EXPERT TUITION

Maths Questions By Topic:

Sequences & Series
Mark Scheme

A-Level Edexcel

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Question	Scheme	Marks	AOs
1 (i)	States that $S = a + (a+d) + \dots\dots + (a+(n-1)d)$	B1	1.1a
	$S = \quad a + \quad . \quad (a+d) + \quad (a+(n-1)d)$ $S = (a+(n-1)d) + (a+(n-2)d) + \dots\dots + a$	M1	3.1a
	Reaches $2S = n \times (2a + (n-1)d)$ And so proves that $S = \frac{n}{2}[2a+(n-1)d]$ *	A1*	2.1
		(3)	
(ii)	(a) $S = 10 + 9.20 + 8.40 + \dots$		
	$64 = \frac{n}{2}(20 - 0.8(n-1))$ o.e	M1	3.1b
	$128 = 20n - 0.8n^2 + 0.8n$ $0.8n^2 - 20.8n + 128 = 0$ $n^2 - 26n + 160 = 0$ *	A1*	2.1
		(2)	
	(b) $n = 10, 16$	B1	1.1b
		(1)	
	(c) 10 weeks with a minimal correct reason. E.g. • He has saved up the amount by 10 weeks so he would not save for another 6 weeks • You would choose the smaller number • He starts saving negative amounts (in week 14) so 16 does not make sense	B1	2.3
		(1)	
			(7 marks)
Notes:			

(i)

B1: Correctly writes down an expression for the key terms S or S_n including $S =$ or $S_n =$

Allow a minimum of 3 correct terms including the first and last terms, and no incorrect terms.

Score for S or $S_n = a + (a+d) + \dots\dots + (a+(n-1)d)$ with + signs, not commas

If the series contains extra terms that should not be there E.g

$S = a + (a+d) + \dots\dots(\boldsymbol{a+nd}) + (a+(n-1)d)$ score B0

M1: For the key step in reversing the terms and adding the two series.

Look for a minimum of two terms, including a and $a+(n-1)d$, the series reversed with evidence of adding, for example $2S =$ Condone the extra incorrect terms (see above) appearing.

Can be scored when terms are separated by commas

A1*: Shows correct work (no errors) with all steps shown leading to given answer.

There should be no incorrect terms. A minimum of 3 terms should be shown in each sum

The solution below is a variation of this.

$S = a + (a+d) + \dots\dots\dots\dots\dots\dots\dots\dots\dots + l$

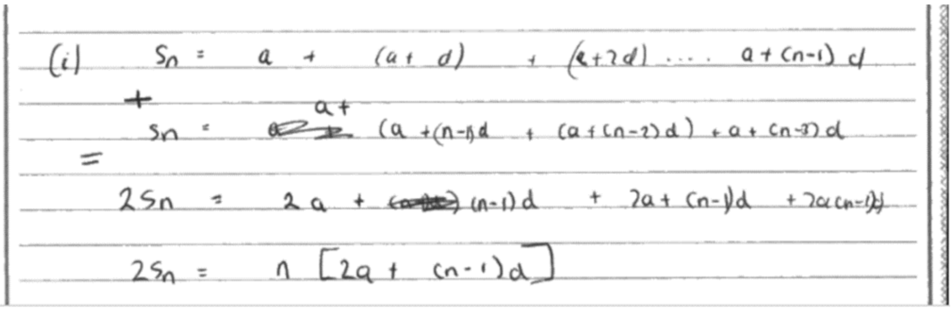
$S = l + (l-d) + \dots\dots\dots\dots\dots\dots\dots\dots\dots + a$

$2S = n(a+l)$

$S = \frac{n}{2}(a+l) = \frac{n}{2}(a+a+(n-1)d) = \frac{n}{2}(2a+(n-1)d)$

B1 and A1 are not scored until the last line, M scored on line 3

The following scores B1 M0 A0 as the terms in the second sum are not reversed


(i) $S_n = a + (a+d) + (a+2d) \dots a+(n-1)d$

$+$

$S_n = a + (a+(n-1)d + (a+(n-2)d) + a+(n-3)d$

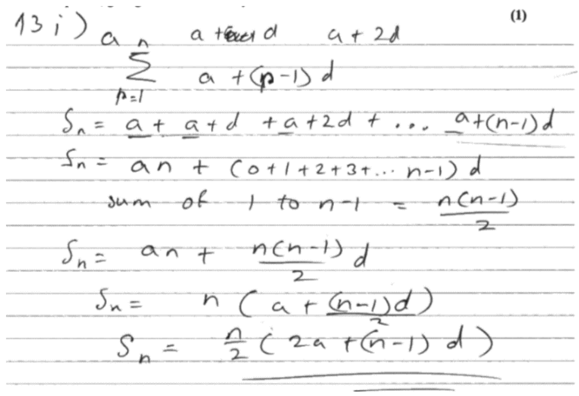
$=$

$2S_n = 2a + (n-1)d + 2a+(n-1)d + 2a(n-1)d$

$2S_n = n[2a+(n-1)d]$

SC in (a) Scores B1 M0 A0.

They use $0+1+2+\dots+(n-1) = \frac{n(n-1)}{2}$ which relies on the quoted proof.


13 i) a_n $a+d$ $a+2d$ (1)

$\sum_{p=1}^{n} a+(p-1)d$

$S_n = a + a+d + a+2d + \dots a+(n-1)d$

$S_n = an + (0+1+2+3+\dots n-1)d$

sum of 1 to $n-1$ = $\frac{n(n-1)}{2}$

$S_n = an + \frac{n(n-1)}{2}d$

$S_n = n(a + \frac{(n-1)d}{2})$

$S_n = \frac{n}{2}(2a+(n-1)d)$

(ii) (a)

M1: Uses the information given to set up a correct equation in n.

The values of S, a and d need to be correct and used within a correct formula

Possible ways to score this include unsimplified versions $64 = \frac{n}{2}(2\times10+(n-1)\times-0.8)$,

$64 = \frac{n}{2}(10+10+(n-1)\times-0.8)$ or versions using pence rather than £'s $6400 = \frac{n}{2}(2000+(n-1)\times-80)$

Allow recovery for both marks following $64 = \frac{n}{2}(2\times10+(n-1)-0.8)$ with an invisible ×

A1*: Proceeds without error to the given answer. (Do not penalise a missing final trailing bracket)
Look for at least a line with the brackets correctly removed as well as a line with the terms in n correctly combined

E.g. $64 = \frac{n}{2}(20+(n-1)\times-0.8) \Rightarrow 64 = 10n - 0.4n^2 + 0.4n \Rightarrow 0.4n^2 - 10.4n + 64 = 0 \Rightarrow n^2 - 26n + 160 = 0$

(ii)(b)

B1: $n = 10, 16$

(ii)(c)

B1: Chooses 10 (weeks) and gives a minimal acceptable reason. The reason must focus on why the answer is 10 (weeks) rather than 16(weeks) or alternatively why it would not be 16 weeks.

Question	Scheme	Marks	AOs
2(a)	3^8 or 6561 as the constant term	B1	1.1b
	$\left(3-\frac{2x}{9}\right)^8 = ...+{}^8C_1(3)^7\left(-\frac{2x}{9}\right)+{}^8C_2(3)^6\left(-\frac{2x}{9}\right)^2+{}^8C_3(3)^5\left(-\frac{2x}{9}\right)^3+...$ $=...+8\times(3)^7\left(-\frac{2x}{9}\right)+28\times(3)^6\left(-\frac{2x}{9}\right)^2+56(3)^5\left(-\frac{2x}{9}\right)^3$	M1 A1	1.1b 1.1b
	$=6561-3888x+1008x^2-\frac{448}{3}x^3+...$	A1	1.1b
		(4)	
(b)	Coefficient of x^2 is $\frac{1}{2}\times"1008"-\frac{1}{2}\times"-\frac{448}{3}"$	M1	3.1a
	$=\frac{1736}{3}$ (or $578\frac{2}{3}$)	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)

B1: Sight of 3^8 or 6561 as the constant term.

