EXPERT TUITION

Maths Questions By Topic:

Integration

A-Level Edexcel

0207 060 4494
www.expert-tuition.co.uk
online.expert-tuition.co.uk
enquiries@expert-tuition.co.uk
The Foundry, 77 Fulham Palace Road, W6 8JA

Table Of Contents

1. (a) Express $\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x$ as an integral.

(1)

(b) Hence show that

$$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x = \ln k$$

where k is a constant to be found.

(2)

Question 1 continued

(Total for Question 1 is 3 marks)

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Show that

$$\int_1^{e^2} x^3 \ln x \, dx = ae^8 + b$$

where a and b are rational constants to be found.

(5)

Question 2 continued

(Total for Question 2 is 5 marks)

3.

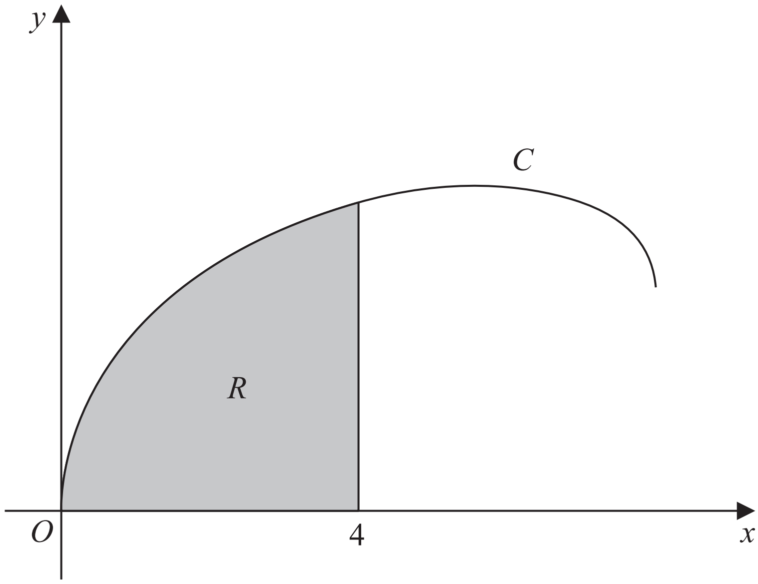


Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 8\sin^2 t \qquad y = 2\sin 2t + 3\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 6, is bounded by C, the x-axis and the line with equation $x = 4$

(a) Show that the area of R is given by

$$\int_0^a \left(8 - 8\cos 4t + 48\sin^2 t\cos t\right)\mathrm{d}t$$

where a is a constant to be found.

(5)

(b) Hence, using algebraic integration, find the exact area of R.

(4)

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

EXPERT TUITION

4. Find

$$\int\left(8x^3 - \frac{3}{2\sqrt{x}} + 5\right)\mathrm{d}x$$

giving your answer in simplest form.

(4)

Question 4 continued

(Total for Question 4 is 4 marks)

5.

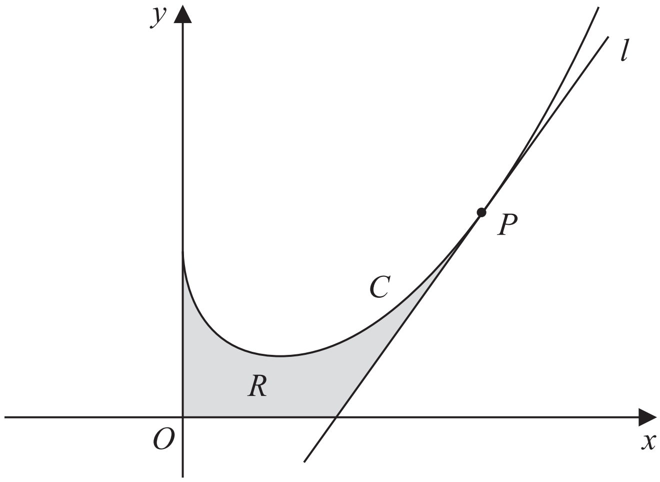


Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{1}{3}x^2 - 2\sqrt{x} + 3 \qquad x \geqslant 0$$

The point P lies on C and has x coordinate 4

The line l is the tangent to C at P.

(a) Show that l has equation

$$13x - 6y - 26 = 0$$

(5)

The region R, shown shaded in Figure 2, is bounded by the y-axis, the curve C, the line l and the x-axis.

(b) Find the exact area of R.

(5)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6.

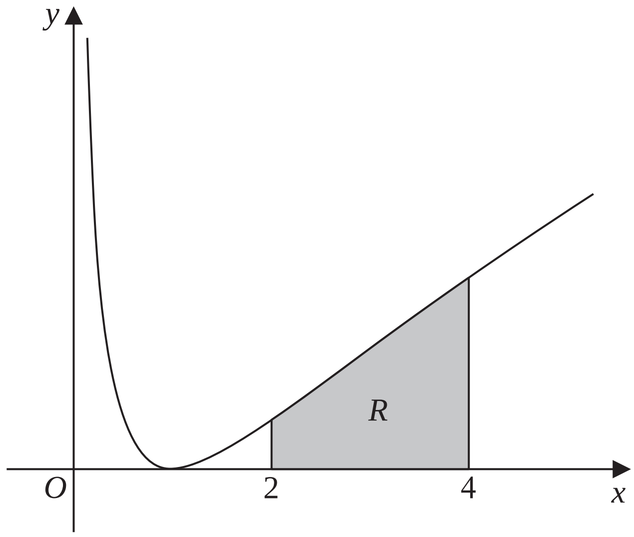


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = (\ln x)^2 \qquad x > 0$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

The table below shows corresponding values of x and y, with the values of y given to 4 decimal places.

x	2	2.5	3	3.5	4
y	0.4805	0.8396	1.2069	1.5694	1.9218

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of R, giving your answer to 3 significant figures.

(3)

(b) Use algebraic integration to find the exact area of R, giving your answer in the form

$$y = a(\ln 2)^2 + b\ln 2 + c$$

where a, b and c are integers to be found.

(5)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 8 marks)

7. Find

$$\int \frac{3x^4 - 4}{2x^3}\, dx$$

writing your answer in simplest form.

(4)

Question 7 continued

(Total for Question 7 is 4 marks)

8. Find the value of the constant k, $0 < k < 9$, such that

$$\int_k^9 \frac{6}{\sqrt{x}}\,dx = 20$$

(4)

Question 8 continued

(Total for Question 8 is 4 marks)

9. A curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = -3x^2 + 12x + 8$$

(a) Write $\mathrm{f}(x)$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The curve C has a maximum turning point at M.

(b) Find the coordinates of M.

(2)

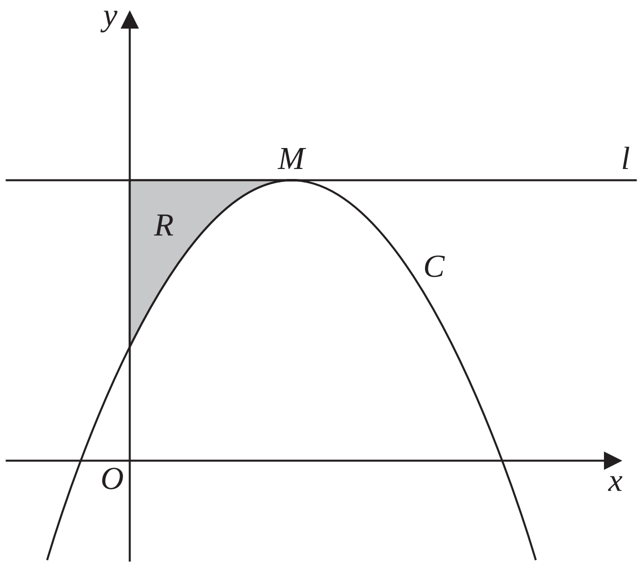


Figure 3

Figure 3 shows a sketch of the curve C.

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 3, is bounded by C, l and the y-axis.

(c) Using algebraic integration, find the area of R.

(5)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 10 marks)

10. (a) Use the substitution $x = u^2 + 1$ to show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \int_p^q \frac{6\,\mathrm{d}u}{u(3+2u)}$$

where p and q are positive constants to be found.

(4)

(b) Hence, using algebraic integration, show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \ln a$$

where a is a rational constant to be found.

(6)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 10 marks)

11. Given that k is a positive constant and $\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) \mathrm{d}x = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) \mathrm{d}x = 4$$

(4)

Question 11 continued

(Total for Question 11 is 8 marks)

12.

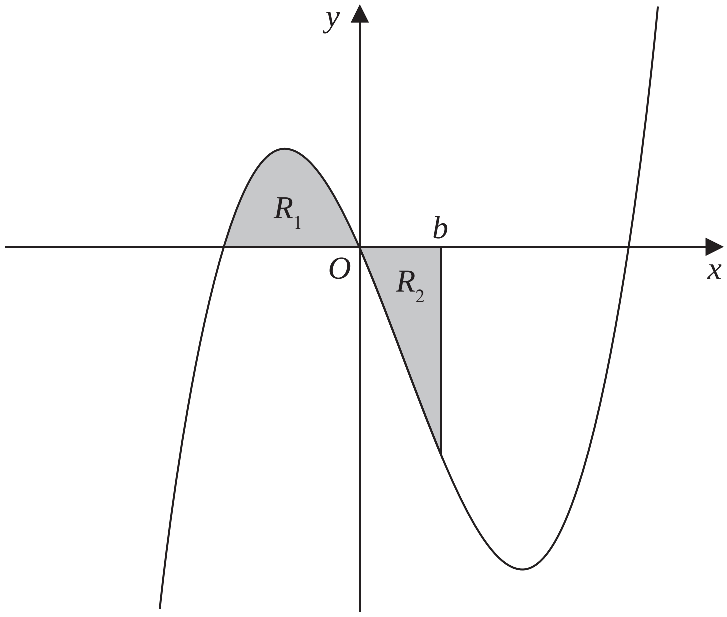


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\dfrac{20}{3}$

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places. The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

Question 12 continued

Question 12 continued

Question 12 continued

(Total for Question 12 is 10 marks)

13. The curve C with equation

$$y = \frac{p - 3x}{(2x - q)(x + 3)} \qquad x \in \mathbb{R}, x \neq -3, x \neq 2$$

where p and q are constants, passes through the point $\left(3, \frac{1}{2}\right)$ and has two vertical asymptotes with equations $x = 2$ and $x = -3$

(a) (i) Explain why you can deduce that $q = 4$

(ii) Show that $p = 15$

(3)

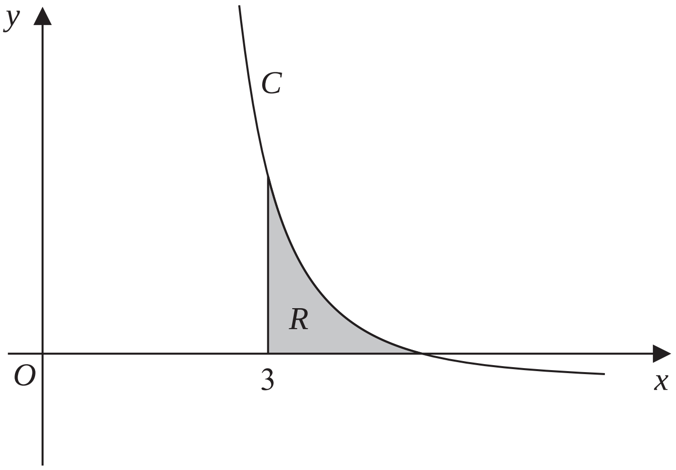


Figure 4

Figure 4 shows a sketch of part of the curve C. The region R, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the line with equation $x = 3$

(b) Show that the exact value of the area of R is $a\ln 2 + b\ln 3$, where a and b are rational constants to be found.

(8)

Question 13 continued

Question 13 continued

Question 13 continued

(Total for Question 13 is 11 marks)

14. (a) Given that k is a constant, find

$$\int\left(\frac{4}{x^3}+kx\right)\mathrm{d}x$$

simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2}\left(\frac{4}{x^3}+kx\right)\mathrm{d}x = 8$$

(3)

Question 14 continued

(Total for Question 14 is 6 marks)

15.

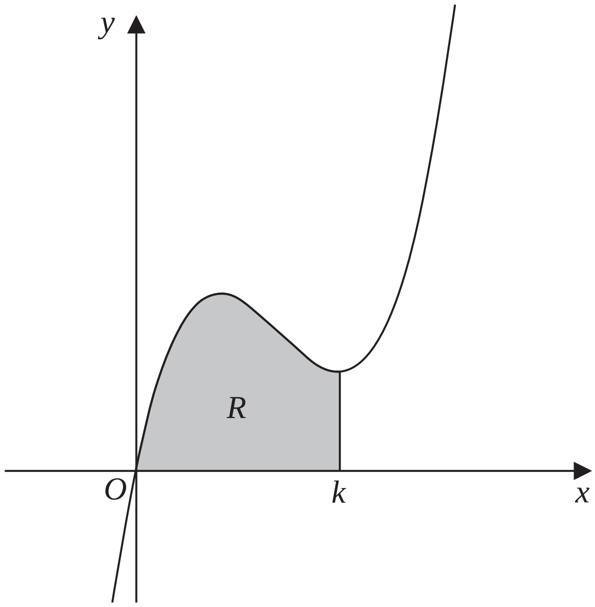


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

Show that the area of R is $\dfrac{256}{3}$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

Question 15 continued

Question 15 continued

Question 15 continued

(Total for Question 15 is 7 marks)

16. Given that $k \in \mathbb{Z}^+$

(a) show that $\displaystyle\int_k^{3k} \frac{2}{(3x-k)}\,dx$ is independent of k,

(4)

(b) show that $\displaystyle\int_k^{2k} \frac{2}{(2x-k)^2}\,dx$ is inversely proportional to k.

(3)

Question 16 continued

Question 16 continued

Question 16 continued

(Total for Question 16 is 7 marks)

17. The height above ground, H metres, of a passenger on a roller coaster can be modelled by the differential equation

$$\frac{dH}{dt} = \frac{H\cos(0.25t)}{40}$$

where t is the time, in seconds, from the start of the ride.

Given that the passenger is 5 m above the ground at the start of the ride,

(a) show that $H = 5e^{0.1\sin(0.25t)}$

(5)

(b) State the maximum height of the passenger above the ground.

(1)

The passenger reaches the maximum height, for the second time, T seconds after the start of the ride.

(c) Find the value of T.

(2)

Question 17 continued

Question 17 continued

Question 17 continued

(Total for Question 17 is 8 marks)

18. Show that

$$\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x = \frac{32}{15}\left(2+\sqrt{2}\right)$$

(7)

Question 18 continued

Question 18 continued

Question 18 continued

(Total for Question 18 is 7 marks)

19. Find

$$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x$$

giving your answer in its simplest form.

(4)

Question 19 continued

(Total for Question 19 is 4 marks)

20.

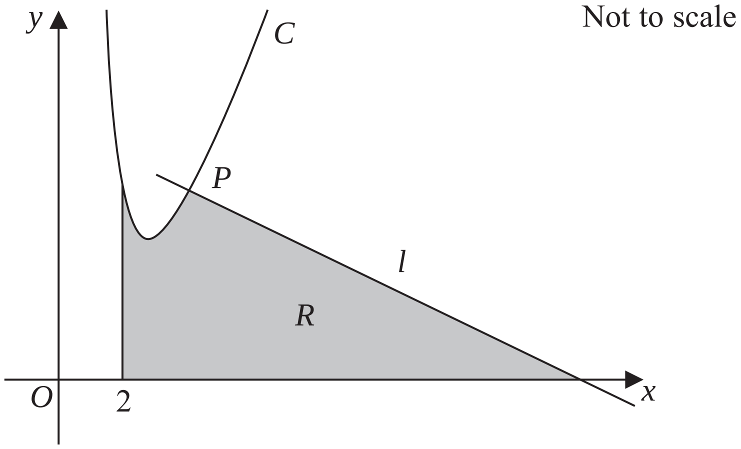


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{32}{x^2} + 3x - 8, \qquad x > 0$$

The point P (4, 6) lies on C.
The line l is the normal to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the line l, the curve C, the line with equation $x = 2$ and the x-axis.

Show that the area of R is 46
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

Question 20 continued

Question 20 continued

Question 20 continued

Question 20 continued

(Total for Question 20 is 10 marks)

21.

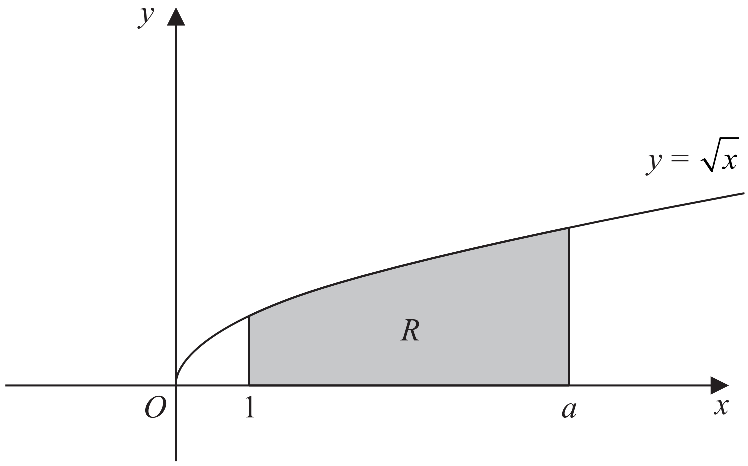


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{x}, \quad x \geqslant 0$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 1$, the x-axis and the line with equation $x = a$, where a is a constant.

Given that the area of R is 10

(a) find, in simplest form, the value of

(i) $\int_1^a \sqrt{8x}\, \mathrm{d}x$

(ii) $\int_0^a \sqrt{x}\, \mathrm{d}x$

(4)

(b) show that $a = 2^k$, where k is a rational constant to be found.

(4)

Question 21 continued

(Total for Question 21 is 8 marks)

22.

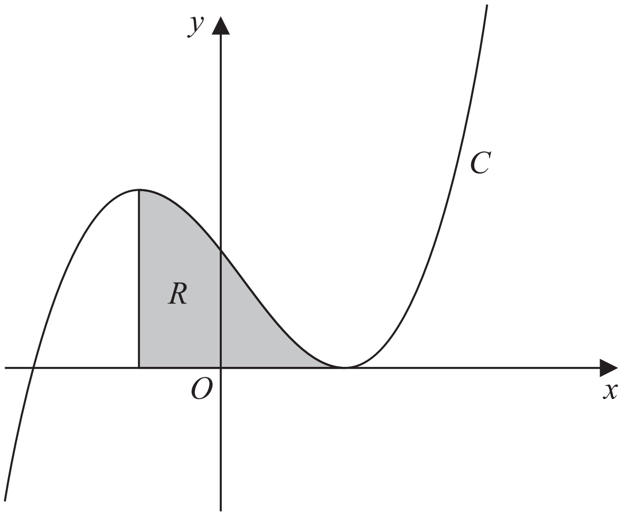


Figure 5

Figure 5 shows a sketch of the curve C with equation $y = (x - 2)^2(x + 3)$

The region R, shown shaded in Figure 5, is bounded by C, the vertical line passing through the maximum turning point of C and the x-axis.

