Maths Questions By Topic:

Moments
Mark Scheme

A-Level Edexcel

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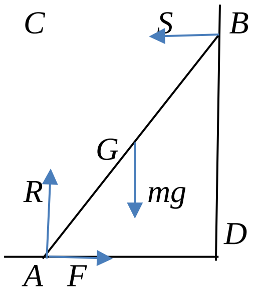
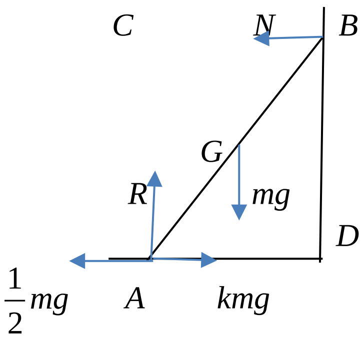
Question	Scheme	Marks	AOs
1(a)	The horizontal component of T acts to the left and since the **only** other horizontal force is friction, it must act to the right oe	B1	2.4
		(1)	
1(b)	Take moments about A or any other complete method to obtain **an equation in *T*, *M* and θ only.** (see possible equations below that they may use)	M1	3.1b
	$T.2a = Mga\cos\theta + 2Mg \times 1.5a\cos\theta$ (A0 if a's missing)	A1	1.1b
	Other possible equations but F and R would need to be eliminated. $(\nwarrow)$, $R\cos\theta + T = F\sin\theta + Mg\cos\theta + 2Mg\cos\theta$ $(\nearrow)$, $R\sin\theta + F\cos\theta = Mg\sin\theta + 2Mg\sin\theta$ $(\rightarrow)$, $F = T\sin\theta$ M(B), $R.2a\cos\theta = Mga\cos\theta + 2Mg \times 0.5a\cos\theta + F.2a\sin\theta$ M(G), $Fa\sin\theta + Ta = Ra\cos\theta + 2Mg \times 0.5a\cos\theta$ M(C), $R \times 1.5a\cos\theta = T \times 0.5a + Mg \times 0.5a\cos\theta + F \times 1.5a\sin\theta$		
	$T = 2Mg\cos\theta$ *	A1*	1.1b
		(3)	
1(c)	e.g. Resolve vertically	M1	3.4
	$(\uparrow)$, $R + T\cos\theta = Mg + 2Mg$	A1	1.1b
	$R = \dfrac{57Mg}{25}$ *	A1*	1.1b
		(3)	
	Other possible equations but F would need to be eliminated. $(\nwarrow)$, $R\cos\theta + T = F\sin\theta + Mg\cos\theta + 2Mg\cos\theta$ $(\nearrow)$, $R\sin\theta + F\cos\theta = Mg\sin\theta + 2Mg\sin\theta$ $(\rightarrow)$, $F = T\sin\theta$ M(B), $R.2a\cos\theta = Mga\cos\theta + 2Mg \times 0.5a\cos\theta + F.2a\sin\theta$ M(G), $Fa\sin\theta + Ta = Ra\cos\theta + 2Mg \times 0.5a\cos\theta$ M(C), $R \times 1.5a\cos\theta = T \times 0.5a + Mg \times 0.5a\cos\theta + F \times 1.5a\sin\theta$		
1(d)	Find an equation containing F e.g. Resolve horizontally	M1	3.4
	$(\rightarrow)$, $F = T\sin\theta$	A1	1.1b
	Other possible equations		

	$(\nwarrow), R\cos\theta + T = F\sin\theta + Mg\cos\theta + 2Mg\cos\theta$ $(\nearrow), R\sin\theta + F\cos\theta = Mg\sin\theta + 2Mg\sin\theta$ $(\rightarrow),\ F = T\sin\theta$ M(B), $R.2a\cos\theta = Mga\cos\theta + 2Mg \times 0.5a\cos\theta + F.2a\sin\theta$ M(G), $Fa\sin\theta + Ta = Ra\cos\theta + 2Mg \times 0.5a\cos\theta$ M(C), $R \times 1.5a\cos\theta = T \times 0.5a + Mg \times 0.5a\cos\theta + F \times 1.5a\sin\theta$		
	$F = \mu R$ used i.e. both F and R are substituted.	M1	3.1b
	$\mu = \dfrac{8}{19}$ *	A1*	2.2a
		(4)	
			(11 marks)

Notes:

1a	B1	Any equivalent explanation
1b	M1	Correct no. of terms, dimensionally correct, condone sin/cos confusion and sign errors
	A1	Correct equation, trig does not need to be substituted (Allow: $T.2a = Mga\cos\theta + 3Mga\cos\theta$)
	A1*	Given answer correctly obtained with <u>no wrong working seen</u>. Allow $2Mg\cos\theta = T$ But not $T = 2\cos\theta Mg$
1c	M1	For an equation in R, M, T and θ **only** Correct no. of terms, dimensionally correct, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation, T and trig do not need to be substituted
	A1*	Given answer correctly obtained with <u>no wrong working seen</u>
1d	M1	For any equation with F in it Correct no. of terms, dimensionally correct, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation, trig does not need to be substituted
	M1	Must be used i.e M0 if merely quoting it.
	A1*	Given answer correctly obtained with <u>no wrong working seen</u>

Question	Scheme	Marks	AOs
	Part (a) is a 'Show that..' so equations need to be given in full to earn A marks		
2(a)			
	Moments equation: (M1A0 for a moments inequality)	M1	3.3
	M(A), $mga\cos\theta = 2Sa\sin\theta$ M(B), $mga\cos\theta + 2Fa\sin\theta = 2Ra\cos\theta$ M(C), $F\times 2a\sin\theta = mga\cos\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + 2Sa\sin\theta$ M(G), $Ra\cos\theta = Fa\sin\theta + Sa\sin\theta$.	A1	1.1b
	($\updownarrow$) $R = mg$ **OR** ($\leftrightarrow$) $F = S$	B1	3.4
	Use their equations <u>(they must have enough)</u> and $F \le \mu R$ to give an inequality in μ and θ **only** (allow DM1 for use of $F = \mu R$ to give an *equation* in μ and θ only)	DM1	2.1
	$\mu \ge \frac{1}{2}\cot\theta$ *	A1*	2.2a
		(5)	
2(b)			
	Moments equation:	M1	3.4
	M(A), $mga\cos\theta = 2Na\sin\theta$ M(B), $mga\cos\theta + 2kmga\sin\theta = 2Ra\cos\theta + \frac{1}{2}mg2a\sin\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + N2a\sin\theta$ M(G), $kmga\sin\theta + Na\sin\theta = \frac{1}{2}mga\sin\theta + Ra\cos\theta$	A1	1.1b

	S.C. M(C), $mga\cos\theta + \frac{1}{2}mg2a\sin\theta = kmg2a\sin\theta$ **M1A1B1** $1+\frac{5}{4}=\frac{5k}{2}$ **M1** $k=0.9$ **A1**		
	$N = kmg - F$ **OR** $R = mg$	B1	3.3
	Use their equations (they must have enough) to solve for k (numerical)	DM1	3.1b
	$k = 0.9$ oe	A1	1.1b
		(5)	
			(10 marks)

Notes:

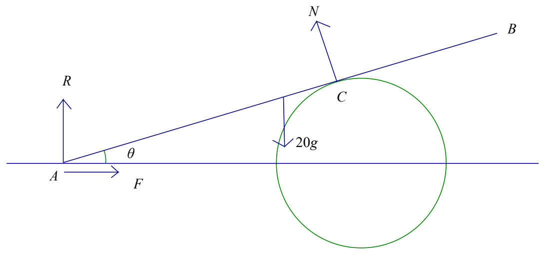
2a	M1	Any moments equation with correct terms, condone sign errors and sin/cos confusion
	A1	Correct equation
	B1	Correct equation
	DM1	Dependent on M1, for using their equations (they must have enough) and $F \le \mu R$ to give an inequality in μ and θ only (allow M1 for use of $F = \mu R$ to give an equation in μ and θ only)
	A1*	Given answer correctly obtained with no wrong working seen (e.g. if they use $F = \mu R$ anywhere, A0)
2b	M1	Any moments equation with correct terms, condone sign errors
	A1	Correct equation
	B1	Correct equation
	DM1	Dependent on M1, for using their equations (they must have enough) with trig substituted, to solve for k, which must be numerical.
	A1	cao

Question	Scheme	Marks	AOs
3(a)	Take moments about A	M1	3.3
	$N \times \dfrac{4a}{\sin\alpha} = Mg \times 3a\cos\alpha$	A1	1.1b
	$\dfrac{9Mg}{25}$ *	A1*	1.1b
		(3)	
3(b)	Resolve horizontally	M1	3.4
	$(\rightarrow)\ F = \dfrac{9Mg}{25}\sin\alpha$	A1	1.1b
	Resolve vertically	M1	3.4
	$(\uparrow)\, R + \dfrac{9Mg}{25}\cos\alpha = Mg$	A1	1.1b
	Other possible equations: $(\nwarrow),\, R\cos\alpha + \dfrac{9Mg}{25} = Mg\cos\alpha + F\sin\alpha$ $(\nearrow),\, Mg\sin\alpha = F\cos\alpha + R\sin\alpha$ $\text{M}(C),\, Mg.2a\cos\alpha + F.5a\sin\alpha = R.5a\cos\alpha$ $\text{M}(G),\, \dfrac{9Mg}{25}.2a + F.3a\sin\alpha = R.3a\cos\alpha$ $\text{M}(B),\, Mg.3a\cos\alpha + F.6a\sin\alpha = R.6a\cos\alpha + \dfrac{9Mg}{25}a$ $(F = \dfrac{36Mg}{125}\ ,\ R = \dfrac{98Mg}{125})$		
	$F = \mu R$ used	M1	3.4
	Eliminate R and F and solve for μ	M1	3.1b
	Alternative equations if they have at A: X horizontally and Y perpendicular to the rod. $(\nwarrow),\, Y + \dfrac{9Mg}{25} = Mg\cos\alpha + X\sin\alpha$ $(\nearrow),\, Mg\sin\alpha = X\cos\alpha$ $(\uparrow),\, \dfrac{9Mg}{25}\cos\alpha + Y\cos\alpha = Mg$ $(\rightarrow),\, Y\sin\alpha + \dfrac{9Mg}{25}\sin\alpha = X$		

	$\mathrm{M}(C), Mg.2a\cos\alpha + X.5a\sin\alpha = Y.5a$ $\mathrm{M}(G), \frac{9Mg}{25}.2a + X.3a\sin\alpha = Y.3a$ M1A1 M1A1 $\mathrm{M}(B), Mg.3a\cos\alpha + X.6a\sin\alpha = Y.6a + \frac{9Mg}{25}a$ $(X = \frac{4Mg}{3}, Y = \frac{98Mg}{75})$ Then $F = \mu R$ becomes: $X - Y\sin\alpha = \mu Y\cos\alpha$ M1 Eliminate X and Y and solve for μ M1		
	$\mu = \frac{18}{49}$ (0.3673…..accept 0.37 or better)	A1	2.2a
		(7)	
			(10 marks)

Notes:

3a	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors for an equation in N and Mg only. For perp distance allow any of : $\frac{4a}{\sin\alpha}, \frac{4a}{\cos\alpha}, 5a$ but use of any of : $6a, 5a\sin\alpha, 4a\cos\alpha, \ldots$ or anything involving $\tan\alpha$ is M0 Also M0 if no a's in their first equation.
	A1	Correct equation, trig does not need to be substituted
	A1*	Given answer correctly obtained.
3b	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation, trig does not need to be substituted but N does.
	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation, trig does not need to be substituted but N does.
		N.B. The above 4 marks are for any two equations, either resolutions or moments or one of each. Mark best two equations. Equations may appear in part (a) but must be used in (b) to earn marks.
	M1	Must be used, e.g. seen on the diagram. i.e. M0 if merely quoting it. (M0 if $F = \mu \times \frac{9Mg}{25}$ used)
	M1	Must have 3 equations (<u>and all 3 previous M marks</u>)
	A1	Accept 0.37 or better

Question	Scheme	Marks	AO
4(a)	Drum **smooth**, or no friction, (therefore reaction is perpendicular to the ramp)	B1	2.4
		(1)	
(b)	**N.B.** In (b), for a moments equation, if there is an extra $\sin\theta$ or $\cos\theta$ on a length, give M0 for the equation e.g. $\text{M}(A)$: $20g\times 4\cos\theta = 5N\sin\theta$ would be given M0A0		
			
	Possible equns $(\nearrow): F\cos\theta + R\sin\theta = 20g\sin\theta$ $(\nwarrow): N + R\cos\theta = 20g\cos\theta + F\sin\theta$ $(\uparrow) R + N\cos\theta = 20g$ $(\rightarrow): F = N\sin\theta$ $\text{M}(A)$: $20g\times 4\cos\theta = 5N$ $\text{M}(B)$: $3N + R\times 8\cos\theta = F\times 8\sin\theta + 20g\times 4\cos\theta$ $\text{M}(C)$: $R\times 5\cos\theta = F\times 5\sin\theta + 20g\times\cos\theta$ $\text{M}(G)$: $R\times 4\cos\theta = F\times 4\sin\theta + N$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $F = 42.14784$; $R = 51.49312$)		
	Alternative 1: using cpts along ramp (X) and perp to ramp(Y) Possible equations: $(\nearrow): X = 20g\sin\theta$ $(\nwarrow): Y + N = 20g\cos\theta$ $(\uparrow): X\sin\theta + Y\cos\theta + N\cos\theta = 20g$ $(\rightarrow): X\cos\theta = Y\sin\theta + N\sin\theta$ $\text{M}(A): 20g\times 4\cos\theta = 5N$ $\text{M}(B): 20g\times 4\cos\theta = 8Y + 3N$ $\text{M}(C): 20g\times\cos\theta = 5Y$ $\text{M}(G): 4Y = N\times 1$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $X = 54.88$; $Y = 37.632$)		

