EXPERT TUITION

Maths Questions By Topic:

Proofs Mark Scheme

A-Level Edexcel

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General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

General Instructions for Marking

1. The total number of marks for the paper is 100

2. These mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- **bod** – benefit of doubt
- **ft** – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- **cao** – correct answer only
- **cso** - correct solution only. There must be no errors in this part of the question to obtain this mark
- **isw** – ignore subsequent working
- **awrt** – answers which round to
- **SC**: special case
- **o.e.** – or equivalent (and appropriate)
- **d** or **dep** – dependent
- **indep** – independent
- **dp** decimal places
- **sf** significant figures
- ***** The answer is printed on the paper or ag- answer given

4. All M marks are follow through.

A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.

If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but the response is deemed to be valid, examiners must escalate the response for a senior examiner to review.

Question	Scheme	Marks	AOs
1(i)	$n=1,\ 2^3=8,\ 3^1=3,\ (8>3)$ $n=2,\ 3^3=27,\ 3^2=9,\ (27>9)$ $n=3,\ 4^3=64,\ 3^3=27,\ (64>27)$ $n=4,\ 5^3=125,\ 3^4=81,\ (125>81)$	M1	2.1
	So if $n \leqslant 4, n \in \mathbb{N}$ then $(n+1)^3 > 3^n$	A1	2.4
		(2)	
(ii)	Begins the proof by negating the statement. "Let m be odd " or "Assume m is not even"	M1	2.4
	Set $m=(2p\pm1)$ and attempt $m^3+5=(2p\pm1)^3+5=...$	M1	2.1
	$=8p^3+12p^2+6p+6$ AND deduces even	A1	2.2a
	Completes proof which requires reason and conclusion • reason for $8p^3+12p^2+6p+6$ being even • acceptable statement such as "this is a contradiction so if m^3+5 is odd then m must be even"	A1	2.4
		(4)	
			(6 marks)

Notes

(i)

M1: A full and rigorous argument that uses all of n = 1, 2, 3 and 4 in an attempt to prove the given result. Award for attempts at both $(n+1)^3$ and 3^n for **ALL** values with at least 5 of the 8 values correct.

There is no requirement to compare their sizes, for example state that $27>9$

Extra values, say $n=0$, may be ignored

A1: Completes the proof with no errors and an appropriate/allowable conclusion.

This requires

- all the values for n =1, 2, 3 and 4 correct. Ignore other values
- all pairs compared correctly
- a minimal conclusion. Accept ✓ or hence proven for example

(ii)

M1: Begins the proof by negating the statement. See scheme

This cannot be scored if the candidate attempts m both odd and even

M1: For the key step in setting $m=2p\pm1$ and attempting to expand $(2p\pm1)^3+5$

Award for a 4 term cubic expression.

A1: Correctly reaches $(2p+1)^3+5=8p^3+12p^2+6p+6$ and **states** even.

Alternatively reaches $(2p-1)^3+5=8p^3-12p^2+6p+4$ and **states** even.

A1: A full and complete argument that completes the contradiction proof. See scheme.

(1) **A reason** why the expression $8p^3+12p^2+6p+6$ or $8p^3-12p^2+6p+4$ is even

Acceptable reasons are

- all terms are even
- sight of a factorised expression E.g. $8p^3-12p^2+6p+4=2(4p^3-6p^2+3p+2)$

(2) Acceptable concluding statement

Acceptable concluding statements are

- "this is a contradiction, so if m^3+5 is odd then m is even"
- "this is contradiction, so proven."
- "So if m^3+5 is odd them m is even"

S.C If the candidate misinterprets the demand and does not use proof by contradiction but states a counter example to the statement "if m^3+5 is odd then m must be even" such as when $m=\sqrt[3]{2}$ then they can score special case mark B1

Question	Scheme	Marks	AOs
2(a)	Selects a correct strategy. E.g uses an odd number is $2k \pm 1$	B1	3.1a
	Attempts to simplify $(2k \pm 1)^3 - (2k \pm 1) = ...$	M1	2.1
	……….and factorise $8k^3 \pm 12k^2 \pm 4k = 4k(2k^2 \pm 3k \pm 1) =$	dM1	1.1b
	Correct work with statement $4 \times ..$ is a multiple of 4	A1	2.4
		(4)	
(b)	Any counter example with correct statement. Eg. $2^3 - 2 = 6$ which is not a multiple of 4	B1	2.4
		(1)	
			(5 marks)
Alt (a)	Selects a correct strategy. Factorises $k^3 - k = k(k-1)(k+1)$	B1	3.1a
	States that if k is odd then both $k-1$ and $k+1$ are even	M1	2.1
	States that $k-1$ multiplied by $k+1$ is therefore a multiple of 4	dM1	1.1b
	Concludes that $k^3 - k$ is a multiple of 4 as it is odd $\times$ multiple of 4	A1	2.4
		(4)	

