqrt{(1+4x)}}\, dx = \int_0^2 3(1+4x)^{-\frac{1}{2}}\, dx$		
	$= \left[\frac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2}.4}\right]_0^2$	***Integrating*** $3(1+4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$.	M1
		$\underline{\text{Correct integration.}}$ Ignore limits.	$\underline{\text{A1}}$
	$= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}}\right]_0^2$		
	$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$	Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round.	M1
	$= \frac{9}{2} - \frac{3}{2} = \underline{3}\,(\text{units})^2$	$\underline{3}$	$\underline{\text{A1}}$
			[4]
	(Answer of 3 with no working scores M0A0M0A0.)		
(b)	Volume $= \underline{\pi \int_0^2 \left(\frac{3}{\sqrt{(1+4x)}}\right)^2}\, dx$	Use of $\underline{V = \pi \int y^2}\, dx$. Can be implied. Ignore limits and dx.	B1
	$= (\pi) \int_0^2 \frac{9}{1+4x}\, dx$		
	$= (\pi)\left[\frac{9}{4}\ln\lvert 1+4x\rvert\right]_0^2$	$\pm k\ln\lvert 1+4x\rvert$	M1
		$\frac{9}{4}\ln\lvert 1+4x\rvert$	A1
	$= (\pi)\left[\left(\frac{9}{4}\ln 9\right) - \left(\frac{9}{4}\ln 1\right)\right]$	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1
	So Volume $= \underline{\frac{9}{4}\pi\ln 9}$	$\underline{\frac{9}{4}\pi\ln 9}$ or $\underline{\frac{9}{2}\pi\ln 3}$ or $\underline{\frac{18}{4}\pi\ln 3}$	A1 oe isw
			[5]
			9 marks

Note the answer must be a one term exact value. Note, also you can ignore subsequent working here.

Note that $\ln 1$ can be implied as equal to 0.

Note that $= \frac{9}{4}\pi\ln 9 + c$ (oe.) would be awarded the final A0.

EXPERT TUITION

Question Number	Scheme		Marks
153. (a)	$\int \tan^2 x \, dx$		
	$\left[NB: \underline{\sec^2 A = 1 + \tan^2 A} \text{ gives } \underline{\tan^2 A = \sec^2 A - 1} \right]$	The correct underlined identity.	M1 oe
	$= \int \sec^2 x - 1 \, dx$		
	$= \underline{\tan x} - x \, (+c)$	Correct integration with/without + c	A1
			[2]
(b)	$\int \frac{1}{x^3} \ln x \, dx$		
	$\begin{cases} u = \ln x & \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^{-3} & \Rightarrow v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{cases}$		
	$= -\frac{1}{2x^2} \ln x - \int -\frac{1}{2x^2} \cdot \frac{1}{x} \, dx$	Use of 'integration by parts' formula in the correct direction.	M1
		Correct expression.	A1
	$= -\frac{1}{2x^2} \ln x + \frac{1}{2} \int \frac{1}{x^3} \, dx$	An attempt to multiply through $\frac{k}{x^n}$, $n \in \square$, $n \ldots 2$ by $\frac{1}{x}$ and an attempt to ...	
	$= \underline{-\frac{1}{2x^2} \ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)} \, (+c)$	… "integrate"(process the result);	M1
		correct solution with/without + c	A1 oe
			[4]

Correct direction means that $u = \ln x$.

Question Number	Scheme		Marks
(c)	$\int \frac{e^{3x}}{1+e^x}\,dx$		
	$\left\{u = 1+e^x \Rightarrow \underline{\frac{du}{dx} = e^x},\ \underline{\frac{dx}{du} = \frac{1}{e^x}},\ \underline{\frac{dx}{du} = \frac{1}{u-1}}\right\}$	Differentiating to find any one of the <u>three underlined</u>	<u>B1</u>
	$= \int \frac{e^{2x}.e^x}{1+e^x}\,dx = \int \frac{(u-1)^2.e^x}{u}.\frac{1}{e^x}\,du$ or $= \int \frac{(u-1)^3}{u}.\frac{1}{(u-1)}\,du$	Attempt to substitute for $e^{2x} = f(u)$, their $\frac{dx}{du} = \frac{1}{e^x}$ and $u = 1+e^x$ or $e^{3x} = f(u)$, their $\frac{dx}{du} = \frac{1}{u-1}$ and $u = 1+e^x$.	M1*
	$= \int \frac{(u-1)^2}{u}\,du$	$\underline{\int \frac{(u-1)^2}{u}\,du}$	A1
	$= \int \frac{u^2-2u+1}{u}\,du$ $= \int u - 2 + \frac{1}{u}\,du$	An attempt to multiply out their numerator to give at least three terms and divide through each term by u	dM1*
	$= \frac{u^2}{2} - 2u + \ln u\ (+c)$	Correct integration with/without +c	A1
	$= \frac{(1+e^x)^2}{2} - 2(1+e^x) + \ln(1+e^x) + c$	Substitutes $u = 1+e^x$ back into their integrated expression with at least two terms.	dM1*
	$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$		
	$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$		
	$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) - \frac{3}{2} + c$		
	$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k$ **AG**	$\underline{\frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k}$ must use a $+c$ and "$-\frac{3}{2}$" combined.	A1 **cso**
			[7]
			13 marks