	Alternative 2: using horizontal cpt (*H*) and cpt perp to ramp (*S*)		
	$(\nearrow): H\cos\theta = 20g\sin\theta$	M1	3.3
	$(\nwarrow): S + N = H\sin\theta + 20g\cos\theta$	A1	1.1b
	$(\uparrow): S\cos\theta + N\cos\theta = 20g$ $(\rightarrow): H = S\sin\theta + N\sin\theta$	M1	3.4
	$\text{M}(A): 20g \times 4\cos\theta = 5N$	A1	1.1b
	$\text{M}(B): 20g \times 4\cos\theta + H \times 8\sin\theta = 8S + 3N$ $\text{M}(C): 20g \times \cos\theta + H \times 5\sin\theta = 5S$	M1	3.4
	$\text{M}(G): 4S = N \times 1 + H \times 4\sin\theta$	A1	1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $H = 57.1666\ldots$; $S = 53.638666\ldots$)		
	Solve their 3 equations for F and R **OR** X and Y **OR** H and S	M1	1.1b
	$\lvert\text{Force}\rvert = \sqrt{R^2 + F^2}$ Main scheme **OR** $= \sqrt{X^2 + Y^2}$ Alternative 1 **OR** $= \sqrt{(H^2 + S^2 - 2HS\cos(90^o - \theta)}$ Alternative 2	M1	3.1b
	Magnitude = 67 or 66.5 (N)	A1	2.2a
		(9)	
(c)	Magnitude of the normal reaction (at C) will **decrease**.	B1	3.5a
		(1)	
		(11)	

Marks		Notes
4a	B1	Ignore any extra incorrect comments.
		Generally 3 independent equations required so **at least one moments equation.**: M1A1M1A1M1A1. More than 3 equations, give marks for the best 3. For each: M1 All terms required. Must be dimensionally correct so if a length is missing from a moments equation it's M0 Condone sin/cos confusion. A1 For a correct equation (trig ratios do not need to be substituted and allow e.g. cos(24/25) if they recover <u>Enter marks on ePEN in order in which equations appear</u>. **N.B.** If reaction at *C* is not perpendicular to the ramp, can only score marks for M(*C*) Allow use of (μR) for *F*
4b	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required, dim correct, condone sin/cos confusion
	A1	Correct unsimplified equation
		N.B. They can find *F* and *R* using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
Alt 1	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
		N.B. They can find *X* and *Y* using only TWO equations, the 1^{st} and 7^{th} in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
Alt 2	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.

	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct.
	A1	Correct unsimplified equation
		N.B. They can find *H* and *S* using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
	M1	Substitute for trig and solve for their two cpts. This is an independent mark but must use 3 equations (unless it's the special case when 2 is sufficient)
	M1	Use Pythagoras to find magnitude (this is an independent M mark but must have found a value for *F* (or *X*) and a value for *R* (or *Y*)) **OR** a complete method to find magnitude e.g. cosine rule but must have found a value for *H* and a value for *S*
	A1	Correct answer only
	B1	Ignore reasons

Question	Scheme	Marks	AOs
5(a)	Moments about A (or any other complete method)	M1	3.3
	$T2a\sin\alpha = Mga + 3Mgx$	A1	1.1b
	$T = \dfrac{Mg(a+3x)}{2a\times\frac{3}{5}} = \dfrac{5Mg(3x+a)}{6a}$ * GIVEN ANSWER	A1*	2.1
		(3)	
(b)	$\dfrac{5Mg(3x+a)}{6a}\cos\alpha = 2Mg$ **OR** $2Mg.2a\tan\alpha = Mga + 3Mgx$	M1	3.1b
	$x = \dfrac{2a}{3}$	A1	2.2a
		(2)	
(c)	Resolve vertically **OR** Moments about B	M1	3.1b
	$Y = 3Mg + Mg - \dfrac{5Mg(3.\frac{2a}{3}+a)}{6a}\sin\alpha$ $2aY = Mga + 3Mg(2a - \frac{2a}{3})$ **Or**: $Y = 3Mg + Mg - \left(\dfrac{2Mg}{\cos\alpha}\right)\sin\alpha$	A1**ft**	1.1b
	$Y = \dfrac{5Mg}{2}$ **N.B.** May use $R\sin\beta$ for Y and/or $R\cos\beta$ for X throughout	A1	1.1b
	$\tan\beta = \dfrac{Y}{X}$ or $\dfrac{R\sin\beta}{R\cos\beta}$ $= \dfrac{\frac{5Mg}{2}}{2Mg}$	M1	3.4
	$= \dfrac{5}{4}$	A1	2.2a
		(5)	
(d)	$\dfrac{5Mg(3x+a)}{6a} \le 5Mg$ and solve for x	M1	2.4
	$x \le \dfrac{5a}{3}$	A1	2.4
	For rope not to break, block can't be more than $\dfrac{5a}{3}$ from A oe Or just: $x \le \dfrac{5a}{3}$, if no incorrect statement seen. **N.B. If the correct inequality is not found,** their comment must mention 'distance from A'.	B1 **A1**	2.4
		(3)	
		(13 marks)	

Notes:
(a) **M1**: Using M(A), with usual rules, or any other complete method to obtain an equation in a, M, x and T only. **A1:** Correct equation **A1*:** Correct PRINTED ANSWER, correctly obtained, need to see $\sin\alpha = \frac{3}{5}$ used.
(b) **M1:** Using an appropriate strategy to find x. e.g. Resolve horizontally with usual rules applying OR Moments about C. Must use the <u>given</u> expression for T. **A1:** Accept $0.67a$ or better
(c) **M1:** Using a complete method to find Y (or $R\sin\beta$) e.g. resolve vertically or Moments about B, with usual rules **A1 ft:** Correct equation <u>with their x substituted in T expression</u> or using $T = \frac{2Mg}{\cos\alpha}$ **A1:** Y (or $R\sin\beta$) $= \frac{5Mg}{2}$ or $2.5Mg$ or $2.50Mg$ **M1:** For finding an equation **in tan** β **only** using $\tan\beta = \frac{Y}{X}$ or $\tan\beta = \frac{X}{Y}$ This is independent but must have found a Y. **A1:** Accept $\frac{-5}{4}$ if it follows from their working.
(d) **M1:** Allow $T = 5Mg$ or $T < 5Mg$ and solves for x, showing all necessary steps (M0 for $T > 5Mg$) **A1:** Allow $x = \frac{5a}{3}$ or $x < \frac{5a}{3}$. Accept $1.7a$ or better. **B1: Treat as A1.** For any appropriate equivalent fully correct comment or statement. E.g. maximum value of x is $\frac{5a}{3}$

Question	Scheme	Marks	AOs
6(a)	Moments about A (or any other complete method)	M1	3.3
	$T\cos30^{\circ} \times (1\sin30^{\circ}) = 20g \times 1.5$	A1	1.1.b
	$T\cos30^{\circ} \times (1\sin30^{\circ}) = 20g \times 1.5$	A1	1.1.b
	$T =$ 679 or 680 (N)	A1	1.1.b
		(4)	
(b)	Resolve horizontally	M1	3.1b
	$X = T\cos60^{\circ}$	A1	1.1b
	Resolve vertically	M1	3.1b
	$Y = T\cos30^{\circ} - 20g$	A1	1.1b
	Use of $\tan q = \dfrac{Y}{X}$ and sub for T	M1	3.4
	49° (or better), below horizontal, away from wall	A1	2.2a
		(6)	
(c)	Tension would increase as you move from D to C	B1	3.5a
	Since each point of the rope has to support the length of rope below it	B1	2.4
		(2)	
(d)	Take moments about G, $1.5Y = 0$	M1	3.3
	$Y = 0$ hence force acts horizontally.*	A1*	2.2a
		(2)	
		(14 marks)	

Notes:

(a)
M1: Correct overall strategy e.g. M(A), with usual rules, to give equation in T only
A1: (A1A0 one error) Condone 1 error
A1: (A0A0 two or more errors)
A1: Either 679 or 680 (since $g = 9.8$ used)

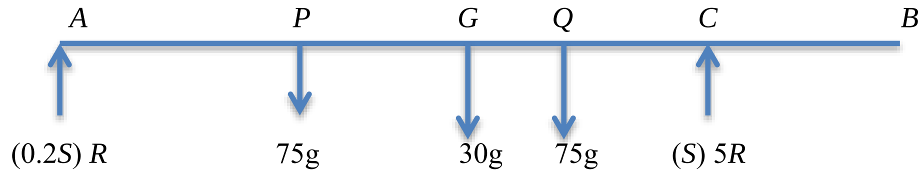
(b)
M1: Using an appropriate strategy to set up first of two equations, with usual rules applying e.g. Resolve horiz. or M(C)
A1: Correct equation in X only
M1: Using an appropriate strategy to set up second of two equations, with usual rules applying e.g. Resolve vert. or M(D)
A1: Correct equation in Y only

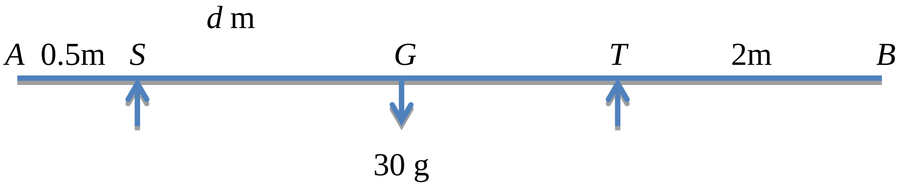
M1: Using the model and their X and Y **A1:** 49 or better (since g cancels) Need all three bits of answer to score this mark or any other appropriate angle e.g 41° to wall, downwards and away from wall
(c) **B1:** Appropriate equivalent comment **B1:** Appropriate equivalent reason
(d) **M1:** Using the model and any other complete method e.g. the three force condition for equilibrium **A1*:** Correct conclusion GIVEN ANSWER

Question	Scheme	Marks	AOs
7(a)	Take moments about A (or any other complete method to produce an equation in S, W and α only)	M1	3.3
	$Wa\cos\alpha + 7W2a\cos\alpha = S\,2a\sin\alpha$	A1 A1	1.1b 1.1b
	Use of $\tan\alpha = \frac{5}{2}$ to obtain S	M1	2.1
	$S = 3W$ *	A1*	2.2a
		(5)	
(b)	$R = 8W$	B1	3.4
	$F = \frac{1}{4}R\ (= 2W)$	M1	3.4
	$P_{\text{MAX}} = 3W + F$ or $P_{\text{MIN}} = 3W - F$	M1	3.4
	$P_{\text{MAX}} = 5W$ or $P_{\text{MIN}} = W$	A1	1.1b
	$W \leq P \leq 5W$	A1	2.5
		(5)	
(c)	M(A) shows that the reaction on the ladder at B is unchanged	M1	2.4
	also R increases (resolving vertically)	M1	2.4
	which increases max F available	M1	2.4
		(3)	
			(13 marks)

Question 7 continued
Notes:
(a) **1st M1:** for producing an equation in S, W and α only **1st A1:** for an equation that is correct, or which has one error or omission **2nd A1:** for a fully correct equation **2nd M1:** for use of $\tan\alpha = \frac{5}{2}$ to obtain S in terms of W only **3rd A1*:** for given answer $S = 3W$ correctly obtained
(b) **B1:** for $R = 8W$ **1st M1:** for use of $F = \frac{1}{4}R$ **2nd M1:** for either $P = (3W + \text{their } F)$ or $P = (3W - \text{their } F)$ **1st A1:** for a correct max or min value for a correct range for P **2nd A1:** for a correct range for P
(c) **1st M1:** for showing, by taking moments about A, that the reaction at B is unchanged by the builder's assistant standing on the bottom of the ladder **2nd M1:** for showing, by resolving vertically, that R increases as a result of the builder's assistant standing on the bottom of the ladder **3rd M1:** for concluding that this increases the limiting friction at A

