EXPERT TUITION

Maths Questions By Topic:

Probability
Mark Scheme

A-Level Edexcel

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Qu	Scheme	Mark	AO
1. (a)(i)	Require $R = 3$ and $G = 4$ so probability is $\frac{3}{4}\times\frac{1}{3}$	M1	2.1
	$= \underline{\frac{1}{4}}$ or **<u>0.25</u>**	A1	1.1b
(ii)	[R must be 2 and $G = 1$so $\frac{1}{4}\times\frac{2}{3}$] = $\underline{\frac{1}{6}}$	A1	1.1b
		(3)	
(b)	P($X = 50$) = 0.25 must mean $R = 3$ and $G = 4$	M1	3.1a
	so $3m + 4n = 50$	A1	1.1b
	P($X = 20$) = $\frac{1}{6}$ $\Rightarrow$ $R = 2$, $G = 1$ so $2m + n = 20$	A1	2.1
	Solving: $3m + 4(20 - 2m) = 50$ (o.e.)	M1	1.1b
	<u>***m* = 6**</u> and <u>***n* = 8**</u>	A1	3.2a
		(5)	
		(8 marks)	

	Notes
(a)(i)	M1 for sight of $\frac{3}{4}\times\frac{1}{3}$ or $\frac{1}{4}\times\frac{2}{3}$ as a single product BUT allow e.g. $\frac{3}{4}\times\frac{1}{3}+\frac{1}{3}\times\frac{3}{4}$ to score M1 However if the products are later added e.g. $\frac{3}{4}\times\frac{1}{3}+\frac{1}{4}\times\frac{2}{3}$ it is M0 May be implied by one correct answer to (i) or (ii)
	A1 for $\frac{1}{4}$ or 0.25 or exact equivalent (allow 25%)
(ii)	A1 for $\frac{1}{6}$ or exact equivalent
(b)	**For the 1st 4 marks condone incorrect labelling e.g. *R* for *m* or *G* for *n* if intention is clear**
	1st M1 for identifying either set of cases ($R = 2$, $G = 1$, $X = 20$) or ($R = 3$, $G = 4$, $X = 50$) Allow 1st M1 for P($X = 20$) = $\frac{1}{4}\times\frac{2}{3}$ **or** P($X = 50$) = $\frac{3}{4}\times\frac{1}{3}$ **NOT** just P($X = 20$) = $\frac{1}{6}$ etc
	<u>or</u> $\frac{1}{4}m+\frac{2}{3}n=20$ **or** $\frac{3}{4}m+\frac{1}{3}n=50$ and might score 2nd M1 (answer is $m = 64$, $n = 6$)
	<u>or</u> $\frac{1}{4}m+\frac{2}{3}n=\frac{1}{6}$ **or** $\frac{3}{4}m+\frac{1}{3}n=\frac{1}{4}$ and might score 2nd M1 (answer is $m = \frac{4}{15}$, $n = \frac{3}{20}$)
	<u>or</u> $2m+n=\frac{1}{6}$ **or** $3m+4n=\frac{1}{4}$ and might score 2nd M1 (answer is $m = \frac{1}{12}$, $n = 0$)
	<u>or</u> $2m+n=50$ **and** $3m+4n=20$ and might score 2nd M1 (answer is $m = 36$, $n = -22$)
	1st A1 for one correct equation
	2nd A1 for **both** correct equations and no incorrect equations, unless they attempt to solve the correct 2 equations only
	2nd M1 for attempt to solve <u>their</u> two linear equations in *m* and *n* (reduce to an equation in one variable, condone one sign error). May be implied by $m = 6$ and $n = 8$.
Calc	If they use one of the 4 sets of equations for 1st M1 and use a calculator to write down the answer, we will allow this mark for sight of the correct answers to those equations as given above.
	3rd A1 $m = 6$ and $n = 8$ only (no incorrect labelling here) Correct answer by trial can score 5/5 if no incorrect working seen.

Qu	Scheme	Marks	AO
2 (a)	[$p = 1 - (0.2 + 0.2 + 0.1 + 0.2)] =$ **<u>0.3</u>**	B1 **(1)**	1.1b
(b)	A and C are mutually exclusive. [NOT P(A) and P(C)]	B1 **(1)**	1.2
		(2 marks)	

	Notes
(a)	B1 for
(b)	B1 for A and C [NB $A \cap C$ or $A \cap C = \varnothing$ is B0] If more than one case given they must <u>all</u> be correct e.g. $A \cap B$ and C

Qu	Scheme	Marks	AO
3	Must end up with 3 of each colour or 4 of each colour	M1	3.1b
	***n* = 2** requires 1st red and 2nd green or red from **A** and green from **B**	M1	2.2a
	P(1st red and 2nd green) $= \frac{4}{9}\times\frac{1}{10}=\frac{4}{90}$ or $\frac{2}{45}$ $\quad \underline{\underline{p = \frac{2}{45}}}$	A1	1.1b
	***n* = 5** requires 1st green and 2nd yellow or green from **A** and yellow from **B**	M1	2.2a
	P(1st green and 2nd yellow) $= \frac{5}{12}\times\frac{3}{10}=\frac{15}{120}$ or $\frac{1}{8}$ $\quad \underline{\underline{p = \frac{1}{8}}}$	A1	1.1b
		(5)	
		(5 marks)	

	Notes
	1st M1 for an overall strategy realising there are 2 options. Award when evidence of both cases (3 of each colour or 4 of each colour) seen.
	2nd M1 for $n = 2$ and attempt at 1st red and 2nd green May be implied by e.g. $\frac{4}{9}\times\frac{1}{9}$
	1st A1 for $p = \underline{\underline{\frac{2}{45}}}$ or exact equivalent
	3rd M1 for $n = 5$ and attempt at 1st green and 2nd yellow May be implied by e.g. $\frac{5}{12}\times\frac{3}{9}$
	2nd A1 for $p = \underline{\underline{\frac{1}{8}}}$ or exact equivalent
NB	If both correct values of p are found and then added (get $\frac{61}{360}$) , deduct final A1 only (i.e. 4/5)

Question	Scheme	Marks	AOs
4	Overall method	M1	2.1
	$a+b=2c+0.5$ oe **or** $a+b=2(1-a-b)$	B1	2.2a
	$a+b+c=0.75$ oe	B1	1.1b
	$3c=0.25 \quad \left[c=0.0833... \text{ or } \frac{1}{12}\right]$	M1	1.1b
	P(scoring 2,4 or 4,2 or 3,3) = $2\times"\frac{1}{12}"\times 0.15+0.1^2$	M1	3.1b
	$=0.035$ oe	A1cso	1.1b
		(6)	
			(6 marks)

Notes		
4	**M1:**	A fully correct method with all the required steps. For gaining 2 correct equations with at least one correct(allow if unsimplified). Attempting to solve to find a value of c followed by **correct method** to find the probability
	B1:	Forming a correct equation from the information given in the question
	B1:	A correct equation using the sum of the probabilities equals 1
	M1:	Correct method for solving 2 equations to find c Implied by $c=\frac{1}{12}$
	M1:	Recognising the ways to get a total of 6. Condone missing arrangments or repeats. Do not ignore extras written unless ignored in the calculation. May be implied by $m\times"\frac{1}{12}"\times 0.15+n\times 0.1^2$ where m and n are positive integers
	A1cso:	Cao 0.035, $\frac{7}{200}$ oe

Question	Scheme	Marks	AOs
5	$x = 0$	B1	2.2a
	$\text{P}(A) = 0.1 + z + y \quad \text{P}(C) = 0.39 + z[+x] \quad \text{P}(A \text{ and } C) = z$	M1	2.1
	$\text{P}(A \text{ and } C) = \text{P}(A) \times \text{P}(C) \rightarrow z = (0.1 + z + y) \times (0.39 + z[+x])$	M1	1.1b
	$\left[\sum p = 1\right]$ $0.06 + 0.3 + 0.39 + 0.1 + z + y[+x] = 1 \rightarrow \quad [z + y[+x] = 0.15]$	M1	1.1b
	Solving (simultaneously) leading to $\underline{z = 0.13} \quad \underline{y = 0.02}$	A1	1.1b
			(5 marks)

Notes

B1: for $x = 0$, may be seen on Venn diagram

M1: Identifying the probabilities required for independence and at least 2 correct
These must be labelled
If there are no labels, then this may be implied by $z = (0.1 + z + y)(0.39 + z\ [+x])$, allow one numerical slip

Allow e.g.
$\text{P}(A') = 0.39 + 0.30 + 0.06[+x] \quad \text{P}(C) = 0.39 + z[+x] \quad \text{P}(A' \text{ and } C) = 0.39$
[Not on spec. but you may see use of conditional probabilities]

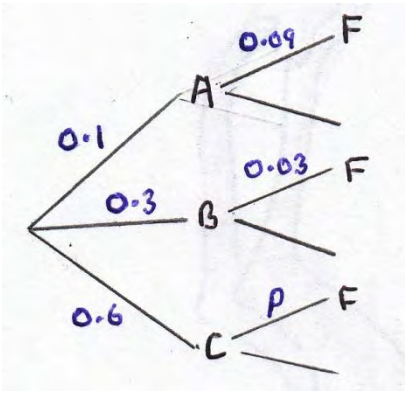
M1: Use of independence equation with their labelled probabilities in terms y, z [and x]
All their probabilities must be substituted into a correct formula
Sight of a correct equation e.g. $z = (0.1 + z + y)(0.39 + z\ [+x])$ scores M1M1

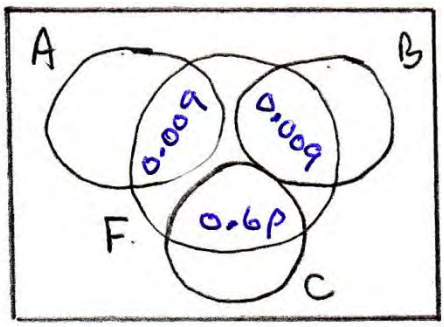
M1: Using $\Sigma p = 1$
Implied by $[x +]\ y + z = 0.15$
or their $x + y + z = 0.15$ where x, y, and z are all probabilities
or e.g. $\text{P}(A) = 0.25$

A1: both $y = 0.02$ and $z = 0.13$

Qu	Scheme	Marks	AO
6 (a)	[Let $p = \text{P}(F \mid C)$]		
	Tree diagram or some other method to find an equation for p	M1	2.1
	$0.1\times0.09+0.3\times0.03+0.6\times p=0.06$	A1	1.1b
	$p = 0.07$ i.e. **7%**	A1	1.1b
		(3)	
(b)	e.g. $\text{P}(B \text{ and } F) = 0.3\times0.03 = 0.009$ but		
	$\text{P}(B)\times\text{P}(F) = 0.3\times0.06 = 0.018$	B1	2.4
	These are not equal so not independent		
		(1)	
		(4 marks)	

	Notes
(a)	M1 for selecting a suitable method to find the missing probability e.g. sight of tree diagram with 0.1, 0.3, 0.6 and 0.09, 0.03, p suitably placed e.g. sight of VD with 0.009 for $A\cap F$ and $B\cap F$ and $0.6p$ suitably placed or attempt an equation with at least one correct numerical and one "p" product (not necessarily correct) on LHS or for sight of $0.06 - (0.009 + 0.009)$ (o.e. e.g. $6 - 1.8 = 4.2\%$) 1st A1 for a correct equation for p (May be implied by a correct answer) or for the expression $\dfrac{0.06-(0.009+0.009)}{0.6}$ (o.e.) 2nd A1 for 7% (accept 0.07) **Correct Ans:** Provided there is no incorrect working seen award 3/3 e.g. may just see tree diagram with 0.07 for p (probably from trial and improv')
(b)	B1 for a suitable explanation…may talk about 2nd branches on tree diagram and point out that $0.03 \neq 0.06$ but need some supporting calculation/words Can condone incorrect use of set notation (it is not on AS spec) provided the rest of the calculations and words are correct.





Question	Scheme	Marks	AOs
7(a)	S and A since there is no intersection between A and S or the probability of S and A happening is zero	B1	1.2
		(1)	
(b)	$(0.1+p)'0.25=0.1$ $[p=0.3]$	M1	3.1b
	$q=0.15$ or $1-q=0.85$	M1	1.1b
	$r=1-"p"-"q"-0.25$	M1dd	3.1b
	$=0.3$	A1	1.1b
		(4)	
(c)	Independent since $0.25\times"0.2"=0.05$	B1	2.2a
		(1)	
(d)	The teacher's belief would appear not to be justified as D and S are independent	B1ft	2.4
		(1)	
			(7 marks)

Notes:

(a) B1: For S and A and a sensible reason

(b) M1: For forming a correct equation in terms of p using the information given.

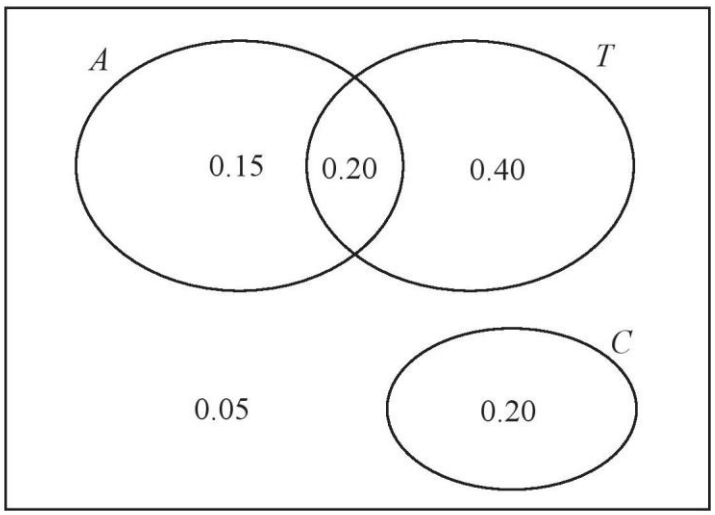
M1: Writing or using $q=0.15$ or $1-q=0.85$

M1dd: dependent on both previous M marks being awarded. For using their values for p and q to form a correct equation to enable them to find r

A1: cao

(c) B1: Yes and a suitable reason to support their answer bringing together the two pieces of information to draw the correct conclusion

(d) B1: A correct comment following their answer to part (c) with reference to the teachers belief.

