EXPERT TUITION

Maths Questions By Topic:

Kinematics
Mark Scheme

A-Level Edexcel

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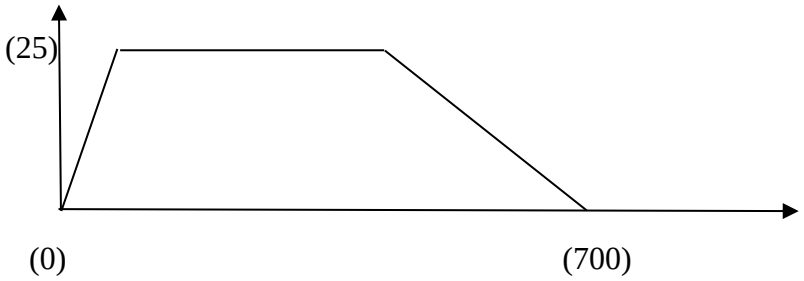
Question	Scheme	Marks	AOs
1(a)	Complete method to produce an equation in U only	M1	3.4
	e.g. $10^2 = U^2 + 2\times g\times 1.8$ oe	A1	1.1b
	OR a **complete** method where they find T first and use it to find an equation in U only M1		
	A correct equation in U only. A1		
	$U = 8$ (only this answer)	A1	1.1b
		(3)	
(b)	Complete method to find an equation in T only: $10 = -8 + gT$ or $1.8 = 10T - \frac{1}{2}gT^2$ or $1.8 = \frac{(-8+10)}{2}T$ or $1.8 = -8T + \frac{1}{2}gT^2$ **OR** a **complete** method if they split the time. In both cases, the M1 is only earned on the final line when they try to add the two times to give an equation in T. **ALT 1**: time up + time down e.g. $0 = 8 - gt_{\text{UP}}$ $(\Rightarrow t_{\text{UP}} = 0.8)$ $h_{\text{UP}} = \frac{(8+0)}{2}\times 0.8$ $(= 3.2)$ $(h_{\text{UP}} + 1.8) = \frac{(0+10)}{2}\times t_{\text{DOWN}}$ $(\Rightarrow t_{\text{DOWN}} = 1)$ $T = t_{\text{UP}} + t_{\text{DOWN}}$ **ALT 2:** time to A + time from A to ground e.g. $8 = -8 + gt_A$ $(\Rightarrow t_A = 1.6)$ $1.8 = \frac{(8+10)}{2}\times t_{AG}$ $(\Rightarrow t_{AG} = 0.2)$ $T = t_A + t_{AG}$	M1	3.4
	$T = 1.8$ oe e.g. 9/5	A1	1.1b
		(2)	
(c)	e.g. Use a more accurate (less rounded) value for g (or gravity), use $g = 9.8$ or $g = 9.81$, allow for wind effects, allow for the spin of the stone, include dimensions of stone (not a particle), shape and/or size of stone, allow for variable acceleration. If air resistance is mentioned as an extra, ignore it.	B1	3.5c

			(1)	
(d)		*U* would be greater. Allow without *U*, e.g it would be greater, or just ‘greater’ oe ISW	B1	3.5a
			(1)	
				(7 marks)

Notes:		
1a	M1	Use the model to obtain an equation in *U* only, condone sign errors, but M0 if using an incorrect formula.
	A1	A correct equation in *U* only, *g* does not need to be substituted (so allow *g* = 9.8 or 9.81)
	A1	cao (A0 if *g* = 10 has not been used)
1b	M1	Use the model to obtain an equation in *T* only , *g* does not need to be substituted (so allow *g* = 9.8 or 9.81) condone sign errors, but M0 if using an incorrect formula. Follow through on their *U* where necessary
	A1	cao (A0 if *g* = 10 has not been used) A0 if they give two answers.
1c	B1	Any appropriate refinement. B0 if an incorrect extra is given e.g. the mass or weight is mentioned
1d	B1	cao

Question	Scheme	Marks	AOs
2(a)	 shape	B1	1.1b
		(1)	
(b)	Using *total* area = 15000 to set up an *equation* in **one unknown** Or they may use *suvat* on one or more sections (but must still be considering *all* sections) Allow an attempt at a clear explicit verification using $t = 40$ e.g. the following would score M1A1A1*: $4\times 40 = 160$ then $700 - 40 - 160 = 500$ $\frac{(700+500)}{2}\times 25 = 15000 = 15$ km Withhold A1* if they don't include = 15 km **N.B.** M0 if a single *suvat* formula is used for the whole journey.	M1	3.4
	$\frac{1}{2}(700+700-t-4t)\times 25 = 15000$ **OR** $\frac{1}{2}\times 25\times t + 25(700-t-4t) + \frac{1}{2}\times 25\times 4t = 15000$	A1	1.1b
	$t = 40$ (s)*	A1*	1.1b
		(3)	
(c)	0.63 or 0.625 or $\frac{5}{8}$ oe (m s^{-2}) isw	B1	1.1b/ (2.2a)
		(1)	
(d)	Complete method to find the speed or velocity at $t = 572$ e.g $\pm\left(25-(32\times\frac{5}{32})\right)$ or $\pm\left(128\times\frac{5}{32}\right)$ oe	M1	3.1b
	20 (m s^{-1})	A1	1.1b
		(2)	
(e)	e.g. (the train) cannot instantaneously change acceleration, (the train) won't move with <u>constant</u> acceleration , (the train) won't move with <u>constant</u> speed Allow negatives of these:	B1	3.5b

	e.g. (The train) moving at constant speed, or just 'constant speed' or 'constant acceleration' (is a limitation of the model) Must be a limitation of the model, so friction or air resistance or size of train is B0. **N.B.** Ignore incorrect **reasons** following a correct answer.		
		(1)	
			(8 marks)

Notes:		
2a	B1	Overall shape of graph, starting at the origin, with deceleration phase *longer* than the acceleration phase if nothing on the *t*-axis but ignore the relative lengths and allow if *t* (or 40) and 4*t* (or 160) are clearly and correctly marked. Ignore incorrect figs on the axes. This mark can be earned if the graph appears anywhere in qu 2.
2b	M1	Need *all* sections to be included, with correct structure for each section, with $\frac{1}{2}$'s where appropriate. Allow = 15 or 150 or 1500 etc instead of 15000
	A1	A correct equation **in their *t* only, seen or implied** (or with $t = 40$ for verification)
	A1*	cso. At least one line of working with brackets removed and *t*'s collected, or equivalent
2c	B1	cao
2d	M1	Any complete method, must have correct figs, but condone sign errors
	A1	cao. Must be positive and exact i.e must not come from rounding.
2e	B1	Any appropriate limitation of the model. B0 if any incorrect extra **answers.**

Question	Scheme	Marks	AOs
3(a)	Differentiate s wrt t	M1	3.1a
	$(v=)\ t^2-5t+6$	A1	1.1b
	Equate their v to 0 and solve	M1	1.1b
	$t = 2$ or 3	A1	1.1b
	$(a=)\ 2t-5$	B1**ft**	2.1
	$a =$ 1 and -1 (m s^{-2}) isw (A0 if extras)	A1	1.1b
		(6)	
(b)	Attempt to find values of s for $t = 2, 3$ *and* 4 oe Correct values are $\left(s_2=\frac{14}{3}, s_3=\frac{9}{2}\ \textit{and}\ s_4=\frac{16}{3}\right)$ Could be implied by correct values for: s_2 , (s_3-s_2) and (s_4-s_3) which are $\frac{14}{3}$, $(-\frac{1}{6})$ and $\frac{5}{6}$	**DM**1	1.1b
	Total distance travelled $= s_2+(s_2-s_3)+s_4-s_3$ **OR** $s_2-(s_3-s_2)+s_4-s_3$ **OR** $\left[\frac{1}{3}t^3-\frac{5}{2}t^2+6t\right]_0^2-\left[\frac{1}{3}t^3-\frac{5}{2}t^2+6t\right]_2^3+\left[\frac{1}{3}t^3-\frac{5}{2}t^2+6t\right]_3^4$ **OR** $\frac{14}{3}\quad -\quad (-\frac{1}{6})\quad +\quad \frac{5}{6}$ **OR** $s_2+2(s_2-s_3)+s_4-s_2$ $(=2s_2-2s_3+s_4)$ oe	M1	2.1
	$5\frac{2}{3}$ oe (m) Accept 5.7 or better	A1	1.1b
		(3)	
			(9 marks)

Notes:

3a	M1	Differentiate, with at least 2 powers decreasing by 1
	A1	Correct expression
	M1	Must have attempted to differentiate s to find v and be solving a 3 term quadratic
	A1	Both values needed
	B1**ft**	Follow their v (must be differentiating)

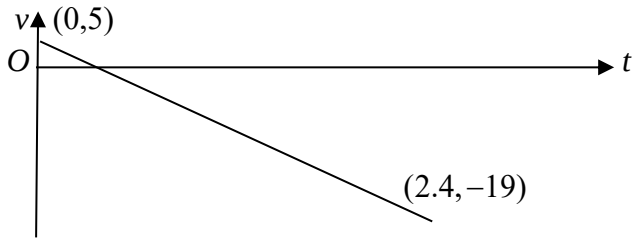
	A1	cao
3b	**DM** 1	<u>This mark is dependent on the 2nd M1 in part (a) and their *t* values are between 0 and 4</u>. Clear attempt to find all three *s* values (may integrate their *v* incorrectly) **N.B.** No penalty for extra values.
	M1	Complete method using their *s* values Do NOT condone sign errors.
	A1	Any equivalent fraction, 5.7 or better.
		S.C. Correct answer, with no working, scores all 3 marks, since $\int_0^4 \left\|t^2 - 5t + 6\right\| \, \mathrm{d}t$ entered on a calculator will give $\frac{17}{3}$

Question	Scheme	Marks	AOs
4.(a)	$14.7 = -14.7 + 9.8T$ or $0 = 14.7T - \frac{1}{2} \times 9.8T^2$ or $0 = 14.7 - 9.8 \times \left(\frac{1}{2}T\right)$ oe	M1	3.4
	$T = 3$	A1	1.1b
		(2)	
(b)	$s_1 = \frac{(14.7 + 0)}{2} \times 1.5$ (11.025 or $\frac{441}{40}$)	M1	1.1b
	$s_2 = \frac{1}{2} \times 9.8 \times 2.5^2$ (30.625 or $\frac{245}{8}$) **OR** $s_3 = 14.7 \times 1 + \frac{1}{2} \times 9.8 \times 1^2$ (19.6 or $\frac{98}{5}$) **OR** $-s_3 = 14.7 \times 4 - \frac{1}{2} \times 9.8 \times 4^2$ (- 19.6) (allow omission of – on LHS)	M1	1.1b
	Total distance = $s_1 + s_2$ **OR** $2s_1 + s_3$	M1	2.1
	= 41.7 m or 42 m	A1	1.1b
		(4)	
(c)	e.g. Take account of the dimensions of the stone (e.g. allow for spin), do not model the stone as a particle, use a more accurate value for g	B1	3.5c
		(1)	
			(7 marks)

Notes: If they use g = 9.81 or 10, penalise once for whole question.		
4a	M1	Complete method to find T, condone sign errors (M0 if they only find time to top)
	A1	$T = 3$ correctly obtained.
4b	M1	Complete method to find one key distance
	M1	Correct method to find another key distance
	M1	Complete method to find the total distance
	A1	41.7 or 42 (after use of $g = 9.8$)
4c	B1	B0 if there are incorrect extra refinements but ignore extra incorrect statements.

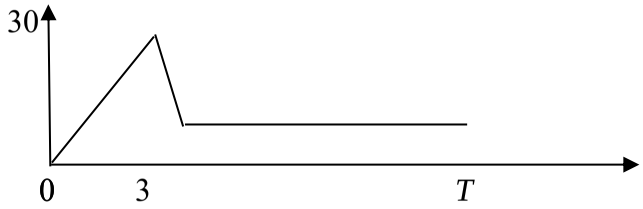
Question	Scheme	Marks	AOs
5(a)	Differentiate v w.r.t. t	M1	3.1a
	$a=\frac{\mathrm{d}v}{\mathrm{d}t}=10-2t$ isw	A1	1.1b
		(2)	
5(b)	Solve problem using $v=0$ when $t=6$	M1	3.1a
	$0=10t-t^2-24$	A1	1.1b
	Solve quadratic oe to find other value of t	M1	1.1b
	$t=4$	A1	1.1b
		(4)	
5(c)	Integrate v or $-v$ w.r.t. t	M1	3.1a
	$5t^2-\frac{1}{3}t^3-24t$	A1	1.1b
	Total distance $=-\left[5t^2-\frac{1}{3}t^3-24t\right]_0^4+\left[5t^2-\frac{1}{3}t^3-24t\right]_4^6$	M1	2.1
	$\frac{116}{3}$ (m)	A1	1.1b
		(4)	
			(10 marks)

Notes:		
5a	M1	Differentiate, with both powers decreasing by 1
	A1	Correct expression
5b	M1	Put $t=6$ **OR** use $(t-6)(t-x)=t^2-10t+k$ oe
	A1	Correct expression (unsimplified) for v **OR** $v=(t-6)(t-4)$
	M1	Put $v=0$ to give quadratic in t and solve for other value of t
	A1	$t=4$
5c	M1	Integrate, with at least two powers increasing by 1 (allow if only two terms integrated)
	A1	Correct expression
	M1	Complete method to find the total distance
	A1	Accept 39(m) or better

Question	Scheme	Marks	AOs
6.(a)	$19^2 = (-U)^2 + 2\times10\times16.8$ (Allow use of $g = 9.8$ for this M mark)	M1	2.1
	$U = 5$ *	A1*	1.1b
		(2)	
	For consistent use of $g = 9.8$ in parts (b), (c) and (d), treat as a MR. i.e. max (b) M1A0 (c)M1A0M(A)0A1ft (d)B1B1ft		
(b)	$19 = -5 + 10T$ **OR** $16.8 = \frac{(-5+19)}{2}T$ **OR** $16.8 = -5T + \frac{1}{2}\times10T^2$ **OR** $16.8 = 19T - \frac{1}{2}\times10T^2$	M1	2.1
	$T = 2.4$	A1	1.1b
		(2)	
(c)	$1.2 = -5t + \frac{1}{2}\times10\times t^2$	M1	2.1
	$5t^2 - 5t - 1.2 = 0$	A1	1.1b
		M(A)1	1.1b
	$t = 1.2$ (s)	A1	1.1b
		(4)	
(d)		B1 shape	1.1b
	(0,5) and $(2.4, -19)$ Allow these to be marked on the axes.	B1ft	1.1b
		(2)	
(e)	Greater since air resistance would slow the ball down.	B1	3.5a
		(1)	
(f)	Take into account: spin, wind effects, use a more accurate value of g, not model the ball as a particle	B1	3.5c
		(1)	

(12 marks)

Notes:		
(a)	M1	Complete method to find U, condone sign errors and use of $g = 9.8$
	A1*	$U = 5$ cao correctly obtained – allow U^2 instead of $(-U)^2$. Allow verification.
(b)	M1	Complete method to find T, condone sign errors
	A1	$T = 2.4$
(c)	M1	Complete method to find t, condone sign errors
	A1	Correct equation with at most one error
	(A)1	Correct equation
	A1	$t = 1.2$ (s)
(d)	B1	Graph could be reflected in the t-axis.
	B1**ft**	Follow through on their T value. If graph is reflected, $(0, -5)$ and $(2.4, 19)$
(e)	B1	Any similar appropriate comment
(f)	B1	B0 if any incorrect extras e.g. weight/mass included

Question	Scheme	Marks	AOs	Notes
7 (a)	$V = 30\ (\text{m s}^{-1})$	B1	3.4	cao
		(1)		
(b)	 shape	B1	1.1b	Overall shape of the graph, starting at the origin. Dotted vertical line at end is OK but solid vertical line is B0
	figs	B1**ft**	1.1b	3, 5 and T marked on the t-axis, and **ft** on their 30 marked on the speed axis. 3 must be where graph reaches a peak. Allow delineators: 3, 2 and $T - 5$ or a mixture
		(2)		
(c)	Using total area = 550 to set up an equation in **one unknown,** Or they may use *suvat* on one or more of the sections (but must still be considering all sections) M0 if they use one *suvat* equation for the whole motion	M1	2.1	Need all sections to be included, with correct structure for each section. e.g. triangle + trapezium + rectangle oe = 550 to give an equation in **one unknown (may not be *T*)**
	$\frac{1}{2}\times 3\times 30+\frac{(30+6)}{2}\times 2+6(T-5)=550$ **OR**: $\frac{1}{2}\times 3\times 30+\frac{1}{2}\times 2\times 24+6(T-3)=550$ **OR:** $\frac{1}{2}\times 3\times 30+\frac{1}{2}\times 2\times 24+(2\times 6)+6(T-5)=550$	A2 **ft**	1.1b	**ft** on their answer to (a). -1 each error. **N.B.** If '6' is incorrect, treat as one error, unless it is correct ft from their 30.

	Solve for T	M1	1.1b	Attempt to solve for T provided they have tried to find the area using at least 3 sections. (M0 if they only solve for their unknown and never try to find T)
	$T = 83$ (nearest whole number)	A1	1.1b	83 is the only answer
		(5)		
(d)	New value of T would be bigger (ignore their reasons whether correct or not)	B1	3.5a	Clear statement about the value of T Allow '*it* would increase, get larger etc' B0 for 'Takes longer' or 'the value of T would be longer'
		(1)		
(e)	e.g. effect of wind; allow for dimensions of parachutist; use a more accurate value for g; parachutist does not fall vertically after chute opens; smooth changes in v; time for parachute to open; deceleration not constant (but B0 if they say *acceleration* not constant); smooth changes in a; B0 for: moves horizontally; mass/weight of parachutist; upthrust; air pressure;air resistance; terminal velocity	B1	3.5c	Any appropriate refinement of the model. B0 if incorrect (or irrelevant) extras
		(1)		
			(10 marks)	

Question	Scheme	Marks	AOs	Notes
8(a)	$v = 12 + 4t - t^2 = 0$ and solving	M1	3.1a	Equating v to 0 and solving the quadratic If no evidence of solving, and at least one answer wrong, M0
	$t = 6$ (or -2)	A1	1.1b	6 but allow -2 as well at this stage
	Differentiate v wrt t	M1	1.1a	For differentiation (both powers decreasing by 1)
	$(a = \frac{dv}{dt} =) \ 4 - 2t$	A1	1.1b	Cao; only need RHS
	When $t = 6$, $a = -8$; Magnitude is 8 (m s^{-2})	A1	1.1b	Substitute in $t = 6$ and get 8 (m s^{-2}) as the answer . Must be **positive.** (A0 if two answers given)
		(5)		
(b)	Integrate v wrt t	M1	3.1a	For integration (at least two powers increasing by 1)
	$(s =) 12t + 2t^2 - \frac{1}{3}t^3 (+C)$	A1	1.1b	Correct expression (ignore C) only need RHS Must be used in part (b)
	$t = 3$ => distance = 45 (m)	A1	1.1b	Correct distance. Ignore units
		(3)		
		(8 marks)		

Question	Scheme	Marks	AOs
9.	Equation in t only	M1	2.1
	$-2 = 9t - \frac{1}{2} \times 10t^2$	A1	1.1b
	$5t^2 - 9t - 2 = 0 = (5t+1)(t-2)$	DM1	1.1b
	$T = 2$ (only)	A1	1.1b
		(4)	
			(4 marks)

Notes:

M1: Complete method to give equation in t only. This mark is for a complete method for the TOTAL time i.e. for finding sufficient equations, with usual rules, correct no. of terms in each equation but condone sign errors and g does not need to be substituted

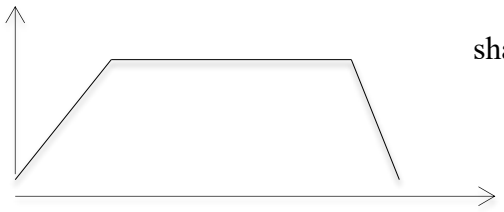
A1: A correct equation **or** correct equations (e.g. if they find the speed, 11 ms^{-1}, when the ball strikes the ground and then use that to find the total time **or** if they split the time (e.g. 0.9s up and 1.1s down or 0.9s + 0.9s + 0.2s))

N.B. $g = 10$ must be substituted in all equations used.

DM1: Dependent on first M1, for solving a 3 term quadratic to find T **or** for solving their equations to find T **or** for solving their equations and adding their split times to find T

A1: $T = 2$ only (i.e. A0 if they give two times)

N.B. If solving a correct quadratic, the DM1 can be implied by a correct answer i.e. the method does not need to be shown, but if there is no method shown and the answer is wrong then award DM0 A0.

Question	Scheme	Marks	AOs
10(a) (i)	24 ($m\,s^{-1}$)	B1	1.1b
(ii)	48 (s)	B1	1.1b
(iii)	 shape	B1	1.1b
		(3)	
(b)	Equating area under graph to 4800 to give equation in one unknown	M1	3.1b
	$\frac{1}{2}(T+T+80+48)\times 24=4800$ **OR** $(\frac{1}{2}\times 80\times 24)+24T+(\frac{1}{2}\times 48\times 24)=4800$ **oe**	A1**ft**	1.1b
	T = 136 so total time is 264 (s)	A1	1.1b
		(3)	
(c)	Accept **Either**: a smooth change from acceleration to constant velocity or from constant velocity to deceleration. **Or** have train accelerating and/or decelerating at a variable rate Do not accept e.g. Comments on air resistance or resistive forces, straightness of track, horizontal track, friction, length of train, mass of train, not having train moving with constant velocity. B0 if either an incorrect extra is included or an incorrect reason for a valid improvement is included. N.B. Variable acceleration due to air resistance is B0 **BUT** Variable acceleration due to **variable** air resistance is B1	B1	3.5c
		(1)	
			(7 marks)

Notes:
(a) **(i) B1:** 24 (m s^{-1})Must be stated i.e. not just inserted on the graph **(ii) B1:** 48 (s) (Allow – 48 changed to 48) Must be stated i.e. not just inserted on the graph **(iii) B1:** A trapezium starting at the origin and ending on the *t*-axis.
(b) **M1:** Complete method to find area of trapezium using trapezium rule with correct structure or using two triangles and a rectangle and equate to 4800 to give equation in *one* unknown *N.B.* $\frac{1}{2}(T+80+48)\times 24 = 4800$ is M0 (equivalent to using three triangles) **OR** they may use *suvat* on one or more sections (must have $a = 0$ for middle section) and equate total distance travelled to 4800 to give equation in *one* unknown **A1ft:** For a correct equation in their unknown **ft** on their 24 and 48 (but must be positive times) **A1:** For 264 (s)
(c) **B1:** **Either**: Include time to change from constant accln to constant velocity and/or time to change from constant velocity to constant deceleration oe **Or**: Have train accelerating and/or decelerating at a variable rate

Question	Scheme	Marks	AOs
11(a)	Multiply out and differentiate *wrt* to time (or use of product rule i.e. must have two terms with correct structure)	M1	1.1a
	$v = 2t^3 - 3t^2 + t$	A1	1.1b
	$2t^3 - 3t^2 + t = 0$ and solve: $t(2t - 1)(t - 1) = 0$	**DM**1	1.1b
	$t = 0$ or $t = \frac{1}{2}$ or $t = 1$; any two	A1	1.1b
	All three	A1	1.1b
		(5)	
(b)	Find x when $t = 0$, $\frac{1}{2}$, 1 and 2 : $(0, \frac{1}{32}, 0, 2)$	M1	2.1
	Distance = $\frac{1}{32} + \frac{1}{32} + 2$	M1	2.1
	$2\frac{1}{16}$ (m) oe or 2.06 or better	A1	1.1b
		(3)	
(c)	$x = \frac{1}{2}t^2(t - 1)^2$	M1	3.1a
	$\frac{1}{2}$ perfect square so $x \geq 0$ i.e. never negative	A1 cso	2.4
		(2)	
			(10 marks)

Notes:

(a)

M1: Must have 3 terms and at least two powers going down by 1

A1: A correct expression

DM1: Dependent on first M, for equating to zero and attempting to solve a cubic

A1: Any two of the three values (Two correct answers can imply a correct method)

A1: The third value

(b)

M1: For attempting to find the values of x (at least two) at their t values found in (a) or at $t = 2$ or equivalent e.g. they may integrate their v and sub in at least two of their t values

M1: Using a correct strategy to combine their distances (must have at least 3 distances)

A1: $2\frac{1}{16}$ (m) oe or 2.06 or better
(c) **M1:** Identify strategy to solve the problem such as: (i) writing x as $\frac{1}{2} \times$ perfect square (ii) or using x values identified in (b). (iii) or using calculus i.e. identifying min points on x–t graph. (iv) or using x-t graph. **A1 cso :** Fully correct explanation to show that $x \geq 0$ i.e. never negative

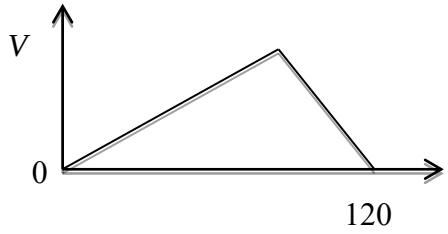
Question	Scheme	Marks	AOs
12(a)	Use of $s = vt - \frac{1}{2}at^2$	M1	2.1
	$19.6 = 4v - \frac{1}{2} \times 9.8 \times 4^2$	A1	1.1b
	$v = 24.5$ or 25 (m s^{-1})	A1	1.1b
		(3)	
(b)	$0 = 14.7^2 - 2 \times 9.8h$	M1	2.1
	$h = 11.0$ or 11 (m)	A1	1.1b
		(2)	
(c)	New value of speed would be lower.	B1	3.5a
		(1)	
			(6 marks)

Notes:

(a)
M1: Complete method to give equation in v only (could involve 2 or more *suvat* equations and then elimination) with usual rules
A1: Correct equation
A1: Correct answer

(b)
M1: Complete method to find h
A1: 11.0 or 11 (m)

(c)
B1: New value of speed will be lower

Question	Scheme	Marks	AOs
13(a)	 Shape	B1	1.1b
	V, 120	B1	1.1b
		(2)	
(b)	$\frac{1}{2} \times 120V = 1500$	M1	3.1b
	$V = 25$	A1	1.1b
		(2)	
(c)	Area of triangle = Distance travelled = $(\frac{1}{2} \times 120V) = 1500$	B1	2.4
	This does not depend on T so T can take any value where $0 < T < 120$	B1	2.4
		(2)	
(d)	Include a constant speed phase in the motion	B1	3.5c
		(1)	
			(7 marks)

Notes:

(a)
B1: Triangle, starting at the origin with base on axis and apex between $t = 0$ and $t = 120$
B1: V and 120 correctly marked (allow a delineator)

(b)
M1: Identifying correct strategy to solve problem to give equation in V only
A1: $V = 25$

(c)
B1: Area of triangle only depends on base and height
B1: So T can take any value $0 < T < 120$

(d)
B1: e.g. Include a *smooth* change from acceleration phase to deceleration phase.
e.g. Have a variable acceleration and/or deceleration phase

Question	Scheme	Marks	AOs
14(a)	$s = \int_0^1 16 - 3t^2 \, dt$	M1	1.1a
	$= \left[16t - t^3\right]_0^1$	A1	1.1b
	$= 15$ (m)	A1	1.1b
		(3)	
(b)	$16 - 3t^2 = 0$	M1	3.1b
	$t = \sqrt{\frac{16}{3}}$ oe	A1	1.1b
		(2)	
(c)	$16t - t^3 = 0$	M1	3.1b
	$t(16 - t^2) = 0$	M1	1.1b
	$t = 4$	A1	1.1b
		(3)	
			(8 marks)

Notes:

(a)

M1: Attempt to integrate, one power going up

A1: Correct integral and limits or indefinite integral with $C = 0$ and $t = 1$.

