EXPERT TUITION

Maths Questions By Topic:

Numerical Methods
Mark Scheme

A-Level Edexcel

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New Spec

Old Spec

Question	Scheme	Marks	AOs
1 (a)	25	B1	3.4
		(1)	
(b)	Attempts to differentiate using the product rule $\frac{dv}{dt} = \ln(t+1)\times -0.4 + \frac{(10-0.4t)}{t+1}$	M1 A1	3.1b 1.1b
	Sets their $\frac{dv}{dt} = 0 \Rightarrow \frac{(10-0.4\boldsymbol{t})}{(\boldsymbol{t}+1)} = 0.4\ln(t+1)$ **and** then makes progress towards making "$\boldsymbol{t}$" the subject (See notes for this)	dM1	1.1b
	$t = \frac{25-\ln(t+1)}{1+\ln(t+1)}$ $t = \frac{26}{1+\ln(t+1)} - 1$ *	A1*	2.1
		(4)	
(c)	(i) Attempts $t_2 = \frac{26}{1+\ln 8} - 1$	M1	1.1b
	awrt 7.298	A1	1.1b
	(ii) awrt 7.33 seconds	A1	3.2a
		(3)	
			(8 marks)
Notes:			

(a)

B1: 25 but condone 25 seconds. If another value is given (apart from 0) it is B0

(b)

M1: Attempts to use the product rule in an attempt to differentiate $v = (10-0.4t)\ln(t+1)$

Look for $(10-0.4t)\times\frac{1}{(t+1)} \pm k\ln(t+1)$, where k is a constant, condoning slips.

If you see direct evidence of an incorrect rule used e.g. $vu' - uv'$ it is M0

You will see attempts from $v = 10\ln(t+1) - 0.4t\ln(t+1)$ which can be similarly marked.

In this case look for $\frac{a}{t+1} \pm \frac{bt}{t+1} \pm c\ln(t+1)$

A1: Correct differentiation. Condone a missing left hand or it seen as v', $\frac{dy}{dx}$ or even = 0

$\left(\frac{dv}{dt}\right) = \ln(t+1)\times -0.4 + \frac{(10-0.4t)}{t+1}$ or equivalent such as $\left(\frac{dv}{dt}\right) = \frac{10}{t+1} - \frac{0.4t}{(t+1)} - 0.4\ln(t+1)$

dM1: Score for setting their dV/dt = 0 (which must be in an appropriate form) and proceeding to an equation where the variable t occurs only once – ignoring ln(t + 1).

See two examples of how this can be achieved below. It is dependent upon the previous M.

Look for the following steps

- An allowable derivative set (or implied) = 0 E.g. $\ln(t+1)\times 0.4=\frac{(10-0.4t)}{t+1}$
- Cross multiplication (or division) and rearrangement to form an equation where the variable t only occurs once.

E.g.1.

$$\ln(t+1)\times 0.4=\frac{(10-0.4t)}{t+1}$$
$$\Rightarrow \ln(t+1)=\frac{25-t}{t+1}$$
$$\Rightarrow \ln(t+1)=-1+\frac{26}{t+1}$$

E.g 2

$$\ln(t+1)\times 0.4=\frac{(10-0.4t)}{t+1}$$
$$\Rightarrow 0.4t\ln(t+1)+0.4\ln(t+1)=10-0.4t$$
$$\Rightarrow 0.4t\left(1+\ln(t+1)\right)=10-0.4\ln(t+1)$$

A1*: Correctly proceeds to the given answer of $t=\frac{26}{1+\ln(t+1)}-1$ showing all key steps.

The key steps must include

- use of $\frac{\mathrm{d}v}{\mathrm{d}t}$ or v' which must be correct
- a correct line preceding the given answer, usually $t=\frac{25-\ln(t+1)}{1+\ln(t+1)}$ or $\frac{26}{t+1}-1=\ln(t+1)$

(c) (i)

M1: Attempts to use the iteration formula at least once.

Usually to find $t_2=\frac{26}{1+\ln 8}-1$ which may be implied by awrt 7.44

A1: awrt 7.298. This alone will score both marks as iteration is implied. ISW after sight of this value. As t_3 is the only value that rounds to 7.298 just score the rhs, it does not need to be labelled t_3

(c)(ii)

A1: Uses repeated iteration until value established as awrt 7.33 **seconds**. Allow awrt 7.33 **s**

Requires units. It also requires some evidence of iteration which will be usually be awarded from the award of the M

Question	Scheme	Marks	AOs
2(a)	$\mathrm{f}'(x) = 2x + \dfrac{4x-4}{2x^2-4x+5}$	M1 A1	1.1b 1.1b
	$2x + \dfrac{4x-4}{2x^2-4x+5} = 0 \Rightarrow 2x\left(2x^2-4x+5\right)+4x-4=0$	dM1	1.1b
	$2x^3-4x^2+7x-2=0*$	A1*	2.1
		(4)	
(b)	(i) $x_2 = \dfrac{1}{7}\left(2+4(0.3)^2-2(0.3)^3\right)$	M1	1.1b
	$x_2 = 0.3294$	A1	1.1b
	(ii) $x_4 = 0.3398$	A1	1.1b
		(3)	
(c)	$\mathrm{h}(x) = 2x^3-4x^2+7x-2$ $\mathrm{h}(0.3415) = 0.00366...$ $\mathrm{h}(0.3405) = -0.00130...$	M1	3.1a
	States: • there is a change of sign • $\mathrm{f}'(x)$ is continuous • $\alpha = 0.341$ to 3dp	A1	2.4
		(2)	
		(9 marks)	

Notes

(a)

M1: Differentiates $\ln\left(2x^2-4x+5\right)$ to obtain $\frac{g(x)}{2x^2-4x+5}$ where g(x) could be 1

A1: For $f'(x)=2x+\frac{4x-4}{2x^2-4x+5}$

dM1: Sets their f′(x) $=ax+\frac{g(x)}{2x^2-4x+5}$ = 0 and uses "**correct**" algebra, condoning slips, to obtain a cubic equation. E.g Look for $ax\left(2x^2-4x+5\right)\pm g(x)=0$ o.e. , condoning slips, followed by some attempt to simplify

A1*: Achieves $2x^3-4x^2+7x-2=0$ with no errors. (The dM1 mark must have been awarded)

(b)(i)

M1: Attempts to use the iterative formula with $x_1 = 0.3$. If no method is shown award for x_2 = awrt 0.33

A1: x_2 = awrt 0.3294 Note that $\frac{1153}{3500}$ is correct

Condone an incorrect suffix if it is clear that a correct value has been found

(b)(ii)

A1: x_4 = awrt 0.3398 Condone an incorrect suffix if it is clear that a correct value has been found

(c)

M1: Attempts to substitute x = 0.3415 and x = 0.3405 into a suitable function and gets one value correct (rounded or truncated to 1 sf). It is allowable to use a tighter interval that contains the root 0.340762654

Examples of suitable functions are $2x^3-4x^2+7x-2,\ x-\frac{1}{7}\left(4x^2-2x^3+2\right)$ and f '(x) as this has been found in part (a) with f '(0.3405)= - 0.00067.., f '(0.3415)= (+) 0.0018

There must be sufficient evidence for the function, which would be for example, a statement such as $h(x)=2x^3-4x^2+7x-2$ or sight of embedded values that imply the function, not just a value or values even if both are correct. Condone $h(x)$ being mislabelled as f

$h(0.3415)=2\times0.3415^3-4\times0.3415^2+7\times0.3415-2$

A1: Requires

- both calculations correct (rounded or truncated to 1sf)
- a statement that there is a change in sign and that the function is continuous
- a minimal conclusion e.g. ✓, proven, α = 0.341, root

Question	Scheme	Marks	AOs
3 (a)	Attempts $\text{f}(3)=$ and $\text{f}(4)=$ where $\text{f}(x)=\pm\left(2\ln(8-x)-x\right)$	M1	2.1
	$\text{f}(3)=\left(2\ln(5)-x\right)=(+)0.22$ and $\text{f}(4)=\left(2\ln(4)-4\right)=-1.23$ Change of sign and function continuous in interval $\left[3,4\right]\Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	For annotating the graph by drawing a cobweb diagram starting at $x_1=4$ It should have at least two spirals	M1	2.4
	Deduces that the iteration formula **can be used** to find an approximation for α because **the cobweb spirals inwards** for the cobweb diagram	A1	2.2a
		(2)	
			(4 marks)

Notes:

(a)

M1: Attempts $\text{f}(3)=$ and $\text{f}(4)=$ where $\text{f}(x)=\pm\left(2\ln(8-x)-x\right)$ or alternatively **compares** $2\ln 5$ to 3 and $2\ln 4$ to 4. This is not routine and cannot be scored by substituting 3 and 4 in both functions

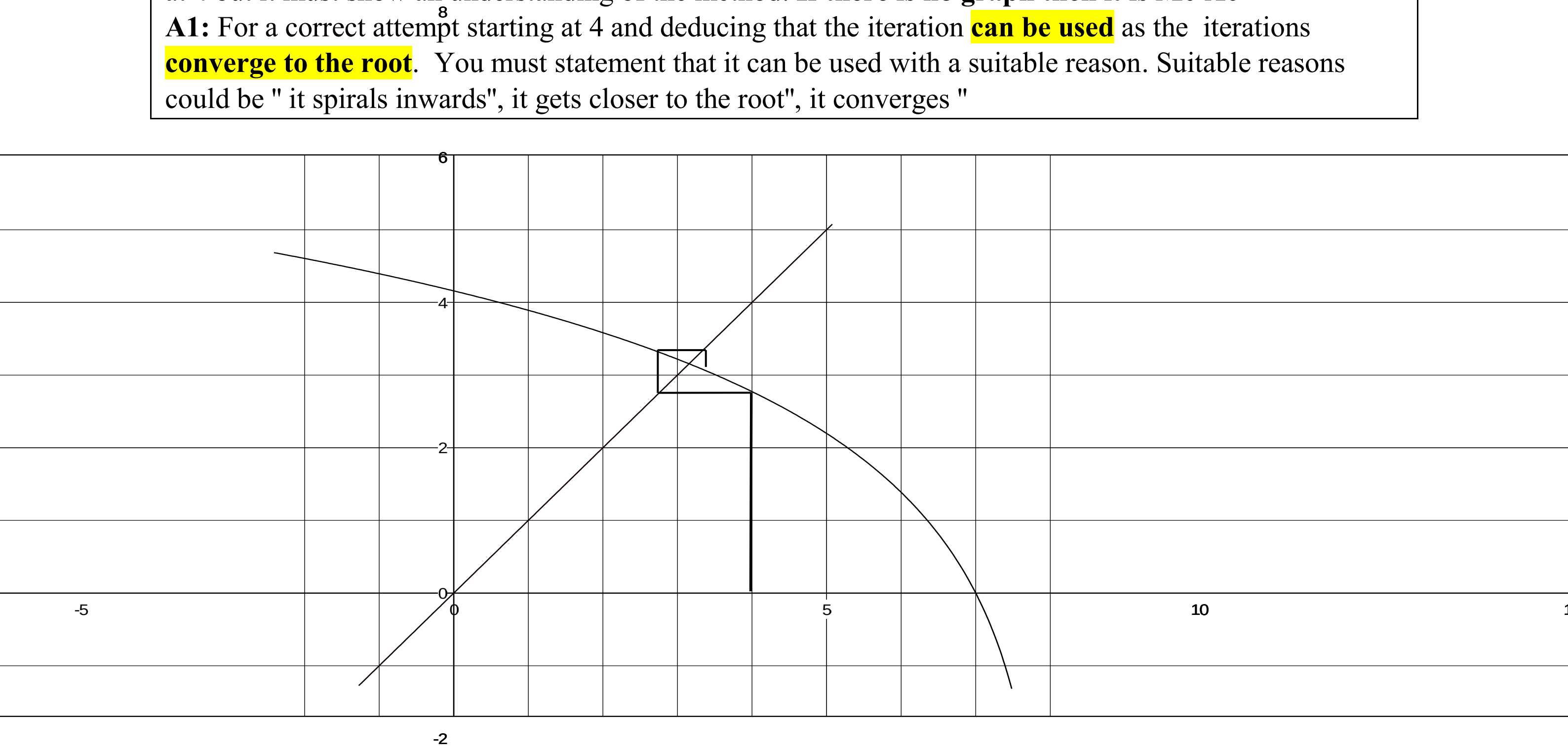
A1: Both values (calculations) correct to at least 1 sf with correct explanation and conclusion. (See underlined statements)

When comparing terms, allow reasons to be $2ln8=3.21>3,\ 2\ln4=2.77<4$ or similar

(b)

M1: For an attempt at using a cobweb diagram. Look for 5 or more correct straight lines. It may not start at 4 but it must show an understanding of the method. **If there is no graph then it is M0 A0**

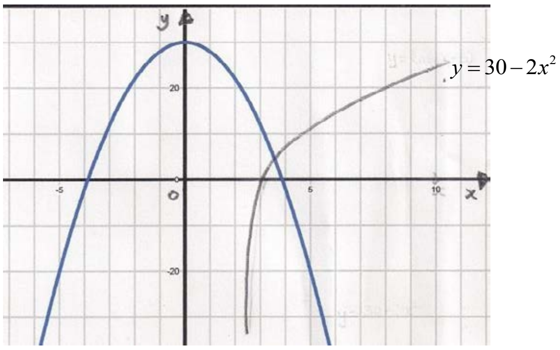
A1: For a correct attempt starting at 4 and deducing that the iteration **can be used** as the iterations **converge to the root**. You must statement that it can be used with a suitable reason. Suitable reasons could be " it spirals inwards", it gets closer to the root", it converges "



Question	Scheme	Marks	AOs
4 (a)	$\text{Area}(R) \approx \frac{1}{2} \times 0.5 \times \left[\underline{0.5 + 2\left(0.6742 + 0.8284 + 0.9686\right) + 1.0981}\right]$	B1 <u>M1</u>	1.1b 1.1b
	$\left\{= \frac{1}{4} \times 6.5405 \quad = \quad 1.635125\right\} = 1.635 \text{ (3 dp)}$	A1	1.1b
		(3)	
(b)	Any valid reason, for example • Increase the number of strips • Decrease the width of the strips • Use more trapezia between $x = 1$ and $x = 3$	B1	2.4
		(1)	
(c)(i)	$\left\{\int_1^3 \frac{5x}{1+\sqrt{x}}\,\mathrm{d}x\right\} = 5(\text{"}1.635\text{"}) = 8.175$	B1ft	2.2a
(c)(ii)	$\left\{\int_1^3 \left(6 + \frac{x}{1+\sqrt{x}}\right)\mathrm{d}x\right\} = 6(2) + (\text{"}1.635\text{"}) = 13.635$	B1ft	2.2a
		(2)	
			(6 marks)

Question **4** Notes:

(a)	
B1:	Outside brackets $\frac{1}{2} \times 0.5$ or $\frac{0.5}{2}$ or 0.25 or $\frac{1}{4}$
M1:	For structure of trapezium rule $[\ \underline{\ldots\ldots\ldots\ldots}\ ]$.
	No errors are allowed, e.g. an omission of a *y*-ordinate or an extra *y*-ordinate or a repeated *y*-ordinate.
A1:	Correct method leading to a correct answer only of 1.635
(b)	
B1:	See scheme
(c)	
B1:	8.175 or a value which is 5 × their answer to part (a)
	Note: Allow B1ft for 8.176 (to 3 dp) which is found from $5(1.63125) = 8.175625$
	Note: Do not allow an answer of 8.1886… which is found directly from integration
(d)	
B1:	13.635 or a value which is 12 + their answer to part (a)
	Note: Do not allow an answer of 13.6377… which is found directly from integration

Question	Scheme	Marks	AOs
5 (a)	$f(3.5) = -4.8,\ f(4) = (+)3.1$	M1	1.1b
	Change of sign and function continuous in interval $[3.5, 4] \Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	Attempts $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow \quad x_1 = 4 - \frac{3.099}{16.67}$	M1	1.1b
	$x_1 = 3.81$	A1	1.1b
	$y = \ln(2x - 5)$	**(2)**	
(c)	 Attempts to sketch both $y = \ln(2x - 5)$ and $y = 30 - 2x^2$	M1	3.1a
	States that $y = \ln(2x - 5)$ meets $y = 30 - 2x^2$ in just one place, therefore $y = \ln(2x - 5) = 30 - 2x$ has just one root $\Rightarrow f(x) = 0$ has just one root	A1	2.4
		(2)	
			(6 marks)

Notes:

(a)

M1: Attempts f(x) at both $x = 3.5$ and $x = 4$ with at least one correct to 1 significant figure

A1*: f (3.5) and f(4) correct to 1 sig figure (rounded or truncated) with a correct reason and conclusion. A reason could be change of sign, or $f(3.5) \times f(4) < 0$ or similar with f(x) being continuous in this interval. A conclusion could be 'Hence root' or 'Therefore root in interval'

(b)

M1: Attempts $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ evidenced by $x_1 = 4 - \frac{3.099}{16.67}$

A1: Correct answer only $x_1 = 3.81$

(c)

M1: For a valid attempt at showing that there is only one root. This can be achieved by

- Sketching graphs of $y = \ln(2x - 5)$ and $y = 30 - 2x^2$ on the same axes
- Showing that $f(x) = \ln(2x - 5) + 2x^2 - 30$ has no turning points
- Sketching a graph of $f(x) = \ln(2x - 5) + 2x^2 - 30$

A1: Scored for correct conclusion

Question	Scheme	Marks	AOs
6(a)	Uses or implies $h = 0.5$	B1	1.1b
	For correct form of the trapezium rule =	M1	1.1b
	$\frac{0.5}{2}\{3+2.2958+2(2.3041+1.9242+1.9089)\}=4.393$	A1	1.1b
		(3)	
(b)	Any valid statement reason, for example • Increase the number of strips • Decrease the width of the strips • Use more trapezia	B1	2.4
		(1)	
(c)	For integration by parts on $\int x^2 \ln x \, dx$	M1	2.1
	$=\frac{x^3}{3}\ln x-\int\frac{x^2}{3}\,dx$	A1	1.1b
	$\int -2x+5\,dx=-x^2+5x \quad (+c)$	B1	1.1b
	All integration attempted and limits used Area of $S=\int_1^3 \frac{x^2\ln x}{3}-2x+5\,dx=\left[\frac{x^3}{9}\ln x-\frac{x^3}{27}-x^2+5x\right]_{x=1}^{x=3}$	M1	2.1
	Uses correct ln laws, simplifies and writes in required form	M1	2.1
	Area of $S=\frac{28}{27}+\ln 27 \quad (a=28, b=27, c=27)$	A1	1.1b
		(6)	
			(10 marks)

Question 6 continued
Notes:
(a) **B1:** States or uses the strip width $h = 0.5$. This can be implied by the sight of $\frac{0.5}{2}\{...\}$ in the trapezium rule **M1:** For the correct form of the bracket in the trapezium rule. Must be y values rather than x values $\{\text{first } y \text{ value} + \text{last } y \text{ value} + 2\times(\text{sum of other } y \text{ values})\}$ **A1:** 4.393 **(b)** **B1:** See scheme **(c)** **M1:** Uses integration by parts the right way around. Look for $\int x^2 \ln x \, dx = Ax^3 \ln x - \int Bx^2 \, dx$ **A1:** $\frac{x^3}{3}\ln x - \int \frac{x^2}{3} \, dx$ **B1:** Integrates the $-2x+5$ term correctly $= -x^2 + 5x$ **M1:** All integration completed and limits used **M1:** Simplifies using ln law(s) to a form $\frac{a}{b} + \ln c$ **A1:** Correct answer only $\frac{28}{27} + \ln 27$

Question	Scheme	Marks	AOs
7(a)	States or uses $h = 1.5$	B1	1.1a
	Full attempt at the trapezium rule $= \frac{...}{2}\{1.63 + 2.63 + 2 \times (2 + 2.26 + 2.46)\}$	M1	1.1b
	= awrt 13.3 or $\frac{531}{40}$	A1	1.1b
		(3)	
(b)(i)	$\int_3^9 \log_3 (2x)^{10}\, dx = 10 \times "13.3" = \text{awrt}\,133$ or e.g. $\frac{531}{4}$	B1ft	2.2a
(ii)	$\int_3^9 \log_3 18x\, dx = \int_3^9 \log_3 (9 \times 2x)\, dx = \int_3^9 2 + \log_3 2x\, dx$ $= [2x]_3^9 + \int_3^9 \log_3 2x\, dx = 18 - 6 + \int_3^9 \log_3 2x\, dx = ...$	M1	3.1a
	Awrt 25.3 or $\frac{1011}{40}$	A1ft	1.1b
		(3)	
			(6 marks)
Notes:			

(a)

B1: States or uses $h = 1.5$

M1: A full attempt at the trapezium rule.

Look for $\frac{\text{their } h}{2}\{1.63 + 2.63 + 2 \times (2 + 2.26 + 2.46)\}$ but condone copying slips

Note that $\frac{\text{their } h}{2} 1.63 + 2.63 + 2 \times (2 + 2.26 + 2.46)$ scores M0 unless the missing brackets are recovered or implied by their answer. You may need to check.

Allow this mark if they add the areas of individual trapezia e.g.

$\frac{\text{their } h}{2}\{1.63 + 2\} + \frac{\text{their } h}{2}\{2 + 2.26\} + \frac{\text{their } h}{2}\{2.26 + 2.46\} + \frac{\text{their } h}{2}\{2.46 + 2.63\}$

Condone copying slips but must be a complete method using all the trapezia.

A1: awrt 13.3 (Note full accuracy is 13.275) or exact equivalent.

Note that the calculator answer is 13.324 so you must see correct working to award awrt 13.3

Use of $h = -1.5$ leading to a negative area can score B1M1A0 but allow full marks if then stated as positive.

