EXPERT TUITION

Maths Questions By Topic:

Numerical Methods

A-Level Edexcel

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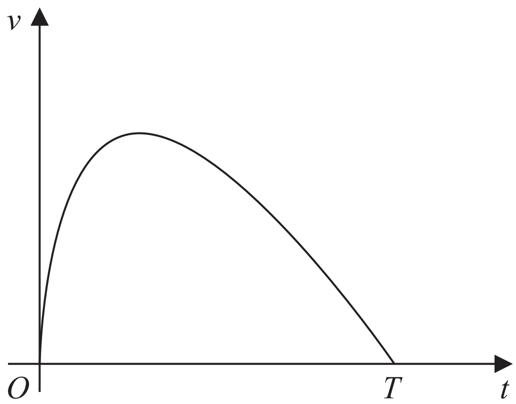

Figure 2

A car stops at two sets of traffic lights.

Figure 2 shows a graph of the speed of the car, $v\,\text{ms}^{-1}$, as it travels between the two sets of traffic lights.

The car takes T seconds to travel between the two sets of traffic lights.

The speed of the car is modelled by the equation

$$v = (10 - 0.4t)\ln(t + 1) \qquad 0 \leqslant t \leqslant T$$

where t seconds is the time after the car leaves the first set of traffic lights.

According to the model,

(a) find the value of T

(1)

(b) show that the maximum speed of the car occurs when

$$t = \frac{26}{1 + \ln(t + 1)} - 1$$

(4)

Using the iteration formula

$$t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$$

with $t_1 = 7$

(c) (i) find the value of t_3 to 3 decimal places,

(ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

(3)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2. The curve with equation $y = f(x)$ where

$$f(x) = x^2 + \ln(2x^2 - 4x + 5)$$

has a single turning point at $x = \alpha$

(a) Show that α is a solution of the equation

$$2x^3 - 4x^2 + 7x - 2 = 0$$

(4)

The iterative formula

$$x_{n+1} = \frac{1}{7}(2 + 4x_n^{\ 2} - 2x_n^{\ 3})$$

is used to find an approximate value for α.

Starting with $x_1 = 0.3$

(b) calculate, giving each answer to 4 decimal places,

(i) the value of x_2

(ii) the value of x_4

(3)

Using a suitable interval and a suitable function that should be stated,

(c) show that α is 0.341 to 3 decimal places.

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3. The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$

(2)

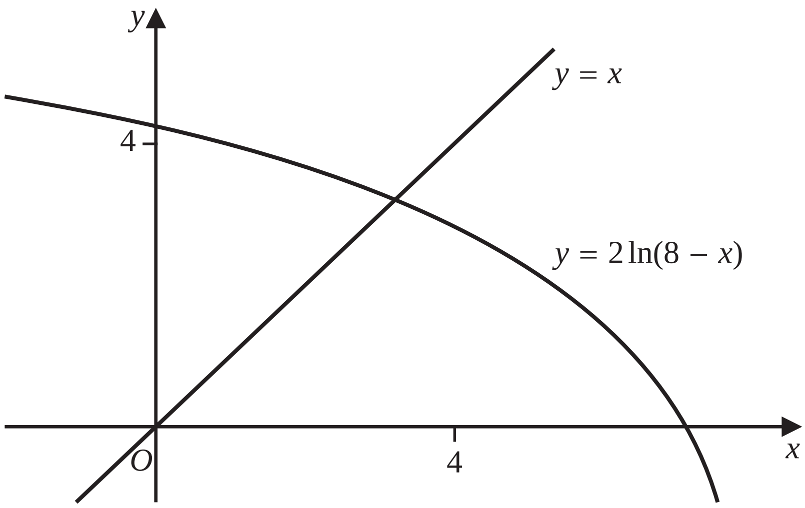

Figure 2

Figure 2 shows the graph of $y = 2\ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2\ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for α.

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for α, justifying your answer.

(2)

Question 3 continued

(Total for Question 3 is 4 marks)

4.

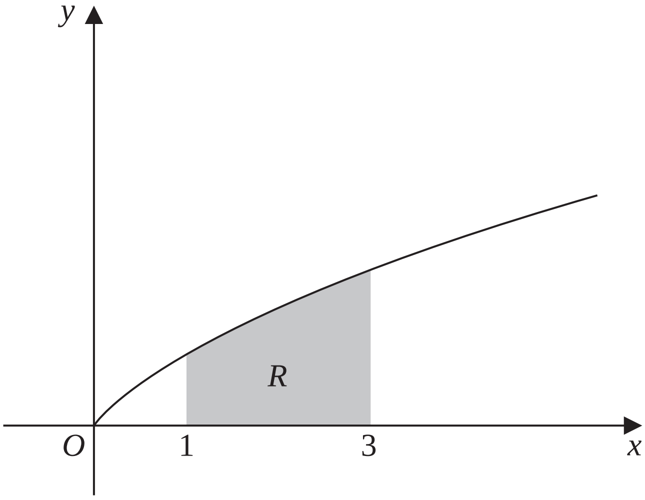

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \dfrac{x}{1+\sqrt{x}}, \quad x \geqslant 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$

The table below shows corresponding values of x and y for $y = \dfrac{x}{1+\sqrt{x}}$

x	1	1.5	2	2.5	3
y	0.5	0.6742	0.8284	0.9686	1.0981

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for the area of R, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule can be used to give a better approximation for the area of R.

(1)

(c) Giving your answer to 3 decimal places in each case, use your answer to part (a) to deduce an estimate for

(i) $\displaystyle\int_1^3 \frac{5x}{1+\sqrt{x}}\,dx$

(ii) $\displaystyle\int_1^3 \left(6 + \frac{x}{1+\sqrt{x}}\right)dx$

(2)

Question 4 continued

(Total for Question 4 is 6 marks)

5. $$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5$$

(a) Show that $f(x) = 0$ has a root α in the interval [3.5, 4]

(2)

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(2)

(c) Show that α is the only root of $f(x) = 0$

(2)

Question 5 continued

(Total for Question 5 is 6 marks)

6.

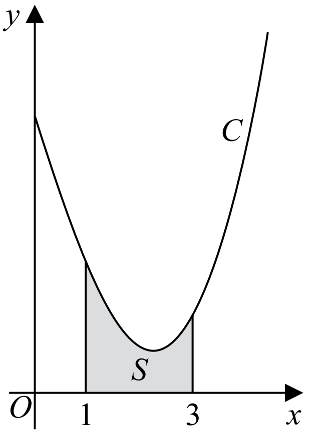

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$

The table below shows corresponding values of x and y with the values of y given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

(c) Show that the exact area of S can be written in the form $\frac{a}{b} + \ln c$, where a, b and c are integers to be found.

(6)

(In part c, solutions based entirely on graphical or numerical methods are not acceptable.)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 10 marks)

7. The table below shows corresponding values of x and y for $y = \log_3 2x$

The values of y are given to 2 decimal places as appropriate.

x	3	4.5	6	7.5	9
y	1.63	2	2.26	2.46	2.63

(a) Using the trapezium rule with all the values of y in the table, find an estimate for

$$\int_3^9 \log_3 2x \, dx$$

(3)

Using your answer to part (a) and making your method clear, estimate

(b) (i) $\int_3^9 \log_3 (2x)^{10} \, dx$

(ii) $\int_3^9 \log_3 18x \, dx$

(3)

Question 7 continued

(Total for Question 7 is 6 marks)

8.

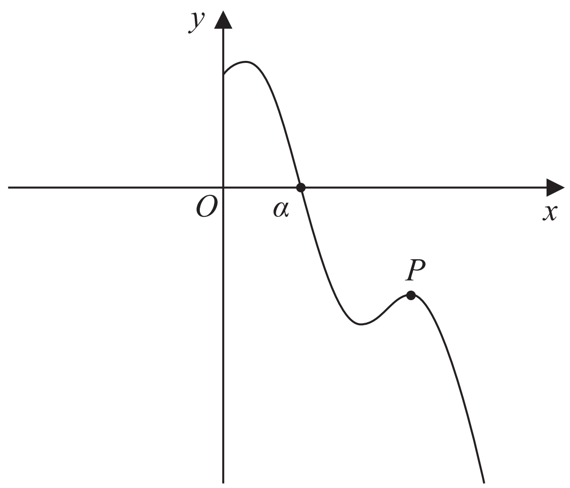

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 8\sin\left(\frac{1}{2}x\right) - 3x + 9 \qquad x > 0$$

and x is measured in radians.

The point P, shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

(a) find the x coordinate of P, giving your answer to 3 significant figures.

(4)

The curve crosses the x-axis at $x = \alpha$, as shown in Figure 2.

Given that, to 3 decimal places, $\mathrm{f}(4) = 4.274$ and $\mathrm{f}(5) = -1.212$

(b) explain why α must lie in the interval $[4, 5]$

(1)

(c) Taking $x_0 = 5$ as a first approximation to α, apply the Newton-Raphson method once to $\mathrm{f}(x)$ to obtain a second approximation to α.

Show your method and give your answer to 3 significant figures.

(2)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 7 marks)

9. The table below shows corresponding values of x and y for $y = \sqrt{\frac{x}{1+x}}$

The values of y are given to 4 significant figures.

x	0.5	1	1.5	2	2.5
y	0.5774	0.7071	0.7746	0.8165	0.8452

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}}\,dx$$

giving your answer to 3 significant figures.

(3)

(b) Using your answer to part (a), deduce an estimate for $\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\,dx$

(1)

Given that

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\,dx = 4.535 \text{ to 4 significant figures}$$

(c) comment on the accuracy of your answer to part (b).

(1)

Question 9 continued

(Total for Question 9 is 5 marks)

10.

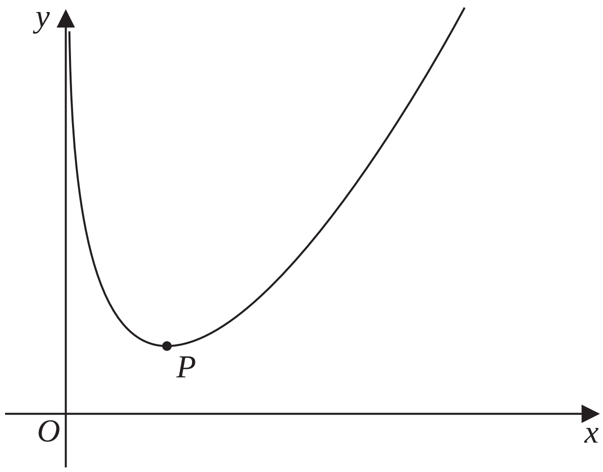

Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \qquad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

(4)

The point P, shown in Figure 1, is the minimum turning point on C.

(b) Show that the x coordinate of P is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$$

(3)

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12}\right)^{\frac{2}{3}} \qquad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,

(ii) the x coordinate of P to 5 decimal places.