A1: 15 (m)

(b)

M1: Identifying correct strategy to solve problem of finding direction change by equating v to 0 and solving for t

A1: correct answer – any surd or decimal equivalent to at least 2 sf

(c)

M1: Identifying correct strategy to solve problem by using use $s = 0$ and equating their integral to 0

M1: Attempt to solve

A1: $t = 4$

Question	Scheme	Marks	AOs
15.	Using distance = total area under graph (e.g. area of rectangle + triangle **or** trapezium **or** rectangle – triangle)	M1	2.1
	e.g. $D = UT + \frac{1}{2} Th$, where h is height of triangle	A1	1.1b
	Using gradient = acceleration to substitute $h = aT$	M1	1.1b
	$D = U T + \frac{1}{2} aT^2$ *	A1 *	1.1b
		(4)	
			(4 marks)

Notes:

M1: For use of distance = total area to give an equation in D, U, T and one other variable
A1: For a correct equation
M1: For using gradient = a to eliminate the other variable to give an equation in D, U, T and a *only*
A1*: For a correct given answer

Question	Scheme	Marks	AOs
16(i)(ii)	Using a correct strategy for solving the problem by setting up two equations in a and u only and solving for either	M1	3.1b
	Equation in a and u only	M1	3.1b
	$22 = 2u + \frac{1}{2} a\, 2^2$	A1	1.1b
	Another equation in a and u only	M1	3.1b
	$126 = 6u + \frac{1}{2} a\, 6^2$	A1	1.1b
	5 m s^{-2}	A1	1.1b
	6 m s^{-1}	A1ft	1.1b
			(7 marks)

Notes:

M1: For solving the problem by setting up two equations in a and u only and solving for either
M1: Use of (one or more) *suvat* formulae to produce an equation in u and a only
A1: For a correct equation
M1: Use of (one or more) *suvat* formulae to produce another equation in u and a only
A1: For a correct equation
A1: For correct accln 5 m s^{-2}
A1: For correct speed 6 m s^{-1} (The second of these A marks is an **ft** mark, following an incorrect value for u or a, depending on which has been found first)

N.B. Do not award the ft mark for absurd answers e.g. $a > 15$, $u > 50$

See alternative on the next page

Question	Scheme	Marks	AOs
16(i)(ii)	Using a correct strategy for solving the problem by obtaining actual speeds at two times and using a = change in speed / time taken	M1	3.1b
	Actual speed at $t = 1$ = Average speed over interval	M1	3.1b
	$22/2 = 11$	A1	1.1b
	Actual speed at $t = 4$ = Average speed over interval	M1	3.1b
	$104/4 = 26$	A1	1.1b
	$5\ \text{m s}^{-2}$	A1	1.1b
	$6\ \text{m s}^{-1}$	A1ft	1.1b
			(7 marks)

Notes:

M1: For solving the problem by obtaining two actual speeds and use of $a = (v - u)/t$
M1: Use of speed at half-time = av speed over interval to produce a speed at $t = 1$
A1: For a correct speed
M1: Use of speed at half-time = av speed over interval to produce a speed at $t = 4$
A1: For a correct speed
A1: For correct accln $5\ \text{m s}^{-2}$
A1: ft for correct speed $6\ \text{m s}^{-1}$ (This is an ft mark, following an incorrect value of a)

N.B. Do not award the ft mark for absurd answers e.g. $a > 15$, $u > 50$

Question	Scheme	Marks	AOs
17(a)	Substitution of both $t = 0$ and $t = 10$	M1	2.1
	$s = 0$ for both $t = 0$ and $t = 10$	A1	1.1b
	Explanation ($s > 0$ for $0 < t < 10$) since $s = \frac{1}{10}t^2(t-10)^2$	A1	2.4
		(3)	
(b)	Differentiate displacement s w.r.t. t to give velocity, v	M1	1.1a
	$v = \frac{1}{10}(4t^3 - 60t^2 + 200t)$	A1	1.1b
	Interpretation of 'rest' to give $v = \frac{1}{10}(4t^3 - 60t^2 + 200t) = \frac{2}{5}t(t-5)(t-10) = 0$	M1	1.1b
	$t = 0, 5, 10$	A1	1.1b
	Select $t = 5$ and substitute their $t = 5$ into s	M1	1.1a
	Distance = 62.5 m	A1ft	1.1b
		(6)	
			(9 marks)

Notes:

(a)

M1: For substituting $t = 0$ and $t = 10$ into s expression

A1: For noting that $s = 0$ at both times

A1: Since s is a perfect square, $s > 0$ for all other t- values

(b)

M1: For differentiating s w.r.t. t to give v (powers of t reducing by 1)

A1: For a correct v expression in any form

M1: For equating v to 0 and factorising

A1: For correct t values

M1: For substituting their intermediate t value into s

A1: ft following an incorrect t-value

Question	Scheme	Marks	AOs
18(a)	Put $t = 2$ in **v** and use Pythagoras: $\sqrt{12^2 + (-6\sqrt{2})^2}$	M1	3.1a
	$\sqrt{216}, 6\sqrt{6}$ or 15 or better (m s^{-1})	A1	1.1b
		(2)	
18(b)	Differentiate **v** wrt t to obtain **a**	M1	3.4
	$6t\mathbf{i} - 3t^{-\frac{1}{2}}\mathbf{j}$ oe (m s^{-2}) isw	A1	1.1b
		(2)	
18(c)	Integrate **v** wrt t to obtain **r**	M1	3.4
	$\mathbf{r} = t^3\mathbf{i} - 4t^{\frac{3}{2}}\mathbf{j}\ (+\mathbf{C})$	A1	1.1b
	$(\mathbf{i} - 4\mathbf{j}) = 4^3\mathbf{i} - 4\times 4^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$	M1	3.1a
	$(-62\mathbf{i} + 24\mathbf{j})$ (m) isw e.g. if they go on to find the distance.	A1	1.1b
		(4)	
			(8 marks)

Notes: Accept column vectors throughout apart from the answer to (b).

18a	M1	Need square root but -ve sign not required. Allow **i**'s and/or **j**'s to go missing from their **v** at $t = 2$, provided they have applied Pythagoras correctly.
	A1	cao **N.B**. Correct answer with no working can score 2 marks.
18b	M1	Both powers decreasing by 1. Allow a column vector. M0 if **i** or **j** is missing but allow recovery in (b).
	A1	cao. Do not accept a column vector.
18c	M1	Both powers increasing by 1 M0 if **i** or **j** is missing but allow recovery.
	A1	(**r** =) not required
	M1	Putting **r** = $(\mathbf{i} - 4\mathbf{j})$ and $t = 4$ into their displacement **vector** expression which must have **C** (allow *C*) to give an equation in **C** only, seen or implied. Must have attempted to integrate **v** for this mark to be available. **N.B**. **C** does not need to be found and this is a method mark, so allow slips.
	A1	cao

Question	Scheme	Marks	AOs
19(a)	Using horizontal motion	M1	3.3
	Whole Motion **Half way**		
	$U\cos\alpha \times t = 120$ $U\cos\alpha \times t = 60$	A1	1.1b
	Using vertical motion **OR**	M1	3.4
	$U\sin\alpha \times t - \frac{1}{2}gt^2 = 0$ $0 = U\sin\alpha - gt$	A1	1.1b
	Attempt to solve problem by eliminating t	DM1	3.1b
	$U^2\sin\alpha\cos\alpha = 588$ *	A1*	2.2a
		(6)	
	N.B. No credit given if they use the given answer from (b).		
19(b)	Using vertical motion	M1	3.4
	$0^2 = (U\sin\alpha)^2 - 2g\times 10$	A1	1.1b
	ALTERNATIVE 1: If t is time to top: use of $10 = \frac{1}{2}gt^2$ oe ($t = \frac{10}{7}$) to obtain an equation in U and α only M1 $U\sin\alpha = 14$ or $U\cos\alpha = 42$ A1 **ALTERNATIVE 2:** If t is time to top: use of : $10 = U\sin\alpha\ t - \frac{1}{2}gt^2$ with $t = \frac{60}{U\cos\alpha}$ substituted to obtain an equation in U and α only : M1 $10 = U\sin\alpha \times \frac{60}{U\cos\alpha} - \frac{1}{2}g\left(\frac{60}{U\cos\alpha}\right)^2$ A1		
	Attempt to solve problem by eliminating α : e.g. $U\sin\alpha = 14$ => $U\cos\alpha = 42$, from part (a) or from using $t = \frac{10}{7}$, then square and add to give result **OR:** $U^2\sin^2\alpha = 20g = 196$ and $U^2\sin\alpha\cos\alpha = 588$, divide to give $\tan\alpha = \frac{1}{3}$ then $\sin^2\alpha = \frac{1}{10}$, hence result **OR in ALTERNATIVE 2**: sub for U^2 using part (a), to give $\tan\alpha = \frac{1}{3}$ then $\sin^2\alpha = \frac{1}{10}$, hence result	DM1	3.1b

	N.B. Just stating that $\sin^2\alpha = \frac{1}{10}$, with no working is DM0A0.		
	$U^2 = 1960$ *	A1*	2.2a
	N.B. Verification (i.e. starting with $U^2 = 1960$ and trying to work backwards) is not an acceptable method for this question.		
		(4)	
19(c)	V, since air resistance has to be overcome, or just 'because of <u>air resistance' isw</u>	B1	3.5a
		(1)	
19(d)	e.g. wind effects, more accurate value of g, spin of ball, size of ball, shape of ball, dimensions of ball, not a particle, variable acceleration, surface area of ball, humidity. Allow wind resistance and rotational resistance (Ignore any mention of air resistance or drag)	B1	3.5c
		(1)	
			(12 marks)

Notes:

19a		**N.B**. Could score 2/6 for any one of the 4 given equations if there is no corresponding second equation or there is an attempt but it's incorrect.
	M1	Complete method to give equation in U, α and t only, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation
	M1	Complete method to give equation in U, α and t only, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation
	DM 1	Eliminate t, dependent on first and second M1's
	A1*	Given answer correctly obtained, <u>with no wrong working seen.</u> Allow $588 = U^2 \sin\alpha\cos\alpha$ but nothing else
19b	M1	Complete method to give equation in U and α only with correct no. of terms, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation
	DM 1	Eliminate α and rearrange, dependent on first M1
	A1*	Given answer correctly obtained with <u>no wrong working seen</u> (**N.B**. If they use a value for α (18.43.°.) they lose the final A1*)
19c	B1	Clear statement isw
19d	B1	B0 if there is an incorrect extra e.g. mass or weight

Question	Scheme	Marks	AOs
20(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $t = 2$: $\mathbf{v} = 4\mathbf{i} + 2(2\mathbf{i} - 3\mathbf{j})$ **OR integration**: $\mathbf{v} = (2\mathbf{i} - 3\mathbf{j})t + 4\mathbf{i}$, with $t = 2$	M1	3.1a
	$\mathbf{v} = 8\mathbf{i} - 6\mathbf{j}$	A1	1.1b
		(2)	
20(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ at $t = 3$: $(\mathbf{i} + \mathbf{j}) + \left[3 \times 4\mathbf{i} + \frac{1}{2} \times (2\mathbf{i} - 3\mathbf{j}) \times 3^2\right]$ **OR**: find $\mathbf{v}$ at $t = 3$: $4\mathbf{i} + 3(2\mathbf{i} - 3\mathbf{j}) = (10\mathbf{i} - 9\mathbf{j})$ then use $\mathbf{r} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$ $(\mathbf{i} + \mathbf{j}) + \left[\frac{1}{2}\left[4\mathbf{i} + (10\mathbf{i} - 9\mathbf{j})\right] \times 3\right]$ or $\mathbf{r} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ $(\mathbf{i} + \mathbf{j}) + \left[3 \times (10\mathbf{i} - 9\mathbf{j}) - \frac{1}{2} \times (2\mathbf{i} - 3\mathbf{j}) \times 3^2\right]$ **OR integration:** $\mathbf{r} = (\mathbf{i} + \mathbf{j}) + \left[(2\mathbf{i} - 3\mathbf{j})\frac{1}{2}t^2 + 4t\mathbf{i}\right]$, with $t = 3$	M1	3.1a
	$\mathbf{r} = 22\mathbf{i} - 12.5\mathbf{j}$	A1	2.2a
		(2)	
			(4 marks)

Notes: Accept column vectors throughout		
20a	M1	Complete method to find **v**, using **ruva***t* or integration (M0 if **i** and/or **j** is missing)
	A1	Apply isw if they also find the speed
20b	M1	Complete method to find the p.v. but this mark can be scored if they omit **(i + j)** i.e. the M1 is for the expression in the square bracket If they integrate, the M1 is earned once the expression in the square bracket is seen with $t = 3$ (M0 if **i** and/or **j** is missing)
	A1	cao

Question	Scheme	Marks	AOs
	Note that g = 10; penalise once for whole question if g = 9.8		
21(a)	Use $s = ut + \frac{1}{2}at^2$ vertically or any complete method to give an equation in t only	M1	3.4
	$-70 = 65\sin\alpha \times t - \frac{1}{2} \times g \times t^2$	A1	1.1b
		M (A)1	1.1b
	t = 7 (s)	A1	1.1b
		(4)	
21(b)	Horizontal velocity component at A = $65\cos\alpha$ (60)	B1	3.4
	Complete method to find vertical velocity component at A	M1	3.4
	$65\sin\alpha - g \times 7$ **OR** $\sqrt{(-25)^2 + 2g \times 70}$ (45)	A1**ft**	1.1b
	Sub for trig and square, add and square root : $\sqrt{60^2 + (-45)^2}$	M1	3.1b
	75 Accept 80 (m s^{-1})	A1	1.1b
		(5)	
21(c)	e.g. an approximate value of g has been used, the dimensions of the stone could affect its motion, spin of the stone, g = 10 instead of 9.8 has been used, g has been assumed to be constant, wind effect, shape of the stone	B1	3.5b
		(1)	
			(10 marks)

Notes:		
21a	M1	Complete method, correct no. of terms, condone sign errors and sin/cos confusion
	A1	Correct equation in t only with at most one error
	M(A)1	Correct equation in t only
		N.B. For 'up and down' methods etc, the two A marks are for all the equations that they use, lose a mark for each error.
	A1	Cao (g = 9.8 , 7.1 or 7.11) (g = 9.81 , 7.1 or 7.12)
21b	B1	Seen, including on a diagram.
	M1	Condone sign errors and sin/cos confusion
	A1**ft**	Correct expression; accept negative of this, follow their t
	M1	Sub for trig and use Pythagoras
	A1	Cao (g = 9.8 or 9.81 , 75 or 74.8)
21c	B1	B0 if incorrect extras

Question	Scheme	Marks	AOs
	Allow column vectors throughout this question		
22(a)	Differentiate **v** wrt *t*	M1	3.1a
	$\frac{3}{2}t^{-\frac{1}{2}}\mathbf{i}-2\mathbf{j}$ isw	A1	1.1b
		(2)	
22(b)	$3t^{\frac{1}{2}}=2t$	M1	2.1
	Solve for *t*	DM1	1.1b
	$t=\frac{9}{4}$	A1	1.1b
		(3)	
22(c)	Integrate **v** wrt *t*	M1	3.1a
	$\mathbf{r}=2t^{\frac{3}{2}}\mathbf{i}-t^2\mathbf{j}(+\mathbf{C})$	A1	1.1b
	$t = 1$, $\mathbf{r} = -\mathbf{j}$ => $\mathbf{C} = -2\mathbf{i}$ so $\mathbf{r}=2t^{\frac{3}{2}}\mathbf{i}-t^2\mathbf{j}-2\mathbf{i}$	A1	2.2a
		(3)	
22(d)	$\sqrt{(3t^{\frac{1}{2}})^2+(2t)^2}=10$ or $(3t^{\frac{1}{2}})^2+(2t)^2=10^2$	M1	2.1
	$9t+4t^2=100$	M(A)1	1.1b
	$t = 4$	A1	1.1b
	$\mathbf{r}=14\mathbf{i}-16\mathbf{j}$	M1	1.1b
	$\sqrt{14^2+(-16)^2}$	M1	3.1a
	$\sqrt{452}$ $(2\sqrt{113})$ (m)	A1	1.1b
		(6)	
			(14 marks)

Notes:		
22a	M1	Both powers decreasing by 1 (M0 if vector(s) disappear but allow recovery)
	A1	cao
22b	M1	Complete method, using **v**, to obtain an equation in *t* only, allow a sign error
	DM1	Dependent on M1,solve for *t*

	A1	cao
22c	M1	Both powers increasing by 1 (M0 if vectors disappear but allow recovery)
	A1	Correct expression without **C**
	A1	cao
22d	M1	Use of Pythagoras on **v** and 10 to set up equation in *t*
	M(A)1	Correct 3 term quadratic in *t*
	A1	cao
	M1	Substitute their numerical *t* value into their **r**
	M1	Use of Pythagoras to find the magnitude of their **r**
	A1	cso

Question	Scheme	Marks	AOs
23(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ or integrate to give: $\mathbf{v} = (-2\mathbf{i} + 2\mathbf{j}) + 2(4\mathbf{i} - 5\mathbf{j})$	M1	3.1a
	$(6\mathbf{i} - 8\mathbf{j})\ (\text{m s}^{-1})$	A1	1.1b
		(2)	
23(b)	Solve problem through use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ or integration (M0 if $\mathbf{u} = \mathbf{0}$) **Or** any other complete method e.g use $\mathbf{v} = \mathbf{u} + \mathbf{a}T$ and $\mathbf{r} = \frac{(\mathbf{u} + \mathbf{v})T}{2}$:	M1	3.1a
	$-4.5\mathbf{j} = 2t\mathbf{j} - \frac{1}{2}t^2 5\mathbf{j}$ (**j** terms only)	A1	1.1b
	The first two marks could be implied if they go straight to an algebraic equation.		
	Attempt to equate **j** components to give equation in T only $(-4.5 = 2T - \frac{5}{2}T^2)$	M1	2.1
	$T = 1.8$	A1	1.1b
		(4)	
23(c)	Solve problem by substituting <u>their</u> T value (M0 if $T < 0$) into the **i** component equation to give an equation in λ only: $\lambda = -2T + \frac{1}{2}T^2 \times 4$	M1	3.1a
	λ = 2.9 or 2.88 or $\frac{72}{25}$ oe	A1	1.1b
		(2)	

Notes: Accept column vectors throughout		**(8 marks)**
23a	M1	For any complete method to give a **v** expression with correct no. of terms with $t = 2$ used, so if integrating, must see the initial velocity as the constant. Allow sign errors.
	A1	Cao isw if they go on to find the speed.
23b	M1	For any complete method to give a vector expression for **j** component of displacement in t (or T) only, using $\mathbf{a} = (4\mathbf{i} - 5\mathbf{j})$, so if integrating, RHS of equation must have the correct structure. Allow sign errors.
	A1	Correct **j** vector equation in t or T. Ignore **i** terms.
	M1	Must have earned 1st M mark.

		Equate **j** components to give equation in T (allow t) only (no **j**'s) which has come from a displacement. Equation must be a 3 term quadratic in T.
	A1	cao
23c	M1	Must have earned 1st M mark in (b) Complete method - must have an equation in λ only (no **i**'s) which has come from an appropriate displacement.. (e.g M0 if **a** = **0** has been used) Expression for λ must be a quadratic in T
	A1	cao

Question	Scheme	Marks	AOs
24(i)(a)	Integrate **a** wrt t to obtain velocity	M1	3.4
	$\mathbf{v} = (t - 2t^2)\mathbf{i} + \left(3t - \frac{1}{3}t^3\right)\mathbf{j}$ $(+\mathbf{C})$	A1	1.1b
	$8\mathbf{i} - \frac{28}{3}\mathbf{j}$ (m s^{-1})	A1	1.1b
		(3)	
24(i)(b)	Equate **i** component of **v** to zero	M1	3.1a
	$t - 2t^2 + 36 = 0$	A1**ft**	1.1b
	$t = 4.5$ (ignore an incorrect second solution)	A1	1.1b
		(3)	
24(ii)	Differentiate **r** wrt to t to obtain velocity	M1	3.4
	$\mathbf{v} = (2t - 1)\mathbf{i} + 3\mathbf{j}$	A1	1.1b
	Use magnitude to give an equation in t only	M1	2.1
	$(2t - 1)^2 + 3^2 = 5^2$	A1	1.1b
	Solve problem by solving this equation for t	M1	3.1a
	$t = 2.5$	A1	1.1b
		(6)	
			(12 marks)

Notes: Accept column vectors throughout		
24(i)(a)	M1	At least 3 terms with powers increasing by 1 (but M0 if clearly just multiplying by t)
	A1	Correct expression
	A1	Accept $8\mathbf{i} - 9.3\mathbf{j}$ or better. Isw if speed found.
24(i)(b)	M1	Must have an equation in t only (Must have integrated to find a velocity vector)
	A1 **ft**	Correct equation follow through on their **v** but must be a 3 term quadratic
	A1	cao
24(ii)	M1	At least 2 terms with powers decreasing by 1 (but M0 if clearly just dividing by t)
	A1	Correct expression
	M1	Use magnitude to give an equation in t only, must have differentiated to find a velocity (M0 if they use $\sqrt{x^2 - y^2}$)

	A1	Correct equation $\sqrt{(2t-1)^2+3^2}=5$
	M1	Solve a 3 term quadratic for *t* which has come from differentiating and using a magnitude. This M mark can be implied by a correct answer with no working.
	A1	2.5

Question	Scheme	Marks	AOs
25(a)	Using horizontal motion	M1	3.3
	$U\cos 45^\circ t = 100$	A1	1.1b
	Using vertical motion	M1	3.4
	$U\sin 45^\circ t - \frac{1}{2}gt^2 = -25$	A1	1.1b
	Solve problem by eliminating t and solving for U	M1	3.1b
	$U = 28$*	A1*	1.1b
		(6)	
25(b)	Using vertical motion	M1	3.4
	$0^2 = (28\sin 45^\circ)^2 - 2gh$	A1	1.1b
	Greatest height = 45 m	A1	1.1b
		(3)	
25(c)	New value > 28	B1	3.5a
		(1)	
25(d)	e.g. wind effects, more accurate value of g, spin of ball, include size of the ball, not model as a particle, shape of ball	B1	3.5c
		(1)	
			(11 marks)

Notes:		
25a	M1	Complete method to give equation in U and t only, condone sin/cos confusion and sign errors
	A1	Correct equation
	M1	Complete method to give equation in U and t only, condone sin/cos confusion and sign errors
	A1	Correct equation (g does not need to be substituted)
	M1	Must have earned the previous two M marks. Eliminate t and solve for U. **N.B**. They may solve for t first $(100 - \frac{1}{2}gt^2 = -25)$ and then use it to find U.
	A1*	Exact given answer correctly obtained with no wrong working (e.g. g = 9.81 used) or approximation seen.
25b	M1	Complete method to give equation in h only (allow if U not substituted), condone sin/cos confusion and sign errors

	A1	Correct equation (g does not need to be substituted) (A0 if U is used instead of 28)
	A1	cao
25c	B1	Clear statement
25d	B1	Penalise incorrect extras i.e. B0 if there are incorrect extras. The ground being horizontal, the cliff being vertical, .. are not part of the model so B0 Include weight/mass of the ball B0

Question	Scheme	Marks	AO
26(a)	Differentiate **v**	M1	1.1a
	$(\mathbf{a}=)6\mathbf{i}-\frac{15}{2}t^{\frac{1}{2}}\mathbf{j}$	A1	1.1b
	$=6\mathbf{i}-15\mathbf{j}$ (m s^{-2})	A1	1.1b
		(3)	
26(b)	Integrate **v**	M1	1.1a
	$(\mathbf{r}=)(\mathbf{r}_0)+3t^2\mathbf{i}-2t^{\frac{5}{2}}\mathbf{j}$	A1	1.1b
	$=(-20\mathbf{i}+20\mathbf{j})+(48\mathbf{i}-64\mathbf{j})=28\mathbf{i}-44\mathbf{j}$ (m)	A1	2.2a
		(3)	
		(6)	

Marks		Notes
		N.B. Accept column vectors throughout and condone missing brackets in working but they must be there in final answers
26a	M1	Use of $\mathbf{a}=\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$ with attempt to differentiate (both powers decreasing by 1) M0 if **i**'s and **j**'s omitted and they don't recover
	A1	Correct differentiation in any form
	A1	Correct and simplified. Ignore subsequent working (ISW) if they go on and find the magnitude.
26b	M1	Use of $\mathbf{r}=\int\mathbf{v}\mathrm{d}t$ with attempt to integrate (both powers increasing by 1) M0 if **i**'s and **j**'s omitted and they don't recover
	A1	Correct integration in any form. Condone $\mathbf{r}_0$ not present
	A1	Correct and simplified.