Find the exact area of R.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(9)

Question 22 continued

Question 22 continued

(Total for Question 22 is 9 marks)

23. Given that

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\int_1^{2\sqrt{2}} f(x)\,dx = 16 + 3\sqrt{2}$

(5)

(Total for Question 23 is 5 marks)

24. Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\,dt = \ln 7$$

show that $a = \ln k$, where k is a constant to be found.

(4)

(Total for Question 24 is 4 marks)

25. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

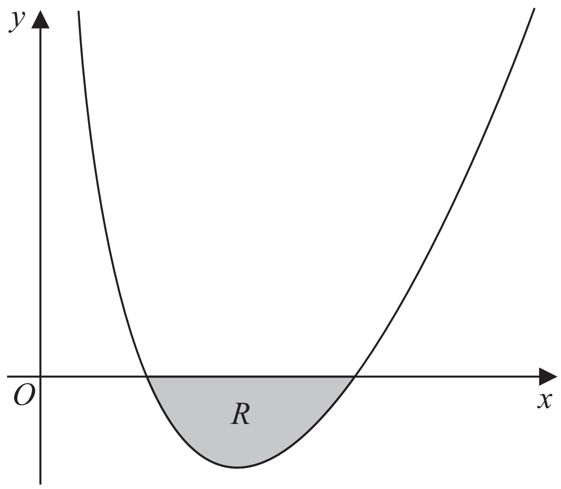


Figure 3

Figure 3 shows a sketch of part of a curve with equation

$$y = \frac{(x-2)(x-4)}{4\sqrt{x}} \qquad x > 0$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

Find the exact area of R, writing your answer in the form $a\sqrt{2} + b$, where a and b are constants to be found.

(6)

Question 25 continued

Question 25 continued

Question 25 continued

(Total for Question 25 is 6 marks)

26. (a) Express $\dfrac{3}{(2x-1)(x+1)}$ in partial fractions.

(3)

When chemical A and chemical B are mixed, oxygen is produced.

A scientist mixed these two chemicals and measured the total volume of oxygen produced over a period of time.

The total volume of oxygen produced, $V\,\text{m}^3$, t hours after the chemicals were mixed, is modelled by the differential equation

$$\frac{dV}{dt} = \frac{3V}{(2t-1)(t+1)} \qquad V \geqslant 0 \qquad t \geqslant k$$

where k is a constant.

Given that exactly 2 hours after the chemicals were mixed, a total volume of $3\,\text{m}^3$ of oxygen had been produced,

(b) solve the differential equation to show that

$$V = \frac{3(2t-1)}{(t+1)}$$

(5)

The scientist noticed that

- there was a **time delay** between the chemicals being mixed and oxygen being produced
- there was a **limit** to the total volume of oxygen produced

Deduce from the model

(c) (i) the **time delay** giving your answer in minutes,

(ii) the **limit** giving your answer in m^3

(2)

Question 26 continued

Question 26 continued

Question 26 continued

(Total for Question 26 is 10 marks)

27. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

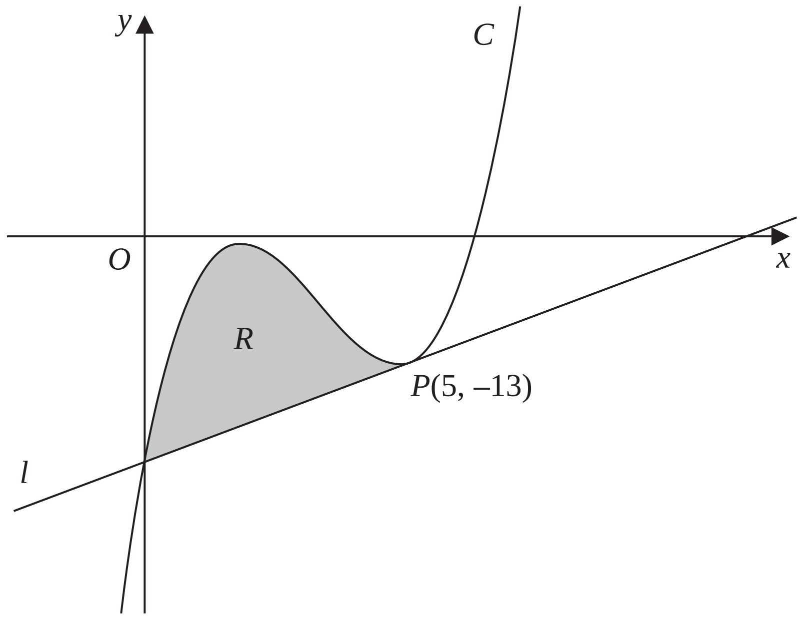


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on C

The line l is the tangent to C at P

(a) Use differentiation to find the equation of l, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(4)

(b) Hence verify that l meets C again on the y-axis.

(1)

The finite region R, shown shaded in Figure 2, is bounded by the curve C and the line l.

(c) Use algebraic integration to find the exact area of R.

(4)

Question 27 continued

Question 27 continued

Question 27 continued

(Total for Question 27 is 9 marks)

28. (a) Use the substitution $u = 1 + \sqrt{x}$ to show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\, dx = \int_p^q \frac{2(u-1)^3}{u}\, du$$

where p and q are constants to be found.

(3)

(b) Hence show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\, dx = A - B \ln 5$$

where A and B are constants to be found.

(4)

Question 28 continued

Question 28 continued

Question 28 continued

(Total for Question 28 is 7 marks)

29.

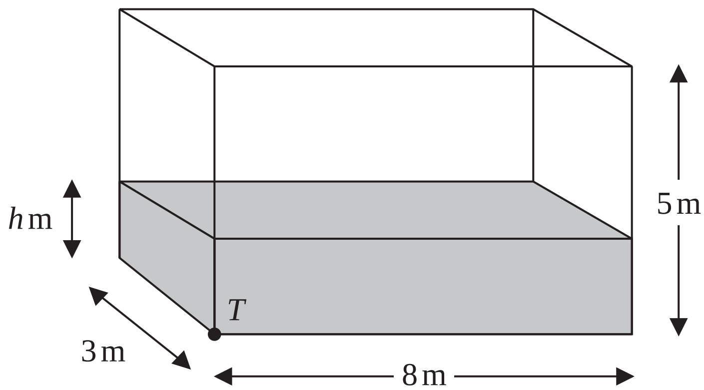


Figure 5

Water flows at a constant rate into a large tank.

The tank is a cuboid, with all sides of negligible thickness.

The base of the tank measures 8 m by 3 m and the height of the tank is 5 m.

There is a tap at a point T at the bottom of the tank, as shown in Figure 5.

At time t minutes after the tap has been opened

- the depth of water in the tank is h metres
- water is flowing into the tank at a constant rate of $0.48\,\text{m}^3$ per minute
- water is modelled as leaving the tank through the tap at a rate of $0.1h\,\text{m}^3$ per minute

(a) Show that, according to the model,

$$1200\frac{\mathrm{d}h}{\mathrm{d}t} = 24 - 5h$$

(4)

Given that when the tap was opened, the depth of water in the tank was 2 m,

(b) show that, according to the model,

$$h = A + B\mathrm{e}^{-kt}$$

where A, B and k are constants to be found.

(6)

Given that the tap remains open,

(c) determine, according to the model, whether the tank will ever become full, giving a reason for your answer.

(2)

Question 29 continued

Question 29 continued

Question 29 continued

(Total for Question 29 is 12 marks)

30. (a) Given that

$$\frac{x^2 + 8x - 3}{x + 2} \equiv Ax + B + \frac{C}{x + 2} \qquad x \in \mathbb{R} \;\; x \neq -2$$

find the values of the constants A, B and C

(3)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2 + 8x - 3}{x + 2}\, dx$$

giving your answer in the form $a + b\ln 2$ where a and b are integers to be found.

(4)

Question 30 continued

Question 30 continued

Question 30 continued

(Total for Question 30 is 7 marks)

31. A curve C has equation $y = \mathrm{f}(x)$

Given that

- $\mathrm{f}'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x + 4)$ is a factor of $\mathrm{f}(x)$

find, in simplest form, $\mathrm{f}(x)$

(6)

Question 31 continued

Question 31 continued

Question 31 continued

(Total for Question 31 is 6 marks)

32.

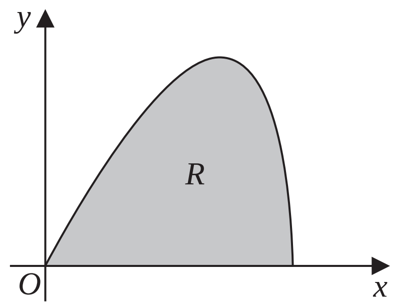


Figure 3

The curve shown in Figure 3 has parametric equations

$$x = 6\sin t \qquad y = 5\sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

(a) (i) Show that the area of R is given by $\displaystyle\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t \, dt$

(3)

(ii) Hence show, by algebraic integration, that the area of R is exactly 20

(3)

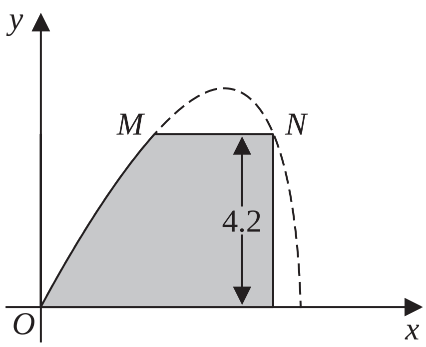


Figure 4

Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4.
Using the model and given that

- x and y are in metres
- the vertical wall of the dam is 4.2 metres high
- there is a horizontal walkway of width MN along the top of the dam

(b) calculate the width of the walkway.

(5)

Question 32 continued

Question 32 continued

Question 32 continued

(Total for Question 32 is 11 marks)

33.

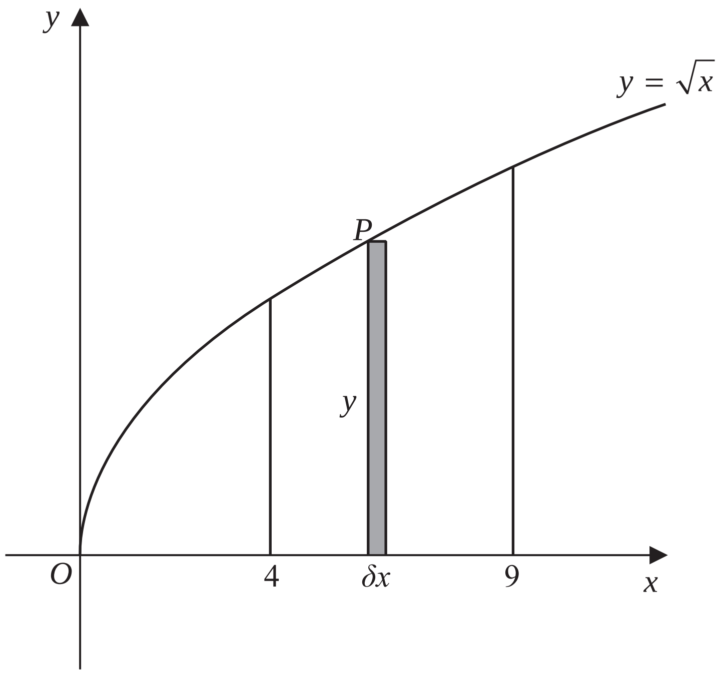


Figure 3

Figure 3 shows a sketch of the curve with equation $y = \sqrt{x}$

The point $P(x, y)$ lies on the curve.

The rectangle, shown shaded on Figure 3, has height y and width δx.

Calculate

$$\lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\, \delta x$$

(3)

Question 33 continued

(Total for Question 33 is 3 marks)

34. (a) Use the substitution $u = 4 - \sqrt{h}$ to show that

$$\int \frac{\mathrm{d}h}{4 - \sqrt{h}} = -8\ln\left|4 - \sqrt{h}\right| - 2\sqrt{h} + k$$

where k is a constant

(6)

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}\left(4 - \sqrt{h}\right)}{20}$$

where h is the height in metres and t is the time, measured in years, after the tree is planted.

(b) Find, according to the model, the range in heights of trees in this species.

(2)

One of these trees is one metre high when it is first planted.

According to the model,

(c) calculate the time this tree would take to reach a height of 12 metres, giving your answer to 3 significant figures.

(7)

Question 34 continued

Question 34 continued

Question 34 continued

Question 34 continued

(Total for Question 34 is 15 marks)

35.

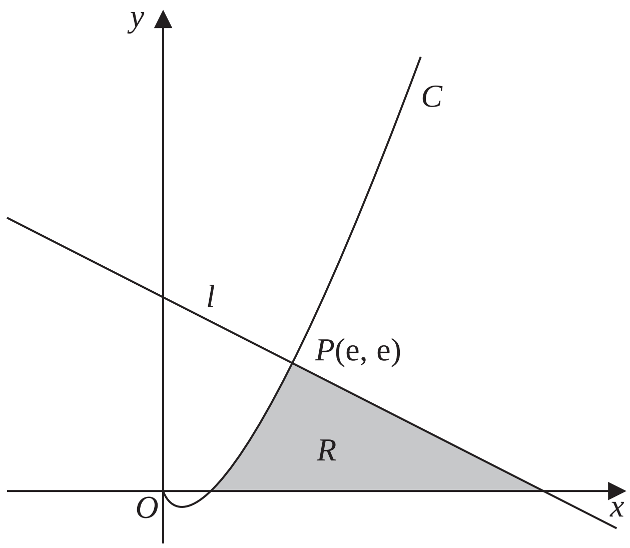


Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = x\ln x, \quad x > 0$

The line l is the normal to C at the point $P(\mathrm{e}, \mathrm{e})$

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the x-axis.

Show that the exact area of R is $A\mathrm{e}^2 + B$ where A and B are rational numbers to be found.

(10)

Question 35 continued

Question 35 continued

Question 35 continued

(Total for Question 35 is 10 marks)

36.

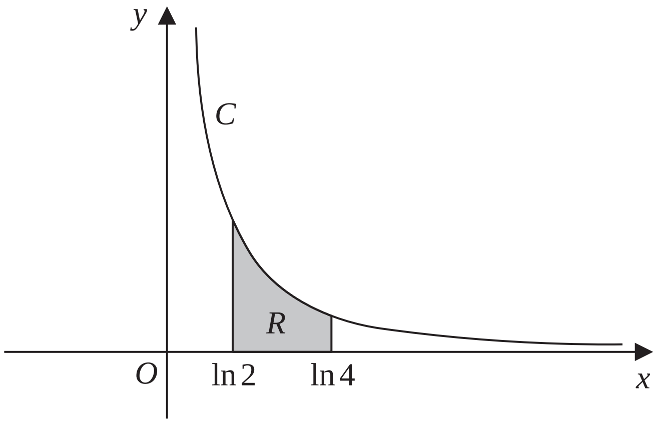


Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = \ln(t + 2), \quad y = \frac{1}{t+1}, \qquad t > -\frac{2}{3}$$

(a) State the domain of values of x for the curve C.

(1)

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = \ln 2$, the x-axis and the line with equation $x = \ln 4$

(b) Use calculus to show that the area of R is $\ln\left(\frac{3}{2}\right)$.

(8)

Question 36 continued

(Total for Question 36 is 9 marks)

37. Given that A is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right) \mathrm{d}x = 2A^2$$

show that there are exactly two possible values for A.

(5)

(Total for Question 37 is 5 marks)

38.

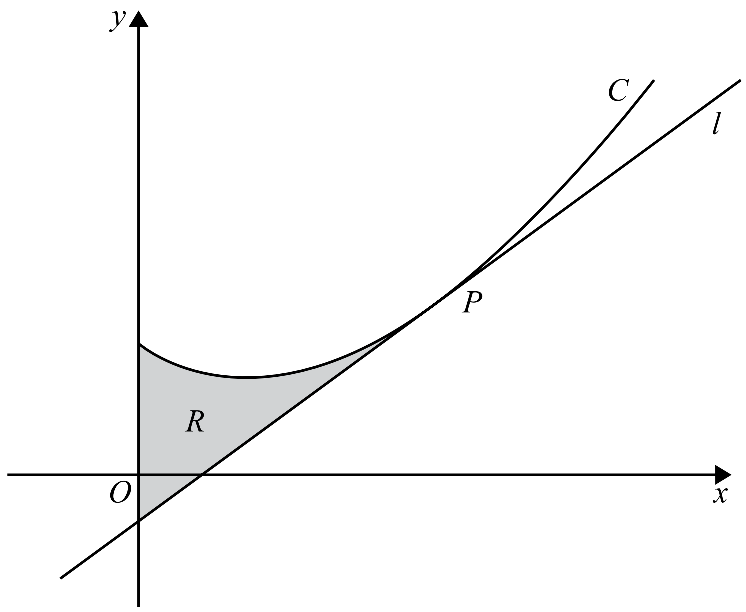


Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 5x^{\frac{3}{2}} - 9x + 11, \; x \geqslant 0$$

The point P with coordinates (4, 15) lies on C.

The line l is the tangent to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the curve C, the line l and the y-axis.

Show that the area of R is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

Question 38 continued

(Total for Question 38 is 10 marks)

39. (a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

(3)

A population of meerkats is being studied.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{22}P(11 - 2P), \quad t \geqslant 0, \qquad 0 < P < 5.5$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(6)

(c) show that

$$P = \frac{A}{B + Ce^{-\frac{1}{2}t}}$$

where A, B and C are integers to be found.

(3)

Question 39 continued

Question 39 continued

(Total for Question 39 is 12 marks)

Leave blank

40. Given

$$y = 3\sqrt{x} - 6x + 4, \qquad x > 0$$

find $\int y \, dx$, simplifying each term.

(3)

Leave blank

Question 40 continued

(Total 3 marks)

Leave blank

41. The curve C has equation $y = f(x)$, where

$$f'(x) = (x - 3)(3x + 5)$$

Given that the point P (1, 20) lies on C,

(a) find $f(x)$, simplifying each term.

(5)

(b) Show that

$$f(x) = (x - 3)^2(x + A)$$

where A is a constant to be found.

(3)

(c) Sketch the graph of C. Show clearly the coordinates of the points where C cuts or meets the x-axis and where C cuts the y-axis.

(4)

Leave blank

Question 41 continued

Leave blank

Question 41 continued

Leave blank

Question 41 continued

(Total 12 marks)

Leave blank

42. Find

$$\int\left(2x^5 - \frac{1}{4x^3} - 5\right)\mathrm{d}x$$

giving each term in its simplest form.

(4)

Leave blank

Question 42 continued

(Total 4 marks)

Leave blank

43. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, where

$$\mathrm{f}'(x) = 30 + \frac{6 - 5x^2}{\sqrt{x}}$$

Find $\mathrm{f}(x)$, giving each term in its simplest form.

(5)

Leave blank

Question 43 continued

(Total 5 marks)

Leave blank

44. Find

$$\int \left(2x^4 - \frac{4}{\sqrt{x}} + 3 \right) \mathrm{d}x$$

giving each term in its simplest form.

(4)

(Total 4 marks)

Leave blank

45. Given that $y = 4x^3 - \dfrac{5}{x^2}$, $x \neq 0$, find in their simplest form

$$\int y \, dx$$

(3)

(Total 3 marks)

Leave blank

46. A curve with equation $y = \mathrm{f}(x)$ passes through the point (4, 9).

Given that

$$\mathrm{f}'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2, \quad x > 0$$

find $\mathrm{f}(x)$, giving each term in its simplest form.