(3)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 10 marks)

11. The speed of a small jet aircraft was measured every 5 seconds, starting from the time it turned onto a runway, until the time when it left the ground.

The results are given in the table below with the time in seconds and the speed in m s^{-1}.

Time (s)	0	5	10	15	20	25
Speed (m s^{-1})	2	5	10	18	28	42

Using all of this information,

(a) estimate the length of runway used by the jet to take off.

(3)

Given that the jet accelerated smoothly in these 25 seconds,

(b) explain whether your answer to part (a) is an underestimate or an overestimate of the length of runway used by the jet to take off.

(1)

Question 11 continued

(Total for Question 11 is 4 marks)

12.

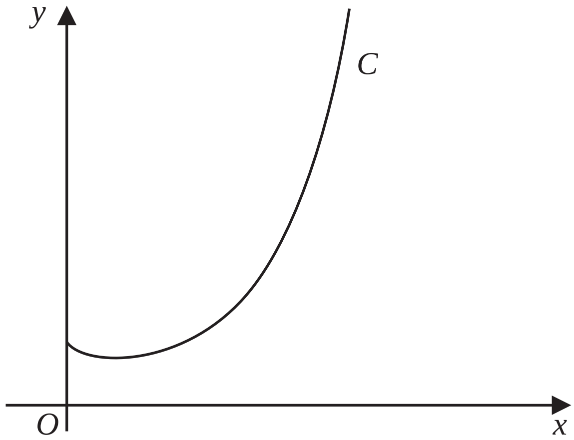

Figure 8

Figure 8 shows a sketch of the curve C with equation $y = x^x, \ x > 0$

(a) Find, by firstly taking logarithms, the x coordinate of the turning point of C.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

The point $P(\alpha, 2)$ lies on C.

(b) Show that $1.5 < \alpha < 1.6$

(2)

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{1-x_n}$$

Using this formula with $x_1 = 1.5$

(c) find x_4 to 3 decimal places,

(2)

(d) describe the long-term behaviour of x_n

(2)

Question 12 continued

Question 12 continued

Question 12 continued

(Total for Question 12 is 11 marks)

13. The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$$

(3)

Using the formula given in part (a) with $x_1 = 1$

(b) find the values of x_2 and x_3

(2)

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$

(1)

Question 13 continued

(Total for Question 13 is 6 marks)

14.

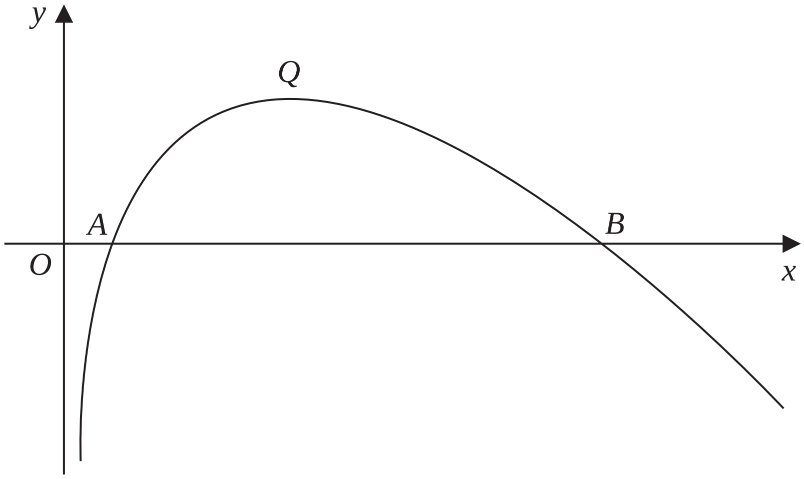

Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = (8 - x)\ln x, \qquad x > 0$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 2.

(a) Find the x coordinate of A and the x coordinate of B.

(1)

(b) Show that the x coordinate of Q satisfies

$$x = \frac{8}{1 + \ln x}$$

(4)

(c) Show that the x coordinate of Q lies between 3.5 and 3.6

(2)

(d) Use the iterative formula

$$x_{n+1} = \frac{8}{1 + \ln x_n} \qquad n \in \mathbb{N}$$

with $x_1 = 3.5$ to

(i) find the value of x_5 to 4 decimal places,

(ii) find the x coordinate of Q accurate to 2 decimal places.

(2)

Question 14 continued

(Total for Question 14 is 9 marks)

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15.

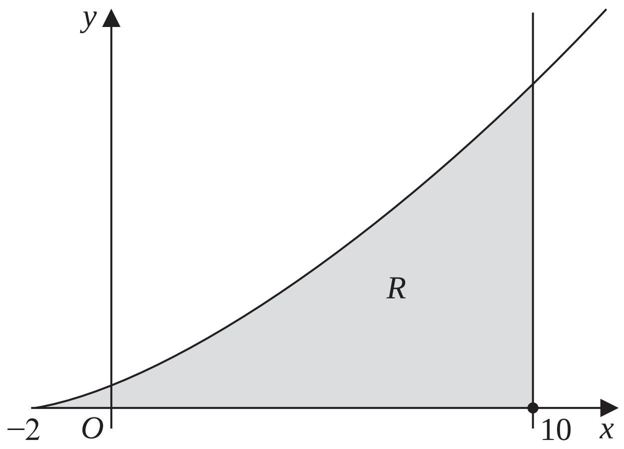

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \frac{(x+2)^{\frac{3}{2}}}{4}, \qquad x \geqslant -2$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line with equation $x = 10$

The table below shows corresponding values of x and y for $y = \frac{(x+2)^{\frac{3}{2}}}{4}$

(a) Complete the table, giving values of y corresponding to $x = 2$ and $x = 6$

x	-2	2	6	10
y	0			$6\sqrt{3}$

(1)

(b) Use the trapezium rule, with all the values of y from the completed table, to find an approximate value for the area of R, giving your answer to 3 decimal places.

(4)

Leave blank

Question 15 continued

(Total 5 marks)

Leave blank

16. (a) $$y = 5^x + \log_2(x + 1), \qquad 0 \leqslant x \leqslant 2$$

Complete the table below, by giving the value of y when $x = 1$

x	0	0.5	1	1.5	2
y	1	2.821		12.502	26.585

(1)

(b) Use the trapezium rule, with all the values of y from the completed table, to find an approximate value for

$$\int_0^2 (5^x + \log_2(x + 1))\,dx$$

giving your answer to 2 decimal places.

(4)

(c) Use your answer to part (b) to find an approximate value for

$$\int_0^2 (5 + 5^x + \log_2(x + 1))\,dx$$

giving your answer to 2 decimal places.

(1)

Leave blank

Question 16 continued

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Leave blank

Question 16 continued

Leave blank

Question 16 continued

(Total 6 marks)

Leave blank

17. The curve C has equation

$$y = 8 - 2^{x-1}, \qquad 0 \leqslant x \leqslant 4$$

(a) Complete the table below with the value of y corresponding to $x = 1$

x	0	1	2	3	4
y	7.5		6	4	0

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for $\int_0^4 \left(8 - 2^{x-1}\right) \mathrm{d}x$

(3)

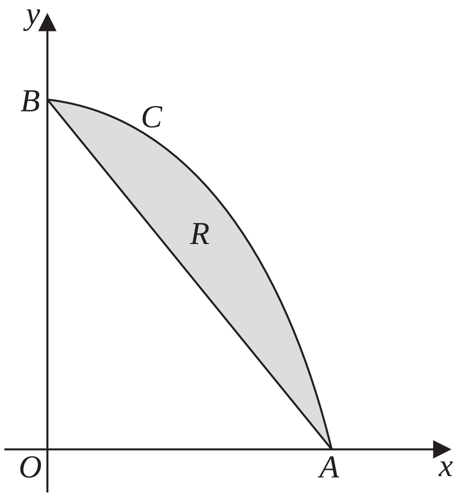

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = 8 - 2^{x-1}$, $\; 0 \leqslant x \leqslant 4$

The curve C meets the x-axis at the point A and meets the y-axis at the point B.

The region R, shown shaded in Figure 1, is bounded by the curve C and the straight line through A and B.

(c) Use your answer to part (b) to find an approximate value for the area of R.

(2)

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Question 17 continued

(Total 6 marks)

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18.

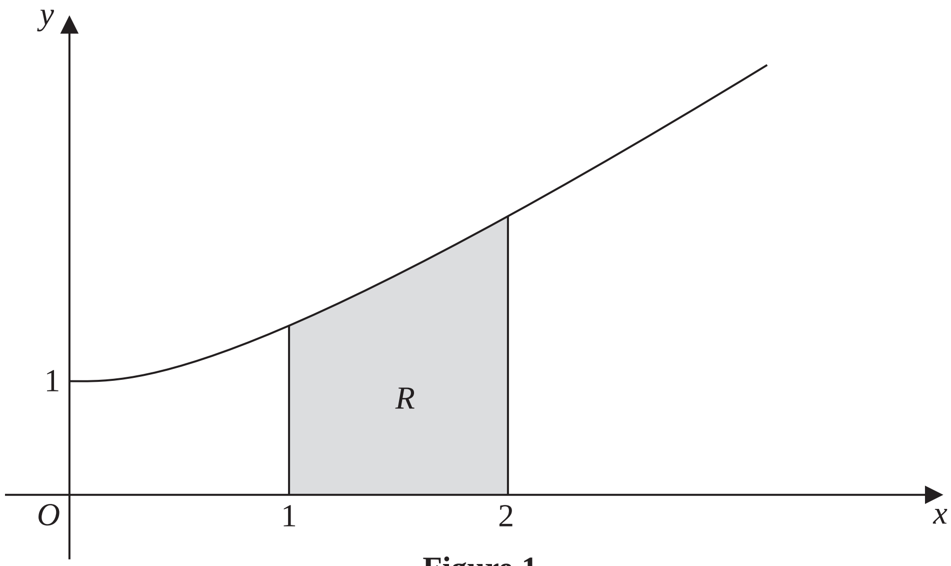

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{(x^2 + 1)}$, $x \geqslant 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$

The table below shows corresponding values for x and y for $y = \sqrt{(x^2 + 1)}$.

x	1	1.25	1.5	1.75	2
y	1.414		1.803	2.016	2.236

(a) Complete the table above, giving the missing value of y to 3 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(Total 5 marks)

Leave blank

19.