Question	Scheme	Marks	AO
27(a)	$(\mathbf{v}=)\mathbf{C}+(2\mathbf{i}-3\mathbf{j})t$	M1	3.1a
	$(\mathbf{v}=)(-\mathbf{i}+4\mathbf{j})+(2\mathbf{i}-3\mathbf{j})t$	A1	1.1b
	$\frac{4-3T}{-1+2T}=\frac{-4}{3}$ oe	M1	3.1a
	$T=8$	A1	1.1b
		(4)	
(b)	$(\mathbf{s}=)\mathbf{C}t+(2\mathbf{i}-3\mathbf{j})\frac{1}{2}t^2$ $(+\mathbf{D})$	M1	3.1a
	$(\mathbf{s}=)(-\mathbf{i}+4\mathbf{j})t+\frac{1}{2}(2\mathbf{i}-3\mathbf{j})t^2$ $(+\mathbf{D})$	A1	1.1b
	$AB=\sqrt{12^2+8^2}$ **N.B. Beware you may see 4(2i – 3j) which leads to** $\sqrt{(8^2+12^2)}$ this is M0A0M0A0.	M1	3.1a
	$=4\sqrt{13}\,(=14.422051....)$ (m)	A1**cso**	1.1b
		(4)	
		(8)	

Marks		Notes
27a	M1	Use of $\mathbf{v}=\mathbf{u}+\mathbf{a}t$ **OR** integration to give an expression of the form $\mathbf{C}+(2\mathbf{i}-3\mathbf{j})t$ **, where C is a non-zero constant <u>vector</u>** M0 if **u** and **a** are reversed Condone use of **a** = (2**i** + 3**j**) for this M mark
	A1	Any correct unsimplified expression seen or implied
	M1	Correct use of ratios, <u>using a velocity vector</u> (must be using $\frac{-4}{3}$) to give equation <u>in T only</u> M0 if they equate $4-3T=-4$ and/or $-1+2T=3$ and therefore M0 if they then divide to produce their equation
	A1	Correct only
		N.B. (i) Can score the second M1A1 if they get $T=8$, using a calculator to solve two simultaneous equations, but if answer is wrong, and no equation in T only, second M0 (ii) Can score M1A1 M1A1 if they get $T=8$, using trial and error, but if they don't get $T=8$, can only score max M1A1M0A0

27b	M1	Use of $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with **a = (2i – 3j)** **OR** integration to give an expression of the form $\mathbf{C}t + (2\mathbf{i} - 3\mathbf{j})\frac{1}{2}t^2$ **, where C is their non-zero constant <u>vector</u> from (a)** Condone use of **a** = (2**i** + 3**j**) for this M mark **OR** any other complete method using vector **suva***t* equations
	A1	Correct unsimplified expression seen or implied
	M1	Use of $t = 4$ in their **s** (which must be a **displacement vector**) and then Pythagoras with the root sign **N.B.** This M mark can be implied by a correct answer, otherwise we need to see Pythagoras used, with the root sign, for the M mark.
	A1**cso**	Any surd form or 14 or better

Question	Scheme	Marks	AO
	In this question mark parts (a) and (b) together.		
28(a)	Horizontal speed $= 20\cos 30°$	B1	3.4
	Vertical velocity <u>at $t = 2$</u>	M1	3.4
	$= 20\sin 30° - 2g$	A1	1.1b
	$\theta = \tan^{-1}\left(\pm\dfrac{9.6}{10\sqrt{3}}\right)$	M1	1.1b
	Speed $= \sqrt{100\times 3 + 9.6^2}$ or e.g. speed $= \dfrac{9.6}{\sin\theta}$	M1	1.1b
	19.8 or 20 $\left(\text{m s}^{-1}\right)$ at 29.0° or 29° to the horizontal oe	A1	2.2a
		(6)	
(b)	Using sum of horizontal distances $= 50$ at $t = 2$	M1	3.3
	$(u\cos\theta)\times 2 + (20\cos 30°)\times 2 = 50$ $(u\cos\theta = 25 - 20\cos 30°)$	A1	1.1b
	Vertical distances equal	M1	3.4
	$\Rightarrow (20\sin 30°)\times 2 - \dfrac{g}{2}\times 4 = (u\sin\theta)\times 2 - \dfrac{g}{2}\times 4$ $(20\sin 30° = u\sin\theta)$	A1	1.1b
	Solving for both θ and u	M1	3.1b
	$\theta = 52°$ or better (52.47756849….°) $u = 13$ or better (12.6085128…)	A1	2.2a
		(6)	
(c)	It does not take account of the fact that they are not particles (moving freely under gravity) It does not take account of the size(s) of the balls It does not take account of the spin of the balls It does not take account of the wind g is not exactly 9.8 m s^{-2} **N.B.** If they refer to the mass or weight of the balls give B0	B1	3.5b
		(1)	
		(13)	

Marks		**Notes**		
28a	B1	Seen or implied, possibly on a diagram		
	M1	Use of $v = u + at$ or any other complete method <u>using $t = 2$</u> Condone sign errors and sin/cos confusion.		
	A1	Correct unsimplified equation in v or v^2		
	M1	Correct use of trig to find a relevant angle for the direction. Must have found a horizontal and a vertical velocity component		
	M1	Use Pythagoras or trig to find the magnitude Must have found a horizontal and a vertical velocity component		
	A1	Or equivalent. Need magnitude **and** direction stated or implied in a diagram. (0.506 or 0.51 rads)		
28b	M1	First equation,in terms of u and θ (could be implied by subsequent working), using the horizontal motion with $t = 2$ used Condone sign errors and sin/cos confusion		
	A1	Correct unsimplified equation – any equivalent form		
	M1	Second equation, in terms of u and θ (could be implied by subsequent working), using the vertical motion – equating distances or just vertical components of velocities. Condone sign errors and sin/cos confusion		
	A1	Correct unsimplified equation – any equivalent form		
	M1	Complete strategy: all necessary equations formed and solve for u and θ **N.B.** This is an independent method mark but can only be earned if 50 m has been used in their solution.		
	A1	Both values correct. (Here we accept 2SF or better, since the g's cancel) Allow radians for θ: 0.92 or better (0.915906..) rads.		
28c	B1	Any factor **related to the model** as stated in the question. Penalise incorrect extras but ignore consequences e.g. 'AB (or the ground) is not horizontal' should be penalised or 'they do not move in a vertical plane' should be penalised		

Question	Scheme	Marks	AOs
29.	Integrate **v** w.r.t. time	M1	1.1a
	$\mathbf{r} = 2t^{\frac{1}{2}}\mathbf{i} - 2t^2\mathbf{j}\ (+\ \mathbf{C})$	A1	1.1b
	Substitute $t = 4$ and $t = 1$ into their **r**	M1	1.1b
	$t = 4, \mathbf{r} = 4\mathbf{i} - 32\mathbf{j}(+\ \mathbf{C});\ t = 1, \mathbf{r} = 2\mathbf{i} - 2\mathbf{j}(+\ \mathbf{C})$ or $(4,-32); (2,-2)$	A1	1.1b
	$\sqrt{2^2 + (-30)^2}$	M1	1.1b
	$\sqrt{904} = 2\sqrt{226}$	A1	1.1b
		(6)	
			(6 marks)

Notes: Allow column vectors throughout

M1: At least one power increasing by 1.

A1: Any correct (unsimplified) expression

M1: Must have attempted to integrate **v**. Substitute $t = 4$ and $t = 1$ into their **r** to produce 2 vectors (or 2 points if just working with coordinates).

A1: $4\mathbf{i} - 32\mathbf{j}(+\ \mathbf{C})$ and $2\mathbf{i} - 2\mathbf{j}(+\ \mathbf{C})$ or $(4,-32)$ and $(2,-2)$. These can be seen or implied.

M1: Attempt at distance of form $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ for their points. Must have 2 non zero terms.

A1: $\sqrt{904} = 2\sqrt{226}$ or any equivalent surd (exact answer needed)

Question	Scheme	Marks	AOs
30(a)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$: $(7\mathbf{i} - 10\mathbf{j}) = 2(2\mathbf{i} - 3\mathbf{j}) + \frac{1}{2}\mathbf{a}2^2$	M1	3.1b
	$\mathbf{a} = (1.5\mathbf{i} - 2\mathbf{j})$	A1	1.1b
	$\lvert\mathbf{a}\rvert = \sqrt{1.5^2 + (-2)^2}$	M1	1.1b
	= 2.5 m s^{-2} * GIVEN ANSWER	A1*	2.1
		(4)	
(b)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2\mathbf{i} - 3\mathbf{j}) + 2(1.5\mathbf{i} - 2\mathbf{j})$	M1	3.1b
	$= (5\mathbf{i} - 7\mathbf{j})$	A1	1.1b
	$\mathbf{v} = (5\mathbf{i} - 7\mathbf{j}) + t(4\mathbf{i} + 8.8\mathbf{j}) = (5 + 4t)\mathbf{i} + (8.8t - 7)\mathbf{j}$ and $(5 + 4t) = (8.8t - 7)$	M1	3.1b
	t = 2.5 (s)	A1	1.1b
		(4)	
			(8 marks)

Notes: Allow column vectors throughout

(a)

No credit for individual component calculations

M1: Using a complete method to obtain the acceleration. **N.B.** Equation, in **a** only, could be obtained by two integrations

ALTERNATIVE

M1: Use velocity at half-time (t = 1) = Average velocity over time period

So at $t = 1$, $\mathbf{v} = \frac{1}{2}(7\mathbf{i} - 10\mathbf{j})$ so $\mathbf{a} = \frac{1}{2}(7\mathbf{i} - 10\mathbf{j}) - (2\mathbf{i} - 3\mathbf{j})$

N.B. could see $(7\mathbf{i} - 10\mathbf{j}) = (4\mathbf{i} - 6\mathbf{j}) + 2\mathbf{a}$ as first line of working

A1: Correct **a** vector

M1: Attempt to find magnitude of their **a** using form $\sqrt{a^2 + b^2}$

A1*: Correct GIVEN ANSWER obtained correctly

(b)

M1: Using a complete method to obtain the velocity at A e.g.by use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with t = 2 and **u** = 2**i** – 3**j** and their **a**

OR: by use of $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

OR: by integrating their **a,** with addition of **C** = 2**i** – 3**j,** and putting t = 2

A1: correct vector

M1: Complete method to find equation in t only

e.g. by using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, with their **u** and equating **i** and **j** components

OR: by integrating $(4\mathbf{i} + 8.8\mathbf{j})$, with addition of a constant, and equating **i** and **j** components.

N.B. Must be equating **i** and **j** components of a velocity vector and must be their velocity at *A*, to give an equation in *t* only for this M mark

A1: 2.5 (s)

Question	Scheme	Marks	AOs
31(a)	Using the model and vertical motion: $0^2 = (U\sin\alpha)^2 - 2g\times(3-2)$	M1	3.3
	$U^2 = \dfrac{2g}{\sin^2\alpha}$ * GIVEN ANSWER	A1*	2.2a
		(2)	
(b)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \dfrac{1}{2}at^2$	M1	3.4
	$-\dfrac{5}{4} = Ut\sin\alpha - \dfrac{1}{2}gt^2$	A1	1.1b
	sub for t: $-\dfrac{5}{4} = U\sin\alpha\left(\dfrac{20}{U\cos\alpha}\right) - \dfrac{1}{2}g\left(\dfrac{20}{U\cos\alpha}\right)^2$	M1 (I)	3.1b
	sub for U^2	M1(II)	3.1b
	$-\dfrac{5}{4} = 20\tan\alpha - 100\tan^2\alpha$	A1(I)	1.1b
	$(4\tan\alpha - 1)(100\tan\alpha + 5) = 0$	M1(III)	1.1b
	$\tan\alpha = \dfrac{1}{4} \Rightarrow \alpha = 14^0$ or better	A1(II)	2.2a
		(9)	
	N.B. For the last 5 marks, they may set up a quadratic in t, by substituting for $U\sin\alpha$ first, then solve the quadratic to find the value of t, then use $20 = Ut\cos\alpha$ to find α. The marks are the same but earned in a different order. Enter on ePen in the corresponding M and A boxes above, as indicated below.		
	Sub for $U\sin\alpha$ to give equation in t only	M1(II)	
	$-\dfrac{5}{4} = \sqrt{2g}t - \dfrac{1}{2}gt^2$	A1(I)	
	Solve for t	M1(III)	
	$t = \dfrac{5}{\sqrt{2g}}$ or 1.1 or 1.13 and use $20 = Ut\cos\alpha$	M1(I)	
	$\alpha = 14^0$ or better	A1(II)	
(b)	**ALTERNATIVE**		

	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos\alpha$	A1	1.1b
	A to top: $s = vt - \frac{1}{2}at^2$ and top to T: $s = ut + \frac{1}{2}at^2$		
	$1 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2}{g}}$ and $\frac{9}{4} = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	M1	3.4
	$= \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}} \left(= \frac{5}{\sqrt{2g}}\right)$	A1	1.1b
	$20 = U\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for t)	M1	3.1b
	$20 = \sqrt{\frac{2g}{\sin^2\alpha}}\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for U)	M1	3.1b
	$\tan\alpha = \frac{1}{4}$	A1	1.1b
	Solve for α	M1	1.1b
	$\Rightarrow \alpha = 14^0$ or better	A1	2.2a
		(9)	
(c)	The target will have dimensions so in practice there would be a range of possible values of α **Or** There will be air resistance **Or** The ball will have dimensions **Or** Wind effects **Or** Spin of the ball	B1	3.5b
		(1)	
(d)	Find U using their α e.g. $U = \sqrt{\frac{2g}{\sin^2\alpha}}$	M1	3.1b
	Use $20 = Ut\cos\alpha$ (or use vertical motion equation)	A1 **M1**	1.1b
	$t = \frac{5}{\sqrt{2g}}$ or 1.1 or 1.13	B1 **A1**	1.1b
		(3)	
(d)	**ALTERNATIVE**		

	A to top: $s = vt - \frac{1}{2}at^2$ and top to T: $s = ut + \frac{1}{2}at^2$	M1	3.1b
	$1 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2}{g}}$ and $\frac{9}{4} = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	A1 **M1**	1.1b
	$= = \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}}$ $(= \frac{5}{\sqrt{2g}})$ = 1.1 or 1.13 (s)	B1 **A1**	1.1b
		(3)	
			(15 marks)

Notes:

(a)

M1: Or any other complete method to obtain an equation in U, g and α **only**

A1*: Correct GIVEN ANSWER

(b)

M1: Using horizontal motion

A1: Correct equation

M1: Using vertical motion . N.B. M0 if they use $s = \pm 2$ or ± 3, but allow $s = \pm 1.25$ or ± 0.75 or ± 2.25 or ± 2.75

A1: Correct equation

M1: Using $20 = Ut\cos\alpha$ to sub. for t

M1: Substituting for U^2 using (a)

A1: Correct quadratic equation (in $\tan\alpha$ **or** $\cot\alpha$ **)**

M1: Solve a 3 term quadratic, either by factorisation or formula (or by calculator (implied) if answer is correct) **and find** α

A1: $\alpha = 14^o$ or better (No restriction on accuracy since g's cancel)

N.B. If answer is correct, previous M mark can be implied, but if answer is incorrect, an explicit attempt to solve must be seen to earn the previous M mark.

(b) ALTERNATIVE

M1: Using the model with the usual rules applying to the equation

A1: Correct equation

M1: Using the model to obtain the **total** time from A to T

A1: Correct **total** time t

M1: Substitute for t in $20 = Ut\cos\alpha$

M1: Substitute for U in $20 = Ut\cos\alpha$, using part (a)

A1: Correct equation in $\tan\alpha$ **only**

M1: Solve equation for α

A1: $\alpha = 14^o$ or better (No restriction on accuracy since g's cancel)

N.B. If they quote the equation of the trajectory $y = x\tan\alpha - \dfrac{gx^2}{2U^2\cos^2\alpha}$ oe **AND** put in values for x and y, could score first 5 marks, M1A1M1A1M1 (nothing for the equation only); wrong x value loses first A mark and wrong y value loses second A mark
(c) **B1:** Give one limitation of the model e.g. the ball will have dimensions, or there will be air resistance or wind effects or spin N.B. B0 if any incorrect extra(s) but ignore extra consequences.
(d) **M1:** Using their a to find a value for U **A1: Treat as M1:** Using their U to find a value for t **B1: Treat as A1 :** $t = 1.1$ or 1.10 (since depends on $g = 9.8$) **(d) ALTERNATIVE** **M1:** Using their a to find a value for U **A1: Treat as M1:** Using their U to find a value for t **B1: Treat as A1 :** $t = 1.1$ or 1.10 (since depends on $g = 9.8$)

Question	Scheme	Marks	AOs
32	$\mathbf{r} = (-4.5\mathbf{i} + 3\mathbf{j})$	B1	1.1b
	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	3.1b
	$(-4.5\mathbf{i} + 3\mathbf{j}) = 3\mathbf{u} + 0.5(\mathbf{i} - 2\mathbf{j})\ 3^2$	A1**ft**	1.1b
	$\mathbf{u} = (-3\mathbf{i} + 4\mathbf{j})$	A1	1.1b
		(4)	
			(4 marks)

Notes:

B1: Correct displacement vector

M1: Use of correct strategy and/or formula to give equation in **u** only (could be obtained by two integrations)

A1ft: Correct equation in **u** only, following their displacement vector

A1: Correct answer

Question	Scheme	Marks	AOs
33(a)	Using the model and horizontal motion: $s = ut$	M1	3.3
	$12 = T \times 45\cos 10^o$	A1	1.1b
	$T = 0.2707..$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$s = 45T\sin 10^o + 4.9T^2$	A1	1.1b
	Correct strategy: sub for T and find s	M1	3.1b
	$d = 3.5 - 2.4752 - 1$	M1	3.1b
	$= 2.5$ (cm) (2 SF)	A1	2.2a
		(8)	
(b)	Using the model and vertical motion: $v = u + at$	M1	3.3
	$v = 45\sin 10^o + 9.8T$	A1	1.1b
	Speed $= ((45\cos 10^o)^2 + v^2)^{0.5}$	M1	3.1b
	46 (m s^{-1}) (2 SF)	A1	1.1b
		(4)	
(c)	Model does not take account of air resistance.	B1	3.5b
	Model does not take account of the size of the tennis ball	B1	3.5b
		(2)	
			(14 marks)

Notes:

(a)
M1: Using the model and correct strategy
A1: Correct equation in T only
A1: 0.271 or better
M1: Using the model and correct strategy
A1: Correct equation
M1: Sub for T and solve for s
M1: Correct method to find d using their s
A1: 2.5 is the only correct answer

(b)
M1: Using the model and correct strategy
A1: Correct equation
M1: Must have found a v and usual rules apply. Square root is needed.

A1: 46 (2 SF) is only correct answer

(c)

B1: Other appropriate answer e.g. spin of the ball, wind effect

B1: Other appropriate answer e.g. spin of the ball, wind effect

Question	Scheme	Marks	AOs
34	Integrate **a** w.r.t. time	M1	1.1a
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ (allow omission of **C**)	A1	1.1b
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
	When $t = 4$, $\mathbf{v} = 60\mathbf{i} - 80\mathbf{j}$	M1	1.1b
	Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
	Speed = 100 m s^{-1}	A1**ft**	1.1b
			(6 marks)

Notes:

1st M1: for integrating **a** w.r.t. time (powers of t increasing by 1)
1st A1: for a correct **v** expression without **C**
2nd A1: for a correct **v** expression including **C**
2nd M1: for putting $t = 4$ into their **v** expression
3rd M1: for finding magnitude of their **v**
3rd A1: ft for 100 m s^{-1}, follow through on an incorrect **v**

Question	Scheme	Marks	AOs
35(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $(10.5\mathbf{i} - 0.9\mathbf{j}) = 0.6\mathbf{j} + 15\mathbf{a}$	M1	3.1b
	$\mathbf{a} = (0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2} Given answer	A1	1.1b
		(2)	
(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	3.1b
	$\mathbf{r} = 0.6\mathbf{j}\,t + \frac{1}{2}(0.7\mathbf{i} - 0.1\mathbf{j})\,t^2$	A1	1.1b
		(2)	
(c)	Equating the **i** and **j** components of **r**	M1	3.1b
	$\frac{1}{2} \leftarrow 0.7\,t^2 = 0.6\,t - \frac{1}{2} \leftarrow 0.1\,t^2$	A1ft	1.1b
	$t = 1.5$	A1	1.1b
		(3)	
(d)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $\mathbf{v} = 0.6\mathbf{j} + (0.7\mathbf{i} - 0.1\mathbf{j})\,t$	M1	3.1b
	Equating the **i** and **j** components of **v**	M1	3.1b
	$t = 0.75$	A1 **ft**	1.1b
		(3)	
			(10 marks)

Notes:

(a)
M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$
A1: for given answer correctly obtained

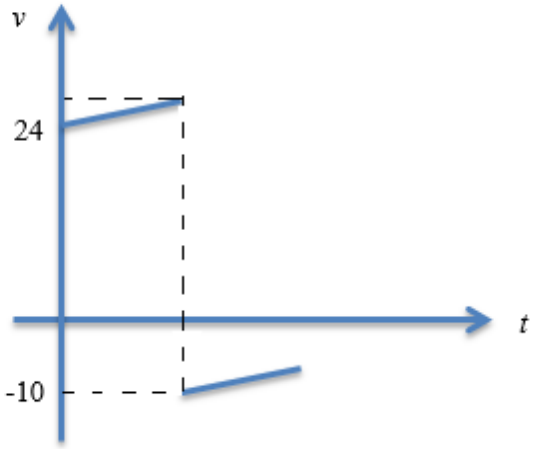
(b)
M1: for use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
A1: for a correct expression for **r** in terms of t

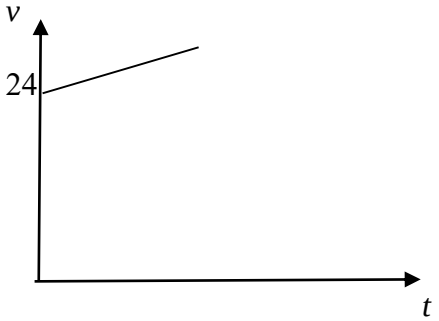
(c)
M1: for equating the **i** and **j** components of their **r**
A1ft: for a correct equation following their **r**
A1: for $t = 1.5$

(d)
M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ for a general t
M1: for equating the **i** and **j** components of their **v**
A1ft: for $t = 0.75$, or a correct follow through answer from an incorrect equation

Question	Scheme	Marks	AOs
36(a)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$36 = U t\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-18 = Ut\sin\alpha - \frac{1}{2}gt^2$	A1	1.1b
	Correct strategy for solving the problem by setting up two equations in t and U and solving for U	M1	3.1b
	$U = 15$	A1	1.1b
		(6)	
(b)	Using the model and horizontal motion: $U\cos\alpha$ (12)	B1	3.4
	Using the model and vertical motion: $v^2 = (U\sin\alpha)^2 + 2(-10)(-7.2)$	M1	3.4
	$v = 15$	A1	1.1b
	Correct strategy for solving the problem by finding the horizontal and vertical components of velocity and combining using Pythagoras: Speed = $\sqrt{(12^2 + 15^2)}$	M1	3.1b
	$\sqrt{369} = 19$ m s^{-1} (2sf)	A1 **ft**	1.1b
		(5)	
(c)	Possible improvement (see below in notes)	B1	3.5c
	Possible improvement (see below in notes)	B1	3.5c
		(2)	
			(13 marks)

Question 36 continued
Notes:
(a) **1st M1:** for use of $s = ut$ horizontally **1st A1:** for a correct equation **2nd M1:** for use of $s = ut + \frac{1}{2}at^2$ vertically **2nd A1:** for a correct equation **3rd M1:** for correct strategy (need both equations) **2nd A1:** for $U = 15$
(b) **B1:** for $U\cos\alpha$ used as horizontal velocity component **1st M1:** for attempt to find vertical component **1st A1:** for 15 **2nd M1:** for correct strategy (need both components) **2nd A1ft:** for 19 m s $^{-1}$ (2sf) following through on incorrect component(s)
(c) **B1, B1:** for any two of e.g. Include air resistance in the model of the motion e.g. Use a more accurate value for g in the model of the motion e.g. Include wind effects in the model of the motion e.g. Include the dimensions of the stone in the model of the motion

Question Number	Scheme	Marks
37.(a)	$V^2 = U^2 + 2g \times 2.5$	M1A1
	Eliminate V *and* solve for U	A1 **(DM1)**
	$7 = 0.2(10 - -V)$	M1A1
	$U = 24$	A1 (6)
37.(b)	$1 = 10t - 4.9t^2$ **OR** e.g. $v^2 = 10^2 - 2 \times 9.8 \times 1$ and $v = 10 - 9.8t$	
	$1 = 10t - 4.9t^2$ to give $\sqrt{80.4} = 10 - 9.8t$	M1 A1
	$t = \dfrac{10 \pm \sqrt{100 - 19.6}}{9.8}$ so $t = \dfrac{10 - \sqrt{10^2 - 2 \times 9.8 \times 1}}{9.8}$	**DM**1
	$t = 0.11$ s or 0.105 s	A1 (4)
37(c)		B1**ft**1st line B1 2nd line B1 ,-10 (3)
		(13)
	Notes for Qu 37	
	37(a) First M1 for complete method, using *suvat*, to find equation in U and V only First A1 for a correct equation Second A1 – **treat as third DM1**, dependent on the other two M's, for eliminating V and solving for U Second M1 for using Impulse = Change in Momentum of ball (must have 0.2 in both terms and be using 10 as one of the velocities) (M0 if *clearly* adding momenta or if g is included) but condone sign errors. Third A1 for a correct equation, 7 and 10 must have the same sign but equation may have V instead of $-V$ Fourth A1 for $U = 24$ (must appear here) **N.B.** If they use U instead of V in the impulse-momentum equation, can score max M1A0/6 for part (a). **N.B.** If they go from $V^2 = U^2 + 49$ to $V = U + 7$, can score max 5/6	