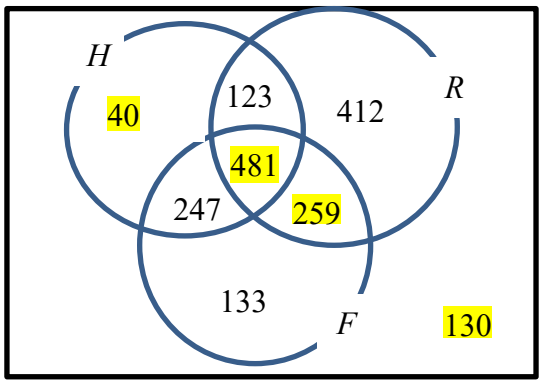
Question	Scheme	Marks	AOs
8(a)	$p = [1 - 0.75 - 0.05 =]$ **<u>0.20</u>**	B1	1.1b
		(1)	
(b)	$q =$ **<u>0.15</u>**	B1ft	1.1b
	P(A) = 0.35 P(T) = 0.6 P(A and T) = 0.20 P(A) × P(T) = 0.21	M1	2.1
	Since 0.20 ≠ 0.21 therefore A and T are **not** independent	A1	2.4
		(3)	
			
(c)	P(not [A or C]) = **<u>0.45</u>**	B1	1.1b
		(1)	
			(5 marks)

Notes:

(a)
B1: cao for $p = 0.20$

(b)
B1: Ft for use of their p and P(A or T) to find q i.e. 0.75 – "p" – 0.40 **or** $q = 0.15$
M1: For the statement of all probabilities required for a suitable test and sight of any appropriate calculations required

(c)
A1: All probabilities correct, correct comparison and suitable comment
B1: cao for 0.45

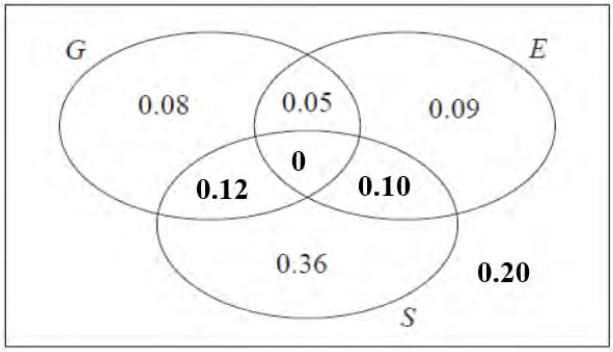
Question	Scheme	Marks	AOs
9(a)	$\frac{365}{1825}$ or $\frac{1}{5}$ or 0.2 oe	B1	1.1b
		(1)	
(b)	$\frac{170}{1825}$ or $\frac{34}{365}$ or awrt 0.093	B1	1.1b
		(1)	
(c)	$90\times0.4+80\times0.05[=40]$ or $90\times0.6+80\times0.95[=130]$ or $740\times0.65[=481]$ or $740\times0.35[=259]$ 	M1 B1 B1 A1	3.1b 1.1b 1.1b 1.1b
		(4)	
(d)	$P(R'\cap F)=\frac{380}{1825}\left[=\frac{76}{365}=0.208...\right]$ oe awrt 0.208	B1	1.1b
		(1)	
(e)	$\left[\frac{133\ +\ "130"}{1825}=\right]\frac{"263"}{1825}$ awrt 0.144	B1ft	1.1b
		(1)	
(f)	$\frac{247+"481"}{247+"481"+123+"40"}$	M1	3.4
	$=\frac{728}{891}$ awrt 0.817	A1	1.1b
		(2)	

Notes: **(10 marks)**

		Look out for answers given in the question. If you see answers in the question and in the answer space those in the answer space take precedence.
(a)	**B1**	Allow equivalent
(b)	**B1**	Allow equivalent
(c)	**M1**	Correct method to find one of the values 40 or 130 or 481or 259 Implied by 40, 481, 259 or130 seen in correct place on diagram
	B1	One of the highlighted correct
	B1	A second value highlighted correct or their $("259"+"\ 481")=740$ or their $("40"+"\ 481")=521$ or their $("40"+"130")=170$
	A1	Fully correct
(d)	**B1**	380/1825oe or awrt 0.208
(e)	**B1ft**	Correct answer or Ft their 130 (> 0) do not allow if blank Allow ft correct to 3 sf.
(f)	**M1**	For a single fraction with the numerator < denominator and *n* is an integer we will award for *n*/ 891 **or** *n*/(sum of their 4 values in *H*, each > 0) or awrt 0.817
	A1	728/891 oe or awrt 0.817

Qu 10	Scheme	Marks	AO
(a)	$0.08 + 0.09 + 0.36 =$ **0.53**	B1 **(1)**	1.1b
(b)(i)	$\left[\mathrm{P}(G \cap E \cap S) = 0 \Rightarrow\right]$ **$p = 0$**	B1	1.1b
(ii)	$[\mathrm{P}(G) = 0.25 \Rightarrow]\ 0.08 + 0.05 + q + "p" = 0.25$	M1	1.1b
	$q = 0.12$	A1 **(3)**	1.1b
(c)(i)	$\left[\mathrm{P}(S \mid E) = \frac{5}{12} \Rightarrow\right] \frac{r + "p"}{r + "p" + 0.09 + 0.05} = \frac{5}{12}$	M1	3.1a
		A1ft	1.1b
	$[12r = 5r + 5 \times 0.14 \Rightarrow]$ **$r = 0.10$**	A1	1.1b
(ii)	$[0.08 + 0.05 + "0.12" + "0" + 0.09 + "0.10" + 0.36 + t = 1 \Rightarrow]$ **$t = 0.20$**	B1ft **(4)**	1.1b
(d)	$\mathrm{P}(S \cap E') =$ 0.36 + "q" [= 0.48]	B1ft	1.1b
	$\mathrm{P}\left(\left[(S \cap E')\right] \cap G\right) =$ "q"[=0.12] **and** P(G) = 0.25 **and**		
	$\mathrm{P}(S \cap E') \times \mathrm{P}(G) =$ "0.48" $\times \frac{1}{4}$ or 0.12	M1	2.1
	$\mathrm{P}(S \cap E') \times \mathrm{P}(G) = 0.12 = \mathrm{P}\left(\left[(S \cap E')\right] \cap G\right)$ so **are independent**	A1 **(3)**	2.2a
		(11 marks)	

	Notes
(a)	B1 for 0.53 (or exact equivalent) [Allow 53%]
(b)(i)	B1 for $p = 0$ (may be placed in Venn diagram)
(ii)	M1 for a linear equation for q (ft letter "p" or their value if $0 \leqslant p \leqslant 0.12$) $\Rightarrow$ by $p + q = 0.12$
	A1 for $q = 0.12$ (may be placed in Venn diagram)
(c)(i)	M1 for a ratio of probabilities (r on num and den) (on LHS) with num < den **and** num or den correct ft. Allow ft of letter "p" or their p where $0 \leqslant p < 0.86$ but "+ 0" is not required.
	1st A1ft for a correct ratio of probabilities (on LHS) allowing ft of their p where $0 \leqslant p < 0.86$
	2nd A1 for $r = 0.1(0)$ or exact equivalent (may be in Venn diagram) **Ans only** 3/3
(ii)	B1ft for $t = 0.2(0)$ (o.e.) or correct ft i.e. $0.42 - (p + q + r)$ where p, q, r and t are all probs
(d)	B1ft for $\mathrm{P}(S \cap E') = 0.48$ (with label) (ft letter "q" or their value if $0 \leqslant q \leqslant 0.12$)
	M1 for attempting all required probs (labelled) and using them in a correct test (allow ft of q)
	A1 for **all probs correct** and a correct deduction (no ft deduction here)
SC	**No "P"** If correct argument seen apart from P for probability for all 3 marks, award (B0M1A1)
	If unsure about an attempt using conditional probabilities, please send to review.



Qu 11	Scheme	Marks	AO
(a)	*A*, *C* or *D*, *B* or *D*,*C*	B1 **(1)**	1.2
(b)	[$p = 0.4 - 0.07 - 0.24$ =] **0.09**	B1 **(1)**	1.1b
(c)	*A* and *B* independent implies		1.1b
	$P(A)\times 0.4 = 0.24$ or $(q + 0.16 + 0.24)\times 0.4 = 0.24$	M1	
	so P(*A*) = 0.6 and *q* = **0.20**	A1cso **(2)**	1.1b
(d)(i)	$P(B' \mid C) = 0.64$ gives $\frac{r}{r+p} = 0.64$ or $\frac{r}{r+"0.09"} = 0.64$	M1	3.1a
	$r = 0.64r + 0.64$ "p" so $0.36r = 0.0576$ so r = **0.16**	A1	1.1b
(ii)	Using sum of probabilities = 1 e.g. "0.6" + 0.07 + "0.25" + s =1	M1	1.1b
	so s = **0.08**	A1 **(4)**	1.1b
		(8 marks)	

	Notes
(a)	B1 for one correct pair. If more than one pair they must all be correct. Condone in a correct probability statement such as $P(A \cap C) = 0$ or correct use of set notation e.g. $A \cap C = \varnothing$ BUT e.g. "P(*A*) and P(*C*) are mutually exclusive" alone is B0
(b)	B1 for $p = 0.09$ (Maybe stated in Venn Diagram [VD]) [If values in VD and text conflict, take text or a value used in a later part]
(c)	M1 for a correct equation in one variable for P(*A*) or q using independence or for seeing **both** $P(A \cap B) = P(A) \times P(B)$ and $0.24 = 0.6 \times 0.4$
	A1cso for $q = 0.20$ or exact equivalent (dep on correct use of independence)
Beware	Use of P(*A*) = 1 – P(*B*) = 0.6 leading to $q = 0.2$ scores M0A0
(d)(i)	1st M1 for use of $P(B' \mid C) = 0.64$ leading to a correct equation in r and possibly p. Can ft their p provided $0 < p < 1$
	1st A1 for $r = 0.16$ or exact equivalent
(ii)	2nd M1 for use of total probability = 1 to form a linear equation in s. Allow p, q, r etc Can follow through their values provided each of p, q, r are in [0, 1)
	2nd A1 for $s = 0.08$ or exact equivalent

Qu 12	Scheme	Marks	AO
(a)	$\frac{k}{10}+\frac{k}{20}+\frac{k}{30}+\frac{k}{40}+\frac{k}{50}=1$ or $\frac{1}{600}\left(60k+30k+20k+15k+12k\right)=1$	M1	1.1b
	So $k=\frac{600}{137}$ (*)	A1cso	1.1b
		(2)	
(b)	(Cases are:) $D_1=30, D_2=50$ and $D_1=50, D_2=30$ and $D_1=40, D_2=40$	M1	2.1
	$\mathrm{P}\left(D_1+D_2=80\right)=\frac{k}{50}\times\frac{k}{30}\times 2 + \left(\frac{k}{40}\right)^2$	M1	3.4
	= 0.0375619… awrt **0.0376**	A1	1.1b
		(3)	
(c)	Angles are: $a,\quad a+d,\quad a+2d,\quad a+3d$	M1	3.1a
	$S_4 = a + (a + d) + (a + 2d) + (a + 3d) = 360$	M1	2.1
	$2a + 3d = 180$ (o.e.)	A1	2.2a
	Smallest angle is $a > 50$ consider cases:	M1	3.1b
	$d = 10$ so $a = 75$ or $d = 20$ so $a = 60$ [$d = 30$ gives $a = 45$ no good]		
	$\mathrm{P}(D = 10 \text{ or } 20) = \frac{3k}{20} = \frac{90}{137}$	A1	1.1b
		(5)	
		(10 marks)	

Notes

(a)

M1 for clear use of sum of probabilities = 1 (all terms seen)

A1 cso (*) M1 scored and no incorrect working seen.

Verify **(Assume $k = \frac{600}{137}$)** to score the final A1 they must have a final comment "$\therefore k = \frac{600}{137}$"

(b)

1st M1 for selecting at least 2 of the relevant cases (may be implied by their correct probs)
e.g. allow 30, 50 and 50,30 i.e. D_1 and D_2 labels not required

2nd M1 for using the model to obtain a correct expression for two different probabilities.
May use letter k or their value for k.

Allow for $\frac{k}{50}\times\frac{k}{30} + \left(\frac{k}{40}\right)^2$ or $2\times\left(\frac{k}{50}\times\frac{k}{30} + \left(\frac{k}{40}\right)^2\right)$

A1 for awrt 0.0376 (exact fraction is $\frac{705}{18769}$)

(c)

1st M1 for recognising the 4 angles and finding expressions in terms of d and their a

2nd M1 for using property of quad with these 4 angles (equation can be un-simplified)
Allow these two marks for use of a (possible) value of d
e.g. $a + a + 10 + a + 20 + a + 30 = 360$ (If at least 3 cases seen allow A1 for e.g. $4a = 300$)
or allow M1M1 for a set of 4 angles with sum 360 and possible value of d (3 cases for A1)
e.g. (for $d = 20$) 60, 80, 100, 120

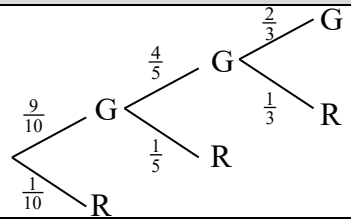
1st A1 for $2a + 3d = 180$ condition (o.e.) [Must be in the form $pa + qd = N$]

3rd M1 for examining cases and getting $d = 10$ and $d = 20$ only

2nd A1 for $\frac{90}{137}$ or exact equivalent

The correct answer and no obviously incorrect working will score 5/5

A final answer of awrt 0.657 (0.65693…) with no obviously incorrect working scores 4/5

Question	Scheme	Marks	AOs
13(a)		B1 dB1	1.1b 1.1b
		(2)	
(b)	$\frac{9}{10}\times\frac{4}{5}\times\frac{2}{3}$	M1	1.1b
	$=\frac{12}{25}(=0.48)$	A1	1.1b
		(2)	
(c)	$\frac{9}{10}\times\frac{1}{5}+\frac{9}{10}\times\frac{4}{5}\times\frac{1}{3}$ or $1-\left(\frac{1}{10}+\frac{9}{10}\times\frac{4}{5}\times\frac{2}{3}\right)$	M1	3.1b
	$=\frac{21}{50}$ $(=0.42)$	A1	1.1b
		(2)	
(d)	$[\text{P(Red from } B\text{}\vert\text{Red selected)}\ ]=\frac{\frac{9}{10}\times\frac{1}{5}}{\frac{1}{10}+\frac{9}{10}\times\frac{1}{5}+\frac{9}{10}\times\frac{4}{5}\times\frac{1}{3}}\left[=\frac{\frac{9}{50}}{\frac{13}{25}}\right]$	M1	3.1b
	$=\frac{9}{26}$	A1	1.1b
		(2)	
			(8 marks)

Notes	
	Allow decimals or percentages throughout this question.
(a)	**B1:** for correct shape (3 pairs) and at least **one** label on **at least two** pairs G(reen) and R(ed) allow G and G' **or** R and R' as labels, etc. condone 'extra' pairs if they are labelled with a probability of 0 **dB1:** (dep on previous B1) all correct i.e. for all 6 correct probabilities on the correct branches with at least one label on **each** pair
(b)	**M1:** Multiplication of 3 correct probabilities (allow ft from their tree diagram) **A1:** $\frac{12}{25}$ oe
(c)	**M1:** Either addition of only two correct products (product of two probs + product of three probs) which may ft from their tree diagram or for $1-(\text{'}\frac{1}{10}\text{'}+\text{'}(b)\text{'})$ **A1:** $\frac{21}{50}$ oe
(d)	**M1:** Correct ratio of probabilities **or** correct ft ratio of probabilities e.g. $\frac{\text{'}\frac{9}{10}\text{'}\times\text{'}\frac{1}{5}\text{'}}{1-\text{'}(b)\text{'}}$ or $\frac{\text{'}\frac{9}{10}\text{'}\times\text{'}\frac{1}{5}\text{'}}{\text{'}\frac{1}{10}\text{'}+\text{'}(c)\text{'}}$ with num < den **A1:** $\frac{9}{26}$ (allow awrt 0.346)

Question	Scheme	Marks	AOs
14	$\frac{132}{184} = 0.71739...$ awrt **0.717**	B1	1.1b
		(1)	
			(1 mark)
	Notes		
	Allow fractions, decimals or percentages throughout this question.		
	Allow equivalent fraction, e.g. $\frac{33}{46}$		

Qu 15	Scheme	Marks	AO
(a)	c: 0 1 2 3 4 5 6 7 8 P($C = c$): $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$	B1 B1ft (2)	1.2 1.2
(b)	P($C < 4$) = $\frac{4}{9}$ (accept 0.444 or better)	B1 (1)	3.4
(c)	Probability lower than expected suggests model is <u>not</u> good	B1ft (1)	3.5a
(d)	e.g. Cloud cover will vary from month to month and place to place So e.g. use a non-uniform distribution	B1 (1)	3.5c
		(5 marks)	

Notes

(a) 1st B1 for a correct set of values for c. Allow $\left\{\frac{1}{8}, \frac{2}{8}, \ldots \frac{8}{8}\right\}$

2nd B1ft for correct probs from their values for c, consistent with discrete uniform distrib'n
Maybe as a prob. function. Allow P($X = x$) = $\frac{1}{9}$ for $0 \leqslant x \leqslant 8$ provided $x = \{0, 1, 2, \ldots, 8\}$ is clearly defined somewhere.

(b) B1 for using correct model to get $\frac{4}{9}$ (o.e.)

SC **Sample space {1, …, 8}** If scored B0B1 in (a) for this allow P($C < 4$) = $\frac{3}{8}$ to score B1 in (b)

(c) B1ft for comment that states that the model proposed is or is not a good one based on their model in part (a) and their probability in (b)
|(b) – 0.315| > 0.05 Allow e.g. "it is not suitable"; "it is not accurate" etc
|(b) – 0.315| ⩽ 0.05 Allow a comment that suggests it <u>is</u> suitable
No prob in (b) Allow a comparison that mentions 50% or 0.5 and rejects the model
No prob in (b) and no 50% or 0.5 or **(b) > 1** scores B0
Ignore any comments about location or weather patterns.