Question Number	Scheme		Marks
154. (a)	$\left\{\begin{matrix} u = x & \Rightarrow & \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x & \Rightarrow & v = e^x \end{matrix}\right\}$		
	$\int x e^x \, dx = x e^x - \int e^x.1 \, dx$	Use of 'integration by parts' formula in the **correct direction**. (See note.) Correct expression. (Ignore dx)	M1 A1
	$= x e^x - \int e^x \, dx$		
	$= x e^x - e^x \; (+ c)$	Correct integration with/without + c	A1
			[3]
(b)	$\left\{\begin{matrix} u = x^2 & \Rightarrow & \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x & \Rightarrow & v = e^x \end{matrix}\right\}$		
	$\int x^2 e^x \, dx = x^2 e^x - \int e^x.2x \, dx$	Use of 'integration by parts' formula in the **correct direction**. Correct expression. (Ignore dx)	M1 A1
	$= x^2 e^x - 2\int x e^x \, dx$		
	$= x^2 e^x - 2\left(x e^x - e^x\right) + c$	Correct expression **including + c**. **(seen at any stage! in part (b))** You can ignore subsequent working.	A1 **ISW**
			[3]
	$\left\{\begin{matrix} = x^2 e^x - 2x e^x + 2e^x + c \\ = e^x\left(x^2 - 2x + 2\right) + c \end{matrix}\right\}$	*Ignore subsequent working*	
			6 marks

Note integration by parts in the **correct direction** means that u and $\frac{dv}{dx}$ must be assigned/used as $u = x$ and $\frac{dv}{dx} = e^x$ in part (a) for example

+ c is not required in part (a).

Question Number	Scheme		Marks
155. (a)	$\dfrac{2}{4-y^2} \equiv \dfrac{2}{(2-y)(2+y)} \equiv \dfrac{A}{(2-y)} + \dfrac{B}{(2+y)}$		
	$2 \equiv A(2+y) + B(2-y)$	Forming this identity. **NB**: A & B are not assigned in this question	M1
	Let $y=-2$, $2 = B(4) \Rightarrow B = \frac{1}{2}$		
	Let $y=2$, $2 = A(4) \Rightarrow A = \frac{1}{2}$	Either one of $A=\frac{1}{2}$ or $B=\frac{1}{2}$	A1
	giving $\underline{\dfrac{\frac{1}{2}}{(2-y)} + \dfrac{\frac{1}{2}}{(2+y)}}$	$\underline{\dfrac{\frac{1}{2}}{(2-y)} + \dfrac{\frac{1}{2}}{(2+y)}}$, aef	A1 **cao**
	(If n**o working seen**, but candidate writes down ***correct partial fraction*** then award all three marks. If no working is seen but one of A or B is incorrect then M0A0A0.)		**[3]**

EXPERT TUITION

<table>
<tr><th>Question Number</th><th>Scheme</th><th></th><th>Marks</th></tr>
<tr><td>155. (b)</td><td>$\int \frac{2}{4-y^2}\,dy = \int \frac{1}{\cot x}\,dx$</td><td>Separates variables as shown. Can be implied. Ignore the integral signs, and the '2'.</td><td>B1</td></tr>
<tr><td></td><td>$\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}\,dy = \int \tan x\,dx$</td><td></td><td></td></tr>
<tr><td></td><td>$\therefore\ -\tfrac{1}{2}\ln(2-y) + \tfrac{1}{2}\ln(2+y) = \ln(\sec x)\ \ + (c)$</td><td>$\ln(\sec x)$ or $-\ln(\cos x)$
Either $\pm a\ln(\lambda - y)$ or $\pm b\ln(\lambda + y)$
their $\int \frac{1}{\cot x}\,dx =$ LHS correct with ft for their A and B and no error with the "2" with or without $+c$</td><td>B1
M1;