(5)

Leave blank

Question 46 continued

(Total 5 marks)

Leave blank

47. Find

$$\int (8x^3 + 4)\,dx$$

giving each term in its simplest form.

(3)

(Total 3 marks)

Leave blank

48. A curve with equation $y = f(x)$ passes through the point (4, 25).

Given that

$$f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1, \qquad x > 0$$

find $f(x)$, simplifying each term.

(5)

Leave blank

Question 48 continued

(Total 5 marks)

Leave blank

49. Given that $y = 2x^5 + \frac{6}{\sqrt{x}}$, $x > 0$, find in their simplest form

$\int y \, dx$

(3)

(Total 3 marks)

Leave blank

50.

$$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + x\sqrt{x}, \quad x > 0$$

Given that $y = 37$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

(7)

(Total 7 marks)

Leave blank

51. Find

$$\int \left(10x^4 - 4x - \frac{3}{\sqrt{x}}\right) \mathrm{d}x$$

giving each term in its simplest form.

(4)

(Total 4 marks)

Leave blank

52.

$$f'(x) = \frac{(3 - x^2)^2}{x^2}, \quad x \neq 0$$

(a) Show that $f'(x) = 9x^{-2} + A + Bx^2$,

where A and B are constants to be found.

(3)

(b) Find $f''(x)$.

(2)

Given that the point (–3, 10) lies on the curve with equation $y = f(x)$,

(c) find $f(x)$.

(5)

Leave blank

Question 52 continued

(Total 10 marks)

EXPERT TUITION

Leave blank

53. Find

$$\int \left(3x^2 - \frac{4}{x^2}\right) dx$$

giving each term in its simplest form.

(4)

(Total 4 marks)

Leave blank

54. A curve has equation $y = f(x)$. The point P with coordinates (9, 0) lies on the curve.

Given that

$$f'(x) = \frac{x+9}{\sqrt{x}}, \qquad x > 0$$

find f(x).

(6)

(Total 6 marks)

Leave blank

55.

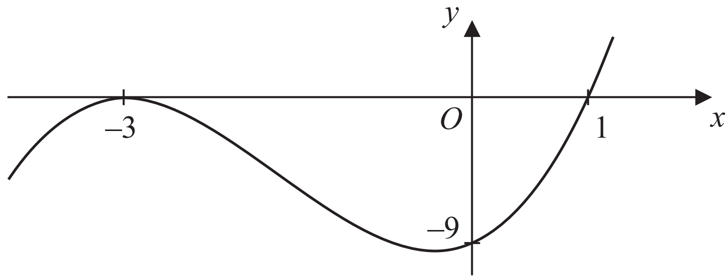


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (x + 3)^2 (x - 1), \qquad x \in \mathbb{R}.$$

The curve crosses the x-axis at $(1, 0)$, touches it at $(-3, 0)$ and crosses the y-axis at $(0, -9)$

(a) In the space below, sketch the curve C with equation $y = \mathrm{f}(x + 2)$ and state the coordinates of the points where the curve C meets the x-axis.

(3)

(b) Write down an equation of the curve C.

(1)

(c) Use your answer to part (b) to find the coordinates of the point where the curve C meets the y-axis.

(2)

Leave blank

Question 55 continued

(Total 6 marks)

Leave blank

56. Find

$$\int \left(6x^2 + \frac{2}{x^2} + 5 \right) dx$$

giving each term in its simplest form.

(4)

(Total 4 marks)

Leave blank

57. The point P (4, –1) lies on the curve C with equation $y = \mathrm{f}(x)$, $x > 0$, and

$$\mathrm{f}'(x) = \frac{1}{2}x - \frac{6}{\sqrt{x}} + 3$$

Find $\mathrm{f}(x)$.

(4)

(Total 4 marks)

Leave blank

58. Given that $y = x^4 + 6x^{\frac{1}{2}}$, find in their simplest form

$\int y \, dx$

(3)

(Total 3 marks)

EXPERT TUITION

Leave blank

59. A curve with equation $y = f(x)$ passes through the point (2, 10). Given that

$$f'(x) = 3x^2 - 3x + 5$$

find the value of $f(1)$.

(5)

(Total 5 marks)

Leave blank

60. Given that $y = 2x^5 + 7 + \frac{1}{x^3}$, $x \neq 0$, find, in their simplest form,

$\int y \, dx$.

(4)

(Total 4 marks)

Leave blank

61. Given that $\dfrac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^{p}+3x^{q}$,

(a) write down the value of p and the value of q.

(2)

Given that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$, and that $y = 90$ when $x = 4$,

(b) find y in terms of x, simplifying the coefficient of each term.

(5)

(Total 7 marks)

Leave blank

62. Find

$$\int (12x^5 - 3x^2 + 4x^{\frac{1}{3}})\ \mathrm{d}x$$

giving each term in its simplest form.

(5)

(Total 5 marks)

Leave blank

63. The curve with equation $y = \mathrm{f}(x)$ passes through the point $(-1, 0)$.

Given that

$$\mathrm{f}'(x) = 12x^2 - 8x + 1$$

find $\mathrm{f}(x)$.

(5)

(Total 5 marks)

Leave blank

64. Find

$$\int (8x^3 + 6x^{\frac{1}{2}} - 5)\,dx$$

giving each term in its simplest form.

(4)

(Total 4 marks)

Leave blank

65. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, where

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x - \frac{5}{\sqrt{x}} - 2$$

Given that the point $P(4, 5)$ lies on C, find

(a) $\mathrm{f}(x)$,

(5)

(b) an equation of the tangent to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Leave blank

Question 65 continued

(Total 9 marks)

Leave blank

66.
$$\frac{dy}{dx} = 5x^{-\frac{1}{2}} + x\sqrt{x}, \qquad x > 0$$

Given that $y = 35$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

(7)

(Total 7 marks)

Leave blank

67. Given that $y = 2x^3 + \frac{3}{x^2}$, $x \neq 0$, find

$\int y \, \mathrm{d}x$, simplifying each term.

(3)

(Total 3 marks)

Leave blank

68. Find $\int (12x^5 - 8x^3 + 3)\, dx$, giving each term in its simplest form.

(4)

(Total 4 marks)

Leave blank

69. A curve has equation $y = f(x)$ and passes through the point (4, 22).

Given that

$$f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find $f(x)$, giving each term in its simplest form.

(5)

(Total 5 marks)

Leave blank

70. Find $\int (2 + 5x^2)\,dx$.

(3)

(Total 3 marks)

Leave blank

71. The gradient of a curve C is given by $\frac{dy}{dx} = \frac{(x^2+3)^2}{x^2}$, $x \neq 0$.

(a) Show that $\frac{dy}{dx} = x^2 + 6 + 9x^{-2}$.

(2)

The point (3, 20) lies on C.

(b) Find an equation for the curve C in the form $y = f(x)$.

(6)

Leave blank

Question 71 continued

(Total 8 marks)

Leave blank

72. Find $\int (3x^2 + 4x^5 - 7)\,dx$.

(4)

(Total 4 marks)

Leave blank

73. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, and $\mathrm{f}'(x) = 4x - 6\sqrt{x} + \dfrac{8}{x^2}$.

Given that the point $P(4, 1)$ lies on C,

(a) find $\mathrm{f}(x)$ and simplify your answer.

(6)

(b) Find an equation of the normal to C at the point $P(4, 1)$.

(4)

Leave blank

Question 73 continued

(Total 10 marks)

Leave blank

74.

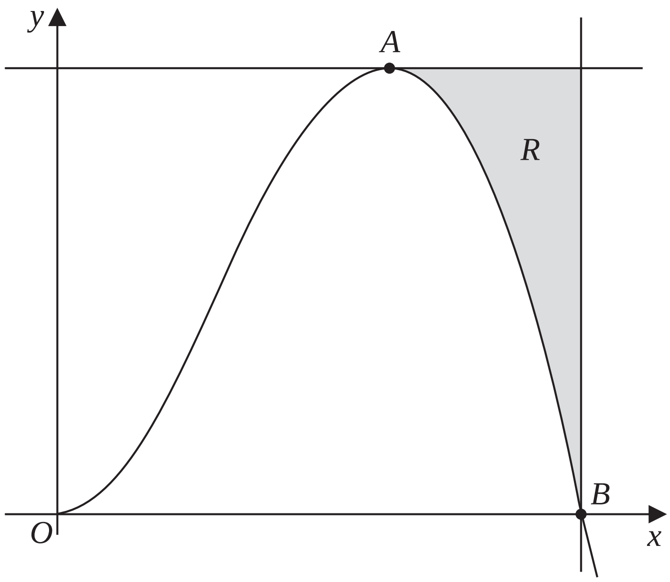


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 7x^2(5 - 2\sqrt{x}), \qquad x \geqslant 0$$

The curve has a turning point at the point A, where $x > 0$, as shown in Figure 3.

(a) Using calculus, find the coordinates of the point A.

(5)

The curve crosses the x-axis at the point B, as shown in Figure 3.

(b) Use algebra to find the x coordinate of the point B.

(2)

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line through A parallel to the x-axis and the line through B parallel to the y-axis.

(c) Use integration to find the area of the region R, giving your answer to 2 decimal places.

(5)

Leave blank

Question 74 continued

Leave blank

Question 74 continued

Leave blank

Question 74 continued

Leave blank

Question 74 continued

(Total 12 marks)

Leave blank

75.

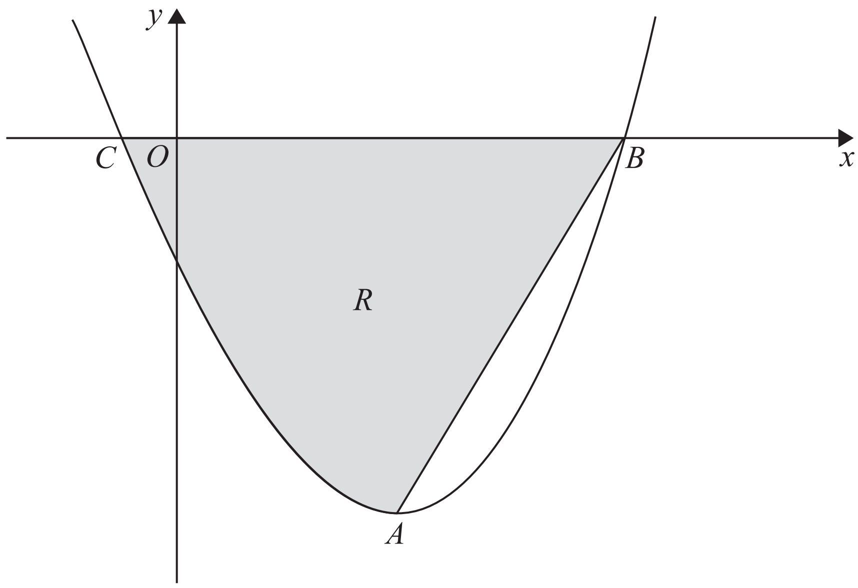


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 4x^3 + 9x^2 - 30x - 8, \quad -0.5 \leqslant x \leqslant 2.2$$

The curve has a turning point at the point A.

The curve crosses the x-axis at the points $B(2, 0)$ and $C\left(-\frac{1}{4}, 0\right)$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line AB, and the x-axis.

Use integration to find the area of the finite region R, giving your answer to 2 decimal places.

(7)

Leave blank

Question 75 continued

Leave blank

Question 75 continued

Leave blank

Question 75 continued

Leave blank

Question 75 continued

(Total 7 marks)

Leave blank

76.

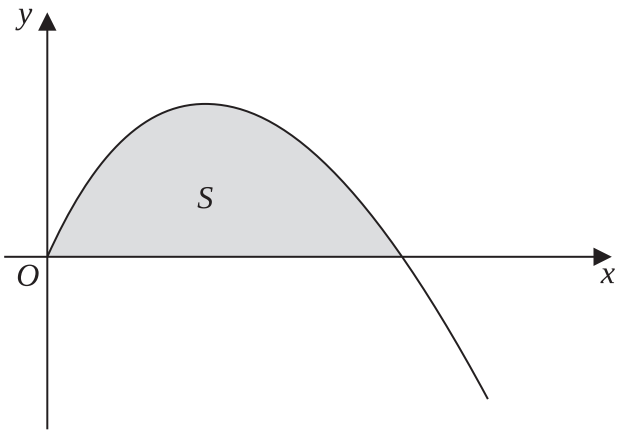


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 3x - x^{\frac{3}{2}}, \qquad x \geqslant 0$$

The finite region S, bounded by the x-axis and the curve, is shown shaded in Figure 3.

(a) Find

$$\int \left(3x - x^{\frac{3}{2}}\right) dx$$

(3)

(b) Hence find the area of S.

(3)

Leave blank

Question 76 continued

(Total 6 marks)

Leave blank

77. (a) Find

$$\int 10x(x^{\frac{1}{2}} - 2)\,dx$$

giving each term in its simplest form.

(4)

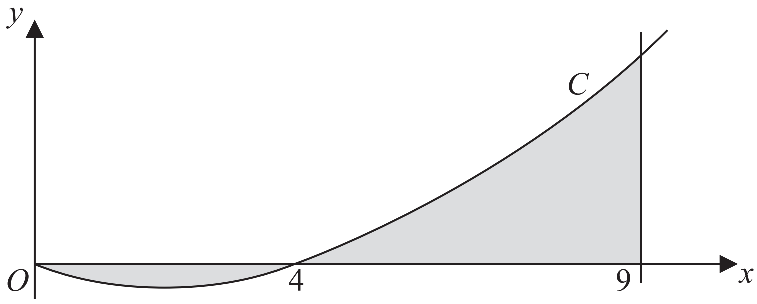


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 10x(x^{\frac{1}{2}} - 2), \qquad x \geqslant 0$$

The curve C starts at the origin and crosses the x-axis at the point (4, 0).

The area, shown shaded in Figure 2, consists of two finite regions and is bounded by the curve C, the x-axis and the line $x = 9$

(b) Use your answer from part (a) to find the total area of the shaded regions.

(5)

Leave blank

Question 77 continued

(Total 9 marks)

Leave blank

78. Use integration to find

$$\int_1^{\sqrt{3}} \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) dx$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(5)

(Total 5 marks)

Leave blank

79.

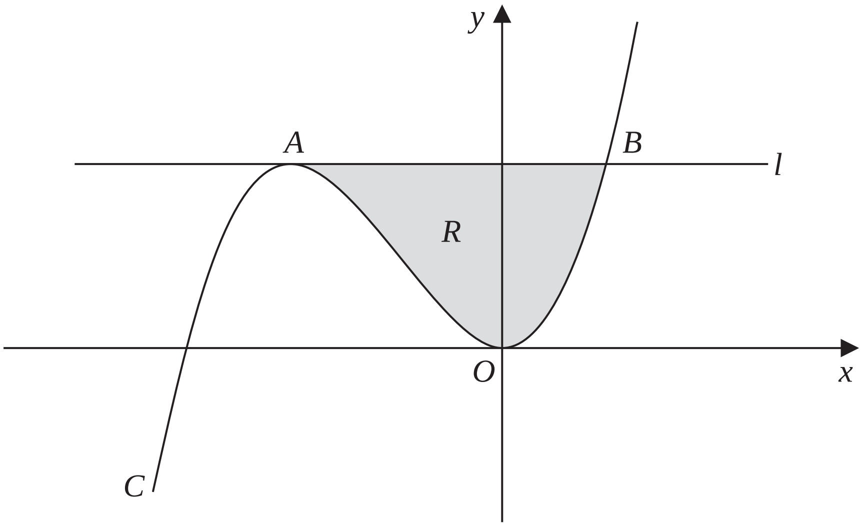


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{1}{8}x^3 + \frac{3}{4}x^2, \quad x \in \mathbb{R}$$

The curve C has a maximum turning point at the point A and a minimum turning point at the origin O.

The line l touches the curve C at the point A and cuts the curve C at the point B.

The x coordinate of A is -4 and the x coordinate of B is 2.

The finite region R, shown shaded in Figure 3, is bounded by the curve C and the line l.

Use integration to find the area of the finite region R.

(7)

Leave blank

Question 79 continued

(Total 7 marks)

Leave blank

80.

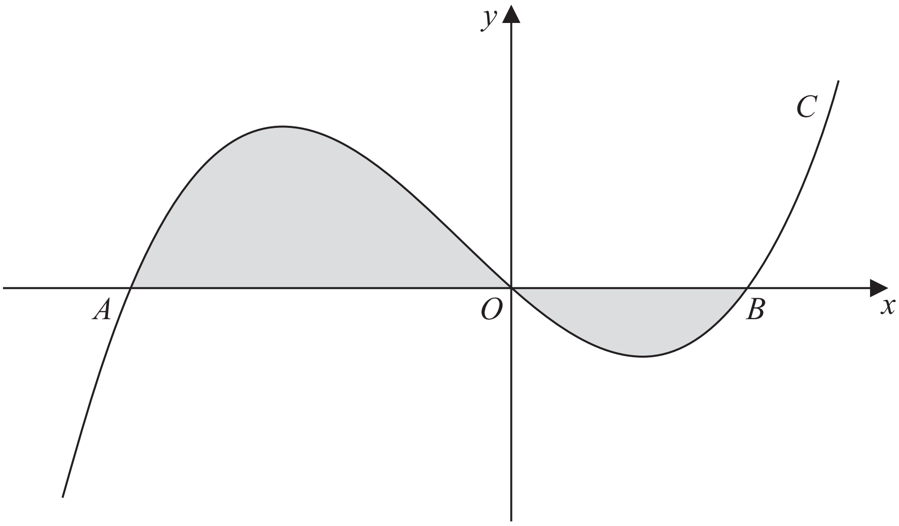


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2)$$

The curve C crosses the x-axis at the origin O and at the points A and B.

(a) Write down the x-coordinates of the points A and B.

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

Leave blank

Question 80 continued

(Total 8 marks)

Leave blank

81.