(d) B1 for a sensible refinement considering variations in month or location
Just saying "not uniform" is B0
Context & "non-uniform" Allow mention of different locations, months <u>and</u> non-uniform
<u>or</u> use more locations to form a new distribution with probabilities based on frequencies
Context & "binomial" Allow mention of different locations, months <u>and</u> binomial
Just refined model Model must be outlined and discrete and non-uniform
e.g. higher probabilities for more cloud cover <u>or</u> lower probabilities for less cloud cover
Continuous model Any model that is based on a continuous distribution. e.g. normal is B0

Question	Scheme	Marks	AOs
16(a)	$P(S \cap D') = 0$	B1	1.1b
		(1)	
(b)	$P(C \mid S \cap D) = \frac{0.27}{0.6} = \frac{9}{20} = 0.45$	M1	3.1b
	$\therefore 80 \times "0.45"$	M1	1.1b
	$=36$	A1	1.1b
		(3)	
(c)	$[P(C) \times P(S) = P(C \cap S)]$		
	$P(S) = 0.6$, $P(C) = 0.27 + v + u$, $P(S \cap C) = 0.27$	M1	3.1a
	$0.6 \times (0.27 + u + v) = 0.27$ or $u + v = 0.18$ o.e	A1	1.1b
	$\left[P(D \mid C) = \frac{P(D \cap C)}{P(C)} \right]$ $P(D \cap C) = 0.27 + v$	M1	3.1a
	$\frac{14}{15} = \frac{0.27 + v}{0.27 + v + u}$ or $14u - v = 0.27$ o.e	A1	1.1b
	$15u = 0.45$	M1dd	1.1b
	$u = 0.03$ $v = 0.15$	A1	1.1b
	$w = 0.22$	A1ft	1.1b
		(7)	
			(11 marks)

Notes:

(a) B1: correct answer only

(b) M1: for a correct ratio of probabilities formula with at least one correct value and multiplying by 80

A1: a correct answer

(c) M1: for translating the problem and realising the equation $P(C) \times P(S) = P(C \cap S)$ needs to be used with at least 2 parts correct.

A1: a correct equation

M1: for a correct probability formula with $P(D \cap C) = 0.27 + v$

A1: a second correct equation

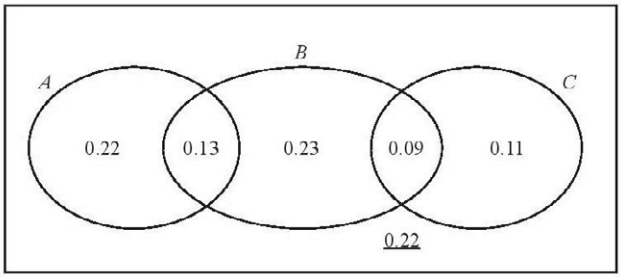
M1dd: dependent on the previous 2 method marks being awarded. Solving the two simultaneous equations by eliminating one variable. May be implied by either *u* or *v* correct

A1: *u* correct

A1: *v* correct

A1ft: w = 0.22, ft *their* u, v provided that $u + v + w < 0.4$

Question	Scheme	Marks	AOs
17	e.g. It requires extrapolation so will be unreliable (o.e.)	B1	1.2
		(1)	
			(1 mark)
Notes:			
B1: for a correct statement (unreliable) with a suitable reason			

Question	Scheme	Marks	AOs
18(a)	$\mathrm{P}(A' \mid B') = \dfrac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')}$ or $\dfrac{0.33}{0.55}$	M1	3.1a
	$= \dfrac{3}{5}$ or 0.6	A1	1.1b
		(2)	
(b)	e.g. $\mathrm{P}(A) \times \mathrm{P}(B) = \frac{7}{20} \times \frac{9}{20} = \frac{63}{400} \neq \mathrm{P}(A \cap B) = 0.13 = \frac{52}{400}$ or $\mathrm{P}(A' \mid B') = 0.6 \neq \mathrm{P}(A') = 0.65$	B1	2.4
		(1)	
(c)		B1	2.5
		M1	3.1a
		A1	1.1b
		M1	1.1b
		A1	1.1b
		(5)	
(d)	$\mathrm{P}(B \cup C)' = 0.22 + 0.22$ or $1-[0.56]$ or $1-[0.13+0.23+0.09+0.11]$ o.e.	M1	1.1b
	= 0.44	A1	1.1b
		(2)	
			(10 marks)

Notes:

(a)

M1: for a correct ratio of probabilities formula and at least one correct value.

A1: a correct answer

(b)

for a fully correct explanation: correct probabilities and correct comparisons.

(c)

B1: for box with B intersecting A and C but C not intersecting A.(Or accept three intersecting circles, but with zeros entered for $A \cap C$ and $A \cap B \cap C$)No box is B0

M1: for method for finding P($B \cap C$)

A1: for 0.09

M1: for 0.13 and their 0.09 in correct places and method for their 0.23

A1: fully correct

(d)

M1: for a correct expression – ft their probabilities from their Venn diagram.

A1: cao

Question Number	Scheme	Marks
19. (a)	$P(G_1) + P(R_1 \cap G_2) + P(Y_1 \cap G_2)$ or P(*GY*) + P(*GR*) + P(*RG*) + P(*YG*) (o.e.)	M1
	$= \frac{1}{64} + \frac{r}{64} \times \frac{1}{63} + \frac{y}{64} \times \frac{1}{63} = \frac{1}{64} + \frac{r+y}{64 \times 63}$ or $2 \times \frac{r+y}{64 \times 63}$	A1
	$= \frac{1}{64} + \frac{63}{64 \times 63}$ or $\frac{2 \times 63}{64 \times 63}$ or $\frac{1}{64} + \frac{1}{64}$ or	M1
	$= \frac{1}{32}$ or 0.03125	A1
		(4)
(b)	$P(R_1 \cap R_2) = \frac{r}{64} \times \frac{r-1}{63} = \frac{5}{84}$	M1A1
	$r(r-1) = 5 \times 64 \times 63 \div 84 = 240$ hence $r^2 - r - 240 = 0$ or $r^2 - r = 240$ (*****)	A1cso
		(3)
(c)	$r^2 - r - 240 = (r-16)(r+15)\{=0\}$ or $16^2 - 16 - 240 = 256 - 256$	M1
	or $\frac{16}{64} \times \frac{15}{63} = \frac{5}{84}$	
	so $r = 16$ and rejecting -15 (*****)	A1cso
		(2)
(d)	P(⩾1red) = P(*RG*) + P(*GR*) + P(*RY*) + P(*YR*) + P(*RR*) or $\frac{2}{252} + \frac{2y}{252} + \frac{15}{252}$ (o.e.)	M1,
	or $P(R_1) + P(R_1' \cap R_2)$ or $\frac{16}{64} + \frac{48}{64} \times \frac{16}{63}$ or $1 - \frac{48}{64} \times \frac{47}{63}$, $= \frac{37}{84}$	A1
	Require: $\frac{P(R_1 \cap R_2)}{P(\text{at least one red})} = \frac{\frac{5}{84}}{"\frac{37}{84}"}$, $= \frac{5}{37}$ or $0.\dot{1}3\dot{5}$	M1, A1
		(4) **[Total 13]**

Notes

(a)
1st M1 for at least 2 correct cases. May be in symbols or probs. May be in tree diagram
Use of $r = 16$ or $y = 47$ can score maximum of 1st M1 then A0M0A0
1st A1 for all cases and their assosciated probs added
2nd M1 for combining probabilities and using $r + y = 63$
2nd A1 for $\frac{1}{32}$ or an exact equivalent (correct answer only 4/4)

(b)
M1 for $\frac{r}{64} \times g(r) = ...$ where g(r) is any linear function of r
1st A1 for any correct equation in r
2nd A1cso for correctly simplifying to the given equation with no incorrect working seen.
There should be at least 1 intermediate step seen

(c)
M1 for correct factors or completing square or use of formula or substitution
A1cso for concluding $r = 16$ and rejecting -15 (e.g. crossing out etc)

(d)
1st M1 for a correct expression for at least one red. May be in symbols or probs. or in a tree
1st A1 for $\frac{37}{84}$ (o.e.) as a single fraction or awrt 0.440 [May be implied by correct answer]
2nd M1 for a ratio of probabilities (denom may be in symbols) with numerator of $\frac{5}{84}$ (o.e.)
2nd A1 for $\frac{5}{37}$ or an exact equivalent

Question Number	Scheme	Marks
20. (a)	$p = \text{P}(B \cap C) = \text{P}(B) \times \text{P}(C) = 0.6 \times 0.25 = \underline{\underline{\mathbf{0.15}}}$ $q = [\text{P}(C) - p\,] = \underline{\underline{\mathbf{0.10}}}$	M1 A1 (2)
(b)	$r = 1 - 0.08 - [\text{P}(B) + q] = 1 - 0.08 - 0.6 - 0.1$ (o.e.) <u>or</u> $1 - 0.08 - (0.6 + 0.25 - p)$ $= \underline{\underline{\mathbf{0.22}}}$	M1 A1cao (2)
(c)	$s = [\text{P}(A) - r\,] = \underline{\underline{\mathbf{0.28}}}$ $t = [\text{P}(B) - p - s$ <u>or</u> use $\text{P}(B \cap C') - s = 0.6 \times 0.75 - \text{"0.28"}\,] = \underline{\underline{\mathbf{0.17}}}$	B1ft B1ft (2)
(d)	$\text{P}(A) \times \text{P}(B) = 0.5 \times 0.6 = 0.3$ which is <u>not</u> equal to s (= 0.28) So A and B are <u>not</u> independent	M1 A1 (2)
(e)	$\dfrac{(s+p) \text{ or } (0.6-t)}{\text{P}(A \cup C) \text{ or } [\text{P}(A)+\text{P}(C)] \text{ or } (r+s+p+q)} = \dfrac{(\text{"0.28"}+\text{"0.15"}) \text{ or } (0.6-\text{"0.17"})}{0.5+0.25}$ $= \underline{\underline{\frac{43}{75}}}$	M1, A1ft A1 (3)
		[11]

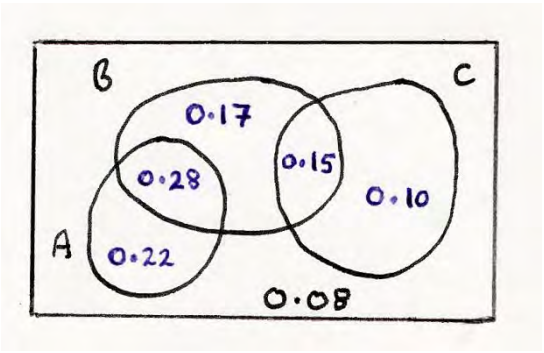
Notes

(a)
M1 for a correct expression (using independence) for p <u>or</u> 0.15
A1 for $q = 0.10$ (both correct 2/2)

Mark (b) & (c) together

(b)
M1 for a correct expression for r using $\text{P}(B \cup C)$. Can ft their $q \in [0, 0.32]$
A1cao for $r = 0.22$ (correct ans only 2/2)

(c)
1st B1ft for $s = 0.28$ <u>or</u> 0.5 – their "0.22"
2nd B1ft for $t = 0.17$ <u>or</u> 0.6 – their "0.15" – their "0.28"



Fully correct Venn diagram will score the first 6 marks
If text and VD disagree use <u>text</u> values

ALT **Find *t* then *s* then *r***
(c) 2nd B1 for $t = 0.17$ [from $1 - 0.08 - \text{P}(A) - \text{P}(C)$]
1st B1ft for $s = 0.28$ <u>or</u> $\text{P}(B)$ – "0.17" – "0.15"
(b) M1 for $r = \text{P}(A) - s$ and the A1 for 0.22

$s = 0.3$ **They assume *A* and *B* are independent and get $s = 0.3$ [from $\text{P}(A) \times \text{P}(B)$]**
(c) 1st B0 for $s = 0.3$ BUT can get 2nd B1ft for either case in the scheme
(b) M1 for $r = \text{P}(A) - s$ BUT then A0cao for $r = 0.2$

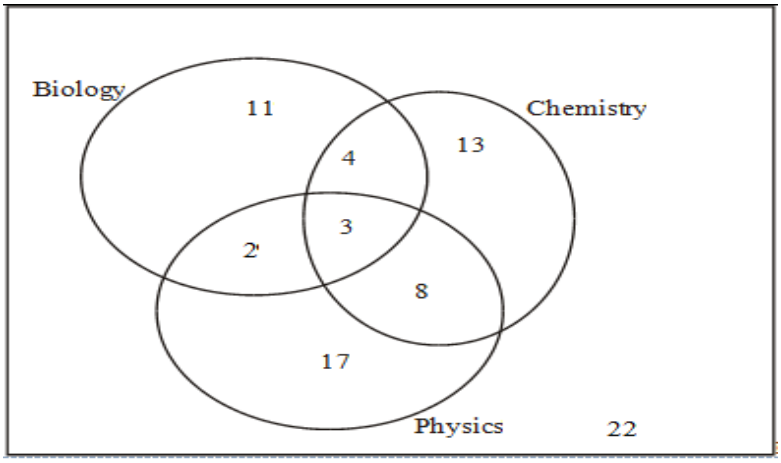
(d)
M1 for a correct $\text{P}(A) \times \text{P}(B) = 0.5 \times 0.6$ <u>or</u> 0.3 **and** a clear comparison with their $s (\neq 0.3)$
<u>Or</u> calculation of $\text{P}(A \mid B) = \frac{7}{15}$ <u>or</u> 0.467 <u>or</u> $\dfrac{\text{their } s}{0.6}$ **and** comparison with $\text{P}(A) = 0.5$ (o.e.)
A1 dep. on M1 being earned and clear statement that A and B are <u>not</u> independent

SC $s = 0.3$ dep on 1st B1ft for $s = 0.5 - 0.2$ in (c); for correct calc. <u>and</u> conclusion seen (B1). On epen M0A1

(e)
M1 for a correct ratio expression of probs: num. < den. Allow 1 – (0.08+their "t") on den.
Any sight of multiplication on the numerator e.g. 0.6×0.75 is M0
1st A1ft for correct ratio or ft using their values in numerator but correct denominator.
2nd A1 for $\frac{43}{75}$ or accept awrt 0.573

Question Number	Scheme	Marks
21.(a)	$[\mathrm{P}(B \cap R') =]$ **0**	B1 (1)
(b)	$\mathrm{P}(B) = 0.27+0.33 = 0.6$, $\mathrm{P}(D) = 0.27 + 0.15 + t$, $\mathrm{P}(B \cap D) = 0.27$	M1
	$[\mathrm{P}(B) \times \mathrm{P}(D) = \mathrm{P}(B \cap D)$ gives$]$ $\quad 0.6 \times (0.42 + t) = 0.27$	M1
	$0.42 + t = \dfrac{0.27}{0.6}$ or $0.6t = 0.018$	A1
	$t =$ **0.03**	A1 (4)
(c)	$[u =]$ $\quad 1 - (0.6 + 0.15 + t)$	M1
	$u =$ **0.22**	A1ft (2)
(d)(i)	$\left[\dfrac{\mathrm{P}(D \cap R \cap B)}{\mathrm{P}(R \cap B)} =\right] = \dfrac{0.27}{0.27+0.33}$ or $\mathrm{P}(D \mid R \cap B) = \mathrm{P}(D \mid B) = \mathrm{P}(D)$	M1
	$=$ **0.45**	A1
(ii)	$\left[\dfrac{\mathrm{P}(D \cap [R \cap B'])}{\mathrm{P}(R \cap B')} =\right] = \dfrac{0.15}{0.15+u}$	M1
	$= \dfrac{15}{37}$	A1 (4)
(e)	$40 \times$"0.45" and $37 \times$"$\dfrac{15}{37}$"	M1
	$=$ **33**	A1 (2)
		[13 marks]