M1: An attempt at the binomial expansion. This can be awarded for the correct structure of the 2nd, 3rd or 4th term. The correct binomial coefficient must be associated with the correct power of 3 and the correct power of $(\pm)\frac{2x}{9}$. Condone invisible brackets

eg ${}^8C_2(3)^6-\frac{2x}{9}^2$ for this mark.

A1: For a correct simplified or unsimplified **second** or **fourth term** (with binomial coefficients evaluated).

$+8\times(3)^7\left(-\frac{2x}{9}\right)$ or $+56(3)^5\left(-\frac{2x}{9}\right)^3$

A1: $6561-3888x+1008x^2-\frac{448}{3}x^3$ Ignore any extra terms and allow the terms to be listed.

Allow the exact equivalent to $-\frac{448}{3}$ eg $-149.\dot{3}$ but not -149.3.

Condone x^1 and eg $+-3888x$. Do not isw if they multiply all the terms by eg 3

Alt(a)

B1: Sight of $3^8(1+.....)$ or 6561 as the constant term

M1: An attempt at the binomial expansion $\left(1-\frac{2}{27}x\right)^8$. This can be awarded for the correct structure of the 2nd, 3rd or 4th term. The correct binomial coefficient must be associated with the correct power of $(\pm)\frac{2x}{27}$. Condone invisible brackets for this mark.

Score for any of:

$8\times-\frac{2}{27}x,\quad \frac{8\times7}{2}\times\left(-\frac{2}{27}x\right)^2,\quad \frac{8\times7\times6}{6}\times\left(-\frac{2}{27}x\right)^3$ which may be implied by any of

$-\frac{16}{27}x,\ +\frac{112}{729}x^2,\ -\frac{448}{19683}x^3$

A1: For a correct simplified or unsimplified **second** or **fourth** term including being multiplied by 3^8

A1: $6561-3888x+1008x^2-\frac{448}{3}x^3$ Ignore any extra terms and allow the terms to be listed.

Allow the exact equivalent to $-\frac{448}{3}$ eg $-149.\dot{3}$ but not -149.3.

Condone x^1 and eg $+-3888x$

(b)

M1: Adopts a correct strategy for the required coefficient. This requires an attempt to calculate $\pm\frac{1}{2}$ their coefficient of x^2 from part (a) $\pm\frac{1}{2}$ their coefficient of x^3 from part (a).

There must be an attempt to bring these terms together to a single value. ie they cannot just circle the relevant terms in the expansion for this mark. The strategy may be implied by their answer.

Condone any appearance of x^2 or x^3 appearing in their intermediate working.

A1: $\frac{1736}{3}$ or $578\frac{2}{3}$ Do not accept $578.\dot{6}$ or $\frac{1736}{3}x^2$

Question	Scheme	Marks	AOs
3(a)	$u_2 = k - 12,\ u_3 = k - \frac{24}{k-12}$	M1	1.1b
	$u_1 + 2u_2 + u_3 = 0 \Rightarrow 2 + 2(k-12) + k - \frac{24}{k-12} = 0$	dM1	1.1b
	$\Rightarrow 3k - 22 - \frac{24}{k-12} = 0 \Rightarrow (3k-22)(k-12) - 24 = 0$ $\Rightarrow 3k^2 - 36k - 22k + 264 - 24 = 0$ $\Rightarrow 3k^2 - 58k + 240 = 0$ *	A1*	2.1
		(3)	
(b)	$k = 6, \left(\frac{40}{3}\right)$	M1	1.1b
	$k = 6$ as k must be an integer	A1	2.3
		(2)	
(c)	$(u_3 =)10$	B1	2.2a
		(1)	
		(6 marks)	

Notes

(a)

M1: Attempts to apply the sequence formula once for either u_2 **or** u_3.

Usually for $u_2 = k - \frac{24}{2}$ o.e. but could be awarded for $u_3 = k - \frac{24}{their\ "u_2"}$

dM1: Award for

- attempting to apply the sequence formula to find both u_2 **and** u_3
- using $2 + 2"u_2" + "u_3" = 0 \Rightarrow$ an equation in k. The u_3 may have been incorrectly adapted

A1*: Fully correct work leading to the printed answer.

There must be

- (at least) one correct intermediate line between $2 + 2(k-12) + k - \frac{24}{k-12} = 0$ (o.e.) and the given answer that shows how the fractions are "removed". E.g. $(3k-22)(k-12) - 24 = 0$
- no errors in the algebra. The = 0 may just appear at the answer line.

(b)

M1: Attempts to solve the quadratic which is implied by sight of $k = 6$.

This may be awarded for any of

- $3k^2 - 58k + 240 = (ak \pm c)(bk \pm d) = 0$ where $ab = 3, cd = 240$ followed by $k =$
- an attempt at the correct quadratic formula (or completing the square)
- a calculator solution giving at least $k = 6$

A1: Chooses $k = 6$ and gives a minimal reason

Examples of a minimal reason are

- 6 because it is an integer
- 6 because it is a whole number
- 6 because $\frac{40}{3}$ or $13.\dot{3}$ is not an integer

(c)

B1: Deduces the correct value of u_3.

Question	Scheme	Marks	AOs
4(a)	$u_3 = £20\,000 \times 1.08^2 = (£)23\,328*$	B1*	1.1b
		(1)	
(b)	$20\,000 \times 1.08^{n-1} > 65\,000$	M1	1.1b
	$1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \frac{\ln(3.25)}{\ln(1.08)}$ or e.g. $1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \log_{1.08}\left(\frac{13}{4}\right)$	M1	3.1b
	Year 17	A1	3.2a
		(3)	
(c)	$S_{20} = \frac{20\,000(1-1.08^{20})}{1-1.08}$	M1	3.4
	Awrt (£) 915 000	A1	1.1b
		(2)	
			(6 marks)
Notes			

(a)

B1*: Uses a correct method to show that the Profit in Year 3 will be £23 328. Condone missing units

E.g. $£20\,000 \times 1.08^2$ or $£20\,000 \times 108\% \times 108\%$

This may be obtained in two steps. E.g $\frac{8}{100} \times 20000 = 1600$ followed by $\frac{8}{100} \times 21600 = 1728$ with the calculations $21600 + 1728 = 23328$ seen.

Condone calculations seen as 8% of 20000 = 1600.

This is a show that question and the method must be seen.

It is not enough to state Year 1 = £21 600, Year 2 = £ 23 328

(b)

M1: Sets up an inequality or an equation that will allow the problem to be solved.

Allow for example N or n for $n-1$. So award for $20\,000\times1.08^{n-1} > 65\,000$,

$20\,000\times1.08^{n} = 65\,000$ or $20\,000\times(108\%)^{n} \geqslant 65\,000$ amongst others.

Condone **slips** on the 20 000 and 65 000 but the 1.08 o.e. must be correct

M1: Uses a correct strategy involving logs in an attempt to solve a type of equation or inequality of the form seen above. It cannot be awarded from a sum formula

The equation/inequality must contain an index of $n-1$, N, n etc.