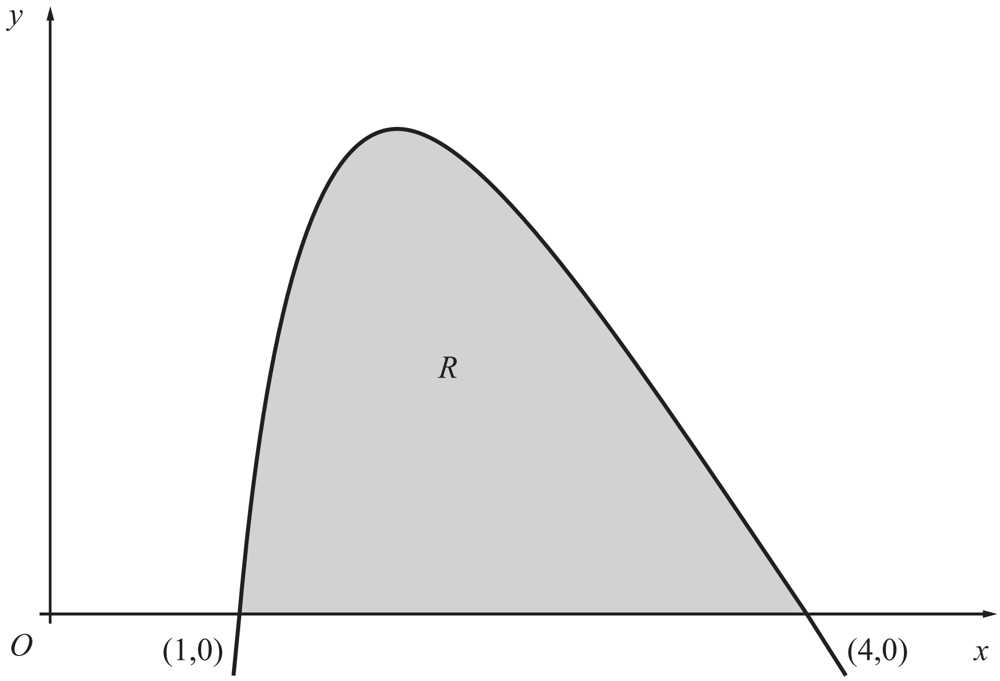


Figure 2

The finite region R, as shown in Figure 2, is bounded by the x-axis and the curve with equation

$$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}, \qquad x > 0$$

The curve crosses the x-axis at the points (1, 0) and (4, 0).

Use integration to find the exact value for the area of R.

(6)

Leave blank

Question 81 continued

(Total 6 marks)

Leave blank

82.

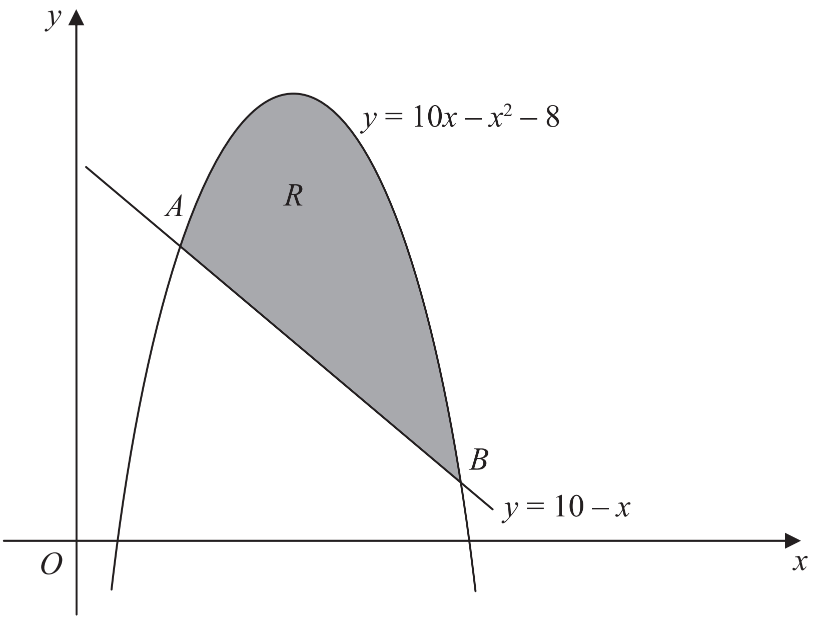


Figure 2

Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$

The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

(5)

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(b) Calculate the exact area of R.

(7)

Leave blank

Question 82 continued

(Total 12 marks)

Leave blank

83.

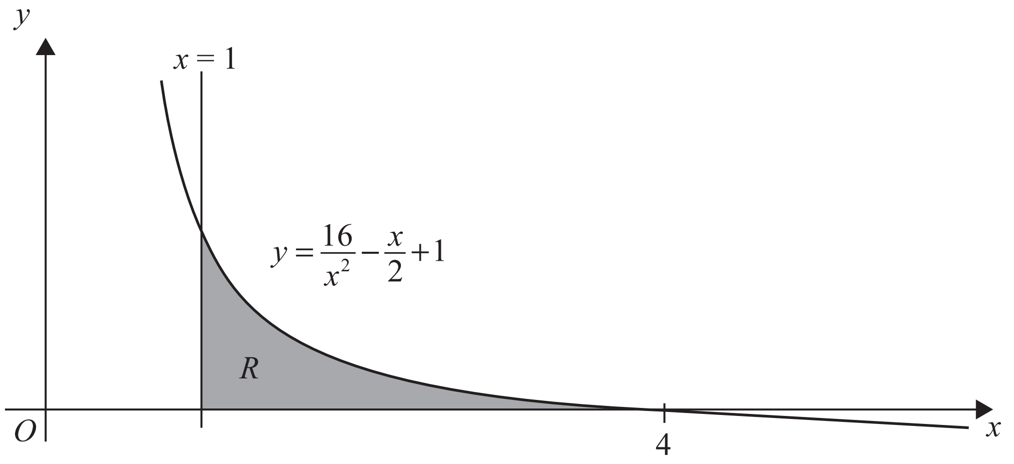


Figure 1

Figure 1 shows the graph of the curve with equation

$$y=\frac{16}{x^2}-\frac{x}{2}+1, \qquad x>0$$

The finite region R, bounded by the lines $x=1$, the x-axis and the curve, is shown shaded in Figure 1. The curve crosses the x-axis at the point $(4, 0)$.

(a) Complete the table with the values of y corresponding to $x=2$ and 2.5

x	1	1.5	2	2.5	3	3.5	4
y	16.5	7.361			1.278	0.556	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(c) Use integration to find the exact value for the area of R.

(5)

Leave blank

Question 83 continued

(Total 11 marks)

Leave blank

84.

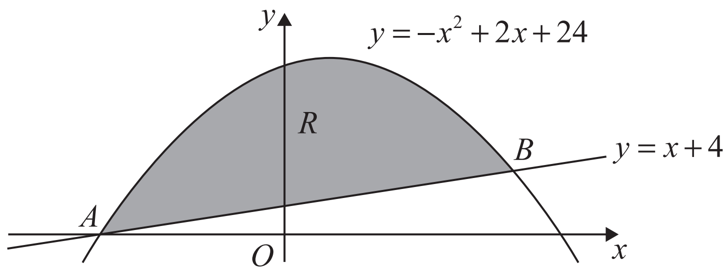


Figure 3

The straight line with equation $y = x + 4$ cuts the curve with equation $y = -x^2 + 2x + 24$ at the points A and B, as shown in Figure 3.

(a) Use algebra to find the coordinates of the points A and B.

(4)

The finite region R is bounded by the straight line and the curve and is shown shaded in Figure 3.

(b) Use calculus to find the exact area of R.

(7)

Leave blank

Question 84 continued

(Total 11 marks)

Leave blank

85.

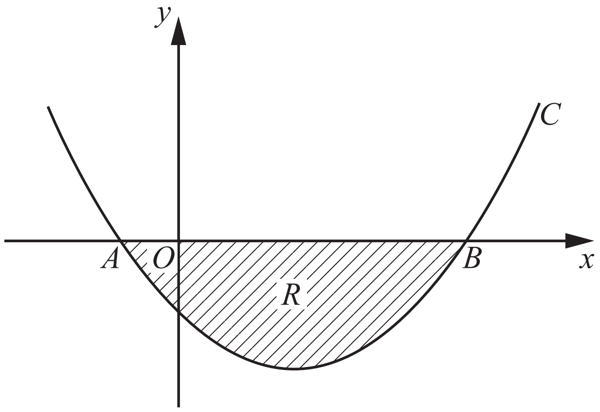


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x+1)(x-5)$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

Leave blank

Question 85 continued

(Total 7 marks)

Leave blank

86. $$y = \frac{5}{3x^2 - 2}$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	2	2.25	2.5	2.75	3
y	0.5	0.38			0.2

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_2^3 \frac{5}{3x^2 - 2}\, dx$.

(4)

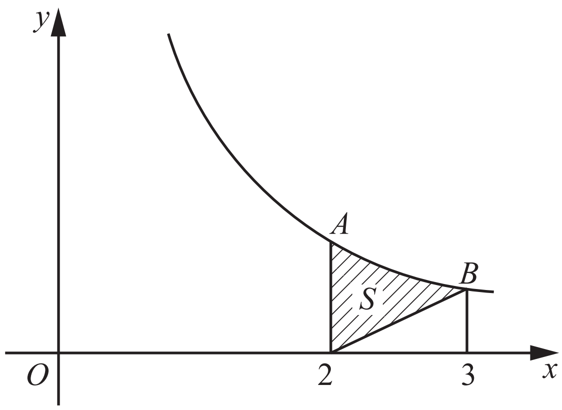


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \frac{5}{3x^2 - 2}$, $x > 1$.

At the points A and B on the curve, $x = 2$ and $x = 3$ respectively.

The region S is bounded by the curve, the straight line through B and (2, 0), and the line through A parallel to the y-axis. The region S is shown shaded in Figure 2.

(c) Use your answer to part (b) to find an approximate value for the area of S.

(3)

Leave blank

Question 86 continued

(Total 9 marks)

Leave blank

87.

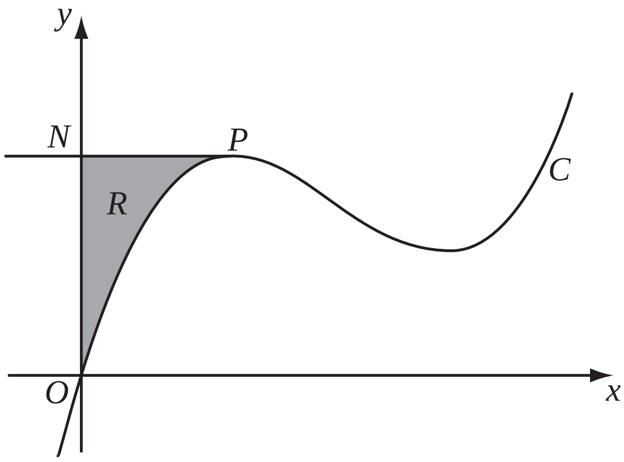


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point N.
The region R is bounded by C, the y-axis and PN, as shown shaded in Figure 2.

(b) Use calculus to find the exact area of R.

(6)

Leave blank

Question 87 continued

(Total 9 marks)

Leave blank

88.

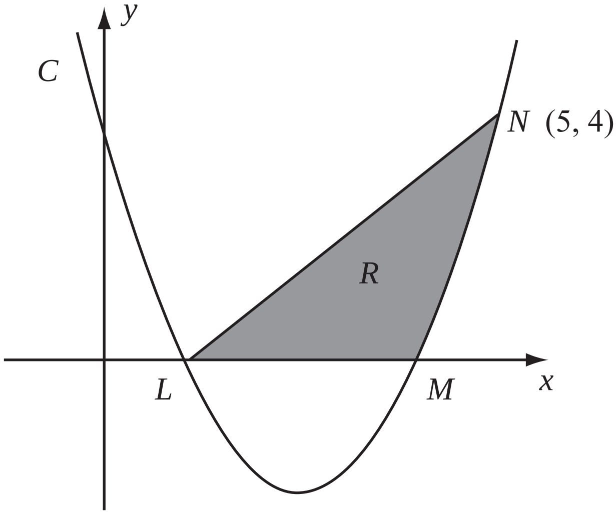


Figure 2

The curve C has equation $y = x^2 - 5x + 4$. It cuts the x-axis at the points L and M as shown in Figure 2.

(a) Find the coordinates of the point L and the point M.

(2)

(b) Show that the point N (5, 4) lies on C.

(1)

(c) Find $\int (x^2 - 5x + 4)\,dx$.

(2)

The finite region R is bounded by LN, LM and the curve C as shown in Figure 2.

(d) Use your answer to part (c) to find the exact value of the area of R.

(5)

Leave blank

Question 88 continued

(Total 10 marks)

Leave blank

89. Use calculus to find the value of

$$\int_1^4 \left(2x + 3\sqrt{x}\right) \mathrm{d}x \, .$$

(5)

(Total 5 marks)

Leave blank

90.

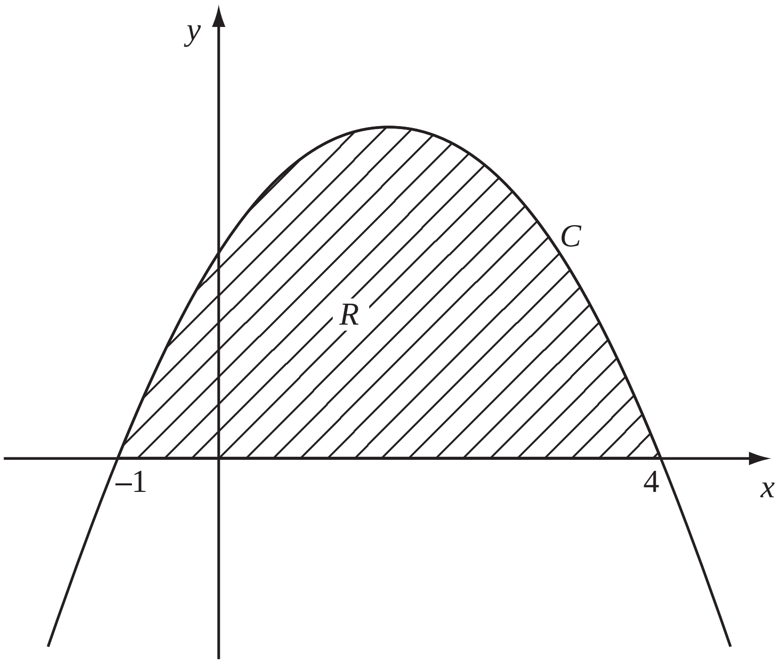


Figure 1

Figure 1 shows part of the curve C with equation $y = (1 + x)(4 - x)$.

The curve intersects the x-axis at $x = -1$ and $x = 4$. The region R, shown shaded in Figure 1, is bounded by C and the x-axis.

Use calculus to find the exact area of R.

(5)

(Total 5 marks)

91.

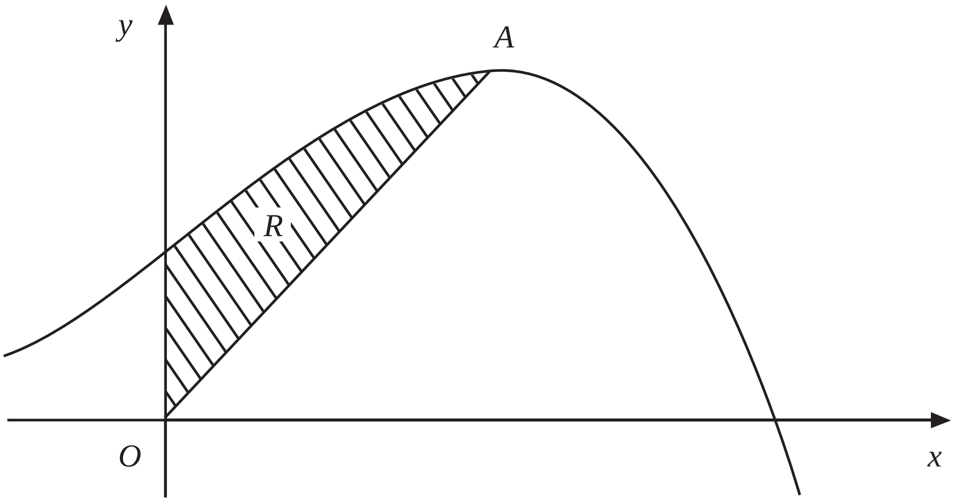


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$.

The curve has a maximum turning point *A*.

The region *R*, shown shaded in Figure 2, is bounded by the curve, the *y*-axis and the line from *O* to *A*, where *O* is the origin.

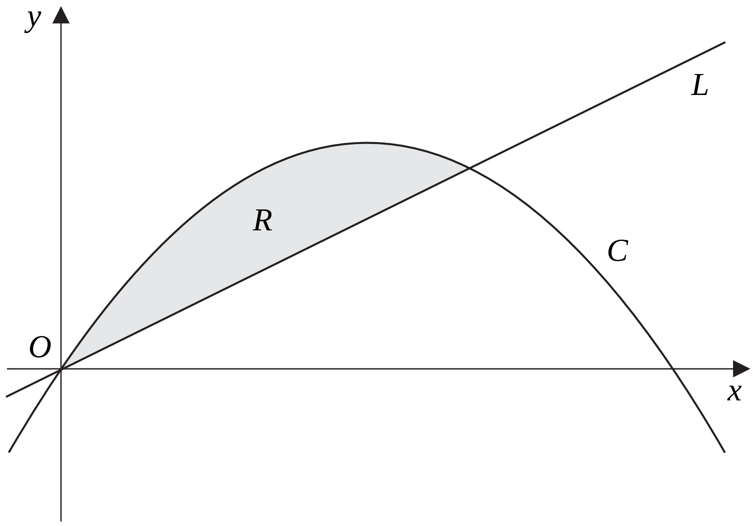
Using calculus, find the exact area of *R*.

(8)

(Total 8 marks)

Leave blank

92. **Figure 2**



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects the x-axis at $x = 0$ and $x = 6$.

(1)

(b) Show that the line L intersects the curve C at the points (0, 0) and (4, 8).

(3)

The region R, bounded by the curve C and the line L, is shown shaded in Figure 2.

(c) Use calculus to find the area of R.

(6)

Leave blank

Question 92 continued

(Total 10 marks)

Leave blank

93. (i) Given that

$$\frac{13-4x}{(2x+1)^2(x+3)} \equiv \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} + \frac{C}{(x+3)}$$

(a) find the values of the constants A, B and C.

(4)

(b) Hence find

$$\int \frac{13-4x}{(2x+1)^2(x+3)}\,\mathrm{d}x, \quad x > -\frac{1}{2}$$

(3)

(ii) Find

$$\int (\mathrm{e}^x + 1)^3\,\mathrm{d}x$$

(3)

(iii) Using the substitution $u^3 = x$, or otherwise, find

$$\int \frac{1}{4x + 5x^{\frac{1}{3}}}\,\mathrm{d}x, \quad x > 0$$

(4)

Leave blank

Question 93 continued

Leave blank

Question 93 continued

Leave blank

Question 93 continued

(Total 14 marks)

Leave blank

94. Given that $y = 2$ when $x = -\frac{\pi}{8}$, solve the differential equation

$$\frac{dy}{dx} = \frac{y^2}{3\cos^2 2x} \qquad -\frac{1}{2} < x < \frac{1}{2}$$

giving your answer in the form $y = f(x)$.

(6)

Leave blank

Question 94 continued

Leave blank

Question 94 continued

Leave blank

Question 94 continued

(Total 6 marks)

Leave blank

95.