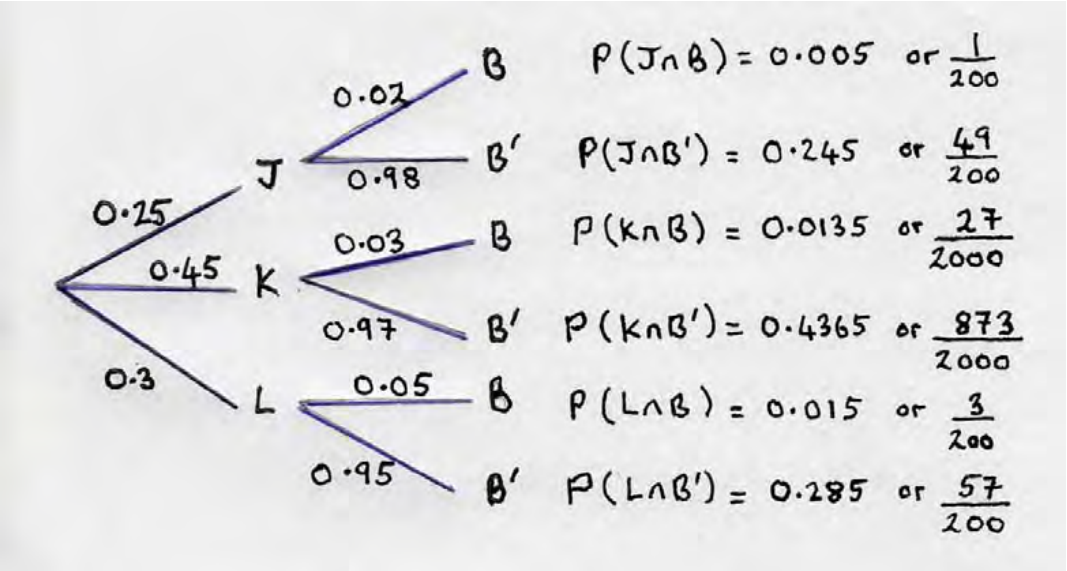
	Notes
(b)	1st M1 for attempting 3 suitable probabilities, one involving t (at least 2 correct) e.g. sight of 0.6, 0.27, 0.42 + t correctly labelled in terms of B,D,R or in a correct equation. May see e.g. $\mathrm{P}(B\|D) = \dfrac{0.27}{0.42+t}$
	2nd M1 for using the independence to form a linear equation in t. ft their probs if stated.
	1st A1 for solving leading to a correct equation as far as $p + t = q$ or $pt = q$
	2nd A1 for 0.03 or exact equivalent
(c)	M1 for a correct expression for u . Allow their t or just letter t in a correct expression
	A1ft for 0.22 (or exact equivalent) or ft their t. i.e. $u = 0.25 - t$ provided u & t are probs
	Can score M1A1ft provided their u + their t = 0.25 where u and t are both in [0, 1]
(d)(i)	M1 for a correct numerical ratio of probabilities
	A1 for 0.45 or exact equivalent (Answer only 2/2)
(ii)	M1 for a correct numerical ratio of probabilities, ft their u, provided u is a probability
	A1 for $\frac{15}{37}$ or $0.\dot{4}0\dot{5}$ or allow awrt 0.41 following a correct expression (Ans only 2/2)
(e)	M1 for a correct method for both 18 and 15 ft their 0.45 and their $\frac{15}{37}$ provided both in [0,1]
	NB $\mathrm{P}(D) \times 77$ is M0
	A1 for 33 only
	NB $\frac{27}{33} \times 40 = 32.7\ldots$ which rounds to 33 but scores M0A0. (**Ans only send to review**)

Question	Scheme	Marks
22. (a)		B1 M1 A1 A1 B1 (5)
(b)	$\frac{'13'}{80}$ or 0.1625	B1ft (1)
(c)	$\frac{28+30-11}{80}$ or $\frac{2+3+4+8+13+17}{80}$ or $1-\frac{(11+22)}{80}=\frac{47}{80}$ or 0.5875	M1 A1 (2)
(d)	$\frac{"17+8+13"}{"47"}$ or $\frac{\frac{"38"}{80}}{\frac{"47"}{80}}$ or $1-\frac{"2+3+4"}{"47"}=\frac{38}{47}$ (condone awrt 0.809)	M1 A1cao (2)
(e)	$P(B\|C)=\frac{7}{28}$, $P(B)=\frac{20}{80}$ $P(C\|B)=\frac{7}{20}$, $P(C)=\frac{28}{80}$ $P(B\cap C)=\frac{7}{80}$, $P(B)=\frac{20}{80}P(C)=\frac{28}{80}$ $P(B\|C)=P(B)$, $P(C\|B)=P(C)$ these may be implied by correct conclusion $P(B\cap C)=P(B)\times P(C)$ this approach requires the product to be seen So, they are independent.	M1 M1 A1 (3) (13 marks)

	Notes
(a)	B1 for 3 intersecting circles with 3 in the centre. **Allow probs. or integers in diagram.** M1 for some correct subtraction e.g. at least one of 2, 4, 8 or for *B*: 20 – their(2+3+4) etc A1 for 2, 4 and 8 (ignore labels) A1 for 11, 13 and 17 (must be in compatible regions with 2, 4, 8 if no labels) B1 for correct labels and 22 and box (Do not treat "blank" as 0 so can't use 0 for ft in (c))
(c)	M1 for a correct expression seen in (c) (or ft their diagram). Correct ans M1A1
(d)	M1 for denominator of 47 or ft their numerator from part (c) and numerator of 38 or their (17 + 8 + 13) or (their 47) – their (2 + 3 + 4). Correct ans M1A1
(e)	M1 for stating at least the required probs.& labelled for a correct test (can ft their diagram) M1 for use of a correct test with *B* and *C* Must see product attempted for $P(B\cap C)$ test. A1 for a correct test with all probabilities correct and a correct concluding statement. NB M0M1A0 should be possible but A1 requires both Ms

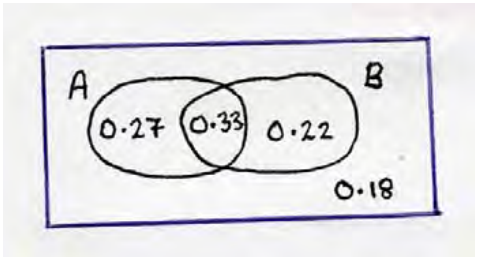
Question	Scheme	Marks
23. (a)	To score 15 points, 2 correct and 1 not correct $[0.6\times0.6\times0.4]+[0.6\times0.4\times0.6]+[0.4\times0.6\times0.6]$ <u>or</u> $3\times(0.6\times0.6\times0.4)$ $= 0.432$ **(*)**	M1 A1cso (2)
(b)	$1 - (0.216 + 0.432 + 0.064) =$ **<u>0.288</u>** or $3\times0.6\times(0.4)^2$	B1 (1)
(c)	[(30, 0), (0, 30) or (15, 15)] $0.216\times'0.288'+'0.288'\times0.216+0.432\times0.432$ **awrt <u>0.311</u>**	M1 A1ft A1 (3)
		(6 marks)

	Notes
(a)	M1 for $0.6^2\times0.4$ may be $\Rightarrow$ by tree diagram with 0.6 & 0.4 but just 3×0.144 or 2×0.216 is M0 A1 cso for $3\times0.6^2\times0.4$ (seen) and no incorrect working seen
(b)	0.288 or $\frac{36}{125}$ answer may be seen in table. [NB Fractions: $\frac{27}{125},\frac{54}{125},\frac{36}{125}$ and $\frac{8}{125}$]
	Correct answers to (c), (d) and (e) score full marks for these parts.
(c)	M1 for either $0.216\times'0.288'$ = (0.062208) <u>or</u> $0.432\times0.432 = 0.186624$ (ft (b) provided their (b) is a probability) 1st A1ft for a fully correct expression 2nd A1 for awrt 0.311 or $\frac{972}{3125}$
SC	**6 questions 4 correct** Award M1&1st A1 for $6C4\times0.6^4\times0.4^2$ or $15\times0.6^4\times0.4^2$

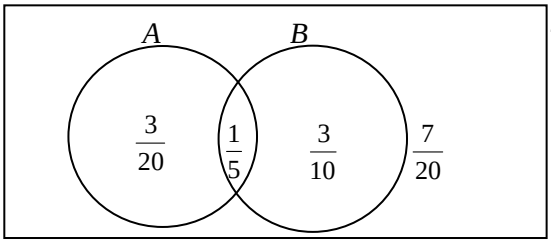
Question Number	Scheme	Marks
24 (a)		M1 A1 **(2)**
(b)	0.25×0.98, = **0.245** (or exact equiv. e.g. $\frac{49}{200}$)	M1A1 **(2)**
(c)	$0.25\times0.02+0.45\times0.03+0.3\times0.05$, = **0.0335** (or exact equiv. e.g. $\frac{67}{2000}$)	M1A1 **(2)**
(d)	$[\,P(J\cup L\,\|\,B)\,] = \frac{0.25\times0.02+0.3\times0.05}{0.0335}$ or $\frac{0.0335-0.45\times0.03}{0.0335}$ $=0.5970...$ awrt **0.597** (or $\frac{40}{67}$ or exact equiv.)	M1A1ft A1 **(3)**
	Notes	**Total 9**
	Allow fractions or percentages throughout this question	
(a)	Allow 3+6 tree diagram with the 6 correct "end" probs and labels to get 2/2 (1st, 3rd, 5th gets M1) M1 for (3+6) tree drawn with 0.25, 0.45, 0.02, 0.03, 0.05 on correct branches A1 for 0.3, 0.98, 0.97, 0.95 on the correct branches and labels, condone missing B's **Correct answer only scores full marks for parts (b), (c) and (d)** **When using "their probability *p*" for M1 and A1ft they must have $0 < p < 1$**	
(b)	M1 for $0.25\times$ 'their 0.98' o.e.	
(c)	M1 for $0.25\times$their $0.02+0.45\times$their $0.03+$ their $0.3\times$their 0.05 Condone 1 transcription error. Or $1-(0.25\times$their $0.98+0.45\times$their $0.97+$ their $0.3\times$their $0.95)$	
(d)	M1 for use of conditional probability with their (c) as denominator. Also exactly 2 products on num' and at least one correct (or correct ft) or their (c) – one of the products from their (c). Ignore an incorrect expression inside their probability statement A1ft for $\frac{0.25\times\text{their }0.02+\text{their }0.3\times\text{their }0.05}{\text{their(c)}}$ or $\frac{\text{their (c)}-0.45\times\text{their }0.03}{\text{their (c)}}$ or $\frac{0.02}{\text{their (c)}}$ A1 awrt 0.597 or exact fraction e.g. $\frac{40}{67}$	

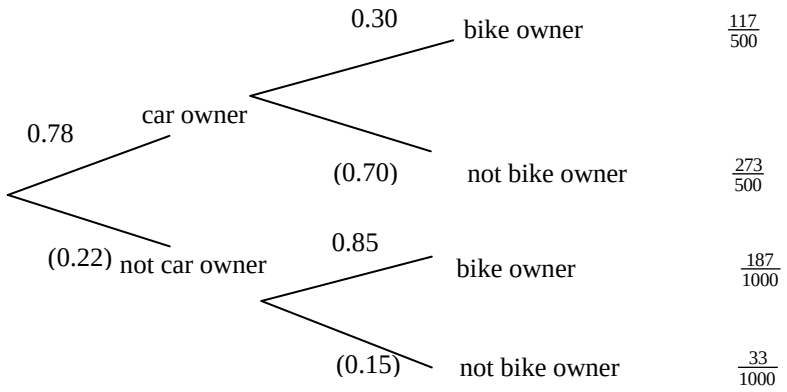
Question Number		Scheme	Marks
25	(a)	$[\,P(A) = 1 - 0.18 - 0.22\,]$ = **0.6** (or exact equivalent)	B1 **(1)**
	(b)	$P(A \cup B) = "0.6" + 0.22$ = **0.82** (or exact equivalent)	B1ft **(1)**
	(c)	$x = P(A \cap B)$ | Use $P(B)P(A' \mid B) = P(A' \cap B)$ | **Establish** independence before or after 1st M1and score marks for (d) (RH ver)	M1
		$\frac{x}{x+0.22} = 0.6$ | $P(B) \times [1 - 0.6] = 0.22$ | **Find** $P(B)$	
		$x = 0.6x + 0.132$ | Use $P(A \cap B) = P(A \mid B)P(B)$ | Use $P(B)P(A) = P(A \cap B)$	dM1
		$0.4x = 0.132$ | $P(A \cap B) = 0.6 \times 0.55$ | $P(A \cap B) = 0.6 \times 0.55$	
		x = **0.33** (or exact equivalent)	A1cso **(3)**
	(d)	$P(B) = 0.55$	
		$P(B) \times P(A) = 0.55 \times 0.6$ = 0.33 **or** stating $P(A) = P(A\|B)$ [= 0.6]	M1
		$P(B) \times P(A) = P(A \cap B)$ therefore (statistically) independent **or** $P(A) = P(A\|B)$ therefore (statistically) independent	A1cso **(2)**
			Total 7

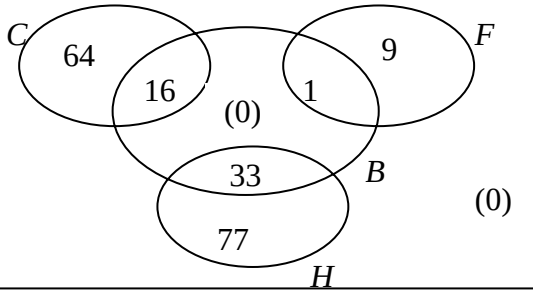
	Notes
(b)	B1ft for their (a) + 0.22 or $1 - P(A' \cap B')$ Do not ft their (a) if it is > 0.78
	NB 3 versions for (c). Check carefully that Ms are genuinely scored.
	Look out for <u>assuming independence</u> and if you see P(*B*) = 0.55 check it is <u>derived</u> properly
(c)	1st M1 for a correct equation for x e.g. $\frac{x}{x+0.22} = 0.6$ <u>or</u> a correctly derived equation for $P(B)$
	2nd dM1 for solving to get in form $kx = L$ <u>or</u> <u>correct</u> use of $P(B)$ to find $P(A \cap B)$ [2nd or 3rd ver] <u>or</u> $P(A \cap B) = P(B) - 0.22$
	A1cso for 0.33 Dep. on <u>both</u> Ms and no incorrect working seen.
(d)	M1 for finding $P(B) \times P(A) = 0.33$ (values needed) <u>or</u> stating $P(A) = P(A\|B)$ (= 0.6 not needed)
	A1cso for a correct statement: $P(B) \times P(A) = P(A \cap B)$ or $P(A) = P(A\|B)$ <u>and</u> stating independent
	NB The M1 in (d) using $P(A \cap B)$ requires $P(B) = 0.55$ There is no ft of an incorrect $P(B)$ Full marks in (d) is OK even if 0/3 in (c)
	{This Venn diagram may be helpful.}



Question Number	Scheme	Marks
26. (a)	(Discrete) Uniform	B1 **(1)**
(b)	(i) $P(X = 10) = \frac{1}{10}$	B1
	(ii) $P(X < 10) = \frac{9}{10}$	B1 **(2)** **[Total 3]**
	Notes	
(a)	B1 for seeing the word **uniform** Condone "continuous" uniform	

Question Number		Scheme	Marks
27.	**(a)**	$P(A \cap B) = P(A \mid B) \times P(B)$ $P(A \cap B) = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$	M1 A1 **(2)**
	(b)	 2 intersecting circles and 'P($A \cap B$)' $\frac{3}{20}$ and $\frac{3}{10}$ Box and $\frac{7}{20}$	B1ft B1 B1 **(3)**
	(c)	$\left[P(A) = \frac{3}{20} + \frac{1}{5} \right] = \frac{7}{20}$ or 0.35	B1ft **(1)**
	(d)	$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{5}}{\frac{7}{20}}$ $= \frac{4}{7}$	M1 A1 cao **(2)**
	(e)	0.3	B1ft **(1)** **[Total 9]**
		Notes	
	(a)	M1 for $\frac{2}{5} \times \frac{1}{2}$ or a correct probability product expression and one correct prob. Ans only 2/2	
	(b)	1st B1 for 2 intersecting circles labelled *A* and *B* and ft their prob. for intersection Condone missing labels for 2nd and 3rd B marks	
	(c)	B1ft for 0.35 (o.e.) if no Venn diagram or correct follow through from their diagram or allow 0.35 (or correct ft) from correct working e.g. 0.65 – 0.5 + (a) B0 for 0.35 if their diagram does not give 0.35 unless it comes from correct work Don't insist on P(*A*) =... but do not award for $P(A' \cap B') = \frac{7}{20}$	
	(d)	M1 for $\frac{\text{their (a)}}{\text{their (c)}}$ or a correct ratio of probabilities from their diagram NB incorrect use of $P(A' \cap B') = \frac{7}{20}$ scores M0 and num $\geq$ denom scores M0 A1 for $\frac{4}{7}$ only	
	(e)	B1ft for 0.3 or correct ft from their Venn diagram or ft from $\frac{13}{20}$ – their (c)	