A1 $\sqrt{}$</td></tr>
<tr><td></td><td>$y = 0,\ x = \frac{\pi}{3} \ \Rightarrow\ -\tfrac{1}{2}\ln 2 + \tfrac{1}{2}\ln 2 = \ln\left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right) + c$</td><td>Use of $y = 0$ and $x = \frac{\pi}{3}$ in an integrated equation containing c ;</td><td>M1*</td></tr>
<tr><td></td><td>$\left\{0 = \ln 2 + c \ \Rightarrow\ \underline{c = -\ln 2}\right\}$</td><td></td><td></td></tr>
<tr><td></td><td>$-\tfrac{1}{2}\ln(2-y) + \tfrac{1}{2}\ln(2+y) = \ln(\sec x) - \ln 2$</td><td></td><td></td></tr>
<tr><td></td><td>$\frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$</td><td>Using either the quotient (or product) or power laws for logarithms CORRECTLY.</td><td>M1</td></tr>
<tr><td></td><td>$\ln\left(\frac{2+y}{2-y}\right) = 2\ln\left(\frac{\sec x}{2}\right)$</td><td></td><td></td></tr>
<tr><td></td><td>$\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)^2$</td><td>Using the log laws correctly to obtain a single log term on both sides of the equation.</td><td>dM1*</td></tr>
<tr><td></td><td>$\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$</td><td></td><td></td></tr>
<tr><td></td><td>Hence, $\underline{\sec^2 x = \frac{8+4y}{2-y}}$</td><td>$\underline{\sec^2 x = \frac{8+4y}{2-y}}$</td><td>A1 aef

[8]</td></tr>
<tr><td></td><td></td><td></td><td>11 marks</td></tr>
</table>

Question Number	Scheme		Marks
156. (a)	At $P(4, 2\sqrt{3})$ either $\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	$\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	M1
	$\Rightarrow$ only solution is $\underline{t = \frac{\pi}{3}}$ where $0 \leqslant t \leqslant \frac{\pi}{2}$.	$\underline{t = \frac{\pi}{3}}$ or $\underline{\text{awrt } 1.05}$ (radians) only stated in the range $0 \leqslant t \leqslant \frac{\pi}{2}$.	A1
			[2]
(b)	$x = 8\cos t$, $y = 4\sin 2t$		
	$\frac{dx}{dt} = -8\sin t$, $\frac{dy}{dt} = 8\cos 2t$	Attempt to differentiate both x and y wrt t to give $\pm p\sin t$ and $\pm q\cos 2t$ respectively	M1
		Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	At P, $\frac{dy}{dx} = \underline{\frac{8\cos\left(\frac{2\pi}{3}\right)}{-8\sin\left(\frac{\pi}{3}\right)}}$	Divides in correct way round and attempts to substitute their value of t (in degrees or radians) into their $\frac{dy}{dx}$ expression.	M1*
	$\left\{= \frac{8\left(-\frac{1}{2}\right)}{(-8)\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58\right\}$	You may need to check candidate's substitutions for M1* **Note the next two method marks are dependent on M1***	
	Hence $\text{m}(\mathbf{N}) = -\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	Uses $\text{m}(\mathbf{N}) = -\frac{1}{\text{their m}(\mathbf{T})}$.	dM1*
	N: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$	Uses $y - 2\sqrt{3} = (\text{their } m_N)(x - 4)$ or finds c using $x = 4$ and $y = 2\sqrt{3}$ and uses $y = (\text{their m}_N)x + "c"$.	dM1*
	N: $\underline{y = -\sqrt{3}x + 6\sqrt{3}}$ **AG**	$\underline{y = -\sqrt{3}x + 6\sqrt{3}}$	A1 **cso AG**
	or $2\sqrt{3} = -\sqrt{3}(4) + c \Rightarrow c = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$ so **N**: $\left[\underline{y = -\sqrt{3}x + 6\sqrt{3}}\right]$		
			[6]

Question	Scheme		Marks
156. (c)	$A = \int_0^4 y \,\mathrm{d}x = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t.(-8\sin t)\,\mathrm{d}t$	attempt at $A = \int y \frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t$	M1
		correct expression (ignore limits and $\mathrm{d}t$)	A1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t.\sin t\,\mathrm{d}t = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2\sin t\cos t).\sin t\,\mathrm{d}t$	Seeing $\sin 2t = 2\sin t\cos t$ anywhere in PART (c).	M1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64.\sin^2 t\cos t\,\mathrm{d}t$ $A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64.\sin^2 t\cos t\,\mathrm{d}t$	Correct proof. Appreciation of how the negative sign affects the limits. **Note that the answer is given in the question.**	A1 **AG**
			[4]
(d)	{Using substitution $u = \sin t \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = \cos t$} {change limits: when $t = \frac{\pi}{3}$, $u = \frac{\sqrt{3}}{2}$ & when $t = \frac{\pi}{2}$, $u = 1$}		
	$A = 64\left[\frac{\sin^3 t}{3}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $A = 64\left[\frac{u^3}{3}\right]_{\frac{\sqrt{3}}{2}}^{1}$	$k\sin^3 t$ or ku^3 with $u = \sin t$	M1
		Correct integration ignoring limits.	A1
	$A = 64\left[\frac{1}{3} - \left(\frac{1}{3}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\right)\right]$	Substitutes limits of either $\left(t = \frac{\pi}{2} \text{ and } t = \frac{\pi}{3}\right)$ or $\left(u = 1 \text{ and } u = \frac{\sqrt{3}}{2}\right)$ and subtracts the correct way round.	dM1
	$A = 64\left(\frac{1}{3} - \frac{1}{8}\sqrt{3}\right) = \underline{\frac{64}{3} - 8\sqrt{3}}$	$\underline{\frac{64}{3} - 8\sqrt{3}}$ Aef in the form $a + b\sqrt{3}$, with awrt 21.3 and anything that cancels to $a = \frac{64}{3}$ and $b = -8$.	A1 aef **isw**
			[4]
	(Note that $a = \frac{64}{3}$, $b = -8$)		
			16 marks