Notes:

(a)
Note: May be in any variable (condone use of *n*)

B1: Selects a correct strategy. E.g uses an odd number is $2k \pm 1$

M1: Attempts $(2k \pm 1)^3 - (2k \pm 1) = ...$ Condone errors in multiplying out the brackets and invisible brackets for this mark. Either the coefficient of the *k* term or the constant of $(2k \pm 1)^3$ must have changed from attempting to simplify.

dM1: Attempts to take a factor of 4 or 4*k* from their cubic

A1: Correct work with statement $4 \times ..$ is a multiple of 4

(b)

B1: Any counter example with correct statement.

Question	Scheme	Marks	AOs
3	Sets up the contradiction and factorises: There are positive integers p and q such that $$(2p+q)(2p-q)=25$$	M1	2.1
	If true then $\begin{matrix} 2p+q=25 \\ 2p-q=1 \end{matrix}$ or $\begin{matrix} 2p+q=5 \\ 2p-q=5 \end{matrix}$ **Award for deducing either of the above statements**	M1	2.2a
	Solutions are $p=6.5, q=12$ or $p=2.5, q=0$ Award for one of these	A1	1.1b
	This is a contradiction as there are no integer solutions hence there are no positive integers p and q such that $4p^2-q^2=25$	A1	2.1
		(4)	
	(4 marks)		
Notes:			

M1: For the key step in setting up the contradiction and factorising

M1: For deducing that for p and q to be integers then either $\begin{matrix} 2p+q=25 \\ 2p-q=1 \end{matrix}$ or $\begin{matrix} 2p+q=5 \\ 2p-q=5 \end{matrix}$ must be true.

Award for deducing either of the above statements.

You can ignore any reference to $\begin{matrix} 2p+q=1 \\ 2p-q=25 \end{matrix}$ as this could not occur for positive p and q.

A1: For correctly solving one of the given statements,

For $\begin{matrix} 2p+q=25 \\ 2p-q=1 \end{matrix}$ candidates only really need to proceed as far as $p=6.5$ to show the contradiction.

For $\begin{matrix} 2p+q=5 \\ 2p-q=5 \end{matrix}$ candidates only really need to find either p or q to show the contradiction.

Alt for $\begin{matrix} 2p+q=5 \\ 2p-q=5 \end{matrix}$ candidates could state that $2p+q \neq 2p-q$ if p, q are positive integers.

A1: For a complete and rigorous argument with both possibilities and a correct conclusion.

Question	Scheme	Marks	AOs
3 Alt 1	Sets up the contradiction, attempts to make q^2 or $4p^2$ the subject and states that either $4p^2$ is even(*) , or that q^2 (or q) is odd (**) Either There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow q^2 = 4p^2 - 25$ with * or ** Or There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow 4p^2 = q^2 + 25$ with * or **	M1	2.1
	Sets $q = 2n \pm 1$ and expands $(2n \pm 1)^2 = 4p^2 - 25$	M1	2.2a
	Proceeds to an expression such as $4p^2 = 4n^2 + 4n + 26 = 4(n^2 + n + 6) + 2$ $4p^2 = 4n^2 + 4n + 26 = 4(n^2 + n) + \frac{13}{2}$ $p^2 = n^2 + n + \frac{13}{2}$	A1	1.1b
	States This is a contradiction as $4p^2$ must be a multiple of 4 Or p^2 must be an integer And concludes there are no positive integers p and q such that $4p^2 - q^2 = 25$	A1	2.1
		(4)	

Alt 2

An approach using odd and even numbers is unlikely to score marks.

To make this consistent with the Alt method, score

M1: Set up the contradiction and start to consider one of the cases below where q is odd, $m \neq n$.

Solutions using the same variable will score no marks.

M1: Set up the contradiction and start to consider BOTH cases below where q is odd, $m \neq n$.