	37(b) First M1 for complete method, using one or more *suvat* formulae, to produce an equation in t only using $s = 1$ or -1 First A1 for a correct equation in t only Second DM1, dependent on first M1, for solving their equation (this mark can be implied by a correct answer) Second A1 for either 0.105 (s) or 0.11 (s) (must be only ONE answer)	
	37(c) First B1**ft** for a straight line, with positive gradient, starting at their U value (or just at U) on the positive v-axis. Second B1 for a parallel (approx.) line placed correctly (B0 if a continuous vertical line is included) i.e. starting at a point where the t coordinate is equal to the t coordinate of the point where the first line stopped, and the v coordinate is negative. Third B1 for second line, placed correctly, starting on $v = -10$ N.B. Whole graph could be reflected in the t-axis **SC: If second line is placed correctly but extends up to the t-axis, or beyond, lose second B1 but can score the third B1.**	
37(b)	**ALTERNATIVE :** "the instant when the ball first passes through B" is taken to be when the ball is on the way down from A.	
	$s = vt - \frac{1}{2}at^2$ **OR** $v_B^{\,2} = 24^2 + 2\times 9.8\times 1.5$ and $25 = v_B + 9.8t$	
	$1 = 25t - 4.9t^2$ to give $25 = \sqrt{605.4} + 9.8t$	M1 A1
	$t = \frac{25 \pm \sqrt{625 - 19.6}}{9.8}$ so $t = \frac{25 - \sqrt{625 - 19.6}}{9.8}$	**DM1**
	$t = 0.040$ (s) or 0.0403 (s) or 0.04 (s) (must only be ONE answer)	A1 (4)
37(c)	**ALTERNATIVE :** again "when it first passes through B" is taken to be when the ball is on the way down from A.	
		B2 line B1**ft** 24 (3)

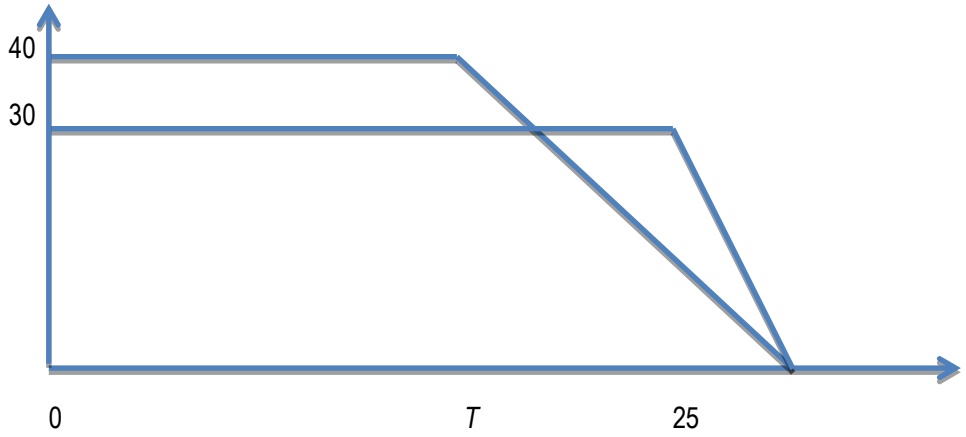
	Notes for Qu 37 continued	
	37(b) First M1 for complete method, using one or more *suvat* formulae, to produce an equation in t only using $s = 1$ or -1 First A1 for a correct equation in t only Second DM1, dependent on first M1, for solving their equation (this mark can be implied by a correct answer) Second A1 $t = 0.040$ (s) or 0.0403 (s)	
	37(c) B2 for a straight line, with positive gradient, starting on the positive v-axis. B1**ft** starting at their U value (or just at U)	

Question Number	Scheme	Marks
38(a)	$s = vt - \frac{1}{2}at^2$ $40 = 10 \times 5 - \frac{1}{2}a5^2$ $a = 0.8$	M1 A2 A1 (4)
(b)	Finding u (= 6) $s = ut + \frac{1}{2}at^2$ (A to M) $20 = 6t + \frac{1}{2}0.8t^2$ $t = \frac{-15 \pm \sqrt{225 + 200}}{2}$ $= 2.8$ or 2.81 or better	M1 M1 A1 **DM**1 A1 (5)
	Alternative : Finding v (= $\sqrt{68}$) $s = vt - \frac{1}{2}at^2$ (A to M) $20 = \sqrt{68}t - \frac{1}{2}0.8t^2$ $t = \frac{\sqrt{68} \pm \sqrt{68 - 32}}{0.8}$ $= 2.8$ or 2.81 or better	M1 M1 A1 **DM1** A1 (5)
	Alternative : $s = vt_1 - \frac{1}{2}at_1^2$ (M to B) $20 = 10t_1 - \frac{1}{2}0.8t_1^2$ $t_1 = \frac{10 \pm \sqrt{100 - 32}}{0.8}$ $= 2.192$ $t = 5 - t_1 = 2.8$ or 2.81 or better	M2 A1 **DM**1 A1 (5) **9**

	Notes	
38(a)	First M1 for a complete method to produce a value for *a*. They may use two (or more equations) and solve for *a*.(see possible equations) A2 if all correct, A1A0 for one error Third A1 for 0.8 (m s^{-2}) Possible equations: $40 = 5u + \frac{1}{2}a.5^2$ $10^2 = u^2 + 2a.40$ $10 = u + 5a$ $40 = \frac{(u+10)}{2}.5$	
38(b)	First M1 for attempt to find a value for u (This may have been done in part (a) but MUST be used in (b)) Second M1 for a complete method (may involve 2 or more *suvat* equations) for finding an equation in *t only* First A1 for a correct equation Third M1, **dependent** on previous M, for solving their equation for *t* Second A1 for 2.8 (s) or better or $\frac{5(2\sqrt{17} - 6)}{4}$; $\frac{40}{6+2\sqrt{17}}$	

Question Number	Scheme	Marks
39(a)	$\tan q = \frac{2}{9}$ $q = 12.5^\circ$ bearing 103°	M1 A1 A1 (3)
(b) **(i)** **(ii)**	$\mathbf{p} = (9\mathbf{i} + 10\mathbf{j}) + t(9\mathbf{i} - 2\mathbf{j})$ $\mathbf{q} = (\mathbf{i} + 4\mathbf{j}) + t(4\mathbf{i} + 8\mathbf{j})$	M1 A1 A1 (3)
(c)	$QP = (8 + 5t)\mathbf{i} + (6 - 10t)\mathbf{j}$	M1 A1 (2)
(d)	$D^2 = (8 + 5t)^2 + (6 - 10t)^2$ $= 125t^2 - 40t + 100$ $100 = 125t^2 - 40t + 100$ $0 = 5t(25t - 8)$ $t = 0$ or 0.32	M1 A1 M1 M1 A1 A1 (6) **14**
	Notes	
39(a)	M1 for $\tan q = \pm\frac{2}{9}$ or $\pm\frac{9}{2}$ or use $\sin q$ or $\cos q$	
	First A1 for $q = \pm 13^\circ$ or $\pm 77^\circ$ or $\pm 12.5^\circ$ or $\pm 77.5^\circ$ or better	
	Second A1 for 103°	
39(b)	M1 for clear attempt at $\mathbf{p} = (9\mathbf{i} + 10\mathbf{j}) + t(9\mathbf{i} - 2\mathbf{j})$ or $\mathbf{q} = (\mathbf{i} + 4\mathbf{j}) + t(4\mathbf{i} + 8\mathbf{j})$ (Allow slips but must be a '+' sign and **r** + *t* **v**)	
(i)	First A1 for $\mathbf{p} = (9\mathbf{i} + 10\mathbf{j}) + t(9\mathbf{i} - 2\mathbf{j})$oe	
(ii)	Second A1 for $\mathbf{q} = (\mathbf{i} + 4\mathbf{j}) + t(4\mathbf{i} + 8\mathbf{j})$oe	
39(c)	M1 for **p** – **q** or **q** – **p** with their **p** and **q** substituted A1 for correct answer $QP = (8 + 5t)\mathbf{i} + (6 - 10t)\mathbf{j}$ (don't need QP but on R.H.S must be **identical** coefficients of **i** and **j** but allow column vectors)	
39(d)	First M1 for attempt to find QP or QP^2 in terms of *t only*, using correct formula First A1 for a correct expression (with or without √) $125t^2 - 40t + 100$ Second M1 for √ (3 term quadratic) = 10 or (3 term quadratic) =100. Third M1 for quadratic expression = 0 and attempt to solve (e.g. factorising or using formula) Second A1 for $t = 0$ (if they divide by *t* and lose this value but get 0.32, M1A0A1) Third A1 for $t = 0.32$ oe	

Question Number	Scheme	Marks
40(a)	$\tan q = \frac{5}{20}$ $q = 14.036..^0$ $q = 104^0$ nearest degree	M1 A1 A1 (3)
(b)	$\mathbf{p} = 400\mathbf{i} + t(15\mathbf{i} + 20\mathbf{j})$ $\mathbf{q} = 800\mathbf{j} + t(20\mathbf{i} - 5\mathbf{j})$	M1 A1 A1 (3)
(c)	Equate their **j** components: $20t(\mathbf{j}) = (800 - 5t)(\mathbf{j})$ $t = 32$ $\mathbf{s} = 800\mathbf{j} + 32(20\mathbf{i} - 5\mathbf{j})$ $= 640\mathbf{i} + 640\mathbf{j}$	M1 A1 M1 A1 (4) **10**
40(a)	**Notes** **Allow column vectors throughout** M1 for $\tan q = \pm\frac{5}{20}$ or $\pm\frac{20}{5}$ (or any other complete method) First A1 for $\pm 14.04^o$ or $\pm 75.96^o$ Second A1 for 104^o	
40(b) **(i)** **(ii)**	M1 for clear attempt at either **p** or **q** (allow slip but *t* must be attached to the velocity vector and position vector and velocity vector must be paired up correctly) First A1 $400\mathbf{i} + t(15\mathbf{i} + 20\mathbf{j})$ "**p** =" not needed but must be clear it's *P* Second A1 $800\mathbf{j} + t(20\mathbf{i} - 5\mathbf{j})$ "**q** =" not needed but must be clear it's *Q*	
40(c)	First M1 for equating their **j** components; allow **j**'s on both sides First A1 for $t = 32$ Second M1 independent for substituting their *t* value into their **q** from (b) Second A1 for $640\mathbf{i} + 640\mathbf{j}$	

Question Number	Scheme	Marks
41(a)		B1 shape (*M*) B1 figs (40,*T*) B1 shape (*N*) B1 figs (30,25) (4)
(b)	**For N**: $\frac{1}{2}(25+25+t).30=975$ OR $\frac{1}{2}(25+t_1).30=975$ $t=15$ $t_1=40$ **For M**: $\frac{1}{2}(25+t+T).40=975$ OR $\frac{1}{2}(t_1+T).40=975$ $T=8.75$ ($8\frac{3}{4}$ or $\frac{35}{4}$ oe) ALTERNATIVE: They may find t or t_1 , in terms of T, from their (*M*) equation , and substitute for t or t_1 in their (*N*) equation, and then solve for T: **For M**: $\frac{1}{2}(25+t+T).40=975$ OR $\frac{1}{2}(t_1+T).40=975$ $t=(\frac{1950}{40}-25-T)$ $t_1=(\frac{1950}{40}-T)$ **For N** : $\frac{1}{2}(25+25+t).30=975$ OR $\frac{1}{2}(25+t_1).30=975$ sub for t or sub for t_1 $T=8.75$ ($8\frac{3}{4}$ or $\frac{35}{4}$ oe)	M1 A1 **DM**1 A1 M1 A1 **DM**1 A1 (8) **12** M1 A1 **DM**1 A1 M1 A1 **DM**1 A1 (8) **12**
	Notes	
41(a)	First B1 (*M*) for correct shape – *must start and finish on the axes*. Second B1 for 40 and *T* marked clearly (if delineators omitted B0) and correctly Third B1 (*N*) for correct shape – *must start and finish on the axes*. Fourth B1 for 30 and 25 (if delineators omitted B0) marked clearly and correctly **N.B.** If graphs do not cross and/or do not finish at the same point, max score is B1B1B0B1.	

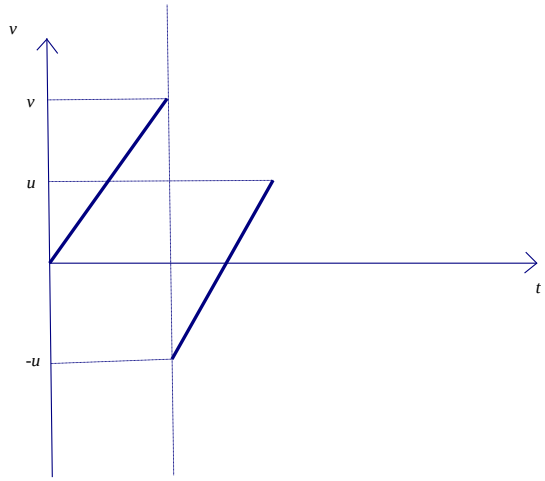
	N.B. If graphs done on separate diagrams, mark each and award the higher mark i.e. can score max 2/4 for part (a).	
41(b)	**N.B.** When attempting to find the area of a triangle, must see ½ x …. to be able to award an M mark i.e. M0 if ½ is missing **N.B.** When attempting to find the area of a trapezium, must see something of the form : ½ x $(a + b)h$ to be able to award an M mark i.e. M0 if ½ is missing and bracket is not a **sum** First M1 for attempt at using 975m distance travelled by N to obtain an equation in one unknown *time* (usually extra time t after 25 s, but could, for example, be whole time t_1) .They may use the area under their graph or use *suvat* (**N.B.** Any single *suvat* equn using $s = 975$ is M0). First A1 for a correct equation in their unknown *time* e.g. $(30 \times 25) + \frac{1}{2}\, 30t = 975$ **OR** $(30 \times 25) + \frac{1}{2}\, 30\,(t_1 - 25) = 975$ Second M1, dependent on first M, for solving their equation Second A1 for a correct value for their unknown. Third M1 for attempt at using 975m distance travelled by M to obtain an equation in T and possibly one other unknown *time* (usually extra time t after 25 s, but could, for example, be whole time t_1). They may use the area under their graph or use *suvat* (**N.B.** Any *suvat* equn using $s = 975$ is M0) Third A1 for a correct equation in T and possibly their unknown. This A1 can be earned if they just have a letter for their unknown :- e.g. $40T + \frac{1}{2}\, 40.(25 + t - T) = 975$ **OR** $40T + \frac{1}{2}\, 40.(t_1 - T) = 975$ or <u>for an incorrect numerical value in place of t or t_1</u>. Fourth M1, dependent on first, second and third M's, for solving for T. Fourth A1 for 8.75 or 35/4 or any other equivalent SEE MARKS FOR ALTERNATIVE ABOVE.	

Question Number	Scheme	Marks
42(a)	$0^2 = 19.6^2 - 2\times gH$ $H = 19.6\text{m}$ (20)	M1 A1 (2)
(b)	$14.7 = 19.6t - \frac{1}{2}gt^2$ $t^2 - 4t + 3 = 0$ $(t-1)(t-3) = 0$ $t = 1$ or 3; Answer 2 s	M1 A1 **DM**1 A1; A1 (5) **7**
42(b) **ALT 1**	(their h - 14.7) = ½ g t^2 **OR** $v^2 = 19.6^2 - 2g$ x 14.7 => v = (±) 9.8 t =1 and 0 9.8 – 9.8 t => t = 1 Total = 2 x their 1 = 2 s	M1 A1 A1 **DM** 1 A1
42(b) **ALT 2/3**	$v^2 = 19.6^2 - 2g$ x 14.7 $v = \pm 9.8$ **EITHER:** - 9.8 = 9.8 – gT T = 2 **OR:** 0 = 9.8t - ½ g t^2 t = (0) or 2	M1 A1 **DM**1 A1 A1 **DM**1 A1 A1
	Notes	
42(a)	M1 is for a complete method (which could involve use of two *suvat* equations) for finding H i.e. for an equation in H *only*, condone sign errors A1 for 19.6 or 20 correctly obtained (2g is A0)	
42(b)	First M1 is for a quadratic equation in t only (where t is time at 14.7 above O) First A1 for a correct equation Second DM1, dependent on first M1, for solving for t Second A1 for both values of t , 1 and 3. **N.B**. If answer(s) are wrong or have come from an incorrect quadratic, and the quadratic formula has been used, M1 can only be awarded if there is clear evidence that the correct formula has been used. If their expression is not correct for their quadratic, allow a slip but only if we see an attempt to substitute into a stated correct formula. Third A1 for 2 s **N.B.** Obtaining t = 1 at s = 14.7 (above O) only, can score max M1 A1	

Question Number	Scheme	Marks
43(a)	$\mathbf{r} = (-3\mathbf{i} + 4\mathbf{j})$ m	B1 (1)
(b)	$3.4 = 2T - 3$ or $-12 = 4 - 5T$ $T = 3.2$	M1 A1 A1 (3)
(c)	$\mathbf{r} = (-3\mathbf{i} + 4\mathbf{j}) + t(2\mathbf{i} - 5\mathbf{j})$ $\mathbf{v} = (2\mathbf{i} - 5\mathbf{j})$ speed $= \sqrt{\left(2^2 + (-5)^2\right)} = \sqrt{29} = 5.4\text{m s}^{-1}$ or better	M1 A1 M1 A1 (4) **8**
Alt (c)	$\lvert\mathbf{s}\rvert = \sqrt{6.4^2 + (-16)^2} = 17.23...$ $\therefore$ speed $= \frac{17.23}{3.2} = 5.4$ or better	M1 A1 M1 A1 (4)
	Notes	
43(a)	**Allow column vectors throughout.** B1 for $(-3\mathbf{i} + 4\mathbf{j})$ (m)	
(b)	M1 for a clear attempt at either 3.4 (**i**)= $(2T - 3)$ (**i**) or -12(**j**) = $(4 - 5T)$ (**j**) First A1 for a correct equation (either) without **i**'s and **j**'s A1 for 3.2 oe **N.B.** $T = \frac{6.4\mathbf{i} - 16\mathbf{j}}{2\mathbf{i} - 5\mathbf{j}}$ = 3.2 scores M1A1A1 BUT if RHS is not a single number, then M0. Also, if they get 3.2 and another value and don't clearly choose 3.2 then A0	
(c)	First M1 for a complete method for finding **v** e.g. $\mathbf{r} = (-3\mathbf{i} + 4\mathbf{j}) + t(2\mathbf{i} - 5\mathbf{j})$ so **v** = 2**i** – 5**j** OR: $\mathbf{v} = \frac{(3.4\mathbf{i} - 12\mathbf{j}) - (-3\mathbf{i} + 4\mathbf{j})}{\text{their } T}$ OR: $\mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = 2\mathbf{i} - 5\mathbf{j}$ First A1 for $2\mathbf{i} - 5\mathbf{j}$; M1A1 can be awarded for $2\mathbf{i} - 5\mathbf{j}$ only. Second M1 for attempt to find magnitude of their **v**, i.e. $\sqrt{2^2 + (-5)^2}$ Second A1 for √29 or 5.4 or better **OR** First M1 for attempt to find distance travelled: $d = \sqrt{(-3 - 3.4)^2 + (4 - -12)^2}$ First A1 if correct Second M1 for their d / their T Second A1 for √29 or 5.4 or better	

Question Number	Scheme	Marks
44(a)		B1 (shape) B1 (V) (2)
(b) **(i) (ii)**	$\frac{V}{t_1}=\frac{1}{2} \Rightarrow t_1=2V$ s; $t_2=4V$ s	M1 A1; A1
(iii)	$t_3=300-2V-4V=300-6V$ s	M1 A1 (5)
(c)	$6300=\frac{V(300+300-6V)}{2}$ or $\frac{1}{2}2V.V+(300-6V).V+\frac{1}{2}4V.V$ $V^2-100V+2100=0$ $(V-30)(V-70)=0$ $V=30$ or 70 $V=30\ (<50)$	M1 **A1 ft** A1 M1 A1 A1 (6) **13**
	Notes	
44(a)	B1 for a trapezium with line starting and finishing on the t-axis B1 for V correctly marked	
(b)	First M1 for a correct method First A1 for $V/0.5$ oe Second A1 for $V/0.25$ oe Second M1 for (300 – sum of previous answers) Allow 5 instead of 300. Third A1 for $300-6V$ oe	
(c)	First M1 for using the area under the curve (distance travelled) to form an equation in V only. (Allow use of 6.3 but must see ½ used at least once in their expression.) First A1 **ft on their answers in (b)** for a correct equation so must have used 6300 not 6.3 Second A1 for correct equation in form $aV^2+bV+c=0$ Second M1 for solving a 3 term quadratic. (Can be implied by correct answers) Second A1 for either 30 or 70	

	Third A1 for 30 as final answer. **N.B**. If answer(s) are wrong or have come from an incorrect quadratic, and the quadratic formula is used, M1 can only be awarded if there is clear evidence that the correct formula has been used. i.e. we need to see numbers substituted into a stated correct formula.	

Question Number	Scheme	Marks
45a	Using $v^2 = u^2 + 2as$: $v^2 = 4g$, $v = \sqrt{4g}$ or 6.3 or 6.26 (m s^{-1})	M1,A1 (2)
b	Rebounds to 1.5 m, $0 = u^2 - 3g$, $u = \sqrt{3g}$, 5.4 or 5.42 (m s^{-1})	M1A1 (2)
c	If speed downwards is taken to be positive:  First line Second line -*u*,*u*,	 B1 B1 B1 (3)
d.	Use of suvat to find t_1 or t_2, $\sqrt{4g} = gt_1 \quad t_1 = \sqrt{\frac{4}{g}} = 0.64$ s $\sqrt{3g} = gt_2 \quad t_2 = \sqrt{\frac{3}{g}} = 0.55$ s Total time = $t_1 + 2t_2 = 1.7$ s or 1.75 s	M1A1 (t_1 or t_2) **DM**1A1 (4) **[11]**

Notes for Question 45

N.B. Deduct only 1 mark in **whole question** for not giving an answer to either 2 sf or 3 sf, following use of g = 9.8 or use of g = 9.81

Question 45(a)

M1 is for a complete method for finding speed (usually $v^2=u^2+2as$)

A1 for v = 6.3 (ms^{-1}) or 6.26 (ms^{-1}) or $\sqrt{4g}$ (ms^{-1}) (must be positive)

Allow 0= $u^2 - 4g$ or $v^2 = 4g$ but not 0= $u^2 + 4g$ or $v^2 = -4g$

Question 45(b)

M1 is for a complete method for finding speed

Allow 0= $u^2 - 3g$ or $v^2 = 3g$ but not 0= $u^2 + 3g$ or $v^2 = -3g$

A1 for 5.4 (ms^{-1}) or 5.42 (ms^{-1}) or $\sqrt{3g}$ (ms^{-1}) (must be positive)

Question 45(c)

First B1 for a straight line from origin to their *v which must be marked on the axis.*

Second B1 for a parallel straight line correctly positioned (*if continuous vertical lines are clearly included as part of the graph then B0)*

Third B1 for their *–u* and *u* correctly marked, *provided their second line is correctly positioned*

N.B. A reflection of the graph in the *t*-axis (upwards +ve) is also acceptable

Question 45(d)

First M1 for use of *suvat* or area under their *v-t* graph to find either t_1 or t_2 or $2t_2$

First A1 for correct value for either t_1 or t_2 *(can be in terms of g at this stage or surds or unsimplified e.g.6.3/9.8)*

Second M1 **dependent on the first M1** for their $t_1 + 2t_2$

Second A1 for 1.7 (s) or 1.75 (s).