EXPERT TUITION

Question Number	Scheme		Marks
157.	Volume $= \underline{\pi \int_a^b \left(\frac{1}{2x+1}\right)^2} \mathrm{d}x = \pi \int_a^b \frac{1}{(2x+1)^2} \mathrm{d}x$	Use of $V = \underline{\pi \int y^2} \mathrm{d}x$. Can be implied. Ignore limits.	B1
	$= \pi \int_a^b (2x+1)^{-2} \mathrm{d}x$		
	$= (\pi)\left[\frac{(2x+1)^{-1}}{(-1)(2)}\right]_a^b$		
	$= (\pi)\left[\underline{-\tfrac{1}{2}(2x+1)^{-1}}\right]_a^b$	Integrating to give $\underline{\pm p(2x+1)^{-1}}$ $\underline{-\tfrac{1}{2}(2x+1)^{-1}}$	M1 A1
	$= (\pi)\left[\left(\frac{-1}{2(2b+1)}\right) - \left(\frac{-1}{2(2a+1)}\right)\right]$	Substitutes limits of b and a and subtracts the correct way round.	dM1
	$= \frac{\pi}{2}\left[\frac{-2a-1+2b+1}{(2a+1)(2b+1)}\right]$		
	$= \frac{\pi}{2}\left[\frac{2(b-a)}{(2a+1)(2b+1)}\right]$		
	$= \frac{\pi(b-a)}{(2a+1)(2b+1)}$	$\underline{\frac{\pi(b-a)}{(2a+1)(2b+1)}}$	A1 aef
			[5]
			5 marks

Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b - \pi a}{4ab+2a+2b+1}.$$

Note that π is not required for the middle three marks of this question.

Question Number	Scheme		Marks
***Aliter* 157. Way 2**	Volume $= \pi \int_a^b \left(\frac{1}{2x+1}\right)^2 \mathrm{d}x = \pi \int_a^b \frac{1}{(2x+1)^2} \mathrm{d}x$	Use of $V = \pi \int y^2 \,\mathrm{d}x$. Can be implied. Ignore limits.	B1
	$= \pi \int_a^b (2x+1)^{-2} \,\mathrm{d}x$		
	Applying substitution $u = 2x+1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2$ and changing limits $x \to u$ so that $a \to 2a+1$ and $b \to 2b+1$, gives		
	$= (\pi) \int_{2a+1}^{2b+1} \frac{u^{-2}}{2} \,\mathrm{d}u$		
	$= (\pi)\left[\frac{u^{-1}}{(-1)(2)}\right]_{2a+1}^{2b+1}$		
	$= (\pi)\left[-\frac{1}{2}u^{-1}\right]_{2a+1}^{2b+1}$	Integrating to give $\pm pu^{-1}$	M1
		$-\frac{1}{2}u^{-1}$	A1
	$= (\pi)\left[\left(\frac{-1}{2(2b+1)}\right) - \left(\frac{-1}{2(2a+1)}\right)\right]$	Substitutes limits of $2b+1$ and $2a+1$ and subtracts the correct way round.	dM1
	$= \frac{\pi}{2}\left[\frac{-2a-1+2b+1}{(2a+1)(2b+1)}\right]$		
	$= \frac{\pi}{2}\left[\frac{2(b-a)}{(2a+1)(2b+1)}\right]$		
	$= \frac{\pi(b-a)}{(2a+1)(2b+1)}$	$\frac{\pi(b-a)}{(2a+1)(2b+1)}$	A1 aef
			[5]
			5 marks

Note that π is not required for the middle three marks of this question.

Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b - \pi a}{4ab+2a+2b+1}.$$

Question Number	Scheme		Marks
158. (i)	$\int \ln\left(\frac{x}{2}\right) \mathrm{d}x = \int 1.\ln\left(\frac{x}{2}\right) \mathrm{d}x \Rightarrow \left\{ \begin{array}{ll} u = \ln\left(\frac{x}{2}\right) & \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{\frac{1}{2}}{\frac{x}{2}} = \frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = 1 & \Rightarrow v = x \end{array} \right\}$		
	$\int \ln\left(\frac{x}{2}\right) \mathrm{d}x = x\ln\left(\frac{x}{2}\right) - \int x.\frac{1}{x}\, \mathrm{d}x$	Use of 'integration by parts' formula in the correct direction. Correct expression.	M1 A1
	$= x\ln\left(\frac{x}{2}\right) - \int \underline{1}\, \mathrm{d}x$	An attempt to multiply x by a candidate's $\frac{a}{x}$ or $\frac{1}{bx}$ or $\frac{1}{x}$.	dM1
	$= x\ln\left(\frac{x}{2}\right) - x + c$	Correct integration with + c	A1 aef **[4]**
(ii)	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\, \mathrm{d}x$		
	$\left[\text{NB}: \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \text{ or } \underline{\sin^2 x = \frac{1}{2}(\pm 1 \pm \cos 2x)}\right]$	Consideration of double angle formula for $\cos 2x$	M1
	$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2}\, \mathrm{d}x = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\underline{1 - \cos 2x})\, \mathrm{d}x$		
	$= \frac{1}{2}\left[\underline{x - \frac{1}{2}\sin 2x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	<u>Integrating to give</u> $\underline{\pm ax \pm b\sin 2x}$; $a, b \neq 0$ Correct result of anything equivalent to $\frac{1}{2}x - \frac{1}{4}\sin 2x$	dM1 A1
	$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin(\pi)}{2}\right) - \left(\frac{\pi}{4} - \frac{\sin\left(\frac{\pi}{2}\right)}{2}\right)\right]$ $= \frac{1}{2}\left[\left(\frac{\pi}{2} - 0\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)\right]$	Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi}{8} + \frac{1}{4}$	$\underline{\frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)}$ or $\underline{\frac{\pi}{8} + \frac{1}{4}}$ or $\underline{\frac{\pi}{8} + \frac{2}{8}}$ Candidate must collect their π term and constant term together for A1 No fluked answers, hence **cso**.	A1 **aef**, **cso** **[5]**
			9 marks

Note: $\int \ln\left(\frac{x}{2}\right) \mathrm{d}x = (\text{their } v)\ln\left(\frac{x}{2}\right) - \int (\text{their } v).\left(\text{their } \frac{\mathrm{d}u}{\mathrm{d}x}\right) \mathrm{d}x$ for M1 in part (i).

Note $\frac{\pi}{8} + \frac{1}{4} = 0.64269...$

Question Number	Scheme		Marks
Aliter **158.** (i) **Way 2**	$\int \ln\left(\frac{x}{2}\right) \mathrm{d}x = \int (\ln x - \ln 2)\, \mathrm{d}x = \int \ln x \,\mathrm{d}x - \int \ln 2 \,\mathrm{d}x$		
	$\int \ln x \,\mathrm{d}x = \int 1.\ln x \,\mathrm{d}x \Rightarrow \left\{ \begin{array}{ll} u = \ln x & \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = 1 & \Rightarrow v = x \end{array} \right\}$		
	$\int \ln x \,\mathrm{d}x = x\ln x - \int x.\frac{1}{x}\,\mathrm{d}x$	Use of 'integration by parts' formula in the correct direction.	M1
	$= x\ln x - x + c$	Correct integration of $\ln x$ with or without + c	A1
	$\int \ln 2 \,\mathrm{d}x = x\ln 2 + c$	Correct integration of $\ln 2$ with or without + c	M1
	Hence, $\int \ln\left(\frac{x}{2}\right) \mathrm{d}x = x\ln x - x - x\ln 2 + c$	Correct integration with + c	A1 aef
			[4]

Note: $\int \ln x \,\mathrm{d}x = (\text{their } v)\ln x - \int (\text{their } v).\left(\text{their } \frac{\mathrm{d}u}{\mathrm{d}x}\right) \mathrm{d}x$ for M1 in part (i).

