EXPERT TUITION

Maths Questions By Topic:

Kinematics

A-Level Edexcel

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1. The point A is 1.8 m vertically above horizontal ground.

At time $t = 0$, a small stone is projected vertically upwards with speed U m s^{-1} from the point A.

At time $t = T$ seconds, the stone hits the ground.

The speed of the stone as it hits the ground is 10 m s^{-1}

In an initial model of the motion of the stone as it moves from A to where it hits the ground

- the stone is modelled as a particle moving freely under gravity

- **the acceleration due to gravity is modelled as having magnitude 10 m s^{-2}**

Using the model,

(a) find the value of U,

(3)

(b) find the value of T.

(2)

(c) Suggest one refinement, apart from including air resistance, that would make the model more realistic.

(1)

In reality the stone will not move freely under gravity and will be subject to air resistance.

(d) Explain how this would affect your answer to part (a).

(1)

Question 1 continued

Question 1 continued

(Total for Question 1 is 7 marks)

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2. A train travels along a straight horizontal track from station P to station Q.

In a model of the motion of the train, at time $t = 0$ the train starts from rest at P, and moves with constant acceleration until it reaches its maximum speed of $25\,\text{m s}^{-1}$

The train then travels at this constant speed of $25\,\text{m s}^{-1}$ before finally moving with constant deceleration until it comes to rest at Q.

The time spent decelerating is four times the time spent accelerating.

The journey from P to Q takes $700\,\text{s}$.

Using the model,

(a) sketch a speed-time graph for the motion of the train between the two stations P and Q.

(1)

The distance between the two stations is $15\,\text{km}$.

Using the model,

(b) show that the time spent accelerating by the train is $40\,\text{s}$,

(3)

(c) find the acceleration, in m s^{-2}, of the train,

(1)

(d) find the speed of the train $572\,\text{s}$ after leaving P.

(2)

(e) State one limitation of the model which could affect your answers to parts (b) and (c).

(1)

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Question 2 continued

Question 2 continued

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(Total for Question 2 is 8 marks)

3. A fixed point O lies on a straight line.

A particle P moves along the straight line.

At time t seconds, $t \geqslant 0$, the distance, s metres, of P from O is given by

$$s = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t$$

(a) Find the acceleration of P at each of the times when P is at instantaneous rest.

(6)

(b) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(3)

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Question 3 continued

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 9 marks)

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4. At time $t = 0$, a small stone is thrown vertically upwards with speed $14.7\,\text{m s}^{-1}$ from a point A.

At time $t = T$ seconds, the stone passes through A, moving downwards.

The stone is modelled as a particle moving freely under gravity throughout its motion.

Using the model,

(a) find the value of T,

(2)

(b) find the total distance travelled by the stone in the first 4 seconds of its motion.

(4)

(c) State one refinement that could be made to the model, apart from air resistance, that would make the model more realistic.

(1)

Question 4 continued

(Total for Question 4 is 7 marks)

5. A particle P moves along a straight line.

At time t seconds, the velocity $v\,\mathrm{m\,s}^{-1}$ of P is modelled as

$$v = 10t - t^2 - k \qquad t \geqslant 0$$

where k is a constant.

(a) Find the acceleration of P at time t seconds.

(2)

The particle P is instantaneously at rest when $t = 6$

(b) Find the other value of t when P is instantaneously at rest.

(4)

(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 6$

(4)

Question 5 continued

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 10 marks)

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6. At time $t = 0$, a small ball is projected vertically upwards with speed $U \, \text{m s}^{-1}$ from a point A that is 16.8 m above horizontal ground.

The speed of the ball at the instant immediately before it hits the ground for the first time is $19 \, \text{m s}^{-1}$

The ball hits the ground for the first time at time $t = T$ seconds.

The motion of the ball, from the instant it is projected until the instant just before it hits the ground for the first time, is modelled as that of a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude $10 \, \text{m s}^{-2}$

Using the model,

(a) show that $U = 5$

(2)

(b) find the value of T,

(2)

(c) find the time from the instant the ball is projected until the instant when the ball is 1.2 m below A.

(4)

(d) Sketch a velocity-time graph for the motion of the ball for $0 \leqslant t \leqslant T$, stating the coordinates of the start point and the end point of your graph.

(2)

In a refinement of the model of the motion of the ball, the effect of air resistance on the ball is included and this refined model is now used to find the value of U.

(e) State, with a reason, how this new value of U would compare with the value found in part (a), using the initial unrefined model.

(1)

(f) Suggest one further refinement that could be made to the model, apart from including air resistance, that would make the model more realistic.

(1)

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Question 6 continued

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Question 6 continued

Question 6 continued

(Total for Question 6 is 12 marks)

7. At time $t = 0$, a parachutist falls vertically from rest from a helicopter which is hovering at a height of 550 m above horizontal ground.

The parachutist, who is modelled as a particle, falls for 3 seconds before her parachute opens.

While she is falling, and before her parachute opens, she is modelled as falling freely under gravity.

The acceleration due to gravity is modelled as being $10\,\text{m}\,\text{s}^{-2}$.

(a) Using this model, find the speed of the parachutist at the instant her parachute opens.

(1)

When her parachute is open, the parachutist continues to fall vertically.

Immediately after her parachute opens, she decelerates at $12\,\text{m}\,\text{s}^{-2}$ for 2 seconds before reaching a constant speed and she reaches the ground with this speed.

The total time taken by the parachutist to fall the 550 m from the helicopter to the ground is T seconds.

(b) Sketch a speed-time graph for the motion of the parachutist for $0 \leqslant t \leqslant T$.

(2)

(c) Find, to the nearest whole number, the value of T.

(5)

In a refinement of the model of the motion of the parachutist, the effect of air resistance is included before her parachute opens and this refined model is now used to find a new value of T.

(d) How would this new value of T compare with the value found, using the initial model, in part (c)?

(1)

(e) Suggest one further refinement to the model, apart from air resistance, to make the model more realistic.

(1)

Question 7 continued

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Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

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8. A particle, P, moves along a straight line such that at time t seconds, $t \geqslant 0$, the velocity of P, $v\,\mathrm{m\,s^{-1}}$, is modelled as

$$v = 12 + 4t - t^2$$

Find

(a) the magnitude of the acceleration of P when P is at instantaneous rest,

(5)

(b) the distance travelled by P in the interval $0 \leqslant t \leqslant 3$

(3)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 8 marks)

Unless otherwise indicated, wherever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

9. A man throws a tennis ball into the air so that, at the instant when the ball leaves his hand, the ball is 2 m above the ground and is moving vertically upwards with speed $9\,\text{m s}^{-1}$

   The motion of the ball is modelled as that of a particle moving freely under gravity and the acceleration due to gravity is modelled as being of constant magnitude $10\,\text{m s}^{-2}$

   The ball hits the ground T seconds after leaving the man's hand.

   Using the model, find the value of T. (4)

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Question 9 continued

(Total for Question 9 is 4 marks)

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10. A train travels along a straight horizontal track between two stations, *A* and *B*.

In a model of the motion, the train starts from rest at *A* and moves with constant acceleration $0.3 \, \text{m s}^{-2}$ for $80 \, \text{s}$.
The train then moves at constant velocity before it moves with a constant deceleration of $0.5 \, \text{m s}^{-2}$, coming to rest at *B*.

(a) For this model of the motion of the train between *A* and *B*,

(i) state the value of the constant velocity of the train,

(ii) state the time for which the train is decelerating,

(iii) sketch a velocity-time graph.

(3)

The total distance between the two stations is $4800 \, \text{m}$.

(b) Using the model, find the total time taken by the train to travel from *A* to *B*.

(3)

(c) Suggest one improvement that could be made to the model of the motion of the train from *A* to *B* in order to make the model more realistic.

(1)

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Question 10 continued

Question 10 continued

Question 10 continued

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11. A particle, P, moves along the x-axis. At time t seconds, $t \geqslant 0$, the displacement, x metres, of P from the origin O, is given by $x = \dfrac{1}{2}t^2(t^2 - 2t + 1)$

(a) Find the times when P is instantaneously at rest.

(5)

(b) Find the total distance travelled by P in the time interval $0 \leqslant t \leqslant 2$

(3)

(c) Show that P will never move along the negative x-axis.

(2)

EXPERT TUITION

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 10 marks)

12. A small ball is projected vertically upwards from a point *A* which is 19.6 m above the ground. The ball strikes the ground, for the first time, 4 s later.

The motion of the ball is modelled as that of a particle moving freely under gravity.

(a) Use the model to find the speed of the ball as it hits the ground for the first time.

(3)

The ball rebounds from the ground with a vertical speed of 14.7 m s⁻¹ and next comes to instantaneous rest at the point *B*.

(b) Use the model to find the height of *B* above the ground.

(2)

In a refined model of the motion of the ball, the effect of air resistance is included and this refined model is now used to find the speed of the ball as it hits the ground for the first time

(c) How would this new value of the speed of the ball as it hits the ground for the first time compare with the value found using the initial model in part (a)?

(1)

(Total 6 marks)

(Total for Question 12 is 6 marks)

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13. A car travels along a straight horizontal road between two sets of traffic lights. The distance between the two sets of traffic lights is 1500 m.

In a model of the journey, the car leaves the first set of traffic lights, accelerating uniformly from rest until it reaches a speed of V m s^{-1}, then immediately decelerates uniformly until it comes to rest at the second set of traffic lights. The car completes the journey between the two sets of lights in 120 s.

(a) Sketch a velocity-time graph which represents the above model of the journey of the car between the two sets of traffic lights.

(2)

(b) Using the model, find the value of V.

(2)

It is given that the car accelerates uniformly for T seconds.

(c) Explain why there is a range of possible values for T which satisfy the requirements of the model.

(2)

(d) Suggest one improvement to the model that would make it more realistic.

(1)

(Total 7 marks)

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(Total for Question 13 is 7 marks)

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14. A particle P moves along a straight line such that at time t seconds, $t \geq 0$, its velocity, v m s^{-1}, is given by

$$v = 16 - 3t^2$$

Find

(a) the distance travelled by P in the first second,

(3)

(b) the value of t at the instant when P changes its direction of motion,

(2)

(c) the value of t at the instant when P returns to its starting point.

(3)

(Total 8 marks)

Question 14 continued

(Total for Question 14 is 8 marks)

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

15.
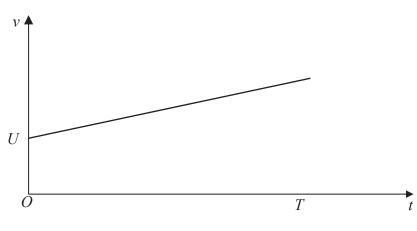
Figure 1

A car moves along a straight horizontal road. At time $t = 0$, the velocity of the car is U m s^{-1}. The car then accelerates with constant acceleration a m s^{-2} for T seconds. The car travels a distance D metres during these T seconds.