Question Number	Scheme	Marks
46a	$\mathbf{F} = m\boldsymbol{a}$ **:** $3\mathbf{i} - 2\mathbf{j} = \mathbf{0}.5\boldsymbol{a}$ $\boldsymbol{a} = 6\mathbf{i} - 4\mathbf{j}$ $\lvert\boldsymbol{a}\rvert = \sqrt{6^2 + (-4)^2} = 2\sqrt{13}$ **(m s^{-2}) ****	M1 A1 M1A1 (4)
b	$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{at}$**:** $\boldsymbol{v} = (\mathbf{i} + 3\mathbf{j}) + 2(6\mathbf{i} - 4\mathbf{j})$	M1A1 **ft**
	$= 13\mathbf{i} - 5\mathbf{j}$ m s^{-1}	A1 (3)
c	Distance $= 2\lvert\mathbf{v}\rvert = 2\sqrt{4+1} = 2\sqrt{5} = 4.47$ **(m)**	M1A1
		(2)
d	When $t = 3.5$**,** velocity of P is $(\mathbf{i} + 3\mathbf{j}) + 3.5(6\mathbf{i} - 4\mathbf{j}) = 22\mathbf{i} - 11\mathbf{j}$ Given conclusion reached correctly. E.g. $22\mathbf{i} - 11\mathbf{j} = 11(2\mathbf{i} - \mathbf{j})$	M1A1 **ft** A1 **(3)**
		[12]

Notes for Question 46

<u>Question 46(a)</u>

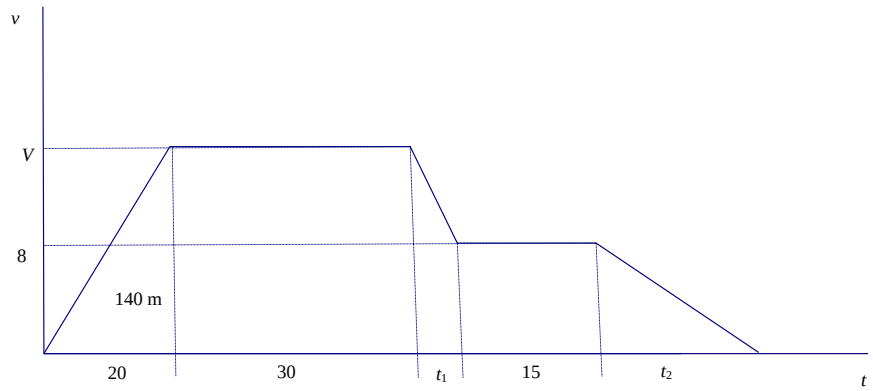
Either:
First M1 for use of **F** = *m* **a**
First A1 for **a** = 6**i** – 4**j**
Second M1 for $a = \sqrt{(6^2 + (-4)^2)}$ (Allow $\sqrt{(6^2 + 4^2)}$)
Second A1 for $a = 2\sqrt{13}$ (ms^{-2}) **Given answer**

Or:
First M1 for $F = \sqrt{(3^2 + (-2)^2)}$ (Allow $\sqrt{(3^2 + 2^2)}$)
First A1 $F = \sqrt{13}$
Second M1 for $\sqrt{13} = 0.5\,a$
Second A1 for $a = 2\sqrt{13}$ (ms^{-2}) **Given answer**

<u>Question 46(b)</u>
M1 for **(i + 3j) + (**2 x their **a**)
First A1 **ft** for a correct expression
Second A1 for 13**i** – 5**j** ; isw if they go on to find the speed

<u>Question 46(c)</u>
M1 for $2\sqrt{(2^2 + (-1)^2)}$ or $\sqrt{(4^2 + (-2)^2)}$
A1 for $2\sqrt{5}$ or $\sqrt{20}$ or 4.5 or 4.47 or better

<u>Question 46(d)</u>
M1 for **(i + 3j)** + (3.5 x their **a**) ,or possibly, their (b) + (1.5 x their **a**)
First A1 ft for a correct expression *of form a***i** + *b***j**
Second A1 for given conclusion reached correctly *e.g.* 22**i** -11**j**=11**(**2**i** - **j**) oe **Given answer**

Question Number	Scheme	Marks
47(a)		B1 $0 < t < 50$ B1 $50 < t$ B1 (V,8,15, 20,30) (3)
(b)	Use area under graph or *suvat* to form an equation in V only. $140 = \frac{1}{2} \times 20 \times V$ $V = 14$	M1 A1 (2)
(c)	$8 = V - \frac{1}{2}t_1$ (and /or $0 = 8 - \frac{1}{3}t_2$) $t_1 = 12$, (and/or $t_2 = 24$) Total time = $20 + 30 + t_1 + 15 + t_2 = 101$(seconds)	M1 A1 **DM**1 A1 (4)
(d)	Total distance = $140 + 30V + \frac{V+8}{2}t_1 + 15 \times 8 + \frac{1}{2} \times 8 \times t_2$ $= 140 + 30 \times 14 + 11 \times 12 + 15 \times 8 + 24 \times 4$ =908 (m)	M1A2 **ft** A1 (4)
		[13]

Notes for Question 47

Question 47(a)
First B1 for shape of graph for $0 \leq t \leq 50$
Second B1 for shape of graph for $t > 50$
Third B1 for V, 8, 15, 20, 30 appropriately used

Question 47(b)
M1 for use of area under graph (must have '1/2') or *suvat* to obtain an equation in V only.
A1 for $V = 14$

Question 47(c)
First M1 for use of either $8 = V - ½\,t_1$ or $0 = 8 - {}^1/_3\,t_2$
First A1 for either $t_1 = 12$ or $t_2 = 24$
Second M1, **dependent on the first M1**, for $20 + 30 + t_1 + 15 + t_2$ (must include all 5 times)
Second A1 for 101 (s)

Question 47(d)
First M1 for an expression for the total area (distance) **including all parts of the motion**. Where a triangle or trapezium is used , a '1/2' must be seen.
Second A2 **ft** on their V, t_1 and t_2, -1 each error.
Fourth A1 for 908 (m).

Question Number	Scheme	Marks
48(a)	Max ht $v = 0$. $v = u - gt \Rightarrow T = \frac{u}{g}$	M1A1 (2)
(b)	Max ht $H = ut + \frac{1}{2}at^2 = \frac{u^2}{g} - \frac{u^2}{2g} = \frac{u^2}{2g}$ * Given answer* Or use of $v^2 = u^2 + 2as$	M1A1 (2)
(c)	$-3 \times \frac{u^2}{2g} = ut - \frac{1}{2}gt^2$ $-3u^2 = 2ugt - g^2t^2$ $g^2t^2 - 2ugt - 3u^2 = 0, \quad gt = \frac{2u \pm \sqrt{4u^2 + 12u^2}}{2}$ $t = \frac{3u}{g} = 3T$	M1 **DM**1 A1 A1 (4)
(c) alt	$-4H = -\frac{1}{2}gt^2$ Total time = $T + \sqrt{\frac{8H}{g}} = T + \sqrt{\frac{8u^2}{2g^2}}$ $= T + 2T = 3T$	M1 **DM**1A1 A1 (4)
		[8]

Notes for Question 48

Question 48

In this question, condone sign errors in a *suvat* equation for the M mark, but a missing term is M0 or an incorrect term is M0. An incorrect *suvat* formula is M0

Allow use of symmetry of motion.

e.g. in (a), using $v = u + at$, *either* $0 = u - gT$ *or* $u = 0 + gT$

Question 48(a)

M1 for use of *suvat* to obtain an equation in T, u and g only.

A1 for $T = u/g$ correctly obtained.

Question 48(b)

M1 for use of *suvat* to obtain an equation in H, u and g only.

A1 for $H = u^2/2g$ correctly obtained **(given answer)**

Question 48(c) *Watch out for t / T confusion* (**N.B.** if only T's used, M0DM0)

First M1 for a complete method to find the *total* time in terms of u, g, H or T:-

either: $3H = -ut + \frac{1}{2}gt^2$

or: $4H = \frac{1}{2}gt^2$ and $t + T$

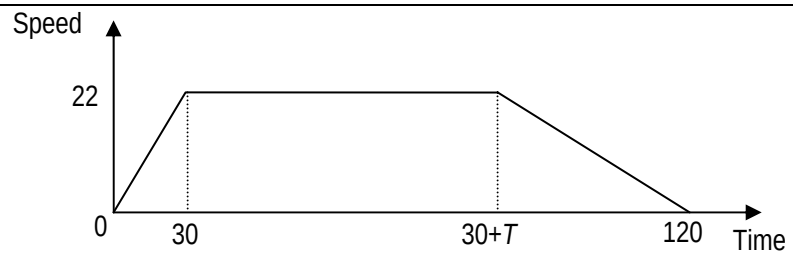
or: $v^2 = u^2 + 6gH$ and $v = -u + gt$,with v eliminated

Second M1, **dependent on first M1**, for producing an expression, in terms of u, g, H or T, for the total time, by solving a quadratic

First A1 for any correct expression for the total time in terms of u, g, H or T.

Second A1 for $3T$ cso

Question Number	Scheme	Marks
49.		
(a)	$240 = \frac{1}{2}(u+34)10$	M1 A1
	$u = 14$	A1
		(3)
(b)	$34 = 14 + 10a \quad => \quad a = 2$	M1 A1
	$120 = 14t + \frac{1}{2} \times 2 \times t^2$	M1 A1
	$t^2 + 14t - 120 = 0$	
	Solving, $t = -20$ or 6	**DM**1
	$t = 6$	A1
	OR	
	$34 = 14 + 10a \quad => \quad a = 2$	M1 A1
	$v^2 = 14^2 + 2 \times 2 \times 120 \quad \Rightarrow v = 26$	
	AND $26 = 14 + 2t$	M1 A1
	$t = 6$	**DM**1 A1
		(6)
		[9]
Notes for Question 49		
Q49(a)	First M1 for a complete method to produce an equation in u only. First A1 for a correct equation. ($u^2 - 48u + 476 = 0$ oe is possible). Second A1 for $u = 14$.	
Q49(b)	**EITHER** First M1 for an equation in a only. (M0 if $v = 34$ when $s = 120$ is used) First A1 for $a = 2$. (This may have been found in part (a)) Second M1 for a 3-term quadratic equation in t only, allow sign errors (must have found a value of a. (M0 if $v = 34$ when $s = 120$ is used) Second A1 for a correct equaton. Third M1 dependent on previous M1 for solving for t. Third A1 for $t = 6$ **OR** First M1 for an equation in a only. First A1 for $a = 2$. (This may have been found in part (a)) Second M1 for a complete method to obtain an equation in t only, allow sign errors. (must have found a value of a) Second A1 for a correct equaton. Third M1 dependent on previous M1 for solving for t. Third A1 for $t = 6$	

Question Number	Scheme	Marks
50.		
(a)	 Shape Figures	B1 B1 **(2)**
(b)	$\frac{(120+T)22}{2}=2145$	M1 A1
	$T=75$	A1
		(3)
(c)	$\frac{(t+t-30)22}{2}=990$	M1 A1
	$t=60$	A1
	Answer $=60-10=50$	A1
		(4)
(d)	$990=0.5a50^2$	M1
	a = 0.79, 0.792, 99/125 oe	A1
		(2)
		[11]
Notes for Question 50		
Q50(a)	First B1 for a trapezium starting at the origin and ending on the *t*-axis. Second B1 for the figures marked (allow missing 0 and a delineator oe for *T*) (allow if they have used $T = 75$ correctly on their graph)	
Q50(b)	First M1 for producing an equation in their *T only* by equating the area of the trapezium to 2145, with the correct no. of terms. If using a single trapezium, we need to see evidence of using ½ the sum of the two parallel sides or if using triangle(s), need to see ½ base x height. Second A1 cao for a correct equation in *T* (<u>This is not f.t. on their *T*</u>) Third A1 for $T = 75$. N.B. Use of a single *suvat* equation for the whole motion of the car e.g. $s = t(u+v)/2$ is M0	
Q50(c)	First M1 for producing an equation in *t only* (they may use $(t – 30)$ oe as their variable) by equating the area of the trapezium to 990, with the correct no. of terms. If using a trapezium, we need to see evidence of using ½ the sum of the two parallel sides or if using triangle(s), need to see ½ base x height. First A1 for a correct equation. Second A1 for $t = 60$ (Allow 30 + 30). Third A1 for answer of 50. N.B. Use of a single *suvat* equation for the whole motion of the car e.g. $s = t(u+v)/2$ is M0. Use of the motion of the motorcycle is M0 (insufficient information). Use of $v = 22$ for the motorcycle is M0.	
Q50(d)	First M1 for an equation in *a* only. First A1 for a = 0.79, 0.792, 99/125 oe N.B. Use of $v = 22$ for the motorcycle is M0.	

Question Number	Scheme	Marks
51.		
(a)	$t = 0$ gives $\mathbf{v} = \mathbf{i} - 3\mathbf{j}$	B1
	speed $= \sqrt{1^2 + (-3)^2}$	M1
	$= \sqrt{10} = 3.2$ or better	A1
		(3)
(b)	$t = 2$ gives $\mathbf{v} = (-3\mathbf{i} + 3\mathbf{j})$	M1
	Bearing is 315°	A1
		(2)
(c)(i)	$1 - 2t = 0 \Rightarrow t = 0.5$	M1 A1
(ii)	$-(3t - 3) = -3(1 - 2t)$	M1 A1
	Solving for t	**DM**1
	t = 2/3, 0.67 or better	A1
		(6)
		[11]
Notes for Question 51		
51(a)	B1 for **i** – 3**j.** M1 for √ (sum of squares of cpt.s) A1 for √10, 3.2 or better	
51(b)	M1 for clear attempt to sub $t = 2$ into given expression. A1 for 315.	
51(c)	**(i)** First M1 for $1 - 2t = 0$. First A1 for $t = 0.5$. N.B. If they offer two solutions, by equating both the **i** and **j** components to zero, give M0. **(ii)** First M1 for $\frac{1-2t}{3t-3} = \pm(\frac{-1}{-3})$ o.e. (Must be an equation in t *only*) First A1 for a correct equation (the + sign) Second M1, dependent on first M1, for solving for t. Second A1 for 2/3, 0.67 or better.	

Question Number	Scheme	Marks
52. (a)	Use of $s = ut + \frac{1}{2}at^2$	M1
	$-2t + \frac{1}{2}gt^2$ (+ or – 50)	A1
	$20t - \frac{1}{2}gt^2$ (+ or – 50)	A1
	$50 = -2T + \frac{1}{2}gT^2 + 20T - \frac{1}{2}gT^2 = 18T$	M1
	$T = \frac{50}{18} = 2.777.... = 2.8$ or better	A1 **(5)**
(b)	$h = 20 \times T - 4.9 \times T^2 = 17.74.... \approx 17.7$ (18 to 2 s.f.) (use of 2.8 gives 17.584)	M1A1 **(2)** **[7]**
	Notes on Question 52	
52(a)	First M1 for use of $s = ut + 1/2at^2$ (or use of 2 *suvat* formulae AND eliminating *v*, to give an equation in *s* and *t*). N.B. M0 if they use $s = 50$ or $u = 0$ or $v = 0$) First A1 with $u = 2$ and $a = -g$ or -9.8 to obtain a distance, possibly with 50 added or subtracted. (2 and 4.9 must have *opposite* signs) Second A1 with $u = 20$ and $a = -g$ or -9.8 to obtain a distance, possibly with 50 added or subtracted. (2 and 4.9 must have *opposite* signs) Second M1 dependent on first M1 for a *correct* equation obtained correctly in *T* only. Third A1 for 25/9 oe, 2.8 or better	
52(b)	First M1 for substituting their *T* value (allow –ve changed to +ve but A mark is then unavailable) into an appropriate equation First A1 for 17.7 or 18 (m). (A0 if they then add 50)	

Question Number	Scheme	Marks
53. **(a)**	$s = \frac{u+v}{2}t \qquad 10 = \frac{2+v}{2} \times 3.5$	M1A1
	$v = \frac{20}{3.5} - 2 = \frac{26}{7} = 3.71 \quad (\text{m s}^{-1})$	A1 **(3)**
(b)	$a = \frac{v-u}{t} = \frac{\frac{26}{7} - 2}{3.5} = \frac{24}{49} = 0.490 \ (\text{m s}^{-2})$	M1A1 **(2)**
		[5]
	Notes for Question 53	
53(a)	First M1 for producing an equation in *v only.* First A1 for a correct equation Second A1 for 26/7 oe, 3.7 or better (ms^{-1})	
53(b)	M1 for producing an equation in *a only.* A1 for 24/49, 0.49 or better (ms^{-2})	

Question Number	Scheme	Marks
54. **(a)**	Use of $\mathrm{r} = \mathrm{r}_0 + \mathrm{v}t$ $(-4\mathrm{i} + 2\mathrm{j}) + (3\mathrm{i} + 3\mathrm{j})t = (-4 + 3t)\mathrm{i} + (2 + 3t)\mathrm{j}$	M1 A1 **(2)**
(b)	$(6\mathrm{i} + \mathrm{j}) + (-2\mathrm{i} + n\mathrm{j})t = (6 - 2t)\mathrm{i} + (1 + nt)\mathrm{j}$ Position vectors identical $\Rightarrow -4 + 3t = 6 - 2t$ **AND** $5t = 10$, Either equation $2 + 3 \times 2 = 1 + 2n$, $n = 3.5$	B1 M1 A1 **DM1** A1 **(5)**
(c)	Position vector of P is $(-4 + 6)\mathrm{i} + (2 + 6)\mathrm{j} = 2\mathrm{i} + 8\mathrm{j}$ Distance OP $= \sqrt{2^2 + 8^2} = \sqrt{68} = 8.25\,(\mathrm{km})$	M1A1 M1A1 **(4)** **[11]**

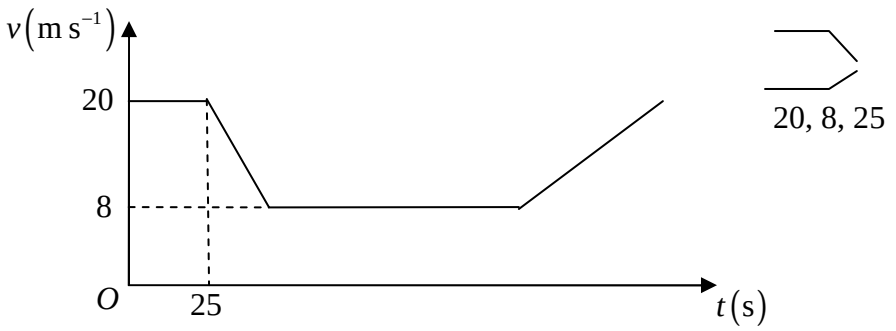
Notes for Question 54

54(a)	M1 for clear attempt to use $\mathbf{r}_0 + t\mathbf{v}$ (M0 if $\mathbf{r}_0$ and $\mathbf{v}$ reversed) A1 for answer in any form.
54(b)	B1 for $(6\mathbf{i} + \mathbf{j}) + (-2\mathbf{i} + n\mathbf{j})t$ seen or implied First M1 for equating their **i**- cpts *and* their **j**- cpts. (must have *both* equations in terms of *same t*) First A1 for a correct equation (either) Second M1 dependent on first M1 for producing an equation in *n* only. Second A1 for $n = 3.5$ oe
54(c)	First M1 for clear attempt to find pv of *P*, using their *t* and/or *n* value(s) First A1 for $2\mathbf{i} + 8\mathbf{j}$ Second M1 for attempt to find magnitude of their **p** Second A1 for $\sqrt{68}$, $2\sqrt{17}$, 8.2 or better (km)

Question Number	Scheme	Marks
55 **(a)**	Use of $v^2 = u^2 + 2as$ $14^2 = 20^2 - 2a \times 100$ Deceleration is 1.02(m s^{-2})	M1 A1 A1 **(3)**
(b)	Horizontal forces on the car: $\pm T\cos\theta - 300 = 750 \times -1.02 = -765$ T = - 1550/3 The force in the tow-bar is 1550/3, 520 (N) or better (allow –ve answer)	M1A2 **f.t.** A1 **(4)**
(c)	Horizontal forces on the truck: $\pm T\cos\theta - 500 - R = 1750 \times -1.02$ Braking force R = 1750 (N)	M1A2 **f.t.** A1 **(4)** **[11]**
	ALT: Whole system: $800 + R = 2500 \times 1.02$ R = 1750	M1A2 **f.t.** A1
	Notes for Question 55	
55(a)	M1 for a complete method to produce an equation in *a* only. First A1 for a correct equation. Second A1 for 1.02 (ms^{-2}) oe. must be POSITIVE.	
55(b)	M1 for considering *the car ONLY* horizontally to produce an equation in *T* only, with usual rules. i.e. correct no. of terms AND *T* resolved: $\pm T\cos\theta - 300 = 750$ x -1.02 A2 **ft** on their *a* for a correct equation (300 and *a* must have *same* sign); -1 each error (treat cos 0.9 as an A error) A1 for 1550/3 oe, 520 or better (N) N.B. Allow a negative answer.	
55(c)	M1 for considering *the truck ONLY* horizontally to produce an equation, with usual rules. i.e. correct no. of terms AND *T* resolved: $\pm T\cos\theta - 500 - R = 1750$ x -1.02 A2 **ft** on their *T* and *a* for a correct equation (500, *a* and *R* must have *same* sign); -1 each error (treat cos 0.9 as an A error) A1 for 1750 (N). **OR** M1 for considering *the whole system* to produce an equation in *R* only, with usual rules. i.e. correct no. of terms. A2 **ft** on their *a* for a correct equation (*a* and *R* must have *same* sign) -1 each error A1 for 1750 (N). N.B. If 300 and 500 are given separately, penalise any sign errors only ONCE.	

56. (a)	$30^2 = 2a.300$ $a = 1.5$	M1 A1 (2)
(b)	$0^2 = 30^2 - 2 \text{ x } 1.25s$ OR $0 = 30 - 1.25t_2$ $s = 360$ $t_2 = 24$ $300 + 30T + 360 = 1500$ $\frac{(20 + T + 24 + T)}{2} \text{ x } 30 = 1500$ $T = 28$ $T = 28$	M1 A1 M1 A1 A1 (5)
(c)	triangle, *drawn on the diagram*, with base coinciding with base of trapezium, top vertex above line $v = 30$ and meeting trapezium at least once V marked correctly	B1 DB1 (2)
(d)	$30 = 1.5t_1 \Rightarrow \quad t_1 = 20$ $30 = 1.25t_2 \Rightarrow t_2 = 24$ $\frac{1}{2}(20 + 28 + 24)V = 1500$ $V = \frac{750}{18} = 41.67$ $= \frac{125}{3}$ (oe) 0r 42 (or better)	M1 A1 A1 M1 A1 A1 (6)
		15

57(a)	$\frac{(\mathbf{i} - 4\mathbf{j}) - (4\mathbf{i} - 8\mathbf{j})}{0.5}; (\pm 6\mathbf{i} \pm 8\mathbf{j})$ $\sqrt{(\pm 6)^2 + (\pm 8)^2} = 10$	M1 A1 M1 A1 (4)
(b)	$\mathbf{r} = (4\mathbf{i} - 8\mathbf{j}) + t(-6\mathbf{i} + 8\mathbf{j})$ $= (4\mathbf{i} - 8\mathbf{j}) - 6t\mathbf{i} + 8t\mathbf{j}$ $= (4 - 6t)\mathbf{i} + (8t - 8)\mathbf{j}$ *	M1 A1 (2)
(c)	At 10 am, $\mathbf{r} = -2\mathbf{i}$ At 10.30 am, $\mathbf{r} = -5\mathbf{i} + 4\mathbf{j}$ $\mathbf{l} = k\mathbf{i},\ k < -2$ $k = -5 - 4 = -9$ $\mathbf{l} = -9\mathbf{i}$	M1 A1 A1 DM1 A1 (5) **11**

Question Number	Scheme	Marks
58.	(a) 	B1 B1 B1 **(3)**
	(b) $v = u + at \Rightarrow 8 = 20 - 0.4t$ $t = 30 \ (\text{s})$	M1 A1 **(2)**
	(c) $1960 = (25 \text{ x } 20) + (30 \text{ x } 8) + (\tfrac{1}{2} \text{ x } 30 \text{ x } 12) + (60 \text{ x } 8) + 8 \text{ x } t + \tfrac{1}{2} \text{ x } t \text{ x } 12$	M1A3 **ft** (2 ,1, 0)
	$1960 = 500 + 240 + 180 + 480 + 14t$	DM1 A1
	$T = 115 + 40$ $= 155$	DM1 A1
	N.B. SEE ALTERNATIVES	**(8)** **[13]**

Question 58(a)
First B1 for 1st section of graph
Second B1 for 2nd section
Third B1 for the figures 20, 8 and 25

Question 58(b)
M1 for a complete method to produce an equation in t only; allow (20 – 8)/0.4
A1 for 30 N.B.
Give A0 for t = - 30, even if changed to 30, but then allow use of 30 in part (c), where full marks could then be scored.

Question 58(c)

First M1 (generous) for clear attempt to find whole area under *their* graph (must include at least one "1/2"), in terms of *a single unknown time (t say),*and equate it to 1960.
First A3, ft on their (b), for a correct equation.
Deduct 1 mark for each numerical error, or omission, in each of the 4 *sections of the area* corresponding to each stage of the motion. (they may 'slice' it, horizontally into 3 sections, or a combination of the two)
Second DM1, dependent on first M1, for simplifying to produce an equation with all their *t* terms collected.
Fourth A1 for a correct equation for *t* or *T*
Third DM1, dependent on second M1. for solving for *T*
Fifth A1 155

Please note that any incorrect answer to (b) will lead to an answer of 155 in (c) and can score max 6/8;

Solutions with the correct answer of 155 will need to be checked carefully.

Solutions to 58 (c) **N.B.** $t = T - 115$

A. $1960 = (25 \times 20) + (30 \times 8) + (\frac{1}{2} \times 30 \times 12) + (60 \times 8) + 8 \times t + \frac{1}{2} \times t \times 12$ M1 A3 **ft**
$1960 = 500 + 240 + 180 + 480 + 14t$ M1 A1
$T = 115 + 40$ M1
$= 155$ A1

B. $1960 = (25 \times 20) + \frac{1}{2} \times 30 \times (20 + 8) + (60 \times 8) + \frac{1}{2} \times t \times (20 + 8)$ M1 A3 **ft**
$1960 = 500 + 420 + 480 + 14t$ M1 A1
$T = 115 + 40$ M1
$= 155$ A1

C. $1960 = 8T + \frac{1}{2} \times 12 \times (55 + 25) + \frac{1}{2} \times 12 \times (T - 115)$ M1 A3 **ft**
$1960 = 8T + 480 + 6T - 690$
$1960 = 14T - 210$ M1 A1
$155 = T$ M1 A1

D. $1960 = 20T - \frac{1}{2} \times 12 \times (60 + T - 25)$ M1 A3 **ft**
$1960 = 20T - 6T - 210$
$1960 = 14T - 210$ M1 A1
$155 = T$ M1 A1

E. $1960 = (55 \times 20) - \frac{1}{2} \times 30 \times 12 + (60 \times 8) + \frac{1}{2} \times t \times (20 + 8)$ M1 A3 **ft**
$1960 = 1100 - 180 + 480 + 14t$ M1 A1
$T = 115 + 40$ M1
$= 155$ A1

F. $1960 = (8 \times 115) + \frac{1}{2} \times 12 \times (55 + 25) + \frac{1}{2} \times 28 \times (T - 115)$ M1 A3 **ft**
$1960 = 920 + 480 + 14T - 1610$
$1960 = 14T - 210$ M1 A1
$155 = T$ M1 A1

Question Number	Scheme	Marks
59.	(a) $v^2 = u^2 + 2as \Rightarrow 28^2 = u^2 + 2\times 9.8\times 17.5$	M1 A1
	Leading to $u = 21$ ✱ cso	A1 **(3)**
	(b) $s = ut + \frac{1}{2}at^2 \Rightarrow 19 = 21t - 4.9t^2$	M1 A1
	$4.9t^2 - 21t + 19 = 0$	
	$t = \frac{21 \pm \sqrt{21^2 - 4x4.9.x19}}{9.8}$	
	t = 2.99 or 3.0	DM1 A1 A1
	t = 1.30 or 1.3	**(5)**
	(c) N2L $4g - 5000 = 4a$	M1 A1
	$(a = -1240.2)$	
	$v^2 = u^2 + 2as \Rightarrow 0^2 = 28^2 - 2\times 1240.2\times s$	
	Leading to $s = 0.316$ (m) or 0.32	M1 A1 **(4)**
		[12]

Question 59(a)

First M1 for a complete method for finding u e.g.