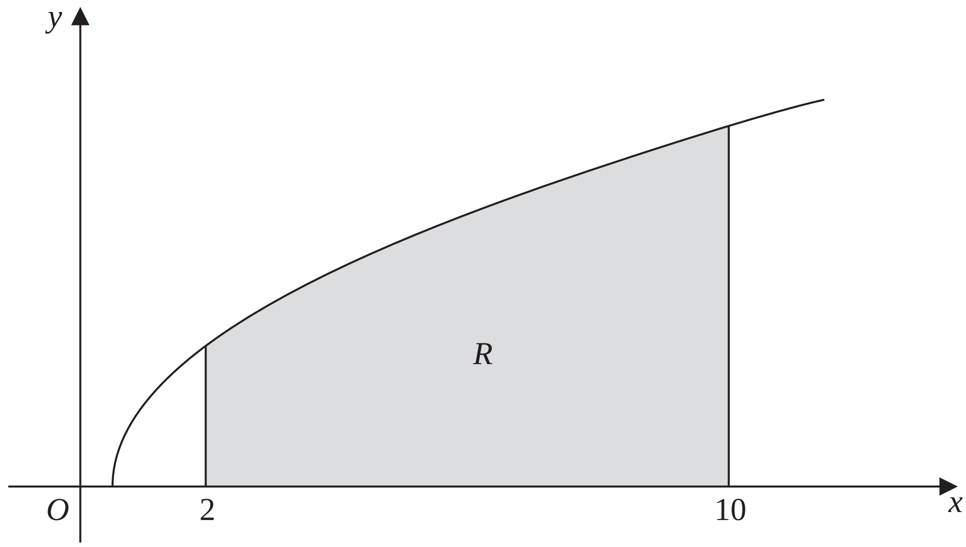

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{(2x - 1)}$, $x \geqslant 0.5$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines with equations $x = 2$ and $x = 10$.

The table below shows corresponding values of x and y for $y = \sqrt{(2x - 1)}$.

x	2	4	6	8	10
y	$\sqrt{3}$		$\sqrt{11}$		$\sqrt{19}$

(a) Complete the table with the values of y corresponding to $x = 4$ and $x = 8$.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 2 decimal places.

(3)

(c) State whether your approximate value in part (b) is an overestimate or an underestimate for the area of R.

(1)

Leave blank

Question 19 continued

(Total 5 marks)

Leave blank

20. $$y = \frac{5}{(x^2 + 1)}$$

(a) Complete the table below, giving the missing value of y to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
y	5	4	2.5		1	0.690	0.5

(1)

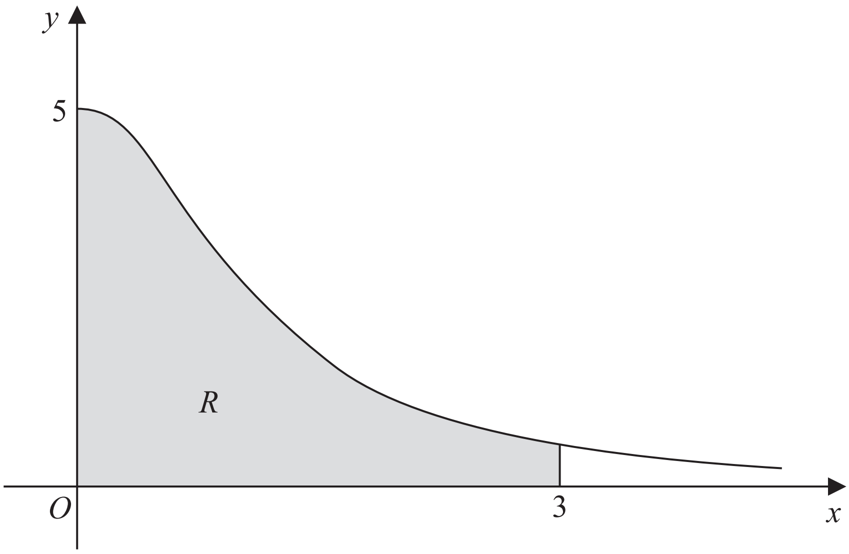

Figure 1

Figure 1 shows the region R which is bounded by the curve with equation $y = \frac{5}{(x^2 + 1)}$, the x-axis and the lines $x = 0$ and $x = 3$

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for the area of R.

(4)

(c) Use your answer to part (b) to find an approximate value for

$$\int_0^3 \left(4 + \frac{5}{(x^2 + 1)}\right) dx$$

giving your answer to 2 decimal places.

(2)

Leave blank

Question 20 continued

(Total 7 marks)

Leave blank

21.
$$y = \frac{x}{\sqrt{(1+x)}}$$

(a) Complete the table below with the value of y corresponding to $x = 1.3$, giving your answer to 4 decimal places.

(1)

x	1	1.1	1.2	1.3	1.4	1.5
y	0.7071	0.7591	0.8090		0.9037	0.9487

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an approximate value for

$$\int_1^{1.5} \frac{x}{\sqrt{(1+x)}}\,\mathrm{d}x$$

giving your answer to 3 decimal places.

You must show clearly each stage of your working.

(4)

(Total 5 marks)

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22.

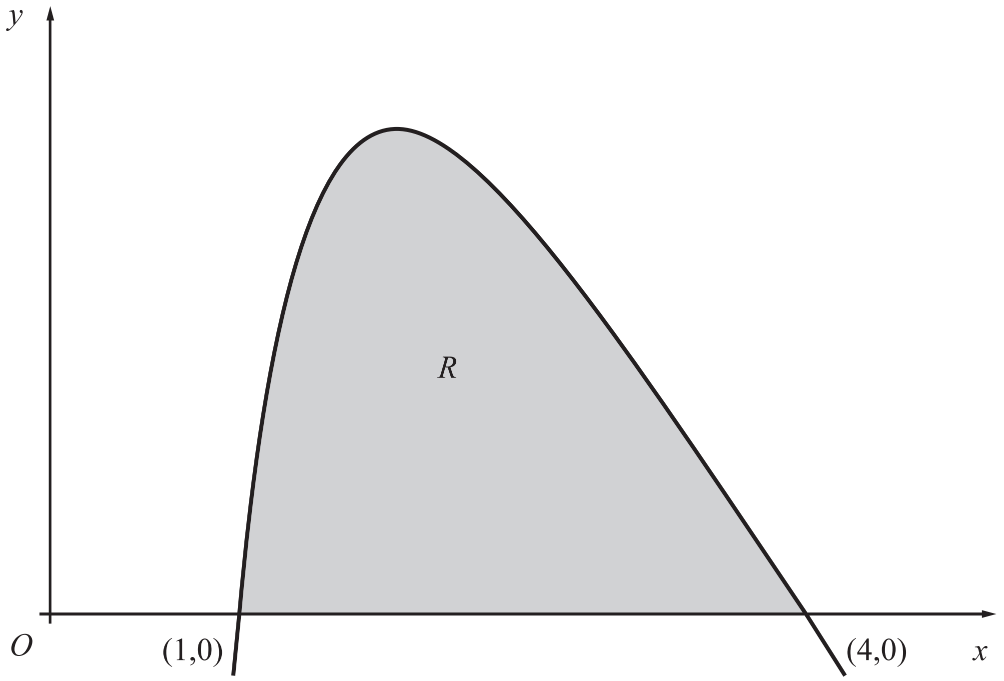

Figure 2

The finite region R, as shown in Figure 2, is bounded by the x-axis and the curve with equation

$$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}, \qquad x > 0$$

The curve crosses the x-axis at the points (1, 0) and (4, 0).

(a) Complete the table below, by giving your values of y to 3 decimal places.

x	1	1.5	2	2.5	3	3.5	4
y	0	5.866		5.210		1.856	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

Leave blank

Question 22 continued

(Total 6 marks)

Leave blank

23. $$y = \sqrt{(3^x + x)}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.25	0.5	0.75	1
y	1	1.251			2

(2)

(b) Use the trapezium rule with all the values of y from your table to find an approximation for the value of $\int_0^1 \sqrt{(3^x + x)}\, dx$

You must show clearly how you obtained your answer.

(4)

(Total 6 marks)

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24.

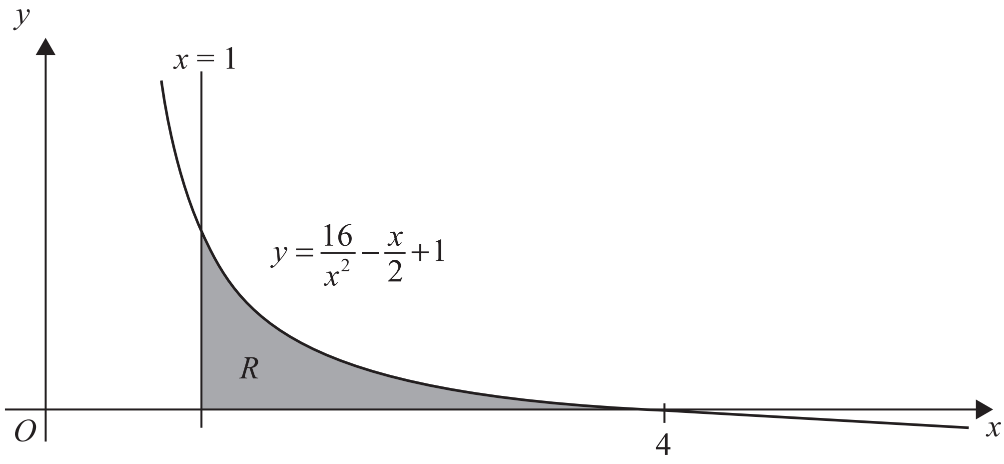


Figure 1

Figure 1 shows the graph of the curve with equation

$$y = \frac{16}{x^2} - \frac{x}{2} + 1, \qquad x > 0$$

The finite region R, bounded by the lines $x = 1$, the x-axis and the curve, is shown shaded in Figure 1. The curve crosses the x-axis at the point $(4, 0)$.

(a) Complete the table with the values of y corresponding to $x = 2$ and 2.5

x	1	1.5	2	2.5	3	3.5	4
y	16.5	7.361			1.278	0.556	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

Leave blank

Question 24 continued

(Total 6 marks)

Leave blank

25. $$y = 3^x + 2x$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	0	0.2	0.4	0.6	0.8	1
y	1	1.65				5

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_0^1 (3^x + 2x)\, dx$.

(4)

(Total 6 marks)

Leave blank

26. (a) Complete the table below, giving values of $\sqrt{(2^x + 1)}$ to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
$\sqrt{(2^x + 1)}$	1.414	1.554	1.732	1.957			3

(2)

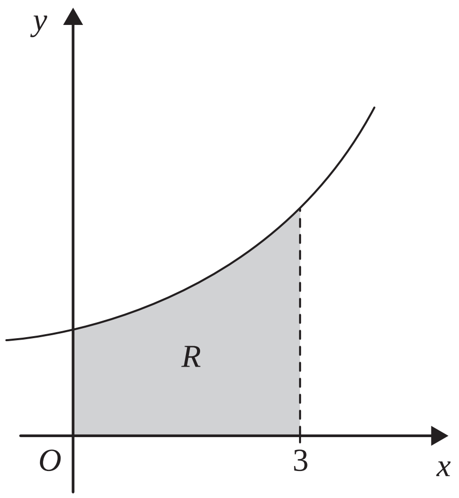


Figure 1

Figure 1 shows the region R which is bounded by the curve with equation $y = \sqrt{(2^x + 1)}$, the x-axis and the lines $x = 0$ and $x = 3$

(b) Use the trapezium rule, with all the values from your table, to find an approximation for the area of R.