(b)(i)

B1ft: Deduces that $\int_3^9 \log_3 (2x)^{10}\, dx = 10 \times "13.3" = \text{awrt } 133$

FT on their 13.3 look for 3sf accuracy but follow through on e.g. their rounded answer to part (a) so if 13 was their answer to part (a) then allow 130 here **following a correct method**.

A correct method must be seen here but a minimum is e.g. 10 × “13.3” = “133”

Note that $\int_3^9 \log_3 (2x)^{10}\, dx = 133.2414316...$ so a correct method must be seen to award marks.

Attempts to apply the trapezium rule again in any way score M0 as the instruction in the question was to use the <u>answer</u> to part (a).

(b)(ii)

M1: Shows correct log work to relate the given question to part (a)

Must reach as far as e.g. $\left[2x\right]_3^9+\int_3^9 \log_3 2x\,\mathrm{d}x=\ldots$ with correct use of limits on $\left[2x\right]_3^9$ which may be implied or equivalent work e.g. finds the area of the rectangle as 2×6

A1ft: **Correct working** followed by awrt 25.3 but ft on their 13.3 so allow for 12 + their answer to part (a) **following correct work** as shown.

Note that $\int_3^9 \log_3 18x\,\mathrm{d}x=25.32414\ldots$ **so a correct method must be seen to award marks**.

Some examples of an acceptable method are:

$$\int_3^9 \log_3 18x\,\mathrm{d}x=\int_3^9 \log_3\left(9\times 2x\right)\mathrm{d}x=\int_3^9 2+\log_3 2x\,\mathrm{d}x=6\times 2+\text{"13.3"}=25.3$$

$$\int_3^9 \log_3 18x\,\mathrm{d}x=\int_3^9 \log_3\left(9\times 2x\right)\mathrm{d}x=\int_3^9 2+\log_3 2x\,\mathrm{d}x=12+\text{"13.3"}=25.3$$

$$\int_3^9 \log_3 18x\,\mathrm{d}x=\int_3^9 \log_3\left(9\times 2x\right)\mathrm{d}x=\int_3^9 2+\log_3 2x\,\mathrm{d}x=\left[2x\right]_3^9+\int_3^9 \log_3 2x\,\mathrm{d}x=25.3$$

BUT just $12+\text{"13.3"}=25.3$ **scores M0**

Attempts to apply the trapezium rule again in any way score M0 as the instruction in the question was to use the <u>answer</u> to part (a).

Question	Scheme	Marks	AOs
8(a)	$\left(\text{f}'(x)=\right)4\cos\left(\frac{1}{2}x\right)-3$	M1 A1	1.1b 1.1b
	Sets $\text{f}'(x)=4\cos\left(\frac{1}{2}x\right)-3=0\Rightarrow x=$	dM1	3.1a
	$x=14.0$ Cao	A1	3.2a
		(4)	
(b)	Explains that $\text{f}(4)>0$, $\text{f}(5)<0$ **and** the function is continuous	B1	2.4
		(1)	
(c)	Attempts $x_1=5-\frac{8\sin 2.5-15+9}{"4\cos 2.5-3"}$ (NB $\text{f}(5)=-1.212\ldots$ and $\text{f}'(5)=-6.204\ldots$)	M1	1.1b
	$x_1=$ awrt 4.80	A1	1.1b
		(2)	
		(7 marks)	
Notes:			

(a)

M1: Differentiates to obtain $k\cos\left(\frac{1}{2}x\right)\pm\alpha$ where α is a constant which may be zero and no other terms. The brackets are not required.

A1: Correct derivative $\text{f}'(x)=4\cos\left(\frac{1}{2}x\right)-3$. Allow unsimplified e.g. $\text{f}'(x)=\frac{1}{2}\times 8\cos\left(\frac{1}{2}x\right)-3x^0$

There is no need for $\text{f}'(x)=\ldots$ or $\frac{\text{d}y}{\text{d}x}=\ldots$ just look for the expression and the brackets are not required.

dM1: For the complete strategy of proceeding to a value for x.

Look for

- $\text{f}'(x)=a\cos\left(\frac{1}{2}x\right)+b=0,\quad a,b\neq 0$
- Correct method of finding a valid solution to $a\cos\left(\frac{1}{2}x\right)+b=0$

Allow for $a\cos\left(\frac{1}{2}x\right)+b=0\Rightarrow\cos\left(\frac{1}{2}x\right)=\pm k\Rightarrow x=2\cos^{-1}\left(\pm k\right)$ where $|k|<1$

If this working is not shown then you may need to check their value(s).

For example $4\cos\left(\frac{1}{2}x\right)-3=0\Rightarrow x=1.4\ldots$ or 11.1… (or 82.8… or 637…. or 803 in degrees) would indicate this method.

A1: Selects the correct turning point $x=14.0$ and not just 14 or unrounded e.g. 14.011…
Must be this value only and no other values unless they are clearly rejected or 14.0 clearly selected. Ignore any attempts to find the y coordinate.

Correct answer with no working scores no marks.

(b)

B1: See scheme. Must be a full reason, (e.g. change of sign and continuous)
Accept equivalent statements for $\text{f}(4)>0$, $\text{f}(5)<0$ e.g. $\text{f}(4)\times\text{f}\left(5\right)<0$, "there is a change of sign", "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because x is continuous" or "because the interval is continuous"

(c)

M1: Attempts $x_1 = 5 - \frac{f(5)}{f'(5)}$ to obtain a value following through on their $f'(x)$ as long as it is a "changed" function.

Must be a correct N-R formula used – may need to check their values.

Allow if attempted in degrees. For reference in degrees f (5) = –5.65… and f ′(5) = 0.996… and gives $x_1 = 10.67...$

There must be clear evidence that $5 - \frac{f(5)}{f'(5)}$ is being attempted.

so e.g. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow x_1 = 4.80$ scores M0 as does e.g. $x_1 = x - \frac{8\sin\left(\frac{1}{2}x\right) - 3x + 9}{4\cos\left(\frac{1}{2}x\right) - 3} = 4.80$

BUT evidence may be provided by the accuracy of their answer. Note that the full N-R accuracy is 4.804624337 so e.g. 4.805 or 4.804 (truncated) with no evidence of incorrect work may imply the method.

A1: $x_1 =$ awrt 4.80 not awrt 4.8 but isw if awrt 4.80 is seen. Ignore any subsequent iterations.

Note that work for part (a) cannot be recovered in part (c)

Note also:

$5 - \frac{f(5)}{f'(5)} =$ awrt 4.80 following a correct derivative scores M1A1

$5 - \frac{f(5)}{f'(5)} \neq$ awrt 4.80 with no evidence that $5 - \frac{f(5)}{f'(5)}$ was attempted scores M0

Question	Scheme	Marks	AOs
9(a)	$h = 0.5$	B1	1.1a
	$A \approx \frac{0.5}{2}\left\{0.5774+0.8452+2\left(0.7071+0.7746+0.8165\right)\right\}$	M1	1.1b
	= awrt 1.50	A1	1.1b
	For reference: **The integration on a calculator gives 1.511549071** **The full accuracy for *y* values gives 1.504726147** **The accuracy from the table gives 1.50475**		
		(3)	

(b)	3 × their (a)
	If (a) is correct, allow awrt 4.50 or awrt 4.51 even with no working.
	Only allow 4.5 if (a) is correct and working is shown e.g. 3 × 1.5
	If (a) is incorrect allow 3 × their (a) given to at least 3sf but do not be too concerned about the accuracy (as they may use rounded or rounded value from (a))
		B1ft	2.2a

	For reference the integration on a calculator gives 4.534647213		
		(1)	
(c)	<u>**This mark depends on the B1 having been awarded in part (b) with awrt 4.5**</u> Look for a sensible comment. Some examples: • The answer is accurate to 2 sf or one decimal place • Answer to (b) is accurate as 4.535 ≈ 4.50 • Very accurate as 4.535 to 2 sf is 4.5 • 4.51425 < 4.535 so my answer is underestimate but not too far off • It is an underestimate but quite close • It is a very good estimate • High accuracy • (Quite) accurate • It is less than 1% out • 4.535 – 4.5 = 0.035 so not far out But **not** just "it is an underestimate" or Calculates percentage error correctly using awrt 4.50 or awrt 4.51 or 4.5 (No comment is necessary in these cases although one may be given) Examples: $\left\lvert\frac{4.535-4.50}{4.535}\right\rvert\times100=0.77\%$ or $\left\lvert\frac{4.535-4.51}{4.535}\right\rvert\times100=0.55\%$ or $\left\lvert\frac{4.535-4.51425}{4.535}\right\rvert\times100=0.46\%$ or $\left\lvert\frac{4.50}{4.535}\right\rvert\times100=99\%$ In these cases don't be too concerned about accuracy e.g. allow 1sf. **This mark should be withheld if there are any contradictory statements**	B1	3.2b
		(1)	
		(5 marks)	

(a)

B1: States or uses $h = 0.5$. May be implied by $\frac{1}{4}\times\{\ldots$ below.

M1: Correct attempt at the trapezium rule.

Look for $\frac{1}{2}h\times\left\{0.5774+0.8452+2\left(0.7071+0.7746+0.8165\right)\right\}$ condoning slips on the terms but must use all y values with no repeats.

There must be a clear attempt at $\frac{1}{2}h\times$(first y + last y + $2\times$"sum of the rest")

Give M0 for $\frac{1}{2}\times\frac{1}{2}0.5774+0.8452+2\left(0.7071+0.7746+0.8165\right)$ unless the missing brackets are implied.

NB this incorrect method gives 5.85…

May be awarded for separate trapezia e.g.

$$\frac{1}{4}\left(0.5774+0.7071\right)+\frac{1}{4}\left(0.7071+0.7746\right)+\frac{1}{4}\left(0.7746+0.8165\right)+\frac{1}{4}\left(0.8165+0.8452\right)$$

May be awarded for using the function e.g. $\frac{1}{2}h\times\left\{\sqrt{\frac{0.5}{1+0.5}}+\sqrt{\frac{2.5}{1+2.5}}+2\left(\sqrt{\frac{1}{1+1}}+\sqrt{\frac{1.5}{1+1.5}}+\sqrt{\frac{2}{1+2}}\right)\right\}$

A1: Awrt 1.50 (Apply isw if necessary)

Correct answers with no working – send to review

(b)

B1ft: See main scheme. Must be considering 3 × (a) and not e.g. attempting trapezium rule again.

(c)

B1: See scheme

10(a)	$\ln x \to \frac{1}{x}$	B1	1.1a
	Method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$ – see notes	M1	1.1b
	E.g. $2\times\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$	A1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=3\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{4}{x}=\frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ *	A1*	2.1
		(4)	
(b)	$12x^2+x-16\sqrt{x}=0\Rightarrow 12x^{\frac{3}{2}}+x^{\frac{1}{2}}-16=0$	M1	1.1b
	E.g. $12x^{\frac{3}{2}}=16-\sqrt{x}$	dM1	1.1b
	$x^{\frac{3}{2}}=\frac{4}{3}-\frac{\sqrt{x}}{12}\Rightarrow x=\left(\frac{4}{3}-\frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$ *	A1*	2.1
		(3)	
(c)	$x_2=\sqrt[3]{\left(\frac{4}{3}-\frac{\sqrt{2}}{12}\right)^2}$	M1	1.1b
	x_2 = awrt 1.13894	A1	1.1b
	x = 1.15650	A1	2.2a
		(3)	
			(10 marks)

Notes:

(a)

B1: Differentiates $\ln x \to \frac{1}{x}$ seen or implied

M1: Correct method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$:

Look for $\frac{4x^2+x}{2\sqrt{x}} \to ...x^{\frac{3}{2}}+...x^{\frac{1}{2}}$ being then differentiated to $Px^{\frac{1}{2}}+...$ **or** $...+Qx^{-\frac{1}{2}}$

Alternatively uses the quotient rule on $\frac{4x^2+x}{2\sqrt{x}}$.

Condone slips but if rule is not quoted expect $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{2\sqrt{x}\left(Ax+B\right)-\left(4x^2+x\right)Cx^{-\frac{1}{2}}}{\left(2\sqrt{x}\right)^2}\left(A,B,C>0\right)$

But a correct rule may be implied by their *u*, *v*, *u*′, *v*′ followed by applying $\frac{vu'-uv'}{v^2}$ **etc.**

Alternatively uses the product rule on $\left(4x^2+x\right)\left(2\sqrt{x}\right)^{-1}$

Condone slips but expect $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=Ax^{-\frac{1}{2}}\left(Bx+C\right)+D\left(4x^2+x\right)x^{-\frac{3}{2}}\left(A,B,C>0\right)$

In general condone missing brackets for the M mark. If they quote *u* = 4*x*^{2} + *x* and v = 2√*x* and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have *v* rather than *v*^{2} in the denominator.

A1: Correct differentiation of $\frac{4x^2+x}{2\sqrt{x}}$ although may not be simplified.

Examples: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{2\sqrt{x}\left(8x+1\right)-\left(4x^2+x\right)x^{-\frac{1}{2}}}{\left(2\sqrt{x}\right)^2}$, $\frac{1}{2}x^{-\frac{1}{2}}\left(8x+1\right)-\frac{1}{4}\left(4x^2+x\right)x^{-\frac{3}{2}}$, $2\times\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$

A1*: Obtains $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ via $3\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{4}{x}$ or a correct application of the quotient or product rule and with sufficient working shown to reach the printed answer.

There must be no errors e.g. missing brackets.

(b)

M1: Sets $12x^2+x-16\sqrt{x}=0$ and divides by $\sqrt{x}$ or equivalent e.g. divides by x and multiplies by $\sqrt{x}$

dM1: Makes the term in $x^{\frac{3}{2}}$ the subject of the formula

A1*: A correct and rigorous argument leading to the given solution.

Alternative - working backwards:

$$x=\left(\frac{4}{3}-\frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}\Rightarrow x^{\frac{3}{2}}=\frac{4}{3}-\frac{\sqrt{x}}{12}\Rightarrow 12x^{\frac{3}{2}}=16-\sqrt{x}\Rightarrow 12x^2=16\sqrt{x}-x\Rightarrow 12x^2-16\sqrt{x}+x=0$$

M1: For raising to power of 3/2 both sides. dM1: Multiplies through by $\sqrt{x}$. A1: Achieves printed answer and makes a minimal comment e.g. tick, #, QED, true etc.

(c)

M1: Attempts to use the iterative formula with $x_1 = 2$. This is implied by sight of $x_2=\left(\frac{4}{3}-\frac{\sqrt{2}}{12}\right)^{\frac{2}{3}}$ or awrt 1.14

A1: x_2 = awrt 1.13894

A1: Deduces that $x = 1.15650$

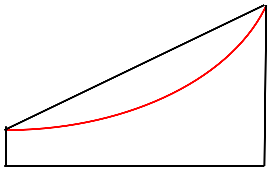
Question	Scheme	Marks	AOs
11			

Time (s)	0	5	10	15	20	25
Speed (m s^{-1})	2	5	10	18	28	42

(a)	Uses an allowable method to estimate the area under the curve. E.g.	M1	3.1a
	Way 1: an attempt at the trapezium rule (see below)		
	Way 2: $\{s=\}\left(\frac{2+42}{2}\right)(25)\ \{=550\}$		
	Way 3: $42=2+25(a)\Rightarrow a=1.6\Rightarrow s=2(25)+(0.5)(1.6)(25)^2\{=550\}$		
	Way 4: $\{d=\}(2)(5)+5(5)+10(5)+18(5)+28(5)\ \left\{=63(5)=315\right\}$		
	Way 5: $\{d=\}\ 5(5)+10(5)+18(5)+28(5)+42(5)\ \left\{=103(5)=515\right\}$		
	Way 6: $\{d=\}\ \frac{315+515}{2}\ \left\{=415\right\}$		
	Way 7: $\{d=\}\left(\frac{2+5+10+18+28+42}{6}\right)(25)\ \left\{=437.5\right\}$		
	$\frac{1}{2}\times(5)\times[2+2(5+10+18+28)+42]$ or $\frac{1}{2}\times[\text{"315"}+\text{"515"}]$	M1	1.1b
	$=415\ \{\text{m}\}$	A1	1.1b
		(3)	
(b) **Alt 1**	**<u>Uses a Way 1, Way 2, Way 3, Way 5, Way 6 or Way 7 method in (a)</u>**		
	Overestimate **and** a relevant explanation e.g. • {top of} trapezia lie above the curve • Area of trapezia > area under curve • An appropriate diagram which gives reference to the extra area • Curve is convex • $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}>0$ • Acceleration is {continually} increasing • The gradient of the curve is {continually} increasing • All the rectangles are above the curve (Way 5)	B1ft	2.4
		(1)	
(b) **Alt 2**	**<u>Uses a Way 4 method in (a)</u>**		
	Underestimate **and** a relevant explanation e.g. • All the rectangles are below the curve	B1ft	2.4
		(1)	
			(4 marks)

Notes for Question **11**

(a)	
M1:	A low-level problem-solving mark for using an allowable method to estimate the area under the curve. E.g.
	Way 1: See scheme. Allow $\lambda(2+2(5+10+18+28)+42);\ \lambda>0$ for 1st M1
	Way 2: Uses $s=\left(\frac{u+v}{2}\right)t$ which is equivalent to finding the area of a large trapezium
	Way 3: Complete method using a uniform acceleration equation.
	Way 4: Sums rectangles lying below the curve. Condone a slip on one of the speeds.
	Way 5: Sums rectangles lying above the curve. Condone a slip on one of the speeds.
	Way 6: Average the result of Way 3 and Way 4. Equivalent to Way 1.
	Way 7: Applies (average speed) × (time)

Notes for Question **11** Continued	
(a)	***continued***
M1:	Correct trapezium rule method with $h=5$. Condone a slip on one of the speeds. The ‘2’ and ‘42’ should be in the correct place in the [......].
A1:	415
Note:	Units do not have to be stated
Note:	Give final A0 for giving a final answer with incorrect units. e.g. give final A0 for 415 km or $415\,\text{ms}^{-1}$
Note:	Only the 1st M1 can only be scored for Way 2, Way 3, Way 4, Way 5 and Way 7 methods
Note:	Full marks in part (a) can only be scored by using a Way 1 or a Way 6 method.
Note:	Give M0 M0 A0 for $\{d=\}\ 2(5)+5(5)+10(5)+18(5)+28(5)+42(5)\ \ \{=105(5)=525\}$ (i.e. using too many rectangles)
Note	Condone M1 M0 A0 for $\left[\frac{(2+10)}{2}(10)+\frac{(10+18)}{2}(5)+\frac{(18+28)}{2}(5)+\frac{(28+42)}{2}(5)\right]=395\text{ m}$
Note:	Give M1 M1 A1 for $5\left[\frac{(2+5)}{2}+\frac{(5+10)}{2}+\frac{(10+18)}{2}+\frac{(18+28)}{2}+\frac{(28+42)}{2}\right]=415\text{ m}$
Note:	Give M1 M1 A1 for $\frac{5}{2}(2+42)+5(5+10+18+28)=415\text{ m}$
Note:	<u>**Bracketing mistake:**</u>
	Unless the final calculated answer implies that the method has been applied correctly
	give M1 M0 A0 for $\frac{5}{2}(2)+2(5+10+18+28)+42\ \ \{=169\}$
	give M1 M0 A0 for $\frac{5}{2}(2+42)+2(5+10+18+28)\ \ \{=232\}$
Note:	Give M0 M0 A0 for a Simpson’s Rule Method
(b)	**Alt 1**
B1ft:	**This mark depends on both an answer to part (a) being obtained and the first M in part (a)** See scheme
Note:	Allow the explanation “curve concaves upwards”
Note:	Do not allow explanations such as “curve is concave” or “curve concaves downwards”
Note:	Do not allow explanation “gradient of the curve is positive”
Note:	Do not allow explanations which refer to “friction” or “air resistance”
Note:	The diagram opposite is sufficient as an explanation. It must show the top of a trapezium lying above the curve. 
(b)	**Alt 2**
B1ft:	**This mark depends on both an answer to part (a) being obtained and the first M in part (a)** See scheme
Note:	Do not allow explanations which refer to “friction” or “air-resistance”

Question	Scheme	Marks	AOs
12 (a) **Way 1**	$\{y = x^x \Rightarrow\}\ \ \ln y = x\ln x$	B1	1.1a
	$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x$	M1	1.1b
		A1	2.1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ \ \frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = \mathrm{e}^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	**(5)**	
(a) **Way 2**	$\{y = x^x \Rightarrow\}\ \ y = \mathrm{e}^{x\ln x}$	B1	1.1a
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{x}{x} + \ln x\right)\mathrm{e}^{x\ln x}$	M1	1.1b
		A1	2.1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ \ \frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = \mathrm{e}^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	**(5)**	
(b) **Way 1**	Attempts both $1.5^{1\ 5} = 1.8...$ and $1.6^{1\ 6} = 2.1...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$1.8... < 2$ and $2.1... > 2$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
(c)	Attempts $x_{n+1} = 2x_n^{1-x_n}$ at least once with $x_1 = 1.5$ Can be implied by $2(1.5)^{1-1\ 5}$ or awrt 1.63	M1	1.1b
	$\{x_4 = 1.67313... \Rightarrow\}\ x_4 = 1.673$ (3 dp) **cao**	A1	1.1b
		(2)	
(d)	Give 1st B1 for any of • oscillates • periodic • non-convergent • divergent • fluctuates • goes up and down • 1, 2, 1, 2, 1, 2 • alternates (condone) Give B1 B1 for any of • periodic {sequence} with period 2 • oscillates between 1 and 2	B1	2.5
	Condone B1 B1 for any of • fluctuates between 1 and 2 • keep getting 1, 2 • alternates between 1 and 2 • goes up and down between 1 and 2 • 1, 2, 1, 2, 1, 2, …	B1	2.5
		(2)	
		(11 marks)	