Diagram not drawn to scale

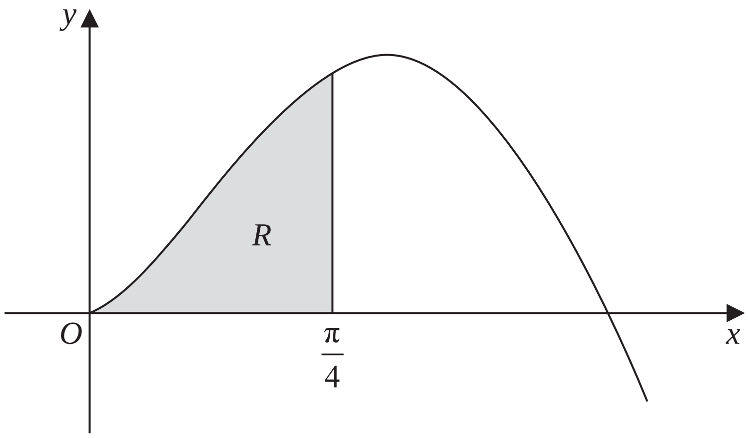


Figure 3

(a) Find $\int x\cos 4x \, dx$

(3)

Figure 3 shows part of the curve with equation $y = \sqrt{x}\sin 2x, \quad x \geqslant 0$

The finite region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = \frac{\pi}{4}$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact value of the volume of this solid of revolution, giving your answer in its simplest form.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(6)

Leave blank

Question 95 continued

Leave blank

Question 95 continued

Leave blank

Question 95 continued

Leave blank

Question 95 continued

(Total 9 marks)

96.

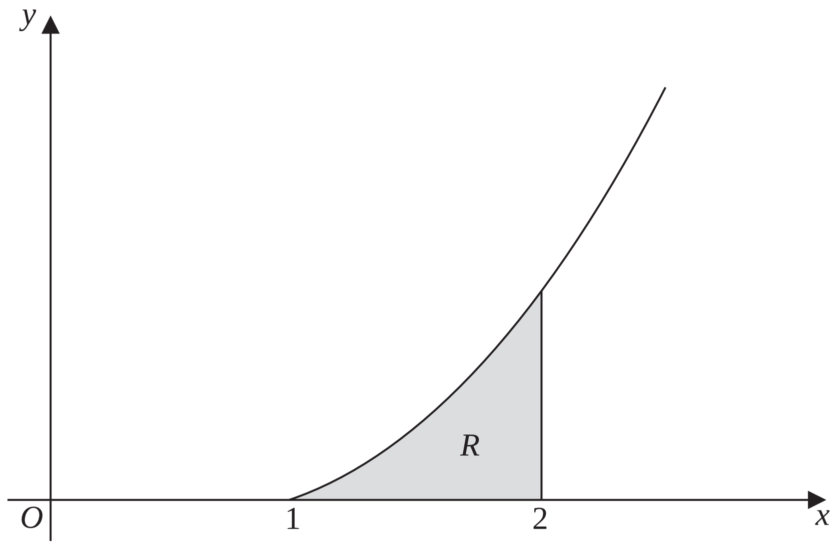


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = x^2 \ln x, \quad x \geqslant 1$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$

Use integration to find the exact value for the area of R.

(5)

Leave blank

Question 96 continued

(Total 5 marks)

Leave blank

97. The rate of decay of the mass of a particular substance is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{5}{2}x, \qquad t \geqslant 0$$

where x is the mass of the substance measured in grams and t is the time measured in days.

Given that $x = 60$ when $t = 0$,

(a) solve the differential equation, giving x in terms of t. You should show all steps in your working and give your answer in its simplest form.

(4)

(b) Find the time taken for the mass of the substance to decay from 60 grams to 20 grams. Give your answer to the nearest minute.

(3)

Leave blank

Question 97 continued

(Total 7 marks)

Leave blank

98. (i) Given that $y > 0$, find

$$\int \frac{3y - 4}{y(3y + 2)}\, dy$$

(6)

(ii) (a) Use the substitution $x = 4\sin^2\theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4 - x}\right)}\, dx = \lambda \int_0^{\frac{\pi}{3}} \sin^2\theta \, d\theta$$

where λ is a constant to be determined.

(5)

(b) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4 - x}\right)}\, dx$$

giving your answer in the form $a\pi + b$, where a and b are exact constants.

(4)

Leave blank

Question 98 continued

(Total 15 marks)

Leave blank

99. (a) Find

$$\int (2x-1)^{\frac{3}{2}}\,dx$$

giving your answer in its simplest form.

(2)

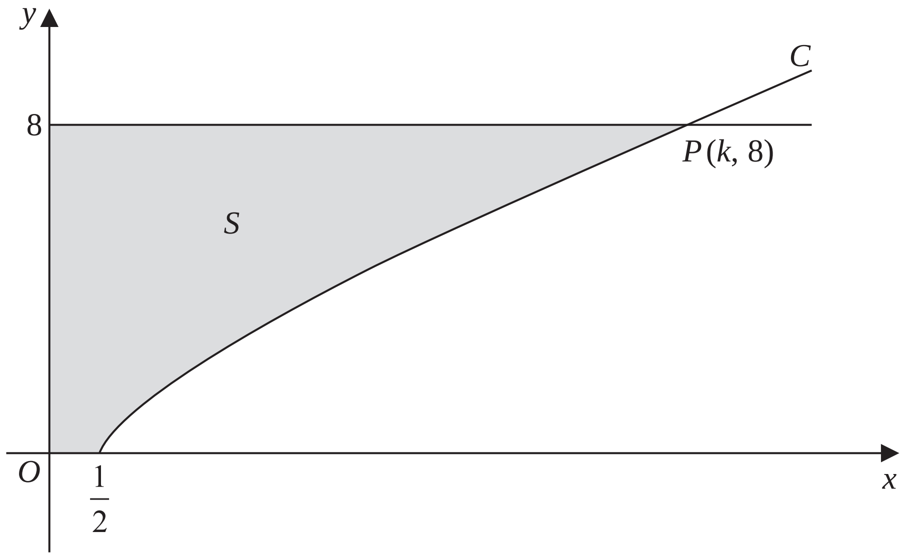


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = (2x-1)^{\frac{3}{4}}, \qquad x \geqslant \frac{1}{2}$$

The curve C cuts the line $y = 8$ at the point P with coordinates $(k, 8)$, where k is a constant.

(b) Find the value of k.

(2)

The finite region S, shown shaded in Figure 3, is bounded by the curve C, the x-axis, the y-axis and the line $y = 8$. This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Find the exact value of the volume of the solid generated.

(4)

Leave blank

Question 99 continued

(Total 8 marks)

Leave blank

100.

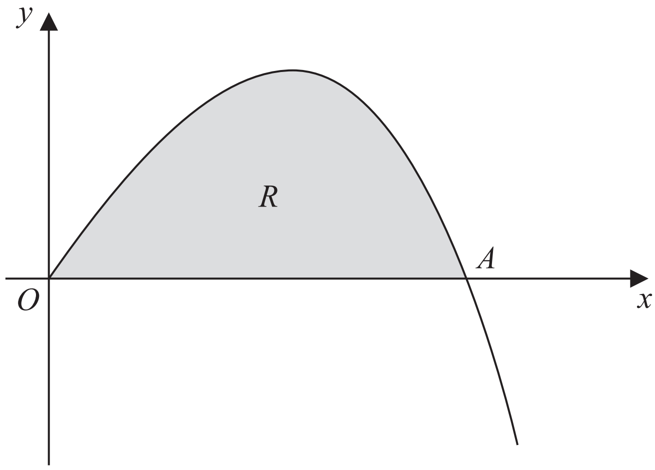


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geqslant 0$

The curve meets the x-axis at the origin O and cuts the x-axis at the point A.

(a) Find, in terms of $\ln 2$, the x coordinate of the point A.

(2)

(b) Find

$$\int xe^{\frac{1}{2}x}\,dx$$

(3)

The finite region R, shown shaded in Figure 1, is bounded by the x-axis and the curve with equation

$$y = 4x - xe^{\frac{1}{2}x}, \quad x \geqslant 0$$

(c) Find, by integration, the exact value for the area of R.
Give your answer in terms of $\ln 2$

(3)

Leave blank

Question 100 continued

(Total 8 marks)

Leave blank

101.

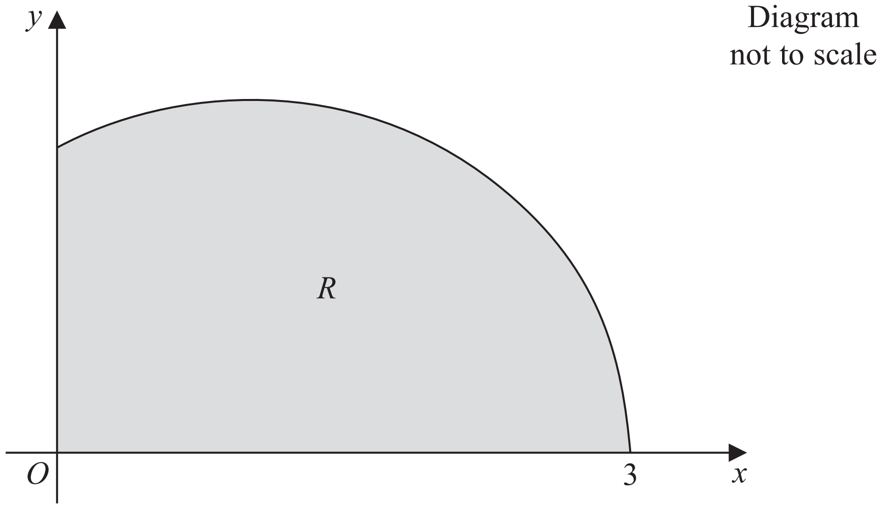


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{(3 - x)(x + 1)}$, $0 \leqslant x \leqslant 3$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis, and the y-axis.

(a) Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3 - x)(x + 1)}\, dx = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$$

where k is a constant to be determined.

(5)

(b) Hence find, by integration, the exact area of R.

(3)

Leave blank

Question 101 continued

(Total 8 marks)

Leave blank

102. (a) Express $\dfrac{2}{P(P-2)}$ in partial fractions.

(3)

A team of biologists is studying a population of a particular species of animal.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{2}P(P-2)\cos 2t, \quad t \geqslant 0$$

where P is the population in thousands, and t is the time measured in years since the start of the study.

Given that $P = 3$ when $t = 0$,

(b) solve this differential equation to show that

$$P = \frac{6}{3 - e^{\frac{1}{2}\sin 2t}}$$

(7)

(c) find the time taken for the population to reach 4000 for the first time.
Give your answer in years to 3 significant figures.

(3)

Leave blank

Question 102 continued

(Total 13 marks)

Leave blank

103.

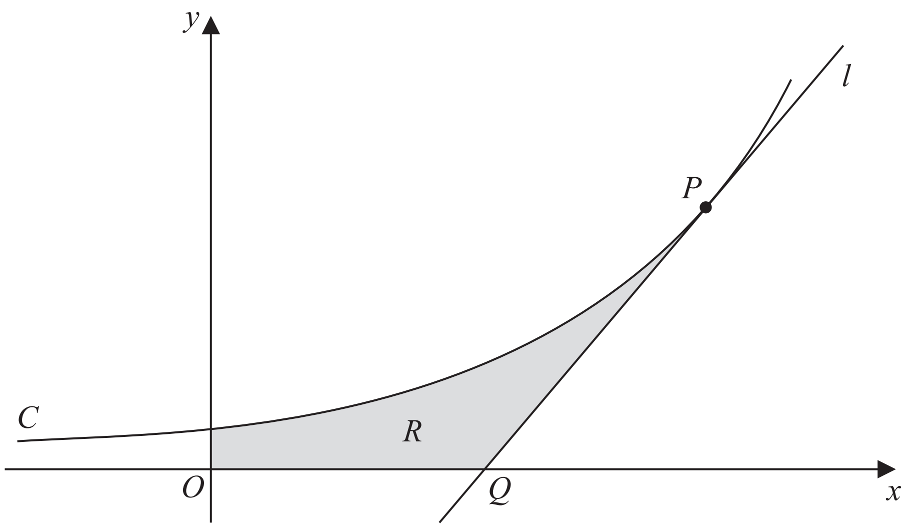


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = 3^x$$

The point P lies on C and has coordinates (2, 9).

The line l is a tangent to C at P. The line l cuts the x-axis at the point Q.

(a) Find the exact value of the x coordinate of Q.

(4)

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the x-axis, the y-axis and the line l. This region R is rotated through 360° about the x-axis.

(b) Use integration to find the exact value of the volume of the solid generated.

Give your answer in the form $\frac{p}{q}$ where p and q are exact constants.

[*You may assume the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

(6)

Leave blank

Question 103 continued

(Total 10 marks)

Leave blank

104.

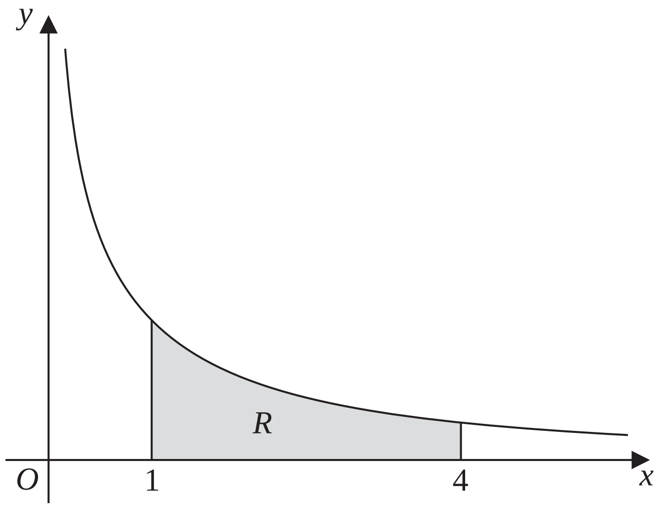


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{10}{2x + 5\sqrt{x}}$, $x > 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, and the lines with equations $x = 1$ and $x = 4$

Use the substitution $u = \sqrt{x}$, or otherwise, to find the exact value of

$$\int_1^4 \frac{10}{2x + 5\sqrt{x}}\, dx$$

(6)

Leave blank

Question 104 continued

(Total 6 marks)

EXPERT TUITION

Leave blank

105. (i) Find

$$\int x e^{4x} \, dx$$

(3)

(ii) Find

$$\int \frac{8}{(2x-1)^3} \, dx, \quad x > \frac{1}{2}$$

(2)

(iii) Given that $y = \frac{\pi}{6}$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y$$

(7)

Leave blank

Question 105 continued

(Total 12 marks)

EXPERT TUITION

Leave blank

106.

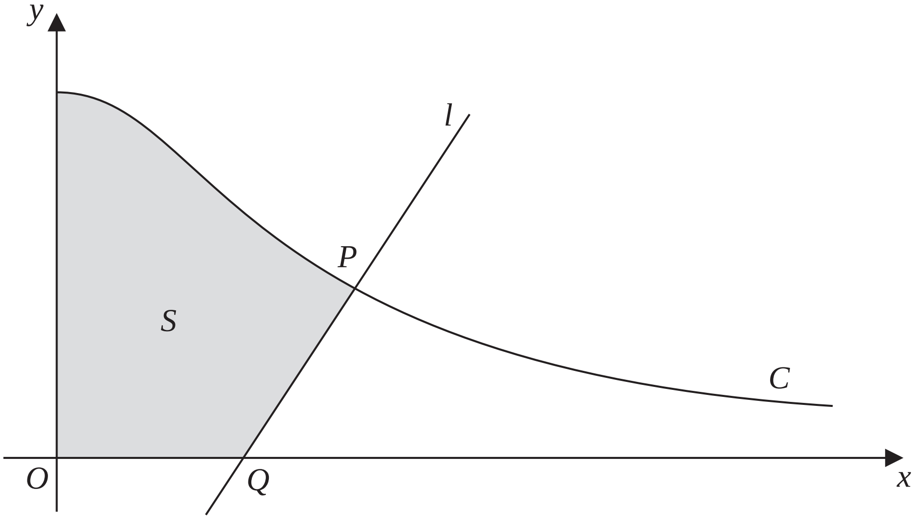


Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$$x = 3\tan\theta, \quad y = 4\cos^2\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates (3, 2).

The line l is the normal to C at P. The normal cuts the x-axis at the point Q.

(a) Find the x coordinate of the point Q.

(6)

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the x-axis, the y-axis and the line l. This shaded region is rotated 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact value of the volume of the solid of revolution, giving your answer in the form $p\pi + q\pi^2$, where p and q are rational numbers to be determined.

[*You may use the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

(9)

Leave blank

Question 106 continued

(Total 15 marks)

Leave blank

107.

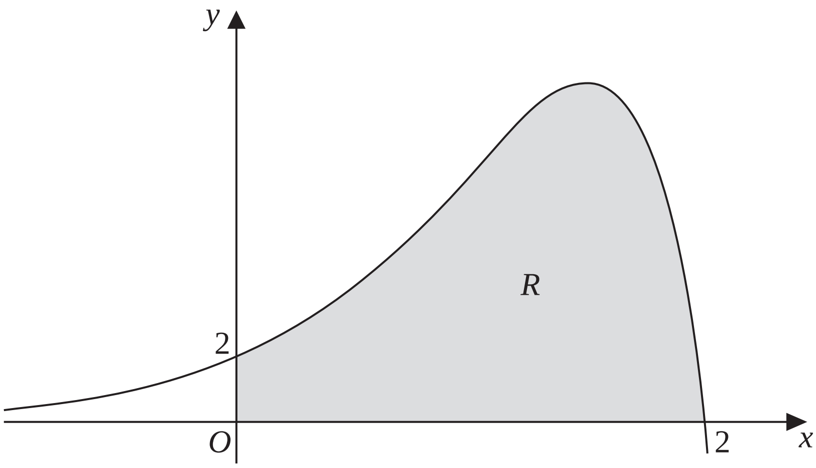


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = (2 - x)e^{2x}, \qquad x \in \mathbb{R}$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the y-axis.

Use calculus, showing each step in your working, to obtain an exact value for the area of R. Give your answer in its simplest form.

(5)

Leave blank

Question 107 continued

(Total 5 marks)

Leave blank

108. (a) Express $\dfrac{25}{x^2(2x+1)}$ in partial fractions.

(4)

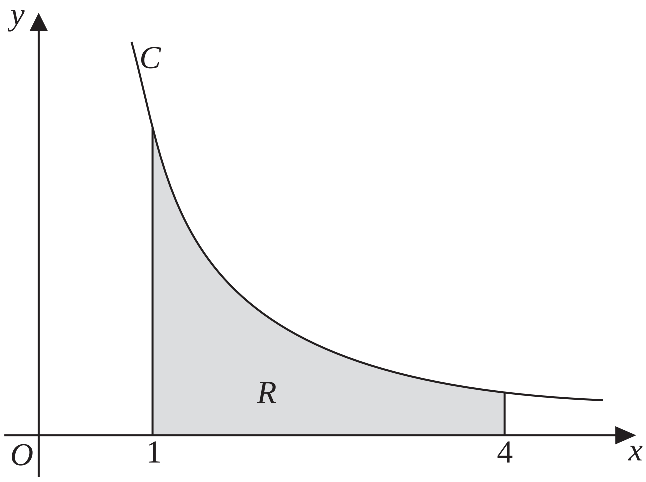


Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = \dfrac{5}{x\sqrt{(2x+1)}}$, $x > 0$

The finite region R is bounded by the curve C, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$

This region is shown shaded in Figure 2

The region R is rotated through 360° about the x-axis.

(b) Use calculus to find the exact volume of the solid of revolution generated, giving your answer in the form $a + b\ln c$, where a, b and c are constants.

(6)

Leave blank

Question 108 continued

(Total 10 marks)

EXPERT TUITION

Leave blank

109. The rate of increase of the number, N, of fish in a lake is modelled by the differential equation

$$\frac{dN}{dt} = \frac{(kt - 1)(5000 - N)}{t} \qquad t > 0, \quad 0 < N < 5000$$

In the given equation, the time t is measured in years from the start of January 2000 and k is a positive constant.