(4)

(c) By reference to the curve in Figure 1 state, giving a reason, whether your approximation in part (b) is an overestimate or an underestimate for the area of R.

(2)

Leave blank

Question 26 continued

(Total 8 marks)

Leave blank

27. $$y = \sqrt{(10x - x^2)}.$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	1	1.4	1.8	2.2	2.6	3
y	3	3.47			4.39	

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximation for the value of $\int_1^3 \sqrt{(10x - x^2)}\, dx$.

(4)

(Total 6 marks)

Leave blank

28. $$y = \sqrt{(5^x + 2)}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.5	1	1.5	2
y			2.646	3.630	

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximation for the value of $\int_0^2 \sqrt{(5^x + 2)}\, dx$.

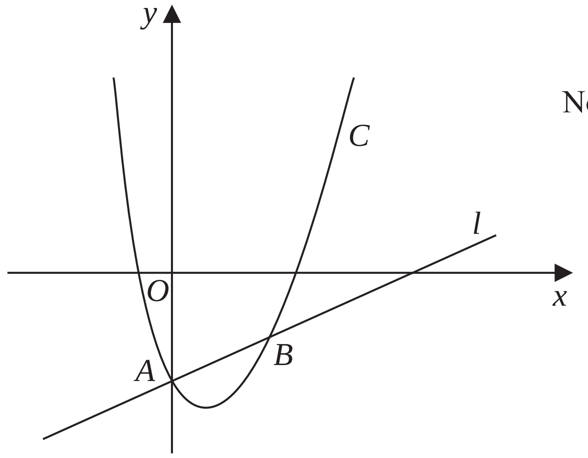
(4)

(Total 6 marks)

EXPERT TUITION

Leave blank

29.



Not to scale

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = e^{-2x} + x^2 - 3$$

The curve C crosses the y-axis at the point A.

The line l is the normal to C at the point A.

(a) Find the equation of l, writing your answer in the form $y = mx + c$, where m and c are constants.

(5)

The line l meets C again at the point B, as shown in Figure 1.

(b) Show that the x coordinate of B is a solution of

$$x = \sqrt{1 + \frac{1}{2}x - e^{-2x}}$$

(2)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{1}{2}x_n - e^{-2x_n}}$$

with $x_1 = 1$

(c) find x_2 and x_3 to 3 decimal places.

(2)

Leave blank

Question 29 continued

Leave blank

Question 29 continued

Leave blank

Question 29 continued

(Total 9 marks)

Leave blank

30.

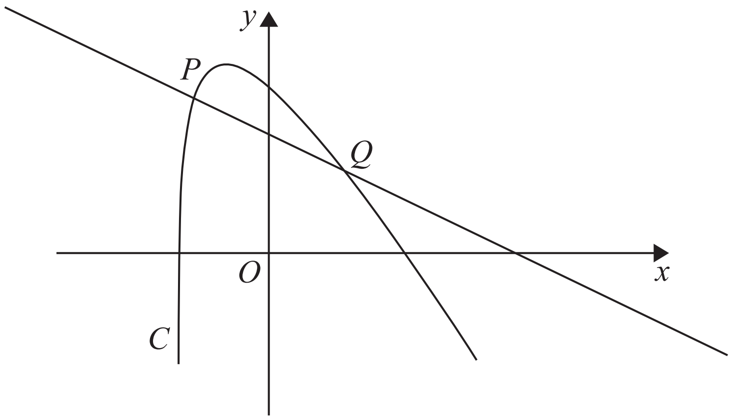


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 2\ln(2x + 5) - \frac{3x}{2}, \qquad x > -2.5$$

The point P with x coordinate -2 lies on C.

(a) Find an equation of the normal to C at P. Write your answer in the form $ax + by = c$, where a, b and c are integers.

(5)

The normal to C at P cuts the curve again at the point Q, as shown in Figure 2.

(b) Show that the x coordinate of Q is a solution of the equation

$$x = \frac{20}{11}\ln(2x + 5) - 2$$

(3)

The iteration formula

$$x_{n+1} = \frac{20}{11}\ln(2x_n + 5) - 2$$

can be used to find an approximation for the x coordinate of Q.

(c) Taking $x_1 = 2$, find the values of x_2 and x_3, giving each answer to 4 decimal places.

(2)

Leave blank

Question 30 continued

Leave blank

Question 30 continued

Leave blank

Question 30 continued

(Total 10 marks)

Leave blank

31.

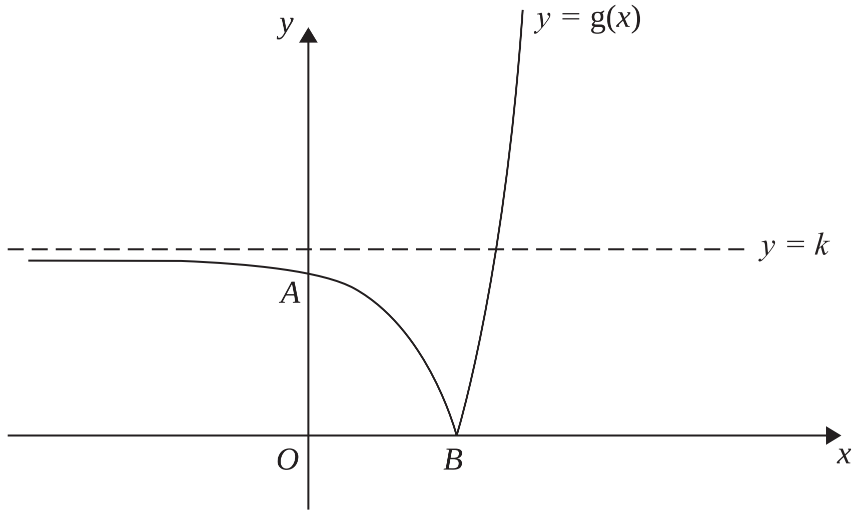


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = |4e^{2x} - 25|, \qquad x \in \mathbb{R}$$

The curve cuts the y-axis at the point A and meets the x-axis at the point B. The curve has an asymptote $y = k$, where k is a constant, as shown in Figure 1

(a) Find, giving each answer in its simplest form,

(i) the y coordinate of the point A,

(ii) the exact x coordinate of the point B,

(iii) the value of the constant k.

(5)

The equation $\mathrm{g}(x) = 2x + 43$ has a positive root at $x = \alpha$

(b) Show that α is a solution of $x = \frac{1}{2}\ln\left(\frac{1}{2}x + 17\right)$

(2)

The iteration formula

$$x_{n+1} = \frac{1}{2}\ln\left(\frac{1}{2}x_n + 17\right)$$

can be used to find an approximation for α

(c) Taking $x_0 = 1.4$ find the values of x_1 and x_2
Give each answer to 4 decimal places.

(2)

(d) By choosing a suitable interval, show that $\alpha = 1.437$ to 3 decimal places.

(2)

Leave blank

Question 31 continued

(Total 11 marks)

Leave blank

32.

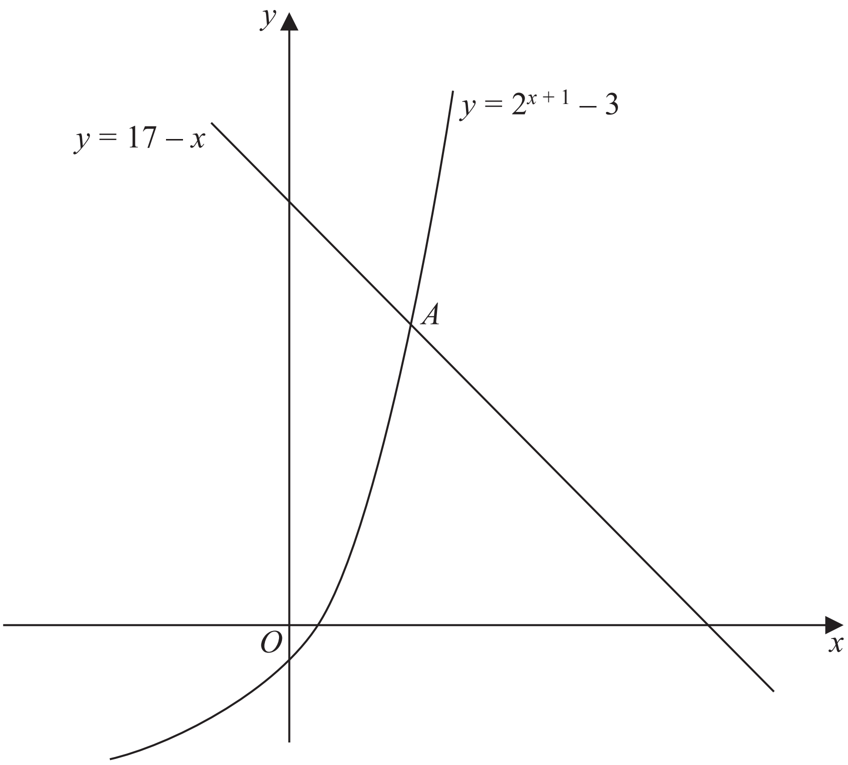


Figure 1

Figure 1 is a sketch showing part of the curve with equation $y = 2^{x+1} - 3$ and part of the line with equation $y = 17 - x$.

The curve and the line intersect at the point A.

(a) Show that the x coordinate of A satisfies the equation

$$x = \frac{\ln(20 - x)}{\ln 2} - 1$$

(3)

(b) Use the iterative formula

$$x_{n+1} = \frac{\ln(20 - x_n)}{\ln 2} - 1, \quad x_0 = 3$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(c) Use your answer to part (b) to deduce the coordinates of the point A, giving your answers to one decimal place.

(2)

Leave blank

Question 32 continued

(Total 8 marks)

Leave blank

33.

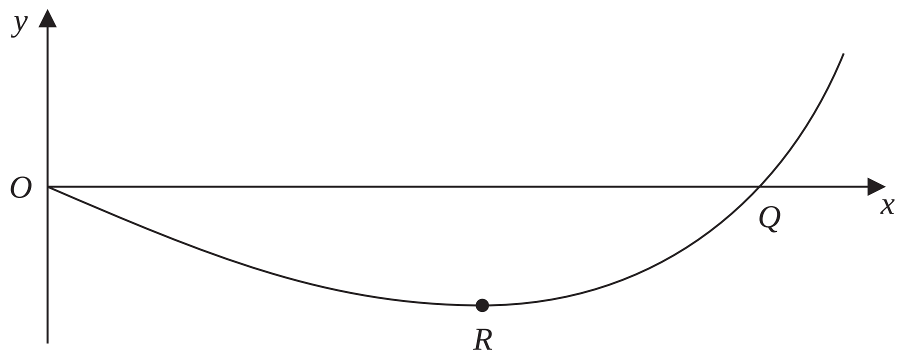


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 2\cos\left(\frac{1}{2}x^2\right) + x^3 - 3x - 2$$

The curve crosses the x-axis at the point Q and has a minimum turning point at R.