Question Number	Scheme	Marks
28. (a)		B1 B1 B1 **(3)**
(b)	P(car or bike but not both)$= 0.78\times 0.70 + 0.22\times 0.85 = 0.733$	M1 A1 **(2)**
(c)	$[\text{P(car}\mid\text{bike)}] = \dfrac{\text{P(car}\cap\text{bike)}}{\text{P(bike)}} = \dfrac{0.78\times 0.30}{0.78\times 0.30 + 0.22\times 0.85}, = 0.555819....$ **awrt** 0.556	M1A1 A1 **(3)**
(d)	$\text{P(bike)} = 0.78\times 0.30 + 0.22\times 0.85 = 0.421$, P(not bike) = 1 – 0.421 $0.421\times 0.579 + 0.579\times 0.421$ = 0.487518 **awrt** 0.488	M1 dM1 A1 **(3)** **[Total 11]**
	Notes	
(a)	1st B1 for a (2+4) tree with 6 branches 2nd B1 for 0.78 with label 3rd B1 for 0.30 and 0.85 with label	
(b)	M1 for correct expression of follow through their correct tree branches A1 for 0.733 or exact equivalent e.g. $\frac{733}{1000}$ and allow 73.3%	
(c)	M1 for a correct expression correct ft or correct formula and $\dfrac{\text{1 product}}{\text{sum of 2 products}}$ With at least 2 products correct or correct ft. Ratio must be smaller than 1 1st A1 for finding the denominator correctly. Fully correct expression or = 0.421 (oe) 2nd A1 for awrt 0.556 or exact equivalent e.g. $\frac{234}{421}$ and allow 55.6%	
(d)	M1 for their $\text{P(bike)}\times(1-\text{P(bike)})$ dM1 for $\times 2$ A1 for awrt 0.488	

Question	Scheme	Marks
29. (a)	$\frac{35+75}{200}=0.55$	M1 A1 **(2)**
(b)	$\frac{200-2}{200}=0.99$	M1 A1 **(2)**
(c)	$\left[\mathrm{P}\left(W \mid C\right)\right]=\frac{\mathrm{P}(W \cap C)}{\mathrm{P}(C)}=\frac{30/200}{80/200}=\frac{30}{80}=0.375$	M1 A1 **(2)**
(d)	 Allow diagrams with intersections between *F*, *C* and *H* provided these are marked with 0. If their diagram indicates extra empty regions do not treat a blank as 0.	M1 B1 for 9, 1 B1 for 77,33 B1 for 64,16 **(4)**
(e)	$\frac{1+16+33}{200}=0.25$	M1 A1 **(2)**
		(12 marks)

	Notes
	Correct answers only score full marks for each part **If a probability is not in [0, 1] award M0**
(a)	M1 for denominator of 200 and attempt to add 2 + 8 or 35 + 75 or 30 + 50 A1 for 0.55 or exact equivalent fraction e.g. $\frac{11}{20}$
(b)	M1 for a fully correct expression (e.g. $1-0.01$) A1 for 0.99 or an exact equivalent fraction
(c)	M1 for a correct ratio or a correct formula and at least one correct prob (i.e. a correct num or denom). BUT award M0 if num is $\mathrm{P}(W)\times\mathrm{P}(C)=\frac{67}{200}\times\frac{80}{200}$ or if num>denom A1 for 0.375 or 3/8 or any exact equivalent.
(d)	M1 for a box and the 3 regions *F*, *C* and *H* labelled or <u>implied</u> and single set *B* labelled. There should be no intersections between *F*, *C* and *H* unless marked by zeros. They may have 3 circles for *F*, *C* and *B* with $H=F'\cap C'$ etc. Condone lack of zero in the <u>given</u> diagram. *F* 1st B1 for the 9 and 1 <u>or</u> 0.045 and 0.005 (o.e.) in the correct regions *H* 2nd B1 for the 77 and 33 <u>or</u> 0.385 and 0.165 (o.e.) in the correct regions *C* 3rd B1 for the 64 and 16 <u>or</u> 0.32 and 0.08 (o.e.) in the correct regions. May have *B* in 3 bits that are disconnected.
(e)	M1 for a numerator made up of their 1 + their 16 + their 33 and a denom of 200 and num < 200 Also allow sum of their probabilities (provided sum < 1) A1 for 0.25 or any exact equivalent

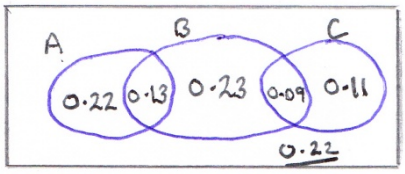
Question	Scheme	Marks
30. (a)	$3a + 2b = 0.7$	M1
	$a + 2a + 3a + 4b + 5b + 1.8 = 4.2$ or $6a + 9b = 2.4$	M1
	$5b = 1$ Attempt to solve	M1
	$b = \mathbf{\underline{0.2}}$ cao	B1
	$a = \mathbf{\underline{0.1}}$ cao	B1
		(5)
(b)	[$5k = 1$ so] $k = \mathbf{\underline{0.2}}$	B1 **(1)**
(c)	$P(Y = 1) = 0.1$	B1
	e.g. $P(Y = 2) = F(2) - F(1) = 0.1$	M1
	y | 1 | 2 | 3 | 4 | 5 $P(Y = y)$ | 0.1 | 0.1 | 0.4 | 0.2 | 0.2 Condone use of $X(x)$ instead of $Y(y)$ Ignore incorrect or no label if table fully correct	A1
		(3)
(d)	$P(X = 1) \times P(Y = 1)$,= $\mathbf{\underline{0.01}}$ cao	M1, A1 **(2)**
		(11 marks)

	Notes
	Probabilities outside [0, 1] should be awarded M0
(a)	1st M1 for an attempt at a linear equation in a and b based on sum of probs. = 1
	2nd M1 for an attempt at a second linear equation in a and b based on $E(X) = 4.2$ Allow one slip.
	3rd M1 for an attempt to solve their 2 linear equations based on sum of probs and $E(X)$. Must reduce to a linear equation in one variable. 1st B1 for b and 2nd B1 for a. Answers only score B1B1 only
	The 3rd M1 may be implied if M2 is scored and both correct answers are given.
ALT	B1B1 for stating b and a.
	1st M1 for showing that sum of probs. = 1
	2nd M1 for showing that $E(X) = 4.2$
	3rd M1 for an overall comment "(therefore) a = …and b = …" No comment loses this mark.
(c)	B1 for $P(Y = 1) = 0.1$
	M1 for correct use of $F(y)$ to find one other prob. Can ft their k if finding $P(Y = y)$ for $y > 2$
	Can be implied by one other prob. correct or correct ft Look out for $P(3) = 3k - 0.2$ or $P(4) = P(5) = k$.
	A1 for a fully correct probability distribution. Correct table only is 3/3
(d)	M1 for a correct expression or answer ft their $P(Y = 1)$ and their $P(X = 1)$
	A1 for 0.01 or exact equivalent only
	Don't ISW here e.g. $0.1\times0.1+0.1\times0.1$ or $2\times0.1\times0.1$ are M0A0

Question	Scheme	Marks
31. (a)	[P(B) = 0.4, P(A) = p + 0.1 so] $0.4\times(p+0.1)=0.1$ or $0.4\times \text{P}(A)=0.1$	M1
	$p=\frac{1}{4}-0.1$ **$\underline{p = 0.15}$**	M1A1 (3)
(b)	$\frac{5}{11}=\left[\frac{\text{P}(B\cap C)}{\text{P}(C)}=\right]\frac{0.2}{0.2+q}$ or $\frac{5}{11}=\frac{0.2}{\text{P}(C)}$	M1
	$11\times 0.2=5\times(0.2+q)$	dM1
	$\underline{q = 0.24}$	A1
	r = 0.6 – (p + q) i.e. **$\underline{r = 0.21}$**	A1ft (4)
(c)	$\left[\frac{\text{P}((A\cup C)\cap B)}{\text{P}(B)}\right]=\frac{0.3}{0.4}$	M1
	= **$\underline{0.75}$**	A1 (2)
		[9]

	Notes
(a)	1st M1 for using independence in an attempt to form an equation in p or P(A) 2nd M1 for a correct attempt to solve their linear equation leading to p = … A1 for 0.15 or exact equivalent
(b)	1st M1 for a clear attempt to use P(B|C) to form an equation for q or P(C). Assuming indep M0 2nd dM1 Dep. on 1st M1 for correctly simplifying to a linear equation in q or P(C) e.g. accept $11\times 0.2=5\times 0.2+q$ or 5P(C) = 2.2 1st A1 for q = 0.24 or exact equivalent 2nd A1ft for 0.6 – their (p + q) Dependent on 1st M1 in (b) only.
(c)	M1 for a correct ratio expression and one correct value (num < denom) <u>or</u> a fully correct ratio. Allow $\frac{\text{P}(A\cup C\cap B)}{\text{P}(B)}$ with one probability correct but only if num < denom. A numerator of $\text{P}(A\cup C)\times\text{P}(B)$ scores M0 A1 for 0.75 or an exact equivalent

Question Number	Scheme	Marks
32. (a)	b | 1 | 3 | 5 P($B = b$) | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ Also allow b values 1,1,3,3,5,5 and probabilities all $\frac{1}{6}$	B1 B1 (2)
(b)	Discrete Uniform {distribution}	B1 (1)
		3
	Notes	
(a)	1st B1 for correctly identifying values of b as 1, 3, 5 or 1,1,3,3,5,5 2nd B1 for probabilities all = $\frac{1}{3}$ or exact equivalent (or of course 6 cases of $\frac{1}{6}$) Any correct probability distribution or probability function is 2/2. Must be in part (a)	
(b)	B1 for "Discrete Uniform" . Both words required.	

Question Number	Scheme	Marks
33.		
(a)	$P(A \cup B) = 0.35 + 0.45 - 0.13$ or $0.22 + 0.13 + 0.32$ $= \mathbf{0.67}$	M1 A1 (2)
(b)	$P(A' \mid B') = \dfrac{P(A' \cap B')}{P(B')}$ or $\dfrac{0.33}{0.55}$ $= \dfrac{3}{5}$ or 0.6	M1 A1 (2)
(c)	$P(B \cap C) = 0.45 \times 0.2$ $= \mathbf{0.09}$	M1 A1 (2)
(d)	 Allow 1st B1 for 3 intersecting circles in a box with zeros in the regions for $A \cap C$ Do not accept "blank" for zero	B1 B1ft B1 B1 (4)
(e)	$P(B \cup C)' = 0.22 + \underline{0.22}$ or $1-[0.56]$ or $1-[0.13+0.23+0.09+0.11]$ o.e. $= \mathbf{0.44}$	M1 A1 (2) **12**
	Notes	
	NB May see Venn diagram for *A* and *B* only used for (a) and (b) but M marks are awarded for correct expressions only. No ft from an incorrect diagram for M marks.	
(a)	M1 for attempt to use the addition rule. Correct substitution i.e. correct expression seen A1 for 0.67 only. Correct answer only scores 2/2	
(b)	M1 for a correct ratio of probabilities or a correct formula and at least one correct prob For a correct formula allow "1 − their (a)" instead of 0.33 but not for correct ratio case. Do not award for assuming independence i.e. $\frac{P(A' \cap B')}{P(B')} = \frac{0.65 \times 0.55}{0.55}$ is M0. M0 if num>denom A1 for 3/5 or any exact equivalent.	
(c)	M1 for correct expression. Need correct values for P(*B*) and P(*C*) seen. A1 for 0.09 or any exact equivalent. Correct answer only is 2/2	
(d)	**No labels *A*, *B*, *C* in (d) loses 1st B1 but can score the other 3 by implication** B1 for box with *B* intersecting *A* and *C* but *C* not intersecting *A*. No box is B0 B1ft for 0.13 and their 0.09 in correct places. [ft $P(B \cap C)$ from (c)] B1 for any 2 of 0.22, 0.22, 0.11 and 0.23 correct B1 for all 4 values correct	
(e)	M1 for a correct expression or follow through from their Venn diagram NB $P(B') \times P(C') = 0.55 \times 0.8$ is OK. Do not ft "blank" for zero and M0 for negative probs. A1 for 0.44 only. Correct answer only is 2/2	

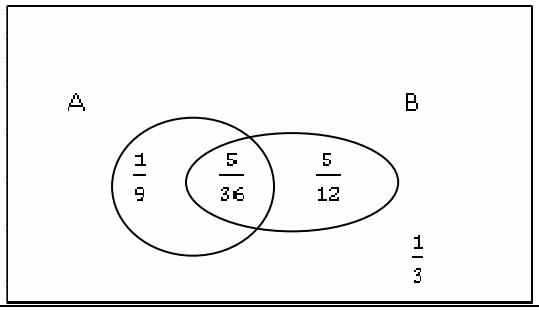
Question Number	Scheme	Marks
34 (a)	(*R* and *S* are mutually) exclusive.	B1 (1)
(b)	$\frac{2}{3} = \frac{1}{4} + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ use of Addition Rule $\frac{2}{3} = \frac{1}{4} + \mathrm{P}(B) - \frac{1}{4} \times \mathrm{P}(B)$ use of independence $\frac{5}{12} = \frac{3}{4}\,\mathrm{P}(B)$ $\mathrm{P}(B) = \frac{5}{9}$	M1 M1 A1 A1 (4)
(c)	$\mathrm{P}(A' \cap B) = \frac{3}{4} \times \frac{5}{9} = \frac{15}{36} = \frac{5}{12}$	M1A1ft (2)
(d)	$\mathrm{P}(B' \mid A) = \frac{(1 - (b)) \times 0.25}{0.25}$ or $\mathrm{P}(B')$ or $\frac{\frac{1}{9}}{\frac{1}{4}}$ $= \frac{4}{9}$	M1 A1 (2)
		Total 9

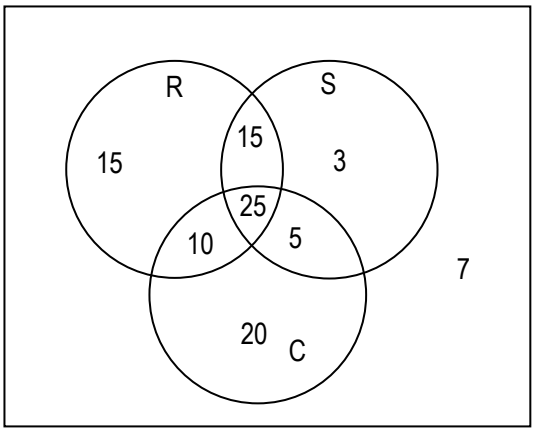
NOTES

(a) B1 for '(mutually) exclusive' or 'cannot occur at the same time' seen or equivalent.
'Intersection is zero' or 'no overlaps' without further explanation is B0.

(b) M1 for use of Addition Formula, including an intersection, with at least one probability substituted. Intersection must be explicitly considered for this mark.
Accept $\frac{2}{3} = \frac{1}{4} + \mathrm{P}(B) - 0$ for M1.
M1 for $\mathrm{P}(A \cap B) = \frac{1}{4}\mathrm{P}(B)$
A1 for completely correct equation or equivalent.
A1 for $\frac{5}{9}$ or exact equivalent..
Venn Diagram with 2 overlapping closed curves and correct values possibly without $\frac{1}{3}$, award M1M1A1.

(c) M1 for $\frac{3}{4}$ x 'their P(*B*)' or 'their P(*B*)'- P($A \cap B$) or P($A \cup B$)-P(*B*) = $\frac{2}{3} - \frac{1}{4}$
Or P($A' \cap B$) = P(A') + 'their P(*B*)' – P($A' \cup B$) = $\frac{3}{4} + \frac{5}{9} - \frac{8}{9}$
A1 for $\frac{5}{12}$ or follow through from their method. Accept exact equivalent.
Correct answer only with no working M1A1 but must be clearly labelled (c).