Question Number	Scheme		Marks
Aliter **158.** (i) **Way 3**	$\int \ln\left(\frac{x}{2}\right)\, dx$		
	$u = \frac{x}{2} \Rightarrow \dfrac{du}{dx} = \dfrac{1}{2}$		
	$\int \ln\left(\frac{x}{2}\right)\, dx = 2\int \ln u\, du$	Applying substitution correctly to give $\int \ln\left(\frac{x}{2}\right)\, dx = 2\int \ln u\, du$ ***Decide to award 2nd M1 here!***	
	$\int \ln u\, dx = \int 1.\ln u\, du$		
	$\int \ln u\, dx = u\ln u - \int u.\frac{1}{u}\, du$	Use of 'integration by parts' formula in the correct direction.	M1
	$= u\ln u - u + c$	Correct integration of $\ln u$ with or without + c	A1
		Decide to award 2nd M1 here!	M1
	$\int \ln\left(\frac{x}{2}\right) dx = 2\left(u\ln u - u\right) + c$		
	Hence, $\int \ln\left(\frac{x}{2}\right)\, dx = x\ln\left(\frac{x}{2}\right) - x + c$	Correct integration with + c	A1 aef
			[4]

Question Number	Scheme		Marks
Aliter **158.** (ii) **Way 2**	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x.\sin x \, dx$ and $I = \int \sin^2 x \, dx$		
	$\left\{\begin{array}{l} u = \sin x \Rightarrow \frac{du}{dx} = \cos x \\ \frac{dv}{dx} = \sin x \Rightarrow v = -\cos x \end{array}\right\}$		
	$\therefore I = \underline{\left\{-\sin x \cos x + \int \cos^2 x \, dx\right\}}$	An attempt to use the correct by parts formula.	M1
	$\therefore I = \left\{-\sin x \cos x + \int (1 - \sin^2 x) dx\right\}$		
	$\int \sin^2 x \, dx = \left\{-\sin x \cos x + \int 1 \, dx - \int \sin^2 x \, dx\right\}$		
	$2\int \sin^2 x \, dx = \left\{-\sin x \cos x + \int 1 \, dx\right\}$	For the LHS becoming $2I$	dM1
	$2\int \sin^2 x \, dx = \{-\sin x \cos x + x\}$		
	$\int \sin^2 x \, dx = \left\{\underline{-\tfrac{1}{2}\sin x \cos x + \tfrac{x}{2}}\right\}$	<u>Correct integration</u>	A1
	$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \left[\left(-\tfrac{1}{2}\sin(\tfrac{\pi}{2})\cos(\tfrac{\pi}{2}) + \tfrac{(\frac{\pi}{2})}{2}\right) - \left(-\tfrac{1}{2}\sin(\tfrac{\pi}{4})\cos(\tfrac{\pi}{4}) + \tfrac{(\frac{\pi}{4})}{2}\right)\right]$ $= \left[(0 + \tfrac{\pi}{4}) - (-\tfrac{1}{4} + \tfrac{\pi}{8})\right]$	Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \frac{\pi}{8} + \frac{1}{4}$	$\underline{\tfrac{1}{2}\left(\tfrac{\pi}{4} + \tfrac{1}{2}\right)}$ or $\underline{\tfrac{\pi}{8} + \tfrac{1}{4}}$ or $\underline{\tfrac{\pi}{8} + \tfrac{2}{8}}$ Candidate must collect their π term and constant term together for A1 No fluked answers, hence **cso**.	A1 **aef cso** **[5]**

Note $\frac{\pi}{8} + \frac{1}{4} = 0.64269...$

Question Number	Scheme		Marks
159. (a)	$\left[x = \ln(t+2),\ \ y = \dfrac{1}{t+1}\right], \quad \Rightarrow \dfrac{dx}{dt} = \dfrac{1}{t+2}$	Must state $\dfrac{dx}{dt} = \dfrac{1}{t+2}$	B1
	$\text{Area}(R) = \displaystyle\int_{\ln 2}^{\ln 4} \frac{1}{t+1}\,dx ;= \int_0^2 \left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right)dt$	Area $= \displaystyle\int \frac{1}{t+1}\,dx$. Ignore limits.	M1;
		$\displaystyle\int \left(\frac{1}{t+1}\right)\times\left(\frac{1}{t+2}\right)dt$. Ignore limits.	A1 **AG**
	Changing limits, when: $x = \ln 2 \ \Rightarrow \ln 2 = \ln(t+2) \ \Rightarrow \ 2 = t+2 \ \Rightarrow t = 0$ $x = \ln 4 \ \Rightarrow \ln 4 = \ln(t+2) \ \Rightarrow \ 4 = t+2 \ \Rightarrow t = 2$	changes limits $x \to t$ so that $\ln 2 \to 0$ and $\ln 4 \to 2$	B1
	Hence, $\text{Area}(R) = \displaystyle\int_0^2 \frac{1}{(t+1)(t+2)}\,dt$		**[4]**
(b)	$\left(\dfrac{1}{(t+1)(t+2)}\right) = \dfrac{A}{(t+1)} + \dfrac{B}{(t+2)}$	$\dfrac{A}{(t+1)} + \dfrac{B}{(t+2)}$ with A and B found	M1
	$1 = A(t+2) + B(t+1)$		
	Let $t = -1,\quad 1 = A(1) \quad \Rightarrow \ \underline{A = 1}$ Let $t = -2,\quad 1 = B(-1) \ \Rightarrow \ \underline{B = -1}$	Finds both A and B correctly. Can be implied. (See note below)	A1
	$\displaystyle\int_0^2 \frac{1}{(t+1)(t+2)}\,dt = \int_0^2 \frac{1}{(t+1)} - \frac{1}{(t+2)}\,dt$		
	$= \left[\ln(t+1) - \ln(t+2)\right]_0^2$	Either $\pm a\ln(t+1)$ or $\pm b\ln(t+2)$ Both ln terms correctly ft.	dM1 A1 $\sqrt{}$
	$= (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	Substitutes ***both*** limits of 2 and 0 and subtracts the correct way round.	ddM1
	$= \ln 3 - \ln 4 + \ln 2 = \ln 3 - \ln 2 = \ln\left(\frac{3}{2}\right)$	$\underline{\ln 3 - \ln 4 + \ln 2}$ or $\underline{\ln\left(\frac{3}{4}\right) - \ln\left(\frac{1}{2}\right)}$ or $\underline{\ln 3 - \ln 2}$ or $\underline{\ln\left(\frac{3}{2}\right)}$ (must deal with ln 1)	A1 aef isw **[6]**