$28^2 = u^2 + 2g\text{x}17.5$

or $28^2 = u^2 + 2(-g)\text{x}(-17.5)$

or $28^2 = 2gs \Rightarrow s = 40$ then $0^2 = u^2 + 2(-g)\text{x}(22.5)$

condone sign errors

First A1 for a correct equation(s) with g = 9.8

Second A1 for "u = 21" PRINTED ANSWER

N.B. Allow a verification method, but they must state, as a conclusion, that "u = 21" , to score the final A1.

Question 59(b)

First M1 for a complete method for finding at least one t value i.e. for producing an equation in t only.
(condone sign errors but not missing terms)

First A1 for a correct quadratic equation in t only or TWO correct linear equations in t only.

Second DM1, dependent on first M1, for attempt to solve the quadratic or one of the linear equations.

Second A1 for 3.0 or 3 or 2.99

Third A1 for 1.3 or 1.30

Question 59(c)

First M1 for resolving vertically with usual rules.

First A1 for a correct equation

Second M1 for use of $v^2 = u^2 + 2as$, with v = 0, u = 28 or u = 0 and v = 28 and their a, (or any other complete method which produces an equation in s,which could be negative)

M0 if they haven't *calculated* a value of a.

Second A1 for 0.32 or 0.316. (must be positive since it's a distance)

Question Number	Scheme	Marks
60.	(a) $\arctan\frac{7.5}{12} = 32°$	M1 A1
	Bearing is 302 (allow more accuracy)	A1 **(3)**
	(b) $\mathbf{s} = 40\mathbf{i} - 6\mathbf{j} + t(-12\mathbf{i} + 7.5\mathbf{j})$	M1 A1 **(2)**
	(c) $t = 3$, $\mathbf{s} = 4\mathbf{i} + 16.5\mathbf{j}$	M1
	$\mathbf{s} - \mathbf{b} = -3\mathbf{i} + 4\mathbf{j}$	M1
	$SB = \sqrt{\left((-3)^2 + 4^2\right)} = 5 \ (\text{km})$	DM1 A1 **(4)**
	(d) Equating **i** components	
	$40 - 12t = 7$ **or** $-33 + 12t = 0$	M1
	$t = 2\frac{3}{4}$	A1
	When $t = 2\frac{3}{4}$, $\mathbf{s} = (7\mathbf{i}) + 14\frac{5}{8}\mathbf{j}$	M1
	$SB = 2\frac{1}{8}$ (km) 2.125, 2.13	A1 **(4)**
		[13]
	OR When $t = 2\frac{3}{4}$, 7.5 t - 18.5 = 2.125, 2.13	M1 A1

Question 60(a)

First M1 for $\arctan\left(\frac{\pm 7.5}{\pm 12}\right)$ either way up

First A1 for a correct value from their expression, usually 32^o or 58^o

Second A1 for 302 (allow more accurate answers)

Question 60(b)

M1 for a clear attempt at $(40\mathbf{i} - 6\mathbf{j}) + t(-12\mathbf{i} + 7.5\mathbf{j})$

A1 for any correct expression

Question 60(c)

First M1 is really B1 for $4\mathbf{i} + 16.5\mathbf{j}$ (seen or implied but can be in unsimplified form)

Second M1 is for a subtraction, $\mathbf{s} - \mathbf{b}$ or $\mathbf{b} - \mathbf{s}$.

Third DM1, dependent on second M1, for finding magnitude of their $\mathbf{s} - \mathbf{b}$ or $\mathbf{b} - \mathbf{s}$

A1 for 5

Question 60(d)

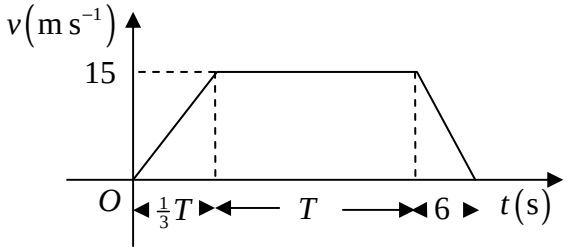
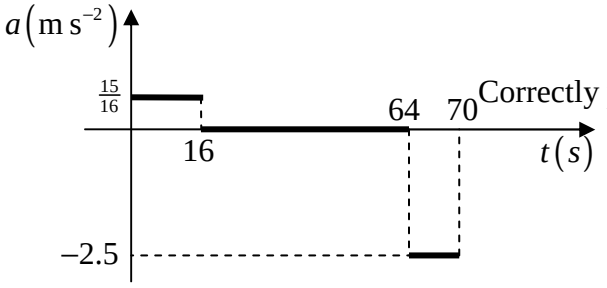
First M1 for equating **i**-component of their answer in part (b) to 7 or
the **i**-component of their $\mathbf{s} - \mathbf{b}$ or $\mathbf{b} - \mathbf{s}$ to zero

First A1 for 2.75 cao

Second M1 (independent) for attempt to find **j**-component of their **s** at their $t = 2.75$

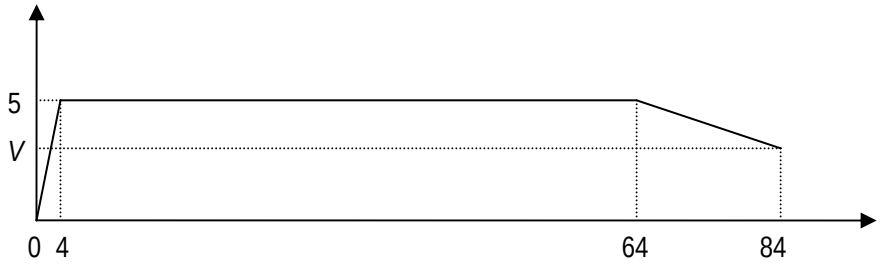
Second A1 2.125 or 2.13 cao

Question Number	Scheme	Marks
61 (a)	$v = u + at(\uparrow) \Rightarrow 0 = u - g(\frac{25}{14})$	M1 M(A)1
	$u = 17\frac{1}{2}$ ✱	A1
		(3)
(b)	$v^2 = u^2 + 2as(\uparrow) \Rightarrow 0^2 = 17.5^2 - 2gs$	M1
	$s = 15.6 \quad (\text{m}) \quad$ or 16 (m)	A1
		(2)
(c)	$s = ut + \frac{1}{2}at^2(\uparrow) \Rightarrow 6.6 = 17.5t - \frac{1}{2}gt^2$	M1
	$4.9t^2 - 17.5t + 6.6 = 0$	A1
	$t = \frac{17.5 \pm \sqrt{(17.5^2 - 129.36)}}{9.8} = \frac{17.5 \pm 13.3}{9.8}$	DM1
	t = 3.142.. (22/7) or 0.428…(3/7)	A1
	$T = t_2 - t_1 = 2.71 \quad (2.7)$	DM1 A1 (6)
	OR	
	$v^2 = u^2 + 2as(\uparrow) \Rightarrow v^2 = 17.5^2 - 2g\text{x}6.6$	
	$v = \pm 13.3$	
	$v = u + at(\uparrow) \Rightarrow \pm 13.3 = 17.5 - gt$	M1A1
	$t = \frac{17.5 \pm 13.3}{9.8}$	DM1
	= 3.14.. (22/7) or 0.428..(3/7)	A1
	T = 3.14.. – 0.428.. = 2.71 or 2.7	DM1 A1 (6)
	OR	
	$v^2 = u^2 + 2as(\uparrow) \Rightarrow v^2 = 17.5^2 - 2g\text{x}6.6$ or $0^2 = u^2 - 2g\text{x}(15.625 - 6.6)$	
	$v = 13.3$ $\qquad$ u = 13.3	
	$v = u + at(\uparrow) \Rightarrow 0 = 13.3 - gt$	M1 A1
	$t = \frac{13.3}{g}$	DM1 A1
	$T = 2 \text{ x } \frac{13.3}{g} = 2.7$ or 2.71	DM1 A1 (6)
		11

Question Number	Scheme	Marks
62 (a)	$v = u + at \Rightarrow 0 = 15 - 2.5t$ $t = 6 \quad (\text{s})$	M1 A1 (2)
(b)	 Shape 15, T	B1 B1 (2)
(c)	$\frac{1}{2}15\left(\frac{4}{3}T + 6 + T\right) = 885$ ft their 6 $\frac{7}{3}T = 118 - 6$ $T = 112 \times \frac{3}{7} = 48$	M1 A1ft M1 A1 (4)
(d)	$a = \frac{15}{\frac{1}{3}T} = \frac{15}{16}$, 0.9375, 0.938, 0.94	M1 A1 (2)
(e)	 3 horizontal lines Correctly placed;no cts vert line –2.5, ft their $\frac{15}{16}$	B1 B1 B1 (3)
		13

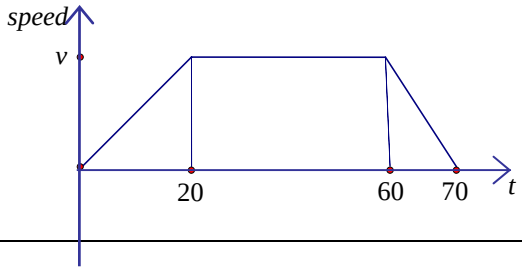
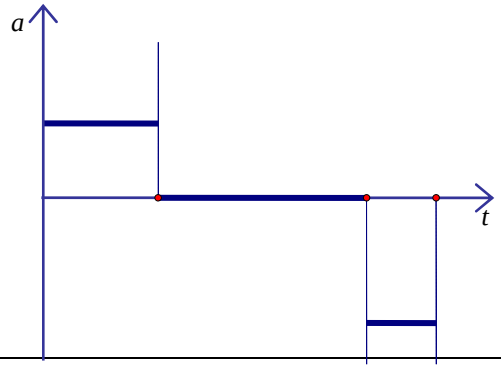
Question Number	Scheme	Marks
63 (a)	$\sqrt{\left((-4)^2+8^2\right)}=\sqrt{80}\quad\left(\text{km h}^{-1}\right)$ accept exact equivalents or 8.9 or better	M1 A1 (2)
(b)	$\mathbf{p}=\left(2\mathbf{i}-8\mathbf{j}\right)+t\left(-4\mathbf{i}+8\mathbf{j}\right)$	B1 (1)
(c)	Equating **j** components $-8+8t=12-8t$ $t=\frac{5}{4}$ oe	M1 A1 A1 (3)
(d)	Using their t from (c) to find the **i**-cpts of **p** and **q and subtract them** $10\frac{1}{2}-(-3)=13\frac{1}{2}\quad(\text{km})$	M1 A1 ft A1 (3) **9**

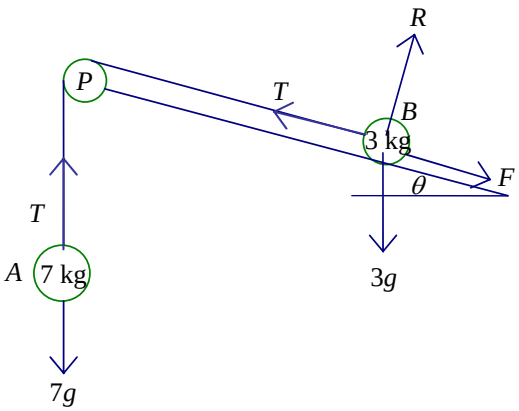
Question Number	Scheme	Marks
64. **(a)**	$0^2 = u^2 - 2\text{x}9.8\text{x}40$ $u = 28$ m s^{-1} ****** GIVEN ANSWER	M1 A1 A1 (3)
(b)	$33.6 = 28t - \frac{1}{2}9.8t^2$ $4.9t^2 - 28t + 33.6 = 0$ $t = \dfrac{28 \pm \sqrt{28^2 - 4\text{x}4.9\text{x}33.6}}{9.8}$ = 4 s or (1.7 s or 1.71 s)	M1 A1 M1 A1 A1 (5) **8**

Question Number	Scheme	Marks
65. **(a)**		B1 shape B1 figs (2)
(b)	$(\frac{1}{2}$x4x5) + 60 x 5 = 310	M1 A1 A1 (3)
(c)	$\frac{(5+V)}{2}$ x 20 = (400-310) $V = 4$	M1 A2 **ft** **DM**1 A1 (5)
(d)	$\frac{5-4}{20} = 0.05$ ms^{-2}	M1 A1 (2) **12**

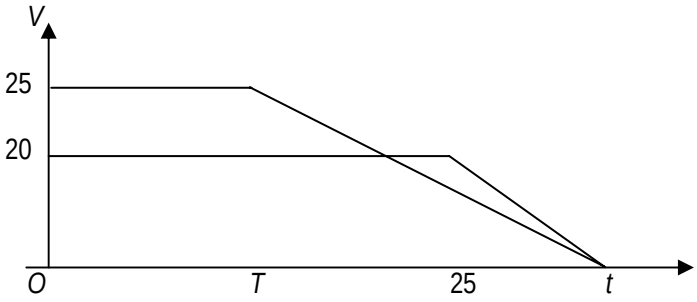
Question Number	Scheme	Marks
66. **(a)**	$\tan\theta = \frac{3}{4}$; bearing is 37^o **(nearest degree)**	M1; A1 (2)
(b)		
(i)	$\mathbf{p} = (\mathbf{i} + \mathbf{j}) + t(2\mathbf{i} - 3\mathbf{j})$	M1 A1
(ii)	$\mathbf{q} = (-2\mathbf{j}) + t(3\mathbf{i} + 4\mathbf{j})$	A1
(iii)	$\mathbf{PQ} = \mathbf{q} - \mathbf{p} = (-\mathbf{i} - 3\mathbf{j}) + t(\mathbf{i} + 7\mathbf{j})$	M1 A1 (5)
(c)		
(i)	$-1 + t = 0$ $t = 1$ or 3pm	M1 A1
(ii)	$-1 + t = -(-3 + 7t)$ $t = \frac{1}{2}$ or 2.30 pm	M1 A1 (4)
		11

Question Number	Scheme	Marks
67. **(a)**	$-6.45 = u - 9.8 \times 0.75$ $0.9 = u$ **	M1 A1 A1 (3)
(b)	$0 = 0.81 - 2 \times 9.8 \times s$ $s = 0.041$ or 0.0413	M1 A1 (2)
(c)	$h = -0.9 \times 0.75 + 4.9 \times 0.75^2$ $h = 2.1$ or 2.08	M1 A1 A1 (3) **[8]**

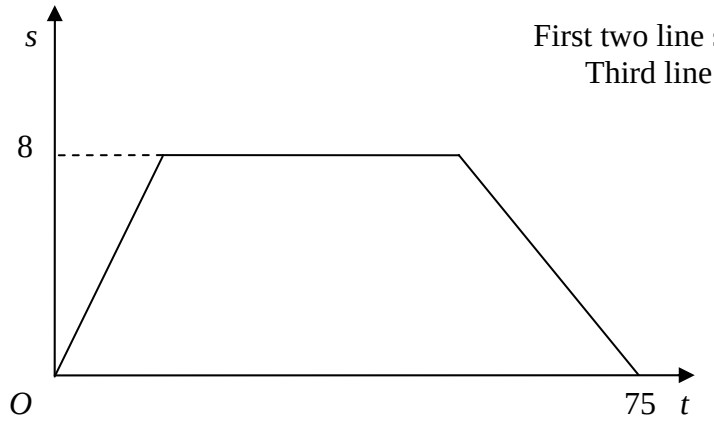
Question Number	Scheme	Marks
68. **(a)** **(i)**	 1st section correct 2nd & 3rd sections correct Numbers and v marked correctly on the axes.	B1 B1 DB1
(ii)	 1st section correct 2nd section correct 3rd section correct and no "extras" on the sketch	B1 B1 B1 (6)
(b)	$\frac{70+40}{2}\times v = 880$ $v = 880\times\frac{2}{110} = 16$	M1 A1 DM1 A1 (4) **[10]**

Question Number	Scheme	Marks
69. **(a)**	 $\tan\theta = \frac{5}{12}$ $\sin\theta = \frac{5}{13}$ $\cos\theta = \frac{12}{13}$	
	For A: $7g - T = 7a$	M1 A1
	For B: parallel to plane $T - F - 3g\sin\theta = 3a$	M1 A1
	perpendicular to plane $R = 3g\cos\theta$	M1 A1
	$F = \mu R = 3g\cos\theta = 2g\cos\theta$	M1
	Eliminating T, $7g - F - 3g\sin\theta = 10a$	DM1
	Equation in g and a: $7g - 2g\times\frac{12}{13} - 3g\frac{5}{13} = 7g - \frac{39}{13}g = 4g = 10a$	DM1
	$a = \frac{2g}{5}$ *oe* or 3.9 or 3.92	A1
		(10)
(b)	After 1 m,	
	$v^2 = u^2 + 2as$, $v^2 = 0 + 2\times\frac{2g}{5}\times 1$	M1
	$v = 2.8$	A1
		(2)
(c)	$-(F + 3g\ \sin\theta) = 3a$	M1
	$\frac{2}{3}\times 3g\times\frac{12}{13} + 3g\times\frac{5}{13} = 3g = -3a,\ a = -g$	A1
	$v = u + at,\ 0 = 2.8 - 9.8t,$	DM1
	$t = \frac{2}{7}$ oe, 0.29. 0.286	A1
		(4) **[16]**

Question Number	Scheme	Marks
70	$(-4\mathbf{i}-7\mathbf{j})=\mathbf{r}+4(-3\mathbf{i}+2\mathbf{j})$ $\mathbf{r}=(8\mathbf{i}-15\mathbf{j})$ $\lvert\mathbf{r}\rvert=\sqrt{8^2+(-15)^2}=17$ m	M1 A1 A1 M1 A1 ft **[5]**

Question Number	Scheme	Marks
71 (a)	 Shape (both) Cross Meet on *t*-axis Figures 25,20,*T*,25	B1 B1 B1 B1 (4)
(b)	For *Q*: $20\left(\frac{t+25}{2}\right)=800$ $t = 55$	M1 A1 **DM1** A1
	For *P*: $25\left(\frac{T+55}{2}\right)=800$ solving for *T:* $T = 9$	M1 A1 **DM1** A1 (8) **[12]**

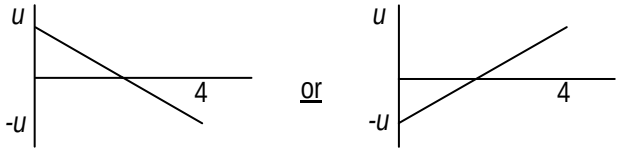
Question Number		Scheme	Marks
72	(a)	$(\uparrow) v^2 = u^2 + 2as$ $0 = 14.7^2 - 2\text{x } 9.8 \text{ x } s$ s = 11.025 (or 11 or 11.0 or 11.03) m Height is 60 m or 60.0 m **ft**	M1A1 A1 A1ft (4)
	(b)	$(\downarrow) v^2 = u^2 + 2as$ $v^2 = (-14.7)^2 + 2\text{x } 9.8 \text{ x } 49$ $v = 34.3$ or 34 m s^{-1}	M1 A1 A1 (3)
	(c)	$(\downarrow) v = u + at$ **OR** $(\downarrow) s = ut + \frac{1}{2}at^2$ $34.3 = -14.7 + 9.8t$ $\quad$ $49 = -14.7t + 4.9t^2$ $t = 5$ $\quad$ $t = 5$	M1 A1 A1 (3) **[10]**

Question Number	Scheme	Marks
73.	(a)  First two line segments Third line segment 8, 75	B1 B1 B1 (3)
	(b) $\frac{1}{2}\times 8\times(T+75)=500$ Solving to $T=50$	M1 A2 (1,0) DM1 A1 (5)
		[8]

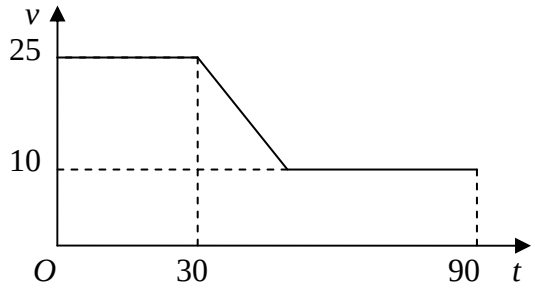
Question Number	Scheme	Marks
74.	(a) $\mathbf{v} = \dfrac{21\mathbf{i}+10\mathbf{j}-(9\mathbf{i}-6\mathbf{j})}{4} = 3\mathbf{i}+4\mathbf{j}$	M1 A1
	speed is $\sqrt{(3^2+4^2)} = 5\,(\text{km h}^{-1})$	M1 A1 (4)
	(b) $\tan\theta = \dfrac{3}{4}\ (\Rightarrow \theta \approx 36.9°)$	M1
	bearing is 37, 36.9, 36.87, …	A1 (2)
	(c) $\mathbf{s} = 9\mathbf{i}-6\mathbf{j}+t(3\mathbf{i}+4\mathbf{j})$	M1
	$= (3t+9)\mathbf{i}+(4t-6)\mathbf{j}$ ✱ cso	A1 (2)
	(d) Position vector of S relative to L is	
	$(3T+9)\mathbf{i}+(4T-6)\mathbf{j}-(18\mathbf{i}+6\mathbf{j}) = (3T-9)\mathbf{i}+(4T-12)\mathbf{j}$	M1 A1
	$(3T-9)^2+(4T-12)^2 = 100$	M1
	$25T^2-150T+125=0$ or equivalent	DM1 A1
	$(T^2-6T+5=0)$	
	$T = 1, 5$	A1 (6)
		[14]

Question Number	Scheme	Marks
75	$45 = 2u + \frac{1}{2}a2^2 \quad \Rightarrow \quad 45 = 2u + 2a$	M1 A1
	$165 = 6u + \frac{1}{2}a6^2 \quad \Rightarrow \quad 165 = 6u + 18a$	M1 A1
	eliminating either u or a	M1
	$u = 20$ and $a = 2.5$	A1 A1 **[7]**

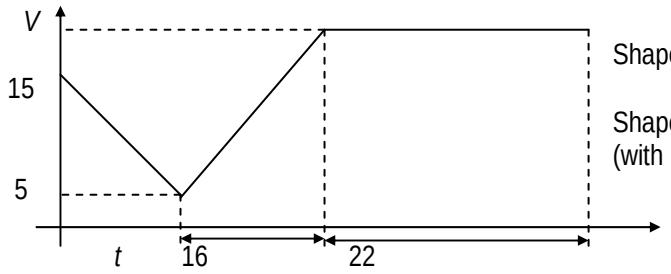
Question Number	Scheme	Marks
76 (a)	$\lvert\mathbf{v}\rvert = \sqrt{1.2^2 + (-0.9)^2} = 1.5 \text{ m s}^{-1}$	M1 A1 (2)
(b)	$(\mathbf{r}_H =) 100\mathbf{j} + t(1.2\mathbf{i} - 0.9\mathbf{j})$ m	M1 A1 (2)
(c)	$(\mathbf{r}_K =) 9\mathbf{i} + 46\mathbf{j} + t(0.75\mathbf{i} + 1.8\mathbf{j})$ m	M1 A1
	$\overrightarrow{HK} = \mathbf{r}_K - \mathbf{r}_H = (9 - 0.45t)\mathbf{i} + (2.7t - 54)\mathbf{j}$ m **Printed Answer**	M1 A1 (4)
(d)	Meet when $\overrightarrow{HK} = \mathbf{0}$	
	$(9 - 0.45t) = 0$ and $(2.7t - 54) = 0$	M1 A1
	$t = 20$ from both equations	A1
	$\mathbf{r}_K = \mathbf{r}_H = (24\mathbf{i} + 82\mathbf{j})$ m	DM1 A1 **cso**
		(5)
		[13]

Question Number	Scheme	Marks
77	$-6\mathbf{i}+\mathbf{j}=\mathbf{u}+3(2\mathbf{i}-5\mathbf{j})$ $\Rightarrow \mathbf{u}=-12\mathbf{i}+16\mathbf{j}$ $\Rightarrow u=\sqrt{(-12)^2+16^2}=20$	M1 A1 A1 cso M1 A1 **[5]**
78 (a)	 shape values	B1 B1 (2)
(b)	$19.6=\frac{1}{2}\times 2\times u$ $u=19.6$	M1 A1 A1 (3) **[5]**

Question Number	Scheme	Marks
79.	(a) $v^2 = u^2 + 2as \Rightarrow 17.5^2 = u^2 + 2 \times 9.8 \times 10$ Leading to $u = 10.5$	M1 A1 A1 **(3)**
	(b) $v = u + at \Rightarrow 17.5 = -10.5 + 9.8T$ $T = 2\frac{6}{7}$ (s)	M1 A1 f.t. DM1 A1 **(4)**
	Alternatives for (b) $s = (\frac{u+v}{2})T \Rightarrow 10 = (\frac{17.5 + -10.5}{2})T$ $\frac{20}{7} = T$	**[7]** M1A1 f.t. DM1A1 (4)
	OR $s = ut + \frac{1}{2}at^2 \Rightarrow -10 = 10.5t - 4.9t^2$ Leading to $T = 2\frac{6}{7}, \left(-\frac{5}{7}\right)$ Rejecting negative	M1 A1 f.t. DM1 A1 **(4)**
	(b) can be done independently of (a) $s = vt - \frac{1}{2}at^2 \Rightarrow -10 = -17.5t + 4.9t^2$ Leading to $T = 2\frac{6}{7}, \frac{5}{7}$	M1 A1 DM1
	For final A1, second solution has to be rejected. $\frac{5}{7}$ leads to a negative *u*.	A1 **(4)**

Question Number	Scheme	Marks
80.	(a)  shape 25, 10, 30, 90	B1 B1 **(2)**
	(b) $30\times 25+\frac{1}{2}(25+10)t+10(60-t)=1410$ $7.5t=60$ $t=8\ (\text{s})$ $a=\frac{25-10}{8}=1.875\ (\text{ms}^{-2})$ $1\frac{7}{8}$	M1 A1 A1 DM1 A1 M1 A1 **(7)** **[9]**