(d)	M1 for using 1-'their P(B)' or (P($A \cup B$) – P(A))/P(A) or (P(A) – P($A \cap B$))/P(A) with a correct attempt at the numerator and denominator. If mutually exclusive is assumed then the last option gives $\frac{\frac{1}{4}}{\frac{1}{4}}$ for M1. A1 for $\frac{4}{9}$ or exact equivalent. For part (c) follow through their stated values; **do not** follow through incorrectly labelled regions on a Venn Diagram. Throughout the question we require probabilities between 0 and 1 for method marks. Venn Diagram: 	

Question Number	Scheme	Marks
35 (a)		
	3 closed curves and 25 in correct place	M1
	15,10,5	A1
	15,3,20	A1
	Labels *R*, *S* ,*C* and box	B1
	All values/100 or equivalent fractions award accuracy marks.	(4)
(b)	7/100 or 0.07 M1 for ('their 7'in diagram or here)/100	M1 A1 (2)
(c)	(3+5)/100 = 2/25 or 0.08	M1A1 (2)
(d)	(25+15+10+5)/100 = 11/20 or 0.55	M1 A1 (2)
(e)	$P(S \cap C' \mid R) = \frac{P(S \cap C' \cap R)}{P(R)}$ Require denominator to be 'their 65' or 'their $\frac{65}{100}$'	M1
	$= \frac{15}{65}$ require 'their 15' and correct denominator of 65	A1
	$= \frac{3}{13}$ or exact equivalents.	A1 (3)
		Total 13

NOTES	
(b)	M1 for 'their 7'/100 seen. A1 Correct answer only In parts (c) and (d) we require "/100" for methods to be awarded. Also check their values and award correct method if they follow from their Venn Diagram.
(c)	M1 For ('their 3'+'their 5')/100. $\frac{8}{48}$ award M0. A1 Correct answer only or equivalent.
(d)	M1 Accept sum of their 4 values from the Venn diagram /100. A1 Correct answer only or equivalent
(e)	M1 Attempt to use correct formula for conditional probability. Award for correct formula and a denominator of 'their 65' or 'their 65/100'. A1 for 'their 15'/65 only. A1 for exact equivalent answers, including 15/65. In all parts correct answers with no working award full marks.

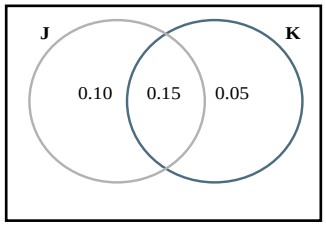
Question Number	Scheme	Marks
36.		
(a)	$P(J \cup K) = 1 - 0.7$ or $0.1 + 0.15 + 0.05 =$ $\underline{0.3}$	B1 (1)
(b)	$P(K) = 0.05 + 0.15$ or "0.3" – 0.25 + 0.15 or "0.3" = 0.25 + $P(K)$ – 0.15	M1
	May be seen on Venn diagram $= \underline{0.2}$	A1 (2)
(c)	$[P(K \mid J)] = \dfrac{P(K \cap J)}{P(J)}$	M1
	$= \dfrac{0.15}{0.25}$	A1
	$= \underline{\dfrac{3}{5} \text{ or } 0.6}$	A1 (3)
(d)	$P(J) \times P(K) = 0.25 \times 0.2 (= 0.05)$, $P(J \cap K) = 0.15$ or $P(K \mid J) = 0.6$, $P(K) = 0.2$ or may see $P(J\|K) = 0.75$ and $P(J) = 0.25$	M1
	not equal therefore not independent	A1ft (2)
(e)	Not independent so confirms the teacher's suspicion or they are linked (This requires a statement about independence in (d) or in (e))	B1ft (1)
		(9 marks)

Notes

(b)

M1 for a complete method, follow through their 0.3, leading to a linear equation for $P(K)$

NB You may see this Venn diagram.



A correct diagram (Venn or table) implies M1 in (b)
Need not include box or 0.7
Correct answer only is 2/2

In parts (c) and (d) they must have defined *A* and *B*

(c)

M1 for a correct expression (including ratio) in symbols.
1st A1 for a correct ratio of probabilities (if this is seen the M1 is awarded by implication)
Must be in (c). Condone no LHS but wrong LHS (e.g. $P(K)$ or $P(J|K)$) is M0A0
2nd A1 for correct answer as printed only. **Correct answer only 3/3**

Mark (d) and (e) together

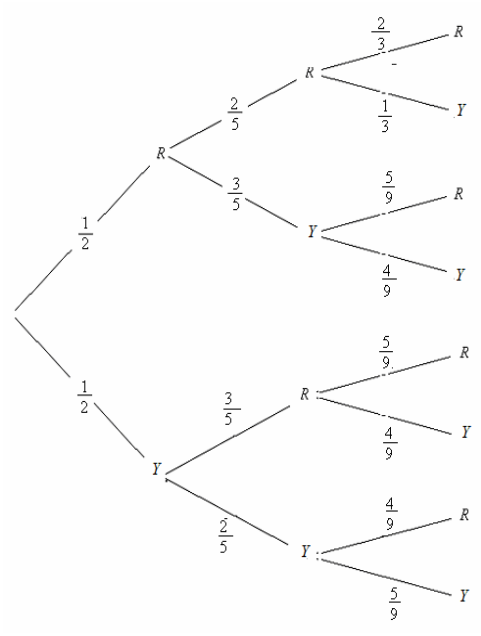
(d)

M1 for a correct comparison of known probabilities for an independence test - ft their values. E.g. $P(J) \times P(K)$ with $P(J \cap K)$ or $P(K|J)$ with $P(K)$ [Must have expressions]
The values of these probabilities should be given unless they are in the question or stated elsewhere.
A1ft for correct calculations and correct comment for their probabilities

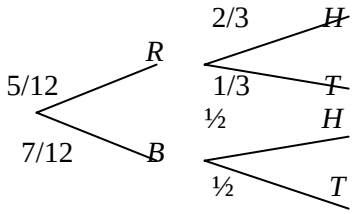
(e)

B1ft ft their conclusion on independence so not independent confirms teacher…independent contradicts teacher.

Methods leading to negative probabilities should score M0

Question Number	Scheme	Marks
37. (a)		B1 B1 B1 B1 (4)
(b)	$P(A) = P(RR) + P(YY) = \frac{1}{2}\times\frac{2}{5} + \frac{1}{2}\times"\frac{2}{5}" = \frac{2}{5}$ B1 for $\frac{1}{2}\times\frac{2}{5}$ (oe) seen at least once	B1 M1 A1 (3)
(c)	$P(B) = P(RRR) + P(RYR) + P(YRR) + P(YYR)$ M1 for at least 1 case of 3 balls identified. (Implied by 2nd M1) $\left(\frac{1}{2}\times\frac{2}{5}\times"\frac{2}{3}"\right)+\left(\frac{1}{2}\times\frac{3}{5}\times\frac{5}{9}\right)+\left(\frac{1}{2}\times"\frac{3}{5}"\times\frac{5}{9}\right)+\left(\frac{1}{2}\times"\frac{2}{5}"\times"\frac{4}{9}"\right) = \underline{\frac{5}{9}}$ **(*)**	M1 M1,A1cso (3)
(d)	$P(A\cap B) = P(RRR) + P(YYR)$ M1 for identifying both cases and + probs. may be implied by correct expressions $=\left(\frac{1}{2}\times\frac{2}{5}\times\frac{2}{3}\right)+\left(\frac{1}{2}\times\frac{2}{5}\times\frac{4}{9}\right) = \underline{\frac{2}{9}}$ **(*)**	M1 A1cso (2)
(e)	$P(A\cup B) = P(A) + P(B) - P(A\cap B)$ Must have some attempt to use $="\frac{2}{5}"+\frac{5}{9}-\frac{2}{9} = \underline{\frac{11}{15}}$	M1 A1cao (2)

Question Number	Scheme		Marks
37. (f)	$\dfrac{\mathrm{P}(RRR)}{\mathrm{P}(RRR)+\mathrm{P}(YYY)}=\dfrac{\frac{1}{2}\times\frac{2}{5}\times\frac{2}{3}}{\left(\frac{1}{2}\times\frac{2}{5}\times\frac{2}{3}\right)+\left(\frac{1}{2}\times\frac{2}{5}\times\frac{5}{9}\right)}=\dfrac{6}{11}$	Probabilities must come from the product of 3 probs. from their tree diagram.	M1 A1ft A1 cao (3) **[17]**
	Notes		
(b)	M1 for both cases, and +, attempted, ft their values from tree diagram. May be 4 cases of 3 balls.		
(c)	2nd M1 for all 4 correct expressions, ft their values from tree diagram. A1 is cso		
(e)	M1 for clear attempt to use the correct formula, must have some correct substitution. ft their (b)		
(f)	M1 for identifying the correct probabilities and forming appropriate fraction of probs. 1st A1ft for a correct expression using probabilities from their tree **Accept exact decimal equivalents. Correct answer only is full marks except in (c) and (d)**		

Question Number	Scheme	Marks
38 (a)	 P(R) and P(B) 2nd set of probabilities	B1 B1 (2)
(b)	P(H) = $\frac{5}{12}\times\frac{2}{3}+\frac{7}{12}\times\frac{1}{2},=\frac{41}{72}$ or awrt 0.569	M1 A1 (2)
(c)	P(R|H) = $\frac{\frac{5}{12}\times\frac{2}{3}}{"\frac{41}{72}"},=\frac{20}{41}$ or awrt 0.488	M1 A1ft A1 (3)
(d)	$\left(\frac{5}{12}\right)^2+\left(\frac{7}{12}\right)^2$ $=\frac{25}{144}+\frac{49}{144}=\frac{74}{144}$ or $\frac{37}{72}$ or awrt 0.514	M1 A1ft A1 (3) **Total 10**

(a) 1st B1 for the probabilities on the first 2 branches. Accept $0.41\dot{6}$ and $0.58\dot{3}$

2nd B1 for probabilities on the second set of branches. Accept $0.\dot{6}$, $0.\dot{3}$, 0.5 and $\frac{1.5}{3}$

Allow exact decimal equivalents using clear recurring notation if required.

(b) M1 for an expression for P(H) that follows through their sum of two products of **probabilities** from their tree diagram

(c)

Formula seen: M1 for $\frac{\text{P}(R\cap H)}{\text{P}(H)}$ with denominator their (b) substituted e.g. $\frac{\text{P}(R\cap H)}{\text{P}(H)}=\frac{\frac{5}{12}}{(\text{their (b)})}$ award M1.

Formula not seen: M1 for $\frac{\text{probability}\times\text{probability}}{\text{their } b}$ but M0 if fraction repeated e.g. $\frac{\frac{5}{12}\times\frac{2}{3}}{\frac{2}{3}}$.

1st A1ft for a fully correct expression or correct follow through

2nd A1 for $\frac{20}{41}$ o.e.

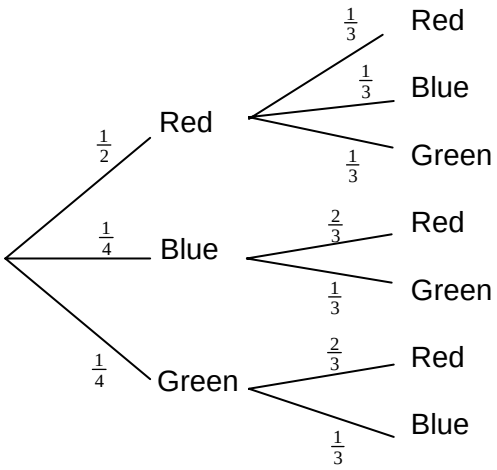
(d) M1 for $\left(\frac{5}{12}\right)^2$ or $\left(\frac{7}{12}\right)^2$ can follow through their equivalent values from tree diagram

1st A1 for both values correct or follow through from their original tree and +

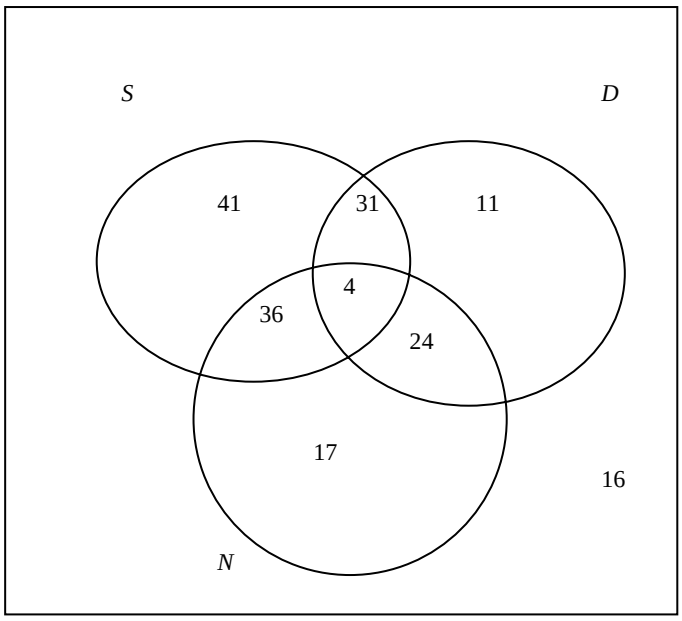
2nd A1 for a correct answer

Special Case $\frac{5}{12}\times\frac{4}{11}$ or $\frac{7}{12}\times\frac{6}{11}$ seen award M1A0A0

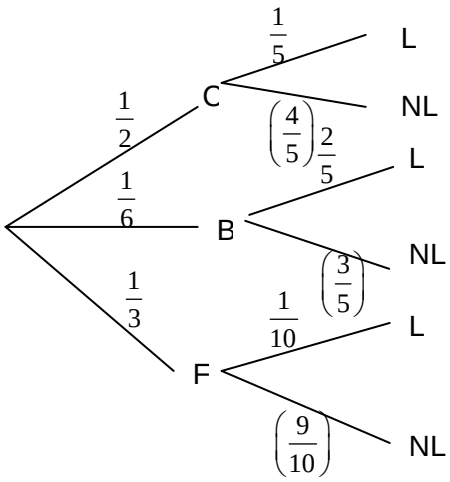
Question Number		Scheme	Marks
39	(a)	$\frac{2+3}{\text{their total}} = \frac{5}{\text{their total}} = \frac{1}{6}$ **(** given answer**)**	M1 A1cso (2)
	(b)	$\frac{4+2+5+3}{\text{total}}, \quad = \frac{14}{30}$ or $\frac{7}{15}$ or $0.4\dot{6}$	M1 A1 (2)
	(c)	$P(A \cap C) = 0$	B1 (1)
	(d)	$P(C \mid \text{reads at least one magazine}) = \frac{6+3}{20} = \frac{9}{20}$	M1 A1 (2)
	(e)	$P(B) = \frac{10}{30} = \frac{1}{3}$, $P(C) = \frac{9}{30} = \frac{3}{10}$, $P(B \cap C) = \frac{3}{30} = \frac{1}{10}$ or $P(B\|C) = \frac{3}{9}$	M1
		$P(B) \times P(C) = \frac{1}{3} \times \frac{3}{10} = \frac{1}{10} = P(B \cap C)$ or $P(B\|C) = \frac{3}{9} = \frac{1}{3} = P(B)$	M1
		So yes they are statistically independent	A1cso (3)
			Total 10
	(a)	M1 for $\frac{2+3}{\text{their total}}$ or $\frac{5}{30}$	
	(b)	M1 for adding at least 3 of "4, 2, 5, 3" and dividing by their total to give a probability Can be written as separate fractions substituted into the completely correct Addition Rule	
	(c)	B1 for 0 or 0/30	
	(d)	M1 for a **denominator of 20** or $\frac{20}{30}$ leading to an answer with denominator of 20 $\frac{9}{20}$ only, 2/2	
	(e)	1st M1 for attempting all the required probabilities for a suitable test 2nd M1 for use of a correct test - must have attempted all the correct probabilities. Equality can be implied in line 2. A1 for fully correct test carried out with a comment	