Question Number	Scheme	Marks
8.(a)	$M(D),\ (150g\times 1)+(60g\times 2.5)=Tc\times 4$	M1 A1
	$Tc=75g$ or 735 N or 740 N Allow omission of N	A1 (3)
(b)	$M(B),\ (150g\times 4.5)+(60g\times 6)=T_D\times 3.5$	M1 A2
	$T_D=2900\text{ N}$ or $\dfrac{2070g}{7}$ Allow omission of N	A1 (4)
		(7)
	Notes for Qu 8	
	8(a) M1 for a complete method to find Tc (M0 if they assume $T_C = T_D$) i.e. for producing an equation in Tc only. Each equation used must have correct no. of terms and be dimensionally correct. First A1 for correct equation. Second A1 for any of the 3 possible answers <u>Other possible equations</u>: $(\uparrow), Tc+T_D=60g+150g$ $M(A),\ (150g\times 4.5)+(60g\times 3)=(Tc\times 1.5)+(T_D\times 5.5)$ $M(C),\ (150g\times 3)+(60g\times 1.5)=T_D\times 4$ $M(B),\ (150g\times 4.5)+(60g\times 6)=(Tc\times 7.5)+(T_D\times 3.5)$ $M(G),\ (T_D\times 1)+(60g\times 1.5)=T_C\times 3$	
	8(b) **N.B.** <u>(M0 if T_C is never equated to 0</u>) M1 for a complete method to obtain an equation in T_D only. If they use more than one equation, each equation used must have correct no. of terms and be dimensionally correct. First and second A1 for a correct equation in T_D only. A1A0 if one error.Consistent omission of *g* is one error except in *M(D)* where it's not an error. Third A1 for either answer <u>Other possible equations</u>: $(\uparrow), T_D=60g+150g+Mg$ $M(A),\ (150g\times 4.5)+(60g\times 3)+9Mg=T_D\times 5.5$ $M(C),\ (150g\times 3)+(60g\times 1.5)+7.5Mg=T_D\times 4$ $M(D),\ (150g\times 1)+(60g\times 2.5)=3.5Mg$ $M(G),\ (T_D\times 1)+(60g\times 1.5)=4.5Mg$	

Question Number	Scheme	Marks
9(a)	 $(-)\ R+5R=75g+30g+75g$ $M(A)\quad 75gx+75g2x+30g\times 3=5R\times 4$ $x=\frac{34}{15}=2.3$ or better (N.B. Or another Moments Equation)	M1 A2 M1 A2 A1 **(**M1 A2) (7)
(b)	uniform – mass is or acts at midpoint of plank; centre of mass is at middle of plank; weight acts at the middle of the plank, centre of gravity is at midpoint rod - plank does not bend, remains straight, is inflexible, is rigid	B1 B1 (2) **9**
	Notes	
(a)	First M1 for either a vertical resolution (with correct of terms) or a moments equation (all terms dim correct and correct no. of terms) First A1 and Second A1 for a correct equation in R (or S where $S = 5R$) only or R and x only or S and x only. (- 1 each error , A1A0 or A0A0) Second M1 for a moments equation (all terms dim correct and correct no. of terms) Third A1 and Fourth A1 for a correct equation in R (or S where $S = 5R$) only or R and x only or S and x only. (- 1 each error, A1A0 or A0A0) Fifth A1 for $x = {}^{34}/_{15}$ oe or 2.3 (or better) **(i)** In a moments equation, if R and $5R$ (or S and $0.2S$) are interchanged, treat as 1 error. **(ii)** Ignore diagram if it helps the candidate. **(iii)** If an equation is correct but contains both R and S, *or S = 5R is never used*, treat as 1 error. **(iv)** Full marks possible if all g's omitted. **(v)** For inconsistent omission of g, penalise each omission. $M(B),\ R\times 6+5R\times 2=75g(6-x)+75g(6-2x)+30g\times 3$ $M(C),\ 75g(4-x)+75g(4-2x)+30g\times 1=R\times 4$ $M(G),\ 75g(3-x)+5R\times 1=R\times 3+75g(2x-3)$ $M(P),\ Rx+30g(3-x)+75gx=5R(4-x)$ $M(Q),\ 75gx+30g(2x-3)+5R(4-2x)=R\times 2x$	
(b)	First B1 for first correct answer seen. Second B1 for the other answer, but only award this second mark if no extras given.	

Question Number	Scheme	Marks
10.	 $M(S)$: $Mg \times 0.5 = 30g(d - 0.5)$ $M(T)$: $Mg \times 2 = 30g(4 - d)$ dividing: $4 = \dfrac{(4 - d)}{(d - 0.5)}$ => (i) $d = 1.2$ => (ii) $M = 42$	M1 A1 M1 A1 **DM1** A1 A1 7
10.	**Notes** **N.B.** They may use a different variable, other than *d*, in their moments equations e.g. say they use $x = SG$ consistently, they can score all the marks for their two equations and if they eliminate x correctly, DM1 A1 (for M), and, if they found x correctly, then added 0.5 to obtain d, the other A1 also.	
	First M1 for moments about S (need correct no. of terms, so if they don't realise that the reaction at T is zero it's M0) *to give an equation in d and M only*.	
	First A1 for a correct first equation *in d and M only*. (A1 for both g's or no g's but A0 if one g is missing)	
	N.B. They may use 2 equations and eliminate to obtain their equation *in d and M only* e.g. $M(A)$ $0.5R_S = 30gd$ and (^) $R_S = 30g + Mg$ and then eliminate R_S. The M mark is only earned once they have produced an equation *in d and M only*, with all the usual rules about correct no. of terms etc applying to all the equations they use to obtain it.	
	Second M1 for moments about T (need correct no. of terms, so if they don't realise that the reaction at S is zero it's M0) *to give an equation in d and M only*	
	Second A1 for a correct second equation *in d and M only*. (A1 for both g's or no g's but A0 if one g is missing)	
	N.B. They may use 2 equations and eliminate to obtain their equation *in d and M only* e.g. $M(B)$ $2R_T = 30g(6 - d)$ and (^) $R_T = 30g + Mg$ and then eliminate R_T. The M mark is only earned once they have produced an equation *in d and M only*, with all the usual rules about correct no. of terms etc applying to all the equations they use to obtain it.	

	Third M1, dependent on 1st and 2nd M marks, for eliminating either M or d to produce an equation in either d only or M only.	
	Third A1 for (d =) 1.2 oe (**N.B.** Neither this A mark nor the next one can be awarded if there are any errors in the equations.) **Beware:** If one g is missing consistently from each of their equations, they can obtain d = 1.2 but award A0	
	Fourth A1 for (M =) 42	
	Scenario 1: Below are the possible equations, (if they don't use *M(S)*), any two of which can be used, by eliminating R_S , to obtain an equation *in d and M only* , for the first M1. N.B. If R_T appears in any of these and doesn't subsequently become zero then it's M0.	
	M(A) $0.5R_S = 30gd$	
	M(B) $5.5R_S = 30g(6 - d) + 6Mg$	
	M(T) $3.5R_S = 30g(4 - d) + 4Mg$	
	(^) $R_S = 30g + Mg$	
	Scenario 2: Below are the possible equations, (if they don't use *M(T)*), any two of which can be used, by eliminating R_T , to obtain an equation *in d and M only* , for the second M1. N.B. If R_S appears in any of these and doesn't subsequently become zero then it's M0.	
	M(A) $4R_T = 30gd + 6Mg$	
	M(B) $2R_T = 30\mathrm{g}(6 - d)$	
	M(S) $3.5R_T = 30g(d - 0.5) + 5.5Mg$	
	(^) $R_T = 30g + Mg$	

Question Number	Scheme	Marks
11(a)	$T_A + T_C = 85g$ OR $M(A)$, $25g \times 2.5 + 60g \times 5 = 4.5 \times T_C$ OR $M(C)$, $T_A \times 4.5 + 60g \times 0.5 = 25g \times 2$ OR $M(B)$, $T_A \times 5 + T_C \times 0.5 = 25g \times 2.5$ OR $M(G)$, $T_A \times 2.5 + 60g \times 2.5 = 2 \times T_C$ $T_A = \frac{40g}{9} = 44\text{N or } 43.6\text{N}$; $T_C = \frac{725g}{9} = 790\text{N or } 789\text{ N}$	M1 A1 M1 A1 A1; A1 (6)
(b)	$\text{M}(C)$, $25g \times 2 = Mg \times 0.5$	M1 A1
(i)	$M = 100$	A1
(ii)	$T_c = 25g + 100g$ $T_c = 125g$ $(1200 \text{ or } 1230)\text{N}$	M1 A1 B1 (6) **12**
	Notes	
11(a)	First M1 for a moments or vertical resolution equation, with correct no. of terms and dimensionally correct. First A1 for a correct equation. Second M1 for a moments equation, with correct no. of terms and dimensionally correct. Second A1 for a correct equation. Third A1 for 44 (N) or 43.6 (N) or 40g/9 Fourth A1 for 790 (N) or 789 (N) or 725g/9 Deduct 1 mark for inexact multiples of *g* **N.B.** If they assume that both tensions are the same, can only score max M1 in (a) for *M(A)* or *M(C)*. If a vertical resolution is used, please give marks for this equation FIRST. If not, enter marks for each moments equation in the order in which they appear.	
11(b)	**SCHEME CHANGE** B1 BECOMES THE FOURTH A1 First M1 for a moments equation with $T_A = 0$ First A1 for a correct equation Second A1 for $M = 100$ Second M1 for a(nother) moments or vertical resolution equation with $T_A = 0$ Third A1 for a correct equation Fourth A1 (B1) for $T_C = 125g$ or 1230 (N) or 1200 (N) *N.B. Some candidates may need to solve 2 simult. equations in M and* T_C *and so will earn the 'equation' marks before they earn Second and Fourth A (B) marks.* If a vertical resolution is used, please give marks for this equation SECOND. If not, enter marks for each moments equation in the order	

	in which they appear. The possible equations are: $T_C = 25g + Mg$ M(C), $25g \times 2 = Mg \times 0.5$ M(A), $25g \times 2.5 + 5Mg = 4.5\ T_C$ M(B), $25g \times 2.5 = T_C \times 0.5$ M(G), $T_C \times 2 = Mg \times 2.5$ Any two of these can each earn M1A1 (M0 if incorrect no. of terms) Then Second A1 for $M = 100$ And Fourth A1 (B1) for T_C = 125g or 1230 or 1200 **N.B.** No marks in (b) if they use any answers from (a) or $M = 60$	

Question Number	Scheme	Marks
12a	Resolving vertically: $T+2T\left(=3T\right)=W$	M1A1
	Moments about A: $2W=2T\times d$	M1A1
	Substitute and solve: $2W=2\frac{W}{3}d$	**DM1**
	$d=3$	A1 (6)
b	Resolving vertically: $T+4T=W+kW \quad \left(5T=W\left(1+k\right)\right)$	M1A1 **ft**
	Moments about A: $2W+4kW=3\times 4T$	M1A1 **ft**
	Substitute and solve: $2W+4kW=\frac{12}{5}W\left(1+k\right)$	**DM1**
	$2+4k=\frac{12}{5}+\frac{12}{5}k$	
	$\frac{8}{5}k=\frac{2}{5}, \qquad k=\frac{1}{4}$	A1 (6)
		[12]

Notes for Question 12

<u>N.B.</u> In moments equations, for the M mark, all terms must be force x distance but take care in the cases when the distance is 1.

<u>Question 12(a)</u>

N.B. If *Wg* is used, mark as a misread. *If T and 2T are reversed, mark as per scheme NOT as a misread.*

First M1 for an equation in *W* and *T* and possibly *d* (either resolve vertically or moments about any point other than the mid-pt), with usual rules.

First A1 for a correct equation.

Second M1 for an equation in *W* and *T* and possibly *d* (either resolve vertically or moments about any point other than the mid-pt), with usual rules.

Second A1 for a correct equation.

Third M1, dependent on first and second M marks, for solving for *d*

Third A1 for d = 3 cso

N.B. If a single equation is used (see below) by taking moments about the mid-point of the rod, $2T = 2T(d-2)$, this scores M2A2 (-1 each error)

Third M1, dependent on first and second M marks, for solving for *d*

Third A1 for d = 3 cso

<u>Question 12(b)</u>

N.B. If *Wg* and *kWg* are used, mark as a misread.

If they use any results from (a), can score max M1A1 in (b) for one equation.

If T and 4T are reversed, mark as per scheme NOT as a misread.

First M1 for an equation in *W* and a tension T_1 and possibly their *d* or their *d* and *k* (either resolve vertically or moments about any point), with usual rules.

First A1 **ft** on their *d*, for a correct equation.

Second M1 for an equation in *W* and **the same tension** T_1 and possibly their *d* or their *d* and *k* (either resolve vertically or moments about any point), with usual rules.

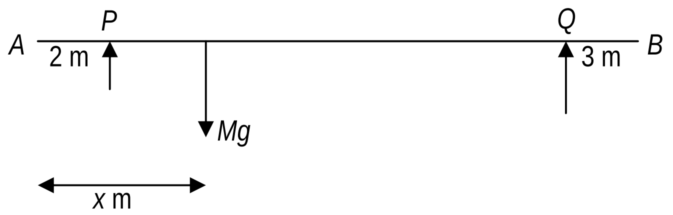
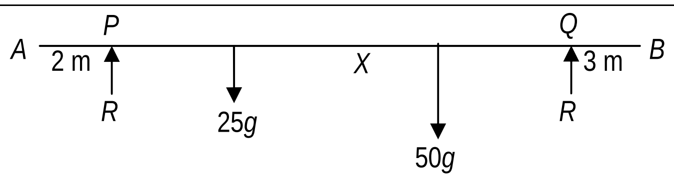
Second A1 **ft** on their *d*, for a correct equation.