Note	**A common solution**
	A maximum of 3 marks (i.e. B1 1st M1 and 2nd M1) can be given for the solution $\log y = x\log x \Rightarrow \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \log x$ $\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ 1 + \log x = 0 \Rightarrow x = 10^{-1}$
	• 1st B1 for $\log y = x\log x$ • 1st M1 for $\log y \to \lambda\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x};\ \lambda \neq 0$ or $x\log x \to 1 + \log x$ or $\frac{x}{x} + \log x$ • 2nd M1 can be given for $1 + \log x = 0 \Rightarrow \log x = k \Rightarrow x = ...;\ \ k \neq 0$

Question	Scheme	Marks	AOs
12 (b) **Way 2**	For $x^x - 2$, attempts both $1.5^{1\ 5} - 2 = -0.16...$ and $1.6^{1\ 6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$-0.16... < 0$ and $0.12... > 0$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
12 (b) **Way 3**	For $\ln y = x\ln x$, attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$0.608... < 0.69...$ and $0.752... > 0.69...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
12 (b) **Way 4**	For $\log y = x\log x$, attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp	M1	1.1b
	$0.264... < 0.301...$ and $0.326... > 0.301...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	

Notes for Question 12	
(a)	**Way 1**
B1:	$\ln y = x\ln x$. Condone $\log_x y = x\log_x x$ or $\log_x y = x$
M1:	For either $\ln y \to \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x$ or $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{x} + \ln x$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = y(1 + \ln x)$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = \mathrm{e}^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give no marks for no working leading to 0.368
Note:	Give M0 A0 M0 A0 for $\ln y = x\ln x \to x = 0.368$ with no intermediate working
(a)	**Way 2**
B1:	$y = \mathrm{e}^{x\ln x}$
M1:	For either $y = \mathrm{e}^{x\ln x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(\ln x)\mathrm{e}^{x\ln x}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{x}{x} + \ln x\right)\mathrm{e}^{x\ln x}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = (1 + \ln x)\mathrm{e}^{x\ln x}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = \mathrm{e}^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give B1 M1 A0 M1 A1 for the following solution: $\{y = x^x \Rightarrow\}$ $\ln y = x\ln x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x \Rightarrow 1 + \ln x = 0 \Rightarrow x = \mathrm{e}^{-1}$ or awrt 0.368

Notes for Question 12 Continued	
(b)	**Way 1**
M1:	Attempts both $1.5^{1\,5} = 1.8...$ and $1.6^{1\,6} = 2.1...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1\,5} =$ awrt 1.8... and $1.6^{1\,6} =$ awrt 2.1..., reason (e.g. $1.8... < 2$ and $2.1... > 2$ or states *C* cuts through $y = 2$), *C* continuous and conclusion
(b)	**Way 2**
M1:	Attempts both $1.5^{1\,5} - 2 = -0.16...$ and $1.6^{1\,6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1\,5} - 2 = -0.16...$ and $1.6^{1\,6} - 2 = 0.12...$ correct to awrt 1 dp, reason (e.g. $-0.16... < 0$ and $0.12... > 0$, sign change or states C cuts through $y = 0$), *C* continuous and conclusion
(b)	**Way 3**
M1:	Attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ correct to awrt 1 dp, reason (e.g. $0.608... < 0.69...$ and $0.752... > 0.69...$ or states they are either side of $\ln 2$), *C* continuous and conclusion.
(b)	**Way 4**
M1:	Attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp
A1:	Both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ correct to awrt 2 dp, reason (e.g. $0.264... < 0.301...$ and $0.326... > 0.301...$ or states they are either side of $\log 2$), *C* continuous and conclusion.
(c)	
M1:	An attempt to use the given or their formula once. Can be implied by $2(1.5)^{1-1\,5}$ or awrt 1.63
A1:	States $x_4 = 1.673$ **cao** (to 3 dp)
Note:	Give M1 A1 for stating $x_4 = 1.673$
Note:	M1 can be implied by stating their final answer $x_4 =$ awrt 1.673
Note:	$x_2 = 1.63299...$, $x_3 = 1.46626...$, $x_4 = 1.67313...$
(d)	
B1:	see scheme
B1:	see scheme
Note:	Only marks of B1B0 or B1B1 are possible in (d)
Note:	Give B0 B0 for "Converges in a cob-web pattern" or "Converges up and down to α"

Question	Scheme	Marks	AOs
13	The equation $2x^3+x^2-1=0$ has exactly one real root		
(a)	$\{\mathrm{f}(x)=2x^3+x^2-1\Rightarrow\}\ \mathrm{f}'(x)=6x^2+2x$	B1	1.1b
	$\left\{x_{n+1}=x_n-\dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}\Rightarrow\right\}\ \{x_{n+1}\}=x_n-\dfrac{2x_n^{\ 3}+x_n^{\ 2}-1}{6x_n^{\ 2}+2x_n}$	M1	1.1b
	$=\dfrac{x_n(6x_n^{\ 2}+2x_n)-(2x_n^{\ 3}+x_n^{\ 2}-1)}{6x_n^{\ 2}+2x_n}\Rightarrow x_{n+1}=\dfrac{4x_n^{\ 3}+x_n^{\ 2}+1}{6x_n^{\ 2}+2x_n}\quad *$	A1*	2.1
		(3)	
(b)	$\{x_1=1\Rightarrow\}\ x_2=\dfrac{4(1)^3+(1)^2+1}{6(1)^2+2(1)}$ or $x_2=1-\dfrac{2(1)^3+(1)^2-1}{6(1)^2+2(1)}$	M1	1.1b
	$\Rightarrow x_2=\dfrac{3}{4},\ x_3=\dfrac{2}{3}$	A1	1.1b
		(2)	
(c)	Accept any reasons why the Newton-Raphson **method** cannot be used with $x_1=0$ which refer or ***allude*** to either the stationary point or the tangent. E.g. • There is a stationary point at $x=0$ • Tangent to the curve (or $y=2x^3+x^2-1$) would not meet the x-axis • Tangent to the curve (or $y=2x^3+x^2-1$) is horizontal	B1	2.3
		(1)	
			(6 marks)

Notes for Question **13**

(a)	
B1:	States that $\mathrm{f}'(x)=6x^2+2x$ or states that $\mathrm{f}'(x_n)=6x_n^{\ 2}+2x_n$ (Condone $\dfrac{\mathrm{d}y}{\mathrm{d}x}=6x^2+2x$)
M1:	Substitutes $\mathrm{f}(x_n)=2x_n^{\ 3}+x_n^{\ 2}-1$ and their $\mathrm{f}'(x_n)$ into $x_{n+1}=x_n-\dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$
A1*:	A correct intermediate step of making a common denominator which leads to the given answer
Note:	Allow B1 if $\mathrm{f}'(x)=6x^2+2x$ is applied as $\mathrm{f}'(x_n)$ (or $\mathrm{f}'(x)$) in the NR formula $\{x_{n+1}\}=x_n-\dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$
Note:	Allow M1A1 for • $x_{n+1}=x-\dfrac{2x^3+x^2-1}{6x^2+2x}=\dfrac{x(6x^2+2x)-(2x^3+x^2-1)}{6x^2+2x}\Rightarrow x_{n+1}=\dfrac{4x_n^{\ 3}+x_n^{\ 2}+1}{6x_n^{\ 2}+2x_n}$
Note	Condone $x=x-\dfrac{2x^3+x^2-1}{\text{"}6x^2+2x\text{"}}$ for M1
Note	Condone $x_n-\dfrac{2x_n^{\ 3}+x_n^{\ 2}-1}{\text{"}6x_n^{\ 2}+2x_n\text{"}}$ or $x-\dfrac{2x^3+x^2-1}{\text{"}6x^2+2x\text{"}}$ (i.e. no $x_{n+1}=\dots$) for M1
Note:	Give M0 for $x_{n+1}=x_n-\dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$ followed by $x_{n+1}=2x_n^{\ 3}+x_n^{\ 2}-1-\dfrac{2x_n^{\ 3}+x_n^{\ 2}-1}{6x_n^{\ 2}+2x_n}$
Note:	Correct notation, i.e. x_{n+1} and x_n must be seen in their final answer for A1*

Notes for Question **13** Continued	
(b)	
M1:	An attempt to use the given or their formula once. Can be implied by $\frac{4(1)^3+(1)^2+1}{6(1)^2+2(1)}$ or 0.75 o.e.
Note:	Allow one slip in substituting $x_1=1$
A1:	$x_2=\frac{3}{4}$ and $x_3=\frac{2}{3}$
Note:	Condone $x_2=\frac{3}{4}$ and x_3 = awrt 0.667 for A1
Note:	Condone $\frac{3}{4},\frac{2}{3}$ listed in a correct order ignoring subscripts
(c)	
B1:	See scheme
Note:	Give B0 for the following isolated reasons: e.g. • You cannot divide by 0 • The fraction (or the NR formula) is undefined at $x=0$ • At $x=0$, $f'(x_1)=0$ • x_1 cannot be 0 • $6x^2+2x$ cannot be 0 • the denominator is 0 which cannot happen • if $x_1=0$, $6x^2+2x=0$

Question	Scheme	Marks	AOs
14(a)	$\mathrm{f}(x) = (8 - x)\ln x,\ x > 0$		
	Crosses x-axis $\Rightarrow \mathrm{f}(x) = 0 \Rightarrow (8 - x)\ln x = 0$		
	x coordinates are 1 and 8	B1	1.1b
		(1)	
(b)	Complete strategy of setting $\mathrm{f}'(x) = 0$ and rearranges to make $x = ...$	M1	3.1a
	$\left\{\begin{matrix} u = (8 - x) & v = \ln x \\ \dfrac{\mathrm{d}u}{\mathrm{d}x} = -1 & \dfrac{\mathrm{d}v}{\mathrm{d}x} = \dfrac{1}{x} \end{matrix}\right\}$		
	$\mathrm{f}'(x) = -\ln x + \dfrac{8-x}{x}$	M1	1.1b
		A1	1.1b
	$-\ln x + \dfrac{8-x}{x} = 0 \Rightarrow -\ln x + \dfrac{8}{x} - 1 = 0$ $\Rightarrow \dfrac{8}{x} = 1 + \ln x \Rightarrow x = \dfrac{8}{1 + \ln x}$ *	A1*	2.1
		(4)	
(c)	Evaluates both $\mathrm{f}'(3.5)$ and $\mathrm{f}'(3.6)$	M1	1.1b
	$\mathrm{f}'(3.5) = 0.032951317...$ and $\mathrm{f}'(3.6) = -0.058711623...$ Sign change and as $\mathrm{f}'(x)$ is continuous, the x coordinate of Q lies between $x = 3.5$ and $x = 3.6$	A1	2.4
		(2)	
(d)(i)	$\{x_5 =\}\ 3.5340$	B1	1.1b
(d)(ii)	$\{x_Q =\}\ 3.54$ (2 dp)	B1	2.2a
		(2)	
			(9 marks)

Question **14** Notes:	
(a)	
B1:	Either • 1 and 8 • on Figure 2, marks 1 next to A and 8 next to B
(b)	
M1:	Recognises that Q is a stationary point (and not a root) and applies a complete strategy of setting $\text{f}'(x) = 0$ and rearranges to make $x = \ldots$
M1:	Applies $vu' + uv'$, where $u = 8 - x,\ v = \ln x$ **Note:** This mark can be recovered for work in part (c)
A1:	$(8 - x)\ln x \rightarrow\ -\ln x\ +\ \dfrac{8-x}{x}$, or equivalent **Note:** This mark can be recovered for work in part (c)
A1*:	Correct proof with no errors seen in working.
(c)	
M1:	Evaluates both $\text{f}'(3.5)$ and $\text{f}'(3.6)$
A1:	$\text{f}'(3.5) = \text{awrt } 0.03$ and $\text{f}'(3.6) = \text{awrt } -0.06$ or $\text{f}'(3.6) = -0.05$ (truncated) **and** a correct conclusion
(d)(i)	
B1:	See scheme
(d)(ii)	
B1:	Deduces (e.g. by the use of further iterations) that the x coordinate of Q is 3.54 accurate to 2 dp **Note:** $3.5 \rightarrow 3.55119 \rightarrow 3.52845 \rightarrow 3.53848 \rightarrow 3.53404 \rightarrow 3.53600 \rightarrow 3.53514$ $(\rightarrow 3.535518\ldots)$

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>15.</td><td><table><tr><td>x</td><td>-2</td><td>2</td><td>6</td><td>10</td></tr><tr><td>y</td><td>0</td><td>2</td><td>$\mathbf{4\sqrt{2}}$</td><td>$6\sqrt{3}$</td></tr></table></td><td></td></tr>
<tr><td>(a)</td><td>$\{\text{At } x=2,\}\ y=2$ and $\{\text{At } x=6,\}\ y=4\sqrt{2}$ or $2\sqrt{8}$ or awrt 5.7</td><td>B1 cao
(1)</td></tr>
<tr><td>(b)</td><td>$\frac{1}{2}\times 4$; or $h=4$</td><td>B1 oe</td></tr>
<tr><td></td><td>$\underline{\left\{0+6\sqrt{3}+2\left(their2+their4\sqrt{2}\right)\right\}}$ <u>For structure of {................}</u></td><td>M1<u>A1ft</u></td></tr>
<tr><td></td><td>$\frac{1}{2}\times 4\underline{\left\{0+6\sqrt{3}+2\left(2+4\sqrt{2}\right)\right\}}$ $\{= 2(25.706) = 51.412..\} =$ awrt 51.412</td><td>A1
(4)
(5 marks)</td></tr>
</table>

Notes

(a) B1: 2 and $4\sqrt{2}$ or $2\sqrt{8}$ or awrt 5.7 (or any correct unsimplified surd equivalent given as the final answer to part (a)) These may be stated as a final answer and not appear in the table, or may appear in the table. If a correct surd appears in the working (unsimplified) and is then simplified to give an incorrect answer to (a) which is used in the table and in part (b) then this is B0.

(b) B1: for using $\frac{1}{2}\times 4$ or 2 or equivalent or for stating h

M1: requires the correct $\{......\}$ bracket structure.

It needs the first bracket to contain first y value (as this is zero it may be omitted) **plus** last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are x values instead of y values

A1ft: for the correct bracket $\{......\}$ following through candidate's y values found in part (a).

A1: for answer which rounds to 51.412 then isw

NB: Separate trapezia may be used : B1 for 4, M1 for $\frac{1}{2}h(a+b)$ used 3 times (and A1ft if it is all correct)

Then A1 as before.

Special case: Bracketing mistake $2\times(0+6\sqrt{3})+2\left(2+4\sqrt{2}\right)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). An answer of 36.098 usually indicates this error.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>16.</td><td>
<table>
<tr><td>x</td><td>0</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td></tr>
<tr><td>y</td><td>1</td><td>2.821</td><td>6</td><td>12.502</td><td>26.585</td></tr>
</table>
</td><td></td></tr>
<tr><td>(a)</td><td>$\{\text{At } x=1,\}\ y=6$ (allow 6.000 or even 6.00)</td><td>B1 cao
(1)</td></tr>
<tr><td>(b)</td><td>$\frac{1}{2}\times 0.5$;
$\underline{\{1+26.585+2(2.821+\text{ their } 6+12.502)\}}$ <u>For structure of $\{\ldots\ldots\ldots\ldots\}$</u> ;
$\frac{1}{2}\times 0.5\ \underline{\{1+26.585+2(2.821+\ 6+12.502)\}}\ \{=\tfrac{1}{4}(70.231)=17.557..\}$ = awrt 17.56</td><td>B1 oe
M1<u>A1ft</u>
A1
(4)</td></tr>
<tr><td>(c)</td><td>10 + “17.56” = “27.56”</td><td>B1ft
(1)
[6]</td></tr>
<tr><td></td><td colspan="2">Notes</td></tr>
<tr><td>(a)
(b)</td><td colspan="2">B1: 6
B1: for using $\frac{1}{2}\times 0.5$ or $\frac{1}{4}$ or equivalent.
M1: requires the correct $\{\ldots\ldots\}$ bracket structure. It needs the first bracket to contain first <i>y</i> value plus last <i>y</i> value and the second bracket to be multiplied by 2 and to be the summation of the remaining <i>y</i> values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are <i>x</i> values instead of <i>y</i> values
A1ft: for the correct bracket $\{\ldots\ldots\}$ following through candidate’s <i>y</i> value found in part (a).
A1: for answer which rounds to 17.56
NB: Separate trapezia may be used: B1 for 0.25, M1 for 1/2 $h(a+b)$ used 3 or 4 times (and A1ft if it is all correct) Then A1 as before.
Special case: Bracketing mistake $0.25\times(1+26.585)+2(2.821+\text{their } 6+12.502)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). An answer of 49.542 usually indicates this error.</td></tr>
<tr><td>(c)</td><td colspan="2">B1ft: 10 + their answer to part (b)
(May be obtained by using the trapezium rule again with all values for <i>y</i> increased by 5)</td></tr>
</table>

Question Number	Scheme	Marks
	$y = 8 - 2^{x-1} \quad 0 \leqslant x \leqslant 4$	
17. (a)	7 7	B1 **cao**
		[1]
(b)	$\left(\int_0^4 \left(8 - 2^{x-1}\right)\mathrm{d}x \approx\right) \frac{1}{2}\times 1; \times \underline{\left\{7.5 + 2\left(\text{"their 7"} + 6 + 4\right) + 0\right\}}$ Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$	B1;
	<u>For structure of trapezium</u> <u>rule</u> $\underline{\{............\}}$ for a candidate's y-ordinates.	<u>M1</u>
	$\left\{= \frac{1}{2}\times 41.5\right\} = 20.75$ o.e. 20.75	A1 **cao**
		[3]
(c)	Area $(R) = \text{"20.75"} - \frac{1}{2}(7.5)(4)$	M1
	$= 5.75$ 5.75	A1 **cao**
		[2]
		6

	Question 17 Notes	
(a)	**B1**	For 7 only
(b)	**B1**	For using $\frac{1}{2}\times 1$ or $\frac{1}{2}$ or equivalent.
	M1	Requires the correct $\{......\}$ bracket structure. It needs the 7.5 stated but the 0 may be omitted. The inner bracket needs to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however (unless it is 0)). M0 is awarded if values used in brackets are x values instead of y values
	A1	For 20.75 or fraction equivalent e.g. $20\frac{3}{4}$ *or* $\frac{83}{4}$
	Note	**NB: Separate trapezia may be used** : B1 for 0.5, M1 for 1/2 $h(a + b)$ used 3 or 4 times Then A1 as before.
	Special case:	Bracketing mistake $0.5\times(7.5 + 0) + 2(\text{ their } 7 + 6 + 4)$ scores B1 M1 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). An answer of 37.75 usually indicates this error.
	Common error:	Many candidates use $\frac{1}{2}\times\frac{4}{5}$ and score B0 Then they proceed with $\underline{\left\{7.5 + 2\left(\text{"their 7"} + 6 + 4\right) + 0\right\}}$ and score M1 This usually gives 16.6 for B0M1A0
(c)	**M1**	their answer to (b) − area of triangle with base 4 and height 7.5 or alternative correct method e.g. their answer to (b) $-\int_0^4 \left(7.5 - \frac{7.5}{4}x\right)\mathrm{d}x$ (Even if this leads to a negative answer) This may be implied by a correct answer or by an answer where they have subtracted 15 from their answer to part (b). Must use answer to part (b).
	A1	5.75 or fraction equivalent e.g. $5\frac{3}{4}$ *or* $\frac{23}{4}$

<table>
<tr><th>Question Number</th><th colspan="3">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="3">18.(a)</td><td colspan="3"><table><tr><td>x</td><td>1</td><td>1.25</td><td>1.5</td><td>1.75</td><td>2</td></tr><tr><td>y</td><td>1.414</td><td>1.601</td><td>1.803</td><td>2.016</td><td>2.236</td></tr></table></td><td></td></tr>
<tr><td>$\{\text{At } x=1.25,\}\ y = 1.601$ (only)</td><td colspan="2">1.601 (May not be in the table and can score if seen as part of their working in (b))</td><td>B1 cao</td></tr>
<tr><td colspan="3"></td><td>[1]</td></tr>
<tr><td rowspan="7">(b)</td><td colspan="3">$\frac{1}{2}\times 0.25;\times\underline{\left\{1.414+2.236+2\left(\text{their } 1.601+1.803+2.016\right)\right\}}$</td><td>B1;
<u>M1 A1ft</u></td></tr>
<tr><td>B1; for using $\frac{1}{2}\times 0.25$ or $\frac{1}{8}$ or equivalent.</td><td><u>M1: Structure of</u>
$\underline{\{\ldots\ldots\ldots\ldots\ldots\}}$</td><td><u>A1ft:</u> for the correct expression as shown following through candidate's y value found in part (a).</td><td></td></tr>
<tr><td colspan="3">M1 requires the correct structure for the y values. It needs to contain first y value plus last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2(…..) bracket this may be regarded as a slip and the M mark can be allowed (nb: an extra repeated term, however, forfeits the M mark). M0 if any values used are x values instead of y values.
A1ft: for the correct underlined expression as shown following through candidate's y value found in part (a).
Bracketing mistakes: e.g.
$\left(\frac{1}{2}\times\frac{1}{4}\right)(1.414+2.236)+2\left(\text{their } 1.601+1.803+2.016\right)(=11.29625)$
$\left(\frac{1}{2}\times\frac{1}{4}\right)1.414+2.236+2\left(\text{their } 1.601+1.803+2.016\right)(=13.25275)$
Both score B1 M1 A0 unless the final answer implies that the calculation has been done correctly (then full marks could be given).