(a) Show that the x coordinate of Q lies between 2.1 and 2.2

(2)

(b) Show that the x coordinate of R is a solution of the equation

$$x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$$

(4)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{2}{3}x_n\sin\left(\frac{1}{2}x_n{}^2\right)}, \quad x_0 = 1.3$$

(c) find the values of x_1 and x_2 to 3 decimal places.

(2)

Leave blank

Question 33 continued

(Total 8 marks)

Leave blank

34. A curve C has equation $y = e^{4x} + x^4 + 8x + 5$

(a) Show that the x coordinate of any turning point of C satisfies the equation

$$x^3 = -2 - e^{4x}$$

(3)

(b) On the axes given on page 5, sketch, on a single diagram, the curves with equations

(i) $y = x^3$,

(ii) $y = -2 - e^{4x}$

On your diagram give the coordinates of the points where each curve crosses the y-axis and state the equation of any asymptotes.

(4)

(c) Explain how your diagram illustrates that the equation $x^3 = -2 - e^{4x}$ has only one root.

(1)

The iteration formula

$$x_{n+1} = (-2 - e^{4x_n})^{\frac{1}{3}}, \qquad x_0 = -1$$

can be used to find an approximate value for this root.

(d) Calculate the values of x_1 and x_2, giving your answers to 5 decimal places.

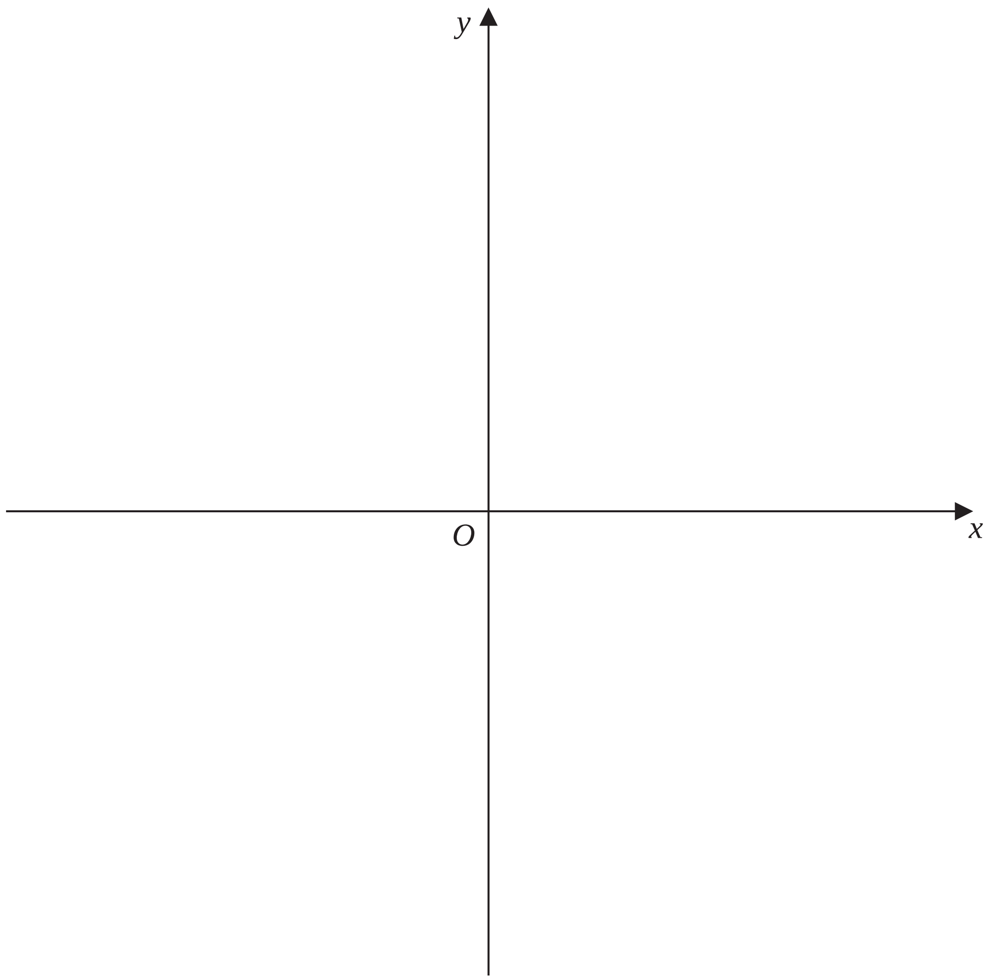
(2)

(e) Hence deduce the coordinates, to 2 decimal places, of the turning point of the curve C.

(2)

Leave blank

Question 34 continued



(Total 12 marks)

EXPERT TUITION

Leave blank

35. $$f(x) = 25x^2e^{2x} - 16, \qquad x \in \mathbb{R}$$

(a) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$.

(5)

(b) Show that the equation $f(x) = 0$ can be written as $x = \pm\frac{4}{5}e^{-x}$

(1)

The equation $f(x) = 0$ has a root α, where $\alpha = 0.5$ to 1 decimal place.

(c) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5}e^{-x_n}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(d) Give an accurate estimate for α to 2 decimal places, and justify your answer.

(2)

Leave blank

Question 35 continued

(Total 11 marks)

Leave blank

36. (a) On the same diagram, sketch and clearly label the graphs with equations

$$y = e^x \quad \text{and} \quad y = 10 - x$$

Show on your sketch the coordinates of each point at which the graphs cut the axes.

(3)

(b) Explain why the equation $e^x - 10 + x = 0$ has only one solution.

(1)

(c) Show that the solution of the equation

$$e^x - 10 + x = 0$$

lies between $x = 2$ and $x = 3$

(2)

(d) Use the iterative formula

$$x_{n+1} = \ln(10 - x_n), \qquad x_1 = 2$$

to calculate the values of x_2, x_3 and x_4.

Give your answers to 4 decimal places.

(3)

Leave blank

Question 36 continued

(Total 9 marks)

37.

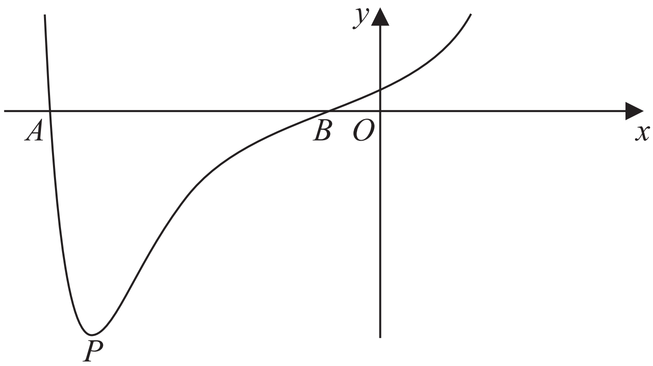


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = (x^2 + 3x + 1)e^{x^2}$$

The curve cuts the x-axis at points A and B as shown in Figure 2.

(a) Calculate the x coordinate of A and the x coordinate of B, giving your answers to 3 decimal places.

(2)

(b) Find $f'(x)$.

(3)

The curve has a minimum turning point at the point P as shown in Figure 2.

(c) Show that the x coordinate of P is the solution of

$$x = -\frac{3(2x^2 + 1)}{2(x^2 + 2)}$$

(3)

(d) Use the iteration formula

$$x_{n+1} = -\frac{3(2x_n^2 + 1)}{2(x_n^2 + 2)}, \qquad \text{with } x_0 = -2.4,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

The x coordinate of P is α.

(e) By choosing a suitable interval, prove that $\alpha = -2.43$ to 2 decimal places.

(2)

Leave blank

Leave blank

Question 37 continued

(Total 13 marks)

Leave blank

38. $$g(x) = e^{x-1} + x - 6$$

(a) Show that the equation $g(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \qquad x < 6$$

(2)

The root of $g(x) = 0$ is α.

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \qquad x_0 = 2$$

is used to find an approximate value for α.

(b) Calculate the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(c) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places.

(3)

(Total 8 marks)

Leave blank

39. $$f(x) = x^3 + 3x^2 + 4x - 12$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \qquad x \neq -3$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, \; n \geqslant 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3.

(3)

The root of $f(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(3)

(Total 9 marks)

Leave blank

40. $$f(x) = x^2 - 3x + 2\cos\left(\tfrac{1}{2}x\right), \quad 0 \leqslant x \leqslant \pi$$

(a) Show that the equation $f(x) = 0$ has a solution in the interval $0.8 < x < 0.9$

(2)

The curve with equation $y = f(x)$ has a minimum point P.

(b) Show that the x-coordinate of P is the solution of the equation

$$x = \frac{3 + \sin\left(\tfrac{1}{2}x\right)}{2}$$

(4)

(c) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin\left(\tfrac{1}{2}x_n\right)}{2}, \quad x_0 = 2$$

find the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(d) By choosing a suitable interval, show that the x-coordinate of P is 1.9078 correct to 4 decimal places.

(3)

Leave blank

Question 40 continued

(Total 12 marks)

Leave blank

41. $$f(x) = 2\sin(x^2) + x - 2, \qquad 0 \leqslant x < 2\pi$$

(a) Show that $f(x) = 0$ has a root α between $x = 0.75$ and $x = 0.85$

(2)

The equation $f(x) = 0$ can be written as $x = \left[\arcsin(1 - 0.5x)\right]^{\frac{1}{2}}$.

(b) Use the iterative formula

$$x_{n+1} = \left[\arcsin(1 - 0.5x_n)\right]^{\frac{1}{2}}, \quad x_0 = 0.8$$

to find the values of x_1, x_2 and x_3, giving your answers to 5 decimal places.

(3)

(c) Show that $\alpha = 0.80157$ is correct to 5 decimal places.

(3)

(Total 8 marks)

Leave blank

42.

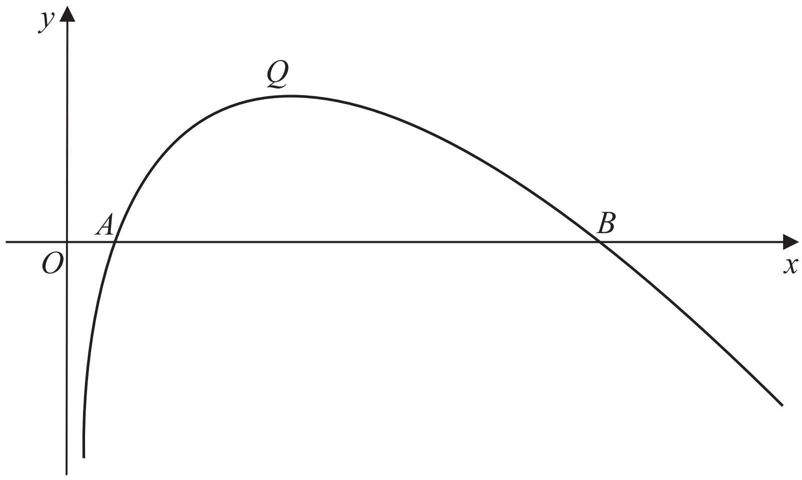


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (8 - x)\ln x, \quad x > 0$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 1.