Third M1, dependent on first and second M marks, for solving to give a numerical value of *k*

Third A1 for k = 1/4 *oe* cso

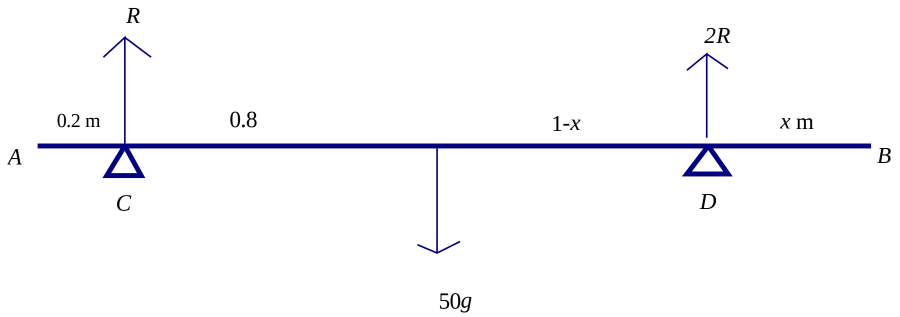
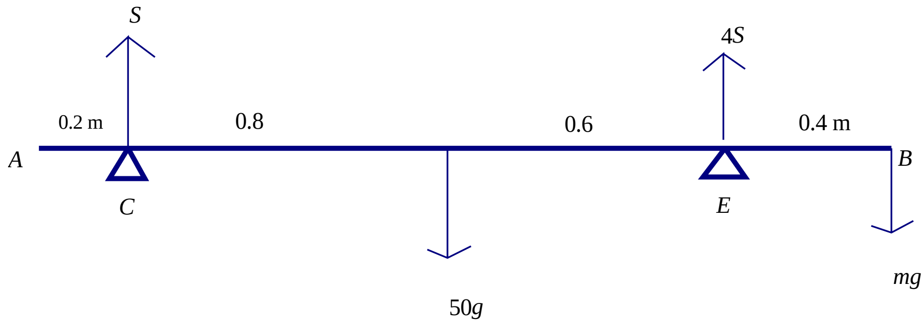
Question Number	Scheme	Marks
13a	Resolving vertically: $T + 2T\left(=3T\right) = W$	M1A1
	Moments about B: $2 \times 2T = \left(d-1\right)W$	M1A1
	Substitute and solve for d : $2 \times 2T = \left(d-1\right)3T$	**D**M1
	$d = \dfrac{7}{3}$ (m)	A1 (6)
13b	Moments about C: $(T_B \times 2) + (kW \times 1) = W \times \dfrac{2}{3}$	M1A1
	$T_B = W\dfrac{\left(2-3k\right)}{6}$ or equivalent	A1 (3)
13c	solving $T_B \geq 0$ or $T_B > 0$ for k.	M1
	$0 < k \leq 2/3$ or $0 < k < 2/3$ only	A1 (2)
		[11]

Notes for Question 13
<u>**Question 13(a)**</u> **N.B.** If Wg is used, mark as a misread. First M1 for an equation in W and T and possibly d (either resolve vertically or moments about any point other than the centre of mass of the rod), with usual rules. First A1 for a correct equation. Second M1 for an equation in W and T and possibly d (either resolve vertically or moments about any point other than the centre of mass of the rod), with usual rules. Second A1 for a correct equation. **N.B.** The above 4 marks can be scored if their d is measured from a different point Third M1, dependent on first and second M marks, for solving for d Third A1 for $d = 7/3$, 2.3 (m) or better **N.B. Alternative** If a single equation is used (see below) by taking moments about the centre of mass of the rod, $2T(3 - d) = T(d - 1)$, this scores M2A2 (-1 each error) Third M1, dependent on first and second M marks, for solving for d Third A1 for $d = 7/3$ <u>**Question 13(b)**</u> First M1 for producing an equation in T_B and W *only*, either by taking moments about C, or using two equations and eliminating First A1 for a correct equation Second A1 for $W(2 - 3k)/6$ oe. <u>**N.B.**</u> **M0** if they use any information about the tension(s) from part (a). <u>**Question 13(c)**</u> M1 for solving $T_B \geq 0$ or $T_B > 0$ for k. A1 for $0 < k \leq 2/3$ or $0 < k < 2/3$ only. **N.B.** $T = 0 \Rightarrow k = 2/3$ then answer is M0. If they also solve $T_C \geq 0$ or $T_C > 0$, can still score M1 and possibly A1.

Question Number	Scheme	Marks
14. **(a)**		
	M(P), $50g\times 2 = Mg\times(x-2)$	M1 A1
	M(Q), $50g\times 3 = Mg\times(12-x)$	M1 A1
(i)	$M = 25$ (kg)	**DM**1 A1
(ii)	$x = 6$ (m)	**DM**1 A1
		(8)
(b)		
	$(\uparrow) R + R = 25g + 50g$	M1 A1 **ft**
	M(A), $2R + 12R = 25g\times 6 + 50g\times AX$	M1 A1 **ft**
	$AX = 7.5$ (m)	**DM**1 A1
		(6)
		[14]

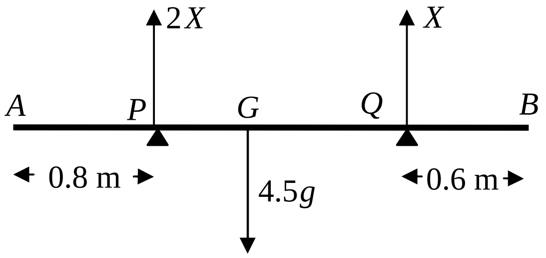
EXPERT TUITION

Notes for Question 14		
Q14(a)	First M1 for moments about P equation with usual rules (or moments about a different point AND vertical resolution and R then eliminated) (M0 if non-zero reaction at Q) Second M1 for moments about Q equation with usual rules (or moments about a different point AND vertical resolution) (M0 if non-zero reaction at P) Second A1 for a correct equation in M and same unknown. Third M1, dependent on first and second M marks, for solving for M Third A1 for 25 (kg) Fourth M1, dependent on first and second M marks, for solving for x Fourth A1 for 6 (m) <u>N.B. No marks available if rod is assumed to be uniform but can score max 5/6 in part (b), provided they have found values for *M* and *x* to f.t. on.</u> If they have just invented values for M and x in part (a), they can score the M marks in part (b) but <u>not</u> the A marks.	
Q14(b)	First M1 for vertical resolution or a moments equation, with usual rules. First A1 **ft** on their M and x from part (a), for a correct equation. (must have *equal reactions* in vertical resolution to earn this mark) Second M1 for a moments equation with usual rules. Second A1 **ft** on their M and x from part (a), for a correct equation in R and same unknown length. Third M1, dependent on first and second M marks, for solving for AX *(not their unknown length)* with $AX \leq 15$ Third A1 for $AX = 7.5$ (m) N.B. If a single equation is used (see below), equating the sum of the moments of the child and the weight about P to the sum of the moments of the child and the weight about Q, this can score M2 A2 **ft** on their M and x from part (a), provided the equation is in one unknown. Any method error, loses both M marks. e.g. $25g.4 + 50g(x-2) = 25g.6 + 50g(12-x)$ oe.	

Question Number	Scheme	Marks
15. **(a)**		
	Vertical equilibrium: $R+2R=50g$, Moments about C: $50g\times 0.8=(1.8-x)\times 2\times R$ $3\times 0.8=3.6-2x$, $x=0.6$	M1A1 M1A1 **DM1**A1 **(6)**
(b)		
	S, $4S$ Vertical equilibrium: $S+4S=(50+m)g=5S$ Moments about B: $50g\times 1=4S\times 0.4+S\times 1.8=3.4S$ $50\times\frac{5}{3.4}=(50+m)$ m = 400/17, 24, 23.5 or better	B1 M1A1 M1A1 **DM1** A1 **(7)** **[13]**

Notes for Question 15		
15(a)	**In both parts consistent omission of g's can score all the marks.** First M1 for vertical resolution or a moments equation, with usual rules. (allow *R* and *N* at this stage) First A1 for a correct equation (with $N = 2R$ substituted) Second M1 for a moments equation in *R* and one unknown length with usual rules. Second A1 for a correct equation. Third M1, dependent on first and second M marks, for solving for x Third A1 for $x = 0.6$. S.C. Moments about centre of rod: $R \times 0.8 = 2R(1 - x)$ M2 A2	
15(b)	B1 for S and $4S$ placed correctly. First M1 for vertical resolution or a moments equation, with usual rules. (allow S and $4S$ reversed) First A1 for a correct equation. Second M1 for a moments equation in S (and m) with usual rules. Second A1 for a correct equation. Third M1, dependent on first and second M marks, for *eliminating S* to give an equation in *m only*. Third A1 for $m = 400/17$ oe or 24 or better. N.B. SC If they use the reaction(s) found in part (a) in their equations, can score max B1M1A0M1A0DM0A0.	

Question Number	Scheme	Marks
16.(a)	$M(D), \quad 8R = (80g \text{ x } 6) + (200g \text{ x } 4)$	M1 A1
	$R = 160g,\ 1600,\ 1570$	A1 (3)
(b)	$(\uparrow), \quad 2S = 80g + 200g$	M1
	$S = 140g,\ 1400,\ 1370$	A1 (2)
(c)	$M(B), \quad Sx + (S \text{ x } 10) = (80g \text{ x } 8) + (200g \text{ x } 6)$	M1 A2
	$140x + 1400 = 640 + 1200$	
	$140x = 440$	
	$x = \frac{22}{7}$	A1 (4)
		9

Question Number	Scheme	Marks
17.		
	(a) $\uparrow$ $2X + X = 4.5g$	M1 A1
	Leading to $X = \dfrac{3g}{2}$ or 14.7 or 15 (N)	A1 **(3)**
	(b) $\mathrm{M}(A)$ $4.5g \times AG = (2X) \times 0.8 + X \times 2.4$	M1 A2 **ft** (1,0)
	$AG = \dfrac{4}{3}$ (m), 1.3, 1.33,...	A1 **(4)**
		[7]

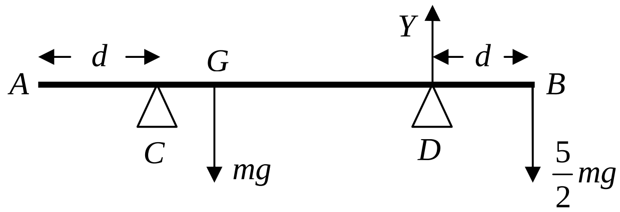
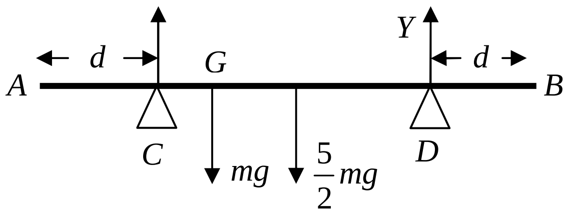
<u>Question 17(a)</u>

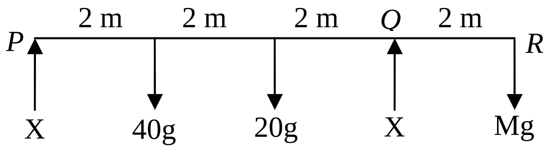
First M1 for a complete method for finding R_Q, either by resolving vertically, or taking moments twice, with usual criteria (allow M1 even if $R_P = 2R_Q$ not substituted)
First A1 for a correct equation in either R_Q or R_P ONLY.
Second A1 for 1.5g or 14.7 or 15 (A0 for a negative answer)

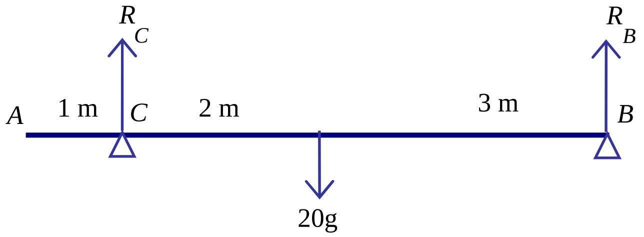
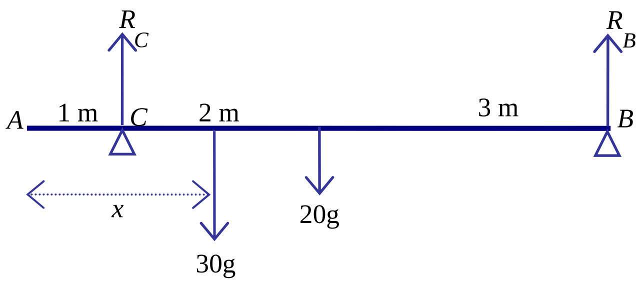
<u>Question 17(b)</u>

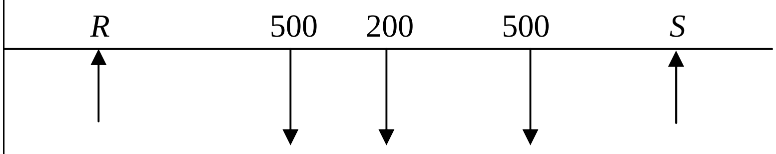
First M1 for taking moments about any point, with usual criteria.
A2 ft for a correct equation (A1A0 one error, A0A0 for two or more errors, ignoring consistent omission of g's) in terms of X and their x (which may not be AG at this stage)
Third A1 for AG = 4/3, 1.3, 1.33,….. (any number of decimal places, since g cancels) need 'AG =' or x marked on diagram

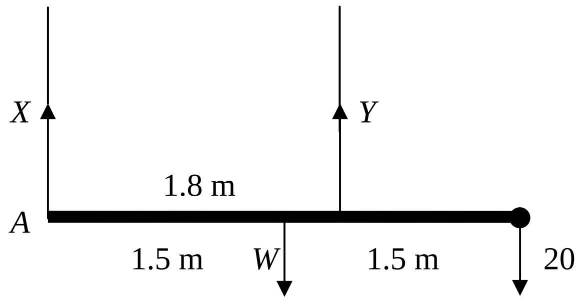
N.B. if $R_Q = 2R_P$ throughout, mark as a misread as follows:
(a) M1A1A0 (resolution method) (b) M1A0A1A1, assuming all work follows through correctly..