Alternative:
Separate trapezia may be used, and this can be marked equivalently.
$\left[\frac{1}{8}(1.414+1.601)+\frac{1}{8}(1.601+1.803)+\frac{1}{8}(1.803+2.016)+\frac{1}{8}(2.016+2.236)\right]$
B1 for $\frac{1}{8}$ (aef), M1 for correct structure, 1st A1ft for correct expression, ft their 1.601</td><td></td></tr>
<tr><td>$\left\{=\tfrac{1}{8}(14.49)\right\}=1.81125$</td><td colspan="2">1.81 or awrt 1.81</td><td>A1</td></tr>
<tr><td colspan="3">Correct answer <u>only</u> in (b) scores no marks
If required accuracy is not seen in (a), full marks can still be scored in (b) (e.g. uses 1.6)</td><td></td></tr>
<tr><td></td><td colspan="2"></td><td>[4]</td></tr>
<tr><td></td><td colspan="2"></td><td>Total 5</td></tr>
</table>

Question Number	Scheme		Marks
19. (a)	$\sqrt{7}$ and $\sqrt{15}$	Both $\sqrt{7}$ and $\sqrt{15}$. Allow awrt 2.65 and 3.87	B1
			[1]
(b)	$\text{Area}\,(R) \approx \frac{1}{2}\times 2;\times\underline{\left\{\sqrt{3}+2\left(\sqrt{7}+\sqrt{11}+\sqrt{15}\right)+\sqrt{19}\right\}}$	Outside brackets $\frac{1}{2}\times 2$ or 1 (may be implied)	B1;
		<u>For structure of</u> $\underline{\{............\}}$	<u>M1</u>
	Note decimal values are $\frac{1}{2}\times 2;\times\underline{\left\{\sqrt{3}+\sqrt{19}+2\left(\sqrt{7}+\sqrt{11}+\sqrt{15}\right)\right\}}=\frac{1}{2}\times 2;\times\underline{\{6.0909..+19.6707...\}}$		
	M1 requires the correct structure for the *y* values. It needs to contain first *y* value **plus** last *y* value and the second bracket to be multiplied by 2 and to be the summation of the remaining *y* values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2(…..) bracket this may be regarded as a slip and the M mark can be allowed (nb: an extra repeated term, however, forfeits the M mark). M0 if any values used are *x* values instead of *y* values. Bracketing mistakes: e.g. $\left(\frac{1}{2}\times 2\right)\times\left(\sqrt{3}+\sqrt{19}\right)+2\left(\sqrt{7}+\sqrt{11}+\sqrt{15}\right)$ $\left(\frac{1}{2}\times 2\right)\times\sqrt{3}+\sqrt{19}+2\left(\sqrt{7}+\sqrt{11}+\sqrt{15}\right)$ Both score B1 M1 **Alternative:** Separate trapezia may be used, and this can be marked equivalently. $\left[\frac{1}{2}\times 2(\sqrt{3}+\sqrt{7})+\frac{1}{2}\times 2(\sqrt{7}+\sqrt{11})+\frac{1}{2}\times 2(\sqrt{11}+\sqrt{15})+\frac{1}{2}\times 2(\sqrt{15}+\sqrt{19})\right]$ B1 for $\frac{1}{2}\times 2$, M1 for correct structure		
	$=1\times 25.76166865...\ =\ 25.76166...\ =\ \underline{25.76}$ (2dp)	$\underline{25.76}$	A1 **cao**
			[3]
(c)	underestimate	Accept ‘under’, ‘less than’ etc.	B1
			[1]
			Total 5

Question Number	Scheme	Marks
20.		

x	0	0.5	1	1.5	2	2.5	3
y	5	4	2.5	1.538	1	0.690	0.5

Question Number	Scheme	Marks
(a)	$\{\text{At } x = 1.5,\}\ y = 1.538$ (only)	B1 **cao**
		[1]
(b)	$\frac{1}{2}\times 0.5$;	B1 oe
	$\underline{\{5 + 0.5 + 2(4 + 2.5 + \text{their } 1.538 + 1 + 0.690)\}}$ <u>For structure of</u> $\underline{\{\ldots\ldots\ldots\ldots\}}$;	M1<u>A1ft</u>
	$\frac{1}{2}\times 0.5 \times \underline{\{(5 + 0.5) + 2(4 + 2.5 + \text{their } 1.538 + 1 + 0.690)\}}$ $\{= \frac{1}{4}(24.956) = 6.239\}$ = awrt 6.24	A1
		[4]
(c)	Adds Area of Rectangle or first integral $= 3\times 4$ or $[4x]_0^3$ to **previous answer**	M1
	So required estimate = $\{\text{"6.239"} + 12 = \text{"18.239"}\}$ = "awrt 18.24" (or 12 + previous answer).	A1ft
	N.B. $7\times 4+$ previous answer is M0A0 (added 4 seven times because 7 numbers in table)	**[2]**
		7
	Notes for Question 20	
(a)	B1: 1.538	
(b)	B1: for using $\frac{1}{2}\times 0.5$ or $\frac{1}{4}$ or equivalent.	
	M1: requires the correct $\{\ldots\ldots\}$ bracket structure. It needs the first bracket to contain first *y* value **plus** last *y* value and the second bracket to be multiplied by 2 and to be the summation of the remaining *y* values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are *x* values instead of *y* values	
	A1ft: for the correct bracket $\{\ldots\ldots\}$ following through candidate's *y* value found in part (a).	
	A1: for answer which rounds to 6.24.	
	NB: Separate trapezia may be used : B1 for 0.25, M1 for 1/2 $h(a + b)$ used 5 or 6 times (and A1ft if it is all correct) Then A1 as before.	
	Special case: Bracketing mistake $0.25\times(5 + 0.5) + 2(4 + 2.5 + \text{their } 1.538 + 1 + 0.690)$ scores B1 M1 A0	
	A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). An answer of 20.831 usually indicates this error.	
(c)	M1: Relates **previous answer (not integral of previous answer**) to this question by integrating 4 between limits, and adding, or by using geometry to find rectangle and adding.	
	A1ft: for 12 + answer to (b)	
Alternative method (c)	Those who do a trapezium rule for part (b)- using the table from (a) with 4 added to each cell of the table Get: M1 for "*their* $\frac{1}{4}$"$\times\underline{\{9 + 4.5 + 2(8 + 6.5 + \text{their } 5.538 + 5 + 4.690)\}}$ = (structure must be correct – allow one copying error only) And A1ft: for awrt 18.24 (or 12 + previous answer).	

Question Number	**Scheme**	**Marks**
21.(a)	$\{x=1.3\}\quad y=0.8572$ (only)	B1 cao
		(1)
(b)	$\frac{1}{2}\times 0.1.........$	B1
	$\{0.7071+0.9487+2(0.7591+0.8090+"0.8572"+0.9037)\}$	M1
	$...\{0.7071+0.9487+2(0.7591+0.8090+"0.8572"+0.9037)\}$	A1ft
	$\{0.05(8.3138)\}=0.41569$ = awrt 0.416	A1
		(4) **Total 5**
	Notes for Question 21	
(a) **(b)**	B1: 0.8572 cao B1 for using $\frac{1}{2}\times 0.1$ or 0.05 or equivalent. M1 It needs the first bracket to contain first *y* value **plus** last *y* value and the second bracket to be multiplied by 2 and to be the summation of the remaining *y* values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are *x* values instead of *y* values A1ft for the correct bracket $\{......\}$ following through candidate's *y* value found in part (a). NB: Separate trapezia may be used : B1 for 0.05, M1 for 1/2 *h*(*a* + *b*) used 4 or 5 times (and A1ft if it is all correct) Then A1 as before. (Equivalent correct formulae may be used) Special case: Bracketing mistake $0.05\times(0.7071+0.9487)+2(0.7591+0.8090+"0.8572"+0.9037)$ scores B1 M1 A0 A0 (usually for 6.74079) unless the final answer implies that the calculation has been done correctly (then full marks can be given).	

22.	$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}$		
(a)	6.272 , 3.634		B1, B1
			(2)
(b)	$\frac{1}{2}\times\frac{1}{2}$ or $\frac{1}{4}$		B1
	$....\left\{(0+0)+2\left(5.866+"6.272"+5.210+"3.634"+1.856\right)\right\}$	Need {} or implied later for A1ft	M1A1ft
	$\frac{1}{2}\times 0.5\left\{(0+0)+2\left(5.866+"6.272"+5.210+"3.634"+1.856\right)\right\}$ $= \frac{1}{4}\times 45.676$		
	= 11.42	cao	A1
			(4)
			[6]

<table>
<tr><th>Question number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>23 (a)</td><td><table><tr><td>x</td><td>0</td><td>0.25</td><td>0.5</td><td>0.75</td><td>1</td></tr><tr><td>y</td><td>1</td><td>1.251</td><td>1.494</td><td>1.741</td><td>2</td></tr></table></td><td>B1, B1
(2)</td></tr>
<tr><td>(b)</td><td>$\frac{1}{2}\times 0.25, \quad \{(1+2)+2(1.251+1.494+1.741)\}$ o.e.
=1.4965</td><td>B1, M1,A1 ft
A1
(4)
6 marks</td></tr>
<tr><td>Notes</td><td>(a) first B1 for 1.494 and second B1 for 1.741 (1.740 is B0)
Wrong accuracy e.g. 1.49, 1.74 is B1B0

(b) B1: Need ½ of 0.25 or 0.125 o.e.
M1: requires first bracket to contain first plus last values and second bracket to include no additional values from the three in the table. If the only mistake is to omit one value from second bracket this may be regarded as a slip and M mark can be allowed (An extra repeated term forfeits the M mark however)
x values: M0 if values used in brackets are x values instead of y values

A1ft follows their answers to part (a) and is for {correct expression}
Final A1: Accept 1.4965, 1.497. or 1.50 only after correct work. (No follow through except one special case below following 1.740 in table)
Separate trapezia may be used : B1 for 0.125, M1 for $\frac{1}{2}h(a+b)$ used 3 or 4 times (and A1ft if it is all correct)
e.g.. 0.125(1+ 1.251) + 0.125(1.251+1.494) + 0.125(1.741 + 2) is M1 A0
equivalent to missing one term in { } in main scheme

Special Case: Bracketing mistake: i.e. 0.125(1+2) +2(1.251+1.494+1.741)
scores B1 M1 A0 A0 for 9.347 If the final answer implies that the calculation has been done correctly i.e. 1.4965 (then full marks can be given).
Need to see trapezium rule – answer only (with no working) is 0/4 any doubts send to review

Special Case; Uses 1.740 to give 1.49625 or 1.4963 or 1.496 or 1.50 gets, B1 B0
B1M1A1ft then A1 (lose 1 mark)

NB Bracket is 11.972</td><td></td></tr>
</table>

<table>
<tr><th>Question number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>24 (a)

(b)</td><td>
<table>
<tr><td>x</td><td>1</td><td>1.5</td><td>2</td><td>2.5</td><td>3</td><td>3.5</td><td>4</td></tr>
<tr><td>y</td><td>16.5</td><td>7.361</td><td>4</td><td>2.31</td><td>1.278</td><td>0.556</td><td>0</td></tr>
</table>

$\frac{1}{2}\times 0.5,\quad \{(16.5+0)+2(7.361+4+2.31+1.278+0.556)\}$
= 11.88 (or answers listed below in note)</td><td>B1, B1
(2)
B1, M1A1ft
A1 (4)
6</td></tr>
<tr><td>Notes</td><td colspan="2">(a) B1 for 4 or any correct equivalent e.g. 4.000 B1 for 2.31 or 2.310
(b) B1: Need 0.25 or ½ of 0.5
M1: requires first bracket to contain first y value plus last y value (0 may be omitted or be at end) and second bracket to include no additional y values from those in the scheme. They may however omit one value as a slip.
N.B. Special Case - Bracketing mistake
$\frac{1}{2}\times 0.5(16.5+0)+2(7.361+4+2.31+1.278+0.556)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks)
A1ft: This should be correct but ft their 4 and 2.31
A1: Accept 11.8775 or 11.878 or 11.88 only</td></tr>
<tr><td>Alternative Method for (b)</td><td colspan="2">Separate trapezia may be used : B1 for 0.25, M1 for $\frac{1}{2}h(a+b)$ used 5 or 6 times (and A1ft all correct for their “4” and “2.31”) final A1 for 11.88 etc. as before</td></tr>
<tr><td></td><td colspan="2">In part (b) Need to use trapezium rule – answer only (with no working) is 0/4 -any doubts send to review In part (c) need to see integration</td></tr>
</table>

Question Number	Scheme	Marks
25.	(a) 2.35, 3.13, 4.01 (One or two correct B1 B0, all correct B1 B1) <u>Important</u>: If part (a) is blank, or if answers have been crossed out and no replacement answers are visible, please send to Review as 'out of clip'.	B1 B1 (2)
	(b) $\frac{1}{2}\times 0.2$ ...... (or equivalent numerical value) $k\{(1+5)+2(1.65+p+q+r)\}$, k constant, $k\neq 0$ (See notes below) = 2.828 (awrt 2.83, allowed even after minor slips in values) The fractional answer $\frac{707}{250}$ (or other fraction wrt 2.83) is also acceptable. Answers with no working score no marks.	B1 M1 A1 A1 (4) **6**
	(a) Answers must be given to 2 decimal places. <u>No marks</u> for answers given to only 1 decimal place. (b) The *p*, *q* and *r* below are positive numbers, none of which is equal to any of: 1, 5, 1.65, 0.2, 0.4, 0.6 or 0.8 M1 A1: $k\{(1+5)+2(1.65+p+q+r)\}$ M1 A0: $k\{(1+5)+2(1.65+p+q)\}$ or $k\{(1+5)+2(p+q+r)\}$ M0 A0: $k\{(1+5)+2(1.65+p+q+r+\textit{other value}(s))\}$ Note that if the only mistake is to <u>omit</u> a value from the second bracket, this is considered as a slip and the M mark is allowed. <u>Bracketing mistake</u>: i.e. $\frac{1}{2}\times 0.2(1+5)+2(1.65+2.35+3.13+4.01)$ instead of $\frac{1}{2}\times 0.2\{(1+5)+2(1.65+2.35+3.13+4.01)\}$, so that only the (1 + 5) is multiplied by 0.1 scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). <u>Alternative</u>: Separate trapezia may be used, and this can be marked equivalently.	

Question Number	Scheme	Marks
26 (a)	$x = 2$ gives 2.236 (allow AWRT) Accept √5 $x = 2.5$ gives 2.580 (allow AWRT) Accept 2.58	B1 B1 (2)
(b)	$\left(\frac{1}{2}\times\frac{1}{2}\right)$, $\left[(1.414+3)+2(1.554+1.732+1.957+2.236+2.580)\right]$ = 6.133 (AWRT 6.13, even following minor slips)	B1,[M1A1ft] A1 (4)
(c)	Overestimate 'Since the trapezia lie <u>above the curve</u>', or an equivalent explanation, or sketch of (one or more) trapezia above the curve on a diagram (or on the given diagram, in which case there should be reference to this). (Note that there must be some reference to a trapezium or trapezia in the explanation or diagram).	B1 dB1 (2) **[8]**
(b)	B1 for $\frac{1}{2}\times\frac{1}{2}$ or equivalent. For the M mark, the first bracket must contain the 'first and last' values, and the second bracket (which must be multiplied by 2) must have no additional values. If the only mistake is to <u>omit</u> one of the values from the second bracket, this can be considered as a slip and the M mark can be allowed. Bracketing mistake: i.e. $\left(\frac{1}{2}\times\frac{1}{2}\right)(1.414+3)+2(1.554+1.732+1.957+2.236+2.580)$ scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). <u>Alternative</u>: Separate trapezia may be used, and this can be marked equivalently. $\left[\frac{1}{4}(1.414+1.554)+\frac{1}{4}(1.554+1.732)+..................+\frac{1}{4}(2.580+3)\right]$ 1st A1ft for correct expression, ft their 2.236 and their 2.580	
(c)	1st B1 for 'overestimate', ignoring earlier mistakes and ignoring any reasons given. 2nd B1 is dependent upon the 1st B1 (overestimate).	

Question Number	Scheme	Marks
27 (a)	3.84, 4.14, 4.58 (Any one correct B1 B0. All correct B1 B1)	B1 B1 (2)
(b)	$\frac{1}{2}\times 0.4,\ \left\{(3+4.58)+2\left(3.47+3.84+4.14+4.39\right)\right\}$ = 7.852 (awrt 7.9)	B1, M1 A1ft A1 (4) **[6]**
Notes (a)	**B1** for one answer correct Second **B1** for all three correct Accept awrt ones given or exact answers so $\sqrt{21}$, $\sqrt{\left(\frac{369}{25}\right)}$ or $\frac{3\sqrt{41}}{5}$, and $\sqrt{\left(\frac{429}{25}\right)}$ or $\frac{\sqrt{429}}{5}$, score the marks.	
(b)	**B1** is for using 0.2 or $\frac{0.4}{2}$ as $\frac{1}{2}h$. **M1** requires first bracket to contain first plus last values and second bracket to include no additional values from those in the table. If the only mistake is to omit one value from 2nd bracket this may be regarded as a slip ar can be allowed (An extra repeated term forfeits the **M** mark however) *x* values: **M0** if values used in brackets are *x* values instead of *y* values. Separate trapezia may be used : **B1** for 0.2, **M1** for $\frac{1}{2}h(a+b)$ used 4 or 5 times (and **A1**ft all e.g.. $0.2(3+3.47)+0.2(3.47+3.84)+0.2(3.84+4.14)+0.2(4.14+4.58)$ is **M1 A0** equivalent to missing one term in { } in main scheme **A1ft** follows their answers to part (a) and is for {correct expression} Final **A1** must be correct. (No follow through)	
Special cases	Bracketing mistake: i.e. $\frac{1}{2}\times 0.4(3+4.58)+2(3.47+3.84+4.14+4.39)$ scores **B1 M1 A0 A0** <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). **Need to see trapezium rule – answer only (with no working) is 0/4.**	

Question number	Scheme	Marks
28.	(a) 1.732, 2.058, 5.196 awrt (One or two correct B1 B0, All correct B1 B1) (b) $\frac{1}{2}\times 0.5$ ....$\{(1.732+5.196)+2(2.058+2.646+3.630)\}$ = 5.899 (awrt 5.9, allowed even after minor slips in values)	B1 B1 (2) B1 M1 A1ft A1 (4) **6**
	(a) Accept awrt (but <u>less</u> accuracy loses these marks). Also accept <u>exact</u> answers, e.g. $\sqrt{3}$ at $x = 0$, $\sqrt{27}$ or $3\sqrt{3}$ at $x = 2$. (b) For the M mark, the first bracket must contain the 'first and last' values, and the second bracket must have no additional values. If the only mistake is to <u>omit</u> one of the values from the second bracket, this can be considered as a slip and the M mark can be allowed. Bracketing mistake: i.e. $\frac{1}{2}\times 0.5(1.732+5.196)+2(2.058+2.646+3.630)$ scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). <u>x values</u>: M0 if the values used in the brackets are x values instead of y values. <u>Alternative</u>: Separate trapezia may be used, and this can be marked equivalently. $\left[\frac{1}{4}(1.732+2.058)+\frac{1}{4}(2.058+2.646)+\frac{1}{4}(2.646+3.630)+\frac{1}{4}(3.630+5.196)\right]$	

Question Number	Scheme	Marks
29.(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=-2\mathrm{e}^{-2x}+2x$	M1A1
	At $x=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-2\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{1}{2}$	M1
	Equation of normal is $y-(-2)=\frac{1}{2}(x-0)\Rightarrow y=\frac{1}{2}x-2$	M1 A1
		(5)
(b)	$y=\mathrm{e}^{-2x}+x^2-3$ meets $y=\frac{1}{2}x-2$ when $\mathrm{e}^{-2x}+x^2-3="\frac{1}{2}x-2"$	
	$x^2=1+\frac{1}{2}x-\mathrm{e}^{-2x}$	M1
	$x=\sqrt{1+\frac{1}{2}x-\mathrm{e}^{-2x}}$ *	A1*
		(2)
(c)	$x_2=\sqrt{1+0.5-\mathrm{e}^{-2}}$	M1
	$x_2=1.168,\ x_3=1.220$	A1
		(2)
		(9 marks)

(a)

M1: Attempts to differentiate with $\mathrm{e}^{-2x}\to A\mathrm{e}^{-2x}$ with any non -zero A, even 1.

Watch for $\mathrm{e}^{-2x}\to A\mathrm{e}^{2x}$ which is M0 A0

A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=-2\mathrm{e}^{-2x}+2x$

M1: A correct method of finding the gradient of the normal at $x=0$

To score this the candidate must find the negative reciprocal of $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0}$

So for example candidates who find $\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{-2x}+2x$ should be using a gradient of -1

Candidates who write down $\frac{\mathrm{d}y}{\mathrm{d}x}=-2$ (from their calculators?) have an opportunity to score this mark and the next.

M1: An attempt at the equation of the normal at $(0,-2)$

To score this mark the candidate must be using the point $(0,-2)$ and a gradient that has been **changed from** $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0}$

Look for $y-(-2)=\mathit{changed}\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0}(x-0)$ or $y=mx-2$ where $m=\mathit{changed}\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0}$

If there is an attempt using $y=mx+c$ then it must proceed using $(0,-2)$ with $m=$ $\mathit{changed}\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0}$

A1: $y=\frac{1}{2}x-2$ cso with as well as showing the correct differentiation.

So reaching $y=\frac{1}{2}x-2$ from $\frac{\mathrm{d}y}{\mathrm{d}x}=-2\mathrm{e}^{2x}+2x$ is A0

If it is not simplified (or written in the required form) you may award this if $y=\frac{1}{2}x-2$ is seen in part (b)

(b)

M1: Equates $y=\mathrm{e}^{-2x}+x^2-3$ and their $y=mx+c, m\neq 0$ and proceeds to $x^2=...$
Condone an attempt for this M mark where the candidate uses an adapted $y=mx+c$ in an attempt to get the printed answer.