Question Number	Scheme	Marks
40 (a)		M1 A1 A1 (3)
(b)	P(Blue bead and a green bead) $=\left(\frac{1}{4}\times\frac{1}{3}\right)+\left(\frac{1}{4}\times\frac{1}{3}\right)=\frac{1}{6}$ (or any exact equivalent)	M1 A1 (2) **Total [5]**

40 (a)	M1 for shape and labels: 3 branches followed by 3,2,2 with some *R*, *B* and *G* seen Allow 3 branches followed by 3, 3, 3 if 0 probabilities are seen implying that 3, 2, 2 intended Allow blank branches if the other probabilities imply probability on blanks is zero Ignore further sets of branches 1st A1 for correct probabilities and correct labels on 1st set of branches. 2nd A1 for correct probabilities and correct labels on 2nd set of branches. (accept 0.33, 0.67 etc or better here)
(b)	M1 for identifying the 2 cases *BG* and *GB* and adding 2 products of probabilities. These cases may be identified by their probabilities e.g. $\left(\frac{1}{4}\times\frac{1}{3}\right)+\left(\frac{1}{4}\times\frac{1}{3}\right)$ NB $\frac{1}{6}$ (or exact equivalent) with no working scores 2/2
Special Case	<u>With Replacement</u> (This oversimplifies so do not apply Mis-Read: max mark 2/5) (a) B1 for 3 branches followed by 3, 3, 3 with correct labels and probabilities of $\frac{1}{2},\frac{1}{4},\frac{1}{4}$ on each. (b) M1 for identifying 2, possibly correct cases and adding 2 products of probabilities but A0 for wrong answer $\left[\left(\frac{1}{4}\times\frac{1}{4}\right)+\left(\frac{1}{4}\times\frac{1}{4}\right)\right]$ will be sufficient for M1A0 here but $\frac{1}{4}\times\frac{1}{2}+\ldots$ would score M0

Question Number	Scheme	Marks
41 (a)	 3 closed curves and 4 in centre Evidence of subtraction 31,36,24 41,17,11 Labels on loops, 16 and box	M1 M1 A1 A1 B1 (5)
(b)	P(None of the 3 options)= $\frac{16}{180}=\frac{4}{45}$	B1ft (1)
(c)	P(Networking only)= $\frac{17}{180}$	B1ft (1)
(d)	P(All 3 options/technician)= $\frac{4}{40}=\frac{1}{10}$	M1 A1 (2) **Total [9]**

41 (a)	2nd M1 There may be evidence of subtraction in "outer" portions, so with 4 in the centre then 35, 40 28 (instead of 31,36,24) along with 33, 9, 3 can score this mark but A0A0 N.B. This is a common error and their "16" becomes 28 but still scores B0 in part (a)
(b)	B1ft for $\frac{16}{180}$ or any exact equivalent. Can ft their "16" from their box. If there is no value for their "16" in the box only allow this mark if they have <u>shown</u> some working.
(c)	B1ft ft their "17". Accept any exact equivalent
(d)	If a probability greater than 1 is found in part (d) score M0A0 M1 for clear sight of $\frac{\mathrm{P}(S\cap D\cap N)}{\mathrm{P}(S\cap N)}$ and an attempt at one of the probabilities, ft their values. Allow P(all 3 | $S\cap N$) = $\frac{4}{36}$ or $\frac{1}{9}$ to score M1 A0. Allow a correct ft from their diagram to score M1A0 e.g. in 33,3,9 case in (a): $\frac{4}{44}$ or $\frac{1}{11}$ is M1A0 A ratio of probabilities with a <u>product</u> of probabilities on top is M0, even with a correct formula. A1 for $\frac{4}{40}$ or $\frac{1}{10}$ or an exact equivalent Allow $\frac{4}{40}$ or $\frac{1}{10}$ to score both marks if this follows from their diagram, otherwise some explanation (method) is required.

Question Number	Scheme	Marks
42 (a)	 Correct tree All labels Probabilities on correct branches	B1 B1 B1 (3)
(b)(i)	$\frac{1}{3}\times\frac{1}{10}=\frac{1}{30}$ or equivalent	M1 A1 (2)
(ii)	$\text{CNL + BNL + FNL} = \frac{1}{2}\times\frac{4}{5}+\frac{1}{6}\times\frac{3}{5}+\frac{1}{3}\times\frac{9}{10}$	M1
	$= \frac{4}{5}$ or equivalent	A1 (2)
(c)	$\text{P}(F'/L)=\frac{\text{P}(F'\cap L)}{\text{P}(L)}$ Attempt correct conditional probability **but see notes**	M1
	$= \frac{\frac{1}{6}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{5}}{1-(ii)}$ $\frac{\text{numerator}}{\text{denominator}}$	$\frac{\text{A1}}{\text{A1ft}}$
	$= \frac{\frac{5}{30}}{\frac{1}{5}} = \frac{5}{6}$ or equivalent cao	A1 (4) **[11]**
Notes	Exact decimal equivalents required throughout if fractions not used e.g. 42(b)(i) $0.0\dot{3}$ Correct path through their tree given in their probabilities award Ms 42(a) All branches required for first B1. Labels can be words rather than symbols for second B1. Probabilities from question enough for third B1 i.e. bracketed probabilities not required. Probabilities and labels swapped i.e. labels on branches and probabilities at end can be awarded the marks if correct. 42(b)(i) Correct answer only award both marks. 42(b)(ii) At least one correct path identified and attempt at adding all three multiplied pairs award M1 42(c) Require probability on numerator and division by probability for M1.Require numerator correct for their tree for M1. Correct formula seen and used, accept denominator as attempt and award M1 No formula, denominator must be correct for their tree or 1-(ii) for M1 1/30 on numerator only is M0, P(L/F') is M0.	

Question Number	Scheme	Marks
43(a) (i)	$P(A \cup B) = a + b$ cao	B1
(ii)	$P(A \cup B) = a + b - ab$ or equivalent	B1 (2)
(b)	$P(R \cup Q) = 0.15 + 0.35$ $= 0.5$ 0.5	B1 (1)
(c)	$P(R \cap Q) = P(R\|Q) \times P(Q)$ $= 0.1 \times 0.35$ M1 $= 0.035$ **0.035** A1 (2)	
(d)	$P(R \cup Q) = P(R) + P(Q) - P(R \cap Q)$ OR $P(R) = P(R \cap Q') + P(R \cap Q)$ $= 0.15$+their (c) $0.5 = P(R) + 0.35 - 0.035$ $= 0.15 + 0.035$ $P(R) = 0.185$ $= 0.185$ 0.185	M1 A1 (2) **[7]**
Notes	43(a) (i) Accept $a + b$ - 0 for B1 **Special Case** If answers to (i) and (ii) are (i) $P(A)+P(B)$ and (ii) $P(A)+P(B)-P(A)P(B)$ award B0B1 43(a)(i) and (ii) answers must be clearly labelled or in correct order for marks to be awarded.	

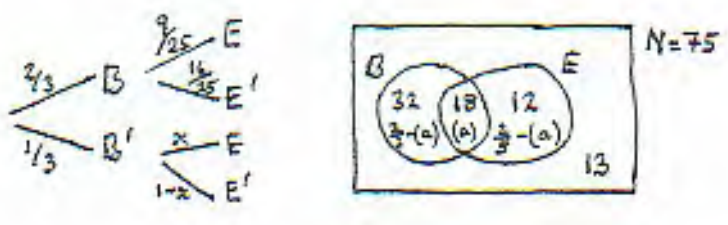
Question Number	Scheme	Marks
44 (a)	E = take regular exercise B = always eat breakfast $\mathrm{P}(E \cap B) = \mathrm{P}(E \mid B) \times \mathrm{P}(B)$ $= \frac{9}{25} \times \frac{2}{3} = 0.24$ or $\frac{6}{25}$ or $\frac{18}{75}$	M1 A1 (2)
(b)	$\mathrm{P}(E \cup B) = \frac{2}{3} + \frac{2}{5} - \frac{6}{25} = \frac{62}{75}$ | or $\mathrm{P}(E' \mid B') = \frac{13}{25}$ | or $\mathrm{P}(B' \cap E) = \frac{12}{75}$ | or $\mathrm{P}(B \cap E') = \frac{32}{75}$ $\mathrm{P}(E' \cap B') = 1 - \mathrm{P}(E \cup B) = \frac{13}{75}$ or $0.17\dot{3}$	M1 A1 M1 A1 (4)
(c)	$\mathrm{P}(E \mid B) = 0.36 \neq 0.40 = \mathrm{P}(E)$ or $\mathrm{P}(E \cap B) = \frac{6}{25} \neq \frac{2}{5} \times \frac{2}{3} = \mathrm{P}(E) \times \mathrm{P}(B)$ So E and B are <u>not</u> statistically independent	M1 A1 (2) **[8]**

(a) M1 for $\frac{9}{25} \times \frac{2}{3}$ or $\mathrm{P}(E|B) \times \mathrm{P}(B)$ <u>and</u> at least one correct value seen. A1 for 0.24 or exact equiv.
NB $\frac{2}{5} \times \frac{2}{3}$ alone or $\frac{2}{5} \times \frac{9}{25}$ alone scores M0A0. Correct answer scores full marks.

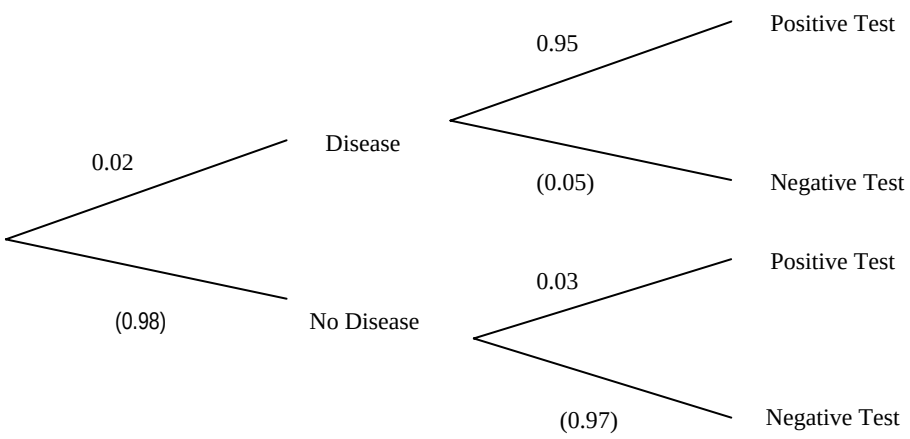
(b) 1st M1 for use of the addition rule. Must have 3 terms and some values, can ft their (a)
<u>Or</u> a full method for P($E'|B'$) requires 1 - P($E|B'$) and equation for P($E|B'$): (a) $+\frac{x}{3} = \frac{2}{5}$
<u>Or</u> a full method for $\mathrm{P}(B' \cap E)$ <u>or</u> $\mathrm{P}(B \cap E')$ [or other valid method]
2nd M1 for a method leading to answer e.g. $1 - \mathrm{P}(E \cup B)$
<u>or</u> $\mathrm{P}(B') \times \mathrm{P}(E' \mid B')$ <u>or</u> $\mathrm{P}(B') - \mathrm{P}(B' \cap E)$ <u>or</u> $\mathrm{P}(E') - \mathrm{P}(B \cap E')$
<u>Venn Diagram</u> 1st M1 for diagram with attempt at $\frac{2}{5} - \mathrm{P}(B \cap E)$ or $\frac{2}{3} - \mathrm{P}(B \cap E)$. Can ft their (a)
1st A1 for a correct first probability as listed or 32, 18 and 12 on Venn Diagram
2nd M1 for attempting 75 - their (18 +32 + 12)

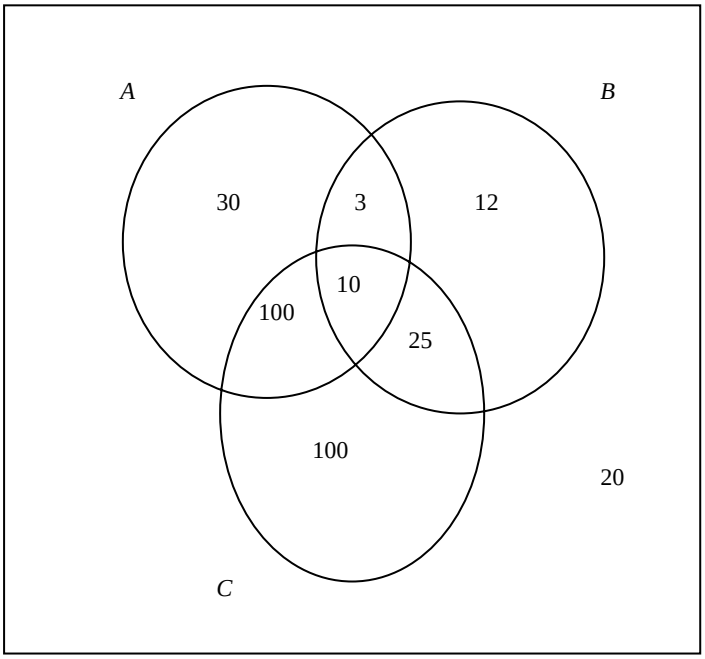
(c) M1 for identifying suitable values to test for independence e.g. P(E) = 0.40 and P($E|B$) = 0.36
<u>Or</u> $\mathrm{P}(E) \times \mathrm{P}(B) = \ldots$ and $\mathrm{P}(E \cap B) =$ their (a) [but their (a) $\neq \frac{2}{5} \times \frac{2}{3}$]. Values seen somewhere
A1 for correct values and a correct comment

<u>Diagrams</u> You may see these or find these useful for identifying probabilities.



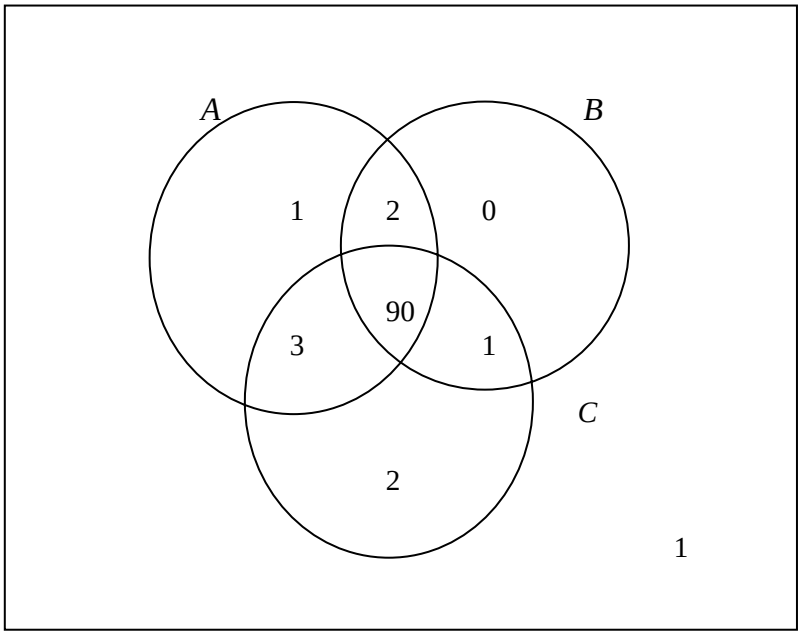
<u>Common Errors</u>
(a) $\frac{9}{25}$ is M0A0
(b) P(EUB)= $\frac{53}{75}$ scores M1A0
$1 - \mathrm{P}(E \cup B) = \frac{22}{75}$ scores M1A0
(b) $\mathrm{P}(B') \times \mathrm{P}(E') = \frac{1}{3} \times \frac{3}{5}$
scores 0/4

Question Number	Scheme	Marks
45 (a)		
	Tree without probabilities or labels	**M1**
	0.02(Disease), 0.95(Positive) on correct branches	**A1**
	0.03(Positive) on correct branch.	**A1**
		[3]
(b)	P(Positive Test) $=0.02\times0.95+0.98\times0.03$	**M1A1ft**
	$=0.0484$	**A1**
		[3]
(c)	P(Do not have disease | Postive test) $=\dfrac{0.98\times0.03}{0.0484}$	**M1**
	$=0.607438..$ awrt 0.607	**A1**
		[2]
(d)	Test not very useful OR High probability of not having the disease for a person with a positive test	**B1**
		[1]
		Total 9
	Notes: (a) M1:All 6 branches. Bracketed probabilities not required. (b) M1 for sum of two products, at least one correct from their diagram A1ft follows from the probabilities on their tree A1 for correct answer only or $\dfrac{121}{2500}$ (c) M1 for conditional probability with numerator following from their tree and denominator their answer to part (b). A1 also for $\dfrac{147}{242}$.	