Again condone **slips** on the 20 000 and 65 000 but additionally condone an error on the 1.08, which may appear as 1.8 for example

E.g. $20\,000\times1.08^{n} = 65\,000 \Rightarrow n\log1.08 = \log\dfrac{65000}{20000} \Rightarrow n = \ldots$

E.g. $2000\times1.8^{n} = 65\,000 \Rightarrow \log2000 + n\log1.8 = \log65000 \Rightarrow n = \ldots$

A1: Interprets their decimal value and gives the correct year number. Year 17

The demand of the question dictates that solutions relying entirely on calculator technology are not acceptable, BUT allow a solution that appreciates **a correct term** formula or the entire set of calculations where you may see the numbers as part of a larger list

E.g. Uses, or implies the use of, an acceptable calculation and finds value(s)

for M1: $(n=16)\Rightarrow P = 20\,000\times1.08^{15}$=awrt 63400 **or** $(n=17)\Rightarrow P = 20\,000\times1.08^{16}$ = awrt 68500

M1: $(n=16)\Rightarrow P = 20\,000\times1.08^{15}$=awrt 63400 **and** $(n=17)\Rightarrow P = 20\,000\times1.08^{16}$ = awrt 68500

A1: 17 years following correct method and both M's

(c)

M1: Attempts to use the model with a **correct** sum formula to find the total profit for the 20 years.

You may see an attempt to find the sum of 20 terms via a list. This is acceptable provided there are 20 terms with $u_n = 1.08\times u_{n-1}$ seen at least 4 times and the sum attempted.

Condone a slip on the 20 000 (e.g appearing as 2 000) and/or a slip on the 1.08 with it being the same "r" as in (b) . Do not condone 20 appearing as 19 for instance

A1: awrt £915 000 but condone missing unit

The demand of the question dictates that all stages of working should be seen. An answer without working scores M0 A0

Question	Scheme	Marks	AOs
5(a)(i)	$50x^2+38x+9\equiv A(5x+2)(1-2x)+B(1-2x)+C(5x+2)^2$ $\Rightarrow B=...$ or $C=...$	M1	1.1b
	$B=1$ and $C=2$	A1	1.1b
(a)(ii)	E.g. $x=0$ $x=0\Rightarrow 9=2A+B+4C$ $\Rightarrow 9=2A+1+8\Rightarrow A=...$	M1	2.1
	$A=0$*	A1*	1.1b
		(4)	
(b)(i)	$\dfrac{1}{(5x+2)^2}=(5x+2)^{-2}=2^{-2}\left(1+\dfrac{5}{2}x\right)^{-2}$ or $(5x+2)^{-2}=2^{-2}+...$	M1	3.1a
	$\left(1+\dfrac{5}{2}x\right)^{-2}=1-2\left(\dfrac{5}{2}x\right)+\dfrac{-2(-2-1)}{2!}\left(\dfrac{5}{2}x\right)^2+...$	M1	1.1b
	$2^{-2}\left(1+\dfrac{5}{2}x\right)^{-2}=\dfrac{1}{4}-\dfrac{5}{4}x+\dfrac{75}{16}x^2+...$	A1	1.1b
	$\dfrac{1}{(1-2x)}=(1-2x)^{-1}=1+2x+\dfrac{-1(-1-1)}{2!}(2x)^2+.$	M1	1.1b
	$\dfrac{1}{(5x+2)^2}+\dfrac{2}{1-2x}=\dfrac{1}{4}-\dfrac{5}{4}x+\dfrac{75}{16}x^2+...+2+4x+8x^2+...$	dM1	2.1
	$=\dfrac{9}{4}+\dfrac{11}{4}x+\dfrac{203}{16}x^2+...$	A1	1.1b
(b)(ii)	$\lvert x\rvert<\dfrac{2}{5}$	B1	2.2a
		(7)	
			(11 marks)

Notes

(a)(i)

M1: **Uses a correct identity** and makes progress using an appropriate strategy (e.g. sub $x=\frac{1}{2}$) to find a value for B or C. May be implied by one correct value (cover up rule).

A1: Both values correct

(a)(ii)

M1: Uses an appropriate method to establish an equation connecting A with B and/or C and uses their values of B and/or C to find a suitable equation in A.

Amongst many different methods are:

Compare terms in x^2 $\Rightarrow 50=-10A+25C$ which would be implied by $50=-10A+25\times"2"$

Compare constant terms or substitute $x=0\Rightarrow 9=2A+B+4C$ implied by $9=2A+1+4\times 2$

A1*: Fully correct proof with no errors.

Note: The second part is a proof so it is important that a suitable proof/show that is seen.

Candidates who write down 3 equations followed by three answers (with no working) will score M1 A1 M0 A0 (b)(i)

M1: Applies the key steps of writing $\frac{1}{(5x+2)^2}$ as $(5x+2)^{-2}$ and takes out a factor of 2^{-2} to form an expression of the form $(5x+2)^{-2} = 2^{-2}(1+*x)^{-2}$ where * is not 1 or 5

Alternatively uses direct expansion to obtain $2^{-2} + \ldots$

M1: Correct attempt at the binomial expansion of $(1+*x)^{-2}$ up to the term in x^2

Look for $1+(-2)*x+\frac{(-2)(-3)}{2}*x^2$ where * is not 5 or 1.

Condone sign slips and lack of $*^2$ on term 3. ....

Alt Look for correct structure for 2nd and 3rd terms by direct expansion. See below

A1: For a fully correct expansion of $(2+5x)^{-2}$ which may be unsimplified. This may have been combined with their '*B*'

A direct expansion would look like $(2+5x)^{-2} = 2^{-2}+(-2)2^{-3}\times 5x+\frac{(-2)(-3)}{2}2^{-4}\times(5x)^2$

M1: Correct attempt at the binomial expansion of $(1-2x)^{-1}$

Look for $1+(-1)*x+\frac{(-1)(-2)}{2}*x^2$ where * is not 1

dM1: Fully correct strategy that is dependent on the previous **TWO** method marks.

There must be some attempt to use their values of *B* and *C*

A1: Correct expression or correct values for *p*, *q* and *r*.

(b)(ii)

B1: Correct range. Allow also other forms, for example $-\frac{2}{5}<x<\frac{2}{5}$ or $x\in\left(-\frac{2}{5},\frac{2}{5}\right)$

Do not allow multiple answers here. The correct answer must be chosen if two answers are offered

6(a)	$(2+ax)^8$ Attempts the term in $x^5 = {}^8C_5 2^3 (ax)^5 = 448a^5x^5$	M1 A1	1.1a 1.1b
	Sets $448a^5 = 3402 \Rightarrow a^5 = \dfrac{243}{32}$	M1	1.1b
	$\Rightarrow a = \dfrac{3}{2}$	A1	1.1b
		(4)	
(b)	Attempts either term. So allow for 2^8 or ${}^8C_4 2^4 a^4$	M1	1.1b
	Attempts the sum of both terms $2^8 + {}^8C_4 2^4 a^4$	dM1	2.1
	= 256 + 5670 = 5926	A1	1.1b
		(3)	
			(7 marks)

Notes

(a)
M1: An attempt at selecting the correct term of the binomial expansion. If all terms are given then the correct term must be used. Allow with a missing bracket ${}^8C_5 2^3 ax^5$ and left without the binomial coefficient expanded

A1: $448a^5x^5$ Allow unsimplified but ${}^8C_5$ must be "numerical"

M1: Sets their $448a^5 = 3402$ and proceeds to $\Rightarrow a^k = ...$ where $k \in \mathbb{N}$ $k \neq 1$

A1: Correct work leading to $a = \dfrac{3}{2}$

(b)
M1: Finds either term required. So allow for 2^8 or ${}^8C_4 2^4 a^4$ (even allowing with a)

dM1: Attempts the sum of both terms $2^8 + {}^8C_4 2^4 a^4$

A1: cso 5926

Question	Scheme	Marks	AOs
7(a)	$(1+8x)^{\frac{1}{2}} = 1+\frac{1}{2}\times 8x+\frac{\frac{1}{2}\times-\frac{1}{2}}{2!}\times(8x)^2+\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\times(8x)^3$	M1 A1	1.1b 1.1b
	$=1+4x-8x^2+32x^3+....$	A1	1.1b
		(3)	
(b)	Substitutes $x=\frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to give $\frac{\sqrt{5}}{2}$	M1	1.1b
	Explains that $x=\frac{1}{32}$ is substituted into $1+4x-8x^2+32x^3$ and you multiply the result by 2	A1ft	2.4
		(2)	
			(5 marks)
Notes:			

(a)

M1: Attempts the binomial expansion with $n=\frac{1}{2}$ and obtains the correct structure for term 3 **or** term 4.