No requirement for evens

A1: Correct work and deduction for one of the two scenarios where q is odd

A1: Correct work and deductions for both scenarios where q is odd with a final conclusion

Options	Example of Calculation	Deduction
p (even) q (odd)	$4p^2 - q^2 = 4 \times (2m)^2 - (2n+1)^2 = 16m^2 - 4n^2 - 4n - 1$	One less than a multiple of 4 so cannot equal 25
p (odd) q (odd)	$4p^2 - q^2 = 4 \times (2m+1)^2 - (2n+1)^2 = 16m^2 + 16m - 4n^2 - 4n + 3$	Three more than a multiple of 4 so cannot equal 25

Question	Scheme	Marks	AOs
4 (a)	States $(2a-b)^2 \geqslant 0$	M1	2.1
	$4a^2+b^2 \geqslant 4ab$	A1	1.1b
	(As $a>0, b>0$) $\dfrac{4a^2}{ab}+\dfrac{b^2}{ab} \geqslant \dfrac{4ab}{ab}$	M1	2.2a
	Hence $\dfrac{4a}{b}+\dfrac{b}{a} \geqslant 4$ * CSO	A1*	1.1b
		(4)	
(b)	$a=5, b=-1 \Rightarrow \dfrac{4a}{b}+\dfrac{b}{a}=-20-\dfrac{1}{5}$ which is less than 4	B1	2.4
		(1)	
			(5 marks)

Notes

(a) (condone the use of $>$ for the first three marks)

M1: For the key step in stating that $(2a-b)^2 \geqslant 0$

A1: Reaches $4a^2+b^2 \geqslant 4ab$

M1: Divides each term by $ab \Rightarrow \dfrac{4a^2}{ab}+\dfrac{b^2}{ab} \geqslant \dfrac{4ab}{ab}$

A1*: Fully correct proof with steps in the correct order and gives the reasons why this is true:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a>0$ and $b>0$

(b)

B1: Provides a counter example and shows it is not true.
This requires values, a calculation or embedded values(see scheme) and a conclusion. The conclusion must be in words eg the result does not hold or not true
Allow 0 to be used as long as they explain or show that it is undefined so the statement is not true.

....

Proof by contradiction: Scores all marks

M1: Assume that there exists an $a, b > 0$ such that $\frac{4a}{b} + \frac{b}{a} < 4$

A1: $4a^2 + b^2 < 4ab \Rightarrow 4a^2 + b^2 - 4ab < 0$

M1: $(2a - b)^2 < 0$

A1*: States that this is not true, hence we have a contradiction so $\frac{4a}{b} + \frac{b}{a} \ldots 4$ with the following reasons given:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a > 0$ and $b > 0$

....

Attempt starting with the left-hand side

M1: $(\text{lhs} =) \frac{4a}{b} + \frac{b}{a} - 4 = \frac{4a^2 + b^2 - 4ab}{ab}$

A1: $= \frac{(2a-b)^2}{ab}$

M1: $= \frac{(2a-b)^2}{ab} \ldots 0$

A1*: Hence $\frac{4a}{b} + \frac{b}{a} - 4 \ldots 0 \Rightarrow \frac{4a}{b} + \frac{b}{a} \ldots 4$ with the following reasons given:

- when you square any (real) number it is always greater than or equal to zero
- ab is positive as $a > 0$ and $b > 0$

....

Attempt using given result: For 3 out of 4

$\frac{4a}{b} + \frac{b}{a} \ldots 4$ M1 $\Rightarrow 4a^2 + b^2 \ldots 4ab \Rightarrow 4a^2 + b^2 - 4ab \ldots 0$

A1 $\Rightarrow (2a - b)^2 \ldots 0$ oe

M1 gives both reasons why this is true

- "square numbers are greater than or equal to 0"
- "multiplying by ab does not change the sign of the inequality because a and b are positive"

Question 5

General points for marking question 5 (i):

- Students who just try random numbers in part (i) are not going to score any marks.
- Students can mix and match methods. Eg you may see odd numbers via logic and even via algebra
- Students who state $4m^2+2$ **cannot be divided** by (instead of is not divisible by) cannot be awarded credit for the accuracy/explanation marks, unless they state correctly that $4m^2+2$ **cannot be divided by 4 to give an integer.**
- Students who write $n^2+2=4k \Rightarrow k=\frac{1}{4}n^2+\frac{1}{2}$ which is not a whole number gains no credit unless they then start to look at odd and even numbers for instance
- Proofs via induction usually tend to go nowhere unless they proceed as in the main scheme
- Watch for unusual methods that are worthy of credit (See below)
- If the final conclusion is $n \in \mathbb{R}$ then the final mark is withheld. $n \in \mathbb{Z}^+$ is correct

Watch for methods that may not be in the scheme that you feel may deserve credit.

If you are uncertain of a method please refer these up to your team leader.

Eg 1. Solving part (i) by modulo arithmetic.