Question Number	Scheme	Marks
46 (a)	 3 closed intersecting curves with labels 100 100,30 12,10,3,25 Box	**M1** **A1** **A1** **B1** **[4]**
(b)	P(Substance C) $= \frac{100+100+10+25}{300} = \frac{235}{300} = \frac{47}{60}$ or exact equivalent	**M1A1ft** **[2]**
(c)	P(All 3 $\|A$) $= \frac{10}{30+3+10+100} = \frac{10}{143}$ or exact equivalent	**M1A1ft** **[2]**
(d)	P(Universal donor) $= \frac{20}{300} = \frac{1}{15}$ or exact equivalent	**M1A1 cao** **[2]** **Total 10**
	Notes: (a) 20 not required. Fractions and exact equivalent decimals or percentages. (b) M1 For adding their positive values in C and finding a probability A1ft for correct answer or answer from their working (c) M1 their 10 divided by their sum of values in A A1ft for correct answer or answer from their working (d) M1 for 'their 20' divided by 300 A1 correct answer only	

47. **Diagram may be drawn with $B \subset (A \cup C)$ or with the 0 for $B \cap (A \cup C)'$ simply left blank**

(a)



Accept decimals or probs. in Venn diagram

3cc	**M1**
90,3,2,1	**A1**
1,(0),2	**M1A1**
1 outside	**A1**
Box	**B1**

(6)

(b) P(none)=0.01 **B1ft**

(1)

(c) P(*A* but not *B*)=0.04 **M1 A1ft**

(2)

(d) P(any wine but *C*)=0.03 **M1A1ft**

(2)

(e) P(exactly two)=0.06 **M1A1ft**

(2)

(f) $P(C|A) = \frac{P(C \cap A)}{P(A)} =, \frac{93}{96}$ or $\frac{31}{32}$ or AWRT 0.969 **M1A1ft,A1**

(3)

Total 16 marks

(a) 1st M1 for 3 closed, labelled curves that overlap. A1 for the 90, 3, 2 and 1
2nd M1 for one of 1, 0 or 2 correct or a correct sum of 4 values for *A, B* or *C*
2nd A1 for all 7 values correct. Accept a blank instead of 0.

NB final mark is a B1 for the box not an A mark as on EPEN

In parts (b) to (f) full marks can be scored for correct answers or correct ft

(b) B1ft Follow through their '1'from outside divided by 100

(c) M1 for correct expression eg $P(A \cup B) - P(B)$ or calculation e.g. 3 + 1 or 4 on top
A1 for a correct probability, follow through with their '3+1' from diagram

(d) M1 for correct expression or calculation e.g. 1+2+0 or 99-96 or 3 on top
A1 for a correct probability, follow through their '2+1+0' from diagram

(e) M1 for a correct expression or calculation e.g. 3+2+1 or 6 on top

For M marks in (c) to (e) they must have a fraction

(f) M1 for a correct expression upto "," and some correct substitution, ft their values. One of these probabilities must be correct or correct ft. If P(*C*) on bottom M0
1st A1ft follow through their $A \cap C$ and their *A* but the ratio must be in (0, 1)
2nd A1 for correct answer only. Answer only scores 3/3, but check working P($A \cap C$)/P(*C*) is M0

48.(a)	$P(R=3 \cap B=0)=\frac{1}{4}\times\frac{1}{4},=\frac{1}{16}$		**M1, A1** **(2)**
(b)			
		All 0s All 1,2,3s All 4,6,9s	**B1** **B1** **B1** **(3)**
(c)	$a=\frac{7}{16}, b=c=d=\frac{1}{16}$		**B1, B1 B1** **(3)**
			Total 8 marks

3	0	3	6	9
2	0	2	4	6
1	0	1	2	3
0	0	0	0	0
B / *R*	0	1	2	3

(a)	M1	for $\frac{1}{4}\times\frac{1}{4}$
(c)	1st B1	for $\frac{7}{16}$,
	2nd B1	for only one error in *b, c, d* ($b=c=d\neq\frac{1}{16}$ or $b=c=\frac{1}{16}\neq d$ etc), 3rd B1 all of $b, c, d = \frac{1}{16}$

Question Number	Scheme		Marks
49.(a)	44, 46, 48, 66, 68, 88 **NB** 64 is the same as 46, 84 is the same as 48, 86 is the same as 68	B1: At least 4 different pairs (ignore incorrect extras) B1: 6 different pairs with no incorrect extras	B1B1 (2)
(b)	$\bar{x}$ | 4 | 5 | 6 | 7 | 8 | $\frac{1}{2}\times\frac{3}{10}\times 2$ | $\frac{3}{10}\times\frac{3}{10}+\frac{1}{2}\times\frac{1}{5}\times 2$ | $\frac{3}{10}\times\frac{1}{5}\times 2$ | $P(\bar{X}=\bar{x})$ | $\frac{1}{4}$ | $\frac{3}{10}$ | $\frac{29}{100}$ | $\frac{3}{25}$ | $\frac{1}{25}$		B1 B1 M1 M1A1
	B1: 4,5,6,7,8 only no extras or omissions		
	B1: Writing or using $P(X=4)=\frac{1}{2}$, $P(X=6)=\frac{3}{10}$ and $P(X=8)=\frac{1}{5}$ May be seen in(a)		
	M1: A correct method for one of P(5), P(6) or P(7) may be implied by correct answer M1: A correct method for two of P(5), P(6) or P(7) may be implied by correct answer		
	A1: fully correct table/list -need 4,5,6,7, 8 and their associated probabilities		
			(5)
(c)	$1-\left(\frac{24}{25}\right)^n > 0.9$ **or** $\left(\frac{24}{25}\right)^n < 0.1$ **oe**	M1: $1-\left(\frac{24}{25}\right)^n > 0.9$ **or** $\left(\frac{24}{25}\right)^n < 0.1$ oe seen or used may use = or ≤ instead of < = or ≥ instead of > Do Not award $\left(\frac{24}{25}\right)^n > 0.1$ **oe**	M1
	$n > 56.4$	A1: Ignore any $n>$, $n<$, $n=$ *etc.* Award if you see awrt 56.4 may be implied by $n = 57$	A1
	$n = 57$	A1: cao $n = 57$ or 57 on its own. Do not allow $n > 57$ or $n < 57$. Do not award if alternative values are given. You must check there is no incorrect working	A1
	Alternative – trial and error 50 | 0.87 | 0.13 51 | 0.865 | 0.125 52 | 0.88 | 0.12 53 | 0.885 | 0.115 54 | 0.89 | 0.11 55 | 0.894 | 0.106 56 | 0.898 | 0.102 57 | 0.902 | 0.098 58 | 0.906 | 0.094 59 | 0.91 | 0.09 60 | 0.94 | 0.086 Allow awrt	M1 at least 2 trials for $50 \le n \le 60$ shown with correct probabilities A1 trial for $n = 56$ and 57 shown with correct probabilities	M1 A 1
	$n = 57$	A1: cao $n = 57$ or 57 on its own. Do not allow $n > 57$ or $n < 57$. Do not award if alternative values are given	A1 (3)
			Total 10

Question Number	Scheme		Marks
		Notes	
50	**NB: If there is a fully correct table award full marks.**		
	P(10) = 0.2, P(20) = 0.4 and P(50) = 0.4	B1: using P(10) = 0.2 (p) P(20) = 0.4(q) and P(50) = 0.4(r) may be seen in calculations or implied by a correct probability.	B1
	Median 10, 20, 50	B1: three correct medians and no extras.	B1
	P(Median 10) = $0.2^3+3\times0.2^2\times0.4+3\times0.2^2\times0.4$ **or** $0.2^3+3\times0.2^2\times0.8$	M1: allow if $(p+q+r)=1$ and use $p^3+3\times p^2\times q+3\times p^2\times r$ **or** $p^3+3\times p^2\times(q+r)$ look for $\frac{1}{125}+\frac{6}{125}+\frac{6}{125}$	See below for how to award
	P(Median 50) = $0.4^3+3\times0.4^2\times0.2+3\times0.4^2\times0.4$ **or** $0.4^3+3\times0.4^2\times0.6$	M1: allow if $(p+q+r)=1$ and use $r^3+3\times r^2\times p+3\times r^2\times q$ **or** $r^3+3\times r^2\times(p+q)$ Look for $\frac{8}{125}+\frac{12}{125}+\frac{24}{125}$	
	P(Median 20) = $3\times0.2\times0.4^2+6\times0.2\times0.4\times0.4+0.4^3+3\times0.4^2\times0.4$	M1: allow if $(p+q+r)=1$ and use $3\times p\times q^2+6\times p\times q\times r+q^3+3\times q^2\times r$ $\frac{12}{125}+\frac{24}{125}+\frac{8}{125}+\frac{24}{125}$	
	<u>How to award the M marks</u> – Allow the use of 1, 2 and 5 for the medians for the method marks **M1** any correct calculation (implied by correct answer) for P(m = 10) or P(m = 20) or P(m = 50) **M1** any 2 correct calculations (implied by 2 correct answers) P(m = 10) or P(m = 20) or P(m = 50) **M1** any 3 correct calculations (implied by 3 correct answers) for P(m = 10) and P(m = 20) and P(m = 50) or 3 probabilities that add up to 1 providing it is 1 – their 2 other calculated probabilities. Do **not** allow $\frac{1}{5}\ \frac{2}{5}\ \frac{2}{5}$ **NB** if they do not have a correct answer their working must be clear including the addition signs.		
	median | 10 | 20 | 50 | 0.104 | 0.544 | 0.352 | Or $\frac{13}{125}$ | Or $\frac{68}{125}$ | Or $\frac{44}{125}$	A1: awrt any 1 correct A2: awrt all 3 correct These do not need to be in a table as long as the correct probablity is with the correct median(10, 20 & 50) **NB: Do Not allow the use of 1,2 and 5 for the medians for the A marks**	A2

Question Number	Scheme	Marks
51(a)	(5,5,5) or (1,5,5) or (2,5,5) (5,5,5) (5,5,1) (5,1,5) (1,5,5) (5,5,2) (5,2,5) (2,5,5) or (5,5,5) **and** (5,5,1) ($\times 3$) **and** (5,5,2) ($\times 3$)	B1 B1 (2)
51(b)	(5,5,5) $\left(\frac{3}{10}\right)^3 = \frac{27}{1000} = 0.027$ (5,5,1) $3\times\frac{1}{2}\times\left(\frac{3}{10}\right)^2 = \frac{135}{1000} or \frac{27}{200} = 0.135$ (5,5,2) $3\times\frac{1}{5}\times\left(\frac{3}{10}\right)^2 = \frac{54}{1000} = \frac{27}{500} = 0.054$ $P(M=5) = \left(\frac{3}{10}\right)^3 + 3\times\frac{1}{2}\times\left(\frac{3}{10}\right)^2 + 3\times\frac{1}{5}\times\left(\frac{3}{10}\right)^2 = \frac{27}{125} = 0.216$ oe	B1 M1 A1A1 (4)
51(c)	$P(M=1) = (0.5)^3 + 3(0.5)^2(0.2) + 3(0.5)^2(0.3)$ $= 0.5$ $P(M=2) = \left(\frac{1}{5}\right)^3 + 3\times\left(\frac{1}{5}\right)^2\times\frac{1}{2} + 3\times\left(\frac{1}{5}\right)^2\times\frac{3}{10} + 6\times\frac{1}{2}\times\frac{1}{5}\times\frac{3}{10}$ $= 0.284$ or $\frac{71}{250}$ oe m | 1 | 2 | 5 $P(M=m)$ | 0.5 | 0.284 | 0.216	M1 A1 M1 A1 A1 (5) Total 11 marks
	Notes	
51(a)	1st B1 for two of the given triples, any order 2nd B1 for all 7 cases. no incorrect extras	
51(b)	B1 $\left(\frac{3}{10}\right)^3 or$ 0.027 oe. This can be a single term in a summation M1 either "3" $\times\frac{1}{2}\times\left(\frac{3}{10}\right)^2$ **or** "3" $\times\frac{1}{5}\times\left(\frac{3}{10}\right)^2$ oe. May omit the 3 × or have another positive integer in place of the 3. These may be seen as a single term in a summation A1 $\left(\frac{3}{10}\right)^3 + 3\times\frac{1}{2}\times\left(\frac{3}{10}\right)^2 + 3\times\frac{1}{5}\times\left(\frac{3}{10}\right)^2$ oe A1 0.216 oe	
51(c)	1st M1 correct calculation for P(M = 1) **or** P(M = 2), working must be shown and **not** implied by a correct answer. 1st A1 either P(M = 1) **or** P(M = 2) correct 2nd M1 correct calculation for both P(M = 1) **and** P(M = 2), **or** their probabilities adding up to 1, but do not allow probabilities of 0.5, 0.2 and 0.3 2nd A1 both P(M = 1) **and** P(M = 2) correct 3rd A1dep on both M marks awarded. All three values written down with their correct probabilities. They must be in part (c) but they do not need to be in a table. **NB** A fully correct table with no working will get M0 A0 M1 A1 A0.	

Question Number	Scheme	Marks
52.		
(a)	(1, 1, 1), (5, 5, 5), (1, 5, 5), (1, 5, 1) (1,1,1); (5,5,5); (1, 5, 5); (5, 1, 5); (5, 5, 1) (5, 1, 1); (1, 5, 1); (1, 1, 5)	B1 B1 **(2)**
(b)	r : 0 and 4 $P(R = 0) = \frac{9}{27}$ or $\frac{1}{3}$ $\quad$ $P(R = 4) = \frac{18}{27}$ or $\frac{2}{3}$	B1 M1d A1 **(3)**
		[5]
	Notes	
(a)	1st B1 for any two of the triples 2nd B1 for all 8 cases. No incorrect extras – condone repeats. Allow (1, 5, 5) (x 3) and (1, 1, 5) (x 3) instead of writing all three cases down	
(b)	B1 for both values of r M1 d dependent on previous B1. For an attempt to evaluate one of the probabilities for r correctly e.g. for $r = 0$; $\left(\frac{2}{3}\right)^3 + \left(\frac{1}{3}\right)^3$ and for $r = 4$; $3\times\left(\frac{1}{3}\right)^2\times\left(\frac{2}{3}\right) + 3\times\left(\frac{1}{3}\right)\times\left(\frac{2}{3}\right)^2$ **Working must be shown.** A1 for both values of r and their correct corresponding probabilities. Allow awrt 0.333 and 0.667 **NB** Correct answer with no working will gain B1M0A0	

Question Number	Scheme		Marks
31	Attempt to write down combinations	at least one seen	**M1**
	(1,1,1), (1,1,2) any order (1,2,2) any order, (2,2,2)	no extra combinations	**A1**
	Range 0 and 1	0 and 1 only	**B1**
	[P(range = 0) =] $(0.65)^3 + (0.35)^3$	either range	**M1**
	$= 0.3175$ or $\frac{127}{400}$		**A1cao**
	[P(range = 1) =] $(0.35)^2(0.65) \times 3 + (0.65)^2(0.35) \times 3$		
	$= 0.6825$ or $\frac{273}{400}$		**A1cao**
			(6)
			Total 6

Notes

First M1 may be implied by either $(0.65)^3$ or $(0.35)^3$or $(0.65)^2(0.35)$ or $(0.35)^2(0.65)$
First A1 may be implied by $(0.65)^3$ **and** $(0.35)^3$ **and** $(0.65)^2(0.35)$ **and** $(0.35)^2(0.65)$
No need for x3
2nd M1 $(p)^3 + (1-p)^3$ or $(1-p)^2(p) \times 3 + (p)^2(1-p) \times 3$
A1 for 0.3175 cao or exact equivalent e.g $\frac{254}{800}$
A1 for 0.6825 cao or exact equivalent e.g $\frac{546}{800}$
NB These probabilities do not need to be associated with the correct range