Figure 1 shows the velocity-time graph for the motion of the car for $0 \leqslant t \leqslant T$.

Using the graph, show that $D = UT + \frac{1}{2} aT^2$.

(4)

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Question 15 continued

(Total for Question 15 is 4 marks)

16. A car is moving along a straight horizontal road with constant acceleration. There are three points A, B and C, in that order, on the road, where $AB = 22$ m and $BC = 104$ m. The car takes 2 s to travel from A to B and 4 s to travel from B to C.

Find

(i) the acceleration of the car,

(ii) the speed of the car at the instant it passes A.

(7)

Question 16 continued

(Total for Question 16 is 7 marks)

17. A bird leaves its nest at time $t = 0$ for a short flight along a straight line.

The bird then returns to its nest.

The bird is modelled as a particle moving in a straight horizontal line.

The distance, s metres, of the bird from its nest at time t seconds is given by

$$s = \frac{1}{10}(t^4 - 20t^3 + 100t^2), \quad \text{where } 0 \leqslant t \leqslant 10$$

(a) Explain the restriction, $0 \leqslant t \leqslant 10$

(3)

(b) Find the distance of the bird from the nest when the bird first comes to instantaneous rest.

(6)

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Question 17 continued

(Total for Question 17 is 9 marks)

18. *[In this question, position vectors are given relative to a fixed origin.]*

At time t seconds, where $t > 0$, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s}^{-1}$ where

$$\mathbf{v} = 3t^2\mathbf{i} - 6t^{\frac{1}{2}}\mathbf{j}$$

(a) Find the speed of P at time $t = 2$ seconds.

(2)

(b) Find an expression, in terms of t, $\mathbf{i}$ and $\mathbf{j}$, for the acceleration of P at time t seconds, where $t > 0$

(2)

At time $t = 4$ seconds, the position vector of P is $(\mathbf{i} - 4\mathbf{j})\,\mathrm{m}$.

(c) Find the position vector of P at time $t = 1$ second.

(4)

Question 18 continued

(Total for Question 18 is 8 marks)
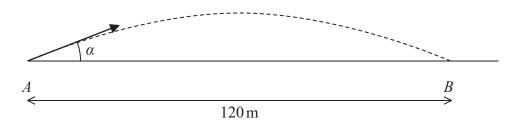
19.

Figure 3

A golf ball is at rest at the point A on horizontal ground.

The ball is hit and initially moves at an angle α to the ground.

The ball first hits the ground at the point B, where $AB = 120\,\text{m}$, as shown in Figure 3.

The motion of the ball is modelled as that of a particle, moving freely under gravity, whose initial speed is $U\,\text{m}\,\text{s}^{-1}$

Using this model,

(a) show that $U^2 \sin\alpha\cos\alpha = 588$

(6)

The ball reaches a maximum height of 10 m above the ground.

(b) Show that $U^2 = 1960$

(4)

In a refinement to the model, the effect of air resistance is included.

The motion of the ball, from A to B, is now modelled as that of a particle whose initial speed is $V\,\text{m}\,\text{s}^{-1}$

This refined model is used to calculate a value for V

(c) State which is greater, U or V, giving a reason for your answer.

(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

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Question 19 continued

Question 19 continued

Question 19 continued

EXPERT TUITION

Question 19 continued

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20. A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s}^{-2}$

At time $t = 0$, P is moving with velocity $4\mathbf{i}\,\mathrm{m\,s}^{-1}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, the position vector of P relative to a fixed origin O is $(\mathbf{i} + \mathbf{j})\,\mathrm{m}$.

(b) Find the position vector of P relative to O at time $t = 3$ seconds.

(2)

Question 20 continued

(Total for Question 20 is 4 marks)

21.
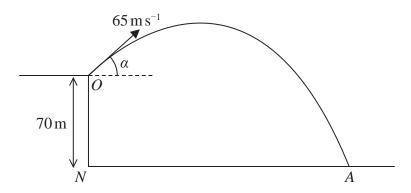
Figure 3

A small stone is projected with speed $65\,\text{m s}^{-1}$ from a point O at the top of a vertical cliff.

Point O is 70 m vertically above the point N.

Point N is on horizontal ground.

The stone is projected at an angle α above the horizontal, where $\tan\alpha = \dfrac{5}{12}$

The stone hits the ground at the point A, as shown in Figure 3.

The stone is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude $10\,\text{m s}^{-2}$

Using the model,

(a) find the time taken for the stone to travel from O to A,

(4)

(b) find the speed of the stone at the instant just before it hits the ground at A.

(5)

One limitation of the model is that it ignores air resistance.

(c) State one other limitation of the model that could affect the reliability of your answers.

(1)

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Question 21 continued

Question 21 continued

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22. At time t seconds, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s}^{-1}$, where

$$\mathbf{v} = 3t^{\frac{1}{2}}\,\mathbf{i} - 2t\mathbf{j} \qquad t > 0$$

(a) Find the acceleration of P at time t seconds, where $t > 0$

(2)

(b) Find the value of t at the instant when P is moving in the direction of $\mathbf{i} - \mathbf{j}$

(3)

At time t seconds, where $t > 0$, the position vector of P, relative to a fixed origin O, is $\mathbf{r}$ metres.

When $t = 1$, $\mathbf{r} = -\mathbf{j}$

(c) Find an expression for $\mathbf{r}$ in terms of t.

(3)

(d) Find the exact distance of P from O at the instant when P is moving with speed $10\,\mathrm{m\,s}^{-1}$

(6)

Question 22 continued

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Question 22 continued

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Question 22 continued

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Question 22 continued

(Total for Question 22 is 14 marks)

23. A particle P moves with acceleration $(4\mathbf{i} - 5\mathbf{j})\,\mathrm{m\,s^{-2}}$

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, P passes through the origin O.

At time $t = T$ seconds, where $T > 0$, the particle P passes through the point A.

The position vector of A is $(\lambda\mathbf{i} - 4.5\mathbf{j})$m relative to O, where λ is a constant.

(b) Find the value of T.

(4)

(c) Hence find the value of λ

(2)

(Total for Question 23 is 8 marks)

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24. (i) At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}\,\mathrm{m\,s^{-2}}$ is given by

$$\mathbf{a} = (1 - 4t)\mathbf{i} + (3 - t^2)\mathbf{j}$$

At the instant when $t = 0$, the velocity of P is $36\mathbf{i}\,\mathrm{m\,s^{-1}}$

(a) Find the velocity of P when $t = 4$

(3)

(b) Find the value of t at the instant when P is moving in a direction perpendicular to $\mathbf{i}$

(3)

(ii) At time t seconds, where $t \geqslant 0$, a particle Q moves so that its position vector $\mathbf{r}$ metres, relative to a fixed origin O, is given by

$$\mathbf{r} = (t^2 - t)\mathbf{i} + 3t\mathbf{j}$$

Find the value of t at the instant when the speed of Q is $5\,\mathrm{m\,s^{-1}}$

(6)

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Question 24 continued

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Question 24 continued

Question 24 continued

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25.

Figure 2
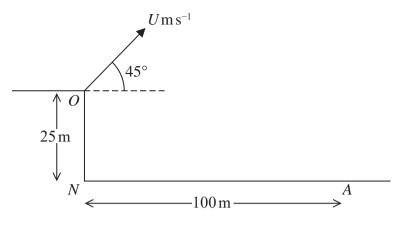
A small ball is projected with speed $U\,\mathrm{m\,s^{-1}}$ from a point O at the top of a vertical cliff.

The point O is 25 m vertically above the point N which is on horizontal ground.

The ball is projected at an angle of 45° above the horizontal.

The ball hits the ground at a point A, where $AN = 100\,\mathrm{m}$, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

Using this initial model,

(a) show that $U = 28$

(6)

(b) find the greatest height of the ball above the horizontal ground NA.

(3)

In a refinement to the model of the motion of the ball from O to A, the effect of air resistance is included.

This refined model is used to find a new value of U.

(c) How would this new value of U compare with 28, the value given in part (a)?

(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

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Question 25 continued

Question 25 continued

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Question 25 continued

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(Total for Question 25 is 11 marks)

[In this question position vectors are given relative to a fixed origin O]

26. At time t seconds, where $t \geqslant 0$, a particle, P, moves so that its velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ is given by

$$\mathbf{v} = 6t\mathbf{i} - 5t^{\frac{3}{2}}\mathbf{j}$$

When $t = 0$, the position vector of P is $(-20\mathbf{i} + 20\mathbf{j})\,\mathrm{m}$.

(a) Find the acceleration of P when $t = 4$

(3)

(b) Find the position vector of P when $t = 4$

(3)

Question 26 continued

(Total for Question 26 is 6 marks)

27. A particle, P, moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-2}}$

At time $t = 0$, the particle is at the point A and is moving with velocity $(-\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$

At time $t = T$ seconds, P is moving in the direction of vector $(3\mathbf{i} - 4\mathbf{j})$

(a) Find the value of T.

(4)

At time $t = 4$ seconds, P is at the point B.

(b) Find the distance AB.

(4)

(Total for Question 27 is 8 marks)

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28.
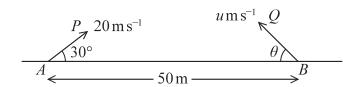
Figure 3

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q, are projected in the vertical plane containing AB.

Ball P is projected from A with speed 20 m s^{-1} at 30° to AB.

Ball Q is projected from B with speed u m s^{-1} at angle θ to BA, as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of P at the instant before it collides with Q.

(6)

(b) Find

    (i) the size of angle θ,

    (ii) the value of u.

(6)

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers.

(1)

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Question 28 continued

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Question 28 continued

Question 28 continued

Question 28 continued

(Total for Question 28 is 13 marks)

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29. At time t seconds, where $t \geqslant 0$, a particle P moves in the x-y plane in such a way that its velocity $\mathbf{v}$ m s^{-1} is given by

$$\mathbf{v} = t^{-\frac{1}{2}}\mathbf{i} - 4t\mathbf{j}$$

When $t = 1$, P is at the point A and when $t = 4$, P is at the point B.

Find the exact distance AB.

(6)

(Total for Question 29 is 6 marks)

30. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to the fixed point O.*]

A particle P moves with constant acceleration.
At time $t = 0$, the particle is at O and is moving with velocity $(2\mathbf{i} - 3\mathbf{j})\,\text{m}\,\text{s}^{-1}$
At time $t = 2$ seconds, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})\,\text{m}$.