(a) By solving the differential equation, show that

$$N = 5000 - Ate^{-kt}$$

where A is a positive constant.

(5)

After one year, at the start of January 2001, there are 1200 fish in the lake.

After two years, at the start of January 2002, there are 1800 fish in the lake.

(b) Find the exact value of the constant A and the exact value of the constant k.

(4)

(c) Hence find the number of fish in the lake after five years. Give your answer to the nearest hundred fish.

(1)

Leave blank

Question 109 continued

(Total 10 marks)

Leave blank

110. (a) Find $\int x^2 e^x \, dx$.

(5)

(b) Hence find the exact value of $\int_0^1 x^2 e^x \, dx$.

(2)

(Total 7 marks)

Leave blank

111.

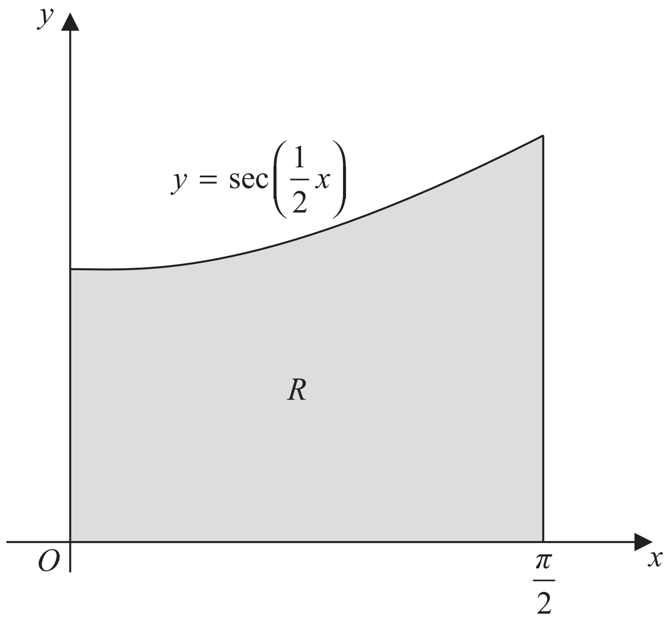


Figure 1

Figure 1 shows the finite region R bounded by the x-axis, the y-axis, the line $x = \frac{\pi}{2}$ and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \quad 0 \leqslant x \leqslant \frac{\pi}{2}$$

Region R is rotated through 2π radians about the x-axis.

Use calculus to find the exact volume of the solid formed.

(4)

Leave blank

Question 111 continued

(Total 4 marks)

Leave blank

112. (a) Use the substitution $x = u^2$, $u > 0$, to show that

$$\int \frac{1}{x(2\sqrt{x} - 1)}\, \mathrm{d}x = \int \frac{2}{u(2u - 1)}\, \mathrm{d}u$$

(3)

(b) Hence show that

$$\int_1^9 \frac{1}{x(2\sqrt{x} - 1)}\, \mathrm{d}x = 2\ln\left(\frac{a}{b}\right)$$

where a and b are integers to be determined.

(7)

Leave blank

Question 112 continued

(Total 10 marks)

Leave blank

113. Water is being heated in a kettle. At time t seconds, the temperature of the water is θ °C.

The rate of increase of the temperature of the water at any time t is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta), \qquad \theta \leqslant 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$,

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$

(8)

When the temperature of the water reaches 100 °C, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off.

(3)

Leave blank

Question 113 continued

(Total 11 marks)

Leave blank

114. Using the substitution $u = 2 + \sqrt{(2x + 1)}$, or other suitable substitutions, find the exact value of

$$\int_0^4 \frac{1}{2 + \sqrt{(2x+1)}}\,dx$$

giving your answer in the form $A + 2\ln B$, where A is an integer and B is a positive constant.

(8)

(Total 8 marks)

Leave blank

115.

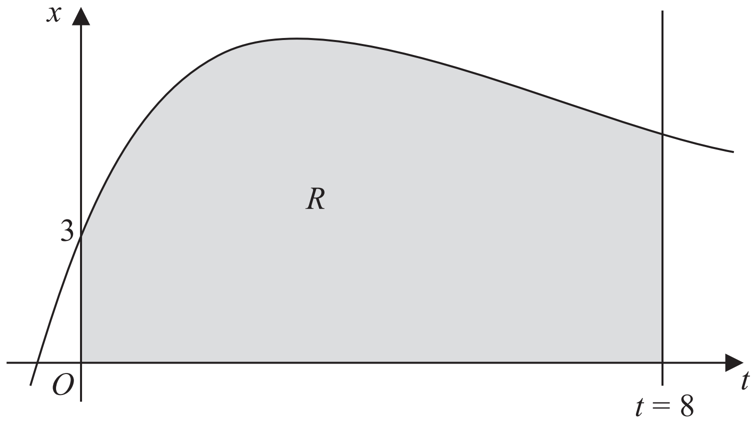


Figure 1

Figure 1 shows part of the curve with equation $x = 4te^{-\frac{1}{3}t} + 3$. The finite region R shown shaded in Figure 1 is bounded by the curve, the x-axis, the t-axis and the line $t = 8$.

(a) Complete the table with the value of x corresponding to $t = 6$, giving your answer to 3 decimal places.

t	0	2	4	6	8
x	3	7.107	7.218		5.223

(1)

(b) Use the trapezium rule with all the values of x in the completed table to obtain an estimate for the area of the region R, giving your answer to 2 decimal places.

(3)

(c) Use calculus to find the exact value for the area of R.

(6)

(d) Find the difference between the values obtained in part (b) and part (c), giving your answer to 2 decimal places.

(1)

Leave blank

Question 115 continued

(Total 11 marks)

Leave blank

116.

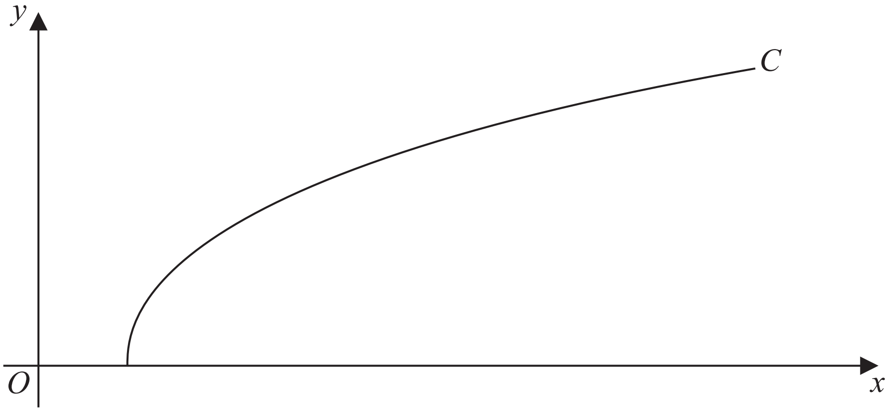


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 27\sec^3 t, \quad y = 3\tan t, \qquad 0 \leqslant t \leqslant \frac{\pi}{3}$$

(a) Find the gradient of the curve C at the point where $t = \dfrac{\pi}{6}$

(4)

(b) Show that the cartesian equation of C may be written in the form

$$y = (x^{\frac{2}{3}} - 9)^{\frac{1}{2}}, \qquad a \leqslant x \leqslant b$$

stating the values of a and b.

(3)

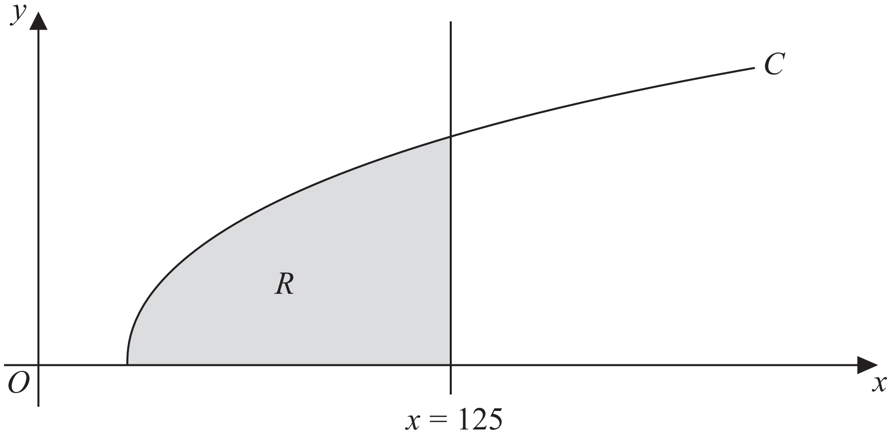


Figure 3

The finite region R which is bounded by the curve C, the x-axis and the line $x = 125$ is shown shaded in Figure 3. This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Use calculus to find the exact value of the volume of the solid of revolution.

(5)

Leave blank

Question 116 continued

(Total 12 marks)

Leave blank

117. In an experiment testing solid rocket fuel, some fuel is burned and the waste products are collected. Throughout the experiment the sum of the masses of the unburned fuel and waste products remains constant.

Let x be the mass of waste products, in kg, at time t minutes after the start of the experiment. It is known that at time t minutes, the rate of increase of the mass of waste products, in kg per minute, is k times the mass of unburned fuel remaining, where k is a positive constant.

The differential equation connecting x and t may be written in the form

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(M - x), \text{ where } M \text{ is a constant.}$$

(a) Explain, in the context of the problem, what $\frac{\mathrm{d}x}{\mathrm{d}t}$ and M represent.

(2)

Given that initially the mass of waste products is zero,

(b) solve the differential equation, expressing x in terms of k, M and t.

(6)

Given also that $x = \frac{1}{2}M$ when $t = \ln 4$,

(c) find the value of x when $t = \ln 9$, expressing x in terms of M, in its simplest form.

(4)

Leave blank

Question 117 continued

(Total 12 marks)

Leave blank

118.

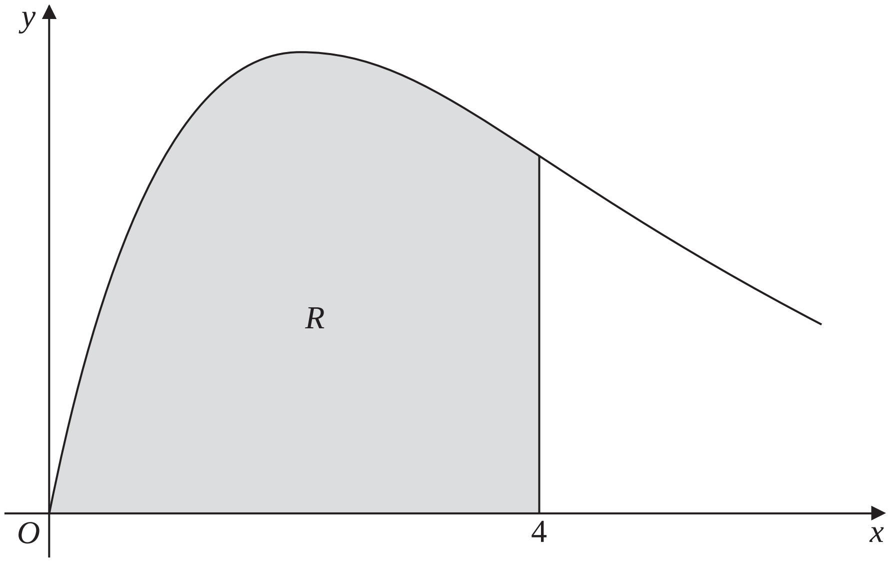


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = x\mathrm{e}^{-\frac{1}{2}x}$, $x \geqslant 0$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, and the line $x = 4$.

(i) Find $\int x\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$.

(ii) Hence find the exact area of R, giving your answer in the form $a + b\mathrm{e}^{-2}$, where a and b are integers.

(6)

Leave blank

Question 118 continued

(Total 6 marks)

Leave blank

119. (i) (a) Express $\dfrac{7x}{(x+3)(2x-1)}$ in partial fractions.

(3)

(b) Given that $x > \dfrac{1}{2}$, find

$$\int \frac{7x}{(x+3)(2x-1)}\,dx$$

(3)

(ii) Using the substitution $u^3 = x$, or otherwise, find

$$\int \frac{1}{x + x^{\frac{1}{3}}}\,dx, \qquad x > 0$$

(5)

Leave blank

Question 119 continued

(Total 11 marks)

Leave blank

120.

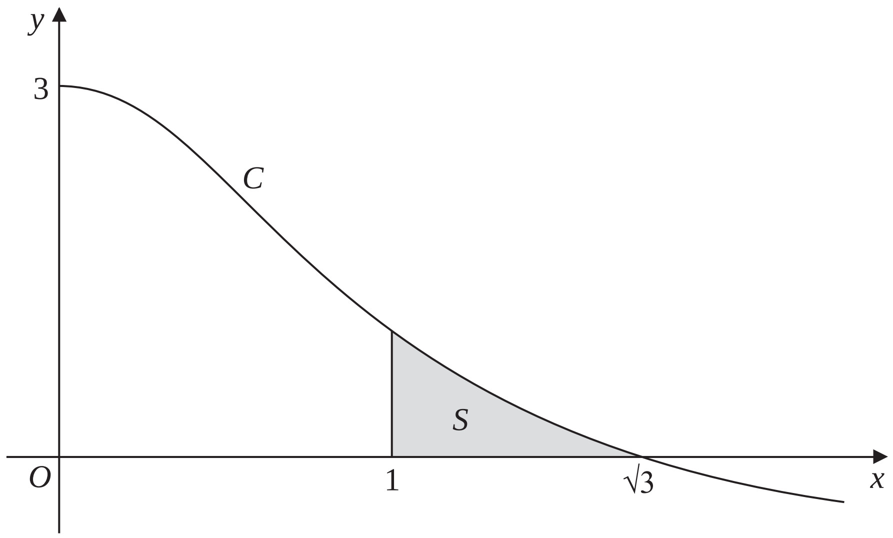


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = 1 + 2\cos 2\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The curve C crosses the x-axis at $(\sqrt{3}, 0)$. The finite shaded region S shown in Figure 2 is bounded by C, the line $x = 1$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the volume of the solid of revolution formed is given by the integral

$$k\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (16\cos^2\theta - 8 + \sec^2\theta)\, d\theta$$

where k is a constant.

(5)

(b) Hence, use integration to find the exact value for this volume.

(5)

Leave blank

Question 120 continued

(Total 10 marks)

Leave blank

121.

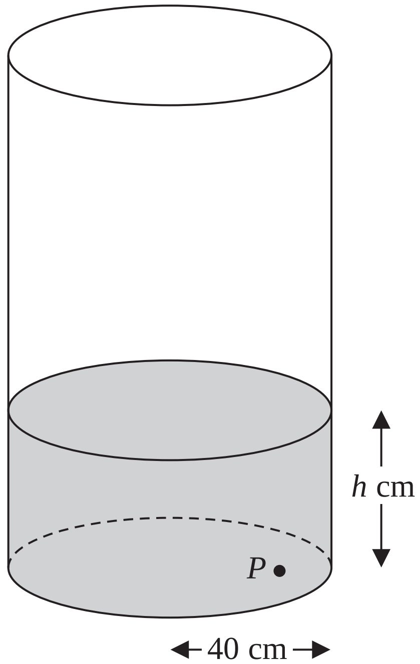


Figure 3

Figure 3 shows a large vertical cylindrical tank containing a liquid. The radius of the circular cross-section of the tank is 40 cm. At time t minutes, the depth of liquid in the tank is h centimetres. The liquid leaks from a hole P at the bottom of the tank.

The liquid leaks from the tank at a rate of $32\pi\sqrt{h}$ cm^3 min^{-1}.

(a) Show that at time t minutes, the height h cm of liquid in the tank satisfies the differential equation

$$\frac{dh}{dt} = -0.02\sqrt{h}$$

(4)

(b) Find the time taken, to the nearest minute, for the depth of liquid in the tank to decrease from 100 cm to 50 cm.

(5)

Leave blank

Question 121 continued

(Total 9 marks)

Leave blank

122. (a) Use integration to find

$$\int \frac{1}{x^3} \ln x \, dx$$

(5)

(b) Hence calculate

$$\int_1^2 \frac{1}{x^3} \ln x \, dx$$

(2)

(Total 7 marks)

123.

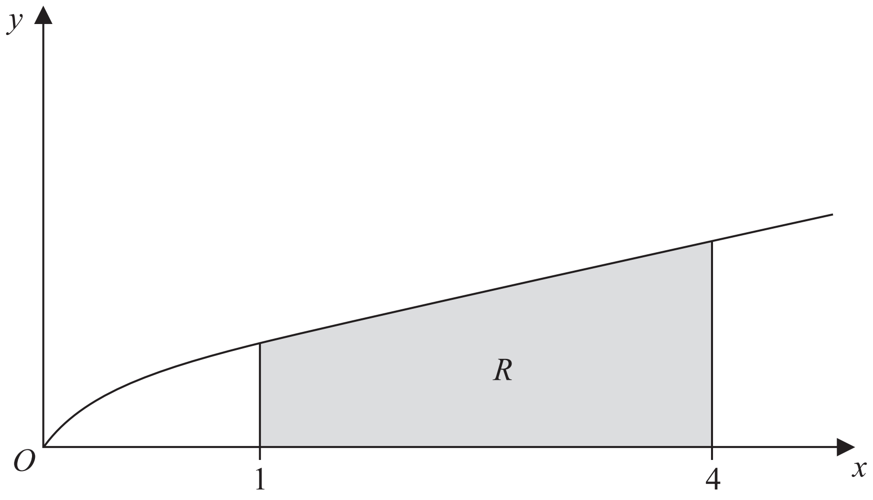


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \frac{x}{1 + \sqrt{x}}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(a) Complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 decimal places.

(1)

x	1	2	3	4
y	0.5	0.8284		1.3333

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R, giving your answer to 3 decimal places.

(3)

(c) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of R.

(8)

Leave blank

Question 123 continued

(Total 12 marks)

124.

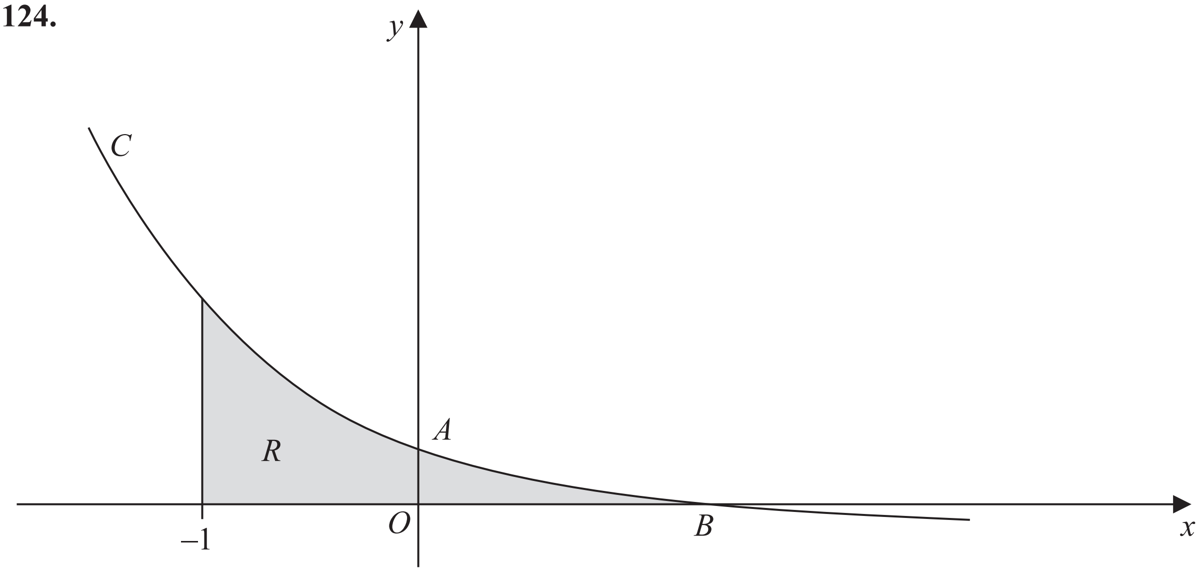


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the y-axis at the point A and crosses the x-axis at the point B.