(a) Write down the coordinates of A and the coordinates of B.

(2)

(b) Find $\mathrm{f}'(x)$.

(3)

(c) Show that the x-coordinate of Q lies between 3.5 and 3.6

(2)

(d) Show that the x-coordinate of Q is the solution of

$$x = \frac{8}{1 + \ln x}$$

(3)

To find an approximation for the x-coordinate of Q, the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(e) Taking $x_0 = 3.55$, find the values of x_1, x_2 and x_3.
Give your answers to 3 decimal places.

(3)

EXPERT TUITION

Leave blank

Question 42 continued

(Total 13 marks)

Leave blank

43. $\mathrm{f}(x) = 4\operatorname{cosec} x - 4x + 1$, where x is in radians.

(a) Show that there is a root α of $\mathrm{f}(x) = 0$ in the interval $[1.2, 1.3]$.

(2)

(b) Show that the equation $\mathrm{f}(x) = 0$ can be written in the form

$$x = \frac{1}{\sin x} + \frac{1}{4}$$

(2)

(c) Use the iterative formula

$$x_{n+1} = \frac{1}{\sin x_n} + \frac{1}{4}, \qquad x_0 = 1.25,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(d) By considering the change of sign of $\mathrm{f}(x)$ in a suitable interval, verify that $\alpha = 1.291$ correct to 3 decimal places.

(2)

Leave blank

Question 43 continued

(Total 9 marks)

EXPERT TUITION

Leave blank

44.

$$f(x) = x^3 + 2x^2 - 3x - 11$$

(a) Show that f(x) = 0 can be rearranged as

$$x = \sqrt{\left(\frac{3x+11}{x+2}\right)}, \quad x \neq -2.$$

(2)

The equation f(x) = 0 has one positive root α.

The iterative formula $x_{n+1} = \sqrt{\left(\frac{3x_n+11}{x_n+2}\right)}$ is used to find an approximation to α.

(b) Taking $x_1 = 0$, find, to 3 decimal places, the values of x_2, x_3 and x_4.

(3)

(c) Show that $\alpha = 2.057$ correct to 3 decimal places.

(3)

(Total 8 marks)

45.

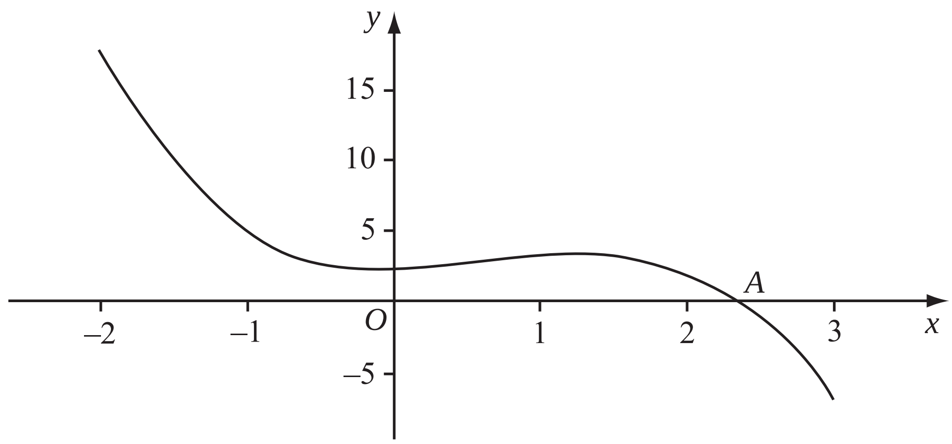


Figure 1

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x-axis at the point A where $x = \alpha$.

To find an approximation to α, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4.
Give your answers to 3 decimal places where appropriate.

(3)

(b) Show that $\alpha = 2.359$ correct to 3 decimal places.

(3)

(Total 6 marks)

Leave blank

EXPERT TUITION

Leave blank

46. $$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P.

(a) Find the exact coordinates of P.

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

(b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3.

(3)

(c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

(Total 11 marks)

Leave blank

47.

$$f(x) = 3x^3 - 2x - 6$$

(a) Show that $f(x) = 0$ has a root, α, between $x = 1.4$ and $x = 1.45$

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(3)

(c) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(d) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)

Leave blank

Question 47 continued

(Total 11 marks)

Leave blank

48. $$f(x) = \ln(x+2) - x + 1, \quad x > -2, x \in \mathbb{R}.$$

(a) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$.

(2)

(b) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \ x_0 = 2.5$$

to calculate the values of x_1, x_2 and x_3 giving your answers to 5 decimal places.

(3)

(c) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

(Total 7 marks)

49.

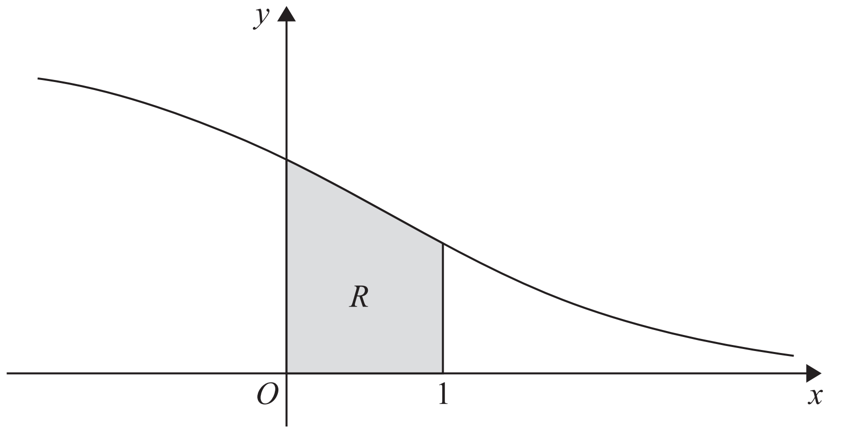


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{6}{(e^x + 2)}$, $x \in \mathbb{R}$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = 1$

The table below shows corresponding values of x and y for $y = \dfrac{6}{(e^x + 2)}$

x	0	0.2	0.4	0.6	0.8	1
y	2		1.71830	1.56981	1.41994	1.27165

(a) Complete the table above by giving the missing value of y to 5 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R, giving your answer to 4 decimal places.

(3)

Leave blank

Leave blank

Question 49 continued

Leave blank

Question 49 continued

EXPERT TUITION

Leave blank

Question 49 continued

(Total 4 marks)

50.

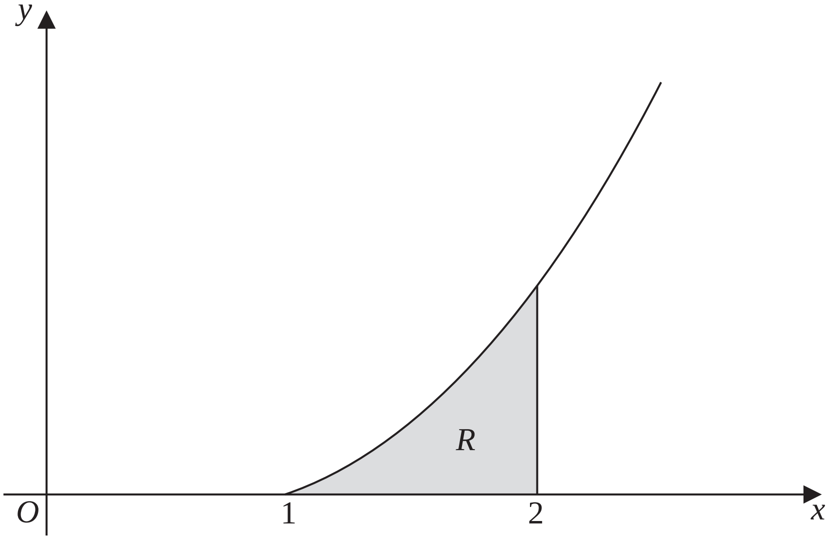


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = x^2 \ln x, \quad x \geqslant 1$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$

The table below shows corresponding values of x and y for $y = x^2 \ln x$

x	1	1.2	1.4	1.6	1.8	2
y	0	0.2625		1.2032	1.9044	2.7726

(a) Complete the table above, giving the missing value of y to 4 decimal places.

(1)

(b) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of R, giving your answer to 3 decimal places.

(3)

Leave blank

Question 50 continued

(Total 4 marks)

Leave blank

51.

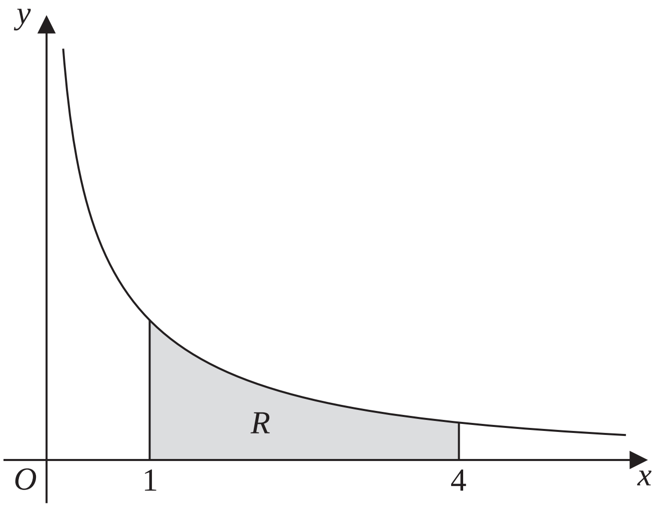


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{10}{2x + 5\sqrt{x}}$, $x > 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, and the lines with equations $x = 1$ and $x = 4$

The table below shows corresponding values of x and y for $y = \dfrac{10}{2x + 5\sqrt{x}}$

x	1	2	3	4
y	1.42857	0.90326		0.55556

(a) Complete the table above by giving the missing value of y to 5 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R, giving your answer to 4 decimal places.

(3)

(c) By reference to the curve in Figure 1, state, giving a reason, whether your estimate in part (b) is an overestimate or an underestimate for the area of R.

(1)

Leave blank

Question 51 continued

(Total 5 marks)

52.