Award for the correct coefficient with the correct power of x. Do not accept ${}^{n}C_{r}$ notation for coefficients.

For example look for term 3 in the form $\frac{\frac{1}{2}\times-\frac{1}{2}}{2!}\times(*x)^2$ or $\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\times(*x)^3$

A1: Correct (unsimplified) expression. May be implied by correct simplified expression

A1: $1+4x-8x^2+32x^3$

Award if there are extra terms (even if incorrect).

Award if the terms are listed $1,\ 4x, -8x^2, 32x^3$

(b)

M1: Score for substituting $x=\frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to obtain $\frac{\sqrt{5}}{2}$ or equivalent such as $\sqrt{\frac{5}{4}}$

Alternatively award for substituting $x=\frac{1}{32}$ into **both sides** and making a connection between the two sides by use of an = or $\approx$.

E.g. $\left(1+\frac{8}{32}\right)^{\frac{1}{2}} = 1+4\times\frac{1}{32}-8\times\left(\frac{1}{32}\right)^2+32\times\left(\frac{1}{32}\right)^3$ following through on their expansion

Also implied by $\frac{\sqrt{5}}{2}=\frac{1145}{1024}$ for a correct expansion

It is not enough to state substitute $x=\frac{1}{32}$ into " the expansion" or just the rhs "$1+4x-8x^2+32x^3$"

A1ft: Requires a full (and correct) **explanation** as to how the expansion can be used to estimate $\sqrt{5}$

E.g. Calculates $1+4\times\frac{1}{32}-8\times\left(\frac{1}{32}\right)^2+32\times\left(\frac{1}{32}\right)^3$ and multiplies by 2.

This can be scored from an incorrect binomial expansion or a binomial expansion with more terms.

The explanation could be mathematical. So $\frac{\sqrt{5}}{2}=\frac{1145}{1024}\rightarrow\sqrt{5}=\frac{1145}{512}$ is acceptable.

SC: For 1 mark, M1,A0 score for a statement such as "substitute $x=\frac{1}{32}$ into both sides of part (a) and make $\sqrt{5}$ the subject"

Question	Scheme	Marks	AOs
8(a)	Uses $115 = 28 + 5d \Rightarrow d = (17.4)$	M1	3.1b
	Uses $28 + 2\times"17.4" = ...$	M1	3.4
	$= 62.8$ (km h $^{-1}$)	A1	1.1b
		(3)	
(b)	Uses $115 = 28r^5 \Rightarrow r = (1.3265)$	M1	3.1b
	Uses $28\times"1.3265^4" = ...$ or $\frac{115}{"1.3265"}$	M1	3.4
	$= 86.7$ (km h $^{-1}$)	A1	1.1b
		(3)	
			(6 marks)
Notes:			

(a)

M1: Translates the problem into maths using n^{th} term $= a + (n-1)d$ and attempts to find d

Look for either $115 = 28 + 5d \Rightarrow d = ...$ or an attempt at $\frac{115-28}{5}$ condoning slips

It is implied by use of $d = 17.4$ Note that $115 = 28 + 6d \Rightarrow d = ...$ is M0

M1: Uses the model to find the fastest speed the car can go in 3rd gear using $28 + 2"d"$ or equivalent. This can be awarded following an incorrect method of finding "d"

A1: 62.8 km/ h Lack of units are condoned. Allow exact alternatives such as $\frac{314}{5}$

(b)

M1: Translates the problem into maths using n^{th} term $= ar^{n-1}$ and attempts to find r

It must use the 1st and 6th gear and not the 3rd gear found in part (a)

Look for either $115 = 28r^5 \Rightarrow r = ...$ o.e. or $\sqrt[5]{\frac{115}{28}}$ condoning slips.

It is implied by stating or using $r =$ awrt 1.33

M1: Uses the model to find the fastest speed the car can go in 5th gear using $28\times"r^4"$ or $\frac{115}{"r"}$ o.e.

This can be awarded following an incorrect method of finding "r"

A common misread seems to be finding the fastest speed the car can go in 3rd gear as in (a).

Providing it is clear what has been done, e.g. $u_3 = 28\times"r^2"$ it can be awarded this mark.

A1: awrt 86.7 km/h Lack of units are condoned. Expressions must be evaluated.

Question	Scheme	Marks	AOs
9(a)	Uses the sequence formula $a_{n+1} = \frac{k(a_n+2)}{a_n}$ once with $a_1 = 2$	M1	1.1b
	$(a_1 = 2), a_2 = 2k, a_3 = k+1, a_4 = \frac{k(k+3)}{k+1}$ Finds four consecutive terms and sets a_4 equal to a_1 (oe)	M1	3.1a
	$\frac{k(k+3)}{k+1} = 2 \Rightarrow k^2 + 3k = 2k + 2 \Rightarrow k^2 + k - 2 = 0$ *	A1*	2.1
		(3)	
(b)	States that when $k = 1$, all terms are the same and concludes that the sequence does not have a period of order 3	B1	2.3
		(1)	
(c)	Deduces the repeating terms are $a_{1/4} = 2, a_{2/5} = -4, a_{3/6} = -1,$	B1	2.2a
	$\sum_{n=1}^{80} a_k = 26 \times (2 + -4 + -1) + 2 + -4$	M1	3.1a
	$= -80$	A1	1.1b
		(3)	
			(7 marks)
Notes:			

(a)

M1: Applies the sequence formula $a_{n+1} = \frac{k(a_n+2)}{a_n}$ seen once.

This is usually scored in attempting to find the second term. E.g. for $a_2 = 2k$ or $a_{1+1} = \frac{k(2+2)}{2}$

M1: Attempts to find $a_1 \to a_4$ and sets $a_1 = a_4$. Condone slips.

Other methods are available. E.g. Set $a_4 = 2$, work backwards to find a_3 and equate to $k+1$

There is no requirement to see either a_1 or any of the labels. Look for the correct terms in the correct order.

There is no requirement for the terms to be simplified

FYI $a_1 = 2, a_2 = 2k, a_3 = k+1, a_4 = \frac{k(k+3)}{k+1}$ and so $2 = \frac{k(k+3)}{k+1}$

A1*: Proceeds to the given answer with accurate work showing all necessary lines. See MS for minimum

(b)

B1: States that when $k = 1$, all terms are the same and concludes that the sequence does not have a period of order 3.

Do not accept "the terms just repeat" or "it would mean all the terms of the sequence are 2"

There must be some reference to the fact that it does not have order 3. Accept it has order 1.

It is acceptable to state $a_2 = a_1 = 2$ and state that the sequence does not have order 3

(c)

B1: Deduces the repeating terms are $a_{1/4} = 2, a_{2/5} = -4, a_{3/6} = -1,$

M1: Uses a clear strategy to find the sum to 80 terms. This will usually be found using multiples of the first three terms.

For example you may see $\sum_{r=1}^{80} a_r = \left(\sum_{r=1}^{78} a_r\right) + a_{79} + a_{80} = 26\times(2+-4+-1)+2+-4$

or $\sum_{r=1}^{80} a_r = \left(\sum_{r=1}^{81} a_r\right) - a_{81} = 27\times(2+-4+-1)-(-1)$

For candidates who find in terms of *k* award for $27\times 2 + 27\times(2k) + 26\times(k+1)$ or $80k+80$

If candidates proceed and substitute $k = -2$ into $80k+80$ to get -80 then all 3 marks are scored.

A1: -80

....

Note: Be aware that we have seen candidates who find the first three terms correctly, but then find

$26\frac{2}{3}\times(2+-4+-1) = 26\frac{2}{3}\times -3$ which gives the correct answer

but it is an incorrect method and should be scored B1 M0 A0

....