All $n \in \mathbb{N}$ mod 4	0	1	2	3
All $n^2 \in \mathbb{N}$ mod 4	0	1	0	1
All $n^2+2 \in \mathbb{N}$ mod 4	2	3	2	3

Hence for all n, n^2+2 is not divisible by 4.

Question 5 (i)	Scheme	Marks	AOs

Notes: Note that M0 A0 M1 A1 and M0 A0 M1 A0 are not possible due to the way the scheme is set up
(i)
M1: Awarded for setting up the proof for either the even or odd numbers.
A1: Concludes correctly with a reason why n^2+2 cannot be divisible by 4 for either n odd or even.
dM1: Awarded for setting up the proof for both even and odd numbers
A1: Fully correct proof with valid explanation and conclusion for all n

Example of an algebraic proof

For $n=2m,\quad n^2+2=4m^2+2$	M1	2.1
Concludes that this number is not divisible by 4 (as the explanation is trivial)	A1	1.1b
For $n=2m+1,\quad n^2+2=(2m+1)^2+2=\ldots$ FYI $(4m^2+4m+3)$	dM1	2.1
Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4(m^2+m)+3$ ....AND states ......hence true for all	A1*	2.4
	(4)	

Example of a very similar algebraic proof

For $n = 2m$, $\dfrac{4m^2+2}{4} = m^2 + \dfrac{1}{2}$	M1	2.1
Concludes that this is not divisible by 4 due to the $\dfrac{1}{2}$ (A suitable reason is required)	A1	1.1b
For $n = 2m+1$, $\dfrac{n^2+2}{4} = \dfrac{4m^2+4m+3}{4} = m^2+m+\dfrac{3}{4}$	dM1	2.1
Concludes that this is not divisible by 4 due to the $\dfrac{3}{4}$ ...AND states ...... hence for all n, n^2+2 is not divisible by 4	A1*	2.4
	(4)	

Example of a proof via logic

When n is odd, "odd $\times$odd" = odd	M1	2.1
so n^2+2 is odd , so (when n is odd) n^2+2 cannot be divisible by 4	A1	1.1b
When n is even, it is a multiple of 2, so "even $\times$even" is a multiple of 4	dM1	2.1
Concludes that when n is even n^2+2 cannot be divisible by 4 because n^2 is divisible by 4…..AND STATES ……..trues for all n.	A1*	2.4
	(4)	

Example of proof via contradiction

Sets up the contradiction ‘Assume that n^2+2 is divisible by 4 $\Rightarrow n^2+2=4k$ ’	M1	2.1
$\Rightarrow n^2 = 4k-2 = 2(2k-1)$ and concludes even Note that the M mark (for setting up the contradiction must have been awarded)	A1	1.1b
States that n^2 is even, then n is even and hence n^2 is a multiple of 4	dM1	2.1
Explains that if n^2 is a multiple of 4 then n^2+2 cannot be a multiple of 4 and hence divisible by 4 Hence there is a contradiction and concludes Hence true for all n.	A1*	2.4
	(4)	

A similar proof exists via contradiction where

A1: $n^2 = 2(2k-1) \Rightarrow n = \sqrt{2}\times\sqrt{2k-1}$

dM1: States that $2k-1$ is odd, so does not have a factor of 2, meaning that n is irrational

Question 5 (ii)	Scheme	Marks	AOs

(ii)

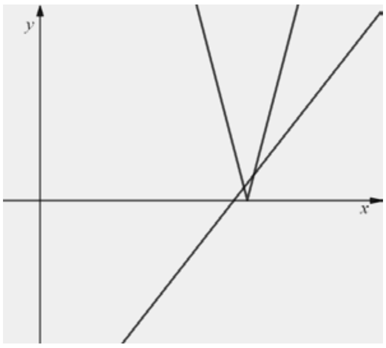
M1: States or implies 'sometimes true' or 'not always true' and gives an example where it is not true.

A1: and gives an example where it is true,

Proof using numerical values

SOMETIMES TRUE and chooses any number $x: 9.25 < x < 9.5$ and shows false Eg $x = 9.4$ $\|3x-28\| = 0.2$ and $x-9=0.4$ ×	M1	2.3
Then chooses a number where it is true Eg $x=12$ $\|3x-28\|=8$ $x-9=3$ ✓	A1	2.4
	(2)	

Graphical Proof

 States or implies ''sometimes true'' Sketches both graphs on the same axes. Expect shapes and relative positions to be correct. V shape on +ve x -axis Linear graph with +ve gradient intersecting twice	M1	2.3
Graphs accurate and explains that as there are points where $\|3x-28\| < x-9$ and points where $\|3x-28\| > x-9$ oe in words like 'above' and 'below' or 'dips below at one point'	A1	2.4
	(2)	