(a) Show that the magnitude of the acceleration of P is $2.5\,\text{m}\,\text{s}^{-2}$

(4)

At the instant when P leaves the point A, the acceleration of P changes so that P now moves with constant acceleration $(4\mathbf{i} + 8.8\mathbf{j})\,\text{m}\,\text{s}^{-2}$

At the instant when P reaches the point B, the direction of motion of P is north east.

(b) Find the time it takes for P to travel from A to B.

(4)

Question 30 continued

Question 30 continued

Question 30 continued

(Total for Question 30 is 8 marks)

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31.
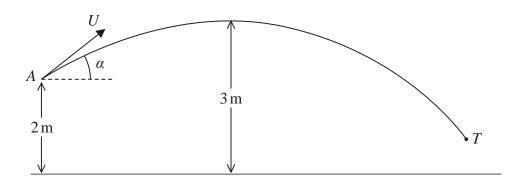
Figure 4

A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point A, the ball is $2\,$m above horizontal ground and is moving with speed U at an angle α above the horizontal.

In the subsequent motion, the highest point reached by the ball is $3\,$m above the ground.
The target is modelled as being the point T, as shown in Figure 4.
The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) show that $U^2 = \dfrac{2g}{\sin^2}$.

(2)

The point T is at a horizontal distance of $20\,$m from A and is at a height of $0.75\,$m above the ground. The ball reaches T without hitting the ground.

(b) Find the size of the angle α

(9)

(c) State one limitation of the model that could affect your answer to part (b).

(1)

(d) Find the time taken for the ball to travel from A to T.

(3)

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Question 31 continued

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Question 31 continued

Question 31 continued

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(Total for Question 31 is 15 marks)

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32. [*In this question position vectors are given relative to a fixed origin O.*]

A particle P moves with constant acceleration $(\mathbf{i} - 2\mathbf{j})$ m s^{-2}.

At time $t = 0$, the particle is at the point A with position vector $(2\mathbf{i} + 5\mathbf{j})$ m and is moving with velocity $\mathbf{u}$ m s^{-1}.

At time $t = 3$ s, P is at the point B with position vector $(-2.5\mathbf{i} + 8\mathbf{j})$ m.

Find $\mathbf{u}$.

(4)

(Total 4 marks)

EXPERT TUITION

33.
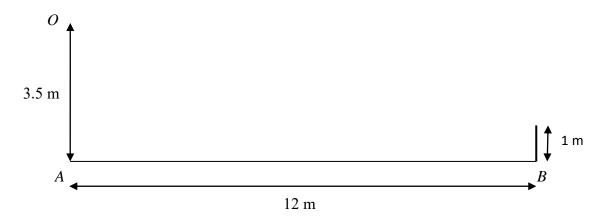
Figure 3

A tennis player serves a ball so as to pass over the net. The ball is given an initial velocity of 45 m s^{-1} in a direction 10° below the horizontal. The ball is struck at a point O which is 3.5 m vertically above the point A which is on horizontal ground. The bottom of the net is the point B which is on the ground and $AB = 12$ m. The height of the net is 1 m, as shown in Figure 3.

The ball is modelled as a particle moving freely under gravity. The ball passes over the net at a point which is vertically above B.

Using the model, find

(a) in centimetres to 2 significant figures, the distance between the ball and the top of the net, as the ball passes over the net,

(8)

(b) to 2 significant figures, the speed of the ball as it passes over the net.

(4)

(c) State two limitations of the model that could affect the reliability of your answers.

(2)

Question 33 continued

EXPERT
TUITION

Question 33 continued

(Total for Question 33 is 14 marks)

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

34. At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\mathbf{i} - 15t^{\frac{1}{2}}\mathbf{j}$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}

Find the speed of P when $t = 4$

(6)

EXPERT TUITION

Question 34 continued

(Total for Question 34 is 6 marks)

35. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is **r** metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2}.

(2)

(b) Find **r** in terms of t.

(2)

(c) Find the value of t when the boat is north-east of O.

(3)

(d) Find the value of t when the boat is moving in a north-east direction.

(3)

Question 35 continued

(Total for Question 35 is 10 marks)

36.
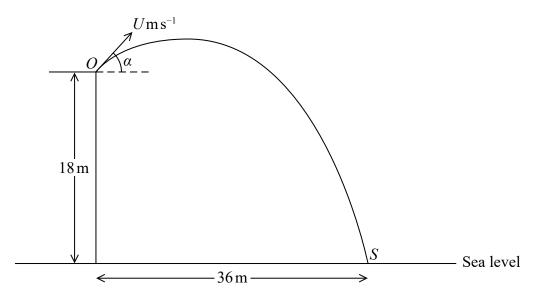
Figure 2

A boy throws a stone with speed $U\,\mathrm{m\,s^{-1}}$ from a point O at the top of a vertical cliff. The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \dfrac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with $g = 10\,\mathrm{m\,s^{-2}}$

Find

(a) the value of U,

(6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

(5)

(c) Suggest two improvements that could be made to the model.

(2)

Question 36 continued

Question 36 continued

Question 36 continued

(Total for Question 36 is 13 marks)

EXPERT
TUITION

37. A ball of mass 0.2 kg is projected vertically downwards with speed U m s^{-1} from a point A which is 2.5 m above horizontal ground. The ball hits the ground. Immediately after hitting the ground, the ball rebounds vertically with a speed of 10 m s^{-1}. The ball receives an impulse of magnitude 7 N s in its impact with the ground. By modelling the ball as a particle and ignoring air resistance, find

(a) the value of U.

(6)

After hitting the ground, the ball moves vertically upwards and passes through a point B which is 1 m above the ground.

(b) Find the time between the instant when the ball hits the ground and the instant when the ball first passes through B.

(4)

(c) Sketch a velocity-time graph for the motion of the ball from when it was projected from A to when it first passes through B. (You need not make any further calculations to draw this sketch.)

(3)

Question 37 continued

EXPERT TUITION

Question 37 continued

EXPERT TUITION

Question 37 continued

(Total 13 marks)

EXPERT TUITION

38. A cyclist is moving along a straight horizontal road and passes a point A. Five seconds later, at the instant when she is moving with speed 10 ms^{-1}, she passes the point B. She moves with constant acceleration from A to B.

Given that $AB = 40 \text{ m}$, find

(a) the acceleration of the cyclist as she moves from A to B,

(4)

(b) the time it takes her to travel from A to the midpoint of AB.

(5)

Question 38 continued

Question 38 continued

EXPERT
TUITION

Question 38 continued

(Total 9 marks)

EXPERT
TUITION

39. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two ships, P and Q, are moving with constant velocities.
The velocity of P is $(9\mathbf{i} - 2\mathbf{j})\,\text{km h}^{-1}$ and the velocity of Q is $(4\mathbf{i} + 8\mathbf{j})\,\text{km h}^{-1}$

(a) Find the direction of motion of P, giving your answer as a bearing to the nearest degree.

(3)

When $t = 0$, the position vector of P is $(9\mathbf{i} + 10\mathbf{j})\,\text{km}$ and the position vector of Q is $(\mathbf{i} + 4\mathbf{j})\,\text{km}$. At time t hours, the position vectors of P and Q are $\mathbf{p}\,\text{km}$ and $\mathbf{q}\,\text{km}$ respectively.

(b) Find an expression for

(i) $\mathbf{p}$ in terms of t,

(ii) $\mathbf{q}$ in terms of t.

(3)

(c) Hence show that, at time t hours,

$$\overrightarrow{QP} = (8 + 5t)\mathbf{i} + (6 - 10t)\mathbf{j}$$

(2)

(d) Find the values of t when the ships are $10\,\text{km}$ apart.

(6)

Question 39 continued

EXPERT TUITION

Question 39 continued

Question 39 continued

(Total 14 marks)

EXPERT TUITION

40. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two cars P and Q are moving on straight horizontal roads with constant velocities. The velocity of P is $(15\mathbf{i} + 20\mathbf{j})$ m s^{-1} and the velocity of Q is $(20\mathbf{i} - 5\mathbf{j})$ m s^{-1}

(a) Find the direction of motion of Q, giving your answer as a bearing to the nearest degree.

(3)

At time $t = 0$, the position vector of P is $400\mathbf{i}$ metres and the position vector of Q is $800\mathbf{j}$ metres. At time t seconds, the position vectors of P and Q are $\mathbf{p}$ metres and $\mathbf{q}$ metres respectively.

(b) Find an expression for

(i) $\mathbf{p}$ in terms of t,

(ii) $\mathbf{q}$ in terms of t.

(3)

(c) Find the position vector of Q when Q is due west of P.

(4)

EXPERT TUITION

Question 40 continued

(Total 10 marks)

EXPERT
TUITION

41. Two trains M and N are moving in the same direction along parallel straight horizontal tracks. At time $t = 0$, M overtakes N whilst they are travelling with speeds 40 m s^{-1} and 30 m s^{-1} respectively. Train M overtakes train N as they pass a point X at the side of the tracks.

After overtaking N, train M maintains its speed of 40 m s^{-1} for T seconds and then decelerates uniformly, coming to rest next to a point Y at the side of the tracks.

After being overtaken, train N maintains its speed of 30 m s^{-1} for 25 s and then decelerates uniformly, also coming to rest next to the point Y.

The times taken by the trains to travel between X and Y are the same.

(a) Sketch, on the same diagram, the speed-time graphs for the motions of the two trains between X and Y.

(4)

Given that $XY = 975$ m,

(b) find the value of T.

(8)

EXPERT TUITION

Question 41 continued

(Total 12 marks)

EXPERT TUITION

42. A small stone is projected vertically upwards from a point O with a speed of $19.6\,\text{ms}^{-1}$.

Modelling the stone as a particle moving freely under gravity,

(a) find the greatest height above O reached by the stone,

(2)

(b) find the length of time for which the stone is more than 14.7 m above O.

(5)

(Total 7 marks)

EXPERT TUITION

43. A particle P is moving with constant velocity. The position vector of P at time t seconds $(t \geqslant 0)$ is $\mathbf{r}$ metres, relative to a fixed origin O, and is given by

$$\mathbf{r} = (2t - 3)\mathbf{i} + (4 - 5t)\mathbf{j}$$

(a) Find the initial position vector of P.

(1)

The particle P passes through the point with position vector $(3.4\mathbf{i} - 12\mathbf{j})\,\text{m}$ at time T seconds.

(b) Find the value of T.

(3)

(c) Find the speed of P.

(4)

EXPERT TUITION

Question 43 continued

(Total 8 marks)

EXPERT TUITION

44. A train travels along a straight horizontal track between two stations, *A* and *B*. The train starts from rest at *A* and moves with constant acceleration 0.5 m s^{-2} until it reaches a speed of *V* m s^{-1}, (*V* < 50). The train then travels at this constant speed before it moves with constant deceleration 0.25 m s^{-2} until it comes to rest at *B*.