Question	Scheme	Marks	AOs
10(a)	$(1+kx)^{10} = 1+\binom{10}{1}(kx)^1+\binom{10}{2}(kx)^2+\binom{10}{3}(kx)^3...$	M1 A1	1.1b 1.1b
	$=1+10kx+45k^2x^2+120k^3x^3...$	A1	1.1b
		(3)	
(b)	Sets $120k^3 = 3\times 10k$	B1	1.2
	$4k^2 = 1 \Rightarrow k = ...$	M1	1.1b
	$k = \pm\frac{1}{2}$	A1	1.1b
		(3)	
			(6 marks)

(a)

M1: An attempt at the binomial expansion. This may be awarded for either the second or third term or fourth term. The coefficients may be of the form ${}^{10}C_1$, $\binom{10}{2}$ etc or eg $\frac{10\times 9\times 8}{3!}$

A1: A correct unsimplified binomial expansion. The coefficients must be numerical so cannot be of the form ${}^{10}C_1$, $\binom{10}{2}$. Coefficients of the form $\frac{10\times 9\times 8}{3!}$ are acceptable for this mark.

The bracketing must be correct on $(kx)^2$ but allow recovery

A1: $1+10kx+45k^2x^2+120k^3x^3...$ or $1+10(kx)+45(kx)^2+120(kx)^3...$
Allow if written as a list.

(b)

B1: Sets their $120k^3 = 3\times$ their $10k$ (Seen or implied)
For candidates who haven't cubed allow $120k = 3\times$ their $10k$
If they write $120k^3x^3 = 3\times$ their $10kx$ only allow recovery of this mark if x disappears afterwards.

M1: Solves a cubic of the form $Ak^3 = Bk$ by factorising out/cancelling the k and proceeding correctly to at least one value for k. Usually $k = \sqrt{\frac{B}{A}}$

A1: $k = \pm\frac{1}{2}$ o.e ignoring any reference to 0

11(a)	$\frac{1}{\sqrt{4-x}} = (4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} \times (1 \pm ......$	M1	2.1
	Uses a "correct" binomial expansion for their $(1+ax)^n = 1 + nax + \frac{n(n-1)}{2}a^2x^2 +$	M1	1.1b
	$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2$	A1	1.1b
	$\frac{1}{\sqrt{4-x}} = \frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2$	A1	1.1b
		(4)	
(b) (i)	States $x = -14$ and gives a valid reason. Eg explains that the expansion is not valid for $\lvert x \rvert > 4$	B1	2.4
		(1)	
(b)(ii)	States $x = -\frac{1}{2}$ and gives a valid reason. Eg. explains that it is closest to zero	B1	2.4
		(1)	
			(6 marks)

(a)

M1: For the strategy of expanding $\frac{1}{\sqrt{4-x}}$ using the binomial expansion.

You must see $4^{-\frac{1}{2}}$ oe and an expansion which may or may not be combined.

M1: Uses a correct binomial expansion for their $(1 \pm ax)^n = 1 \pm nax \pm \frac{n(n-1)}{2}a^2x^2 +$

Condone sign slips and the "*a*" not being squared in term 3. Condone $a = \pm 1$

Look for an attempt at the correct binomial coefficient for their *n*, being combined with the correct power of *ax*

A1: $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2$ unsimplified

FYI the simplified form is $1 + \frac{x}{8} + \frac{3x^2}{128}$ Accept the terms with commas between.

A1: $\frac{1}{\sqrt{4-x}} = \frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2$ Ignore subsequent terms. Allow with commas between.

Note: Alternatively $(4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}4^{-\frac{5}{2}}(-x)^2 + ..$

M1: For $4^{-\frac{1}{2}} + ...$ **M1:** As above but allow slips on the sign of *x* and the value of *n* **A1:** Correct unsimplified (as above) **A1:** As main scheme

(b) Any evaluations of the expansions are irrelevant.

Look for a suitable value and a suitable reason for both parts.

(b)(i)

B1: Requires $x = -14$ with a suitable reason.

Eg. $x = -14$ as the expansion is only valid for $|x| < 4$ or equivalent.

Eg ‘ $x = -14$ as $|-14| > 4$" or ' I cannot use $x = -14$ as $\left|\frac{-14}{4}\right| > 1$ '

Eg. ‘ $x = -14$ as is outside the range $|x| < 4$ ’

Do not allow ‘ -14 is too big’ or ‘ $x = -14, |x| < 4$ ’ either way around without some reference to the validity of the expansion.

(b)(ii)

B1: Requires $x = -\frac{1}{2}$ with a suitable reason.

Eg. $x = -\frac{1}{2}$ as it is ‘the smallest/smaller value’ or ‘ $x = -\frac{1}{2}$ as the value closest to zero’ (that will give the more accurate approximation). The bracketed statement is not required.

Question	Scheme	Marks	AOs
12(a)	Total time for 6 km = 24 minutes + $6\times1.05+6\times1.05^2$ minutes	M1	3.4
	= 36.915 minutes = 36 minutes 55 seconds *	A1*	1.1b
		(2)	
(b)	**5** th km is $6\times1.05=6\times1.05^{\mathbf{1}}$ **6**th km is $6\times1.05\times1.05=6\times1.05^{\mathbf{2}}$ **7**th km is $6\times1.05\times1.05\times1.05=6\times1.05^{\mathbf{3}}$ Hence the time for the r th km is $6\times1.05^{r-4}$	B1	3.4
		(1)	
(c)	Attempts the total time for the race = Eg. 24 minutes + $\sum_{r=5}^{r=20}6\times1.05^{r-4}$ minutes	M1	3.1a
	Uses the series formula to find an allowable sum Eg. Time for 5 th to 20th km $=\frac{6.3\left(1.05^{16}-1\right)}{1.05-1}=(149.04)$	M1	3.4
	Correct calculation that leads to the **total time** Eg. Total time = 24 + $\frac{6.3\left(1.05^{16}-1\right)}{1.05-1}$	A1	1.1b
	Total time = awrt 173 minutes and 3 seconds	A1	1.1b
		(4)	
			(7 marks)

(a)

M1: For using model to calculate the total time.

Sight of 24 minutes + $6\times1.05+6\times1.05^2$ or equivalent is required. Eg 24 + 6.3 + 6.615

Alternatively in seconds 24 minutes + 378 sec (6min 18 s) +396.9 (6 min 37 s)

A1*: 36 minutes 55 seconds following 36.915, 24+ 6.3+6.615 , 24 + $6\times1.05+6\times1.05^2$ or equivalent working in seconds

(b) Must be seen in (b)

B1: As seen in scheme. For making the link between the r th km and the index of 1.05
Or for EXPLAINING that "the time taken per km (6 mins) only starts to increase by 5% after the first 4 km"

(c) The correct sum formula $\frac{a\left(r^n-1\right)}{r-1}$, if seen, must be correct in part (c) for all relevant marks

M1: For the overall strategy of finding the total time.
Award for adding 18, 24, 30.3 or awrt 36.9 and the sum of a geometric sequence

So award the mark for expressions such as $6\times4+\sum6\times1.05^n$ or $24+\frac{6\left(1.05^{20}-1\right)}{1.05-1}$

The geometric sequence formula, must be used with $r=1.05$ oe but condone slips on a and n

M1: For an attempt at using a correct sum formula for a GP to find an allowable sum

The value of r must be 1.05 oe such as 105% but you should allow a slip on the value of n used for their value of a (See below: We are going to allow the correct value of n or one less)

If you don't see a calculation it may be implied by sight of one of the values seen below

Allow for $a = 6, \quad n = 17$ or 16 Eg. $\frac{6(1.05^{17}-1)}{1.05-1} = (155.0)$ or $\frac{6(1.05^{16}-1)}{1.05-1} = (141.9)$

Allow for $a = 6.3, n = 16$ or 15 Eg $\frac{6.3(1.05^{16}-1)}{1.05-1} = (149.0)$ or $\frac{6.3(1.05^{15}-1)}{1.05-1} = (135.9)$

Allow for $a = 6.615, \ n = 15$ or 14 Eg $\frac{6.615(1.05^{15}-1)}{1.05-1} = (142.7)$ or $\frac{6.615(1.05^{14}-1)}{1.05-1} = (129.6)$

A1: For a correct calculation that will find the **total time**. It does not need to be processed

Allow for example, amongst others, $24+\frac{6.3(1.05^{16}-1)}{1.05-1}$, $18+\frac{6(1.05^{17}-1)}{1.05-1}$, $30.3+\frac{6.615(1.05^{15}-1)}{1.05-1}$

A1: For a total time of awrt 173 minutes and 3 seconds

This answer alone can be awarded 4 marks as long as there is some evidence of where it has come from.