Question Number	Scheme	Marks
81.(a)	$27 = 0 + \frac{1}{2}.a.3^2 \Rightarrow a = \underline{6}$	M1 A1 (2)
(b)	$v = 6 \times 3 = \underline{18 \text{ m s}^{-1}}$	M1 A1 f.t. (2)
(c)	From $t = 3$ to $t = 5$, $s = 18 \times 2 - \frac{1}{2} \times 9.8 \times 2^2$	M1 A1 f.t.
	Total ht. $= s + 27 = \underline{43.4 \text{ m}, 43 \text{ m}}$	M1 A1 (4)
		8

Question Number	Scheme	Marks
82.(a)	 Shape 'V' Shape for last 22s (with $V > 15$) Figures	B1 B1 B1 (3)
(b)	$\frac{1}{2}(15 + 5) \times t = 120$ $\Rightarrow t = 12 \rightarrow T = 12 + 16 + 22 = \underline{50\text{ s}}$	M1 M1 A1 (3)
(c)	$120 + \underline{\frac{1}{2}(V + 5).16} + 22V = 1000$ Solve: $30V = 840 \Rightarrow V = \underline{28}$	M1 B1 A1 DM1 A1 (5) **11**

Question Number	Scheme	Marks
83.(a)	Speed $= \sqrt{(5^2 + 8^2)} \approx \underline{9.43 \text{ m s}^{-1}}$	M1 A1 (2)
(b)	Forming arctan 8/5 or arctan 5/8 oe	M1
	Bearing = 360 – arctan 5/8 or 270 + arctan 8/5 $= \underline{328}$	DM1 A1 (3)
(c)	At $t = 3$, p.v. of $P = (7 - 15)\mathbf{i} + (-10 + 24)\mathbf{j} = -8\mathbf{i} + 14\mathbf{j}$	M1 A1
	Hence $-8\mathbf{i} + 14\mathbf{j} + 4(u\mathbf{i} + v\mathbf{j}) = \mathbf{0}$	M1
	$\Rightarrow \underline{u = 2, \quad v = -3.5}$	DM1 A1 (5)
(d)	p.v. of P t secs after changing course $= (-8\mathbf{i} + 14\mathbf{j}) + t(2\mathbf{i} - 3.5\mathbf{j})$	M1
	$= 7\mathbf{i} + \ldots..$	DM1
	Hence total time = $\underline{10.5 \text{ s}}$	A1 (3)
		13

Q.	Scheme	Marks	Notes
84a	$v=0 \Rightarrow 3t^2-16t+21=0$	M1	Set $v=0$ and attempt to solve
	$\left((3t-7)(t-3)=0\right) \quad t_1=\frac{7}{3}, \quad t_2=3$	A1	
		(2)	
84b	$a=\frac{\mathrm{d}}{\mathrm{d}t}\left(3t^2-16t+21\right)$	M1	Differentiate v to obtain a
	$=6t-16$	A1	
	$t=t_1, \quad a=6\times\frac{7}{3}-16=-2$ (m s^{-2}) Magnitude 2 (m s^{-2})	A1	No errors seen. Must be positive - the Q asks for magnitude.
		(3)	
84c	$s=\int\left(3t^2-16t+21\right)\mathrm{d}t$	M1	Integrate v to find s
	$=t^3-8t^2+21t\,(+C)$	A1	
	$\pm\left(\left(3^3-8\times9+21\times3\right)-\left(\left(\frac{7}{3}\right)^3-8\times\frac{49}{9}+21\times\frac{7}{3}\right)\right)$	M1	Correct use of their limits
	$s=0.148$ (m) $\quad\left(\frac{4}{27}\right)$	A1	Final answer must be positive. 0.15 or better
		(4)	
84d	Return to $O \Rightarrow s=0=t\left(t^2-8t+21\right)$	B1	seen or implied
	Discriminant of quadratic $=64-4\times21\left(=-20\right)<0$	M1	Or equivalent. ***given answer so must show some evidence of method***
	No real roots $\Rightarrow$ does not return to O	A1	Sufficient correct working to justify ***given answer***
		(3)	
84dalt	Travels away until $t_1=\frac{7}{3}$, turns back at $t_2=3$ then turns away again	M1	Complete story
	$s_3=18$	B1	Seen or implied
	Complete argument	A1	
		(3)	
84dalt	Distance time graph	B1	
	Locate min turning point	M1	
	Complete argument	A1	
		(3)	
		[12]	

Q	Scheme	Marks	Notes
85a	$t=0,\ v=11\ \Rightarrow r=11$	B1	
	$t=2,\ v=3\ \Rightarrow 4p+2q+11=3,$	M1	Accept $4p+2q+r=3$
	$4p+2q=-8$	A1	Any equivalent unsimplified form with 11 used
	Differentiate to find acceleration	M1	OR use symmetry, $t=4, v=11$
	$a=2pt+q$	A1	$\Rightarrow 11=16p+4q+11\ ,\ 4p+q=0$
	$t=2,\ a=0\ \Rightarrow 4p+q=0$	DM1	2nd eqn in p & q and solve for p & q Dependent on both previous m marks
	$\Rightarrow -q+2q=-8,\ q=-8,\ p=2$	A1	
	$\left(v=2t^2-8t+11\right)$		
	$t=3,\ a=4t-8=4\,(\text{ms}^{-2})$	A1	
		(8)	
85a alt	Min speed at $t=2\ \Rightarrow$ $v=\left(pt^2+qt+r\right)=k\left(t-2\right)^2+c$	B1	
		M1	Completed square form.
	$v=k\left(t-2\right)^2+3$	A1	Correct completed square form
	$t=0,\ v=11\ \Rightarrow 4k+3=11,$	M1	Solve for k
	$k=2$	A1	$v=2\left(t-2\right)^2+3\left(=2t^2-8t+11\right)$
	Differentiate to find acceleration	DM1	Dependent on both previous m marks
	$a=4\left(t-2\right)$	A1	
	$t=3,\ a=4\,(\text{m s}^{-2})$	A1	
		(8)	
85b	Integrate: $\int 2\left(t-2\right)^2+3\text{d}t=\frac{2}{3}\left(t-2\right)^3+3t\left(+C\right)$ or $\int 2t^2-8t+11\text{d}t=\frac{2}{3}t^3-4t^2+11t\left(+C\right)$	M1	follow their coefficients found in (a) Accept in p, q, r
	At most one error seen	A1ft	For their coefficients
	All correct	A1ft	For their coefficients provided $\neq 0$
	$\left[\frac{2}{3}\left(t-2\right)^3+3t\right]_2^3=\left(\frac{2}{3}+9\right)-\left(0+6\right)$ or $\left[\frac{2}{3}t^3-4t^2+11t\right]_2^3$ $=\left(18-36+33\right)-\left(\frac{16}{3}-16+22\right)$	DM1	Use of $t=2, t=3$ as limits on a definite integral (or subtract distances to cancel C). Dependent on having integrated. Allow with p, q, r

Q	Scheme	Marks	Notes
	$3\frac{2}{3}$ (m)	A1	Accept exact equivalent or 3.7 or better
		(5)	
		[13]	

Q	Scheme	Marks	Notes
86a	Horizontal motion: $x = 3t$	B1	
	Vertical motion: $y = 4t - \frac{g}{2}t^2$	M1	Correct use of *suvat*. Condone sign error(s)
		A1	
	$\left(y = 4\times\frac{x}{3} - \frac{g}{2}\times\frac{x^2}{9}\right)$, $\lambda = -\left(\frac{4\lambda}{3} - \frac{g\lambda^2}{18}\right)$	M1	Use $y = -x$ and form an equation in one variable
	, $\frac{7\lambda}{3} = \frac{g\lambda^2}{18}$	M1	solve for λ
	$\lambda = \frac{42}{g}$ or 4.3 (4.29)	A1 (6)	Not $\frac{30}{7}$
alta	Horizontal motion: $x = 3t$	B1	
	Vertical motion: $y = 4t - \frac{g}{2}t^2$	M1	Correct use of *suvat*. Condone sign error(s)
		A1	
	$\Rightarrow -3t = 4t - \frac{1}{2}gt^2$, $\left(t = \frac{14}{g}\right)$	M1	Use $y = -x$ and form an equation in one variable
	$\lambda = 3t$	M1	Solve for λ
	$\lambda = 4.3$ (4.29)	A1 (6)	
86b	At A: $v \rightarrow 3$ (m s^{-1})	B1	
	$v\uparrow \quad 4 - g\times\frac{14}{g}$	M1	Complete method using *suvat* to find $v\uparrow$ with their t or λ
	$= -10$ (m s^{-1})	A1	Accept +10 with direction confirmed by diagram
	Speed $= \sqrt{(\text{their } 10)^2 + (3)^2}$	DM1	Dependent on the first M1 in (b)
	$= \sqrt{109}$ (m s^{-1})	A1	(10.4) Allow for $v\uparrow = 10$
	$\tan^{-1}\left(\frac{\text{their } 10}{3}\right)$ or $\tan^{-1}\left(\frac{3}{\text{their } 10}\right)$	DM1	Use trig to find a relevant angle. Dependent on the first M1 in (b)
	Direction $= 73.3°$ below the horizontal	A1	(1.28 radians) Accept direction $3\mathbf{i} - 10\mathbf{j}$ Do not accept a bearing
		(7)	
Alt 86b	Loss in GPE : $mg\lambda = 42m$	B1	
	Gain in KE : $\frac{1}{2}mv^2 - \frac{1}{2}m\times 25$	M1	Terms must be dimensionally correct. Condone sign error.
		A1	
	Solve for v: $42 = \frac{1}{2}v^2 - \frac{25}{2}$	M1	
	$v = \sqrt{109}$	A1	
	$v\cos\theta = 3$	M1	Use trig. to find a relevant angle
	$\theta = 73.3°$ below the horizontal	A1 (7)	Accept correct angle marked correctly on a diagram.
		[13]	

Question Number	Scheme	Marks	
87a	At rest when $v=0$: $\left(2t^2-9t+4\right)=0$	M1	
	$=\left(2t-1\right)\left(t-4\right)$,	DM1	Solve for t. Dependent on the previous M1
	$t=\frac{1}{2},\ 4$	A1	Incorrect answers with no method shown score M0A0
		[3]	
87b	$a=\frac{\mathrm{d}v}{\mathrm{d}t}=4t-9$	M1	Differentiate v to obtain a (at least one power of t going down)
		A1	Correct derivative
	$t=5,\ a=11\,(\mathrm{m\,s^{-2}})$	A1	
		[3]	
87c	$s=\int v\mathrm{d}t=\frac{2}{3}t^3-\frac{9}{2}t^2+4t\,(+C)$	M1	Integrate v to obtain s (at least one power of t going up)
		A1	
	Use of $t=0$, $t=\frac{1}{2}$, $t=4$, $t=5$ (and $t=0, s=15$) as limits in integrals	DM1	Correct strategy for their limits - requires subtraction of the negative distance. Dependent on the previous M1 and at least one positive solution for t in (0,5) from (a)
	$\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t\left(+15\right)\right]_0^{\frac{1}{2}}-\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t\left(+15\right)\right]_{\frac{1}{2}}^4$ $+\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t\left(+15\right)\right]_4^5$	A1	NB: $\int_0^5 v\mathrm{d}t$ scores M0A0A0
	$(0,\ \frac{23}{24},\ -\frac{40}{3},\ \frac{-55}{6})$ $=\frac{23}{24}+\frac{343}{24}+\frac{100}{24}=19.4$ (m) $\left(15,\ 15\frac{23}{24}\left(\frac{383}{24}\right),\ \frac{5}{3},\ 5.8\dot{3}\left(\frac{35}{6}\right)\right)$	A1 [5]	$19\frac{5}{12}$ $\left(\frac{233}{12}\right)$ or better
		(11)	

Question Number	Scheme	Marks	
88a	After 4 seconds from O, horizontal speed $= u\cos\theta$	B1	
	Vertical component of speed at $A = u + at$	M1	Complete method using *suvat* to find *v*.
	$= u\sin\theta - 4g$	A1	
	At *A*, components are $15\cos 20$ (horizontal) and $15\sin 20$ (vertical)	B1	
	$u\cos\theta = 15\cos 20$ $u\sin\theta = 15\sin 20 + 4g$	DM1	Form simultaneous equations in u and θ and attempt to solve for u or θ. Depends on the previous M1
	$\theta = 72.4 \quad (72)$	A1	Remember - A0 for the first overspecified answer
	$u = 46.5 \quad (47)$	A1	
		[7]	
Alt88a	After 4 seconds from O, horizontal speed $= u\cos\theta$	B1	
	At $t = 4$, $s = vt - \frac{1}{2}gt^2$	M1	Complete method to find the vertical height at *A*
	$= 98.9.....$	A1	
	At A, components are $15\cos 20$ (horizontal) and $15\sin 20$ (vertical)	B1	
	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - 2gh$	DM1	Conservation of energy. The equation needs to include all three terms but condone sign error(s).
	$u = 46.5 \quad (47)$	A1	Remember - A0 for the first overspecified answer
	$\theta = 72.4 \quad (72)$	A1	Beware inappropriate use of *suvat*
88b	$-15\sin 20 = 15\sin 20 - gt$ or $0 = 15\sin 20t - \frac{1}{2}gt^2$	M1	Complete method using *suvat* or otherwise to find the time to travel from *A* to *B*
	$t = 1.05$ (s) or 1.0 (s)	A1	
		[2]	
88c	Total time $= 4 + (1.05) + 4$	B1ft	Follow their t or $\frac{2u\sin\theta}{g}$ for their u, θ
	Range $= 46.5\times\cos 72.4\times(8+1.05)$ (or $15\cos 20\times 9.05$)	M1	Correct method to find *OC* for their *t,u* and θ
	$= 128$ (m) or 127 (m) (130)	A1	
		[3]	
		(12)	

Question Number	Scheme	Marks	Notes
89(a)	Integrate: $\boldsymbol{v} = \left(t^3 - 2t^2\right)\mathbf{i} + \left(3t^2 - 5t\right)\mathbf{j} + \mathbf{C}$	M1	At least 3 powers going up. Condone errors in constants. Must be two separate component equations if not in vector form. Could be in column vector form. Allow with no "+ **C**"
		A2	-1 each integration error. i.e. All correct A1A1 1 error A1A0, 2 or more errors A0A0 Allow with no "+ **C**"
	$t = 3$: $\mathbf{v} = 9\mathbf{i} + 12\mathbf{j} + \mathbf{C} = 11\mathbf{i} + 10\mathbf{j}$ $\quad \mathbf{C} = 2\mathbf{i} - 2\mathbf{j}$	DM1	Substitute given values to find **C**. Dependent on the previous M mark
	$\mathbf{v} = \left(t^3 - 2t^2 + 2\right)\mathbf{i} + \left(3t^2 - 5t - 2\right)\mathbf{j}$	A1 (5)	Correct velocity (any equivalent form)
(b)	Parallel to **i** $\Rightarrow 3t^2 - 5t - 2 = 0$	M1	Set **j** component of their **v** equal to zero and solve for t Correct answers imply method, but incorrect answers need to show method clearly.
	$(3t+1)(t-2) = 0, \quad t = 2$	A1	Correct only. Ignore $-\frac{1}{3}$ if present.
	$\lvert\mathbf{v}\rvert = 8 - 8 + 2 = 2\,(\text{m s}^{-1})$	DM1	Substitute their t to find **v**. Dependent on the previous M mark.
		A1 (4)	The **answer must be a scalar** – the Q asks for speed. Results from negative t must be rejected.
		[9]	
A candidate who has no "+C" can score at most M1A2M0A0 M1A0M1A0			

Question Number	Scheme	Marks	Notes
90(a)	Considering energy: $\frac{1}{2}m\times14^2=\frac{1}{2}m\times10^2+mgh$ $h=\frac{48}{g}=4.90$	M1 A2 A1 (4)	All terms required. Terms need to be of the correct form but condone sign errors. -1 each error in the unsimplified equation Accept $\frac{48}{g}$. Maximum 3 s.f. if they go in to decimals.
alt(a)	Initial $v_y=14\sin\alpha$ Final $v_y=\sqrt{100-14^2\cos^2\alpha}$ $100-196\cos^2\alpha=196\sin^2\alpha-2gh$ $h=\frac{48}{g}=4.90$	M1A2 A1 (4)	Using $v^2=u^2+2as$ on the vertical components of speed. -1 each error in the unsimplified equation Accept in exact form. Maximum 3 s.f. if they go in to decimals.
NB	Using $v^2=u^2+2as$ with 10 and 14 is M0		
NB	In part (a) they must be solving the general case, not using 0.85. However, the marks in (b) are all available if they solve the specific case in (a).		
(b)	Vertical distance: $h=14\sin\alpha t-\frac{1}{2}\times9.8t^2$ $4.9t^2-11.9t+h=0$ $t=\frac{11.9\pm\sqrt{11.9^2-4\times4.9^2}}{9.8}$ $t=1.903.....$ Horizontal distance: $x=14\cos\alpha\times t$ =14.0 (m)	M1 A2 DM1 A1 M1 A1 A1 (8)	A complete method to find an equation in t. Must involve trig condone sin/cos confusion Correct in h or their h. -1 each error Solve a 3 term quadratic for t. Needs their value for h now. 1.9 or better Method for the horizontal distance. Condone consistent sin/cos confusion Correct for their positive t Accept 14

Question Number	Scheme	Marks	Notes
Alt (b)	Vertical speed = $\sqrt{100-(14\cos\alpha)^2}$ (=6.75) $v=u+at=14\times0.85-9.8t\quad(-6.75=11.9-9.8t)$ $t=1.903.....$ Horizontal distance: $x=14\cos\alpha\times t$ =14.0 (m)	M1 A2 DM1 A1 M1 A1 A1 (8)	A complete method to find the vertical component of the speed at B. Correct insimplified. -1 each error. Use their vertical component to find t 1.9 or better Method for the horizontal distance. Correct for their positive t Accept 14
		[12]	
NB	Candidates with a false method leading to 4.9 in (a) score at most M1A1A1DM1A0M1A1A0 if they use their result in (b). This error does not affect the alt (b) approach		

Question Number	Scheme	Marks	Notes
91.(a)	$0=(25\sin\alpha)^2-2gs$	M1	A complete method using *suvat* to find *s*
	$s=400\div19.6$ (20.4)	A1	Correct expression in *s* only
	Height above ground $=10+400\div19.6=30$ or 30.4 m	A1 (3)	30 or 30.4 only
(b)	$10=-25\times\frac{4}{5}t+\frac{1}{2}\times gt^2$	M1	A complete method using *suvat* to find the total time from *A* to *B*. Condone sign slips.
		A1	Correctly substituted equation in *t*
	$4.9t^2-20t-10=0$ $\quad t=\frac{20\pm\sqrt{400+4\times4.9\times10}}{2\times4.9}$	DM1	Dependent on the preceding M1. Solve for *t*
	$t=4.531...$ s	A1	
	Horiz distance $=25\cos\alpha t\,(=15t$ m$)$	M1	
	= 68 m	A1 (6)	68 or 68.0 only
(c)	At *C* horiz speed $=15$ m s^{-1}		
	Vert speed = $\frac{15}{\tan\alpha}$	M1	Use similar triangles, or equivalent, to find vertical speed at C
	$=11.25$	A1	
	$11.25=-20+gt$	DM1	Use *suvat* to find time from *A* to *C*. Dependent on the preceding M1
	$t=\frac{20+11.25}{9.8}=3.2$ or 3.19	A1 (4)	3.2 or 3.19 only
		[13]	

Question Number	Scheme	Marks	Notes
92a	$v = 0 = 2t^2 - 14t + 20$ $= 2\ t-2\ \ t-5$ $t = 2$ or $t = 5$	M1 M1 A1 [3]	Set $v = 0$ Solve for t
	There are many different approaches to part (b). The allocation of the two M marks is M1: A method to find the time when the velocity is a minimum M1: Evaluate the speed at that time		
e.g. b	$t = 0$, $v = 20$ (m s^{-1}) $a = 4t - 14 = 0$ $t = \frac{7}{2}$, $v = 2\times\frac{3}{2}\times\frac{-3}{2} = \frac{-9}{2}$ Max speed = 20 ms^{-1}	B1 M1 M1A1 A1 [5]	 Must see ± 4.5 Clearly stated & correct conclusion. Depends on the two M marks. From correct solution only.
balt1	$t = 0$, $v = 20$ (m s^{-1}) Sketch with symmetry about their $t = 3.5$ v(their 3.5) -4.5 Max speed = 20 ms^{-1}	B1 M1 M1 A1 A1 [5]	 Evaluate v at min. Correct work Clearly stated & correct conclusion. Depends on the two M marks. From correct solution only.
b alt 2	$t = 0$, $v = 20$ (m s^{-1}) Justification of minimum or tabulate sufficient values to confirm location Evaluate v at min. Correct work Correct conclusion. Depends on the two M marks	B1 M1 M1 A1 A1 [5]	 Clearly stated & from correct solution only.

Question Number	Scheme	Marks	Notes
b alt	$t = 0$, $v = 20$ (m s^{-1}) Complete the square as far as $\left(t - \frac{7}{2}\right)^2$ $2\left(t - \frac{7}{2}\right)^2 - \frac{9}{2}$ Max speed = 20 ms^{-1}	B1 M1 M1A1 A1 [5]	 Clearly stated & correct conclusion. Depends on the two M marks. From correct solution only.
c	$\int 2t^2 - 14t + 20\ \mathrm{d}t = \frac{2}{3}t^3 - 7t^2 + 20t(+C)$ Distance $= \left[\frac{2}{3}t^3 - 7t^2 + 20t\right]_0^2 - \left[\frac{2}{3}t^3 - 7t^2 + 20t\right]_2^4$ $= 2\times\left[\frac{2}{3}t^3 - 7t^2 + 20t\right]^2 - \left[\frac{2}{3}t^3 - 7t^2 + 20t\right]_4$ $= 2\left[\frac{16}{3} - 7\times 4 + 40\right] - \left[\frac{2\times 64}{3} - 7\times 16 + 80\right] = 24$ (m)	M1 A1 M1 A1 A1 [5]	Integration. Need to see majority of powers going up All correct. Condone C missing Correct method to find the distance, for their 2 Correct unsimplified

Question Number	Scheme		Notes
93.			
(a)	$\frac{1}{2}t^2 - 3t + 4 = 0$	M1	Set $v = 0$
	$t^2 - 6t + 8 = 0$		
	$(t-2)(t-4) = 0$	DM1	Solve for v
	$t = 2$ s *or* 4 s	A1 A1	
		(4)	
(b)	$\int \frac{1}{2}t^2 - 3t + 4 \mathrm{d}t$	M1	Integration – majority of powers increasing
	$= \frac{1}{6}t^3 - \frac{3}{2}t^2 + 4t(+C)$	A1	Correct (+C not required)
	$s = \int_0^2 \frac{1}{2}t^2 - 3t + 4 \ \mathrm{d}t - \int_2^4 \frac{1}{2}t^2 - 3t + 4 \ \mathrm{d}t$	DM1	Correct strategy for finging the required distance. Follow their "2". Subtraction/swap limits/modulus signs
	$= \left[\frac{1}{6}t^3 - \frac{3}{2}t^2 + 4t\right]_0^2 - \left[\frac{1}{6}t^3 - \frac{3}{2}t^2 + 4t\right]_2^4$		
	$= \frac{8}{6} - 6 + 8 - (\frac{64}{6} - 24 + 16 - (\frac{8}{6} - 6 + 8))$	A1	Correct unsimplified
	$= \frac{10}{3} - \frac{8}{3} + \frac{10}{3}$		
	$= 4$	A1	
		(5)	
		[9]	

Question Number	Scheme		Notes
94. (a)	$(\rightarrow)\sqrt{27ag}\cos\theta.\ t = 9a$	M1	Horizontal motion. Condone trig confusion.
		A1	
	$(\uparrow)\sqrt{27ag}\sin\theta.\ t - \frac{1}{2}gt^2 = 6a$	M1	Vertical motion. Condone sign errors and trig confusion.
		A1	
	$(\uparrow)\sqrt{27ag}\sin\theta.\ \frac{9a}{\sqrt{27ag}\cos\theta} - \frac{1}{2}g\left(\frac{9a}{\sqrt{27ag}\cos\theta}\right)^2 = 6a$	DM1	Substitute for t (unsimplified). Dependent on both previous M marks
	$9a\tan\theta - \frac{1}{2}g.81a^2\frac{(1+\tan^2\theta)}{27ag} = 6a$	DM1	Express all trig terms in terms of tan. Dependent on preceding M.
	$\tan^2\theta - 6\tan\theta + 5 = 0$	A1 **(7)**	
(b)	$\tan^2\theta - 6\tan\theta + 5 = 0$		
	$(\tan\theta - 1)(\tan\theta - 5) = 0$	M1	Method to find one root of the quadratic
	$\tan\theta_2 = 1$ or $\tan\theta_1 = 5$	A1 A1 **(3)**	
(c)	$t = \frac{9a}{\sqrt{27ag}\cos\theta} = \frac{9a}{\sqrt{27ag}} \times \frac{\sqrt{26}}{1}$	M1 A1ft	Use $\tan\theta =$ their 5 to find t. Correct unsimplified. Correct $\cos\theta$ for their $\tan\theta$
	$= \sqrt{\frac{81a^2.26}{27a}} = \sqrt{\frac{78a}{g}}$ ***Answer given***	A1 **(3)**	Given answer →evidence of working is required
(d)	$\frac{1}{2}m(27ag - v^2) = mg6a$	M1 A1	Conservation of energy. Requires all 3 terms. Condone sign error Correct equation
	$v = \sqrt{15ag}$	A1 **(3)**	
Or (d)	$v^2 = \left(\sqrt{27ag}\cos\theta\right)^2 + \left(\sqrt{27ag}\sin\theta - g.\sqrt{\frac{78a}{g}}\right)^2$	M1	Horizontal and vertical components and Pythagoras. Condone trig confusion.
	$= \left(\frac{27ag}{26}\right) + \left(5\sqrt{\frac{27ag}{26}} - \sqrt{78ag}\right)^2 \left(= ag\left(\frac{27}{26} + \frac{363}{26}\right)\right)$	A1	Correctly substituted
	$v = \sqrt{15ag}$	A1 **(3)**	
		[16]	