Question Number	Scheme	Marks
18 (a)		
	$\mathrm{M}(D)$ $\quad mg\times GD=\frac{5}{2}mg\times d$	M1 A1
	$GD=\frac{5}{2}d$ $\ast$	DM1 A1
		(4)
(b)		
	$\mathrm{M}(C)$ $\quad mg\times\frac{d}{2}+\frac{5}{2}mg\times\frac{3}{2}d=Y\times 3d$	M1 A2(1, 0)
	Leading to $\quad Y=\frac{17}{12}mg$	DM1 A1
		(5)
		9

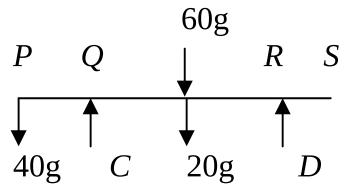
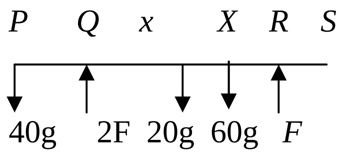
Question Number	Scheme	Marks
19. **(a)**		
(i)	**EITHER** M(R), $8X + 2X = 40g \times 6 + 20g \times 4$	M1 A2
	solving for X, $X = 32g = 314$ or 310 N	M1 A1
(ii)	$(\uparrow)$ $X + X = 40g + 20g + Mg$ (or another moments equation)	M1 A2
	solving for M, $M = 4$	M1 A1
(i)	**OR** M(P), $6X = 40g \times 2 + 20g \times 4 + Mg \times 8$	M1 A2
	solving for X, $X = 32g = 314$ or 310 N	M1 A1
	$(\uparrow)$ $X + X = 40g + 20g + Mg$ (or another moments equation)	M1 A2
(ii)	solving for M, $M = 4$	M1 A1 (10)
(b)	Masses concentrated at a point or weights act at a point	B1 (1) **11**

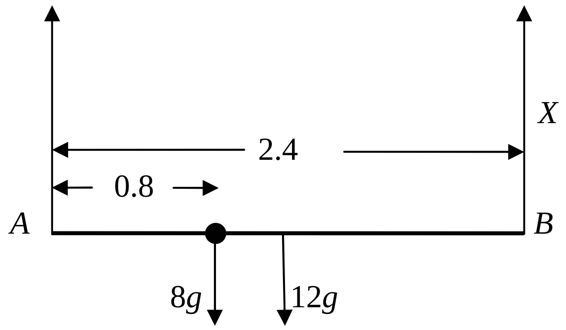
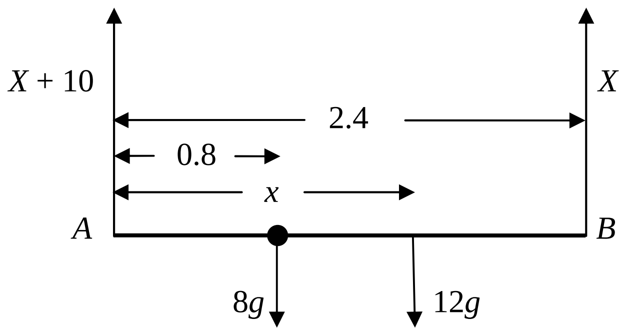
Question Number	Scheme	Marks
20. **(a)**	 Taking moments about B: $5 \times R_C = 20g \times 3$ $R_C = 12g$ or $60g/5$ or 118 or 120 Resolving vertically: $R_C + R_B = 20g$ $R_B = 8g$ or 78.4 or 78	 M1A1 A1 M1 A1 (5)
(b)	 Resolving vertically: 50g = R + R Taking moments about B: $5 \times 25g = 3 \times 20g + (6 - x) \times 30g$ $30x = 115$ $x = 3.8$ or better or 23/6 oe	 B1 M1 A1 A1 A1 (5) **[10]**

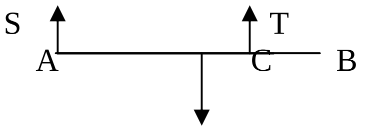
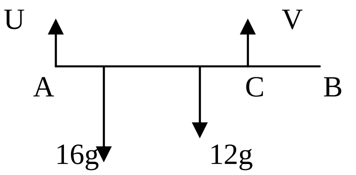
Question Number	Scheme	Marks
21		
	$M(B)$, $500x + 500.2x + 200\text{x}3 = R\text{x}5 \ + \ S\text{x}1$ (or any valid moments equation)	M1 A1 A1
	$(\downarrow)$ $R + S = 500 + 500 + 200 = 1200$ (or a moments equation)	M1 A1
	solving for x; $x = 1.2$ m	M1 A1 cso **[7]**

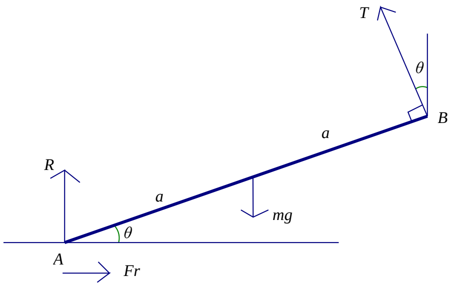
Question Number	Scheme		Marks
22.	(a) 		
	M (A) $\quad W\times1.5+20\times3=Y\times1.8$		M1 A2 (1, 0)
	$Y=\frac{5}{6}W+\frac{100}{3}$ $\ast$	cso	A1 (4)
	(b) $\uparrow$ $\quad X+Y=W+20$	or equivalent	M1 A1
	$X=\frac{1}{6}W-\frac{40}{3}$		A1 (3)
	(c) $\frac{5}{6}W+\frac{100}{3}=8\left(\frac{1}{6}W-\frac{40}{3}\right)$		M1 A1 ft
	$W=280$		A1 (3) **[10]**
	Alternative to (b) M(C) $\quad X\times1.8+20\times1.2=W\times0.3$ $X=\frac{1}{6}W-\frac{40}{3}$		M1 A1 A1

Question Number	Scheme	Marks
23 (a)	$M(Q),\quad 50g(1.4-x)+20g \text{ x } 0.7 = T_P \text{ x } 1.4$	M1 A1
	$T_P = 588 - 350x$ Printed answer	A1 (3)
(b)	$M(P),\quad 50gx + 20g \text{ x } 0.7 = T_Q \text{ x } 1.4$ or $R(\uparrow),\ T_P + T_Q = 70g$	M1 A1
	$T_Q = 98 + 350x$	A1 (3)
(c)	Since $0 < x < 1.4$, $98 < T_P < 588$ and $98 < T_Q < 588$	M1 A1 A1 (3)
(d)	$98 + 350x = 3\,(588 - 350x)$	M1
	$x = 1.19$	DM1 A1 (3) **[12]**

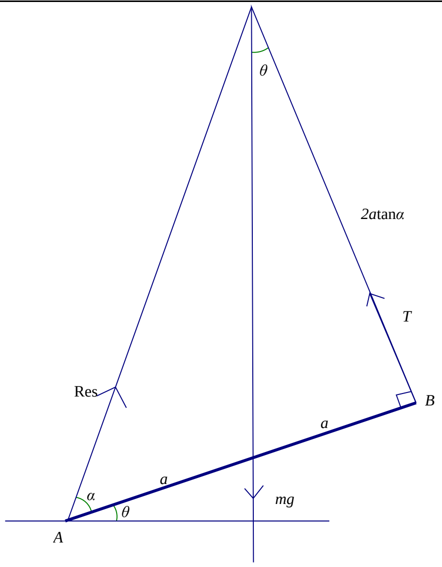
Question Number	Scheme	Marks
24 (a)	 $C + D = 120g$ $M(Q),\ \ 80g.0.8 - 40g.0.4 = D.1.6$ solving $C = 90g; D = 30g$	M1 A1 M1 A1 M1 A1 A1 (7)
(b)	 $2F + F = 40g + 20g + 60g$ $M(Q),\ \ 60gx + 20g.0.8 = 40g.0.4 + F.1.6$ solving $QX = x = \dfrac{16}{15}\ \text{m} = 1.07\text{m}$	M1 A1 M1 A1 M1 A1 (6) **[13]**

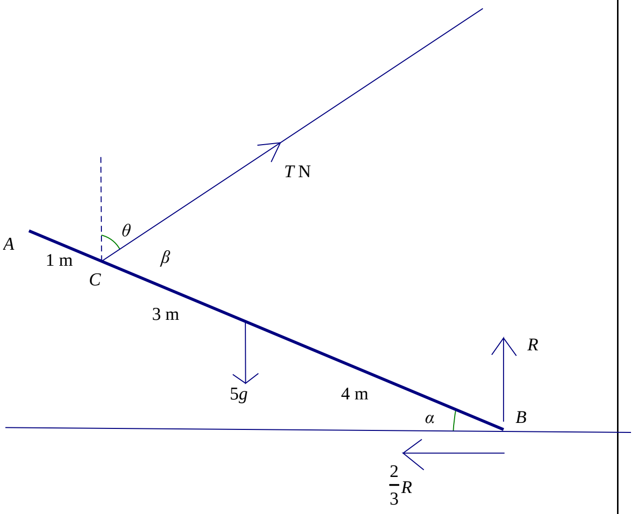
Question Number	Scheme	Marks
25.	(a)	
		
	$M(A)$ $\quad 8g \times 0.8 + 12g \times 1.2 = X \times 2.4$	M1 A1
	$X \approx 85\ (\text{N})$ $\quad$ accept 84.9, $\dfrac{26g}{3}$	DM1 A1 **(4)**
	(b)	
		
	$R(\uparrow)$ $\quad \underline{(X+10)} + \underline{X} = 8g + 12g$	M1 <u>B1</u> A1
	$(X = 93)$	
	$M(A)$ $\quad 8g \times 0.8 + 12g \times x = X \times 2.4$	M1 A1
	$x = 1.4\ (\text{m})$ $\quad$ accept 1.36	A1 **(6)**
		[10]

Question Number	Scheme	Marks
26.(a)	 M(A): $T \times 4 = 12g \times 2.5$	M1 A1
	$T = \underline{7.5g \text{ or } 73.5\text{ N}}$	A1
	R($\uparrow$) $S + T = 12g$	M1
	$\Rightarrow S = \underline{4.5g \text{ or } 44.1\text{ N}}$	A1 (5)
(b)	 M(A) $V \times 4 = 16g \times y + 12g \times 2.5$	M1 A1
	$V = \underline{4gy + 7.5g \text{ or } 39.2y + 73.5\text{ N}}$	A1 (3)
(c)	$V \le 98 \Rightarrow 39.2y + 73.5 \le 98$	M1
	$\Rightarrow y \le 0.625 = 5/8$	DM1
	Hence "load must be no more than 5/8 m from A" (o.e.)	A1 (3)
		11

Q	Scheme	Marks	Notes
27			
	M(A): $2aT = mga\cos\theta$ $\left(T = \frac{1}{2}mg\cos\theta\right)$ M(B): $mga\cos\theta + Fr\times 2a\sin\theta = R\times 2a\cos\theta$	M1A1	First equation Need all terms. Condone sign errors and sin/cos confusion
	Resolve $\leftrightarrow$: $Fr = T\sin\theta\left(=\frac{1}{2}mg\cos\theta\sin\theta\right)$	M1A1	Second equation Need all terms. Condone sign errors and sin/cos confusion
	$\updownarrow$: $R + T\cos\theta = mg$	M1A1	Third equation Need all terms. Condone sign errors and sin/cos confusion
	Use $Fr = \mu R$: $\mu R = T\sin\theta$	B1	Condone correct inequality
	Form equation in μ and θ: $R = mg - \frac{1}{2}mg\cos\theta\cos\theta$ and $\mu R = \frac{1}{2}mg\cos\theta\sin\theta \Rightarrow$	DM1	Eliminate T and R Dependent on first 3 M marks
	$\mu = \dfrac{\frac{1}{2}mg\cos\theta\sin\theta}{mg - \frac{1}{2}mg\cos\theta\cos\theta}$	DM1	Solve for μ Dependent on previous M
	$\mu = \dfrac{\cos\theta\sin\theta}{2-\cos^2\theta}$	A1	Obtain **given answer** from correct working Must explain if inequality becomes equality
		[10]	