A1*: Proceeds to $x=\sqrt{1+\frac{1}{2}x-\mathrm{e}^{-2x}}$. It is a printed answer but you may accept a different order $x=\sqrt{1-\mathrm{e}^{-2x}+\frac{1}{2}x}$

For this mark, the candidate must start with a normal equation of $y=\frac{1}{2}x-2$ oe found in (a). It can be awarded when the candidate finds the equation incorrectly, for example from $\frac{\mathrm{d}y}{\mathrm{d}x}=-2\mathrm{e}^{2x}+2x$

(c)

M1: Sub $x_1=1$ in $x=\sqrt{1+\frac{1}{2}x-\mathrm{e}^{-2x}}$ to find x_2. May be implied by $\sqrt{1+0.5-\mathrm{e}^{-2}}$ oe or awrt 1.17

A1: $x_2=$ awrt 1.168, $x_3=$ awrt 1.220 3dp. Condone 1.22 for x_3
Mark these in the order given, the subscripts are not required and incorrect ones may be ignored.

Question Number	Scheme	Marks
30. (a)	At P $x=-2 \Rightarrow y=3$	B1
	$\frac{dy}{dx}=\frac{4}{2x+5}-\frac{3}{2}$	M1, A1
	$\left.\frac{dy}{dx}\right\|_{x=-2}=\frac{5}{2} \Rightarrow$ Equation of normal is $y-'3'=-\frac{2}{5}(x-(-2))$	M1
	$\Rightarrow 2x+5y=11$	A1
		(5)
(b)	Combines $5y+2x=11$ and $y=2\ln(2x+5)-\frac{3x}{2}$ to form equation in x	
	$5\left(2\ln(2x+5)-\frac{3x}{2}\right)+2x=11$	M1
	$\Rightarrow x=\frac{20}{11}\ln(2x+5)-2$	dM1 A1*
		(3)
(c)	Substitutes $x_1=2 \Rightarrow x_2=\frac{20}{11}\ln 9-2$	M1
	Awrt $x_2=1.9950$ and $x_3=1.9929$.	A1
		(2)
		(10 marks)

(a)

B1 $y=3$ at point *P*. This may be seen embedded within their equation which may be a tangent

M1 Differentiates $\ln(2x+5)\to\frac{A}{2x+5}$ or equivalent. You may see $\ln(2x+5)^2\to\frac{A(2x+5)}{(2x+5)^2}$

A1 $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4}{2x+5}-\frac{3}{2}$ oe. It need not be simplified.

M1 For using a correct method of finding the equation of the normal using their numerical value of $-\left.\frac{\mathrm{d}x}{\mathrm{d}y}\right|_{x=-2}$ as the gradient. Allow for $(y-'3')=-\left.\frac{\mathrm{d}x}{\mathrm{d}y}\right|_{x=-2}(x--2)$, oe.

At least one bracket must be correct for their $(-2,3)$

If the form $y=mx+c$ is used it is scored for proceeding as far as *c* = ..

A1 $\pm k(5y+2x=11)$ It must be in the form $ax+by=c$ as stated in the question

Score this mark once it is seen. Do not withhold it if they proceed to another form, $y=mx+c$ for example

If a candidate uses a graphical calculator to find the gradient they can score a maximum of B1 M0 A0 M1 A1

(b)

M1 For combining 'their' **linear** $5y+2x=11$ with $y=2\ln(2x+5)-\frac{3x}{2}$ to form equation in just *x*, condoning slips on the rearrangement of their $5y+2x=11$. Eg $2\ln(2x+5)-\frac{3x}{2}=\frac{11\pm 2x}{5}$ is OK

dM1 Collects the two terms in *x* and proceeds to $ax=b\ln(2x+5)+c$ Allow numerical slips

A1* This is a given answer. All aspects must be correct including bracketing

(c)

M1 Score for substituting $x_1=2\Rightarrow x_2=\frac{20}{11}\ln(2\times 2+5)-2$ or exact equivalent

This may implied by $x_2=1.99...$

A1 Both values correct. Allow awrt $x_2=1.9950$ and $x_3=1.9929$ but condone $x_2=1.995$

Ignore subscripts. Mark on the first and second values given.

Question	Scheme	Marks
31.(a)	(i) 21	B1
	(ii) $4e^{2x}-25=0 \Rightarrow e^{2x}=\frac{25}{4} \Rightarrow 2x=\ln\left(\frac{25}{4}\right) \Rightarrow x=\frac{1}{2}\ln\left(\frac{25}{4}\right), \Rightarrow x=\ln\left(\frac{5}{2}\right)$	M1A1, A1
	(iii) 25	B1
		(5)
(b)	$4e^{2x}-25=2x+43 \Rightarrow e^{2x}=\frac{1}{2}x+17$	M1
	$\Rightarrow 2x=\ln\left(\frac{1}{2}x+17\right) \Rightarrow x=\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$	A1*
		(2)
(c)	$x_1=\frac{1}{2}\ln\left(\frac{1}{2}\times 1.4+17\right)=awrt\,1.44$	M1
	awrt $x_1=1.4368, x_2=1.4373$	A1
		(2)
(d)	Defines a suitable interval 1.4365 and 1.4375	M1
	...and substitutes into a suitable function Eg $4e^{2x}-2x-68$, obtains correct values with both a reason and conclusion	A1
		(2) **(11 marks)**

In part (a) accept points marked on the graph. If they appear on the graph and in the text, the text takes precedence. If they don't mark (a) as (i) (ii) and (iii) mark in the order given. If you feel unsure then please use the review system and your team leader will advise.

(a) (i)

B1 Sight of 21. Accept $(0,21)$

Do not accept just $|4-25|$ or $(21,0)$

(a) (ii)

M1 Sets $4e^{2x}-25=0$ and proceeds via $e^{2x}=\frac{25}{4}$ or $e^{x}=\frac{5}{2}$ to $x=..$

Alternatively sets $4e^{2x}-25=0$ and proceeds via $(2e^{x}-5)(2e^{x}+5)=0$ to $e^{x}=..$

A1 $\frac{1}{2}\ln\left(\frac{25}{4}\right)$ or awrt 0.92

A1 cao $\ln\left(\frac{5}{2}\right)$ or $\ln 5-\ln 2$. Accept $\left(\ln\left(\frac{5}{2}\right),0\right)$

(a) (iii)

B1 $k=25$ Accept also 25 or $y=25$

Do not accept just $|-25|$ or $x=25$ or $y=\pm 25$

(b)

M1 Sets $4e^{2x}-25=2x+43$ and makes e^{2x} the subject. Look for $e^{2x}=\frac{1}{4}(2x+43+25)$ condoning sign slips.Condone $\left|4e^{2x}-25\right|=2x+43$ and makes $\left|e^{2x}\right|$ the subject. Condone for both marks a solution with $x=a/\alpha$

An acceptable alternative is to proceed to $2e^{2x}=x+34\Rightarrow \ln 2+2x=\ln(x+34)$ using ln laws

A1* Proceeds correctly without errors to the correct solution. This is a given answer and the bracketing must be correct throughout. The solution must have come from $4e^{2x}-25=2x+43$ with the modulus having been taken correctly.

Allow $e^{2x}=\frac{1}{4}(2x+43+25)$ going to $x=\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$ without explanation

Allow $\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$ appearing as $\frac{1}{2}\log_e\left(\frac{1}{2}x+17\right)$ but not as $\frac{1}{2}\log\left(\frac{1}{2}x+17\right)$

……

If a candidate attempts the solution backwards they must proceed from

$x=\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)\Rightarrow e^{2x}=\frac{1}{2}x+17\Rightarrow 4e^{2x}-25=2x+43$ for the M1

For the A1 it must be tied up with a minimal statement that this is $g(x)=2x+43$

……

(c)

M1 Subs 1.4 into the iterative formula in an attempt to find x_1

Score for $x_1=\frac{1}{2}\ln\left(\frac{1}{2}\times 1.4+17\right)$ $x_1=\frac{1}{2}\ln(17.7)$ or awrt 1.44

A1 awrt $x_1=1.4368,\ x_2=1.4373$ Subscripts are not important, mark in the order given please.

(d)

M1 For a suitable interval. Accept 1.4365 and 1.4375 (or any two values of a smaller range spanning the root=1.4373) Continued iteration is M0

A1 Substitutes both values into **a suitable function**, which must be defined or implied by their working calculates both values correctly to 1 sig fig (rounded or truncated)

Suitable functions could be $\pm\left(4e^{2x}-2x-68\right),\ \pm\left(x-\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)\right),\ \pm\left(2x-\ln\left(\frac{1}{2}x+17\right)\right)$.

Using $4e^{2x}-2x-68$ f (1.4365) = -0.1, f (1.4375) = +0.02 or +0.03

Using $2e^{2x}-x-34$ f (1.4365) = -0.05/-0.06, f (1.4375) = +0.01

Using $x-\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$ f (1.4365) =-0.0007 or -0.0008, f (1.4375) =+ 0.0001 or +0.0002

Using $2x-\ln\left(\frac{1}{2}x+17\right)$ f (1.4365) = -0.001 or -0.002, f (1.4375) =+0.0003 or +0.0004

and states a reason (eg change of sign)

and a gives a minimal conclusion (eg root or tick)

It is valid to compare the two functions. Eg
$$\begin{aligned}&g(1.4365)=45.7(6)<2\times 1.4365+43=45.8(73)\\&g(1.4375)=45.90>2\times 1.4375+43=45.8(75)\end{aligned}$$

but the conclusion should be $g(x)=2x+43$ in between, hence root .

Similarly candidates can compare the functions x and $\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$

Question Number	Scheme	Marks
32.(a)	$2^{x+1}-3=17-x \Rightarrow 2^{x+1}=20-x$	M1
	$(x+1)\ln 2=\ln(20-x) \Rightarrow x=\ldots$	dM1
	$x=\dfrac{\ln(20-x)}{\ln 2}-1$	A1*
		(3)
(b)	Sub $x_0=3$ into $x_{n+1}=\dfrac{\ln(20-x_n)}{\ln 2}-1,\Rightarrow$ $x_1=3.087$ (awrt)	M1A1
	$x_2=3.080,\ x_3=3.081$ (awrt)	A1
		(3)
(c)	$A=(3.1,13.9)$ cao	M1,A1
		(2) **(8 marks)**
32.(a)Alt	$2^{x+1}-3=17-x \Rightarrow 2^{x}=\dfrac{20-x}{2}$	M1
	$x\ln 2=\ln\dfrac{20-x}{2} \Rightarrow x=\ldots$	dM1
	$x=\dfrac{\ln(20-x)}{\ln 2}-1$	A1*
		(3)
32.(a) **backwards**	$x=\dfrac{\ln(20-x)}{\ln 2}-1 \Rightarrow (x+1)\ln 2=\ln(20-x)$	M1
	$\Rightarrow 2^{x+1}=20-x$	dM1
	Hence $y=2^{x+1}-3$ meets $y=17-x$	A1*
		(3)

(a)

M1 Setting equations in x equal to each other and proceeding to make 2^{x+1} the subject

dM1 Take ln's or logs of both sides, use the power law and proceed to x =..

A1* This is a given answer and all aspects must be correct including ln or $\log_e$ rather than $\log_{10}$

Bracketing on both $(x+1)$ and $\ln(20-x)$ must be correct.

Eg $x+1\ln 2=\ln(20-x) Þ x=\frac{\ln(20-x)}{\ln 2}-1$ is A0*

Special case: Students who start from the point $2^{x+1}=20-x$ can score M1 dM1A0*

(b)

M1 Sub $x_0=3$ into $x_{n+1}=\frac{\ln(20-x_n)}{\ln 2}-1$ to find $x_1=..$

Accept as evidence $x_1=\frac{\ln(20-3)}{\ln 2}-1$, awrt $x_1=3.1$

Allow $x_0=3$ into the miscopied iterative equation $x_1=\frac{\ln(20-3)}{\ln 2}$ to find $x_1=..$

Note that the answer to this, 4.087, on its own without sight of $\frac{\ln(20-3)}{\ln 2}$ is M0

A1 awrt 3 dp $x_1=3.087$

A1 awrt $x_2=3.080, x_3=3.081$. Tolerate 3.08 for 3.080

Note that the subscripts are not important, just mark in the order seen

(c) Note that this appears as B1B1 on e pen. It is marked M1A1

M1 For sight of 3.1

Alternatively it can be scored for substituting their value of x or a rounded value of x from (b) into either $2^{x+1}-3$ or $17-x$ to find the y coordinate.

A1 $(3.1,13.9)$

Question Number	Scheme	Marks
33.(a)	$y_{2.1} = -0.224$, $y_{2.2} = (+)0.546$	M1
	Change of sign$\Rightarrow Q$ lies between	A1 (2)
(b)	At R $\frac{\mathrm{d}y}{\mathrm{d}x} = -2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$	M1A1
	$-2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3 = 0 \Rightarrow$ $x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$ cso	M1A1*
		(4)
(c)	$x_1 = \sqrt{1 + \frac{2}{3} \times 1.3\sin\left(\frac{1}{2} \times 1.3^2\right)}$	M1
	$x_1 = \text{awrt}\,1.284$ $x_2 = \text{awrt}\,1.276$	A1 (2)
		(8 marks)

(a)

M1 Sub both x = 2.1 and x =2.2 into y and achieve at least one correct to 1 sig fig

In radians $y_{2.1} = \text{awrt} -0.2 \quad y_{2.2} = \text{awrt/truncating to } 0.5$

In degrees $y_{2.1} = \text{awrt } 3 \quad y_{2.2} = \text{awrt } 4$

A1 Both values correct to 1 sf with a reason and a minimal conclusion.

$y_{2.1} = \text{awrt} -0.2 \quad y_{2.2} = \text{awrt/truncating to } 0.5$

Accept change of sign, positive and negative, $y_{2.1} \times y_{2.2} = -1$ as reasons and hence root, Q lies between 2.1 and 2.2, QED as a minimal conclussion.

Accept a smaller interval spanning the root of 2.131528, say 2.13 and 2.14, but the A1 can only be scored when the candidate refers back to the question, stating that as root lies between 2.13 and 2.14 it lies between 2.1 and 2.2

(b)

M1 Differentiating to get $\frac{\mathrm{d}y}{\mathrm{d}x} = ...\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$ where ... is a constant, or a linear function in x.

A1 $\frac{\mathrm{d}y}{\mathrm{d}x} = -2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$

M1 Sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and proceeds to make the x of their $3x^2$ the subject of the formula

Alternatively they could state $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and write a line such as

$2x\sin\left(\frac{1}{2}x^2\right) = 3x^2 - 3$, before making the x of $3x^2$ the subject of the formula

A1* Correct given solution. $x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$

Watch for missing x's in their formula

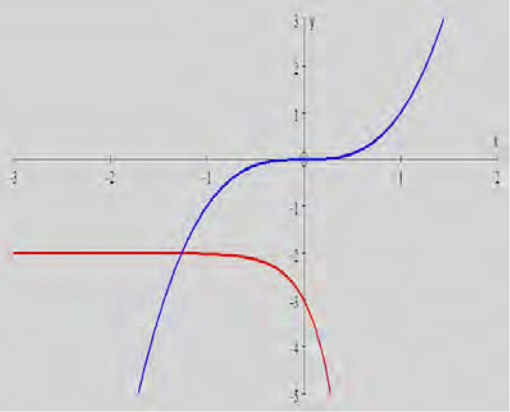
(c)

M1 Subs x= 1.3 into the iterative formula to find at least x_1.

This can be implied by $x_1 = \text{awrt}1.3$ (not just 1.3)

or $x_1 = \sqrt{1 + \frac{2}{3} \times 1.3\sin\left(\frac{1}{2} \times 1.3^2\right)}$ or $x_1 = \text{awrt}1.006$ (degrees)

A1 Both answers correct (awrt 3 decimal places). The subscripts are not important. Mark as the first and second values seen. $x_1 = \text{awrt } 1.284 \quad x_2 = \text{awrt } 1.276$

Question Number	Scheme	Marks
34. (a)	$\frac{dy}{dx} = 4e^{4x} + 4x^3 + 8$ Puts $\frac{dy}{dx} = 0$ to give $x^3 = -2 - e^{4x}$	M1, A1 A1 * (3)
(b)	 $y = x^3$ Shape of $y = -2 - e^{4x}$ $y = -2 - e^{4x}$ cuts y axis at (0,-3) $y = -2 - e^{4x}$ has asymptote at $y = -2$	B1 B1 B1 B1 (4)
(c)	Only one crossing point	B1 (1)
(d)	-1.26376, -1.26126 Accept answers which round to these answers to 5dp	M1 A1 (2)
(e)	α = -1.26 and so turning point is at (-1.26, -2.55)	M1 A1cao (2) **12 marks**

(a)

M1 Two (of the four) terms differentiated correctly

A1 All correct $\frac{\mathrm{d}y}{\mathrm{d}x} = 4\mathrm{e}^{4x} + 4x^3 + 8$

A1* States or sets $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, and proceeds correctly to achieve printed answer $x^3 = -2 - \mathrm{e}^{4x}$.

(b)

B1 Correct shape and position for $y = x^3$. It must appear to go through the origin. It must only appear in Quadrants 1 and 3 and have a gradient that is always $\geqslant 0$. The gradient should appear large at either end. Tolerate slips of the pen.See practice and qualification for acceptable curves.

B1 Correct shape for $y = -2 - \mathrm{e}^{4x}$, its position is not important for this mark. The gradient must be approximately zero at the left hand end and increase negatively as you move from left to right along the curve. See practice and qualification for acceptable curves.

B1 Score for $y = -2 - \mathrm{e}^{4x}$ cutting or meeting the y axis at (0,-3). Its shape is not important.
Accept for the intention of (0,-3), -3 being marked on the y – axis as well as (-3,0)
Do not accept 3 being marked on the negative y axis.

B1 Score for $y = -2 - \mathrm{e}^{4x}$ having an asymptote stated as $y = -2$. This is dependent upon the curve appearing to have an asymptote there. Do not accept the asymptote marked as ‘-2’ or indeed $x = -2$. See practice and qualification for acceptable solutions.

(c)

B1 Score for a statement to the effect that the graphs cross at one point. Accept minimal statements such as ‘one intersection’. Do not award if their diagram shows more than one intersection. They must have a diagram (which may be incorrect)

(d)

M1 Awarded for applying the iteration formula once. Possible ways in which this can be scored are the sight of $\sqrt[3]{-2-\mathrm{e}^{-4}}$, $\left(-2-\mathrm{e}^{4\times -1}\right)^{\frac{1}{3}}$ or awrt -1.264

A1 Both values correct awrt –1.26376, -1.26126 5dps. The subscripts are unimportant for this mark. Score as the first and second values seen.

(e)

M1 Score for EITHER rounding their value in part (c) to 2 dp OR finding turning point of C by substituting a value of x generated from part (d) into $y = \mathrm{e}^{4x} + x^4 + 8x + 5$ in order to find the y value. You may accept the appearance of a y value as evidence of finding the turning point (as long as an x value appears to be generated from part (d) and the correct equation is used.)

A1 (-1.26, -2.55) and correct solution only. It is a deduction and you cannot accept the appearance of a correct answer for two marks.

Question Number	Scheme	Marks
35(a)	$f'(x)=50x^2e^{2x}+50xe^{2x}$ oe.	M1A1
	Puts $f'(x)=0$ to give x = -1 and x = 0 or one coordinate	dM1A1
	Obtains $(0,\text{-}16)$ and $(\text{-}1,\ 25e^{-2}\text{-}16)$ CSO	A1
		(5)
(b)	Puts $25x^2e^{2x}-16=0\Rightarrow x^2=\frac{16}{25}e^{-2x}\Rightarrow x=\pm\frac{4}{5}e^{-x}$	B1*
		(1)
(c)	Subs $x_0=0.5$ into $x=\frac{4}{5}e^{-x}\Rightarrow x_1=\text{awrt }0.485$	M1A1
	$\Rightarrow x_2=\text{awrt }0.492,\ x_3=\text{awrt }0.489$	A1
		(3)
(d)	$\alpha=0.49$	B1
	$f(0.485)=-0.487,\ f(0.495)=(+)0.485$, sign change and deduction	B1
		(2)
		(11 marks)

Notes for Question 35

No marks can be scored in part (a) unless you see differentiation as required by the question.

(a)

M1 Uses $vu'+uv'$. If the rule is quoted it must be correct.
It can be implied by their $u=..,v=...,u'=...,v'=...$ followed by their $vu'+uv'$
If the rule is not quoted nor implied only accept answers of the form $Ax^2e^{2x}+Bxe^{2x}$

A1 $f'(x)=50x^2e^{2x}+50xe^{2x}$.
Allow un simplified forms such as $f'(x)=25x^2\times 2e^{2x}+50x\times e^{2x}$

dM1 Sets $f'(x)=0$, factorises out/ or cancels the e^{2x} leading to at least one solution of x
This is dependent upon the first M1 being scored.

A1 Both x = -1 and x = 0 or one complete coordinate . Accept $(0,\text{-}16)$ and $(\text{-}1,\ 25e^{-2}\text{-}16)$ or $(-1, awrt -12.6)$

A1 CSO. Obtains both solutions from differentiation. Coordinates can be given in any way.
x= - 1, 0 $y=\frac{25}{e^2}-16,\ -16$ or linked together by coordinate pairs $(0,\text{-}16)$ and $(\text{-}1,\ 25e^{-2}\text{-}16)$ but the 'pairs' must be correct and exact.

Notes for Question 35 Continued

(b)

B1 This is a show that question and all elements must be seen

Candidates must 1) State that f(x)=0 or writes $25x^2e^{2x} - 16 = 0$ or $25x^2e^{2x} = 16$

2) Show at least one intermediate (correct) line with either

x^2 or x the subject. Eg $x^2 = \frac{16}{25}e^{-2x}$, $x = \sqrt{\frac{16}{25}e^{-2x}}$ oe

or square rooting $25x^2e^{2x} = 16 \Rightarrow 5xe^x = \pm 4$

or factorising by DOTS to give $(5xe^x + 4)(5xe^x - 4) = 0$

3) Show the given answer $x = \pm\frac{4}{5}e^{-x}$.