(a) Sketch in the space below a speed-time graph for the motion of the train between the two stations *A* and *B*.

(2)

The total time for the journey from *A* to *B* is 5 minutes.

(b) Find, in terms of *V*, the length of time, in seconds, for which the train is

    (i) accelerating,

    (ii) decelerating,

    (iii) moving with constant speed.

(5)

Given that the distance between the two stations *A* and *B* is 6.3 km,

(c) find the value of *V*.

(6)

EXPERT TUITION

Question 44 continued

(Total 13 marks)

EXPERT
TUITION

45. A ball of mass 0.3 kg is released from rest at a point which is 2 m above horizontal ground. The ball moves freely under gravity. After striking the ground, the ball rebounds vertically and rises to a maximum height of 1.5 m above the ground, before falling to the ground again. The ball is modelled as a particle.

(a) Find the speed of the ball at the instant before it strikes the ground for the first time.

(2)

(b) Find the speed of the ball at the instant after it rebounds from the ground for the first time.

(2)

(c) Sketch, in the space provided, a velocity-time graph for the motion of the ball from the instant when it is released until the instant when it strikes the ground for the second time.

(3)

(d) Find the time between the instant when the ball is released and the instant when it strikes the ground for the second time.

(4)

Question 45 continued

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(Total 11 marks)

46. A particle P of mass 0.5 kg is moving under the action of a single force $(3\mathbf{i} - 2\mathbf{j})$ N.

(a) Show that the magnitude of the acceleration of P is $2\sqrt{13}$ m s^{-2}.

(4)

At time $t = 0$, the velocity of P is $(\mathbf{i} + 3\mathbf{j})$ m s^{-1}.

(b) Find the velocity of P at time $t = 2$ seconds.

(3)

Another particle Q moves with constant velocity $\mathbf{v} = (2\mathbf{i} - \mathbf{j})$ m s^{-1}.

(c) Find the distance moved by Q in 2 seconds.

(2)

(d) Show that at time $t = 3.5$ seconds both particles are moving in the same direction.

(3)

Question 46 continued

(Total 12 marks)

EXPERT TUITION

47. A car starts from rest and moves with constant acceleration along a straight horizontal road. The car reaches a speed of V m s^{-1} in 20 seconds. It moves at constant speed V m s^{-1} for the next 30 seconds, then moves with constant deceleration $\frac{1}{2}$ m s^{-2} until it has speed 8 m s^{-1}. It moves at speed 8 m s^{-1} for the next 15 seconds and then moves with constant deceleration $\frac{1}{3}$ m s^{-2} until it comes to rest.

(a) Sketch, in the space below, a speed-time graph for this journey.

(3)

In the first 20 seconds of this journey the car travels 140 m.

Find

(b) the value of V,

(2)

(c) the total time for this journey,

(4)

(d) the total distance travelled by the car.

(4)

EXPERT TUITION

Question 47 continued

(Total 13 marks)

EXPERT TUITION

48. At time $t = 0$, a particle is projected vertically upwards with speed u from a point A. The particle moves freely under gravity. At time T the particle is at its maximum height H above A.

(a) Find T in terms of u and g.

(2)

(b) Show that $H = \dfrac{u^2}{2g}$

(2)

The point A is at a height $3H$ above the ground.

(c) Find, in terms of T, the total time from the instant of projection to the instant when the particle hits the ground.

(4)

EXPERT TUITION

Question 48 continued

(Total 8 marks)

EXPERT TUITION

49. A lorry is moving along a straight horizontal road with constant acceleration. The lorry passes a point A with speed u m s^{-1}, $(u < 34)$, and 10 seconds later passes a point B with speed 34 m s^{-1}. Given that $AB = 240$ m, find

(a) the value of u,

(3)

(b) the time taken for the lorry to move from A to the mid-point of AB.

(6)

(Total 9 marks)

EXPERT TUITION

50. A car is travelling along a straight horizontal road. The car takes 120 s to travel between two sets of traffic lights which are 2145 m apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for 30 s until its speed is 22 m s^{-1}. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.

(a) Sketch, in the space below, a speed-time graph for the motion of the car between the two sets of traffic lights.

(2)

(b) Find the value of T.

(3)

A motorcycle leaves the first set of traffic lights 10 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration, a m s^{-2}, and passes the car at the point A which is 990 m from the first set of traffic lights. When the motorcycle passes the car, the car is moving with speed 22 m s^{-1}.

(c) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point A.

(4)

(d) Find the value of a.

(2)

EXPERT
TUITION

Question 50 continued

(Total 11 marks)

51. *[In this question, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed due east and due north respectively.]*

The velocity, $\mathbf{v}$ m s⁻¹, of a particle P at time t seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}$$

(a) Find the speed of P when $t = 0$

(3)

(b) Find the bearing on which P is moving when $t = 2$

(2)

(c) Find the value of t when P is moving

    (i) parallel to $\mathbf{j}$,

    (ii) parallel to $(-\mathbf{i} - 3\mathbf{j})$.

(6)

Question 51 continued

(Total 11 marks)

52. At time $t = 0$, two balls A and B are projected vertically upwards. The ball A is projected vertically upwards with speed 2 m s^{-1} from a point 50 m above the horizontal ground. The ball B is projected vertically upwards from the ground with speed 20 m s^{-1}. At time $t = T$ seconds, the two balls are at the same vertical height, h metres, above the ground. The balls are modelled as particles moving freely under gravity. Find

(a) the value of T,

(5)

(b) the value of h.

(2)

(Total 7 marks)

53.
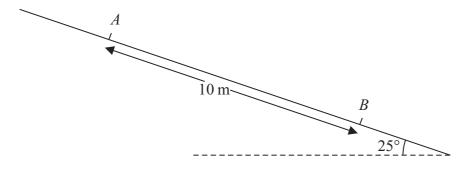
Figure 3

A particle P of mass 0.6 kg slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at 25° to the horizontal. The particle passes through two points A and B, where $AB = 10$ m, as shown in Figure 3. The speed of P at A is 2 m s^{-1}. The particle P takes 3.5 s to move from A to B. Find

(a) the speed of P at B,

(3)

(b) the acceleration of P

(2)

EXPERT TUITION

Question 53 continued

(Total 5 marks)

EXPERT
TUITION

54. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively. Position vectors are given with respect to a fixed origin O.*]

A ship S is moving with constant velocity $(3\mathbf{i} + 3\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of S is $(-4\mathbf{i} + 2\mathbf{j})$ km.

(a) Find the position vector of S at time t hours.

(2)

A ship T is moving with constant velocity $(-2\mathbf{i} + n\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of T is $(6\mathbf{i} + \mathbf{j})$ km. The two ships meet at the point P.

(b) Find the value of n.

(5)

(c) Find the distance OP.

(4)

Question 54 continued

(Total 11 marks)

EXPERT TUITION

55.
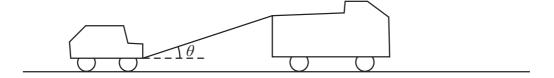
Figure 4

A truck of mass 1750 kg is towing a car of mass 750 kg along a straight horizontal road. The two vehicles are joined by a light towbar which is inclined at an angle θ to the road, as shown in Figure 4. The vehicles are travelling at 20 m s^{-1} as they enter a zone where the speed limit is 14 m s^{-1}. The truck's brakes are applied to give a constant braking force on the truck. The distance travelled between the instant when the brakes are applied and the instant when the speed of each vehicle is 14 m s^{-1} is 100 m.

(a) Find the deceleration of the truck and the car.

(3)

The constant braking force on the truck has magnitude R newtons. The truck and the car also experience constant resistances to motion of 500 N and 300 N respectively. Given that $\cos \theta = 0.9$, find

(b) the force in the towbar,

(4)

(c) the value of R.

(4)

EXPERT TUITION

Question 55 continued

(Total 11 marks)

EXPERT
TUITION

56.

v (m s^{-1})

30

O

T

t (s)

Figure 4

The velocity-time graph in Figure 4 represents the journey of a train P travelling along a straight horizontal track between two stations which are 1.5 km apart. The train P leaves the first station, accelerating uniformly from rest for 300 m until it reaches a speed of 30 m s^{-1}. The train then maintains this speed for T seconds before decelerating uniformly at 1.25 m s^{-2}, coming to rest at the next station.

(a) Find the acceleration of P during the first 300 m of its journey.

(2)

(b) Find the value of T.

(5)

A second train Q completes the same journey in the same total time. The train leaves the first station, accelerating uniformly from rest until it reaches a speed of V m s^{-1} and then immediately decelerates uniformly until it comes to rest at the next station.

(c) Sketch on the diagram above, a velocity-time graph which represents the journey of train Q.

(2)

(d) Find the value of V.

(6)

Question 56 continued

(Total 15 marks)

57. [*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship sets sail at 9 am from a port P and moves with constant velocity. The position vector of P is $(4\mathbf{i} - 8\mathbf{j})$ km. At 9.30 am the ship is at the point with position vector $(\mathbf{i} - 4\mathbf{j})$ km.

(a) Find the speed of the ship in km h^{-1}.

(4)

(b) Show that the position vector **r** km of the ship, t hours after 9 am, is given by
$\mathbf{r} = (4 - 6t)\mathbf{i} + (8t - 8)\mathbf{j}$.

(2)

At 10 am, a passenger on the ship observes that a lighthouse L is due west of the ship. At 10.30 am, the passenger observes that L is now south-west of the ship.

(c) Find the position vector of L.

(5)

EXPERT TUITION

Question 57 continued

(Total 11 marks)

EXPERT
TUITION

58. A car is moving on a straight horizontal road. At time $t = 0$, the car is moving with speed 20 m s^{-1} and is at the point A. The car maintains the speed of 20 m s^{-1} for 25 s. The car then moves with constant deceleration 0.4 m s^{-2}, reducing its speed from 20 m s^{-1} to 8 m s^{-1}. The car then moves with constant speed 8 m s^{-1} for 60 s. The car then moves with constant acceleration until it is moving with speed 20 m s^{-1} at the point B.

(a) Sketch a speed-time graph to represent the motion of the car from A to B.

(3)

(b) Find the time for which the car is decelerating.

(2)

Given that the distance from A to B is 1960 m,

(c) find the time taken for the car to move from A to B.

(8)

Question 58 continued

(Total 13 marks)

59. A particle P is projected vertically upwards from a point A with speed u m s^{-1}. The point A is 17.5 m above horizontal ground. The particle P moves freely under gravity until it reaches the ground with speed 28 m s^{-1}.

(a) Show that $u = 21$

(3)

At time t seconds after projection, P is 19 m above A.