Alt 1	Moments (about B): $mga\cos\theta + Fr \times 2a\sin\theta = R \times 2a\cos\theta$	M1	
		A1	Correct unsimplified
	Resolving (parallel to rod): $Fr\cos\theta + R\sin\theta = mg\sin\theta$	M2	
		A2	-1 each error
	Use of $Fr = \mu R$: $mg\cos\theta + \mu R \times 2\sin\theta = R \times 2\cos\theta$ $\mu R\cos\theta + R\sin\theta = mg\sin\theta$	B1	
	Form equation in μ and θ: $\dfrac{mg\sin\theta}{mg\cos\theta} = \dfrac{\mu R\cos\theta + R\sin\theta}{2R\cos\theta - 2\mu R\sin\theta}$ $\dfrac{\sin\theta}{\cos\theta} = \dfrac{\mu\cos\theta + \sin\theta}{2\cos\theta - 2\mu\sin\theta}$	DM1	
	Solve for μ : $2\cos\theta\sin\theta - 2\mu\sin^2\theta = \mu\cos^2\theta + \cos\theta\sin\theta$	DM1	
	$\mu = \dfrac{\sin\theta\cos\theta}{\cos^2\theta + 2\sin^2\theta} = \dfrac{\sin\theta\cos\theta}{2 - \cos^2\theta}$	A1	Obtain **given answer** from correct working
	NB for alternatives using moments and resolving: e.g. Resolve $\leftrightarrow$: $Fr = T\sin\theta$ M(centre): $aT = a\cos\theta R - a\sin\theta Fr$		First equation M1A1 Sufficient equations to solve M2A2

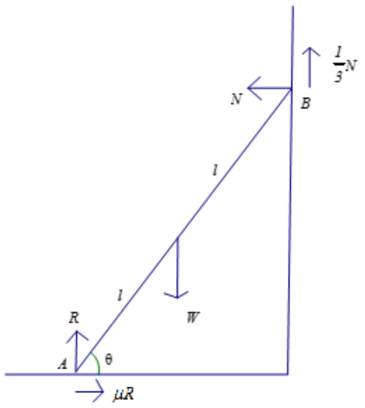
Alt 2			3 concurrent forces
	$\tan(\theta+\alpha)=\dfrac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha}$	M1A1	
	$\tan\theta=\dfrac{a}{2a\tan\alpha}\ \Rightarrow\tan\alpha=\dfrac{1}{2\tan\theta}$	M1	
	$\tan(\theta+\alpha)=\dfrac{\tan\theta+\dfrac{1}{2\tan\theta}}{1-\tan\theta\times\dfrac{1}{2\tan\theta}}$ $=2\left(\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{2\sin\theta}\right)$	M1A1 A1	
	$F=\mu R\ \Rightarrow$ $\mu=\dfrac{1}{\tan(\theta+\alpha)}$	B1 DM1	
	$=\dfrac{1}{2}\left(\dfrac{2\sin\theta\cos\theta}{2\sin^2\theta+\cos^2\theta}\right)\ =\dfrac{\cos\theta\sin\theta}{2-\cos^2\theta}$	DM1 A1	Obtain **given answer** from correct working
		(10)	

Q.	Scheme	Marks	Notes
28a			
	$F=\dfrac{2}{3}R$ seen or implied	B1	Use of $F=\mu R$. Could be on diagram. Allow in (b) if not seen before
	M(C): $5g\times 3\cos\alpha + F\times 7\sin\alpha = 7\cos\alpha\times R$	M1	Moments about C or alternative complete method to find equation in F and R or R only. Dimensionally correct and all terms needed. Condone sin/cos confusion and sign error(s).
		A1	At most one error
		A1	Correct unsimplified equation
	$15g\cos\alpha = R\left(7\cos\alpha - \dfrac{14}{3}\sin\alpha\right)$		
	$15g\times\dfrac{4}{5} = R\left(7\times\dfrac{4}{5}-\dfrac{14}{3}\times\dfrac{3}{5}\right)=\dfrac{14}{5}R$	dM1	Substitute for F and trig and solve for R Dependent on previous M1
	$R=\dfrac{30}{7}g=42$ (N)	A1	
		(6)	
	e.g. of alternative for M1A1A1:		
	M(A): $T\sin\beta + 8R\cos\alpha = 8F\sin\alpha + 20g\cos\alpha$ **and** M(B): $7T\sin\beta = 20g\cos\alpha$	(M1)	
		(A1)	At most 1 error
	$\dfrac{20g}{7}\cos\alpha + 8R\cos\alpha = 8F\sin\alpha + 20g\cos\alpha$	(A1)	Correct unsimplified equation in F and R or R only

Q.	Scheme	Marks	Notes
28b	Resolve $\updownarrow$: $T\cos\theta + R = 5g$ $R + T\sin(\beta-\alpha) = 5g$	M1	Need all terms. Condone sin/cos confusion and sign error(s).
		A1	Correct in R or *their* R
	Resolve $\leftrightarrow$: $T\sin\theta = F\,(=28)$ $F\left(=\frac{2}{3}R\right) = T\cos(\beta-\alpha)$	M1	Need both terms. Condone sin/cos confusion
		A1	Correct in R or *their* R
	Solve simultaneous equations for $\beta-\alpha$		
	$\tan(\beta-\alpha)=4$, $\beta = 50.9°$ $(51°)$	A1	cso . Max 3 s.f.
		(5)	
Alt 28b	M(B): $7\times T\sin\beta = 5g\cos\alpha\times 4$	M1	Moments equation. Dimensionally correct. Condone sin/cos confusion and sign error(s).
	$\left(T\sin\beta = \frac{16}{7}g\right)$	A1	
	OR: resolve perpendicular to the rod: $T\sin\beta + R\cos\alpha = 5g\cos\alpha + \frac{2}{3}R\sin\alpha$	(M1) (A1)	
	Resolve parallel to rod: $T\cos\beta + 5g\sin\alpha = F\cos\alpha + R\sin\alpha$ $\left(=\frac{2}{3}R\cos\alpha + R\sin\alpha\right)$	M1	All terms needed. Condone sin/cos confusion and sign error(s).
	$\left(T\cos\beta = \frac{13}{7}g\right)$	A1	
	Solve simultaneous equations for β		
	$\tan\beta = \frac{16}{13}$, $\beta = 50.9°$ $(51°)$	A1	cso. Max 3 s.f.
		(5)	
		[11]	

Q	Scheme	Marks	Notes
29a	M(A): $d\cos\theta\times 5g = 4P$	M1	Terms must be dimensionally correct. Condone trig confusion
		A1	
	Resolving horizontally: $P\sin\theta = F$	B1	
	Resolving vertically: $P\cos\theta + R = 5g$	M1	Requires all 3 terms. Condone trig confusion and sign errors
		A1	Correct equation
		DM1	Substitute for P to find R or F Dependent on both previous M marks
	$R = 5g - \dfrac{5gd\cos^2\theta}{4}$	A1	One force correct. Accept equivalent forms e.g. $R = \dfrac{20g - 5gd + 20g\tan^2\theta}{4\left(1+\tan^2\theta\right)}$
	$F = \dfrac{5gd\cos\theta\sin\theta}{4}$	A1	Both forces correct. Accept equivalent forms e.g. $F = \dfrac{5gd\tan\theta}{4\sec^2\theta}$
		(8)	
29a alt	M(B): $5g\cos\theta\times\left(4-d\right) + F\sin\theta\times 4 = R\cos\theta\times 4$	M1	Needs all three terms. Terms must be dimensionally correct. Condone trig confusion
		A1	At most one error
	Resolve parallel to the rod: $5g\sin\theta = R\sin\theta + F\cos\theta$	M1	Requires all 3 terms. Condone trig confusion and sign errors
		B1	At most one error
		A1	Correct equation
	$\Rightarrow R = 5g - \dfrac{F\cos\theta}{\sin\theta}$		
	$5g\cos\theta\times\left(4-d\right) + F\sin\theta\times 4 = 4\cos\theta\left(5g - \dfrac{F\cos\theta}{\sin\theta}\right)$	DM1	Eliminate one variable to find F or R Dependent on both previous M marks
	$4F\left(\sin\theta + \dfrac{\cos^2\theta}{\sin\theta}\right) = 20g\cos\theta - 20g\cos\theta + 5gd\cos\theta$		
	$F = \dfrac{5gd\cos\theta\sin\theta}{4}$	A1	One force correct
	$R = 5g - \dfrac{5gd\cos^2\theta}{4}$	A1	Both forces correct
			See next page for part (b)

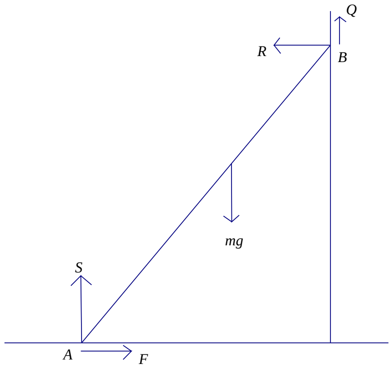
29b	$\mu = \dfrac{\dfrac{5gd\cos\theta\sin\theta}{4}}{5g - \dfrac{5gd\cos^2\theta}{4}}$	M1	Use of $F = \mu R$
	$\dfrac{1}{2}\left(5g - \dfrac{5gd\cos^2\theta}{4}\right) = \dfrac{5gd\cos\theta\sin\theta}{4}$	A1	$\left(4 - d\cos^2\theta = 2d\cos\theta\sin\theta\right)$
	$4\times169 = 120d + 144d$	M1	Use $\tan\theta = \dfrac{5}{12}$ and solve for d
	$d = \dfrac{169}{66}$	A1	(= 2.6 m or better)
		(4)	
29balt	$F = 5gd \times \dfrac{12}{13} \times \dfrac{5}{13} \times \dfrac{1}{4}\left(= \dfrac{75gd}{169}\right)$	M1	Use $\tan\theta = \dfrac{5}{12}$
	$R = 5g - \dfrac{5gd}{4} \times \dfrac{144}{169}$	A1	Both unsimplified expressions
	$75gd = \dfrac{1}{2}\left(5\times169g - 180gd\right)$	M1	Use of $F = \mu R$ and solve for d
	$150gd + 180gd = 845g$, $d = \dfrac{169}{66}$	A1	(= 2.6 m or better)
		(4)	
29balt	$R = 5g - \dfrac{12}{13}P$, $F = \dfrac{5}{13}P$	M1	Substitute trig in their equations from resolving.
	$\dfrac{5}{13}P = \dfrac{1}{2}\left(5g - \dfrac{12}{13}P\right)$	M1	use $F = \mu R$ and solve for d
	$\Rightarrow P = \dfrac{65}{22}g$	A1	
	$d = \dfrac{4P}{5g\cos\theta} = \dfrac{169}{66}$	A1	
		[12]	

Question Number	Scheme	Marks	
30			NB: If μ and $\frac{1}{3}$ are used the wrong way round the candidate loses the first A1 and the final A1.
	Resolve horizontally or vertically:	M1	Allow without friction $= \mu R$
	$\mu R = N$ or $W = R + \frac{1}{3}N$	A1	With coefficient(s) of friction . Condone Wg
	Take moments about A or B.	M1	All terms required but condone sign errors and sin/cos confusion. Terms must be resolved.
	$M(A)$: $2lN\sin\theta + 2l\frac{N}{3}\cos\theta = Wl\cos\theta$ $M(B)$: $2l\cos\theta R = Wl\cos\theta + \mu R 2l\sin\theta$	A2	-1 each error. Could be in terms of Fs. -1 if see Wg in place of W. Any Friction force used should be acting in the right direction. Mark the equation, not what they have called it.
	$\frac{10}{3}N + \frac{2}{3}N = W$ or $2R = W + 2\mu R \times \frac{5}{3}$	M1	Use $\tan\theta = 5/3$ (substitute values for the trig ratios)
	$\Rightarrow 4N = W \Rightarrow 4N - R = \frac{1}{3}N$	DM1	Equation in N and R (Eliminate one unknown) Dependent on the moments equation
	$\frac{11}{3}\mu R = R$	DM1	Solve for μ Dependent on the moments equation
	$\mu = \frac{3}{11} (\simeq 0.273)$	A1	0.27 or better

Alt 1	Resolve horizontally or vertically:	M1	Allow without friction $=\mu R$
	$\mu R = N$ or $W = R + \frac{1}{3}N$	A1	With coefficient(s) of friction
		M1	Take moments about A or B. All terms required but condone sign errors and sin/cos confusion. Terms must be resolved.
	$M(A)$: $2lN\sin\theta + 2l\frac{N}{3}\cos\theta = Wl\cos\theta$ $M(B)$: $2l\cos\theta R = Wl\cos\theta + \mu R 2l\sin\theta$	A2	-1 each error, Could be in terms of Fs. -1 if Wg used. Mark the equation, not what they have called it. Any Friction force used should be acting in the right direction. For this method they need two moments equations – allows the marks for their best equation.
	$2lN\sin\theta + 2l\frac{N}{3}\cos\theta = 2l\cos\theta R - \mu R 2l\sin\theta$	DM1	Use two moments equations to eliminate W Dependent on the moments equation
	Use of $\tan\theta$: $2\mu\times\frac{5}{3} + \frac{2}{3}\mu = 2 - 2\mu\times\frac{5}{3}$	M1	Substitute for the trig ratios
	Solve for μ : $\left(\frac{20}{3}+\frac{2}{3}\right)\mu = 2$,	DM1	Dependent on the moments equation
	$\mu = \frac{3}{11} (\simeq 0.273)$	A1	0.27 or better