Takes out brackets.

Writing down $\dfrac{1}{(t+1)(t+2)} = \dfrac{1}{(t+1)} + \dfrac{1}{(t+2)}$ means first M1A0 in (b).

Writing down $\dfrac{1}{(t+1)(t+2)} = \dfrac{1}{(t+1)} - \dfrac{1}{(t+2)}$ means first M1A1 in (b).

Question Number	Scheme		Marks
	$x=\ln(t+2)$, $\quad y=\dfrac{1}{t+1}$		
159. (c)	$e^x=t+2 \Rightarrow t=e^x-2$	Attempt to make $t=\ldots$ the subject giving $t=e^x-2$	M1 A1
	$y=\dfrac{1}{e^x-2+1} \Rightarrow y=\dfrac{1}{e^x-1}$	Eliminates t by substituting in y giving $y=\dfrac{1}{e^x-1}$	dM1 A1
			[4]
Aliter **7.** (c) **Way 2**	$t+1=\dfrac{1}{y} \Rightarrow t=\dfrac{1}{y}-1$ or $t=\dfrac{1-y}{y}$ $y(t+1)=1 \Rightarrow yt+y=1 \Rightarrow yt=1-y \Rightarrow t=\dfrac{1-y}{y}$	Attempt to make $t=\ldots$ the subject Giving either $t=\dfrac{1}{y}-1$ or $t=\dfrac{1-y}{y}$	M1 A1
	$x=\ln\left(\dfrac{1}{y}-1+2\right)$ or $x=\ln\left(\dfrac{1-y}{y}+2\right)$	Eliminates t by substituting in x	dM1
	$x=\ln\left(\dfrac{1}{y}+1\right)$		
	$e^x=\dfrac{1}{y}+1$		
	$e^x-1=\dfrac{1}{y}$		
	$y=\dfrac{1}{e^x-1}$	giving $y=\dfrac{1}{e^x-1}$	A1
			[4]
(d)	Domain: $\underline{x>0}$	$\underline{x>0}$ or just >0	B1
			[1]
			15 marks

EXPERT TUITION

Question Number	Scheme		Marks
Aliter **159.** (c) **Way 3**	$e^x = t + 2 \;\Rightarrow\; t + 1 = e^x - 1$	Attempt to make $t+1=$ ... the subject	M1
		giving $t + 1 = e^x - 1$	A1
	$y = \dfrac{1}{t+1} \;\Rightarrow\; y = \dfrac{1}{e^x - 1}$	Eliminates t by substituting in y	dM1
		giving $y = \dfrac{1}{e^x - 1}$	A1
			[4]
Aliter **159.** (c) **Way 4**	$t+1 = \dfrac{1}{y} \;\Rightarrow\; t + 2 = \dfrac{1}{y} + 1$ or $t + 2 = \dfrac{1+y}{y}$	Attempt to make $t+2=$... the subject	M1
		Either $t+2 = \dfrac{1}{y} + 1$ or $t+2 = \dfrac{1+y}{y}$	A1
	$x = \ln\left(\dfrac{1}{y} + 1\right)$ or $x = \ln\left(\dfrac{1+y}{y}\right)$	Eliminates t by substituting in x	dM1
	$x = \ln\left(\dfrac{1}{y} + 1\right)$		
	$e^x = \dfrac{1}{y} + 1 \;\Rightarrow\; e^x - 1 = \dfrac{1}{y}$		
	$y = \dfrac{1}{e^x - 1}$	giving $y = \dfrac{1}{e^x - 1}$	A1
			[4]