(b) Find the possible values of t.

(5)

The ground is soft and, after P reaches the ground, P sinks vertically downwards into the ground before coming to rest. The mass of P is 4 kg and the ground is assumed to exert a constant resistive force of magnitude 5000 N on P.

(c) Find the vertical distance that P sinks into the ground before coming to rest.

(4)

Question 59 continued

(Total 12 marks)

EXPERT
TUITION

60. *[In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.]*

A ship S is moving with constant velocity $(-12\mathbf{i} + 7.5\mathbf{j})$ km h^{-1}.

(a) Find the direction in which S is moving, giving your answer as a bearing.

(3)

At time t hours after noon, the position vector of S is **s** km. When $t = 0$, $\mathbf{s} = 40\mathbf{i} - 6\mathbf{j}$.

(b) Write down **s** in terms of t.

(2)

A fixed beacon B is at the point with position vector $(7\mathbf{i} + 12.5\mathbf{j})$ km.

(c) Find the distance of S from B when $t = 3$

(4)

(d) Find the distance of S from B when S is due north of B.

(4)

Question 60 continued

(Total 13 marks)

EXPERT
TUITION

61. A stone is projected vertically upwards from a point A with speed $u\,\mathrm{m\,s^{-1}}$. After projection the stone moves freely under gravity until it returns to A. The time between the instant that the stone is projected and the instant that it returns to A is $3\frac{4}{7}$ seconds.

Modelling the stone as a particle,

(a) show that $u = 17\frac{1}{2}$,

(3)

(b) find the greatest height above A reached by the stone,

(2)

(c) find the length of time for which the stone is at least $6\frac{3}{5}$ m above A.

(6)

EXPERT TUITION

Question 61 continued

(Total 11 marks)

EXPERT TUITION

62. A car moves along a straight horizontal road from a point A to a point B, where $AB = 885\,\text{m}$. The car accelerates from rest at A to a speed of $15\,\text{m s}^{-1}$ at a constant rate $a\,\text{m s}^{-2}$. The time for which the car accelerates is $\frac{1}{3}T$ seconds. The car maintains the speed of $15\,\text{m s}^{-1}$ for T seconds. The car then decelerates at a constant rate of $2.5\,\text{m s}^{-2}$ stopping at B.

(a) Find the time for which the car decelerates.

(2)

(b) Sketch a speed-time graph for the motion of the car.

(2)

(c) Find the value of T.

(4)

(d) Find the value of a.

(2)

(e) Sketch an acceleration-time graph for the motion of the car.

(3)

Question 62 continued

(Total 13 marks)

63. [*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively. Position vectors are relative to a fixed origin O.*]

A boat P is moving with constant velocity $(-4\mathbf{i}+8\mathbf{j})$ km h^{-1}.

(a) Calculate the speed of P.

(2)

When $t = 0$, the boat P has position vector $(2\mathbf{i}-8\mathbf{j})$ km. At time t hours, the position vector of P is **p** km.

(b) Write down **p** in terms of t.

(1)

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is **q** km, where

$$\mathbf{q}=18\mathbf{i}+12\mathbf{j}-t(6\mathbf{i}+8\mathbf{j})$$

Find

(c) the value of t when P is due west of Q,

(3)

(d) the distance between P and Q when P is due west of Q.

(3)

EXPERT TUITION

Question 63 continued

(Total 9 marks)

EXPERT
TUITION

64. At time $t = 0$ a ball is projected vertically upwards from a point O and rises to a maximum height of 40 m above O. The ball is modelled as a particle moving freely under gravity.

(a) Show that the speed of projection is 28 m s^{-1}.

(3)

(b) Find the times, in seconds, when the ball is 33.6 m above O.

(5)

(Total 8 marks)

EXPERT TUITION

65. A girl runs a 400 m race in a time of 84 s. In a model of this race, it is assumed that, starting from rest, she moves with constant acceleration for 4 s, reaching a speed of 5 m s^{-1}. She maintains this speed for 60 s and then moves with constant deceleration for 20 s, crossing the finishing line with a speed of V m s^{-1}.

    (a) Sketch, in the space below, a speed-time graph for the motion of the girl during the whole race.

                                                (2)

    (b) Find the distance run by the girl in the first 64 s of the race.

                                                (3)

    (c) Find the value of V.

                                                (5)

    (d) Find the deceleration of the girl in the final 20 s of her race.

                                              (2)

Question 65 continued

(Total 12 marks)

66. [*In this question **i** and **j** are unit vectors due east and due north respectively. Position vectors are given relative to a fixed origin O.*]

Two ships P and Q are moving with constant velocities. Ship P moves with velocity $(2\mathbf{i} - 3\mathbf{j})$ km h^{-1} and ship Q moves with velocity $(3\mathbf{i} + 4\mathbf{j})$ km h^{-1}.

(a) Find, to the nearest degree, the bearing on which Q is moving.

(2)

At 2 pm, ship P is at the point with position vector $(\mathbf{i} + \mathbf{j})$ km and ship Q is at the point with position vector $(-2\mathbf{j})$ km.

At time t hours after 2 pm, the position vector of P is $\mathbf{p}$ km and the position vector of Q is $\mathbf{q}$ km.

(b) Write down expressions, in terms of t, for

    (i) $\mathbf{p}$,

    (ii) $\mathbf{q}$,

    (iii) $\overrightarrow{PQ}$.

(5)

(c) Find the time when

    (i) Q is due north of P,

    (ii) Q is north-west of P.

(4)

Question 66 continued

EXPERT TUITION

67. A ball is thrown vertically upwards with speed u m s^{-1} from a point P at height h metres above the ground. The ball hits the ground 0.75 s later. The speed of the ball immediately before it hits the ground is 6.45 m s^{-1}. The ball is modelled as a particle.

(a) Show that $u = 0.9$

(3)

(b) Find the height above P to which the ball rises before it starts to fall towards the ground again.

(2)

(c) Find the value of h.

(3)

(Total 8 marks)

EXPERT TUITION

68. A car accelerates uniformly from rest for 20 seconds. It moves at constant speed v m s^{-1} for the next 40 seconds and then decelerates uniformly for 10 seconds until it comes to rest.

(a) For the motion of the car, sketch

    (i) a speed-time graph,

    (ii) an acceleration-time graph.

(6)

Given that the total distance moved by the car is 880 m,

(b) find the value of v.

(4)

Question 68 continued
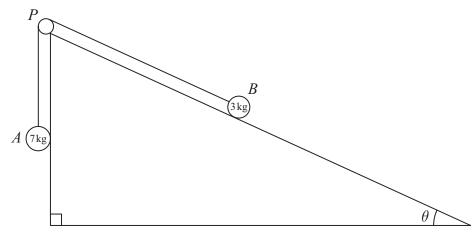
(Total 10 marks)

EXPERT TUITION

69.

Figure 4

Two particles A and B, of mass 7 kg and 3 kg respectively, are attached to the ends of a light inextensible string. Initially B is held at rest on a rough fixed plane inclined at angle θ to the horizontal, where $\tan \theta = \dfrac{5}{12}$. The part of the string from B to P is parallel to a line of greatest slope of the plane. The string passes over a small smooth pulley, P, fixed at the top of the plane. The particle A hangs freely below P, as shown in Figure 4. The coefficient of friction between B and the plane is $\dfrac{2}{3}$. The particles are released from rest with the string taut and B moves up the plane.

(a) Find the magnitude of the acceleration of B immediately after release.

(10)

(b) Find the speed of B when it has moved 1 m up the plane.

(2)

When B has moved 1 m up the plane the string breaks. Given that in the subsequent motion B does not reach P,

(c) find the time between the instants when the string breaks and when B comes to instantaneous rest.

(4)

EXPERT TUITION

Question 69 continued

(Total 16 marks)

EXPERT
TUITION

70. A particle P is moving with constant velocity $(-3\mathbf{i} + 2\mathbf{j})$ m s^{-1}. At time $t = 6$ s P is at the point with position vector $(-4\mathbf{i} - 7\mathbf{j})$ m. Find the distance of P from the origin at time $t = 2$ s.

(5)

(Total 5 marks)

EXPERT TUITION

71. Two cars P and Q are moving in the same direction along the same straight horizontal road. Car P is moving with constant speed 25 m s^{-1}. At time $t = 0$, P overtakes Q which is moving with constant speed 20 m s^{-1}. From $t = T$ seconds, P decelerates uniformly, coming to rest at a point X which is 800 m from the point where P overtook Q. From $t = 25$ s, Q decelerates uniformly, coming to rest at the same point X at the same instant as P.

(a) Sketch, on the same axes, the speed-time graphs of the two cars for the period from $t = 0$ to the time when they both come to rest at the point X.

(4)

(b) Find the value of T.

(8)

EXPERT TUITION

Question 71 continued

(Total 12 marks)

EXPERT TUITION

72. A ball is projected vertically upwards with a speed of 14.7 m s^{-1} from a point which is 49 m above horizontal ground. Modelling the ball as a particle moving freely under gravity, find

(a) the greatest height, above the ground, reached by the ball,

(4)

(b) the speed with which the ball first strikes the ground,

(3)

(c) the total time from when the ball is projected to when it first strikes the ground.

(3)

Question 72 continued

(Total 10 marks)

EXPERT
TUITION

73. An athlete runs along a straight road. She starts from rest and moves with constant acceleration for 5 seconds, reaching a speed of $8\,\text{m}\,\text{s}^{-1}$. This speed is then maintained for T seconds. She then decelerates at a constant rate until she stops. She has run a total of $500\,\text{m}$ in $75\,\text{s}$.

(a) In the space below, sketch a speed-time graph to illustrate the motion of the athlete.

(3)

(b) Calculate the value of T.

(5)

(Total 8 marks)

EXPERT TUITION

74. [*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship S is moving along a straight line with constant velocity. At time t hours the position vector of S is **s** km. When $t = 0$, $\mathbf{s} = 9\mathbf{i} - 6\mathbf{j}$. When $t = 4$, $\mathbf{s} = 21\mathbf{i} + 10\mathbf{j}$. Find

(a) the speed of S,

(4)

(b) the direction in which S is moving, giving your answer as a bearing.

(2)

(c) Show that $\mathbf{s} = (3t + 9)\mathbf{i} + (4t - 6)\mathbf{j}$.

(2)

A lighthouse L is located at the point with position vector $(18\mathbf{i} + 6\mathbf{j})$ km. When $t = T$, the ship S is 10 km from L.

(d) Find the possible values of T.