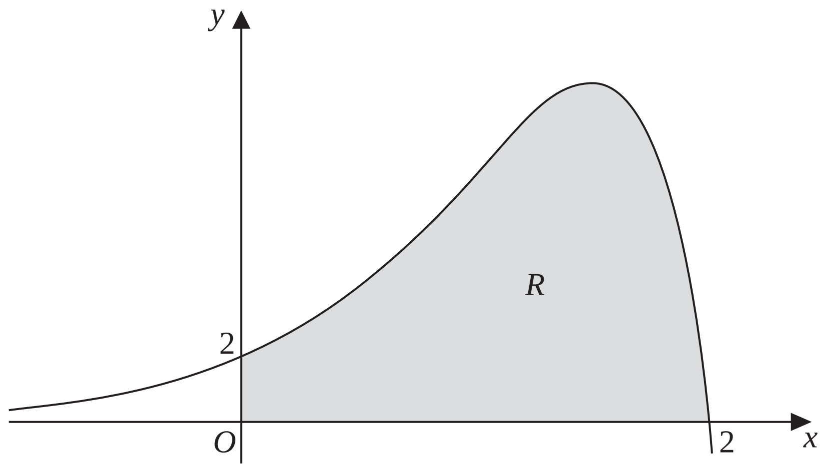


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = (2 - x)e^{2x}, \qquad x \in \mathbb{R}$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the y-axis.

The table below shows corresponding values of x and y for $y = (2 - x)e^{2x}$

x	0	0.5	1	1.5	2
y	2	4.077	7.389	10.043	0

(a) Use the trapezium rule with all the values of y in the table, to obtain an approximation for the area of R, giving your answer to 2 decimal places.

(3)

(b) Explain how the trapezium rule can be used to give a more accurate approximation for the area of R.

(1)

Leave blank

Question 52 continued

(Total 4 marks)

EXPERT TUITION

Leave blank

53.

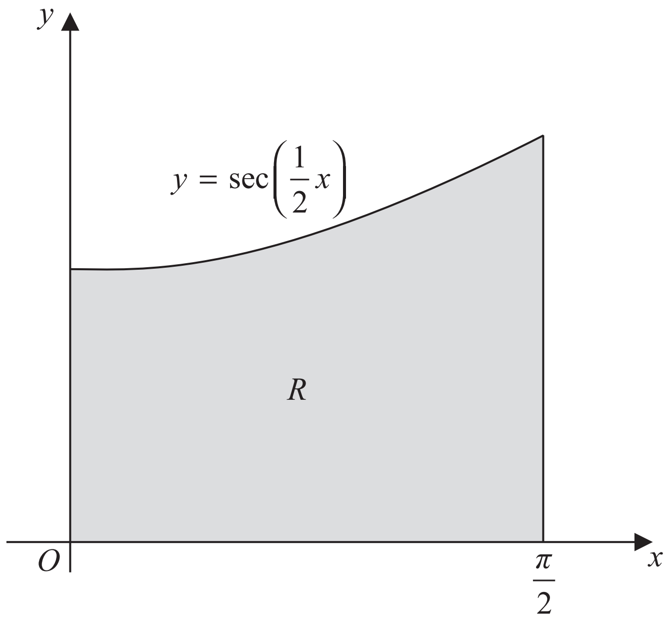


Figure 1

Figure 1 shows the finite region R bounded by the x-axis, the y-axis, the line $x = \frac{\pi}{2}$ and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \quad 0 \leqslant x \leqslant \frac{\pi}{2}$$

The table shows corresponding values of x and y for $y = \sec\left(\frac{1}{2}x\right)$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276		1.414214

(a) Complete the table above giving the missing value of y to 6 decimal places.

(1)

(b) Using the trapezium rule, with all of the values of y from the completed table, find an approximation for the area of R, giving your answer to 4 decimal places.

(3)

Leave blank

Question 53 continued

(Total 4 marks)

54.

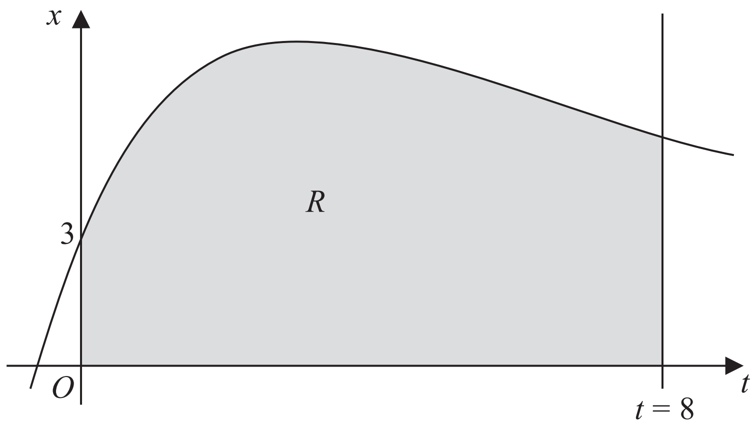


Figure 1

Figure 1 shows part of the curve with equation $x = 4te^{-\frac{1}{3}t} + 3$. The finite region R shown shaded in Figure 1 is bounded by the curve, the x-axis, the t-axis and the line $t = 8$.

(a) Complete the table with the value of x corresponding to $t = 6$, giving your answer to 3 decimal places.

t	0	2	4	6	8
x	3	7.107	7.218		5.223

(1)

(b) Use the trapezium rule with all the values of x in the completed table to obtain an estimate for the area of the region R, giving your answer to 2 decimal places.

(3)

Leave blank

Question 54 continued

(Total 4 marks)

Leave blank

55.

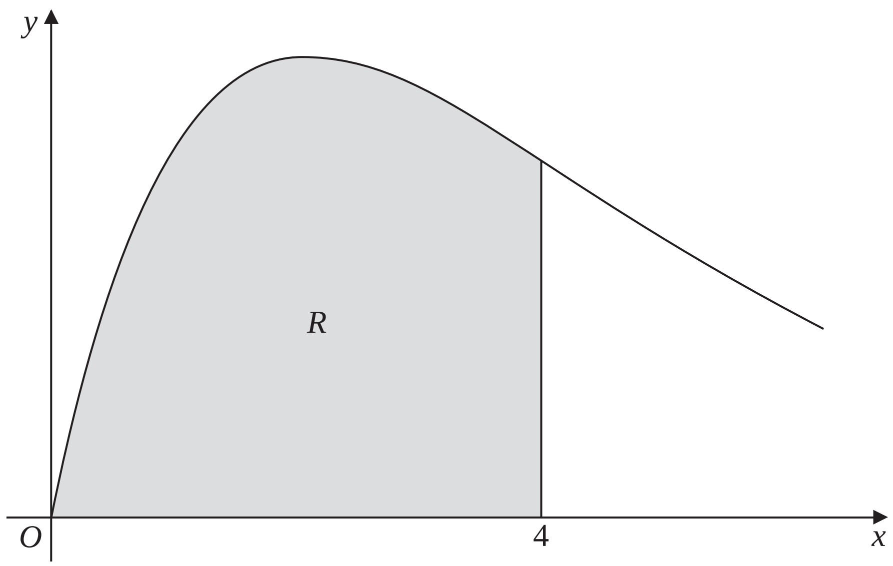


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = x\mathrm{e}^{-\frac{1}{2}x}$, $x \geqslant 0$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, and the line $x = 4$.

The table shows corresponding values of x and y for $y = x\mathrm{e}^{-\frac{1}{2}x}$.

x	0	1	2	3	4
y	0	$\mathrm{e}^{-\frac{1}{2}}$		$3\mathrm{e}^{-\frac{3}{2}}$	$4\mathrm{e}^{-2}$

(a) Complete the table with the value of y corresponding to $x = 2$

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(4)

Leave blank

Question 55 continued

(Total 5 marks)

56.

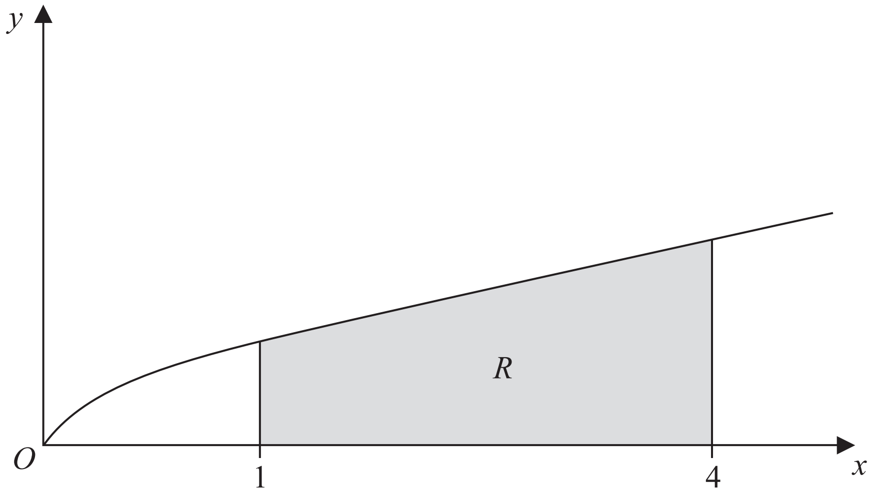


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{x}{1 + \sqrt{x}}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(a) Complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 decimal places.

(1)

x	1	2	3	4
y	0.5	0.8284		1.3333

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R, giving your answer to 3 decimal places.

(3)

Leave blank

Question 56 continued

(Total 4 marks)

57.

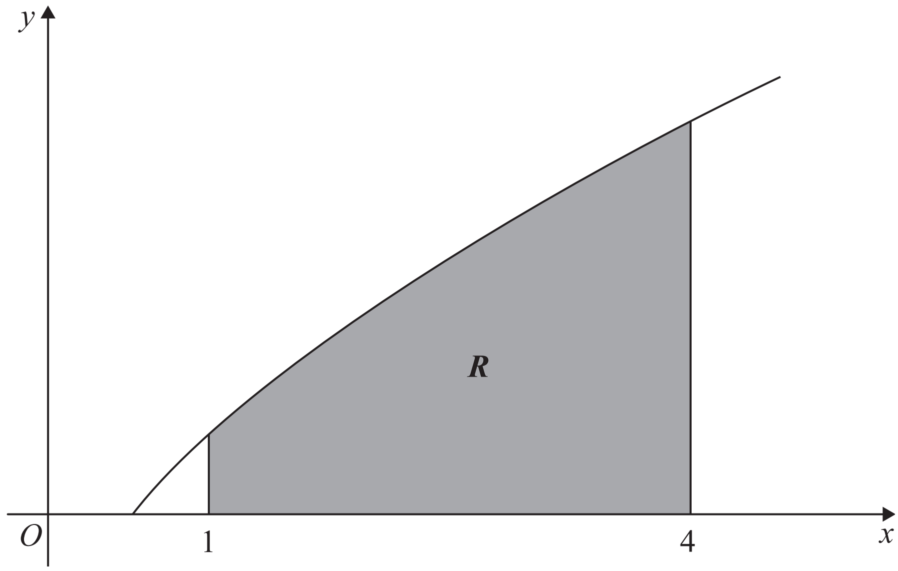


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.