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Candidates that list values: Handy Table for Guidance

M1: For a correct overall strategy which would involve adding four sixes followed by at least 16 other values

The values which may be written in the form 6×1.05^2 or as numbers

Can be implied by $6+6+6+6+(6\times1.05)+.....+(6\times1.05^{16})$

M1: For an attempt to add the numbers from (6×1.05) to (6×1.05^{16}). This could be done on a calculator in which case

expect to see awrt 149

Alternatively, if written out, look for 16 values with 8 correct or follow through correct to 1 dp

A1: Awrt 173 minutes

A1: Awrt 173 minutes and 3 seconds

Km	Time per km	Total Time
1	6.0000	
2	6.0000	12
3	6.0000	18
4	6.0000	24
5	6.3000	30.3
6	6.6150	36.915
7	6.9458	43.86075
8	7.2930	51.15379
9	7.6577	58.81148
10	8.0406	66.85205
11	8.4426	75.29465
12	8.8647	84.15939
13	9.3080	93.46736
14	9.7734	103.2407
15	10.2620	113.5028
16	10.7751	124.2779
17	11.3139	135.5918
18	11.8796	147.4714
19	12.4736	159.945
20	13.0972	173.0422

Question	Scheme	Marks	AOs
13(a)	2^6 or 64 as the constant term	B1	1.1b
	$\left(2+\frac{3x}{4}\right)^6 = ...+ {}^6C_1 2^5\left(\frac{3x}{4}\right)^1 + {}^6C_2 2^4\left(\frac{3x}{4}\right)^2 + ...$ $= ...+6\times 2^5\left(\frac{3x}{4}\right)^1 + \frac{6\times 5}{2}\times 2^4\left(\frac{3x}{4}\right)^2 + ...$	M1 A1	1.1b 1.1b
	$= 64+144x+135x^2+...$	A1	1.1b
		(4)	
(b)	$\frac{3x}{4} = -0.075 \Rightarrow x = -0.1$ So find the value of $64+144x+135x^2$ with $x=-0.1$	B1ft	2.4
		(1)	
			(5 marks)

Notes

(a)

B1: Sight of either 2^6 or 64 as the constant term

M1: An attempt at the binomial expansion. This may be awarded for a correct attempt at either the second **OR** third term. Score for the correct binomial coefficient with the correct power of 2 and the correct power of $\frac{3x}{4}$ condoning slips. Correct bracketing is not essential for this M mark.

Condone ${}^6C_2 2^4 \frac{3x}{4}^2$ for this mark

A1: Correct (unsimplified) second **AND** third terms.

The binomial coefficients must be processed to numbers /numerical expression e.g $\frac{6!}{4!2!}$ or $\frac{6\times 5}{2}$

They cannot be left in the form ${}^6C_1$ and/or $\binom{6}{2}$

A1: $64+144x+135x^2+...$ Ignore any terms after this. Allow to be written $64, 144x, 135x^2$

(b)

B1ft: $x=-0.1$ or $-\frac{1}{10}$ **with** a comment about substituting this into their $64+144x+135x^2$

If they have written (a) as $64, 144x, 135x^2$ candidate would need to say substitute $x=-0.1$ into the sum of the first three terms.

As they do not have to perform the calculation allow

Set $2+\frac{3x}{4}=1.925$, solve for x and then substitute this value into the expression from (a)

If a value of x is found then it must be correct

Alternative to part (a)

$$\left(2+\frac{3x}{4}\right)^6 = 2^6\left(1+\frac{3x}{8}\right)^6 = 2^6\left(1+{}^6C_1\left(\frac{3x}{8}\right)^1 + {}^6C_2\left(\frac{3x}{8}\right)^2 + ...\right)$$

B1: Sight of either 2^6 or 64

M1: An attempt at the binomial expansion. This may be awarded for either the second or third term. Score for the correct binomial coefficient with the correct power of $\frac{3x}{8}$ Correct bracketing is not essential for this mark.

A1: A correct attempt at the binomial expansion on the second and third terms.

A1: $64+144x+135x^2+...$ Ignore any terms after this.

Question	Scheme	Marks	AOs
14 (a)	$\sqrt{\frac{1+4x}{1-x}}=(1+4x)^{0.5}\times(1-x)^{-0.5}$	B1	3.1a
	$(1+4x)^{0.5}=1+0.5\times(4x)+\frac{0.5\times-0.5}{2}\times(4x)^2$ $(1-x)^{-0.5}=1+(-0.5)(-x)+\frac{(-0.5)\times(-1.5)}{2}(-x)^2$ $(1+4x)^{0.5}=1+2x-2x^2$ and $(1-x)^{-0.5}=1+0.5x+0.375x^2$ oe	M1 M1 A1	1.1b 1.1b 1.1b
	$(1+4x)^{0.5}\times(1-x)^{-0.5}=(1+2x-2x^2......)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2...\right)$ $=1+\frac{1}{2}x+\frac{3}{8}x^2+2x+x^2-2x^2+...$ $=A+Bx+Cx^2$	dM1	2.1
	$=1+\frac{5}{2}x-\frac{5}{8}x^2......$ *	A1*	1.1b
		(6)	
(b)	Expression is valid $\lvert x\rvert<\frac{1}{4}$ Should not use $x=\frac{1}{2}$ as $\frac{1}{2}>\frac{1}{4}$	B1	2.3
		(1)	
(c)	Substitutes $x=\frac{1}{11}$ into $\sqrt{\frac{1+4x}{1-x}}\approx1+\frac{5}{2}x-\frac{5}{8}x^2$	M1	1.1b
	$\sqrt{\frac{3}{2}}=\frac{1183}{968}$	A1	1.1b
	(so $\sqrt{6}$ is) $\frac{1183}{484}$ or $\frac{2904}{1183}$	A1	2.1
		(3)	
			(10 marks)

(a)

B1: Scored for key step in setting up the process so that it can be attempted using binomial expansions

This could be achieved by $\sqrt{\frac{1+4x}{1-x}}=(1+4x)^{0.5}\times(1-x)^{-0.5}$ See end for other alternatives

It may be implied by later work.

M1: Award for an attempt at the binomial expansion $(1+4x)^{0.5}=1+0.5\times(4x)+\frac{(0.5)\times(-0.5)}{2}\times(4x)^2$

There must be three (or more terms). Allow a missing bracket on the $(4x)^2$ and a sign slip so the correct application may be implied by $1+2x\pm0.5x^2$

M1: Award for an attempt at the binomial expansion $(1-x)^{-0.5}=1+(-0.5)(-x)+\frac{(-0.5)\times(-1.5)}{2}(-x)^2$

There must be three (or more terms). Allow a missing bracket on the $(-x)^2$ and a sign slips so the method may be awarded on $1\pm0.5x\pm0.375x^2$

A1: Both correct and simplified. This may be awarded for a correct final answer if a candidate does all their simplification at the end

dM1: In the main scheme it is for multiplying their two expansions to reach a quadratic. It is for the key step in adding 'six' terms to produce the quadratic expression. Higher power terms may be seen. Condone slips on the multiplication on one term only. It is dependent upon having scored the first B and one of the other two M's

In the alternative it is for multiplying $\left(1+\frac{5}{2}x-\frac{5}{8}x^2\right)(1-x)^{0.5}$ and comparing it to $(1+4x)^{0.5}$

It is for the key step in adding 'six' terms to produce the quadratic expression.