Q	Scheme	Marks	
95(a)	$t = \frac{5}{4}$	M1 **(1)**	1.25 Integrate the velocity vector
(b)	$\mathbf{r} = \left(2t^2 - 5t\right)\mathbf{i} + 3t\mathbf{j}\left(+\mathbf{c}\right)$	A1	NB Also correct to use suvat with $\boldsymbol{a} = 4\mathbf{i}$ and $\boldsymbol{u} = -5\mathbf{i} + 3\mathbf{j}$.
		DM1	Correct
	$t = 0 \quad 2\mathbf{i} + 5\mathbf{j} = \mathbf{c}$	A1 **(4)**	Use $\mathbf{r}_0$ to find C
	$\mathbf{r} = \left(2t^2 - 5t\right)\mathbf{i} + 3t\mathbf{j} + \left(2\mathbf{i} + 5\mathbf{j}\right)$		oe
	$(2t^2 - 5t + 2)\mathbf{i} + (3t + 5)\mathbf{j}$	B1 **[5]**	

Q.	Scheme	Marks	
96 (a)	$2 = -2u\sin\theta + \frac{1}{2}g\times 4$	M1	Vertical distance. Condone sign errors. Must have used $t = 2$, but could be using $u_y = u\sin\theta$
	$\left(-2 = u\sin\theta t - \frac{1}{2}gt^2\right)$	A1	All correct
	$u\sin\theta = g - 1$		
	$2u\cos\theta = 8 \quad (u\cos\theta = 4)$ $(u\cos\theta t = 8)$	B1	Horizontal distance. Accept $u_x = 4$ o.e.
	$\tan\theta = \frac{g-1}{4} = 2.2$ *	M1	Divide to obtain expression for $\tan\theta$
		A1	**Given answer** It is acceptable to quote and use the equation for the projectile path. Incorrect equation is 0/5.
(b)	$u\cos\theta = 4$	M1	Use the horizontal distance and θ to find u 9.67 or 9.7
	$u = \frac{4}{\cos\theta} = 9.66... = 9.7$	A1	NB $\theta = 65.6°$ leading to 9.68 is an accuracy penalty.
	OR use components from (a) and Pythagoras.		
(c)	$6 = (1-g)T + \frac{1}{2}\times 9.8T^2$	M1	Equation for vertical distance $= \pm 6$ to give a quadratic in T. Allow their u_y
	$4.9T^2 - 8.8T - 6 = 0$		
	$T = \frac{8.8 \pm \sqrt{\left[(-)8.8\right]^2 + 24\times 4.9}}{9.8}$	DM1	Solve a 3 term quadratic
	$T = 2.323... = 2.32$ or 2.3	A1	2.3 or 2.32 only
(d)	$v^2 = 8.8^2 + 2g\times 6$ or v = -8.8+gT	M1	Use *suvat* to find vertical speed
		A1	Correct equation their u_y, T
	$v = 13.96...$		
	Horiz speed = 4		
	$\tan\alpha = \frac{v}{4}$	DM1	Correct trig. with their vertical speed to find the required angle.
		A1	Correct equation
	$\alpha = 74.01... = 74°$	A1	$74°$ or $74.0°$. Allow 106.
	Alternative:		
	$\frac{1}{2}m(9.6664)^2 + 6mg = \frac{1}{2}mv^2$	M1	Conservation of energy to find speed
	$v = 14.52719...$	A1	
	$\cos\alpha = \frac{4}{14.5}$	DM1	Correct method for α
		A1	
	$\alpha = 74.01... = 74°$	A1	Allow 106

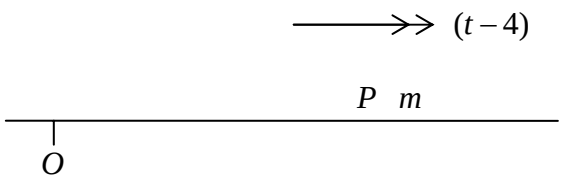
Question Number	Scheme	Marks	Notes
97			
(a)	$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 6t\mathbf{i} + (4-2t)\mathbf{j}$	M1	Differentiate **v** to obtain **a**.
		A1	Accept column vector or **i** and **j** components dealt with separately.
	When $t = 1$, $\mathbf{a} = 6\mathbf{i} + 2\mathbf{j}$	DM1	Substitute $t = 1$ into their **a.** Dependent on 1st M1
	$\lvert\mathbf{a}\rvert = \sqrt{6^2 + 2^2} = \sqrt{40} = 6.32$ (m s^{-2})	DM1	Use of Pythagoras to find the magnitude of their **a**. Allow with their t. Dependent on 1st M1
		A1	Accept awrt 6.32, 6.3 or exact equivalents.
		(5)	
(b)	$\mathbf{r} = \int (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j}\, dt$	M1	Integrate **v** to obtain **r**
	$= (t^3 - t + C)\mathbf{i} + (2t^2 - \frac{1}{3}t^3 + D)\mathbf{j}$	A1	Condone C, D missing
	$t = 0, \mathbf{r} = \mathbf{i} \Rightarrow C = 1, D = 0$	DM1	Use $t = 0$, **r** = **i** to find C & D
	When $t = 3, \mathbf{r} = 25\mathbf{i} + 9\mathbf{j}$ (m)	DM1	Substitute $t = 3$ with their C & D to find **r**. Dependent on both previous Ms.
		A1	cao. Must be a vector.
		(5)	
		10	

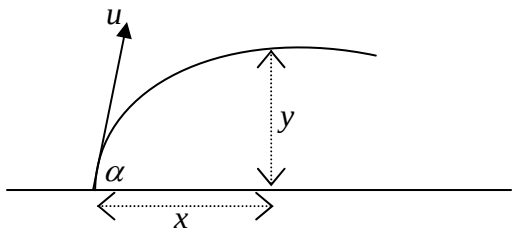
Question Number	Scheme	Marks	
98 (a)	$0^2 = u_V{}^2 - 2 \text{ x } 9.8 \text{ x } 10$	M1	Complete method using *suvat* to form an equation in u_v.
		A1	Correct equation e.g. $0 = u^2 - 20g$
	$u_V = 14$ *	A1	*Answer given* requires equation and working, including 196, seen.
		(3)	
OR	conservation of energy:	M1	Initial KE = gain in GPE + final KE
	$\frac{1}{2}m\left(u_h{}^2 + u_v{}^2\right) = mg \times 10 + \frac{1}{2}mu_h{}^2, \frac{1}{2}u_v{}^2 = 98$	A1	Correct equation
	$u_V = 14$ *	A1	*Answer given*
		(3)	
(b)		M1	Use the vertical distance travelled to find the total time taken.
	$(\uparrow),\ -52.5 = 14t - \frac{1}{2}gt^2$	A1	At most one error
		A1	Correct equation
	$49t^2 - 140t - 525 = 0$	DM1	Solve for *t*. Dependent on the preceding M mark
	$(t-5)(49t+105) = 0 \quad t = 5$	A1	only
	$(\rightarrow),\ 50 = 5u_H$	M1	Use their time of flight to form an equation in u_H
	$u_H = 10$	A1	only
	$u = \sqrt{10^2 + 14^2}$	M1	Use of Pythagoras with two non-zero components, or solution of a pair of simultaneous equations in *u* and *α*.
	$= \sqrt{296}$; 17.2 m s^{-1}	A1	17.2 or 17 (method involves use of g = 9.8 so an exact surd answer is not acceptable)
		(9)	
			See next page for an alternative route to *u*, and (c).

OR			First 3 marks for the quadratic as above.
	$50 = u\cos\alpha t$ or $50 = u_H t$	M1	Used in their quadratic
	$49\left(\frac{50}{u_H}\right)^2 - 140\left(\frac{50}{u_H}\right) - 525 = 0$ $525(u_H)^2 + 140(u_H) - 122500 = 0$	A1	Correct quadratic in u_H
	Solve for u_H	DM1	Dependent on the M mark for setting up the initial quadratic equation in t.
	u_H = 10 etc.	A1	only Complete as above.
(c)	$\tan OBA = \frac{52.5}{50} = 1.05$	B1	Correct direction o.e. (accept reciprocal)
	$v_V = 1.05 \text{ x } 10 = 10.5$	M1	Use trig. with their u_H and correct interpretation of direction to find the vertical component of speed. Working with distances is M0. (condone $10 \div 1.05$)
	$(\uparrow),\ -10.5 = 14 - gt$	DM1 A1	Use suvat to form an equation in t. Dependent on the preceding M. Correct equation for their u_H. For incorrect direction give A0 here.
	$t = 2.5$	A1	only
		(5)	
		17	

Question Number	Scheme	Marks
99		
(a)	Speed = $\sqrt{8^2 + 48^2} = \sqrt{2368} = 48.7\,(\text{ms}^{-1})$	M1 A1 (2)
(b)	$\mathbf{a} = 2\mathbf{i} - 6t\,\mathbf{j}$ When $t = 4$, $\mathbf{a} = 2\mathbf{i} - 24\,\mathbf{j}\ (\text{ms}^{-2})$	M1 A1 A1 (3)
(c)	$\mathbf{r} = t^2\,\mathbf{i} - t^3\,\mathbf{j} + \mathbf{C}$ t = 1, $-4\mathbf{i} + \mathbf{j} = \mathbf{i} - \mathbf{j} + \mathbf{C}$, $\mathbf{C} = -5\mathbf{i} + 2\mathbf{j}$ $\mathbf{r} = (t^2 - 5)\mathbf{i} + \left(-t^3 + 2\right)\mathbf{j}$ When $t = 4$, $\mathbf{r} = (16-5)\mathbf{i} + (-64 + 2)\mathbf{j} = 11\mathbf{i} - 62\mathbf{j}$	M1 A1 DM1 DM1 A1 (5) **10**

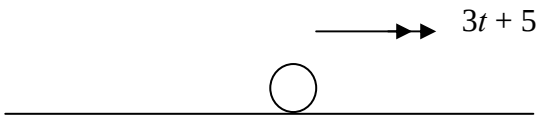
Question Number	Scheme	Marks
100		
(a)	**i** → distance = $6t$	B1
	j ↑ distance = $12t - \frac{1}{2}gt^2$	M1 A1
	At B, $2\left(12t - \frac{1}{2}gt^2\right) = 6t$	M1 A1
	$(24-6)t = gt^2$	DM1
	$18 = gt$, $t = \frac{18}{g} (= 1.84\text{s})$	A1
		(7)
(b)	**i** → speed = 6	B1
	j ↑ velocity = $12 - gt = -6$	M1 A1
	$\therefore$ speed at A	
	$= \sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2} (= 8.49) (\text{ms}^{-1})$	M1 A1
		(5)
(c)	↑ speed = $12 - gt = +6$	M1 A1 ft
	$t = \frac{6}{g} (= 0.61\text{s})$	A1
		(3)
		15

Question Number	Scheme	Marks
101. **(a)**	 $\frac{dv}{dt} = t - 4$ $v = \frac{1}{2}t^2 - 4t(+c)$ $t = 0 \;\; v = 6 \quad \Rightarrow c = 6$ $\therefore v = \frac{1}{2}t^2 - 4t + 6$	 M1 A1 M1 A1 (4)
(b)	$v = 0 \quad 0 = t^2 - 8t + 12$ $(t-6)(t-2) = 0$ $t = 6 \quad t = 2$	M1 DM1 A1 (3)
(c)	$x = \frac{t^3}{6} - 2t^2 + 6t + k$ $x_6 - x_2 = \frac{6^3}{6} - 2\times 6^2 + 6^2 + k$ $-\left(\frac{2^3}{6} - 2\times 2^2 + 6\times 2 + k\right)$ $= -5\frac{1}{3}$ $\therefore$ Distance is $5\frac{1}{3}$m	M1 A1 ft DM1 A1 (4) **11**

Question Number	Scheme	Marks
102. **(a)**		
	Horiz: $x = u\cos\alpha t$	B1
	Vert: $y = u\sin\alpha t - \frac{1}{2}gt^2$	M1
	$y = u\sin\alpha \times \frac{x}{u\cos\alpha} - \frac{1}{2}g \times \frac{x^2}{u^2\cos^2\alpha}$	DM1
	$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$ **	A1
		(4)
(b)	$y = -7$: $-7 = \tan 45x - \frac{gx^2}{2\times 7^2\cos^2 45}$	M1 A1
	$-7 = x - \frac{9.8x^2}{7^2}$	
	$-7 = x - \frac{x^2}{5}$	M1
	$x^2 - 5x - 35 = 0$	
	$x = \frac{5 \pm \sqrt{25 + 4\times 35}}{2}$	M1
	$x = 8.92$ or 8.9	A1
		(5)
(c)	Time to travel 8.922 m horizontally $= \frac{8.922}{7\cos 45} = 1.802...$s	M1
	$v = \frac{8.922}{1.402}$	M1 A1 ft
	$= 6.36$ or 6.4 $\left(\text{m s}^{-1}\right)$	A1
		(4)
		13

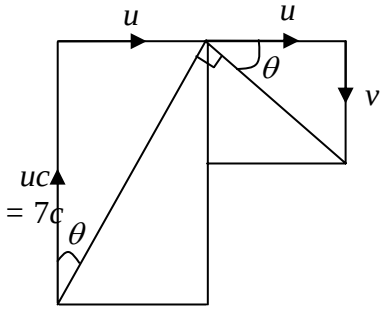
Question Number	Scheme	Marks
103. **(a)**	$a = 4t^3 - 12t$ Convincing attempt to integrate $v = t^4 - 6t^2\,(+c)$ Use initial condition to get $v = t^4 - 6t^2 + 8\left(\text{ms}^{-1}\right)$.	M1 A1 A1 (3)
(b)	Convincing attempt to integrate $s = \frac{t^5}{5} - 2t^3 + 8t\,(+0)$ Integral of their v	M1 A1ft (2)
(c)	Set their $v = 0$ Solve a quadratic in t^2 $(t^2-2)(t^2-4) = 0 \Rightarrow$ at rest when $t = \sqrt{2},\ t = 2$	M1 DM1 A1 (3) **[8]**

Question Number	Scheme	Marks
104. **(a)**	Using $s = ut + \frac{1}{2}at^2$ Method must be clear $\mathbf{r} = (3t)\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}$ Answer given	M1 A1 A1 (3)
(b)	**j** component $= 0$: $10 + 5t - 4.9t^2$ quadratic formula: $t = \frac{5 \pm \sqrt{25 + 196}}{9.8} = \frac{5 \pm \sqrt{221}}{9.8}$ $T = 2.03$(s), 2.0 (s) positive solution only.	M1 DM1 A1 (3)
(c)	Differentiating the position vector (or working from first principles) $\mathbf{v} = 3\mathbf{i} + (5 - 9.8t)\mathbf{j}$ (ms^{-1})	M1 A1 (2)
(d)	At B the **j** component of the velocity is the negative of the **i** component: $5 - 9.8t = -3$, $8 = 9.8t$, $t = 0.82$	M1 A1 (2)
(e)	$\mathbf{v} = 3\mathbf{i} - 3\mathbf{j}$, speed $= \sqrt{3^2 + 3^2} = \sqrt{18} = 4.24$ (m s^{-1})	M1A1 (2) **[12]**

Question Number	Scheme	Marks
105		
	$\frac{\mathrm{d}v}{\mathrm{d}t} = 3t + 5$	
	$v = \int (3t+5)\,\mathrm{d}t$	M1*
	$v = \frac{3}{2}t^2 + 5t \quad (+c)$	A1
	$t = 0 \;\; v = 2 \;\; \Rightarrow \;\; c = 2$	B1
	$v = \frac{3}{2}t^2 + 5t + 2$	
	$t = T \qquad 6 = \frac{3}{2}T^2 + 5T + 2$	DM1*
	$12 = 3T^2 + 10T + 4$	
	$3T^2 + 10T - 8 = 0$	
	$(3T-2)(T+4) = 0$	M1
	$T = \frac{2}{3} \quad (T = -4)$	
	$\therefore T = \frac{2}{3}$ (or 0.67)	A1
		[6]

Question Number	Scheme	Marks
106 (a)	Vertical motion: $v^2 = u^2 + 2as$ $(40\sin\theta)^2 = 2 \times g \times 12$ $(\sin\theta)^2 = \frac{2 \times g \times 12}{40^2}$ $\theta = 22.54 = 22.5°$ (accept 23)	M1 A1 A1 (3)
(b)	Vert motion $P \to R$: $s = ut + \frac{1}{2}at^2$ $-36 = 40\sin\theta t - \frac{g}{2}t^2$ $\frac{g}{2}t^2 - 40\sin\theta t - 36 = 0$ $t = \frac{40\sin 22.54 \pm \sqrt{(40\sin 22.54)^2 + 4 \times 4.9 \times 36}}{9.8}$ $t = 4.694...$ Horizontal P to R: $s = 40\cos\theta t$ $= 173$ m (or 170 m)	M1 A1 A1 A1 M1 A1 (6) **[9]**

Question Number	Scheme	Marks
107.	$\frac{\mathrm{d}v}{\mathrm{d}t} = 6t - 4$	M1 A1
	$6t - 4 = 0 \Rightarrow t = \frac{2}{3}$	M1 A1
	$s = \int 3t^2 - 4t + 3 \,\mathrm{d}t = t^3 - 2t^2 + 3t \ (+c)$	M1 A1
	$t = \frac{2}{3} \Rightarrow s = -\frac{16}{27} + 2$ so distance is $\frac{38}{27}$ m	M1 A1
		[8]

Question Number	Scheme	Marks
108.	(a) $x = ut$	B1
	$y = cut - 4.9t^2$	M1 A1
	eliminating t and simplifying to give $y = cx - \frac{4.9x^2}{u^2}$ **	DM1 A1 (5)
	(b)(i) $0 = cx - \frac{4.9x^2}{u^2}$	M1
	$0 = x(c - \frac{4.9x}{u^2}) \Rightarrow R = \frac{u^2 c}{4.9} = 10c$	M1 A1
	(ii) When $x = 5c$, $y = H$	M1
	$= 5c^2 - \frac{(5c)^2}{10} = 2.5c^2$	M1 A1 (6)
	(c) $\frac{dy}{dx} = c - \frac{9.8x}{u^2} = c - \frac{x}{5}$	M1 A1
	When $x = 0$, $\frac{dy}{dx} = c$	B1
	So, $c - \frac{x}{5} = \frac{-1}{c}$	DM1 A1
	$x = 5(c + \frac{1}{c})$	A1 (6)
		[17]
	Alternative to 108(c)  $\tan\theta = \frac{u}{cu} = \frac{1}{c} = \frac{v}{u}$	B1
	$\Rightarrow v = \frac{u}{c} = \frac{7}{c}$	M1 A1
	$v = u + at$; $-\frac{7}{c} = 7c - 9.8t$	M1
	$t = \frac{7}{9.8}(c + \frac{1}{c})$	A1
	$x = ut = 7t$; $x = 5(c + \frac{1}{c})$	A1

Question Number	Scheme	Marks
109 (a)	$\frac{dv}{dt} = 8 - 2t$ $8 - 2t = 0$ Max $v = 8 \times 4 - 4^2 = 16\,(\text{ms}^{-1})$	M1 M1 M1A1 (4)
(b)	$\int 8t - t^2 dt = 4t^2 - \frac{1}{3}t^3 (+c)$ (t=0, displacement = 0 $\Rightarrow$ c=0) $4T^2 - \frac{1}{3}T^3 = 0$ $T^2(4 - \frac{T}{3}) = 0 \Rightarrow T = 0, 12$ $T = 12$ (seconds)	M1A1 DM1 DM1 A1 (5) **[9]**

Question Number	Scheme	Marks
110 (a)	$v = 10t - 2t^2$, $s = \int v dt$	M1
	$= 5t^2 - \frac{2t^3}{3} (+C)$	A1
	$t = 6 \Rightarrow s = 180 - 144 = \underline{36}$ (m)	A1
		(3)
(b)	$\underline{s} = \int v\,dt = \frac{-432t^{-1}}{-1} (+K) = \frac{432}{t} (+K)$	B1
	$t = 6$, $s =$ "36" $\Rightarrow 36 = \frac{432}{6} + K$	M1*
	$\Rightarrow K = -36$	A1
	At $t = 10$, $s = \frac{432}{10} - 36 = \underline{7.2}$ (m)	d*M1
		A1
		(5)
		[8]

Question Number	Scheme	Marks
111 (a)	Horizontal distance: $57.6 = p \times 3$ $p = 19.2$	M1 A1 (2)
(b)	Use $s = ut + \frac{1}{2}at^2$ for vertical displacement.	M1
	$-0.9 = q \times 3 - \frac{1}{2}g \times 3^2$	A1
	$-0.9 = 3q - \frac{9g}{2} = 3q - 44.1$	
	$q = \frac{43.2}{3} = 14.4$ ***AG***	A1 cso
		(3)
(c)	initial speed $\sqrt{p^2 + 14.4^2}$ (with their p)	M1
	$= \sqrt{576} = \underline{24}$ (m s^{-1})	A1 cao
		(2)
(d)	$\tan\alpha = \frac{14.4}{p} (= \frac{3}{4})$ (with their p)	B1
		(1)
(e)	When the ball is 4 m above ground:	
	$3.1 = ut + \frac{1}{2}at^2$ used	M1
	$3.1 = 14.4t - \frac{1}{2}gt^2$ o.e $(4.9t^2 - 14.4t + 3.1 = 0)$	A1
	$\Rightarrow t = \frac{14.4 \pm \sqrt{(14.4)^2 - 4(4.9)(3.1)}}{2(4.9)}$ seen or implied	M1
	$t = \frac{14.4 \pm \sqrt{146.6}}{9.8} = 0.023389\ldots$ or $2.70488\ldots$ awrt 0.23 and 2.7	A1
	duration $= 2.70488... - 0.23389...$	M1
	$= 2.47$ or 2.5 (seconds)	A1
		(6)
or 111(e)	M1A1M1 as above	
	$t = \frac{14.4 \pm \sqrt{146.6}}{9.8}$	A1
	Duration $2 \times \frac{\sqrt{146.6}}{9.8}$ o.e.	M1
	$= 2.47$ or 2.5 (seconds)	A1
		(6)
(f)	Eg. : Variable 'g', Air resistance, Speed of wind, Swing of ball, The ball is not a particle.	B1
		(1) **[15]**

Question Number	Scheme	Marks
112.	N2L $(6t-5)\mathbf{i}+(t^2-2t)\mathbf{j}=0.5\mathbf{a}$	M1
	$\mathbf{a}=(12t-10)\mathbf{i}+(2t^2-4t)\mathbf{j}$	A1
	$\mathbf{v}=(6t^2-10t)\mathbf{i}+\left(\frac{2}{3}t^3-2t^2\right)\mathbf{j}\quad(+\mathbf{C})$ ft their **a**	M1 A1ft+A1ft
	$\mathbf{v}=(6t^2-10t+1)\mathbf{i}+\left(\frac{2}{3}t^3-2t^2-4\right)\mathbf{j}$	A1 **(6)**

Question Number	Scheme	Marks
113.	(a) $(\downarrow)$ $u_y = 25\sin 30° \ (=12.5)$	B1
	$12 = 12.5t + 4.9t^2$ -1 each error	M1 A2 (1, 0)
	Leading to $t = 0.743$, 0.74	A1 **(5)**
	(b) $(\rightarrow)$ $u_x = 25\cos 30° \left(= \frac{25\sqrt{3}}{2} \approx 21.65\right)$	B1
	$OB = 25\cos 30° \times t \ (\approx 16.094\,58)$ ft their (a)	M1 A1ft
	$TB \approx 1.1$ (m) awrt 1.09	A1 **(4)**
	(c) $(\rightarrow)$ $15 = u_x \times t \Rightarrow t = \frac{15}{u_x} (= \frac{2\sqrt{3}}{5} \approx 0.693$ or $0.69)$	M1 A1
	either $(\downarrow)$ $v_y = 12.5 + 9.8t \ (\approx 19.2896)$	M1
	$V^2 = u_x{}^2 + v_y{}^2 \ (\approx 840.840)$	
	$V \approx 29 \ (\text{ms}^{-1})$, 29.0	M1 A1 **(5)**
		[14]
	or $(\downarrow)$ $s_y = 12.5t + 4.9t^2 \ (\approx 11.0)$	M1
	$\frac{1}{2}m \times 25^2 + mg \times s_y = \frac{1}{2}mv^2$	
	$V \approx 29 \ (\text{ms}^{-1})$, 29.0	M1A1

Question Number	Scheme	Marks
114.	(a) $\dot{\mathbf{p}} = (6t-6)\mathbf{i} + (9t^2-4)\mathbf{j} \quad (\text{m s}^{-1})$	M1 A1 **(2)**
	(b) $9t^2 - 4 = 0$ $t = \frac{2}{3}$	M1 DM1 A1 **(3)** **[5]**

Question Number	Scheme	Marks
115.	(a) $\rightarrow \quad 30 = 2ut$ $\uparrow \quad -47.5 = 5ut - 4.9t^2$ $-47.5 = 75 - 4.9t^2$ eliminating u or t $t^2 = \dfrac{75 + 47.5}{4.9} \left(= 25\right)$ $t = 5$ ✱ cso	B1 M1 A1 DM1 DM1 A1 **(6)**
	(b) $30 = 2ut \Rightarrow 30 = 10u \Rightarrow u = 3$	M1 A1 **(2)**
	(c) $\uparrow \quad \dot{y} = 5u - 9.8t = -34$ M1 requires both $\rightarrow \quad \dot{x} = 2u = 6$ $\dot{x}$ and $\dot{y}$ $v^2 = 6^2 + \left(-34\right)^2$ $v \approx 34.5 \ \left(\text{m s}^{-1}\right)$ accept 35	M1 A1 A1 DM1 A1 **(5)**
		[13]
	Alternative to (c) $\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = m \times g \times 47.5$ with $v_A^2 = 6^2 + 15^2 = 261$ $v_B^2 = 261 + 2 \times 9.8 \times 47.5 \ \left(= 1192\right)$ $v_B \approx 34.5 \ \left(\text{m s}^{-1}\right)$ accept 35 BEWARE : Watch out for incorrect use of $v^2 = u^2 + 2as$	M1 A(2,1,0) DM1 A1 **(5)**