Proof via algebra

States sometimes true and attempts to solve both $3x-28 < x-9$ and $-3x+28 < x-9$ or one of these with the bound $9.\dot{3}$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	

Alt: It is possible to find where it is always true

States sometimes true and attempts to solve where it is just true Solves both $3x-28 \geqslant x-9$ and $-3x+28 \geqslant x-9$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	

Question 6

Score as below so M0 A0 M1 A1 or M1 A0 M1 A1 are not possible

Generally the marks are awarded for
M1: Suitable approach to answer the question for n being even **OR** odd
A1: Acceptable proof for n being even **OR** odd
M1: Suitable approach to answer the question for n being even **AND** odd
A1: Acceptable proof for n being even **AND** odd **WITH** concluding statement.

There is no merit in a

- student taking values, or multiple values, of n and then drawing conclusions.
  So $n=5 \Rightarrow n^3+2=127$ which is not a multiple of 8 scores no marks.
- student using divided when they mean divisible. Eg. "Odd numbers cannot be divided by 8" is incorrect. We need to see either "odd numbers are not divisible by 8" or "odd numbers cannot be divided by 8 **exactly**"
- stating $\frac{n^3+2}{8}=\frac{1}{8}n^3+\frac{1}{4}$ which is not a whole number
- stating $\frac{(n+1)^3+2}{8}=\frac{1}{8}n^3+\frac{3}{8}n^2+\frac{3}{8}n+\frac{3}{8}$ which is not a whole number

There must be an attempt to generalise either logic or algebra.

Example of a logical approach

Logical approach	States that if n is odd, n^3 is odd	M1	2.1
	so n^3+2 is odd and therefore cannot be divisible by 8	A1	2.2a
	States that if n is even, n^3 is a multiple of 8	M1	2.1
	so n^3+2 cannot be a multiple of 8 So (Given $n \in \mathbb{N}$), n^3+2 is not divisible by 8	A1	2.2a
		(4)	
			4 marks

First M1: States the result of cubing an odd or an even number
First A1: Followed by the result of adding two and gives a valid reason why it is not divisible by 8.
So for odd numbers accept for example

"odd number + 2 is still odd and odd numbers are not divisible by 8"

"n^3+2 is odd and cannot be divided by 8 **exactly**"

and for even numbers accept

"a multiple of 8 add 2 is not a multiple of 8, so n^3+2 is not divisible by 8"

"if n^3 is a multiple of 8 then n^3+2 cannot be divisible by 8

Second M1: States the result of cubing an odd and an even number
Second A1: Both valid reasons must be given followed by a concluding statement.

Example of algebraic approaches

Question	Scheme	Marks	AOs
6 **Algebraic approach**	(If n is even,) $n=2k$ and $n^3+2=(2k)^3+2=8k^3+2$	M1	2.1
	Eg.'This is 2 more than a multiple of 8, hence not divisible by 8' Or 'as $8k^3$ is divisible by 8 , $8k^3+2$ isn't'	A1	2.2a
	(If n is odd,) $n=2k+1$ and $n^3+2=(2k+1)^3+2$	M1	2.1
	$=\underline{\underline{8k^3+12k^2+6k+3}}$ which is an even number add 3, therefore odd. Hence it is not divisible by 8 So (given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	
Alt algebraic approach	(If n is even,) $n=2k$ and $\dfrac{n^3+2}{8}=\dfrac{(2k)^3+2}{8}=\dfrac{8k^3+2}{8}$	M1	2.1
	$=k^3+\dfrac{1}{4}$ oe which is not a whole number and hence not divisible by 8	A1	2.2a
	(If n is odd,) $n=2k+1$ and $\dfrac{n^3+2}{8}=\dfrac{(2k+1)^3+2}{8}$	M1	2.1
	$=\dfrac{8k^3+12k^2+6k+3}{8}$ ** The numerator is odd as $\underline{\underline{8k^3+12k^2+6k+3}}$ is an even number +3 hence not divisible by 8 So (Given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	

Notes

Correct expressions are required for the M's. There is no need to state "**If n is even**," $n=2k$ and "**If n is odd,** $n=2k+1$" for the two M's as the expressions encompass all numbers. However the concluding statement must attempt to show that it has been proven for all $n\in\mathbb{N}$

Some students will use $2k-1$ for odd numbers

There is no requirement to change the variable. They may use $2n$ and $2n\pm1$

Reasons must be correct. Don't accept $8k^3+2$ cannot be divided by 8 for example. (It can!)