A1*: Completes proof with no errors or omissions. In the alternative there must be some reference to the fact that both sides are equal.

(b)

B1: States that the expansion may not / is not valid when $|x| > \frac{1}{4}$

This may be implied by a statement such as $\frac{1}{2} > \frac{1}{4}$ or stating that the expansion is only valid when $|x| < \frac{1}{4}$

Condone, for this mark a candidate who substitutes $x = \frac{1}{2}$ into the $4x$ **and** states it is not valid as $2 > 1$ oe

Don't award for candidates who state that $\frac{1}{2}$ is too big without any reference to the validity of the expansion.

As a rule you should see some reference to $\frac{1}{4}$ or $4x$

(c)(i)

M1: Substitutes $x = \frac{1}{11}$ into BOTH sides $\sqrt{\frac{1+4x}{1-x}} \approx 1+\frac{5}{2}x-\frac{5}{8}x^2$ and attempts to find at least one side.

As the left hand side is $\frac{\sqrt{6}}{2}$ they may multiply by 2 first which is acceptable

A1: Finds both sides leading to a correct equation/statement $\sqrt{\frac{15}{10}} = \frac{1183}{968}$ oe $\sqrt{6} = 2\times\frac{1183}{968}$

A1: $\sqrt{6} = \frac{1183}{484}$ or $\sqrt{6} = \frac{2904}{1183}$ $\quad\sqrt{6} = 2\times\frac{1183}{968} = \frac{1183}{484}$ would imply all 3 marks

....

Watch for other equally valid alternatives for 11(a) including

B1: $(1+4x)^{0.5} \approx \left(1+\frac{5}{2}x-\frac{5}{8}x^2\right)(1-x)^{0.5}$ then the M's are for $(1+4x)^{0.5}$ and $(1-x)^{0.5}$

M1: $(1-x)^{0.5} = 1+(0.5)(-x)+\frac{(0.5)\times(-0.5)}{2}(-x)^2$

....

Or

B1: $\sqrt{\frac{1+4x}{1-x}} = \sqrt{1+\frac{5x}{1-x}} = \left(1+5x(1-x)^{-1}\right)^{\frac{1}{2}}$ then the first M1 for one application of binomial and the second would be for both $(1-x)^{-1}$ and $(1-x)^{-2}$

....

Or

B1: $\sqrt{\frac{1+4x}{1-x}} \times \frac{\sqrt{1-x}}{\sqrt{1-x}} = \sqrt{(1+3x-4x^2)} \times (1-x)^{-1} = \left(1+(3x-4x^2)\right)^{\frac{1}{2}} \times (1-x)^{-1}$

Question	Scheme	Marks	AOs
15(a)	$\left(2-\frac{x}{16}\right)^9 = 2^9 + \binom{9}{1}2^8.\left(-\frac{x}{16}\right)+\binom{9}{2}2^7.\left(-\frac{x}{16}\right)^2+...$	M1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = 512+...$	B1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = ... -144x +...$	A1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = ... +... + 18x^2 (+...)$	A1	1.1b
		(4)	
(b)	Sets $'512'a = 128 \Rightarrow a = ...$	M1	1.1b
	$(a=)\frac{1}{4}$ oe	A1 ft	1.1b
		(2)	
(c)	Sets $'512'b + '-144'a = 36 \Rightarrow b = ...$	M1	2.2a
	$(b=)\frac{9}{64}$ oe	A1	1.1b
		(2)	
			(8 marks)
11(a) alt	$\left(2-\frac{x}{16}\right)^9 = 2^9\left(1-\frac{x}{32}\right)^9 = 2^9\left(1+\binom{9}{1}\left(-\frac{x}{32}\right)+\binom{9}{2}\left(-\frac{x}{32}\right)^2+...\right)$	M1	1.1b
	$= 512+...$	B1	1.1b
	$= ... -144x +...$	A1	1.1b
	$= ... +... + 18x^2 (+...)$	A1	1.1b

Notes

(a)

M1: Attempts the binomial expansion. May be awarded on either term two and/or term three Scored for a correct binomial coefficient combined with a correct power of 2 and a correct power of $\left(\pm\frac{x}{16}\right)$ Condone $\binom{9}{2}2^7.\left(-\frac{x^2}{16}\right)$ for term three.

Allow any form of the binomial coefficient. Eg $\binom{9}{2} = {}^9C_2 = \frac{9!}{7!2!} = 36$

In the alternative it is for attempting to take out a factor of 2 (may allow 2^n outside bracket) and having a correct binomial coefficient combined with a correct power of $\left(\pm\frac{x}{32}\right)$

B1: For 512
A1: For $-144x$

A1: For $+\ 18x^2$ Allow even following $\left(+\frac{x}{16}\right)^2$

Listing is acceptable for all 4 marks
(b)
M1: For setting their $512a = 128$ and proceeding to find a value for *a*. Alternatively they could substitute $x = 0$ into both sides of the identity and proceed to find a value for *a*.

A1 ft: $a = \frac{1}{4}$ oe Follow through on $\frac{128}{\text{their } 512}$

(c)
M1: Condone $512b \pm 144 \times a = 36$ following through on their 512, their -144 and using their value of "a" to find a value for "b"

A1: $b = \frac{9}{64}$ oe

Question	Scheme	Marks	AOs
16 (a)	$\left(1+\frac{3}{x}\right)^2 = 1+\frac{6}{x}+\frac{9}{x^2}$	M1 A1	1.1b 1.1b
		(2)	
(b)	$\left(1+\frac{3}{4}x\right)^6 = 1+6\times\left(\frac{3}{4}x\right)+\ldots$	B1	1.1b
	$\left(1+\frac{3}{4}x\right)^6 = 1+6\times\left(\frac{3}{4}x\right)+\frac{6\times5}{2}\times\left(\frac{3}{4}x\right)^2+\frac{6\times5\times4}{3\times2}\times\left(\frac{3}{4}x\right)^3+\ldots$	M1 A1	1.1b 1.1b
	$=1+\frac{9}{2}x+\frac{135}{16}x^2+\frac{135}{16}x^3+\ldots$	A1	1.1b
		(4)	
(c)	$\left(1+\frac{3}{x}\right)^2\left(1+\frac{3}{4}x\right)^6 = \left(1+\frac{6}{x}+\frac{9}{x^2}\right)\left(1+\frac{9}{2}x+\frac{135}{16}x^2+\frac{135}{16}x^3+\ldots\right)$		
	Coefficient of $x = \frac{9}{2}+6\times\frac{135}{16}+9\times\frac{135}{16}=\frac{2097}{16}$	M1 A1	2.1 1.1b
		(2)	
		(8 marks)	

Notes:

(a)

M1: Attempts $\left(1+\frac{3}{x}\right)^2 = A+\frac{B}{x}+\frac{C}{x^2}$

A1: $\left(1+\frac{3}{x}\right)^2 = 1+\frac{6}{x}+\frac{9}{x^2}$

(b)

B1: First two terms correct, may be un-simplified

M1: Attempts the binomial expansion. Implied by the correct coefficient and power of x seen at least once in term 3 or 4

A1: Binomial expansion correct and un-simplified

A1: Binomial expansion correct and simplified.

(c)

M1: Combines all relevant terms for their $\left(1+\frac{A}{x}+\frac{B}{x^2}\right)\left(1+Cx+Dx^2+Ex^3+\ldots\right)$ to find the coefficient of x.