Condone the minus sign just appearing on the final line.

A 'reverse' proof is acceptable as long as there is a statement that f(x)=0

(c)

M1 Substitutes $x_0 = 0.5$ into $x = \frac{4}{5}e^{-x} \Rightarrow x_1 = ...$

This can be implied by $x_1 = \frac{4}{5}e^{-0.5}$, or awrt 0.49

A1 $x_1 =$ awrt 0.485 3dp. Mark as the first value given. Don't be concerned by the subscript.

A1 $x_2 =$ awrt 0.492, $x_3 =$ awrt 0.489 3dp. Mark as the second and third values given.

(d)

B1 States $\alpha = 0.49$

B1 Justifies **by**

either calculating correctly $f(0.485)$ and $f(0.495)$ to awrt 1sf or 1dp,

$f(0.485) = -0.5, f(0.495) = (+)0.5$ rounded

$f(0.485) = -0.4, f(0.495) = (+)0.4$ truncated

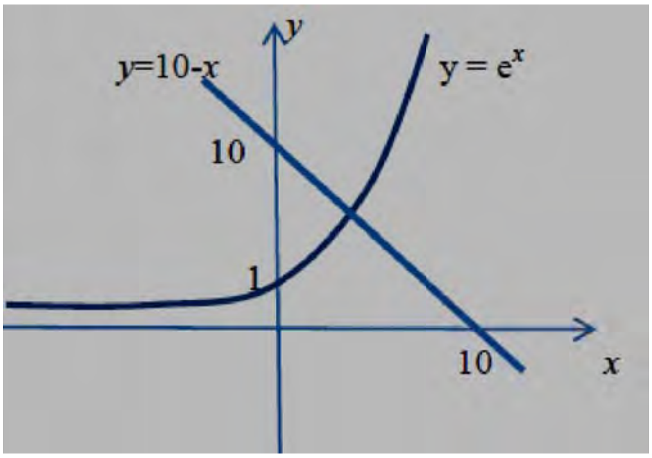
giving a reason – accept change of sign, >0 <0 or $f(0.485) \times f(0.495) < 0$

and giving a minimal conclusion. Eg. Accept hence root or $\alpha = 0.49$

A smaller interval containing the root may be used, eg f (0.49) and f (0.495). Root = 0.49007

or by stating that the iteration is oscillating

or by calculating by continued iteration to at least the value of x_4= awrt 0.491 and stating (or seeing each value round to) 0.49

Question Number	Scheme	Marks
36	(a) 	
	Shape for $y=10-x$	B1
	Shape for $y=\mathrm{e}^x$	B1
	co- ordinates correct (0,10),(10,0) and (0,1)	B1
		(3)
	(b) One solution as there is one point of intersection	B1√
		(1)
	(c) Sub x=2 and x=3 into $\mathrm{f}(x)=\mathrm{e}^x-10+x$	
	f(2)=-0.61, f(3)=(+)13.1	M1
	Both correct to 1sf, reason (change of sign) and conclusion (hence root)	A1
		(2)
	(d) Substitutes $x_1=2$ into $x_{n+1}=\ln(10-x_n)$	M1
	$x_2=2.0794,\quad x_3=2.0695\quad x_4=2.0707$	A1,A1
		(3)
		(9 marks)

Question Number	Scheme	Marks
37.(a)	$f(x)=0 \Rightarrow x^2+3x+1=0$	
	$\Rightarrow x=\frac{-3\pm\sqrt{5}}{2}=$ awrt -0.382, -2.618	M1A1
		(2)
(b)	Uses $vu'+uv'$ $\quad$ $\mathrm{f}'(x)=e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x$	M1A1A1
		(3)
(c)	$e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x=0$	
	$\Rightarrow e^{x^2}\left\{2x^3+6x^2+4x+3\right\}=0$	M1
	$\Rightarrow x(2x^2+4)=-3(2x^2+1)$	M1
	$\Rightarrow x=-\frac{3(2x^2+1)}{2(x^2+2)}$	A1*
		(3)
(d)	Sub $x_0=-2.4$ into $\quad x_{n+1}=-\frac{3(2x_n{}^2+1)}{2(x_n{}^2+2)}$	
	$x_1=awrt-2.420,\ x_2=awrt-2.427\ \ x_3=awrt-2.430$	M1A1,A1
		(3)
(e)	Sub x= - 2.425 and -2.435 into $\mathrm{f}'(x)$ and start to compare signs	
	$\mathrm{f}'(-2.425)=+22.4,\ \mathrm{f}'(-2.435)=-15.02$	**M1**
	Change in sign, hence $\mathrm{f}'(x)=0$ in between. Therefore α = - 2.43 (2dp)	A1
		(2)
		(13 marks)
Alt 7.(c)	$x=-\frac{3(2x^2+1)}{2(x^2+2)}$ $\Rightarrow 2x(x^2+2)=-3(2x^2+1)$ $\Rightarrow 2x^3+6x^2+4x+3=0$	M1
	$\mathrm{f}'(x)=e^{x^2}\left\{2x^3+6x^2+4x+3\right\}=0$ when $2x^3+6x^2+4x+3=0$	M1
	Hence the minimum point occurs when $x=-\frac{3(2x^2+1)}{(2x^2+4)}$	**A1**

Question Number	**Scheme**	**Marks**
Alt 1 37(e)	Sub *x*= - 2.425 and -2.435 into cubic part of $\text{f}'(x) = 2x^3+6x^2+4x+3$ and start to compare signs Adapted $\text{f}'(-2.425)=+0.06, \text{f}'(-2.435)=-0.04$ Change in sign, hence $\text{f}'(x)=0$ in between. Therefore α = - 2.43 (2dp)	 **M1** A1 **(2)**
Alt 2 37 (e)	Sub *x*= - 2.425, -2.43 and -2.435 into $\text{f}(x)=(x^2+3x+1)e^{x^2}$ and start to compare sizes $\text{f}(-2.425)=-141.2, \text{f}(-2.435)=-141.2, \text{f}(-2.43)=-141.3$ $\text{f}(-2.43)<\text{f}(-2.425), \text{f}(-2.43)<\text{f}(-2.435)$. Therefore α = - 2.43 (2dp)	 **M1** A1 **(2)**

Notes for Question 37

(a)

M1 Solves $x^2+3x+1=0$ by completing the square or the formula, producing two 'non integer answers. **Do not accept factorisation here** . Accept awrt -0.4 and -2.6 for this mark

A1 Answers correct. Accept awrt -0.382, -2.618.

Accept just the answers for both marks. Don't withhold the marks for incorrect labelling.

(b)

M1 Applies the product rule $vu'+uv'$ to $(x^2+3x+1)e^{x^2}$.

If the rule is quoted it must be correct and there must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, ie. terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') only accept answers of the form

$$\left(\frac{\text{d}y}{\text{d}x}\right)=\text{f}'(x)=e^{x^2}(Ax+B)+(x^2+3x+1)Cxe^{x^2}$$

A1 One term of $\text{f}'(x)=e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x$ correct.

There is no need to simplify

A1 A fully correct (un simplified) answer $\text{f}'(x)=e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x$

(c)

M1 Sets their $\text{f}'(x)=0$ and either factorises out, or cancels by e^{x^2} to produce a polynomial equation in x

M1 Rearranges the cubic polynomial to $Ax^3+Bx=Cx^2+D$ and factorises to reach $x(Ax^2+B)=Cx^2+D$ or equivalent

A1* Correctly proceeds to $x=-\dfrac{3(2x^2+1)}{2(x^2+2)}$. This is a given answer

Notes on Question 37 Continued

(c) Alternative to (c) working backwards

M1 Moves correctly from $x=-\frac{3(2x^2+1)}{2(x^2+2)}$ to $2x^3+6x^2+4x+3=0$

M1 States or implies that $\mathrm{f}'(x)=0$

A1 Makes a conclusion to tie up the argument

For example, hence the minimum point occurs when $x=-\frac{3(2x^2+1)}{(2x^2+4)}$

(d)

M1 Sub $x_0=-2.4$ into $x_{n+1}=-\frac{3(2x_n{}^2+1)}{2(x_n{}^2+2)}$

This may be implied by awrt -2.42, or $x_{n+1}=-\frac{3(2\times-2.4^2+1)}{2(-2.4^2+2)}$

A1 Awrt. $x_1=-2.420$.

The subscript is not important. Mark as the first value given

A1 awrt $x_2=-2.427$ awrt $x_3=-2.430$

The subscripts are not important. Mark as the second and third values given

(e)

Note that continued iteration is not allowed

M1 Sub x= - 2.425 and -2.435 into $\mathrm{f}'(x)$, starts to compare signs and gets at least one correct to 1 sf rounded or truncated.

A1 Both values correct (1sf rounded or truncated), a reason and a minimal conclusion

Acceptable reasons are change in sign, positive and negative and $\mathrm{f}'(a)\times\mathrm{f}'(b)<0$

Minimal conclusions are hence $\alpha=-2.43$, hence shown, hence root

Alt 1 using adapted $\mathrm{f}'(x)$

(e)

M1 Sub x= - 2.425 and -2.435 into cubic part of $\mathrm{f}'(x)$, starts to compare signs and gets at least one correct to 1 sf rounded or truncated.

A1 Both values correct of adapted $\mathrm{f}'(x)$ correct (1sf rounded or truncated), a reason and a minimal conclusion

Acceptable reasons are change in sign, positive and negative and $\mathrm{f}'(a)\times\mathrm{f}'(b)<0$

Minimal conclusions are hence $\alpha=-2.43$, hence shown, hence root

Alt 2 using $\mathrm{f}(x)$

(e)

M1 Sub x= - 2.425, -2.43 and -2.435 into $\mathrm{f}(x)$, starts to compare sizes and gets at least one correct to 4sf rounded

A1 All three values correct of $\mathrm{f}(x)$ correct (4sf rounded), a reason and a minimal conclusion

Acceptable reasons are $\mathrm{f}(-2.43)<\mathrm{f}(-2.425)$, $\mathrm{f}(-2.43)<\mathrm{f}(-2.435)$, a sketch

Minimal conclusions are hence $\alpha=-2.43$, hence shown, hence root

Question Number	Scheme	Marks
38.	(a) $0=e^{x-1}+x-6\Rightarrow x=\ln(6-x)+1$	M1A1*
		(2)
	(b) Sub $x_0=2$ into $x_{n+1}=\ln(6-x_n)+1\Rightarrow x_1=2.3863$	M1, A1
	AWRT 4 dp. $x_2=2.2847$ $x_3=2.3125$	A1
		(3)
	(c) Chooses interval [2.3065,2.3075]	M1
	g(2.3065)=-0.0002(7), g(2.3075)=0.004(4)	dM1
	Sign change , hence root (correct to 3dp)	A1
		(3)
		(8 marks)

(a) M1 Sets g(x)=0,and using correct *ln* work, makes the *x* of the e^{x-1} term the subject of the formula.
Look for $e^{x-1}+x-6=0\Rightarrow e^{x-1}=\pm6\pm x\Rightarrow x=\ln(\pm6\pm x)\pm1$
Do not accept $e^{x-1}=6-x$ **without firstly seeing** $e^{x-1}+x-6=0$ **or a statement that g(x)=0 ⇒**

A1* cso. $x=\ln(6-x)+1$ **Note that this is a given answer (and a proof).**
'Invisible' brackets are allowed for the M but not the A
Do not accept recovery from earlier errors for the A mark. The solution below scores 0 marks.
$0=e^{x-1}+x-6\Rightarrow\quad 0=x-1+\ln(x-6)\Rightarrow x=\ln(6-x)+1$

(b) M1 Sub $x_0=2$ into $x_{n+1}=\ln(6-x_n)+1$ to produce a numerical value for x_1.
Evidence for the award could be any of $\ln(6-2)+1$, $\ln4+1$, 2.3….. or awrt 2.4

A1 Answer correct to 4 dp $x_1=2.3863$.
The subscript is not important. Mark as the first value given/found.

A1 Awrt 4 dp. $x_2=2.2847$ *and* $x_3=2.3125$
The subscripts are not important. Mark as the second and third values given/found

(c) M1 Chooses the interval [2.3065,2.3075] or smaller containing the root 2.306558641

dM1 Calculates g(2.3065) and g(2.3075) with at least one of these correct to 1sf.
The answers can be rounded or truncated
g(2.3065) = -0.0003 rounded, g(2.3065)= -0.0002 truncated
g(2.3075) = (+) 0.004 rounded and truncated

A1 Both values correct (rounded or truncated),
A reason which could include change of sign, >0 <0, g(2.3065)× g(2.3075) <0
AND a minimal conclusion such as hence root, α=2.307 or □
Do not accept continued iteration as question demands an interval to be chosen.

Alternative solution to (a) working backwards

M1 Proceeds from $x=\ln(6-x)+1$ using correct exp work to …….=0

A1 **Arrives correctly** at $e^{x-1}+x-6=0$ **and** makes a statement to the effect that this is g(x)=0

Alternative solution to (c) using $\text{f}(x)=\ln(6-x)+1-x$ **{Similarly** $\text{h}(x)=x-1-\ln(6-x)$ **}**

M1 Chooses the interval [2.3065,2.3075] or smaller containing the root 2.306558641

dM1 Calculates f(2.3065) and f(2.3075) with at least 1 correct rounded or truncated
f(2.3065) = 0.000074. Accept 0.00007 rounded or truncated. Also accept 0.0001

$f(2.3075) = -0.0011$.. Accept -0.001 rounded or truncated

Question Number	Scheme	Marks
39.	(a) $x^3+3x^2+4x-12=0 \Rightarrow x^3+3x^2=12-4x$	
	$\Rightarrow x^2(x+3)=12-4x$	M1
	$\Rightarrow x^2=\frac{12-4x}{(x+3)} \Rightarrow x=\sqrt{\frac{4(3-x)}{(x+3)}}$	dM1A1*
		(3)
	(b) $x_1=1.41$, *awrt* $x_2=1.20$ $x_3=1.31$	M1A1,A1
		(3)
	(c) Choosing (1.2715,1.2725) or tighter containing root 1.271998323	M1
	$\text{f}(1.2725)=(+)0.00827...$ $\text{f}(1.2715)=-0.00821....$	M1
	Change of sign⇒α=1.272	A1
		(3)
		(9 marks)

Notes

(a) M1 Moves from f(x)=0, which may be implied by subsequent working, to $x^2(x\pm3)=\pm12\pm4x$ by separating terms and factorising in either order. No need to factorise rhs for this mark.

dM1 Divides by '(x+3)' term to make x^2 the subject, then takes square root. No need for rhs to be factorised at this stage

A1* CSO. This is a given solution. Do not allow sloppy algebra or notation with root on just numerator for instance. The 12-4x needs to have been factorised.

(b) **Note that this appears B1,B1,B1 on EPEN**

M1 An attempt to substitute $x_0=1$ into the iterative formula to calculate x_1.

This can be awarded for the sight of $\sqrt{\frac{4(3-1)}{(3+1)}}$, $\sqrt{\frac{8}{4}}$, $\sqrt{2}$ and even 1.4

A1 $x_1=1.41$. The subscript is not important. Mark as the first value found, $\sqrt{2}$ is A0

A1 $x_2=\text{awrt}\,1.20$ $x_3=\text{awrt}\,1.31$. Mark as the second and third values found. Condone 1.2 for x_2

(c) **Note that this appears M1A1A1 on EPEN**

M1 Choosing the interval (1.2715,1.2725) or tighter containing the root 1.271998323. Continued iteration is not allowed for this question and is M0

M1 Calculates f(1.2715) and f(1.2725), or **the** tighter interval with at least 1 correct to 1 sig fig rounded or truncated.
Accept f(1.2715) = -0.008 1sf rounded or truncated. Also accept f(1.2715) = -0.01 2dp
Accept f(1.2725) = (+)0.008 1sf rounded or truncated. Also accept f(1.2725) = (+)0.01 2dp

A1 Both values correct (see above),
A valid reason; Accept change of sign, or >0 <0, or f(1.2715) ×f(1.2725)<0
And a (minimal) conclusion; Accept hence root or α=1.272 or QED or □

Alternative to (a) working backwards

39(a)

	$x=\sqrt{\frac{4(3-x)}{(x+3)}} \Rightarrow x^2=\frac{4(3-x)}{(x+3)} \Rightarrow x^2(x+3)=4(3-x)$	M1
	$x^3+3x^2=12-4x \Rightarrow x^3+3x^2+4x-12=0$	dM1
	States that this is f(x)=0	A1* (3)

Alternative starting with the given result and working backwards

M1 Square (both sides) and multiply by (x+3)

dM1 Expand brackets and collect terms on one side of the equation =0

A1 A statement to the effect that this is f(x)=0

An acceptable answer to (c) with an example of a tighter interval

M1 Choosing the interval (1.2715, 1.2720). This contains the root 1.2719(98323)

M1 Calculates f(1.2715) and f(1.2720), with at least 1 correct to 1 sig fig rounded or truncated.
Accept f(1.2715) = -0.008 1sf rounded or truncated f(1.2715) = -0.01 2dp
Accept f(1.2720) = (+)0.00003 1sf rounded or f(1.2720) = (+)0.00002 truncated 1sf

A1 Both values correct (see above),
A valid reason; Accept change of sign, or >0 <0, or f(1.2715) ×f(1.2720)<0
And a (minimal) conclusion; Accept hence root or α=1.272 or QED or ☐

x	**f(x)**
1.2715	-0.00821362
1.2716	-0.00656564
1.2717	-0.00491752
1.2718	-0.00326927
1.2719	-0.00162088
1.2720	+0.00002765
1.2721	+0.00167631
1.2722	+0.00332511
1.2723	+0.00497405
1.2724	+0.00662312
1.2725	+0.00827233

An acceptable answer to (c) using g(x) where g(x)= $\sqrt{\frac{4(3-x)}{(x+3)}}-x$

2nd M1 Calculates g(1.2715) and g(1.2725), or **the** tighter interval with at least 1 correct to 1 sig fig rounded or truncated.
g(1.2715) = 0.0007559. Accept g(1.2715) = awrt (+)0.0008 1sf rounded or awrt 0.0007 truncated.
g(1.2725)=-0.00076105. Accept g(1.2725) = awrt -0.0008 1sf rounded or awrt -0.0007 truncated.

Question No		Marks
40	(a) f(0.8) = 0.082, f(0.9)= -0.089	M1
	Change of sign ⇒ root (0.8,0.9)	A1
		(2)
	(b)	
	$f'(x) = 2x - 3 - \sin\left(\frac{1}{2}x\right)$	M1 A1
	Sets $f'(x) = 0 \Rightarrow \quad x = \frac{3+\sin(\frac{1}{2}x)}{2}$	M1A1*
		(4)
	(c) Sub x_0=2 into $x_{n+1} = \frac{3+\sin(\frac{1}{2}x_n)}{2}$	M1
	x_1=awrt 1.921, x_2=awrt 1.91(0) and x_3=awrt 1.908	A1,A1
		(3)
	(d) [1.90775,1.90785]	M1
	f’(1.90775)=-0.00016.. AND f’(1.90785)= 0.0000076..	M1
	Change of sign ⇒ x=1.9078	A1
		(3)
		(12 marks)

(a)

M1 Calculates both f(0.8) and f(0.9). Evidence of this mark could be, either, seeing both ‘x’ substitutions written out in the expression, or, one value correct to 1 sig fig, or the appearance of incorrect values of f(0.8)=awrt 0.2 or f(0.9)=awrt 0.1 from use of degrees

A1 This requires both values to be correct as well as a reason and a conclusion.
Accept f(0.8)= 0.08 truncated or rounded (2dp) or 0.1 rounded (1dp) and f(0.9)=-0.08 truncated or rounded as -0.09 (2dp) or -0.1(1dp)
Acceptable reasons are change of sign, <0 >0, +ve –ve, f(0.8)f(0.9)<0. Acceptable conclusion is hence root or

(b)

M1 Attempts to differentiate f(x). Seeing any of 2x,3 or ±Asin(½x) is sufficient evidence.

A1 f’(x) correct. Accept $\frac{dy}{dx} = 2x - 3 - \sin\left(\frac{1}{2}x\right)$

M1 Sets their f’(x)=0 and proceeds to x=….. You must be sure that they are setting what they think is f’(x)=0.
Accept $2x = 3 + \sin\left(\frac{1}{2}x\right)$ going to x=..only if f’(x) =0 is stated first

A1 * $x = \frac{3+\sin(\frac{1}{2}x)}{2}$. This is a given answer so don’t accept just the sight of this answer. It is cso

(c) M1 Substitutes x_0=2 into $x_{n+1} = \frac{3+\sin(\frac{1}{2}x_n)}{2}$. Evidence of this mark could be awrt 1.9 or 1.5 (from degrees)

A1 x_1=awrt 1.921

A1 x_2=awrt 1.91(0) and x_3=awrt 1.908

(d) **Continued iteration is not acceptable for this part. Question states ‘By choosing a suitable interval…’**

M1 Chooses the interval [1.90775,1.90785] or tighter containing the root= 1.907845522

M1 Calculates f’(1.90775) and f’(1.90785) or tighter with at least one correct, rounded or truncated
f’(1.90775)=-0.0001 truncated or awrt -0.0002 rounded
f’(1.90785)= 0.000007 truncated or awrt 0.000008 rounded

Accept versions of g(x)-x where g(x) $=\frac{3+\sin(\frac{1}{2}x)}{2}$.
When x= 1.90775, $g(x) - x = 8 \times 10^{-5}$ rounded and truncated
When x= 1.90785, $g(x) - x = -3 \times 10^{-6}$ truncated or $= -4 \times 10^{-6}$ rounded

A1 Both values correct, rounded or truncated, a valid reason (see part a) and a minimal conclusion (see part a). Saying hence root is acceptable. There is no need to refer to the ‘turning point’.