Also **"$=\dfrac{8k^3+12k^2+6k+3}{8}=k^3+\dfrac{3}{2}k^2+\dfrac{3}{4}k+\dfrac{3}{8}$ which is not whole number" is too vague so A0

Question	Scheme	Marks	AOs
7(i)	$x^2-8x+17=(x-4)^2-16+17$	M1	3.1a
	$=(x-4)^2+1$ with comment (see notes)	A1	1.1b
	As $(x-4)^2\geqslant 0 \Rightarrow (x-4)^2+1\geqslant 1$ hence $x^2-8x+17>0$ for all x	A1	2.4
		(3)	
(ii)	For an explanation that it may not always be true Tests say $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x=5$ $(5+3)^2=64$ whereas $(5)^2=25$ True When $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$ Not true	A1	2.4
		(2)	
			(5 marks)

Notes

(i) Method One: Completing the Square

M1: For an attempt to complete the square. Accept $(x-4)^2$ ...

A1: For $(x-4)^2+1$ with either $(x-4)^2\geqslant 0, (x-4)^2+1\geqslant 1$ or min at (4,1). Accept the inequality statements in words. Condone $(x-4)^2>0$ or a squared number is always positive for this mark.

A1: A fully written out solution, with correct statements and no incorrect statements. There must be a valid reason and a conclusion

....

$x^2-8x+17$

$=(x-4)^2+1\geqslant 1$ as $(x-4)^2\geqslant 0$

Hence $(x-4)^2+1>0$

scores M1 A1 A1

....

$x^2-8x+17>0$

$(x-4)^2+1>0$

This is true because $(x-4)^2\geqslant 0$ and when you add 1 it is going to be positive

scores M1 A1 A1

....

$x^2-8x+17>0$

$(x-4)^2+1>0$

which is true because a squared number is positive

scores M1 A1 A0

incorrect and incomplete

....

$x^2-8x+17=(x-4)^2+1$ scores M1 A1 A0

Minimum is (4,1) so $x^2-8x+17>0$ correct but not explained

....

$x^2-8x+17=(x-4)^2+1$ scores M1 A1 A1

Minimum is (4,1) so as $1>0 \Rightarrow x^2-8x+17>0$ correct and explained

....

$x^2 - 8x + 17 > 0$

$(x-4)^2 + 1 > 0$

scores M1 A0 (no explanation) A0

....

Method Two: Use of a discriminant

M1: Attempts to find the discriminant $b^2 - 4ac$ with a correct *a*, *b* and *c* which may be within a quadratic formula. You may condone missing brackets.

A1: Correct value of $b^2 - 4ac = -4$ **and** states or shows curve is U shaped (or intercept is (0,17)) or equivalent such as $+\text{ve } x^2$ etc

A1: Explains that as $b^2 - 4ac < 0$, there are no roots, and curve is U shaped then $x^2 - 8x + 17 > 0$

....

Method Three: Differentiation

M1: Attempting to differentiate and finding the turning point. This would involve attempting to find $\frac{dy}{dx}$, then setting it equal to 0 and solving to find the *x* value and the *y* value.

A1: For differentiating $\frac{dy}{dx} = 2x - 8 \Rightarrow (4,1)$ is the **turning point**

A1: Shows that $(4,1)$ is the minimum point (second derivative or U shaped), hence $x^2 - 8x + 17 > 0$

....

Method 4: Sketch graph using calculator

M1: Attempting to sketch $y = x^2 - 8x + 17$, U shape with minimum in quadrant one

A1: As above with minimum at (4,1) marked

A1: Required to state that quadratics only have one turning point and as "1" is above the *x*-axis then $x^2 - 8x + 17 > 0$

(ii)

Numerical approach

Do not allow any marks if the student just mentions "positive" and "negative" numbers. Specific examples should be seen calculated if a numerical approach is chosen.

M1: Attempts a value (where it is not true) and shows/implies that it is not true for that value.

For example, for -4 : $(-4+3)^2 > (-4)^2$ and indicates not true (states not true, ✗)

or writing $(-4+3)^2 < (-4)^2$ is sufficient to imply that it is not true

A1: Shows/implies that it can be true for a value **AND** states sometimes true.

For example for +4 : $(4+3)^2 > 4^2$ and indicates true ✓

or writing $(4+3)^2 > 4^2$ is sufficient to imply this is true following $(-4+3)^2 < (-4)^2$

condone incorrect statements following the above such as 'it is only true for positive numbers' as long as they state "sometimes true" and show both cases.