(6)

Question 74 continued

(Total 14 marks)

EXPERT TUITION

75. Three posts P, Q and R, are fixed in that order at the side of a straight horizontal road. The distance from P to Q is 45 m and the distance from Q to R is 120 m. A car is moving along the road with constant acceleration a m s⁻². The speed of the car, as it passes P, is u m s⁻¹. The car passes Q two seconds after passing P, and the car passes R four seconds after passing Q. Find

(i) the value of u,

(ii) the value of a.

(7)

(Total 7 marks)

76. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

A hiker *H* is walking with constant velocity (1.2**i** – 0.9**j**) m s^{-1}.

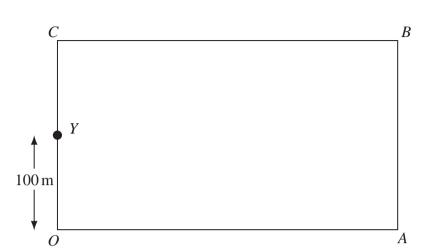(a) Find the speed of *H*.

(2)

Figure 3

A horizontal field *OABC* is rectangular with *OA* due east and *OC* due north, as shown in Figure 3. At twelve noon hiker *H* is at the point *Y* with position vector 100 **j** m, relative to the fixed origin *O*.

(b) Write down the position vector of *H* at time *t* seconds after noon.

(2)

At noon, another hiker *K* is at the point with position vector (9**i** + 46**j**) m. Hiker *K* is moving with constant velocity (0.75**i** + 1.8**j**) m s^{-1}.

(c) Show that, at time *t* seconds after noon,

$$\overrightarrow{HK} = \left[(9 - 0.45t)\,\mathbf{i} + (2.7t - 54)\,\mathbf{j}\right] \text{ metres.}$$

(4)

Hence,

(d) show that the two hikers meet and find the position vector of the point where they meet.

(5)

EXPERT
TUITION

Question 76 continued

(Total 13 marks)

EXPERT
TUITION

77. A particle P moves with constant acceleration $(2\mathbf{i} - 5\mathbf{j})$ m s^{-2}. At time $t = 0$, P has speed u m s^{-1}. At time $t = 3$ s, P has velocity $(-6\mathbf{i} + \mathbf{j})$ m s^{-1}.

Find the value of u.

(5)

(Total 5 marks)

EXPERT TUITION

78. A small ball is projected vertically upwards from ground level with speed u m s^{-1}. The ball takes 4 s to return to ground level.

(a) Draw, in the space below, a velocity-time graph to represent the motion of the ball during the first 4 s.

(2)

(b) The maximum height of the ball above the ground during the first 4 s is 19.6 m. Find the value of u.

(3)

_____

_____

_____

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_____

(Total 5 marks)

EXPERT
TUITION

79. At time $t = 0$, a particle is projected vertically upwards with speed u m s^{-1} from a point 10 m above the ground. At time T seconds, the particle hits the ground with speed 17.5 m s^{-1} . Find

(a) the value of u,

(3)

(b) the value of T.

(4)

(Total 7 marks)

EXPERT TUITION

80. A car is moving along a straight horizontal road. The speed of the car as it passes the point A is 25 m s^{-1} and the car maintains this speed for 30 s. The car then decelerates uniformly to a speed of 10 m s^{-1}. The speed of 10 m s^{-1} is then maintained until the car passes the point B. The time taken to travel from A to B is 90 s and $AB = 1410$ m.

(a) Sketch, in the space below, a speed-time graph to show the motion of the car from A to B.

(2)

(b) Calculate the deceleration of the car as it decelerates from 25 m s^{-1} to 10 m s^{-1}.

(7)

Question 80 continued

(Total 9 marks)

EXPERT TUITION

81. A firework rocket starts from rest at ground level and moves vertically. In the first 3 s of its motion, the rocket rises 27 m. The rocket is modelled as a particle moving with constant acceleration a m s^{-2}. Find

(a) the value of a,

(2)

(b) the speed of the rocket 3 s after it has left the ground.

(2)

After 3 s, the rocket burns out. The motion of the rocket is now modelled as that of a particle moving freely under gravity.

(c) Find the height of the rocket above the ground 5 s after it has left the ground.

(4)

(Total 8 marks)

EXPERT TUITION

82. A car moves along a horizontal straight road, passing two points A and B. At A the speed of the car is 15 m s^{-1}. When the driver passes A, he sees a warning sign W ahead of him, 120 m away. He immediately applies the brakes and the car decelerates with uniform deceleration, reaching W with speed 5 m s^{-1}. At W, the driver sees that the road is clear. He then immediately accelerates the car with uniform acceleration for 16 s to reach a speed of V m s^{-1} ($V > 15$). He then maintains the car at a constant speed of V m s^{-1}. Moving at this constant speed, the car passes B after a further 22 s.

(a) Sketch, in the space below, a speed-time graph to illustrate the motion of the car as it moves from A to B.

(3)

(b) Find the time taken for the car to move from A to B.

(3)

The distance from A to B is 1 km.

(c) Find the value of V.

(5)

Question 82 continued

(Total 11 marks)

83. [*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A particle P is moving with constant velocity $(-5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

(a) the speed of P,

(2)

(b) the direction of motion of P, giving your answer as a bearing.

(3)

At time $t = 0$, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})$ m relative to a fixed origin O. When $t = 3$ s, the velocity of P changes and it moves with velocity $(u\mathbf{i} + v\mathbf{j})$ m s^{-1}, where u and v are constants. After a further 4 s, it passes through O and continues to move with velocity $(u\mathbf{i} + v\mathbf{j})$ m s^{-1}.

(c) Find the values of u and v.

(5)

(d) Find the total time taken for P to move from A to a position which is due south of A.

(3)

Question 83 continued

(Total 13 marks)

EXPERT TUITION

84. At time $t = 0$ a particle P leaves the origin O and moves along the x-axis. At time t seconds, the velocity of P is v m s^{-1} in the positive x direction, where

$$v = 3t^2 - 16t + 21$$

The particle is instantaneously at rest when $t = t_1$ and when $t = t_2$ $(t_1 < t_2)$.

(a) Find the value of t_1 and the value of t_2.

(2)

(b) Find the magnitude of the acceleration of P at the instant when $t = t_1$.

(3)

(c) Find the distance travelled by P in the interval $t_1 \leqslant t \leqslant t_2$.

(4)

(d) Show that P does not return to O.

(3)

EXPERT TUITION

Question 84 continued

EXPERT TUITION

Question 84 continued

EXPERT
TUITION

Question 84 continued

(Total 12 marks)

85. A particle P moves along a straight line. The speed of P at time t seconds ($t \geqslant 0$) is v m s^{-1}, where $v = (pt^2 + qt + r)$ and p, q and r are constants. When $t = 2$ the speed of P has its minimum value. When $t = 0$, $v = 11$ and when $t = 2$, $v = 3$

Find

(a) the acceleration of P when $t = 3$

(8)

(b) the distance travelled by P in the third second of the motion.

(5)

EXPERT TUITION

Question 85 continued

EXPERT TUITION

Question 85 continued

EXPERT
TUITION

Question 85 continued

(Total 13 marks)

EXPERT TUITION

86. [*In this question, **i** is a horizontal unit vector and **j** is an upward vertical unit vector.*]

A particle P is projected from a fixed origin O with velocity $(3\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$. The particle moves freely under gravity and passes through the point A with position vector $\lambda(\mathbf{i} - \mathbf{j})\,\text{m}$, where λ is a positive constant.

(a) Find the value of λ.

(6)

(b) Find

　　(i)　the speed of P at the instant when it passes through A,

　　(ii)　the direction of motion of P at the instant when it passes through A.

(7)

EXPERT
TUITION

Question 86 continued

EXPERT TUITION

Question 86 continued

(Total 13 marks)

87. A particle P moves on the positive x-axis. The velocity of P at time t seconds is $(2t^2 - 9t + 4)$ m s^{-1}. When $t = 0$, P is 15 m from the origin O.

Find

(a) the values of t when P is instantaneously at rest,

(3)

(b) the acceleration of P when $t = 5$

(3)

(c) the total distance travelled by P in the interval $0 \leqslant t \leqslant 5$

(5)

Question 87 continued

EXPERT TUITION

Question 87 continued

EXPERT
TUITION

Question 87 continued

(Total 11 marks)

88.

Figure 3
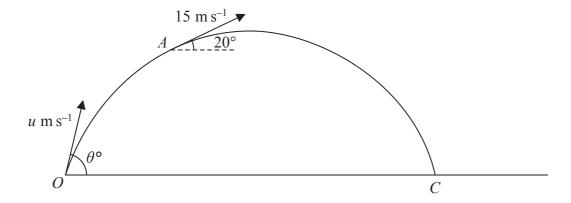
At time $t = 0$, a particle is projected from a fixed point O on horizontal ground with speed $u \text{ m s}^{-1}$ at an angle $\theta°$ to the horizontal. The particle moves freely under gravity and passes through the point A when $t = 4$ s. As it passes through A, the particle is moving upwards at 20° to the horizontal with speed 15 m s^{-1}, as shown in Figure 3.

(a) Find the value of u and the value of θ.

(7)

At the point B on its path the particle is moving downwards at 20° to the horizontal with speed 15 m s^{-1}.

(b) Find the time taken for the particle to move from A to B.

(2)

The particle reaches the ground at the point C.

(c) Find the distance OC.

(3)

EXPERT
TUITION

Question 88 continued

EXPERT TUITION

Question 88 continued

EXPERT TUITION

Question 88 continued

(Total 12 marks)

EXPERT TUITION

89. At time t seconds, where $t \geqslant 0$, a particle P is moving on a horizontal plane with acceleration $[(3t^2 - 4t)\mathbf{i} + (6t - 5)\mathbf{j}]\,\text{m s}^{-2}$.

When $t = 3$ the velocity of P is $(11\mathbf{i} + 10\mathbf{j})\,\text{m s}^{-1}$.

Find

(a) the velocity of P at time t seconds,

(5)

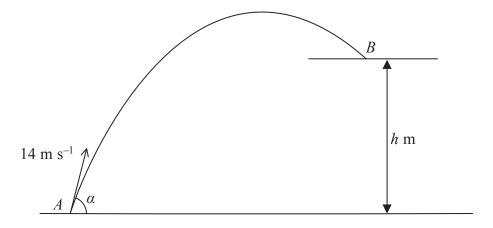(b) the speed of P when it is moving parallel to the vector $\mathbf{i}$.

(4)

Question 89 continued

(Total 9 marks)

EXPERT TUITION

90.

Figure 2

A small ball is projected with speed 14 m s^{-1} from a point A on horizontal ground. The angle of projection is α above the horizontal. A horizontal platform is at height h metres above the ground. The ball moves freely under gravity until it hits the platform at the point B, as shown in Figure 2. The speed of the ball immediately before it hits the platform at B is 10 m s^{-1}.

(a) Find the value of h.