Alt 2	Resolving horizontally or vertically:	M1	Allow without friction $=\mu R$
	$\mu R = N$ or $W = R + \frac{1}{3}N$	A1	With coefficient(s) of friction (condone Wg)
	$l\cos\theta\times R = l\cos\theta\times\frac{1}{3}N + l\sin\theta\times N + l\sin\theta\times\mu R$	M1	Moments about the centre of the rod. All terms required. Terms must be resolved. Condone sign errors and sin/cos confusion. Allow without friction $=\frac{1}{3}N$. Any Friction force used should be acting in the right direction.
		A2	-1 each error. Could be in terms of Fs. -1 if Wg used.
	$l\cos\theta\times R = l\cos\theta\times\frac{1}{3}\mu R + l\sin\theta\times\mu R + l\sin\theta\times\mu R$	DM1	Obtain an equation in μ and θ $\left(\cos\theta = \cos\theta\times\frac{1}{3}\mu + \sin\theta\times\mu + \sin\theta\times\mu\right)$ Dependent on the moments equation
	$\cos\theta\left(1-\frac{1}{3}\mu\right) = 2\mu\sin\theta \Rightarrow \tan\theta = \frac{1-\frac{1}{3}\mu}{2\mu} = \frac{5}{3}$	M1	Use of $\tan\theta$ (substitute values for the trig ratios)
	Solve for μ : $10\mu = 3-\mu$,	DM1	Dependent on the moments equation
	$\mu = \frac{3}{11} (\simeq 0.273)$	A1	0.27 or better
		[9]	

Question Number	Scheme	Marks	Notes
31(a)	Resolving vertically: $Y + P\cos\theta = W$	M1 A1	Needs all 3 terms. Condone sign errors and sin/cos confusion. Condone Wg
	Moments about A: $Wl\cos\theta = 2lP$	M1 A1	Terms need to be of the correct structure, but condone l implied if not seen.
	$P = \dfrac{W\cos\theta}{2} \Rightarrow Y = W - \dfrac{W\cos^2\theta}{2} = \dfrac{W}{2}\left(2-\cos^2\theta\right)$ **	DM1 A1 (6)	Substitute for P to obtain simplified Y Requires both preceding M marks Obtain **given result** correctly.
	NB $W + Y = P\cos\theta$ with correct conclusion is possible		
	They need to find two independent equations that do not include X. If they have equations involving X they need to attempt to eliminate X before they score any marks		
(b)	$\theta = 45° \Rightarrow Y = \dfrac{3W}{4}$	B1	
	$X = P\sin 45$	M1	Resolving horizontally. Accept in terms of θ.
	$= \dfrac{W\cos 45}{2}.\sin 45\left(=\dfrac{W}{4}\right)$	DM1 A1	Express X in terms of W. Accept in terms of θ. Requires preceding M mark. Correct unsimplified but substituted.
	Resultant at A = $\dfrac{W}{4}\sqrt{3^2+1^2} = \dfrac{W\sqrt{10}}{4}$ (0.79W)	DM1 A1 (6)	Use of Pythagoras with X, Y in terms of W only. Dependent on the first M1 Or equivalent (0.79W or better)
	Alternative moments equations: about the centre $Pl + X\sin\theta l = y\cos\theta l$ About the point where the lines of action of P and X intersect $Y\times\dfrac{2l}{\cos\theta} = W\left(\dfrac{2l}{\cos\theta} - l\cos\theta\right)$		

Question Number	Scheme	Marks	Notes
32.			NB As the rod is not uniform, the use of moments equations is not helpful in part (a).
(a)	$R = F$	B1	Re lve horizontally
	$S + Q = mg$	B1	Resolve vertically (requires Q acting upwards)
	$Q = \frac{2}{3}R, \quad F = \frac{1}{4}S$	B1	Use both coefficients of friction
	$Q = \frac{2}{3}R = \frac{2}{3}\times\frac{1}{4}S\,, \qquad S + \frac{1}{6}S = mg,\ S = \frac{6}{7}mg$	M1	Solve to find S in terms of m & g. (Can be scored if Q is acting downwards)
		A1 (5)	
(b)	$\mathrm{M}(A) \quad mg\times x\cos 60 = Q\times 2l\cos 60 + R\times 2l\sin 60$ $\mathrm{M}\ (B) \quad mg(2l - x)\cos 60 + F\times 2l\sin 60 = S\times 2l\cos 60$	M1	Moments equation – must include all terms. Condone sign errors and sin/cos confusion
	M (c of m) $Sx\cos 60 = Fx\sin 60 + R(2l - x)\sin 60 + Q(2l - x)\cos 60$	A2	Correct unsimplified equation (for their S.) -1 each error
	$mgx\cos 60 = \frac{1}{6}\times\frac{6}{7}mg\times 2l\cos 60 + \frac{1}{4}\times\frac{6}{7}mg\times 2l\sin 60$	DM1	Form an equation in x. Depends on the preceding M
	$\frac{1}{2}x = \frac{1}{7}\times 2l\times\frac{1}{2} + \frac{3}{14}\times l\sqrt{3}$		
	$AG = x = 1.028.....l \quad x = 1.03l$	A1 (5)	$1.03l$ or better $\frac{l(2+3\sqrt{3})}{7}$

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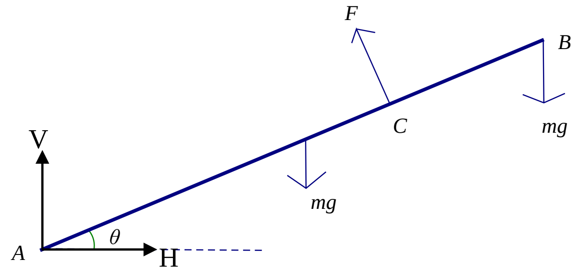
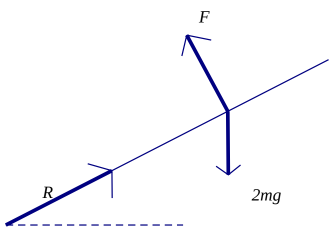
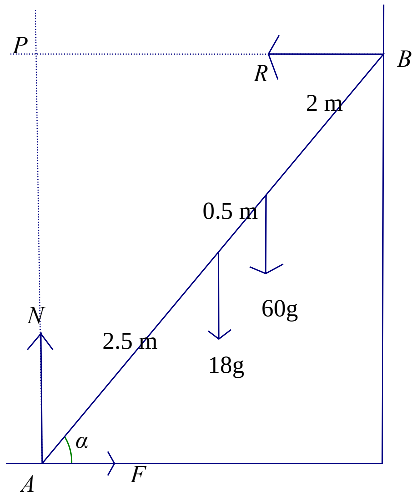
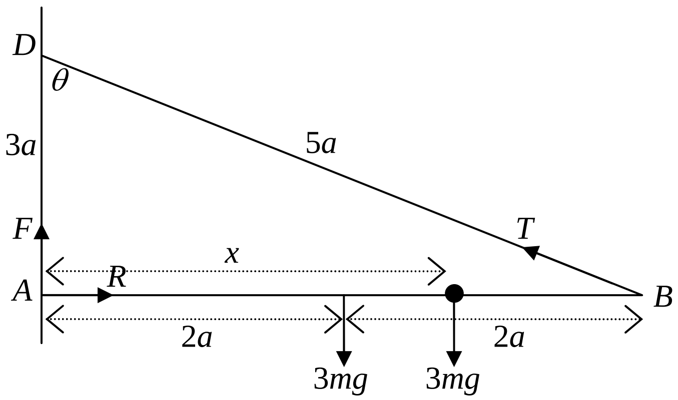
Question Number	Scheme	Marks	Notes
33a			
	Moments about A:	M1	Moments about A. Requires all three terms and terms of correct structure (force x distance). Condone consistent trig confusion
	$bF = a\cos\theta mg + 2a\cos\theta mg (= 3a\cos\theta mg)$	A2	-1 each error
	$F = \dfrac{3amg\cos\theta}{b}$ ***Answer given***	A1 [4]	
33b	$\rightarrow$: $H = F\sin\theta = \dfrac{3amg\cos\theta\sin\theta}{b}$	M1 A1	Resolve horizontally. Condone trig confusion RHS correct. Or equivalent.
	$\uparrow$: $2mg = \pm V + F\cos\theta$	M1 A1	Resolve vertically. Condone sign error and trig confusion Correct equation
	$\pm V = 2mg - \dfrac{3amg\cos\theta}{b} \times \cos\theta \left(= 2mg - \dfrac{3amg\cos^2\theta}{b} \right)$	A1 [5]	RHS correct. Or equivalent

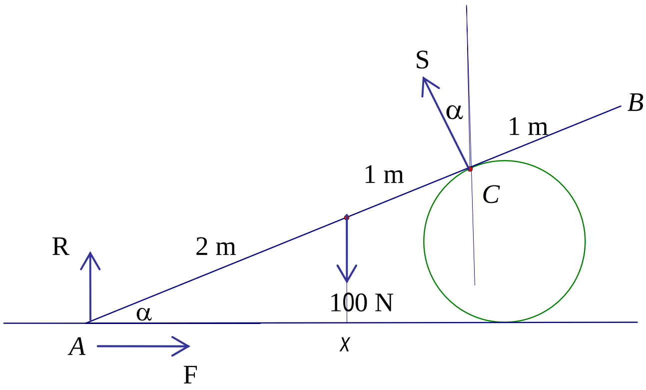
Question Number	Scheme	Marks	Notes
33c	$\dfrac{2mg-\dfrac{3amg\cos^2\theta}{b}}{\dfrac{3amg\cos\theta\sin\theta}{b}}=\tan\theta$ $\dfrac{2b-3a\cos^2\theta}{3a\cos\theta\sin\theta}=\dfrac{\sin\theta}{\cos\theta}$ $\Rightarrow 2b-3a\cos^2\theta=3a\sin^2\theta\Rightarrow 2b=3a\,,\ \ \dfrac{a}{b}=\dfrac{2}{3}$	M1 A1 DM1 A1 [4]	Use of tan, either way up. V, H, F substituted. Correct for their components in θ only Simplify to obtain the ratio of a and b, or equivalent
33c alt 2	The centre of mass of the combined rod + particle is $\dfrac{3}{2}a$ from A  3 forces in equilibrium must be concurrent $\Rightarrow b=\dfrac{3}{2}a$ $\Rightarrow\dfrac{a}{b}=\dfrac{2}{3}$	M1A1 M1 A1 [4]	Not on the spec, but you might see it.
alt c 3	R acts along the rod, so resolve forces perpendicular to the rod. $F=mg\cos\theta+mg\cos\theta$ $2mg\cos\theta=\dfrac{3amg\cos\theta}{b}$ $\Rightarrow\dfrac{a}{b}=\dfrac{2}{3}$	M1 A1 DM1 A1 [4]	Resolve and substitute for F Eliminate θ
alt c 4	R acts along the rod. Take moments about C $mg\cos\theta\ \ 2a-b\ =mg\cos\theta\ \ b-a$ $2a-b=b-a\,,\quad\Rightarrow\dfrac{a}{b}=\dfrac{2}{3}$	M1 A1 DM1A1 [4]	Moments about B gives $2a-b\ \ F=amg\cos\theta$ and substitute for F
c alt 5	Resultant parallel to the rod $\Rightarrow R=2mg\sin\theta$ And $V^2+H^2=R^2$ $2mg\sin\theta\ ^2=\left(\dfrac{3amg\cos\theta\sin\theta}{b}\right)^2+\left(2mg-\dfrac{3amg\cos^2\theta}{b}\right)^2$ Eliminate θ $\Rightarrow\dfrac{a}{b}=\dfrac{2}{3}$	M1 A1 DM1 A1 [4]	Substitute for V, H and R in terms of θ