Algebraic approach

M1: Sets the problem up algebraically Eg. $(x+3)^2 > x^2 \Rightarrow x > k$ Any inequality is fine. You may condone one error for the method mark. Accept $(x+3)^2 > x^2 \Rightarrow 6x + 9 > 0$ oe

A1: States sometimes true a**nd** states/implies true for $x > -\frac{3}{2}$ or states/implies not true for $x \le -\frac{3}{2}$ In both cases you should expect to see the statement "sometimes true" to score the A1

Question	Scheme	Marks	AOs
8(i)	Tries at least one value in the interval Eg $4^2-4-1=11$	M1	1.1b
	States that when $n=8$ it is FALSE and provides evidence $8^2-8-1=55=(11\times5)$ Hence NOT PRIME	A1	2.4
		(2)	
(ii)	Knows that an odd number is of the form $2n+1$	B1	3.1a
	Attempts to simplify $(2n+1)^3-(2n+1)^2$	M1	2.1
	……….and factorise $8n^3+8n^2+2n=2\left(4n^3+4n^2+1n\right)=$	dM1	1.1b
	with statement $2\times..$ is always even	A1	2.4
		(4)	
Alt (ii)	Let the odd number be 'n' and attempts n^3-n^2	B1	3.1a
	Attempts to factorise $n^3-n^2=n^2(n-1)$	M1	2.1
	States that n^2 is odd (odd × odd) and $(n-1)$ is even (odd -1)	dM1	1.1b
	States that the product is even ($\text{odd}\times\text{even}$)	A1	2.4
			(6 marks)

Notes: See above

(i)

M1: Attempts any n^2-n-1 for *n* in the interval. It is acceptable just to show $8^2-8-1=55$

A1: States that when $n=8$ it is FALSE and provides evidence. A comment that $55=11\times5$ and hence not prime is required

(ii)

See scheme for two examples of proof

Note that Alt (i) works equally well with an odd number of the form $2n-1$

For example $(2n-1)^3-(2n-1)^2=(2n-1)^2\{2n-1-1\}=(2n-1)^2\{2n-2\}=2\times(2n-1)^2\{n-1\}$

Question	Scheme	Marks	AOs
9 (a) Way 1	Since x and y are positive, their square roots are real and so $(\sqrt{x}-\sqrt{y})^2 \geq 0$ giving $x - 2\sqrt{x}\sqrt{y} + y \geq 0$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x + y$ provided x and y are positive and so $\sqrt{xy} \leq \frac{x+y}{2}$ *	A1*	2.2a
		(2)	
Way 2 Longer method	Since $(x-y)^2 \geq 0$ for real values of x and y, $x^2 - 2xy + y^2 \geq 0$ and so $4xy \leq x^2 + 2xy + y^2$ i.e. $4xy \leq (x+y)^2$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x + y$ provided x and y are positive and so $\sqrt{xy} \leq \frac{x+y}{2}$ *	A1*	2.2a
		(2)	
(b)	Let $x = -3$ and $y = -5$ then LHS = $\sqrt{15}$ and RHS= -4 so as $\sqrt{15} > -4$ result does not apply	B1	2.4
		(1)	
			(3 marks)

Notes:

(a)

M1: Need two stages of the three stage argument involving the three stages, squaring, square rooting terms and rearranging

A1*: Need all three stages making the correct deduction to achieve the printed result

(b)

B1: Chooses two negative values and substitutes, then states conclusion

Question	Scheme	Marks	AOs
10	NB any natural number can be expressed in the form: $3k$, $3k + 1$, $3k + 2$ or equivalent e.g. $3k – 1$, $3k$, $3k + 1$		
	Attempts to square **any two** distinct cases of the above	M1	3.1a
	Achieves accurate results and makes a valid comment for **any two** of the possible three cases: E.g. $(3k)^2 = 9k^2 \left(= 3\times 3k^2\right)$ is a multiple of 3 $(3k+1)^2 = 9k^2 + 6k + 1 = 3\times\left(3k^2 + 2k\right) + 1$ is one more than a multiple of 3 $(3k+2)^2 = 9k^2 + 12k + 4 = 3\times\left(3k^2 + 4k + 1\right) + 1$ $\left(\text{or } (3k-1)^2 = 9k^2 - 6k + 1 = 3\times\left(3k^2 - 2k\right) + 1\right)$ is one more than a multiple of 3	A1 **M1 on EPEN**	1.1b
	Attempts to square in all 3 distinct cases. E.g. attempts to square $3k$, $3k + 1$, $3k + 2$ or e.g. $3k – 1$, $3k$, $3k + 1$	M1 **A1 on EPEN**	2.1
	Achieves accurate results for all three cases and gives a minimal conclusion (allow tick, QED etc.)	A1	2.4
		(4)	
			(4 marks)

Notes:

M1: Makes the key step of attempting to write the natural numbers in any 2 of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square these expressions.