(4)

Given that $\sin \alpha = 0.85$,

(b) find the horizontal distance from A to B.

(8)

EXPERT TUITION

Question 90 continued

EXPERT
TUITION

Question 90 continued

EXPERT
TUITION

Question 90 continued

(Total 12 marks)

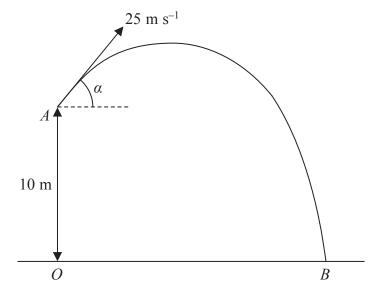EXPERT TUITION

91.

Figure 4

A particle P is projected from a point A with speed 25 m s^{-1} at an angle of elevation α, where $\sin \alpha = \dfrac{4}{5}$. The point A is 10 m vertically above the point O which is on horizontal ground, as shown in Figure 4. The particle P moves freely under gravity and reaches the ground at the point B.

Calculate

(a) the greatest height above the ground of P, as it moves from A to B,

(3)

(b) the distance OB.

(6)

The point C lies on the path of P. The direction of motion of P at C is perpendicular to the direction of motion of P at A.

(c) Find the time taken by P to move from A to C.

(4)

EXPERT TUITION

Question 91 continued

EXPERT
TUITION

Question 91 continued

EXPERT
TUITION

Question 91 continued

(Total 13 marks)

92. A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where

$$v = 2t^2 - 14t + 20, \qquad t \geqslant 0$$

Find

(a) the times when P is instantaneously at rest,

(3)

(b) the greatest speed of P in the interval $0 \leqslant t \leqslant 4$

(5)

(c) the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(5)

Question 92 continued

EXPERT
TUITION

Question 92 continued

EXPERT TUITION

Question 92 continued

(Total 13 marks)

93. A particle P moves along a straight line in such a way that at time t seconds its velocity v m s^{-1} is given by

$$v = \frac{1}{2}t^2 - 3t + 4$$

Find

(a) the times when P is at rest,

(4)

(b) the total distance travelled by P between $t = 0$ and $t = 4$.

(5)

EXPERT TUITION

Question 93 continued

EXPERT
TUITION

Question 93 continued

EXPERT TUITION

Question 93 continued

(Total 9 marks)

EXPERT TUITION

94.
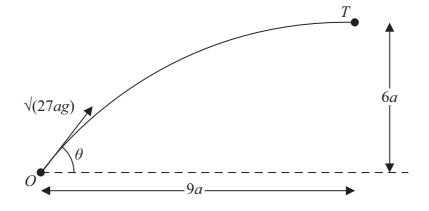
Figure 4

A small ball is projected from a fixed point O so as to hit a target T which is at a horizontal distance $9a$ from O and at a height $6a$ above the level of O. The ball is projected with speed $\sqrt{(27ag)}$ at an angle θ to the horizontal, as shown in Figure 4. The ball is modelled as a particle moving freely under gravity.

(a) Show that $\tan^2 \theta - 6 \tan \theta + 5 = 0$

(7)

The two possible angles of projection are θ_1 and θ_2, where $\theta_1 > \theta_2$.

(b) Find $\tan \theta_1$ and $\tan \theta_2$.

(3)

The particle is projected at the larger angle θ_1.

(c) Show that the time of flight from O to T is $\sqrt{\left(\dfrac{78a}{g} \right)}$.

(3)

(d) Find the speed of the particle immediately before it hits T.

(3)

EXPERT
TUITION

Question 94 continued

EXPERT
TUITION

Question 94 continued

EXPERT TUITION

Question 94 continued

EXPERT
TUITION

Question 94 continued

(Total 16 marks)

95. At time t seconds the velocity of a particle P is $[(4t-5)\mathbf{i} + 3\mathbf{j}]\,\mathrm{m\,s^{-1}}$. When $t = 0$, the position vector of P is $(2\mathbf{i} + 5\mathbf{j})$ m, relative to a fixed origin O.

(a) Find the value of t when the velocity of P is parallel to the vector $\mathbf{j}$.

(1)

(b) Find an expression for the position vector of P at time t seconds.

(4)

EXPERT TUITION

Question 95 continued

EXPERT
TUITION

Question 95 continued

EXPERT TUITION

Question 95 continued
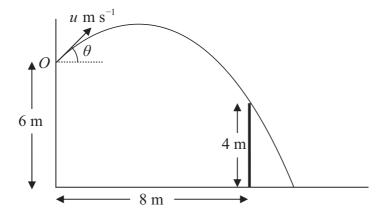
(Total 5 marks)

EXPERT TUITION

96.

Figure 2

A ball is thrown from a point O, which is 6 m above horizontal ground. The ball is projected with speed u m s^{-1} at an angle θ above the horizontal. There is a thin vertical post which is 4 m high and 8 m horizontally away from the vertical through O, as shown in Figure 2. The ball passes just above the top of the post 2 s after projection. The ball is modelled as a particle.

(a) Show that tan θ = 2.2

(5)

(b) Find the value of u.

(2)

The ball hits the ground T seconds after projection.

(c) Find the value of T.

(3)

Immediately before the ball hits the ground the direction of motion of the ball makes an angle α with the horizontal.

(d) Find α.

(5)

Question 96 continued

EXPERT TUITION

Question 96 continued

EXPERT
TUITION

Question 96 continued

(Total 15 marks)

EXPERT TUITION

97. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A particle P moves in such a way that its velocity $\mathbf{v}$ m s^{-1} at time t seconds is given by

$$\mathbf{v} = (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j}$$

(a) Find the magnitude of the acceleration of P when $t = 1$

(5)

Given that, when $t = 0$, the position vector of P is **i** metres,

(b) find the position vector of P when $t = 3$

(5)

EXPERT TUITION

Question 97 continued

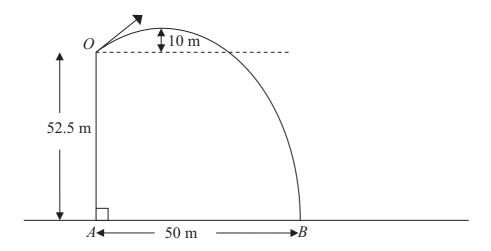**(Total 10 marks)**

EXPERT
TUITION

98.

Figure 4

A small stone is projected from a point O at the top of a vertical cliff OA. The point O is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of O before hitting the sea at the point B, where $AB = 50$ m, as shown in Figure 4. The stone is modelled as a particle moving freely under gravity.

(a) Show that the vertical component of the velocity of projection of the stone is 14 m s^{-1}.

(3)

(b) Find the speed of projection.

(9)

(c) Find the time after projection when the stone is moving parallel to OB.

(5)

EXPERT
TUITION

Question 98 continued

EXPERT
TUITION

Question 98 continued

EXPERT
TUITION

Question 98 continued

EXPERT
TUITION

Question 98 continued

(Total 17 marks)

99. A particle P is moving in a plane. At time t seconds, P is moving with velocity $\mathbf{v}$ m s^{-1}, where $\mathbf{v} = 2t\mathbf{i} - 3t^2\mathbf{j}$.

Find

(a) the speed of P when $t = 4$

(2)

(b) the acceleration of P when $t = 4$

(3)

Given that P is at the point with position vector $(-4\mathbf{i} + \mathbf{j})$ m when $t = 1$,

(c) find the position vector of P when $t = 4$

(5)

EXPERT TUITION

Question 99 continued

EXPERT TUITION

Question 99 continued

EXPERT
TUITION

Question 99 continued

(Total 10 marks)

EXPERT TUITION

100. [*In this question, the unit vectors* **i** *and* **j** *are horizontal and vertical respectively*.]

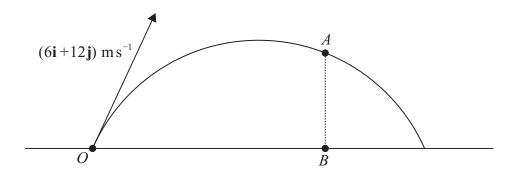**Figure 3**

The point O is a fixed point on a horizontal plane. A ball is projected from O with velocity $(6\mathbf{i} + 12\mathbf{j})$ m s^{-1}, and passes through the point A at time t seconds after projection. The point B is on the horizontal plane vertically below A, as shown in Figure 3. It is given that $OB = 2AB$.

Find

(a) the value of t,

(7)

(b) the speed, V m s^{-1}, of the ball at the instant when it passes through A.

(5)

At another point C on the path the speed of the ball is also V m s^{-1}.

(c) Find the time taken for the ball to travel from O to C.

(3)

EXPERT TUITION

Question 100 continued

EXPERT TUITION

Question 100 continued

EXPERT
TUITION

Question 100 continued

EXPERT TUITION

Question 100 continued

(Total 15 marks)

EXPERT
TUITION

101. A particle P moves on the x-axis. The acceleration of P at time t seconds is $(t-4)$ m s^{-2} in the positive x-direction. The velocity of P at time t seconds is v m s^{-1}. When $t = 0$, $v = 6$.

Find

(a) v in terms of t,

(4)

(b) the values of t when P is instantaneously at rest,

(3)

(c) the distance between the two points at which P is instantaneously at rest.

(4)

EXPERT TUITION

Question 101 continued

EXPERT
TUITION

Question 101 continued

EXPERT
TUITION

Question 101 continued

(Total 11 marks)

102. A particle is projected from a point O with speed u at an angle of elevation α above the horizontal and moves freely under gravity. When the particle has moved a horizontal distance x, its height above O is y.

(a) Show that

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

(4)

A girl throws a ball from a point A at the top of a cliff. The point A is 8 m above a horizontal beach. The ball is projected with speed 7 m s^{-1} at an angle of elevation of 45°. By modelling the ball as a particle moving freely under gravity,

(b) find the horizontal distance of the ball from A when the ball is 1 m above the beach.

(5)

A boy is standing on the beach at the point B vertically below A. He starts to run in a straight line with speed $v\text{ m s}^{-1}$, leaving B 0.4 seconds after the ball is thrown.

He catches the ball when it is 1 m above the beach.

(c) Find the value of v.

(4)

Question 102 continued

EXPERT TUITION

Question 102 continued

EXPERT
TUITION

Question 102 continued

EXPERT
TUITION

Question 102 continued

(Total 13 marks)

103. A particle moves along the x-axis. At time $t = 0$ the particle passes through the origin with speed 8 m s^{-1} in the positive x-direction. The acceleration of the particle at time t seconds, $t \geqslant 0$, is $(4t^3 - 12t)$ m s^{-2} in the positive x-direction.