(a) Show that A has coordinates (0, 3).

(2)

(b) Find the x coordinate of the point B.

(2)

(c) Find an equation of the normal to C at the point A.

(5)

The region R, as shown shaded in Figure 2, is bounded by the curve C, the line $x = -1$ and the x-axis.

(d) Use integration to find the exact area of R.

(6)

Leave blank

Question 124 continued

(Total 15 marks)

125.

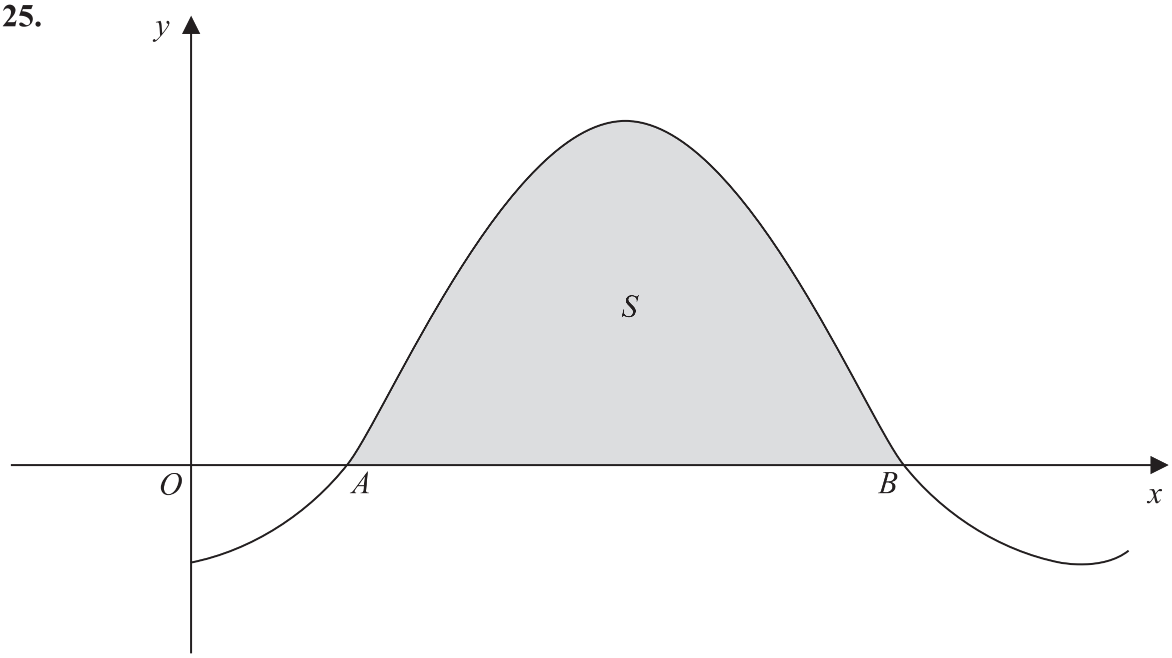


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = 1 - 2\cos x$, where x is measured in radians. The curve crosses the x-axis at the point A and at the point B.

(a) Find, in terms of π, the x coordinate of the point A and the x coordinate of the point B.
(3)

The finite region S enclosed by the curve and the x-axis is shown shaded in Figure 3. The region S is rotated through 2π radians about the x-axis.

(b) Find, by integration, the exact value of the volume of the solid generated.
(6)

Leave blank

Question 125 continued

(Total 9 marks)

Leave blank

126. A bottle of water is put into a refrigerator. The temperature inside the refrigerator remains constant at 3 °C and t minutes after the bottle is placed in the refrigerator the temperature of the water in the bottle is θ °C.

The rate of change of the temperature of the water in the bottle is modelled by the differential equation,

$$\frac{d\theta}{dt} = \frac{(3 - \theta)}{125}$$

(a) By solving the differential equation, show that,

$$\theta = Ae^{-0.008t} + 3$$

where A is a constant.

(4)

Given that the temperature of the water in the bottle when it was put in the refrigerator was 16 °C,

(b) find the time taken for the temperature of the water in the bottle to fall to 10 °C, giving your answer to the nearest minute.

(5)

Leave blank

Question 126 continued

(Total 9 marks)

Leave blank

127.

$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants A, B and C.

(4)

(b) (i) Hence find $\int f(x)\,dx$.

(ii) Find $\int_1^2 f(x)\,dx$, leaving your answer in the form $a + \ln b$, where a and b are constants.

(6)

(Total 10 marks)

Leave blank

128. Given that $y = 2$ at $x = \frac{\pi}{4}$, solve the differential equation

$$\frac{dy}{dx} = \frac{3}{y \cos^2 x}$$

(5)

(Total 5 marks)

Leave blank

129.

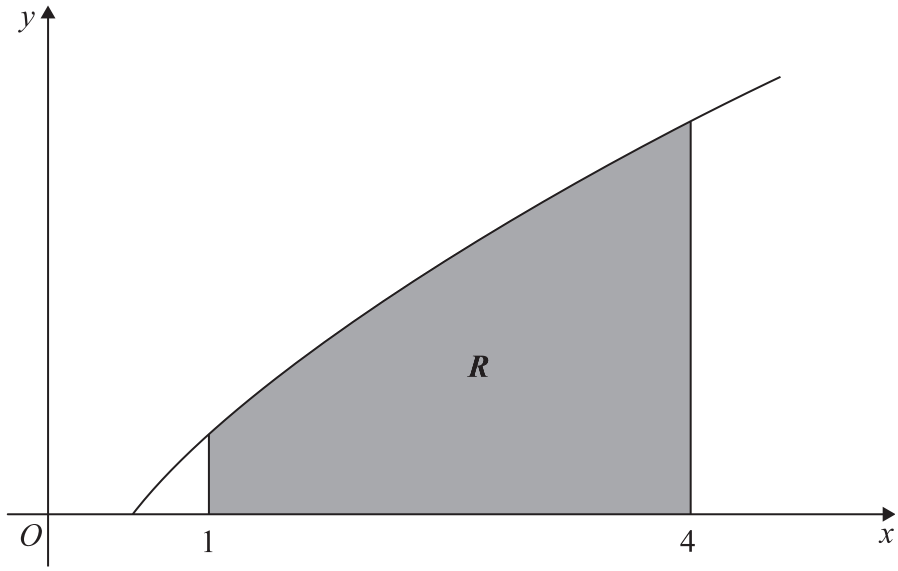


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.

The finite region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$

(a) Find $\int x^{\frac{1}{2}} \ln 2x \, dx$.

(4)

(b) Hence find the exact area of R, giving your answer in the form $a \ln 2 + b$, where a and b are exact constants.

(3)

Leave blank

Question 129 continued

(Total 7 marks)

Leave blank

130. (a) Use integration by parts to find $\int x \sin 3x \, dx$.

(3)

(b) Using your answer to part (a), find $\int x^2 \cos 3x \, dx$.

(3)

(Total 6 marks)

Leave blank

131.

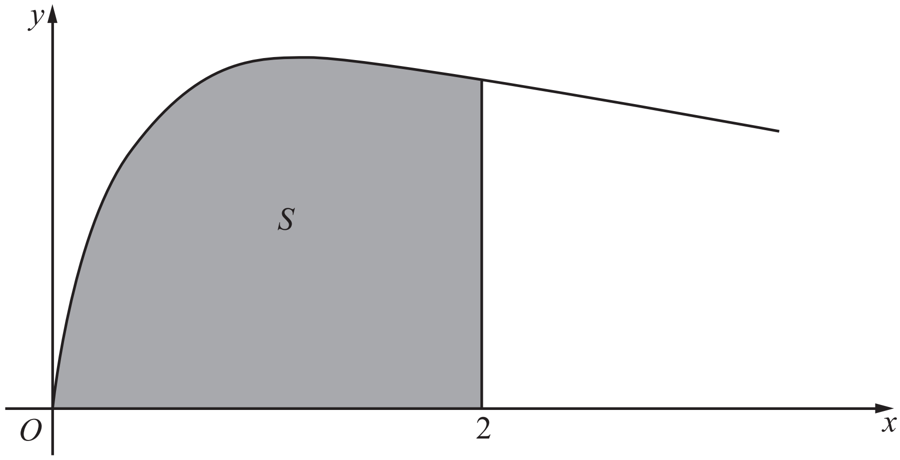


Figure 1

Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2 + 4}\right)}, \quad x \geqslant 0$$

The finite region S, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$

The region S is rotated 360° about the x-axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k \ln a$, where k and a are constants.

(5)

(Total 5 marks)

132.

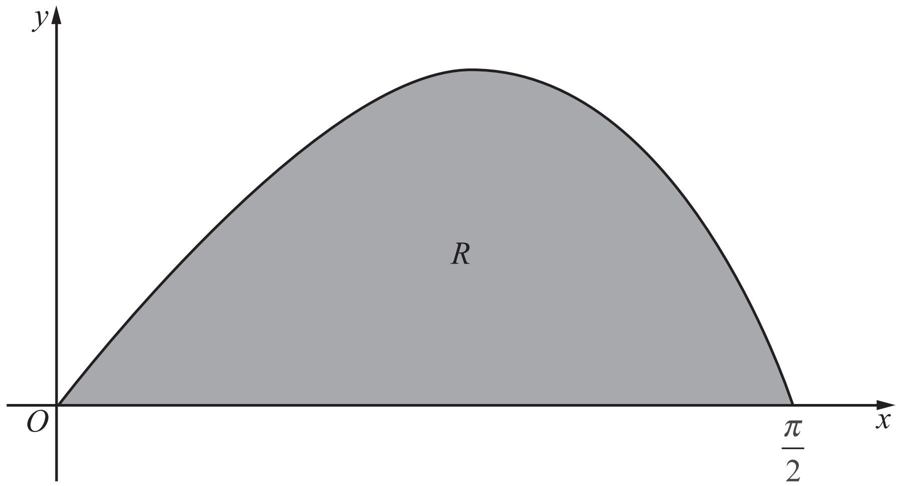


Figure 3

Figure 3 shows a sketch of the curve with equation $y = \dfrac{2\sin 2x}{(1+\cos x)}$, $0 \leqslant x \leqslant \dfrac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \dfrac{2\sin 2x}{(1+\cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(a) Complete the table above giving the missing value of y to 5 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 4 decimal places.

(3)

(c) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k$$

where k is a constant.

(5)

(d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures.

(3)

Leave blank

Question 132 continued

(Total 12 marks)

133. (a) Express $\dfrac{1}{P(5-P)}$ in partial fractions.

(3)

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{15}P(5-P), \qquad t \geqslant 0$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

(b) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + c\mathrm{e}^{-\frac{1}{3}t}}$$

where a, b and c are integers.

(8)

(c) Hence show that the population cannot exceed 5000

(1)

Leave blank

Question 133 continued

(Total 12 marks)

Leave blank

134.

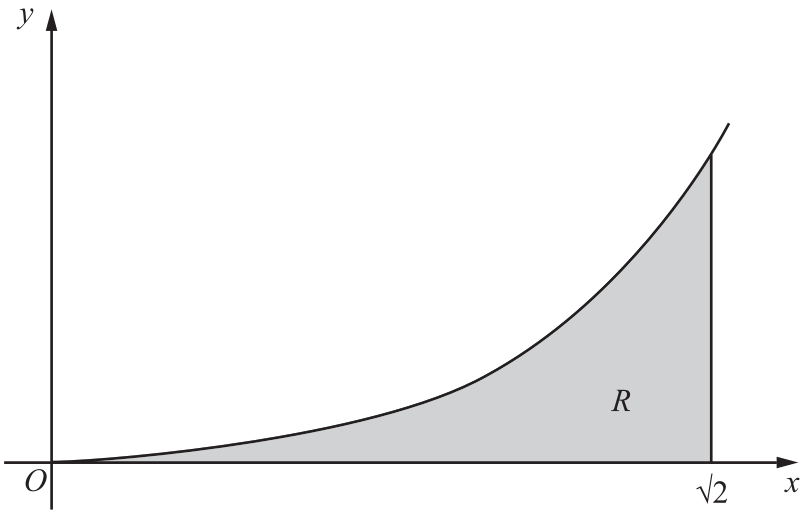


Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2)$, $x \geqslant 0$.
The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln(x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

(a) Complete the table above giving the missing values of y to 4 decimal places.

(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(3)

(c) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2}\int_2^4 (u-2)\ln u \, \mathrm{d}u$$

(4)

(d) Hence, or otherwise, find the exact area of R.

(6)

Leave blank

Question 134 continued

(Total 15 marks)

Leave blank

135.

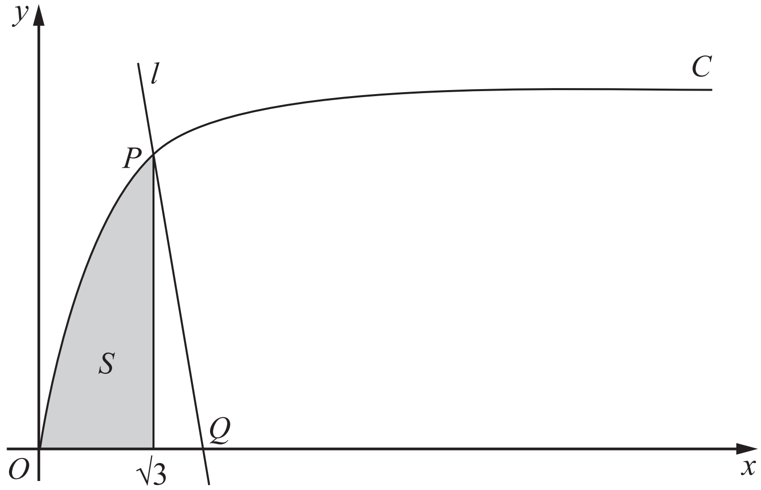


Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan\theta\,, \qquad y = \sin\theta\,, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$.

(a) Find the value of θ at the point P.

(2)

The line l is a normal to C at P. The normal cuts the x-axis at the point Q.

(b) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k.

(6)

The finite shaded region S shown in Figure 3 is bounded by the curve C, the line $x = \sqrt{3}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Find the volume of the solid of revolution, giving your answer in the form $p\pi\sqrt{3} + q\pi^2$, where p and q are constants.

(7)

Leave blank

Question 135 continued

(Total 15 marks)

Leave blank

136. (a) Find $\int (4y+3)^{-\frac{1}{2}}\,\mathrm{d}y$

(2)

(b) Given that $y = 1.5$ at $x = -2$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{(4y+3)}}{x^2}$$

giving your answer in the form $y = \mathrm{f}(x)$.

(6)

(Total 8 marks)

Leave blank

137. Use integration to find the exact value of

$$\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$$

(6)

(Total 6 marks)

Leave blank

138. (a) Express $\dfrac{5}{(x-1)(3x+2)}$ in partial fractions.

(3)

(b) Hence find $\displaystyle\int \frac{5}{(x-1)(3x+2)}\,dx$, where $x > 1$.

(3)

(c) Find the particular solution of the differential equation

$$(x-1)(3x+2)\frac{dy}{dx} = 5y, \quad x > 1,$$

for which $y = 8$ at $x = 2$. Give your answer in the form $y = f(x)$.

(6)

Leave blank

Question 138 continued

(Total 12 marks)

Leave blank

139. The curve C has parametric equations

$$x = \ln t, \qquad y = t^2 - 2, \qquad t > 0$$

Find

(a) an equation of the normal to C at the point where $t = 3$,

(6)

(b) a cartesian equation of C.

(3)

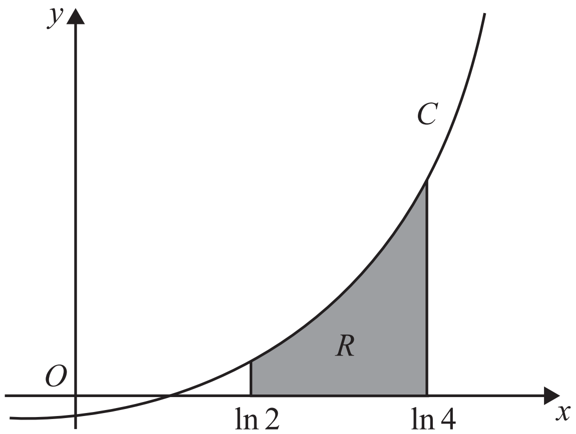


Figure 1

The finite area R, shown in Figure 1, is bounded by C, the x-axis, the line $x = \ln 2$ and the line $x = \ln 4$. The area R is rotated through $360°$ about the x-axis.

(c) Use calculus to find the exact volume of the solid generated.

(6)

Leave blank

Question 139 continued

(Total 15 marks)

Leave blank

140.

$$I = \int_2^5 \frac{1}{4+\sqrt{(x-1)}}\,dx$$

(a) Given that $y = \dfrac{1}{4+\sqrt{(x-1)}}$, complete the table below with values of y corresponding to $x = 3$ and $x = 5$. Give your values to 4 decimal places.

x	2	3	4	5
y	0.2		0.1745	

(2)

(b) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I, giving your answer to 3 decimal places.

(4)

(c) Using the substitution $x = (u-4)^2 + 1$, or otherwise, and integrating, find the exact value of I.

(8)

Leave blank

Question 140 continued

(Total 14 marks)

Leave blank

141. Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\cos x + 1} \sin x \, dx = e(e-1)$$

(6)

(Total 6 marks)

Leave blank

142.

$$f(\theta) = 4\cos^2\theta - 3\sin^2\theta$$

(a) Show that $f(\theta) = \frac{1}{2} + \frac{7}{2}\cos 2\theta$.

(3)

(b) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta f(\theta)\, d\theta$.

(7)

(Total 10 marks)

Leave blank

143.