Question Number	Scheme	Marks
41 (a)	f(0.75)= -0.18… f(0.85) = 0.17…..	M1
	Change of sign, hence root between x=0.75 and x=0.85	A1
		(2)
(b)	Sub x_0=0.8 into ¯ to obtain x_1	M1
	Awrt x_1=0.80219 and x_2=0.80133	A1
	Awrt x_3 = 0.80167	A1
		(3)
(c)	f(0.801565) = -2.7…..$\times10^{-5}$ f(0.801575) = +8.6….$\times10^{-6}$	M1A1
	Change of sign and conclusion	A1
		(3)
	See Notes for continued iteration method	
		8 Marks

Question Number	Scheme		Marks
42.			
(a)	Crosses x-axis $\Rightarrow \mathrm{f}(x) = 0 \Rightarrow (8 - x)\ln x = 0$		
	Either $(8 - x) = 0$ or $\ln x = 0 \Rightarrow x = 8, 1$	Either one of {x}=1 OR x={8}	B1
	Coordinates are $A(1, 0)$ and $B(8, 0)$.	Both $A\left(1, \{0\}\right)$ and $B\left(8, \{0\}\right)$	B1
			(2)
(b)	Apply product rule: $\begin{Bmatrix} u = (8 - x) & v = \ln x \\ \dfrac{\mathrm{d}u}{\mathrm{d}x} = -1 & \dfrac{\mathrm{d}v}{\mathrm{d}x} = \dfrac{1}{x} \end{Bmatrix}$	$vu' + uv'$	M1
	$\mathrm{f}'(x) = -\ln x + \dfrac{8-x}{x}$	Any one term correct	A1
		Both terms correct	A1
			(3)
(c)	$\mathrm{f}'(3.5) = 0.032951317...$ $\mathrm{f}'(3.6) = -0.058711623...$ Sign change (and as $\mathrm{f}'(x)$ is continuous) therefore the x-coordinate of Q lies between 3.5 and 3.6.	Attempts to evaluate **both** $\mathrm{f}'(3.5)$ and $\mathrm{f}'(3.6)$	M1
		both values correct to at least 1 sf, sign change and conclusion	A1
			(2)
(d)	At Q, $\mathrm{f}'(x) = 0 \Rightarrow -\ln x + \dfrac{8-x}{x} = 0$	Setting $\mathrm{f}'(x) = 0$.	M1
	$\Rightarrow -\ln x + \dfrac{8}{x} - 1 = 0$	Splitting up the numerator and proceeding to x=	M1
	$\Rightarrow \dfrac{8}{x} = \ln x + 1 \Rightarrow 8 = x(\ln x + 1)$		
	$\Rightarrow x = \dfrac{8}{\ln x + 1}$ (as required)	For correct proof. No errors seen in working.	A1
			(3)

Question Number	Scheme		Marks
(e)	Iterative formula: $x_{n+1} = \dfrac{8}{\ln x_n + 1}$		
	$x_1 = \dfrac{8}{\ln(3.55) + 1}$	An attempt to substitute $x_0 = 3.55$ into the iterative formula. Can be implied by $x_1 = 3.528(97)...$	M1
	$x_1 = 3.528974374...$ $x_2 = 3.538246011...$ $x_3 = 3.534144722...$	Both $x_1 = \text{awrt } 3.529$ and $x_2 = \text{awrt } 3.538$	A1
	$x_1 = 3.529,\ x_2 = 3.538,\ x_3 = 3.534,$ to 3 dp.	x_1 , x_2 , x_3 all stated correctly to 3 dp	A1 (3) **[13]**

Question Number	Scheme	Marks
43.(a)	$\text{f}(1.2) = 0.49166551...,\quad \text{f}(1.3) = -0.048719817...$	
	Sign change (and as $\text{f}(x)$ is continuous) therefore a root α is such that $\alpha \in [1.2, 1.3]$	M1A1
		(2)
(b)	$4\text{cosec}x - 4x + 1 = 0 \Rightarrow 4x = 4\text{cosec}x + 1$	M1
	$\Rightarrow x = \text{cosec}x + \frac{1}{4} \Rightarrow \underline{x = \frac{1}{\sin x} + \frac{1}{4}}$	A1 *
		(2)
(c)	$x_1 = \frac{1}{\sin(1.25)} + \frac{1}{4}$	M1
	$x_1 = 1.303757858...,\quad x_2 = 1.286745793...$	A1
	$x_3 = 1.291744613...$	A1
		(3)
(d)	$\text{f}(1.2905) = 0.00044566695...,\quad \text{f}(1.2915) = -0.00475017278...$	M1
	Sign change (and as $\text{f}(x)$ is continuous) therefore a root α is such that	
	$\alpha \in (1.2905, 1.2915) \Rightarrow \alpha = 1.291$ (3 dp)	A1
		(2) **[9]**
	(a) M1: Attempts to evaluate both $\text{f}(1.2)$ and $\text{f}(1.3)$ and evaluates at least one of them correctly to awrt (or truncated) 1 sf. A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion. (b) M1: Attempt to make $4x$ or x the subject of the equation. A1: Candidate must then rearrange the equation to give the required result. It must be clear that candidate has made their initial $\text{f}(x) = 0$. (c) M1: An attempt to substitute $x_0 = 1.25$ into the iterative formula Eg $= \frac{1}{\sin(1.25)} + \frac{1}{4}$. Can be implied by $x_1 =$ awrt 1.3 or $x_1 =$ awrt 46°. A1: Both $x_1 =$ awrt 1.3038 and $x_2 =$ awrt 1.2867 A1: $x_3 =$ awrt 1.2917 (d) M1: Choose suitable interval for x, e.g. [1.2905, 1.2915] or tighter and at least one attempt to evaluate f(x). A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion.	

Question Number	Scheme		Marks
44	$\mathrm{f}(x) = x^3 + 2x^2 - 3x - 11$		
(a)	$\mathrm{f}(x) = 0 \Rightarrow x^3 + 2x^2 - 3x - 11 = 0$ $\Rightarrow x^2(x+2) - 3x - 11 = 0$	Sets $\mathrm{f}(x) = 0$ (can be implied) and takes out a factor of x^2 from $x^3 + 2x^2$, or x from $x^3 + 2x$ (slip).	M1
	$\Rightarrow x^2(x+2) = 3x + 11$ $\Rightarrow x^2 = \frac{3x+11}{x+2}$		
	$\Rightarrow x = \sqrt{\left(\frac{3x+11}{x+2}\right)}$	then rearranges to give the quoted result on the question paper.	A1 **AG**
			(2)
(b)	Iterative formula: $x_{n+1} = \sqrt{\left(\frac{3x_n + 11}{x_n + 2}\right)}$, $x_1 = 0$		
	$x_2 = \sqrt{\left(\frac{3(0)+11}{(0)+2}\right)}$	An attempt to substitute $x_1 = 0$ into the iterative formula. Can be implied by $x_2 = \sqrt{5.5}$ ***or 2.35*** or awrt 2.345	M1
	$x_2 = 2.34520788...$ $x_3 = 2.037324945...$	Both $x_2 =$ awrt 2.345 and $x_3 =$ awrt 2.037	A1
	$x_4 = 2.058748112...$	$x_4 =$ awrt 2.059	A1
			(3)
(c)	Let $\mathrm{f}(x) = x^3 + 2x^2 - 3x - 11 = 0$		
	$\mathrm{f}(2.0565) = -0.013781637...$	Choose suitable interval for x, e.g. [2.0565, 2.0575] or tighter	M1
	$\mathrm{f}(2.0575) = 0.0041401094...$	any one value awrt 1 sf	dM1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha \in (2.0565, 2.0575) \Rightarrow \alpha = 2.057$ (3 dp)	both values correct awrt 1sf, sign change and conclusion	A1
			(3)
		As a minimum, both values must be correct to 1 sf, candidate states "change of sign, hence root".	
			[8]

Question Number	Scheme		Marks
45 (a)	Iterative formula: $x_{n+1} = \frac{2}{(x_n)^2} + 2$, $x_0 = 2.5$		
	$x_1 = \frac{2}{(2.5)^2} + 2$	An attempt to substitute $x_0 = 2.5$ into the iterative formula. Can be implied by $x_1 = 2.32$ or 2.320	M1
	$x_1 = 2.32$ $x_2 = 2.371581451...$	Both $x_1 = 2.32(0)$ and $x_2 =$ awrt 2.372	A1
	$x_3 = 2.355593575...$ $x_4 = 2.360436923...$	Both $x_3 =$ awrt 2.356 and $x_4 =$ awrt 2.360 or 2.36	A1 cso
			(3)
(b)	Let $\mathrm{f}(x) = -x^3 + 2x^2 + 2 = 0$		
	$\mathrm{f}(2.3585) = 0.00583577...$	Choose suitable interval for x, e.g. [2.3585, 2.3595] or tighter	M1
	$\mathrm{f}(2.3595) = -0.00142286...$	any one value awrt 1 sf or truncated 1 sf	dM1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)	both values correct, sign change and conclusion	A1
		At a minimum, both values must be correct to 1sf or truncated 1sf, candidate states "change of sign, hence root".	(3)
			[6]

Question Number	Scheme	Marks
46.	(a) $\mathrm{f}'(x)=3\mathrm{e}^x+3x\mathrm{e}^x$	M1 A1
	$3\mathrm{e}^x+3x\mathrm{e}^x=3\mathrm{e}^x(1+x)=0$	
	$x=-1$	M1 A1
	$\mathrm{f}(-1)=-3\mathrm{e}^{-1}-1$	B1 **(5)**
	(b) $x_1=0.2596$	B1
	$x_2=0.2571$	B1
	$x_3=0.2578$	B1 **(3)**
	(c) Choosing $(0.257\,55, 0.257\,65)$ or an appropriate tighter interval.	M1
	$\mathrm{f}(0.257\,55)=-0.000\,379\ ...$	
	$\mathrm{f}(0.257\,65)=0.000\,109\ \ldots$	A1
	Change of sign (and continuity) $\Rightarrow$ root $\in(0.257\,55, 0.257\,65)$ ∗ cso	A1
	($\Rightarrow x=0.2576$, is correct to 4 decimal places)	**(3)**
		[11]
	Note: $x=0.257\,627\,65\ \ldots$ is accurate	

Question Number	Scheme	Marks
47.	(a) $\mathrm{f}(1.4) = -0.568 \ \ldots < 0$	
	$\mathrm{f}(1.45) = 0.245 \ \ldots > 0$	M1
	Change of sign (and continuity) $\Rightarrow \ \alpha \in (1.4, 1.45)$	A1 (2)
	(b) $3x^3 = 2x + 6$	
	$x^3 = \dfrac{2x}{3} + 2$	
	$x^2 = \dfrac{2}{3} + \dfrac{2}{x}$	M1 A1
	$x = \sqrt{\left(\dfrac{2}{x} + \dfrac{2}{3}\right)}$ ∗ cso	A1 (3)
	(c) $x_1 = 1.4371$	B1
	$x_2 = 1.4347$	B1
	$x_3 = 1.4355$	B1 (3)
	(d) Choosing the interval $(1.4345, 1.4355)$ or appropriate tighter interval.	M1
	$\mathrm{f}(1.4345) = -0.01 \ldots$	
	$\mathrm{f}(1.4355) = 0.003 \ \ldots$	M1
	Change of sign (and continuity) $\Rightarrow \ \alpha \in (1.4345, 1.4355)$	
	$\Rightarrow \alpha = 1.435$, correct to 3 decimal places ∗ cso	A1 (3)
		[11]
	Note: $\alpha = 1.435\,304\,553 \ \ldots$	

Question Number	Scheme	Marks	
48.	(a) $\mathrm{f}(2) = 0.38 \ \ldots$		
	$\mathrm{f}(3) = -0.39 \ \ldots$	M1	
	Change of sign (and continuity) $\Rightarrow$ root in $(2, 3)$ ∗ cso	A1	(2)
	(b) $x_1 = \ln 4.5 + 1 \approx 2.50408$	M1	
	$x_2 \approx 2.50498$	A1	
	$x_3 \approx 2.50518$	A1	(3)
	(c) Selecting $[2.5045, 2.5055]$, or appropriate tighter range, and evaluating at both ends.	M1	
	$\mathrm{f}(2.5045) \approx 6 \times 10^{-4}$		
	$\mathrm{f}(2.5055) \approx -2 \times 10^{-4}$		
	Change of sign (and continuity) $\Rightarrow$ root $\in (2.5045, 2.5055)$		
	$\Rightarrow$ root $= 2.505$ to 3 dp ∗ cso	A1	(2)
			[7]
	Note: The root, correct to 5 dp, is 2.50524		

Question Number	Scheme	Notes	Marks
49.	$\begin{array}{c\|c\|c\|c\|c\|c\|c} x & 0 & 0.2 & 0.4 & 0.6 & 0.8 & 1 \\ \hline y & 2 & \boldsymbol{1.8625426...} & 1.71830 & 1.56981 & 1.41994 & 1.27165 \end{array}$	$y = \frac{6}{(2 + e^x)}$	
(a)	$\{\text{At } x = 0.2,\}\ y = 1.86254$ (5 dp)	1.86254	B1 **cao**
	Note: Look for this value on the given table or in their working.		**[1]**
(b)	$\frac{1}{2}(0.2)\underline{\left[2+1.27165+2\left(\text{their } 1.86254+1.71830+1.56981+1.41994\right)\right]}$	Outside brackets $\frac{1}{2}\times(0.2)$ or $\frac{1}{10}$ or $\frac{1}{2}\times\frac{1}{5}$	B1 o.e.
		<u>For structure of</u> $\underline{[\ldots\ldots]}$	M1
	$\left\{= \frac{1}{10}(16.41283)\right\} = 1.641283 = 1.6413$ (4 dp)	anything that rounds to 1.6413	A1
			[3]

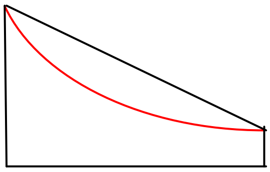
	Question 49 Notes	
49. (b)	**Note**	**M1:** Do not allow an extra y-value ***or*** a repeated y value in their […] Do not allow an omission of a y-ordinate in their […] for M1 **unless** they give the correct answer of awrt 1.6413, in which case both M1 and A1 can be scored.
	Note	**A1:** Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 1.64150274…)
	Note	Full marks can be gained in part (b) for awrt 1.6413 even if B0 is given in part (a)
	Note	Award B1M1A1 for $\frac{1}{10}(2+1.27165) + \frac{1}{5}(\text{their } 1.86254 + 1.71830 + 1.56981 + 1.41994) = \text{awrt } 1.6413$
	<u>Bracketing mistakes:</u> Unless the final answer implies that the calculation has been done correctly Award B1M0A0 for $\frac{1}{2}(0.2) + 2 + 2(\text{their } 1.86254 + 1.71830 + 1.56981 + 1.41994) + 1.27165\ \ (=16.51283)$ Award B1M0A0 for $\frac{1}{2}(0.2)(2 + 1.27165) + 2(\text{their } 1.86254 + 1.71830 + 1.56981 + 1.41994)\ \ (=13.468345)$ Award B1M0A0 for $\frac{1}{2}(0.2)(2) + 2(\text{their } 1.86254 + 1.71830 + 1.56981 + 1.41994) + 1.27165\ \ (=14.61283)$	
	<u>Alternative method: Adding individual trapezia</u> $\text{Area} \approx 0.2 \times \left[\frac{2+\text{"}1.86254\text{"}}{2} + \frac{\text{"}1.86254\text{"}+1.71830}{2} + \frac{1.71830+1.56981}{2} + \frac{1.56981+1.41994}{2} + \frac{1.41994+1.27165}{2}\right]$ $= 1.641283$ **B1** 0.2 and a divisor of 2 on all terms inside brackets **M1** First and last ordinates once and two of the middle ordinates inside brackets ignoring the 2 **A1** anything that rounds to 1.6413	

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td>50.</td><td colspan="2"><table><tr><td>x</td><td>1</td><td>1.2</td><td>1.4</td><td>1.6</td><td>1.8</td><td>2</td></tr><tr><td>y</td><td>0</td><td>0.2625</td><td><i>0.659485...</i></td><td>1.2032</td><td>1.9044</td><td>2.7726</td></tr></table> $y = x^2 \ln x$</td><td></td></tr>
<tr><td>(a)</td><td>$\{\text{At } x = 1.4,\}\ y = 0.6595\ (4\text{ dp})$</td><td>0.6595</td><td>B1 cao</td></tr>
<tr><td></td><td colspan="2"></td><td>[1]</td></tr>
<tr><td>(b)</td><td>$\frac{1}{2}\times(0.2)\times\underline{\left[0+2.7726+2\left(0.2625+\text{ their }0.6595+1.2032+1.9044\right)\right]}$
{Note: The “0” does not have to be included in [……]}</td><td>Outside brackets $\frac{1}{2}\times(0.2)$ or $\frac{1}{10}$
<u>For structure of</u> $\underline{[\ldots\ldots\ldots\ldots]}$</td><td>B1 o.e.
M1</td></tr>
<tr><td></td><td>$\left\{=\frac{1}{10}(10.8318)\right\} = 1.08318 = 1.083\ (3\text{ dp})$</td><td>anything that rounds to 1.083</td><td>A1</td></tr>
<tr><td></td><td colspan="2"></td><td>[3]
4</td></tr>
</table>

<table>
<tr><th></th><th colspan="2">Question 50 Notes</th></tr>
<tr><td>50. (a)</td><td>B1</td><td>0.6595 correct answer only. Look for this on the table or in the candidate’s working.</td></tr>
<tr><td>(b)</td><td>B1</td><td>Outside brackets $\frac{1}{2}\times(0.2)$ or $\frac{1}{2}\times\frac{1}{5}$ or $\frac{1}{10}$ or equivalent.</td></tr>
<tr><td></td><td>M1
Note</td><td><u>For structure of trapezium rule</u> $[\ \underline{\ldots\ldots\ldots\ldots}\ ]$
No errors are allowed [eg. an omission of a <i>y</i>-ordinate or an extra <i>y</i>-ordinate or a repeated <i>y</i> ordinate].</td></tr>
<tr><td></td><td>A1
Note</td><td>anything that rounds to 1.083
Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 1.070614704…)</td></tr>
<tr><td></td><td>Note</td><td>Full marks can be gained in part (b) for using an incorrect part (a) answer of 0.6594</td></tr>
<tr><td></td><td>Note</td><td>Award B1M1A1 for $\frac{1}{10}(2.7726)+\frac{1}{5}(0.2625+\text{ their }0.6595+1.2032+1.9044) = \text{awrt } 1.083$</td></tr>
<tr><td></td><td colspan="2"><u>Bracketing mistake:</u> Unless the final answer implies that the calculation has been done correctly
Award B1M0A0 for $\frac{1}{2}(0.2)+2(0.2625+\text{ their }0.6595+1.2032+1.9044)+2.7726$ (answer of 10.9318)
Award B1M0A0 for $\frac{1}{2}(0.2)(2.7726)+2(0.2625+\text{ their }0.6595+1.2032+1.9044)$ (answer of 8.33646)</td></tr>
<tr><td></td><td colspan="2"><u>Alternative method: Adding individual trapezia</u>
$\text{Area} \approx 0.2\times\left[\frac{0+0.2625}{2}+\frac{0.2625+\text{"}0.6595\text{"}}{2}+\frac{\text{"}0.6595\text{"}+1.2032}{2}+\frac{1.2032+1.9044}{2}+\frac{1.9044+2.7726}{2}\right] = 1.08318...$</td></tr>
<tr><td></td><td>B1</td><td>0.2 and a divisor of 2 on all terms inside brackets</td></tr>
<tr><td></td><td>M1</td><td>First and last ordinates once and two of the middle ordinates inside brackets ignoring the 2</td></tr>
<tr><td></td><td>A1</td><td>anything that rounds to 1.083</td></tr>
</table>

Question Number	Scheme		Marks
51.	$\begin{array}{c\|c\|c\|c\|c} x & 1 & 2 & 3 & 4 \\ \hline y & 1.42857 & 0.90326 & \boldsymbol{0.682116...} & 0.55556 \end{array}$	$y = \dfrac{10}{2x + 5\sqrt{x}}$	
(a)	$\{\text{At } x = 3,\}\ y = 0.68212$ (5 dp)	0.68212	B1 **cao**
			[1]
(b)	$\frac{1}{2} \times 1 \times \underline{\left[1.42857 + 0.55556 + 2\left(0.90326 + \text{their } 0.68212\right)\right]}$	Outside brackets $\frac{1}{2} \times 1$ or $\frac{1}{2}$	B1 aef
		For structure of $\underline{[................]}$	M1
	$\left\{= \tfrac{1}{2}(5.15489)\right\} = 2.577445 = 2.5774$ (4 dp)	anything that rounds to 2.5774	A1
			[3]
(c)	- Overestimate **and** a reason such as - {top of} trapezia lie above the curve - a diagram which gives reference to the extra area - concave or convex - $\dfrac{d^2y}{dx^2} > 0$ (can be implied) - bends inwards - curves downwards		B1
			[1] **5**

	Question 51 Notes	
51. (a)	**B1**	0.68212 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1**	Outside brackets $\frac{1}{2} \times 1$ or $\frac{1}{2}$ or equivalent.
	M1	For structure of trapezium rule$[\ ....\ ]$
	Note	No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate].
	A1	anything that rounds to 2.5774
	Note	Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 2.51314428…)

51. (b) contd	**Note**	Award B1M1A1 for $\frac{1}{2}(1.42857+0.55556)+\left(0.90326+\text{their } 0.68212\right)=2.577445$
		Bracketing mistake: Unless the final answer implies that the calculation has been done correctly award B1M0A0 for $\frac{1}{2}\times 1\ +1.42857+2\left(0.90326+\text{their } 0.68212\right)+0.55556$ (nb: answer of 5.65489). award B1M0A0 for $\frac{1}{2}\times 1\ (1.42857+0.55556)+2\left(0.90326+\text{their } 0.68212\right)$ (nb: answer of 4.162825).
		Alternative method: Adding individual trapezia Area $\approx 1\times\left[\frac{1.42857+0.90326}{2}+\frac{0.90326+\text{"}0.68212\text{"}}{2}+\frac{\text{"}0.68212\text{"}+0.55556}{2}\right]=2.577445$
	B1	**B1:** 1 and a divisor of 2 on all terms inside brackets.
	M1	**M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
	A1	**A1:** anything that rounds to 2.5774
(c)	**B1**	Overestimate **and** either trapezia lie above curve **or** a diagram that gives reference to the extra area **eg.** This diagram is sufficient. It must show the top of a trapezium lying above the curve.  **or** concave or convex **or** $\frac{d^2y}{dx^2}>0$ (can be implied) **or** bends inwards **or** curves downwards.
	Note	Reason of "gradient is negative" by itself is B0.