Find

(a) the velocity of the particle at time t seconds,

(3)

(b) the displacement of the particle from the origin at time t seconds,

(2)

(c) the values of t at which the particle is instantaneously at rest.

(3)

EXPERT TUITION

Question 103 continued

EXPERT TUITION

Question 103 continued

EXPERT TUITION

Question 103 continued

(Total 8 marks)

EXPERT TUITION

104. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

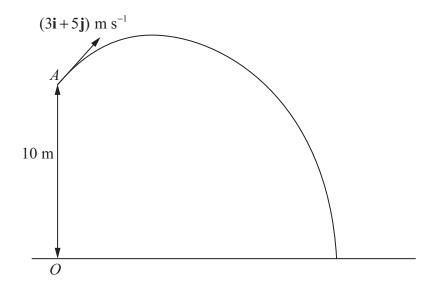$(3\mathbf{i}+5\mathbf{j})$ m s^{-1}

A

10 m

O

Figure 3

At time $t = 0$, a particle P is projected from the point A which has position vector $10\mathbf{j}$ metres with respect to a fixed origin O at ground level. The ground is horizontal. The velocity of projection of P is $(3\mathbf{i}+5\mathbf{j})$ m s^{-1}, as shown in Figure 3. The particle moves freely under gravity and reaches the ground after T seconds.

(a) For $0 \leqslant t \leqslant T$, show that, with respect to O, the position vector, **r** metres, of P at time t seconds is given by

$$\mathbf{r} = 3t\mathbf{i}+(10+5t-4.9t^2)\mathbf{j}$$

(3)

(b) Find the value of T.

(3)

(c) Find the velocity of P at time t seconds $(0 \leqslant t \leqslant T)$.

(2)

When P is at the point B, the direction of motion of P is 45° below the horizontal.

(d) Find the time taken for P to move from A to B.

(2)

(e) Find the speed of P as it passes through B.

(2)

EXPERT TUITION

Question 104 continued

EXPERT TUITION

Question 104 continued

EXPERT
TUITION

Question 104 continued

(Total 12 marks)

EXPERT TUITION

105. A particle P moves on the x-axis. The acceleration of P at time t seconds, $t \geqslant 0$, is $(3t + 5)$ m s^{-2} in the positive x-direction. When $t = 0$, the velocity of P is 2 m s^{-1} in the positive x-direction. When $t = T$, the velocity of P is 6 m s^{-1} in the positive x-direction. Find the value of T.

(6)

EXPERT TUITION

Question 105 continued

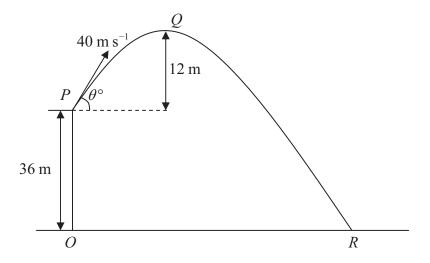**(Total 6 marks)**

EXPERT TUITION

106.

Figure 3

A ball is projected with speed 40 m s^{-1} from a point P on a cliff above horizontal ground. The point O on the ground is vertically below P and OP is 36 m. The ball is projected at an angle $\theta°$ to the horizontal. The point Q is the highest point of the path of the ball and is 12 m above the level of P. The ball moves freely under gravity and hits the ground at the point R, as shown in Figure 3. Find

(a) the value of θ,

(3)

(b) the distance OR

(6)

EXPERT TUITION

Question 106 continued

EXPERT
TUITION

Question 106 continued

(Total 9 marks)

EXPERT TUITION

107. A particle P moves along the x-axis. At time t seconds the velocity of P is $v\,\text{m}\,\text{s}^{-1}$ in the positive x-direction, where $v = 3t^2 - 4t + 3$. When $t = 0$, P is at the origin O. Find the distance of P from O when P is moving with minimum velocity.

(8)

EXPERT TUITION

Question 107 continued

(Total 8 marks)

EXPERT TUITION

108. [*In this question* **i** *and* **j** *are unit vectors in a horizontal and upward vertical direction respectively*]

A particle P is projected from a fixed point O on horizontal ground with velocity $u(\mathbf{i} + c\mathbf{j})\,\mathrm{m\,s}^{-1}$, where c and u are positive constants. The particle moves freely under gravity until it strikes the ground at A, where it immediately comes to rest. Relative to O, the position vector of a point on the path of P is $(x\mathbf{i} + y\mathbf{j})\,\mathrm{m}$.

(a) Show that

$$y = cx - \frac{4.9x^2}{u^2}.$$

(5)

Given that $u = 7$, $OA = R\,\mathrm{m}$ and the maximum vertical height of P above the ground is $H\,\mathrm{m}$,

(b) using the result in part (a), or otherwise, find, in terms of c,

(i) R

(ii) H.

(6)

Given also that when P is at the point Q, the velocity of P is at right angles to its initial velocity,

(c) find, in terms of c, the value of x at Q.

(6)

EXPERT TUITION

Question 108 continued

EXPERT
TUITION

Question 108 continued

EXPERT TUITION

Question 108 continued

EXPERT TUITION

109. At time $t = 0$ a particle P leaves the origin O and moves along the x-axis. At time t seconds the velocity of P is v m s^{-1}, where

$$v = 8t - t^2 .$$

(a) Find the maximum value of v.

(4)

(b) Find the time taken for P to return to O.

(5)

EXPERT TUITION

Question 109 continued

(Total 9 marks)

110. A particle P moves along the x-axis in a straight line so that, at time t seconds, the velocity of P is v m s^{-1}, where

$$v = \begin{cases} 10t - 2t^2, & 0 \leqslant t \leqslant 6, \\ \dfrac{-432}{t^2}, & t > 6. \end{cases}$$

At $t = 0$, P is at the origin O. Find the displacement of P from O when

(a) $t = 6$,

(3)

(b) $t = 10$.

(5)

EXPERT
TUITION

Question 110 continued

(Total 8 marks)

EXPERT
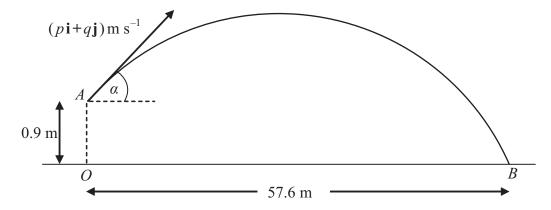TUITION

111.

Figure 3

A cricket ball is hit from a point A with velocity of $(p\mathbf{i} + q\mathbf{j})$ m s^{-1}, at an angle α above the horizontal. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are respectively horizontal and vertically upwards. The point A is 0.9 m vertically above the point O, which is on horizontal ground.

The ball takes 3 seconds to travel from A to B, where B is on the ground and $OB = 57.6$ m, as shown in Figure 3. By modelling the motion of the cricket ball as that of a particle moving freely under gravity,

(a) find the value of p,

(2)

(b) show that $q = 14.4$,

(3)

(c) find the initial speed of the cricket ball,

(2)

(d) find the exact value of $\tan \alpha$.

(1)

(e) Find the length of time for which the cricket ball is at least 4 m above the ground.

(6)

(f) State an additional physical factor which may be taken into account in a refinement of the above model to make it more realistic.

(1)

EXPERT TUITION

Question 111 continued

EXPERT
TUITION

Question 111 continued

(Total 15 marks)

112. A particle P of mass 0.5 kg is moving under the action of a single force $\mathbf{F}$ newtons. At time t seconds,
$$\mathbf{F} = (6t - 5)\,\mathbf{i} + (t^2 - 2t)\,\mathbf{j}.$$

The velocity of P at time t seconds is $\mathbf{v}$ m s^{-1}. When $t = 0$, $\mathbf{v} = \mathbf{i} - 4\mathbf{j}$.

Find $\mathbf{v}$ at time t seconds.

(6)

EXPERT
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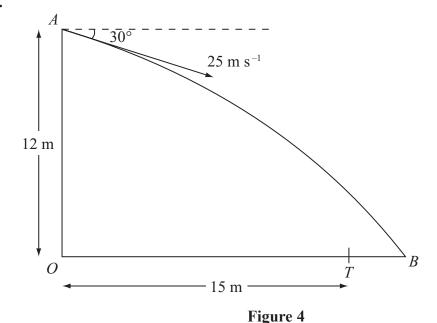
Question 112 continued

Question 112 continued

EXPERT TUITION

Question 112 continued

(Total 6 marks)

113.

Figure 4

A ball is thrown from a point *A* at a target, which is on horizontal ground. The point *A* is 12 m above the point *O* on the ground. The ball is thrown from *A* with speed 25 m s⁻¹ at an angle of 30° below the horizontal. The ball is modelled as a particle and the target as a point *T*. The distance *OT* is 15 m. The ball misses the target and hits the ground at the point *B*, where *OTB* is a straight line, as shown in Figure 4. Find

(a) the time taken by the ball to travel from *A* to *B*,

(5)

(b) the distance *TB*.

(4)

The point *X* is on the path of the ball vertically above *T*.

(c) Find the speed of the ball at *X*.

(5)

EXPERT TUITION

Question 113 continued

EXPERT
TUITION

Question 113 continued

EXPERT
TUITION

Question 113 continued

(Total 14 marks)

EXPERT
TUITION

114. At time t seconds ($t \geqslant 0$), a particle P has position vector $\mathbf{p}$ metres, with respect to a fixed origin O, where

$$\mathbf{p} = (3t^2 - 6t + 4)\mathbf{i} + (3t^3 - 4t)\mathbf{j}.$$

Find

(a) the velocity of P at time t seconds,

(2)

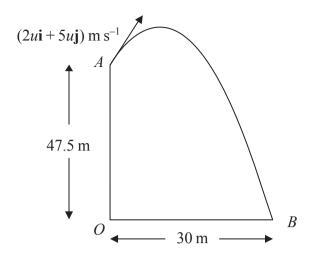(b) the value of t when P is moving parallel to the vector $\mathbf{i}$.

(3)

(Total 5 marks)

EXPERT
TUITION

115.

$(2u\mathbf{i} + 5u\mathbf{j})$ m s⁻¹

A

47.5 m

O

30 m

B

Figure 3

[*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertical.*]

A particle *P* is projected from the point *A* which has position vector 47.5**j** metres with respect to a fixed origin *O*. The velocity of projection of *P* is $(2u\mathbf{i} + 5u\mathbf{j})$ m s⁻¹. The particle moves freely under gravity passing through the point *B* with position vector 30**i** metres, as shown in Figure 3.

(a) Show that the time taken for *P* to move from *A* to *B* is 5 s.

(6)

(b) Find the value of *u*.

(2)

(c) Find the speed of *P* at *B*.

(5)

EXPERT TUITION

Question 115 continued

(Total 13 marks)

EXPERT
TUITION