Question Number	Scheme		Marks
160. (a)	$\frac{dV}{dt} = 1600 - c\sqrt{h}$ or $\frac{dV}{dt} = 1600 - k\sqrt{h}$,	Either of these statements	M1
	$\left(V = 4000h \ \Rightarrow\right) \frac{dV}{dh} = 4000$	$\frac{dV}{dh} = 4000$ or $\frac{dh}{dV} = \frac{1}{4000}$	M1
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dh}}$		
	Either, $\frac{dh}{dt} = \frac{1600 - c\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{c\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$ or $\frac{dh}{dt} = \frac{1600 - k\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{k\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$	Convincing proof of $\frac{dh}{dt}$	A1 **AG**
			[3]
(b)	When $h = 25$ water ***leaks out such that*** $\frac{dV}{dt} = 400$		
	$400 = c\sqrt{h} \Rightarrow 400 = c\sqrt{25} \Rightarrow 400 = c(5) \Rightarrow c = 80$		
	From above; $k = \frac{c}{4000} = \frac{80}{4000} = 0.02$ as required	Proof that $k = 0.02$	B1 **AG**
			[1]
Aliter (b) **Way 2**	$400 = 4000k\sqrt{h}$		
	$\Rightarrow 400 = 4000k\sqrt{25}$		
	$\Rightarrow 400 = k(20000) \Rightarrow k = \frac{400}{20000} = 0.02$	Using 400, 4000 and $h = 25$ or $\sqrt{h} = 5$. Proof that $k = 0.02$	B1 **AG**
			[1]
(c)	$\frac{dh}{dt} = 0.4 - k\sqrt{h} \Rightarrow \int \frac{dh}{0.4 - k\sqrt{h}} = \int dt$	***Separates the variables*** with $\int \frac{dh}{0.4 - k\sqrt{h}}$ and $\int dt$ on either side with integral signs not necessary.	M1 oe
	$\therefore$ time required $= \int_0^{100} \frac{1}{0.4 - 0.02\sqrt{h}}\, dh \quad \frac{\div 0.02}{\div 0.02}$		
	time required $= \int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh$	Correct proof	A1 **AG**
			[2]

EXPERT TUITION

Question Number	Scheme		Marks
160. (d)	$\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h$ with substitution $h=(20-x)^2$		
	$\frac{\mathrm{d}h}{\mathrm{d}x} = 2(20-x)(-1)$ or $\frac{\mathrm{d}h}{\mathrm{d}x} = -2(20-x)$	Correct $\frac{\mathrm{d}h}{\mathrm{d}x}$	B1 aef
	$h=(20-x)^2 \Rightarrow \sqrt{h}=20-x \Rightarrow x=20-\sqrt{h}$		
	$\int \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \int \frac{50}{x}.-2(20-x)\,\mathrm{d}x$	$\pm\lambda\int\frac{20-x}{x}\,\mathrm{d}x$ or $\pm\lambda\int\frac{20-x}{20-(20-x)}\,\mathrm{d}x$ where λ is a constant	M1
	$= 100\int\frac{x-20}{x}\,\mathrm{d}x$		
	$= 100\int\left(1-\frac{20}{x}\right)\mathrm{d}x$		
	$= 100\left(x-20\ln x\right)\ (+c)$	$\pm\alpha x \pm \beta\ln x\ ;\ \alpha,\beta \neq 0$ $100x-2000\ln x$	M1 A1
	change limits: when $h=0$ then $x=20$ and when $h=100$ then $x=10$		
	$\int_0^{100}\frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \left[100x-2000\ln x\right]_{20}^{10}$		
	or $\int_0^{100}\frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \left[100\left(20-\sqrt{h}\right)-2000\ln\left(20-\sqrt{h}\right)\right]_0^{100}$		
	$= \left(1000-2000\ln 10\right)-\left(2000-2000\ln 20\right)$	Correct use of limits, ie. putting them in the correct way round Either $x=10$ and $x=20$ or $h=100$ and $h=0$	ddM1
	$= 2000\ln 20 - 2000\ln 10 - 1000$	Combining logs to give...	
	$= 2000\ln 2 - 1000$	$\boxed{2000\ln 2 - 1000 \text{ or } -2000\ln\left(\tfrac{1}{2}\right)-1000}$	A1 **aef**
			[6]
(e)	Time required $= 2000\ln 2 - 1000 = 386.2943611...$ sec		
	= 386 seconds (nearest second)		
	= 6 minutes and 26 seconds (nearest second)	<u>6 minutes, 26 seconds</u>	B1
			[1]
			13 marks