Question Number	Scheme	Marks
52. (a)	Area $\approx \frac{1}{2}\times 0.5\ ;\times \left[\underline{2+2\left(4.077+7.389+10.043\right)+0}\right]$	B1; M1
	$=\frac{1}{4}\times 45.018\ =\ 11.2545\ = 11.25\,(2\text{ dp})$ 11.25	A1 **cao**
		[3]
(b)	Any one of • Increase the number of strips • Use more trapezia • Make *h* smaller • Increase the **number of** *x* and/or *y* values used • Shorter /smaller intervals for *x* • More values of *y*. • More intervals of *x* • Increase *n*	B1
		[1]
		9

		Question 52 Notes
(a)	**B1**	Outside brackets $\frac{1}{2}\times 0.5$ or $\frac{0.5}{2}$ or 0.25 or $\frac{1}{4}$.
	M1	$\underline{\text{For structure of trapezium rule}}\left[\ \underline{............}\ \right]$. Condone missing 0.
	Note	No errors are allowed [eg. an omission of a *y*-ordinate or an extra *y*-ordinate or a repeated *y* ordinate].
	A1	11.25 **cao**
	Note	Working must be seen to demonstrate the use of the trapezium rule. The actual area is 12.39953751…
	Note	Award B1M1A1 for $\frac{0.5}{2}(2+0)+\frac{1}{2}\left(4.077+7.389+10.043\right) = 11.25$

52. (a) contd	**<u>Bracketing mistake</u>: Unless the final answer implies that the calculation has been done correctly.** Award B1M0A0 for $\frac{1}{2}\times 0.5+2+2\left(4.077+7.389+10.043\right)+0$ (nb: answer of 45.268). **<u>Alternative method for part (a): Adding individual trapezia</u>** Area $\approx$ $0.5\times\left[\frac{2+4.077}{2}+\frac{4.077+7.389}{2}+\frac{7.389+10.043}{2}+\frac{10.043+0}{2}\right]=11.2545=11.25$ (2 dp) cao	
	B1	0.5 and a divisor of 2 on all terms inside brackets.
	M1	First and last ordinates once and the middle ordinates twice inside brackets ignoring the 2.
	A1	11.25 **cao**
(b)	**B0**	Give B0 for • smaller values of *x* and/or *y*. • use more decimal places

Question Number	Scheme	Marks
53. (a)	1.154701	B1 **cao** **[1]**
(b)	Area $\approx\frac{1}{2}\times\frac{\pi}{6};\times\underline{\left[1+2\left(1.035276 + \text{their } 1.154701\right)+1.414214\right]}$	B1; <u>M1</u>
	$=\frac{\pi}{12}\times 6.794168 \;=\; 1.778709023... = 1.7787$ (4 dp) 1.7787 or awrt 1.7787	A1 **[3]** **4**
	Notes for Question 53	
(a)	**B1:** 1.154701 correct answer only. Look for this on the table or in the candidate's working.	
(b)	**B1:** Outside brackets $\frac{1}{2}\times\frac{\pi}{6}$ or $\frac{\pi}{12}$ or awrt 0.262 **M1:** <u>For structure of trapezium rule</u>$\left[\underline{\;............\;}\right]$ **A1:** anything that rounds to 1.7787 **Note:** It can be possible to award : (a) B0 (b) B1M1A1 (awrt 1.7787) **<u>Note:</u>** Working must be seen to demonstrate the use of the trapezium rule. **<u>Note</u>**: actual area is 1.762747174… <u>Note:</u> Award B1M1A1 for $\frac{\pi}{12}(1+1.414214)+\frac{\pi}{6}\left(1.035276 + \text{their } 1.154701\right) = 1.778709023...$ <u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly,** Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{6}+1+2\left(1.035276 + \text{their } 1.154701\right)+1.414214$ (nb: answer of 7.05596...). Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{6}\;(1+1.414214)+\,2\left(1.035276 + \text{their } 1.154701\right)$ (nb: answer of 5.01199...). *<u>Alternative method for part (b): Adding individual trapezia</u>* Area $\approx\frac{\pi}{6}\times\left[\frac{1+1.035276}{2}+\frac{1.035276+1.154701}{2}+\frac{1.154701+1.414214}{2}\right] = 1.778709023...$ **B1:** $\frac{\pi}{6}$ and a divisor of 2 on all terms inside brackets. **M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2. **A1:** anything that rounds to 1.7787	

Question Number	Scheme	Marks
54. (a)	6.248046798... = 6.248 (3dp) 6.248 or awrt 6.248	B1 **[1]**
(b)	Area $\approx \frac{1}{2}\times 2\,;\times\underline{\left[3+2\left(7.107+7.218+\text{their } 6.248\right)+5.223\right]}$	B1; M1
	$= 49.369\ = 49.37\,(2\text{ dp})$ 49.37 or awrt 49.37	A1 **[3]**
		4
	Notes for Question 54	
(a)	**B1:** 6.248 or awrt 6.248. Look for this on the table or in the candidate's working.	
(b)	**B1**: Outside brackets $\frac{1}{2}\times 2$ or 1 **M1:** For structure of trapezium rule$\left[\ \underline{............}\ \right]$. Allow one miscopy of their values. **A1:** 49.37 or anything that rounds to 49.37 **Note:** It can be possible to award : (a) B0 (b) B1M1A1 (awrt 49.37) **Note:** Working must be seen to demonstrate the use of the trapezium rule. **Note**: actual area is 50.828… Bracketing mistake: **Unless the final answer implies that the calculation has been done correctly,** Award B1M0A0 for$1+3+2\left(7.107+7.218+\text{their } 6.248\right)+5.223$ (nb: answer of 50.369).	
54. (b) ctd	*Alternative method for part (b): Adding individual trapezia* Area $\approx 2\times\left[\frac{3+7.107}{2}+\frac{7.107+7.218}{2}+\frac{7.218+6.248}{2}+\frac{6.248+5.223}{2}\right]= 49.369$ **B1:** 2 and a divisor of 2 on all terms inside brackets. **M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2. **A1:** anything that rounds to 49.37	

Question Number	Scheme	Marks
55. (a)	$\begin{array}{c\|ccccc} x & 0 & 1 & 2 & 3 & 4 \\ \hline y & 0 & e^{-\frac{1}{2}} & \underline{2e^{-1}} & 3e^{-\frac{3}{2}} & 4e^{-2} \end{array}$	
	$\underline{2e^{-1}}$ or awrt 0.74	B1
		[1]
	Outside brackets $\frac{1}{2}\times 1$ or 0.5;	B1
(b)	$\text{Area}\,(R) \approx \frac{1}{2}\times 1\,;\times\underline{\left\{0+2\left(e^{-\frac{1}{2}}+2e^{-1}+3e^{-\frac{3}{2}}\right)+4e^{-2}\right\}}$ $\underline{\text{For structure of trapezium rule}\{\ldots\ldots\ldots\ldots\}}$	$\underline{\text{M1}}$
	Correct expression $\underline{\text{inside brackets}}$	A1
	$=\frac{1}{2}\times 4.564701\ldots = 2.282351\ldots = \underline{2.28}$ (2dp) $\underline{2.28}$	A1 **cao**
		[4]
		5
	Notes on Question 55	
(b)	**M1: SC:** Allow either an extra term or one missing term in $\left(e^{-\frac{1}{2}}+2e^{-1}+3e^{-\frac{3}{2}}\right)$.	

Question Number	Scheme	Marks
56. (a)	1.0981	B1 **cao** **[1]**
(b)	Area $\approx \frac{1}{2}\times 1\,;\times \underline{\left[0.5+2\left(0.8284+\text{their }1.0981\right)+1.3333\right]}$ $=\frac{1}{2}\times 5.6863 \;=\; 2.84315 = 2.843$ (3 dp) 2.843 or awrt 2.843	B1; M1 A1 **[3]** **4**
(a)	**B1:** 1.0981 correct answer only. Look for this on the table or in the candidate's working.	
(b)	**B1**: Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$ **M1:** For structure of trapezium rule$[\;\underline{............}\;]$ **A1:** anything that rounds to 2.843 **Note:** Working must be seen to demonstrate the use of the trapezium rule. **Note**: actual area is 2.85573645… Note: Award B1M1 A1 for $\frac{1}{2}(0.5+1.3333)+\left(0.8284+\text{their }1.0981\right)=2.84315$ Bracketing mistake: **Unless the final answer implies that the calculation has been done correctly** Award B1M0A0 for $\frac{1}{2}\times 1\;+0.5+2\left(0.8284+\text{their }1.0981\right)+1.3333$ (nb: answer of 6.1863). Award B1M0A0 for $\frac{1}{2}\times 1\;(0.5+1.3333)+2\left(0.8284+\text{their }1.0981\right)$ (nb: answer of 4.76965).	
56. (b) ctd	*Alternative method for part (b): Adding individual trapezia* Area $\approx 1\times\left[\frac{0.5+0.8284}{2}+\frac{0.8284+1.0981}{2}+\frac{1.0981+1.3333}{2}\right] = 2.84315$ **B1:** 1 and a divisor of 2 on all terms inside brackets. **M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2. **A1:** anything that rounds to 2.843	

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>57.</td><td>

x	1	2	3	4
y	$\ln 2$	$\sqrt{2}\ln 4$	$\sqrt{3}\ln 6$	$2\ln 8$
	0.6931	1.9605	3.1034	4.1589

Area $=\frac{1}{2}\times 1(\ \ldots)$

$\approx\ \ldots\ \left(0.6931+2\left(1.9605+3.1034\right)+4.1589\right)$

$\approx\frac{1}{2}\times 14.97989\ \ldots\ \approx 7.49$ 7.49 cao

</td><td>M1
B1
M1
A1 (4)</td></tr>
</table>

Question Number	Scheme	Marks
58. (a)	0.73508	B1 **cao** **[1]**
(b)	Area $\approx \frac{1}{2}\times\frac{\pi}{8}\,;\times\underline{\left[0+2\left(\text{their }0.73508+1.17157+1.02280\right)+0\right]}$	B1 M1
	$=\frac{\pi}{16}\times 5.8589...\;=\;1.150392325...\;=1.1504\;\;(4\text{ dp})$ awrt 1.1504	A1 **[3]**
		4
(a)	**B1:** 0.73508 correct answer only. Look for this on the table or in the candidate's working.	
(b)	**B1:** Outside brackets $\frac{1}{2}\times\frac{\pi}{8}$ or $\frac{\pi}{16}$ or awrt 0.196 **M1:** For structure of trapezium rule $\left[\;\underline{............}\;\right]$; (0 can be implied). **A1:** anything that rounds to 1.1504 Bracketing mistake: **Unless the final answer implies that the calculation has been done correctly** Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8}\;+2\left(\text{their }0.73508+1.17157+1.02280\right)$ (nb: answer of 6.0552). Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8}\;(0+0)+2\left(\text{their }0.73508+1.17157+1.02280\right)$ (nb: answer of 5.8589). *Alternative method for part (b): Adding individual trapezia* Area $\approx \frac{\pi}{8}\times\left[\frac{0+0.73508}{2}+\frac{0.73508+1.17157}{2}+\frac{1.17157+1.02280}{2}+\frac{1.02280+0}{2}\right]=1.150392325...$ B1: $\frac{\pi}{8}$ and a divisor of 2 on all terms inside brackets. M1: One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2. A1: anything that rounds to 1.1504	

Question Number	Scheme	Marks
59.	(a) 0.0333, 1.3596 awrt 0.0333, 1.3596	B1 B1 **(2)**
	(b) $\text{Area}\left(R\right) \approx \frac{1}{2} \times \frac{\sqrt{2}}{4}\left[\ ...\right]$	B1
	$\approx\ ...\ \left[0 + 2\left(0.0333 + 0.3240 + 1.3596\right) + 3.9210\right]$	M1
	≈ 1.30 Accept 1.3	A1 **(3)** **[5]**

Question Number	Scheme	Marks
60.		
(a)	$x = 3 \;\Rightarrow\; y = 0.1847$ awrt	B1
	$x = 5 \;\Rightarrow\; y = 0.1667$ awrt or $\frac{1}{6}$	B1
		(2)
(b)	$I \approx \frac{1}{\underline{\underline{2}}}\left[0.2 + 0.1667 + 2\left(0.1847 + 0.1745\right)\right]$	$\underline{\underline{\text{B1}}}$ M1 A1ft
	≈ 0.543 0.542 or 0.543	A1 (4)
		[6]

Question Number	Scheme	Marks
61.	(a) $y\left(\frac{\pi}{6}\right) \approx 1.2247, \; y\left(\frac{\pi}{4}\right) = 1.1180$ accept awrt 4 d.p.	B1 B1 **(2)**
	(b)(i) $I \approx \left(\frac{\pi}{12}\right)\left(1.3229 + 2\times 1.2247 + 1\right)$ B1 for $\frac{\pi}{12}$	B1 M1
	≈ 1.249 cao	A1
	(ii) $I \approx \left(\frac{\pi}{24}\right)\left(1.3229 + 2\times\left(1.2973 + 1.2247 + 1.1180\right) + 1\right)$ B1 for $\frac{\pi}{24}$	B1 M1
	≈ 1.257 cao	A1 **(6)** **[8]**

Question Number	Scheme		Marks	
62	(a) 1.386, 2.291	awrt 1.386, 2.291	B1 B1	(2)
	(b) $A \approx \frac{1}{2} \times 0.5(\ \dots\ )$		B1	
	$= \ \dots\ \left(0 + 2\left(0.608 + 1.386 + 2.291 + 3.296 + 4.385\right) + 5.545\right)$		M1	
	$= \ 0.25\left(0 + 2\left(0.608 + 1.386 + 2.291 + 3.296 + 4.385\right) + 5.545\right)$	ft their (a)	A1ft	
	$= 0.25 \times 29.477\ \dots\ \approx 7.37$	cao	A1	(4)
				[6]

Question Number	Scheme		Marks	
63 (a)	1.14805	awrt 1.14805	B1	(1)
(b)	$A \approx \frac{1}{2} \times \frac{3\pi}{8}(\; ... \;)$		B1	
	$= \; ... \; \left(3+2\left(2.77164+2.12132+1.14805\right)+0\right)$	0 can be implied	M1	
	$= \; \frac{3\pi}{16}\left(3+2\left(2.77164+2.12132+1.14805\right)\right)$	ft their (a)	A1ft	
	$= \frac{3\pi}{16} \times 15.08202 \; ... \; = 8.884$	cao	A1	(4)
				[5]

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td>64. (a)</td><td colspan="2"><table>
<tr><td><i>x</i></td><td>0</td><td>0.4</td><td>0.8</td><td>1.2</td><td>1.6</td><td>2</td></tr>
<tr><td><i>y</i></td><td>e^0</td><td>$e^{0.08}$</td><td>$e^{0.32}$</td><td>$e^{0.72}$</td><td>$e^{1.28}$</td><td>e^2</td></tr>
<tr><td>or <i>y</i></td><td>1</td><td>1.08329...</td><td>1.37713...</td><td>2.05443...</td><td>3.59664...</td><td>7.38906...</td></tr>
</table></td><td></td></tr>
<tr><td></td><td></td><td>Either $e^{0.32}$ and $e^{1.28}$ or awrt 1.38 and 3.60 (or a mixture of e's and decimals)</td><td>B1
[1]</td></tr>
<tr><td>(b)
Way 1</td><td>Area $\approx \frac{1}{2}\times 0.4\,;\times\underline{\left[\ e^0+2\left(e^{0.08}+e^{0.32}+e^{0.72}+e^{1.28}\right)+e^2\ \right]}$</td><td>Outside brackets $\frac{1}{2}\times 0.4$ or 0.2
<u>For structure of trapezium rule</u> $\underline{[\ ....\ ]}$;</td><td>B1;
<u>M1</u> $\sqrt{}$</td></tr>
<tr><td></td><td>$= 0.2\times 24.61203164... = 4.922406... = \underline{4.922}$ (4sf)</td><td>$\underline{4.922}$</td><td>A1 cao
[3]</td></tr>
<tr><td><i>Aliter</i>
(b)
Way 2</td><td>Area $\approx 0.4\times\left[\frac{e^0+e^{0.08}}{2}+\frac{e^{0.08}+e^{0.32}}{2}+\frac{e^{0.32}+e^{0.72}}{2}+\frac{e^{0.72}+e^{1.28}}{2}+\frac{e^{1.28}+e^2}{2}\right]$</td><td>0.4 and a divisor of 2 on all terms inside brackets.</td><td>B1</td></tr>
<tr><td></td><td>which is equivalent to:
Area $\approx \frac{1}{2}\times 0.4\,;\times\underline{\left[\ e^0+2\left(e^{0.08}+e^{0.32}+e^{0.72}+e^{1.28}\right)+e^2\ \right]}$</td><td>One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.</td><td><u>M1</u> $\sqrt{}$</td></tr>
<tr><td></td><td>$= 0.2\times 24.61203164... = 4.922406... = \underline{4.922}$ (4sf)</td><td>$\underline{4.922}$</td><td>A1 cao
[3]</td></tr>
<tr><td></td><td></td><td></td><td>4 marks</td></tr>
</table>

Note an expression like Area $\approx \frac{1}{2}\times 0.4 + e^0 + 2\left(e^{0.08}+e^{0.32}+e^{0.72}+e^{1.28}\right)+e^2$ would score B1M1A0

Allow one term missing (slip!) in the ( ) brackets for

The M1 mark for structure is for the material found in the curly brackets ie

$$\underline{\left[\ \text{first } y \text{ ordinate} + 2\left(\text{intermediate ft } y \text{ ordinate}\right) + \text{final } y \text{ ordinate}\ \right]}$$

<table>
<tr><th>Question Number</th><th>Scheme</th><th></th><th>Marks</th></tr>
<tr><td>65. (a)</td><td colspan="2">
<table>
<tr><td>x</td><td>0</td><td>$\frac{\pi}{4}$</td><td>$\frac{\pi}{2}$</td><td>$\frac{3\pi}{4}$</td><td>π</td></tr>
<tr><td>y</td><td>0</td><td>1.844321332…</td><td>4.810477381…</td><td>8.87207</td><td>0</td></tr>
</table>
</td><td></td></tr>
<tr><td></td><td></td><td>awrt 1.84432
awrt 4.81048 or 4.81047</td><td>B1
B1
[2]</td></tr>
<tr><td></td><td>0 can be implied</td><td>Outside brackets awrt 0.39 or $\frac{1}{2}\times$awrt 0.79
$\frac{1}{2}\times\frac{\pi}{4}$ or $\frac{\pi}{8}$</td><td>B1</td></tr>
<tr><td>(b)
Way 1</td><td>Area $\approx\frac{1}{2}\times\frac{\pi}{4}\,;\times\underline{\left\{0+2(1.84432+4.81048+8.87207)+0\right\}}$</td><td><u>For structure of trapezium</u>
<u>rule</u> $\underline{\{............\}}$;</td><td><u>M1</u>$\sqrt{}$</td></tr>
<tr><td></td><td></td><td>Correct expression <u>inside brackets</u> which all must be multiplied by their “outside constant”.</td><td><u>A1</u>$\sqrt{}$</td></tr>
<tr><td></td><td>$=\frac{\pi}{8}\times 31.05374...\;=\;12.19477518...\;=\underline{12.1948}$ (4dp)</td><td><u>12.1948</u></td><td>A1 cao
[4]</td></tr>
<tr><td rowspan="4"><i>Aliter</i>
(b)
Way 2</td><td>Area $\approx\frac{\pi}{4}\times\left\{\frac{0+1.84432}{2}+\frac{1.84432+4.81048}{2}+\frac{4.81048+8.87207}{2}+\frac{8.87207+0}{2}\right\}$</td><td>$\frac{\pi}{4}$ (or awrt 0.79) and a divisor of 2 on all terms inside brackets.</td><td>B1</td></tr>
<tr><td>which is equivalent to:</td><td>One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.</td><td><u>M1</u>$\sqrt{}$</td></tr>
<tr><td>Area $\approx\frac{1}{2}\times\frac{\pi}{4}\,;\times\underline{\left\{0+2(1.84432+4.81048+8.87207)+0\right\}}$</td><td>Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out.</td><td><u>A1</u>$\sqrt{}$</td></tr>
<tr><td>$=\frac{\pi}{4}\times 15.52687...\;=\;12.19477518...\;=\underline{12.1948}$ (4dp)</td><td><u>12.1948</u></td><td>A1 cao
[4]</td></tr>
<tr><td></td><td></td><td></td><td>6 marks</td></tr>
</table>

Note an expression like Area $\approx\frac{1}{2}\times\frac{\pi}{4}+2(1.84432+4.81048+8.87207)$ would score B1M1A0A0