A1(M1 on EPEN): Successfully shows for 2 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 <u>using algebra</u>. This must be made explicit e.g. reaches $3\times\left(3k^2 + 2k\right) + 1$ and makes a statement that this is one more than a multiple of 3 but also allow other rigorous arguments that reason why $9k^2 + 6k + 1$ is one more than a multiple of 3 e.g. "$9k^2$ is a multiple of 3 and $6k$ is a multiple of 3 so $9k^2 + 6k + 1$ is one more than a multiple of 3"

M1(A1 on EPEN): Recognises that all natural numbers can be written in one of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square in all 3 cases.

A1: Successfully shows for all 3 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 using algebra and makes a conclusion

Question	Scheme	Marks	AOs
11	Statement: "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."		
	E.g. $m = \sqrt{3},\, n = \sqrt{12}$	M1	1.1b
	$\{mn =\}\ \left(\sqrt{3}\right)\left(\sqrt{12}\right) = 6$ $\Rightarrow$ statement untrue **or** 6 is not irrational or 6 is rational	A1	2.4
		(2)	

Notes for Question 11

M1:	States or uses any pair of ***different*** numbers that will disprove the statement. E.g. $\sqrt{3},\ \sqrt{12}$; $\sqrt{2},\ \sqrt{8}$; $\sqrt{5}, -\sqrt{5}$; $\frac{1}{\pi}, 2\pi$; $3\mathrm{e}, \frac{4}{5\mathrm{e}}$;
A1:	Uses correct reasoning to disprove the given statement, with a correct conclusion
Note:	Writing $(3\mathrm{e})\left(\frac{4}{5\mathrm{e}}\right) = \frac{12}{5} \Rightarrow$ untrue is sufficient for M1A1

Question	Scheme	Marks	AOs
12 (i)	For an explanation or statement to show when the claim $3^x \ldots 2^x$ fails This could be e.g. • when $x = -1$, $\frac{1}{3} < \frac{1}{2}$ or $\frac{1}{3}$ is not greater than or equal to $\frac{1}{2}$ • when $x < 0$, $3^x < 2^x$ or 3^x is not greater than or equal to 2^x	M1	2.3
	followed by an explanation or statement to show when the claim $3^x \ldots 2^x$ is true. This could be e.g. • $x = 2$, $9 \ldots 4$ or 9 is greater than or equal to 4 • when $x \ldots 0$, $3^x \ldots 2^x$ and a correct conclusion. E.g. • so the claim $3^x \ldots 2^x$ is sometimes true	A1	2.4
		(2)	
(ii)	Assume that $\sqrt{3}$ is a rational number So $\sqrt{3} = \frac{p}{q}$, where p and q integers, $q \neq 0$, and the HCF of p and q is 1	M1	2.1
	$\Rightarrow p = \sqrt{3}q \Rightarrow p^2 = 3q^2$	M1	1.1b
	$\Rightarrow$ p^2 is divisible by 3 and so p is divisible by 3	A1	2.2a
	So where c is an integer From earlier, $p^2 = 3q^2 \Rightarrow (3c)^2 = 3q^2$	M1	2.1
	$\Rightarrow q^2 = 3c^2 \Rightarrow$ q^2 is divisible by 3 and so q is divisible by 3	A1	1.1b
	As both p and q are both divisible by 3 then the HCF of p and q is not 1 This contradiction implies that $\sqrt{3}$ is an irrational number	A1	2.4
		(6)	
			(8 marks)

Question 12 Notes:

(i)	
M1:	See scheme
A1:	See scheme
(ii)	
M1:	Uses a method of proof by contradiction by initially assuming that $\sqrt{3}$ is rational and expresses $\sqrt{3}$ in the form $\frac{p}{q}$, where p and q are correctly defined.
M1:	Writes $\sqrt{3} = \frac{p}{q}$ and rearranges to make p^2 the subject
A1:	Uses a logical argument to prove that p is divisible by 3
M1:	Uses the result that p is divisible by 3, (to construct the initial stage of proving that q is also divisible by 3), by substituting $p = 3c$ into their expression for p^2
A1:	Hence uses a correct argument, in the same way as before, to deduce that q is also divisible by 3
A1:	Completes the argument (as detailed on the scheme) that $\sqrt{3}$ is irrational.
	Note: All the previous 5 marks need to be scored in order to obtain the final A mark.