The finite region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$

Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of R, giving your answer to 2 decimal places.

(4)

Leave blank

Question 57 continued

(Total 4 marks)

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58.

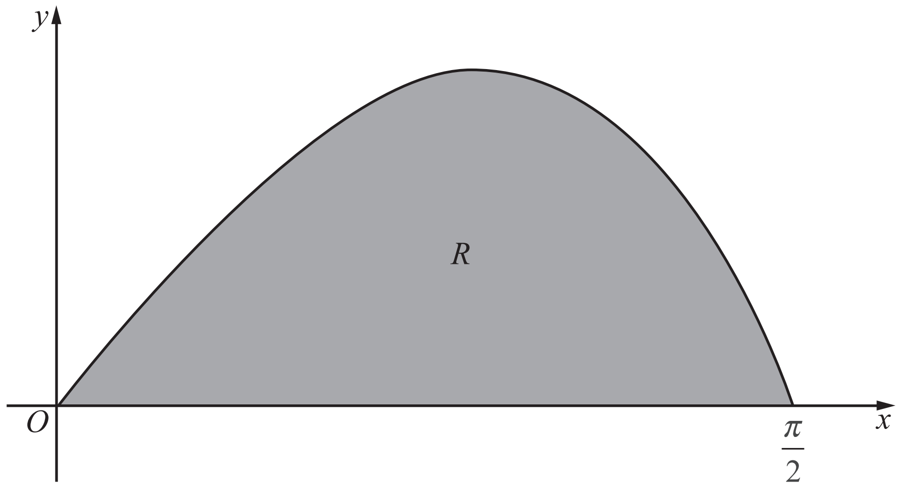

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \dfrac{2\sin 2x}{(1+\cos x)}$, $0 \leqslant x \leqslant \dfrac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \dfrac{2\sin 2x}{(1+\cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(a) Complete the table above giving the missing value of y to 5 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 4 decimal places.

(3)

Leave blank

Question 58 continued

(Total 4 marks)

Leave blank

59.

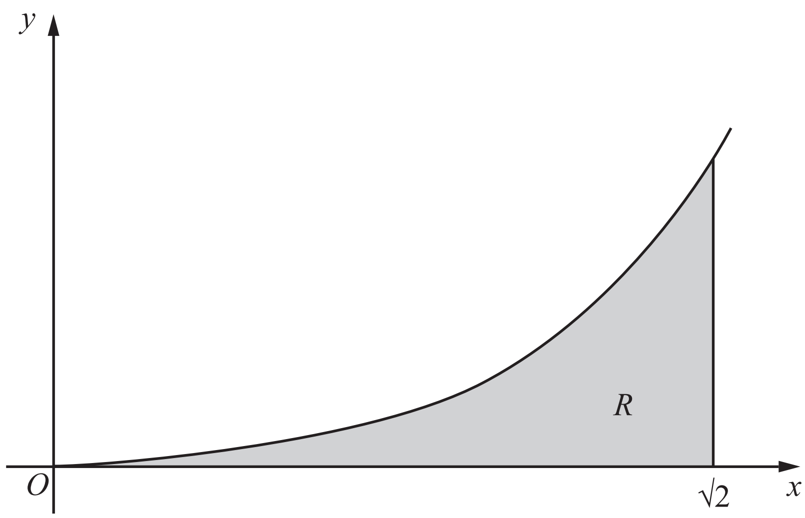

Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2)$, $x \geqslant 0$.
The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln(x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

(a) Complete the table above giving the missing values of y to 4 decimal places.
(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.
(3)

Leave blank

Question 59 continued

(Total 5 marks)

Leave blank

60.

$$I = \int_2^5 \frac{1}{4+\sqrt{(x-1)}}\,\mathrm{d}x$$

(a) Given that $y = \dfrac{1}{4+\sqrt{(x-1)}}$, complete the table below with values of y corresponding to $x = 3$ and $x = 5$. Give your values to 4 decimal places.

x	2	3	4	5
y	0.2		0.1745	

(2)

(b) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I, giving your answer to 3 decimal places.

(4)

Leave blank

Question 60 continued

(Total 6 marks)

Leave blank

61.

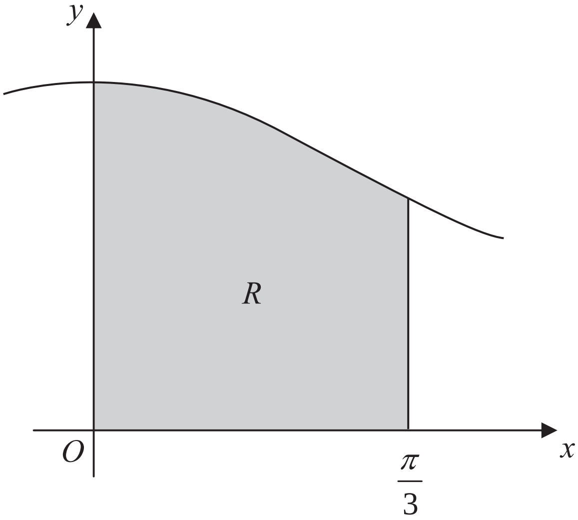

Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(0.75 + \cos^2 x)}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = \frac{\pi}{3}$.

(a) Complete the table with values of y corresponding to $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1.3229	1.2973			1

(2)

(b) Use the trapezium rule

(i) with the values of y at $x = 0$, $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ to find an estimate of the area of R. Give your answer to 3 decimal places.

(ii) with the values of y at $x = 0$, $x = \frac{\pi}{12}$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ to find a further estimate of the area of R. Give your answer to 3 decimal places.

(6)

Leave blank

Question 61 continued

(Total 8 marks)

Leave blank

62.

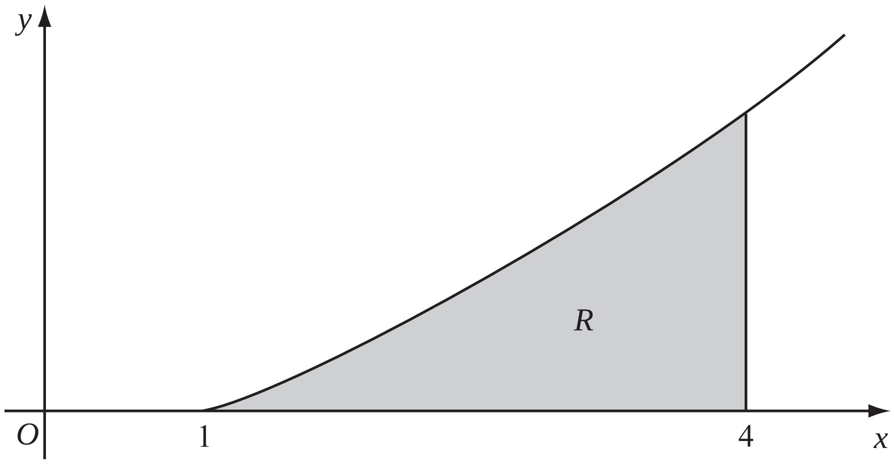

Figure 1

Figure 1 shows a sketch of the curve with equation $y = x \ln x$, $x \geqslant 1$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

(a) Complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places.

(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(4)

Leave blank

Question 62 continued

(Total 6 marks)

Leave blank

63.

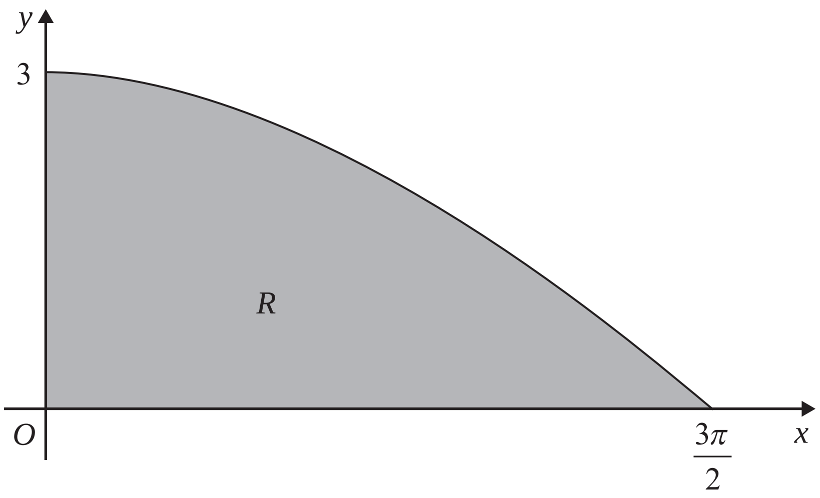

Figure 1

Figure 1 shows the finite region R bounded by the x-axis, the y-axis and the curve with equation $y = 3\cos\left(\frac{x}{3}\right)$, $0 \leqslant x \leqslant \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3\cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

(a) Complete the table above giving the missing value of y to 5 decimal places.

(1)

(b) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R, giving your answer to 3 decimal places.

(4)

Leave blank

Question 63 continued

(Total 5 marks)

Leave blank

64.

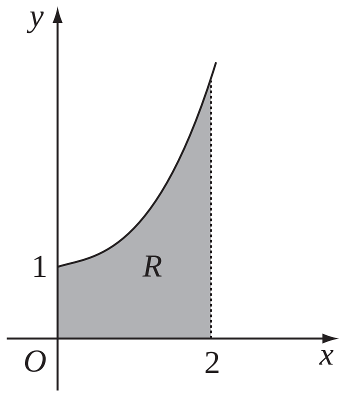

Figure 1

Figure 1 shows part of the curve with equation $y = e^{0.5x^2}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(a) Complete the table with the values of y corresponding to $x = 0.8$ and $x = 1.6$.

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$		$e^{0.72}$		e^2

(1)

(b) Use the trapezium rule with all the values in the table to find an approximate value for the area of R, giving your answer to 4 significant figures.

(3)

(Total 4 marks)

Leave blank

65.

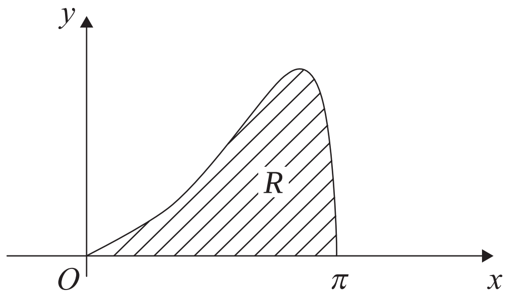

Figure 1

The curve shown in Figure 1 has equation $y = e^x \sqrt{(\sin x)}$, $0 \leqslant x \leqslant \pi$. The finite region R bounded by the curve and the x-axis is shown shaded in Figure 1.

(a) Complete the table below with the values of y corresponding to $x = \frac{\pi}{4}$ and $\frac{\pi}{2}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0			8.87207	0

(2)

(b) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region R. Give your answer to 4 decimal places.

(4)

(Total 6 marks)