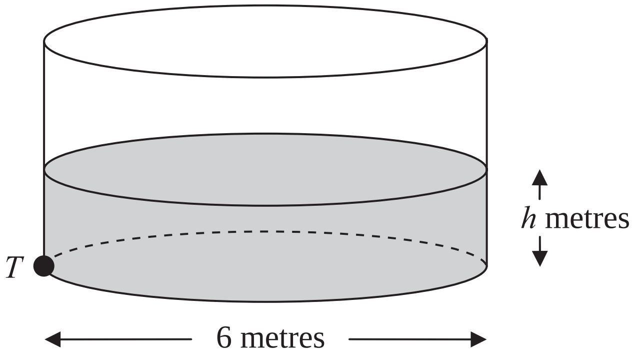


Figure 2

Figure 2 shows a cylindrical water tank. The diameter of a circular cross-section of the tank is 6 m. Water is flowing into the tank at a constant rate of 0.48π m^3 min^{-1}. At time t minutes, the depth of the water in the tank is h metres. There is a tap at a point T at the bottom of the tank. When the tap is open, water leaves the tank at a rate of $0.6\pi h$ m^3 min^{-1}.

(a) Show that t minutes after the tap has been opened

$$75\frac{\mathrm{d}h}{\mathrm{d}t} = (4 - 5h)$$

(5)

When $t = 0$, $h = 0.2$

(b) Find the value of t when $h = 0.5$

(6)

Leave blank

Question 143 continued

(Total 11 marks)

Leave blank

144.

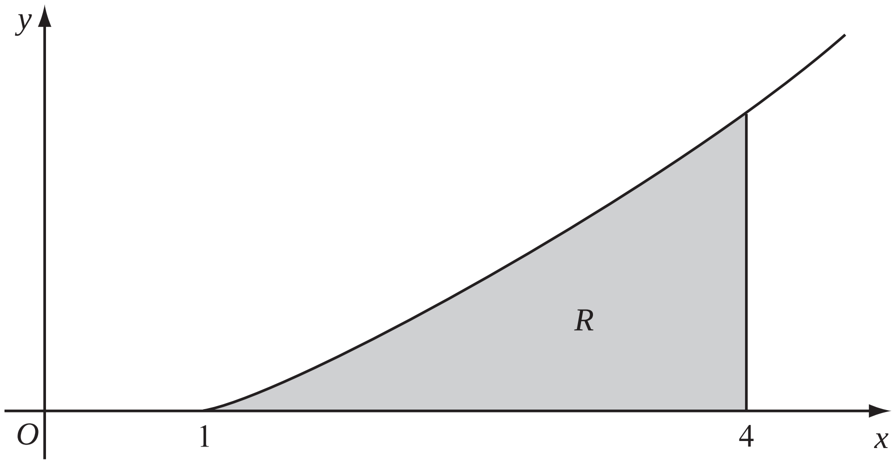


Figure 1

Figure 1 shows a sketch of the curve with equation $y = x\ln x,\ x \geqslant 1$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x\ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

(a) Complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places.

(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(c) (i) Use integration by parts to find $\int x\ln x \, dx$.

(ii) Hence find the exact area of R, giving your answer in the form $\frac{1}{4}(a\ln 2 + b)$, where a and b are integers.

(7)

Leave blank

Question 144 continued

(Total 13 marks)

Leave blank

145. (a) Find $\int \frac{9x+6}{x}\,\mathrm{d}x, \; x > 0.$

(2)

(b) Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(9x+6)\,y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2 = \mathrm{g}(x)$.

(6)

Leave blank

Question 145 continued

(Total 8 marks)

146.

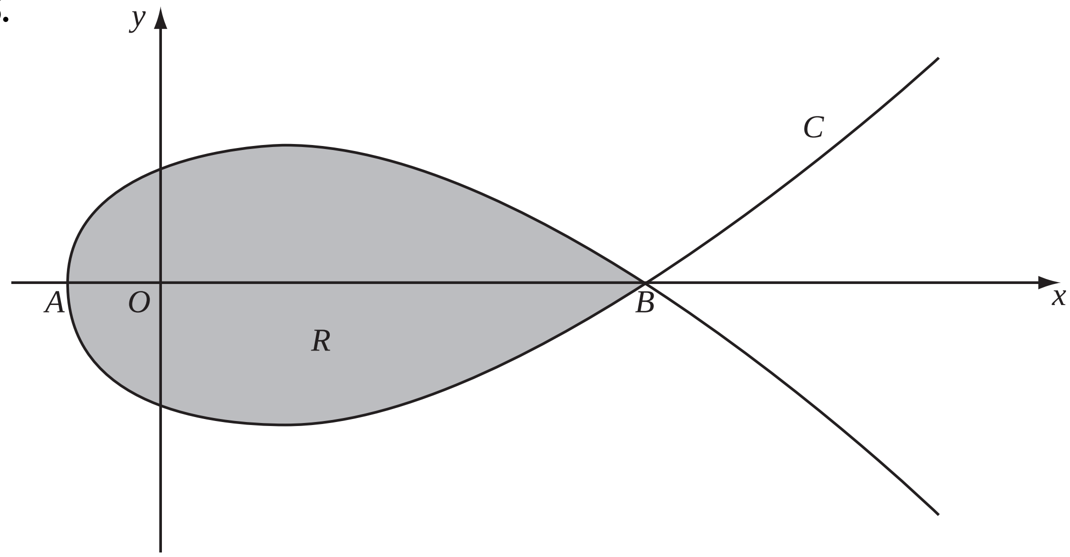


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \quad y = t(9 - t^2)$$

The curve C cuts the x-axis at the points A and B.

(a) Find the x-coordinate at the point A and the x-coordinate at the point B.

(3)

The region R, as shown shaded in Figure 2, is enclosed by the loop of the curve.

(b) Use integration to find the area of R.

(6)

Leave blank

Leave blank

Question 146 continued

(Total 9 marks)

Leave blank

147. (a) Using the substitution $x = 2\cos u$, or otherwise, find the exact value of

$$\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{(4-x^2)}}\,dx$$

(7)

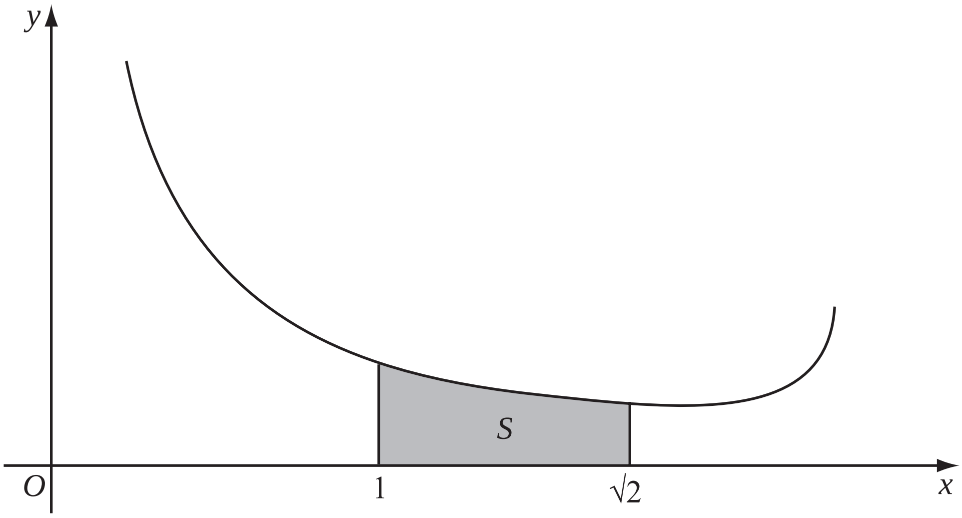


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \dfrac{4}{x(4-x^2)^{\frac{1}{4}}}$, $\quad 0 < x < 2$.

The shaded region S, shown in Figure 3, is bounded by the curve, the x-axis and the lines with equations $x = 1$ and $x = \sqrt{2}$. The shaded region S is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Using your answer to part (a), find the exact volume of the solid of revolution formed.

(3)

Leave blank

Question 147 continued

(Total 10 marks)

Leave blank

148.

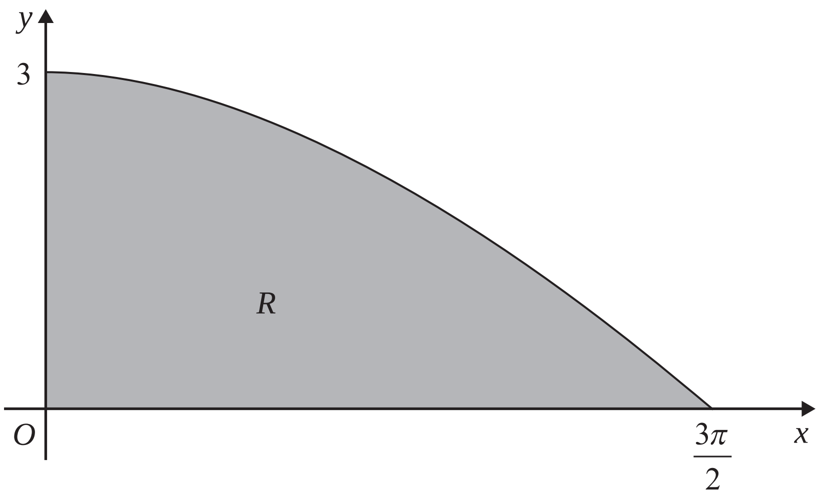


Figure 1

Figure 1 shows the finite region R bounded by the x-axis, the y-axis and the curve with equation $y = 3\cos\left(\frac{x}{3}\right)$, $0 \leqslant x \leqslant \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3\cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

(a) Complete the table above giving the missing value of y to 5 decimal places.
(1)

(b) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R, giving your answer to 3 decimal places.
(4)

(c) Use integration to find the exact area of R.
(3)

Leave blank

Question 148 continued

(Total 8 marks)

Leave blank

149. $$\mathrm{f}(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$$

(a) Find the values of the constants A, B and C.

(4)

(b) (i) Hence find $\int \mathrm{f}(x)\,\mathrm{d}x$.

(3)

(ii) Find $\int_0^2 \mathrm{f}(x)\,\mathrm{d}x$ in the form $\ln k$, where k is a constant.

(3)

Leave blank

Question 149 continued

(Total 10 marks)

Leave blank

150. (a) Find $\int \sqrt{(5-x)}\,dx$.

(2)

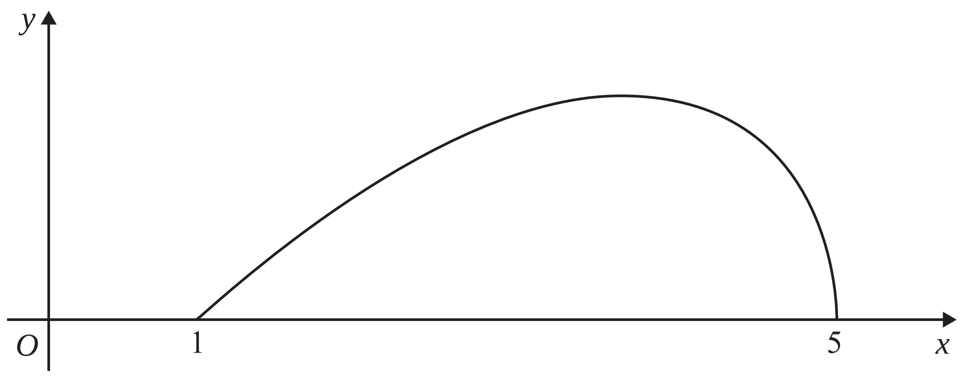


Figure 3

Figure 3 shows a sketch of the curve with equation

$$y = (x-1)\sqrt{(5-x)}, \qquad 1 \leqslant x \leqslant 5$$

(b) (i) Using integration by parts, or otherwise, find

$$\int (x-1)\sqrt{(5-x)}\,dx$$

(4)

(ii) Hence find $\int_1^5 (x-1)\sqrt{(5-x)}\,dx$.

(2)

Leave blank

Question 150 continued

(Total 8 marks)

Leave blank

151. (a) Using the identity $\cos 2\theta = 1 - 2\sin^2\theta$, find $\int \sin^2\theta \, d\theta$.

(2)

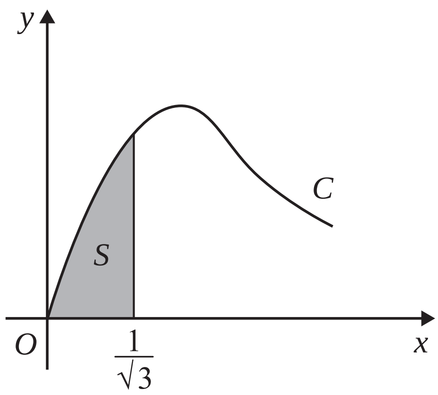


Figure 4

Figure 4 shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = 2\sin 2\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The finite shaded region S shown in Figure 4 is bounded by C, the line $x = \frac{1}{\sqrt{3}}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Show that the volume of the solid of revolution formed is given by the integral

$$k\int_0^{\frac{\pi}{6}} \sin^2\theta \, d\theta$$

where k is a constant.

(5)

(c) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where p and q are constants.

(3)

Leave blank

Question 151 continued

(Total 10 marks)

Leave blank

152.

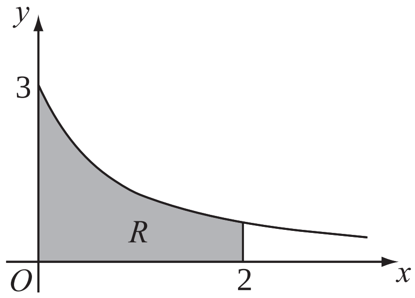


Figure 1

Figure 1 shows part of the curve $y = \dfrac{3}{\sqrt{(1+4x)}}$. The region R is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 2$, as shown shaded in Figure 1.

(a) Use integration to find the area of R.

(4)

The region R is rotated 360° about the x-axis.

(b) Use integration to find the exact value of the volume of the solid formed.

(5)

Leave blank

Question 152 continued

(Total 9 marks)

Leave blank

153. (a) Find $\int \tan^2 x \, dx$.

(2)

(b) Use integration by parts to find $\int \frac{1}{x^3} \ln x \, dx$.

(4)

(c) Use the substitution $u = 1 + e^x$ to show that

$$\int \frac{e^{3x}}{1+e^x} \, dx = \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k,$$

where k is a constant.

(7)

Leave blank

Question 153 continued

(Total 13 marks)

Leave blank

154. (a) Use integration by parts to find $\int x\,e^{x}\,dx$.

(3)

(b) Hence find $\int x^{2}\,e^{x}\,dx$.

(3)

(Total 6 marks)

Leave blank

155. (a) Express $\dfrac{2}{4-y^2}$ in partial fractions.

(3)

(b) Hence obtain the solution of

$$2\cot x \frac{dy}{dx} = (4-y^2)$$

for which $y = 0$ at $x = \frac{\pi}{3}$, giving your answer in the form $\sec^2 x = g(y)$.

(8)

(Total 11 marks)

156.

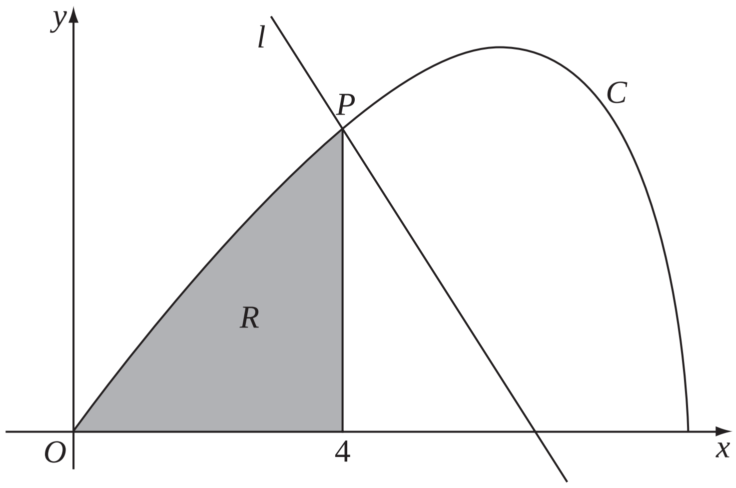


Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

(2)

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64 \sin^2 t \cos t \, dt$.

(4)

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)

Leave blank

Question 156 continued

(Total 16 marks)

Leave blank

157.

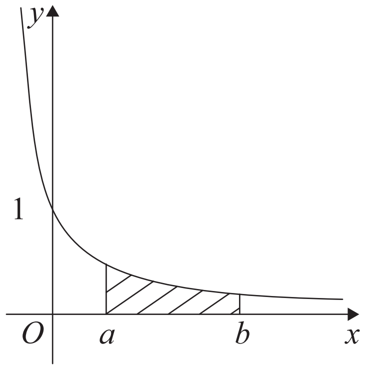


Figure 2

The curve shown in Figure 2 has equation $y = \dfrac{1}{(2x+1)}$. The finite region bounded by the curve, the x-axis and the lines $x = a$ and $x = b$ is shown shaded in Figure 2. This region is rotated through 360° about the x-axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of a and b.

(5)

(Total 5 marks)

Leave blank

158. (i) Find $\int \ln(\frac{x}{2})\, dx$.

(4)

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\, dx$.

(5)

(Total 9 marks)

Leave blank

159.

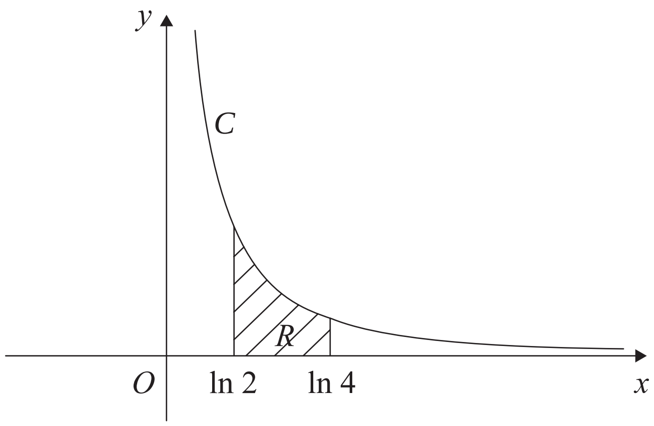


Figure 3

The curve C has parametric equations

$$x = \ln(t+2), \qquad y = \frac{1}{(t+1)}, \qquad t > -1.$$

The finite region R between the curve C and the x-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Show that the area of R is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)} \, dt.$$

(4)

(b) Hence find an exact value for this area.

(6)

(c) Find a cartesian equation of the curve C, in the form $y = f(x)$.

(4)

(d) State the domain of values for x for this curve.

(1)

Leave blank

Question 159 continued

(Total 15 marks)

Leave blank

160. Liquid is pouring into a large vertical circular cylinder at a constant rate of 1600 $\text{cm}^3\,\text{s}^{-1}$ and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 cm^2.

(a) Show that at time t seconds, the height h cm of liquid in the cylinder satisfies the differential equation

$$\frac{dh}{dt} = 0.4 - k\sqrt{h}, \text{ where } k \text{ is a positive constant.}$$

(3)

When $h = 25$, water is leaking out of the hole at 400 $\text{cm}^3\,\text{s}^{-1}$.

(b) Show that $k = 0.02$

(1)

(c) Separate the variables of the differential equation

$$\frac{dh}{dt} = 0.4 - 0.02\sqrt{h},$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}}\,dh.$$

(2)

Using the substitution $h = (20 - x)^2$, or otherwise,

(d) find the exact value of $\int_0^{100} \frac{50}{20 - \sqrt{h}}\,dh$.

(6)

(e) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second.

(1)

Leave blank

Question 160 continued

(Total 13 marks)