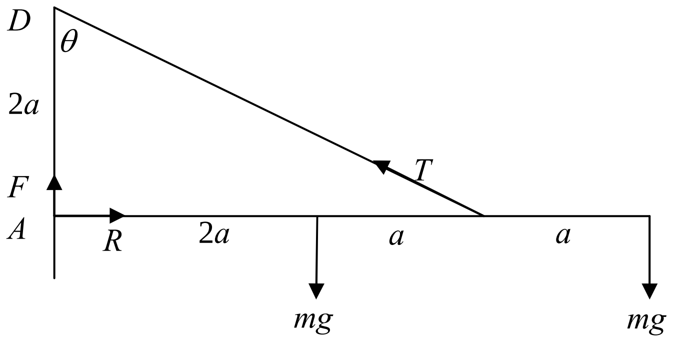
Question Number	Scheme		Notes
34.			
(a)	$AC = 4a\tan 60^o = 4a\sqrt{3}.$	M1 A1	Or $\dfrac{4a}{\tan 30}$ or $\sqrt{(8a)^2-(4a)^2}$
		(2)	
(b)	use of $F=\mu R$ at either A or C	M1	
	3 independent equations required. Award M1A1 for each in the order seen. If more than 3 relevant equations seen, award the marks for the best 3.		
	$M(A)$, $R_C.4a\sqrt{3}=W.3a\sqrt{3}\cos 60^o$	M1 A1	$R_C=\dfrac{3W}{8}$
	$(\uparrow)$, $R_A+R_C\cos 60^o+F_C\cos 30^o=W$	M1 A1	$R_A=\dfrac{5W}{8}$
	$(\rightarrow)$, $F_A-R_C\cos 30^o+F_C\cos 60^o=0$	M1 A1	$F_A=R_C\dfrac{\sqrt{3}}{3}$
	M(C) $a\sqrt{3}\cos 60W+F_A.4a\sqrt{3}\sin 60=R_A.4a\sqrt{3}\cos 60$		
	Parallel: $F_A\cos 60+R_A\cos 30+F_C=W\cos 30$		
	Perpendicular: $R_C+R_A\cos 60=F_A\cos 30+W\cos 60$		
	solving to give $\mu=\dfrac{\sqrt{3}}{5}$; 0.346 or 0.35.	DM1 A1	Equation in μ only. Dependent on 4 M marks for their equations.
	Reactions in the wrong direction(s) – check carefully		
		(9)	
		[11]	

Q	Scheme	Marks	
35			
	$F=\mu N$	B1	Used. Condone an inequality.
	R$(\uparrow)$ $\quad 18g+60g=N$	M1	Resolve vertically
	$=78g$	A1	
	R$(\rightarrow)$ $\quad R=F=\mu N$		
P	$2.5\times 18g\cos\alpha+3\times 60g\cos\alpha=5F\sin\alpha$	M1A2	Moments equation. Condone sign errors. Condone sin/cos confusion -1 each error
A	$18g\times 2.5\cos\alpha+60g\times 3\cos\alpha=R\times 5\sin\alpha$		
C	$\frac{1}{2}\cos\alpha\times 18g+3\sin\alpha F+2\sin\alpha R=3\cos\alpha N$		
B	$5\cos\alpha N=5\sin\alpha F+2.5\cos\alpha\times 18g+2\cos\alpha\times 60$		
W	$60g\times\frac{1}{2}\cos\alpha+2.5N\cos\alpha=2.5R\sin\alpha+2.5F\sin\alpha$		
	$45\times\frac{3}{5}g+180\times\frac{3}{5}g=4R$	DM1	Eliminate α. Dependent on the second M1.
	$R=\frac{135}{4}g$		
	$78g\mu=\frac{135}{4}g$	DM1	Equation in μ only. (Dependent on the first two M marks.)
	$\mu=\frac{135}{4\times 78}=\frac{135}{312}=0.432...=0.43$	A1 **(9)**	NB g cancels. 0.43269..., $\frac{225}{520}$, $\frac{45}{104}$, awrt 0.433 Do not accept an inequality.
	NB If use just two moments equations, M1A2 for the better attempt, M1A1 for the other. Remaining marks as above.		

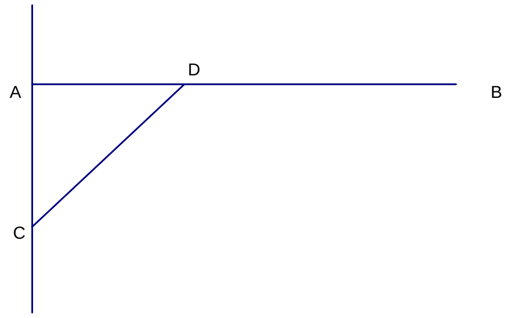
Question Number	Scheme	Marks
36		
(a)	Taking moments about A:	M1
	$4g \times 0.7 \times \cos 20° = 1.4T$	A1 A1
	$T = 18.4$ N	A1
		(4)
(b)	$\uparrow \quad R + T\cos 20 = 4g$	
	$R = 4g - T\cos 20°$	M1 A1
	$\rightarrow \; F = T\sin 20$	M1 A1
	$F = \mu R \Rightarrow T\sin 20° = \mu(4g - T\cos 20°)$	DM1 A1
	$\mu = \dfrac{T\sin 20°}{4g - T\cos 20°} = 0.29$	A1
		(7)
		11

Question Number	Scheme	Marks
37. **(a)**		
	M(A) $3mg\times 2a + 3mgx = T\cos\theta \times 4a$	M1 A2,1,0
	$= \frac{12}{5}aT$	
	$\frac{12}{5}aT = 6mga + 3mgx$	
	$T = \frac{25}{4}mg$ $\quad \frac{12}{5}a \times \frac{25}{4}mg = 6mga + 3mgx$	M1
	$15a = 6a + 3x$	
	$x = 3a$ **	A1
		(5)
(b)	R$(\rightarrow)$ $\quad R = T\sin\theta$	M1
	$= \frac{25}{4}mg \times \frac{4}{5}$	A1
	$= 5mg$ **	A1
		(3)
(c)	R$(\uparrow)$ $\quad F + \frac{25}{4}mg \times \frac{3}{5} = 3mg + 3mg$	M1 A2,1,0
	$F = 6mg - \frac{15}{4}mg = \frac{9}{4}mg$	
	$\mu = \frac{F}{R} = \frac{\frac{9}{4}mg}{5mg} = \frac{9}{20}$	DM1 A1
		(5) **13**

Question Number	Scheme	Marks
38.		
	Taking moments about A: $3S = 100 \times 2 \times \cos\alpha$	M1 A1
	Resolving vertically: $R + S\cos\alpha = 100$	M1 A1
	Resolving horizontally: $S\sin\alpha = F$	M1 A1
	(Most alternative methods need 3 independent equations, each one worth M1A1. Can be done in 2 e.g. if they resolve horizontally and take moments about X then $R \times 2 \times \cos\alpha = S \times (3 - 2 \times \cos^2\alpha)$ scores M2A2)	
	Substitute trig values to obtain correct values for F and R (exact or decimal equivalent).	DM1
	$\left(S = \frac{200\sqrt{8}}{9}\right)$, $R = 100 - \frac{1600}{27} = \frac{1100}{27} \approx 40.74$, $F = \frac{200\sqrt{8}}{27} \approx 20.95...$	A1
	$F \le \mu R,\ 200\sqrt{8} \le \mu \times 1100,\quad \mu \ge \frac{200\sqrt{8}}{1100} = \frac{2\sqrt{8}}{11}.$	M1
	Least possible μ is 0.514 (3sf), or exact.	A1
		[10]

Question Number	Scheme	Marks
39		
(a)	M(A) $3a \times T\cos\theta = 2amg + 4amg$ $\cos\theta = \left(\frac{2}{\sqrt{9+4}} = \right)\frac{2}{\sqrt{13}}$ $\frac{6}{\sqrt{13}}T = 6mg$ $T = mg\sqrt{13}$ *	M1 A1 A1 B1 A1 (5)
(b)	$3a \times T \times \cos\theta = 2amg + 4aMg$ $T = \frac{(2mg + 4Mg)}{6}\sqrt{13} \leq 2mg\sqrt{13}$ $mg + 2Mg < 6mg$ $M \leq \frac{5}{2}m$ * cso	M1 A1 A1 (3) **[8]**

Question Number	Scheme	Marks
40.	$m(B): R \times 4\cos\alpha = F \times 4\sin\alpha + 20g \times 2\cos\alpha$	M1 A2
	Use of $F = \frac{1}{2}R$	M1
	Use of correct trig ratios	B1
	R = 160N or 157N	DM1 A1
		[7]

Question Number	Scheme	Marks
Q41 (a)	 Taking moments about A: $3g \times 0.75 = \frac{T}{\sqrt{2}} \times 0.5$	M1A1A1
	$T = 3\sqrt{2}g \times \frac{7.5}{5} = \frac{9\sqrt{2}g}{2} (= 62.4N)$	A1
		(4)
(b)	$\leftarrow \pm H = \frac{T}{\sqrt{2}} (= \frac{9g}{2} \approx 44.1N)$	B1
	$\uparrow \pm V + \frac{T}{\sqrt{2}} = 3g \quad (\Rightarrow V = 3g - \frac{9g}{2} = \frac{-3g}{2} \approx -14.7\,\text{N})$	M1A1
	$\Rightarrow \lvert R \rvert = \sqrt{81+9} \times \frac{g}{2} \approx 46.5(N)$	M1A1
	at angle $\tan^{-1}\frac{1}{3} = 18.4^\circ$ (0.322 radians) below the line of BA 161.6° (2.82 radians) below the line of AB (108.4° or 1.89 radians to upward vertical)	M1A1
		(7) **[11]**

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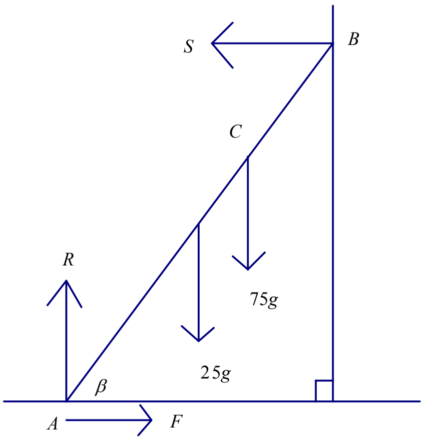
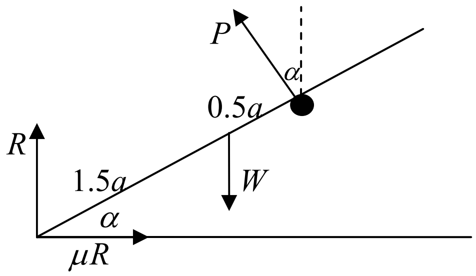
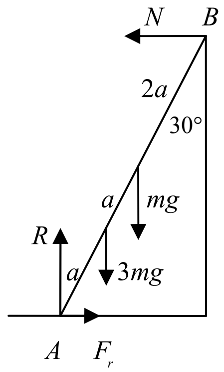
Question Number		Scheme	Marks
42	(a)		
		$\mathrm{R}(\uparrow): R = 25g + 75g (= 100g)$	B1
		$F = \mu R \Rightarrow F = \dfrac{11}{25} \times 100g$	M1
		$= 44\text{g} \ (=431)$	A1
			(3)
	(b)	M(A): $25g \times 2\cos\beta + 75g \times 2.8\cos\beta$ $= S \times 4\sin\beta$	M1 A2,1,0
		$\mathrm{R}(\leftrightarrow): F = S$	
		$176g\sin\beta = 260g\cos\beta$	M1A1
		$\beta = 56(°)$	A1
			(6)
	(c)	So that Reece's weight acts directly at the point C.	B1
			[10]

Question Number	Scheme		Marks
43.	(a) 		
	$R(\uparrow)\quad R + P\cos\alpha = W$		M1 A1
	$M(A)\qquad P \times 2a = W \times 1.5a\cos\alpha$		M1 A1
	$\left(P = \frac{3}{4}W\cos\alpha\right)$		
	$R = W - P\cos\alpha = W - \frac{3}{4}W\cos^2\alpha$		DM1
	$= \frac{1}{4}\left(4 - 3\cos^2\alpha\right)W \quad *$	cso	A1 **(6)**
	(b) Using $\cos\alpha = \frac{2}{3}$, $\qquad R = \frac{2}{3}W$		B1
	$R(\rightarrow)\qquad \mu R = P\sin\alpha$		M1 A1
	Leading to $\mu = \frac{3}{4}\sin\alpha$		
	$\left(\sin\alpha = \sqrt{\left(1 - \frac{4}{9}\right)} = \frac{\sqrt{5}}{3}\right)$		
	$\mu = \frac{\sqrt{5}}{4}$	awrt 0.56	DM1 A1 **(5)**
			[11]

Question Number	Scheme	Marks
44.	(a) 	
	M(A) $\quad N\times 4a\cos 30° = 3mg\times a\sin 30° + mg\times 2a\sin 30°$	M1 A2(1,0)
	$N=\dfrac{5}{4}mg\tan 30°\ \left(=\dfrac{5}{4\sqrt{3}}mg\right.$ $= 7.07\ldots m)$	DM1 A1
	$\rightarrow\ F_r = N\ ,\quad \uparrow\ R=4mg$	B1, B1
	Using $F_r=\mu R$	B1
	$\dfrac{5}{4\sqrt{3}}mg=\mu R$ for their R	M1
	$\mu=\dfrac{5}{16\sqrt{3}}$ awrt 0.18	A1 **(10)**
		[10]
	Alternative method: M(B): $mg\times 2a\sin 30 + 3mg\times 3a\sin 30 + F\times 4a\cos 30 = R\times 4a\sin 30$ $11mga\sin 30 + F\times 4a\cos 30 = R\times 4a\sin 30$	M1A3(2,1,0)
	$\dfrac{11mg}{2}+F\dfrac{4\sqrt{3}}{2}=2R$	DM1A1
	$\uparrow\ R=4mg$,	B1
	Using $F_r=\mu R$	B1
	$8\mu\sqrt{3}=\dfrac{5}{2},\quad \mu=\dfrac{5}{16\sqrt{3}}$	M1 A1