A1: Fully correct

Question	Scheme	Marks	AOs
17 (a)	$(4+5x)^{\frac{1}{2}} = (4)^{\frac{1}{2}}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}} = 2\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}$	B1	1.1b
	$= \{2\}\left[1+\left(\frac{1}{2}\right)\left(\frac{5x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{5x}{4}\right)^2+\dots\right]$	M1	1.1b
		A1ft	1.1b
	$= 2+\frac{5}{4}x-\frac{25}{64}x^2+\dots$	A1	2.1
		(4)	
(b)(i)	$\left\{x=\frac{1}{10}\Rightarrow\right\}(4+5(0.1))^{\frac{1}{2}}$	M1	1.1b
	$=\sqrt{4.5} = \frac{3}{2}\sqrt{2}$ or $\frac{3}{\sqrt{2}}$		
	$\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\frac{3}{\sqrt{2}} = 2+\frac{5}{4}\left(\frac{1}{10}\right)-\frac{25}{64}\left(\frac{1}{10}\right)^2+\dots$ $\{=2.121\dots\}$ $\Rightarrow \frac{3}{2}\sqrt{2}=\frac{543}{256}$ or $\frac{3}{\sqrt{2}}=\frac{543}{256}$ $\Rightarrow \sqrt{2}=\dots$	M1	3.1a
	So, $\sqrt{2}=\frac{181}{128}$ or $\sqrt{2}=\frac{256}{181}$	A1	1.1b
(b)(ii)	$x=\frac{1}{10}$ satisfies $\lvert x\rvert<\frac{4}{5}$ (o.e.), so the approximation is valid.	B1	2.3
		(4)	
			(8 marks)

Question **17** Notes:	
(a)	
B1:	Manipulates $(4+5x)^{\frac{1}{2}}$ by taking out a factor of $(4)^{\frac{1}{2}}$ or 2
M1:	Expands $(...+\lambda x)^{\frac{1}{2}}$ to give at least 2 terms which can be simplified or un-simplified, E.g. $1+\left(\frac{1}{2}\right)(\lambda x)$ or $\left(\frac{1}{2}\right)(\lambda x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(\lambda x)^2$ or $1+...+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(\lambda x)^2$ where λ is a numerical value and **where** $\lambda \neq 1$.
A1ft:	A correct simplified or un-simplified $1+\left(\frac{1}{2}\right)(\lambda x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(\lambda x)^2$ expansion with **consistent** (λx)
A1:	Fully correct solution leading to $2+\frac{5}{4}x+kx^2$, where $k=-\frac{25}{64}$
(b)(i)	
M1:	Attempts to substitute $x=\frac{1}{10}$ or 0.1 into $(4+5x)^{\frac{1}{2}}$
M1:	A complete method of finding an approximate value for $\sqrt{2}$. E.g. • substituting $x=\frac{1}{10}$ or 0.1 into their part (a) binomial expansion and equating the result to an expression of the form $\alpha\sqrt{2}$ or $\frac{\beta}{\sqrt{2}}$; $\alpha, \beta \neq 0$ • followed by re-arranging to give $\sqrt{2}=...$
A1:	$\frac{181}{128}$ **or any equivalent fraction**, e.g. $\frac{362}{256}$ or $\frac{543}{384}$ Also allow $\frac{256}{181}$ **or any equivalent fraction**
(b)(ii)	
B1:	Explains that the approximation is valid because $x=\frac{1}{10}$ satisfies $\lvert x \rvert < \frac{4}{5}$

Question	Scheme	Marks	AOs
18 (a)	$a_1 = 3$, $a_2 = 0$, $a_3 = 1.5$, $a_4 = 3$	M1	1.1b
	$\sum_{r=1}^{100} a_r = 33(4.5) + 3$	M1	2.2a
	$= 151.5$	A1	1.1b
		(3)	
(b)	$\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r = (2)(151.5) - 3 = 300$	B1ft	2.2a
		(1)	
			(4 marks)

Question **18** Notes:

(a)	
M1:	Uses the formula $a_{n+1} = \dfrac{a_n - 3}{a_n - 2}$, with $a_1 = 3$ to generate values for a_2, a_3 and a_4
M1:	Finds $a_4 = 3$ and deduces $\sum_{r=1}^{100} a_r = 33(\text{"3"} + \text{"0"} + \text{"1.5"}) + \text{"3"}$
A1:	which leads to a correct answer of 151.5
(b)	
B1ft:	Follow through on their periodic function. Deduces that either • $\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r = (2)(\text{"151.5"}) - 3 = 300$ • $\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r = \text{"151.5"} + (33)(\text{"3"} + \text{"0"} + \text{"1.5"}) = 151.5 + 148.5 = 300$

Question	Scheme	Marks	AOs
19 (a)	Total amount $= \dfrac{2100(1-(1.012)^{14})}{1-1.012}$ or $\dfrac{2100((1.012)^{14}-1)}{1.012-1}$	M1	3.1b
	$= 31806.9948... = 31800$ (tonnes) (3 sf)	A1	1.1b
		(2)	
	Total Cost $= 5.15(2000(14)) + 6.45(31806.9948... - (2000)(14))$	M1	3.1b
		M1	1.1b
	$= 5.15(28000) + 6.45(3806.9948...) = 144200 + 24555.116...$		
	$= 168755.116... = £169000$ (nearest £1000)	A1	3.2a
		(3)	
		(5 marks)	

Question **19** Notes:

(a)	
M1:	Attempts to apply the correct geometric summation formula with either $n = 13$ or $n = 14$, $a = 2100$ and $r = 1.012$ (Condone $r = 1.12$)
A1:	Correct answer of 31800 (tonnes)
(b)	
M1:	Fully correct method to find the total cost
M1:	For either • $5.15(2000(14))$ $\{= 144200\}$ • $6.45(\text{"}31806.9948...\text{"} - (2000)(14))$ $\{= 24555.116...\}$ • $5.15(2000(13))$ $\{= 133900\}$ • $6.45(\text{"}29354.73794...\text{"} - (2000)(13))$ $\{= 21638.059...\}$
A1:	Correct answer of £169000 **Note:** Using rounded answer in part (a) gives 168710 which becomes £169000 (nearest £1000)

Question	Scheme	Marks	AOs
20(a)	$\left(2-\frac{x}{2}\right)^7 = 2^7 + \binom{7}{1}2^6.\left(-\frac{x}{2}\right)+\binom{7}{2}2^5.\left(-\frac{x}{2}\right)^2 + ...$	M1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = 128 + ...$	B1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = ... -224x + ...$	A1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = ... + ... + 168x^2 (+ ...)$	A1	1.1b
		(4)	
(b)	Solve $\left(2-\frac{x}{2}\right) = 1.995$ so $x = 0.01$ and state that 0.01 would be substituted for x into the expansion	B1	2.4
		(1)	
		(5 marks)	

Notes:

(a)

M1: Need correct binomial coefficient with correct power of 2 and correct power of x. Coefficients may be given in any correct form; e.g. 1, 7, 21 or ${}^7C_0$, ${}^7C_1$, ${}^7C_2$ or equivalent

B1: Correct answer, simplified as given in the scheme

A1: Correct answer, simplified as given in the scheme

A1: Correct answer, simplified as given in the scheme

(b)

B1: Needs a full explanation i.e. to state $x = 0.01$ and that this would be substituted and that it is a solution of $\left(2-\frac{x}{2}\right) = 1.995$

Question	Scheme	Marks	AOs
21(a)(i)	$a_1 = 3,\ a_2 = 5,\ a_3 = 3\ ...$	B1	1.1b
(ii)	2	B1	1.1b
		(2)	
(b)	$\sum_{n=1}^{85} a_n = 42\times(3+5)+3$ o.e.	M1	3.1a
	$= 339$	A1	1.1b
		(2)	
			(4 marks)
Notes:			

(a)(i) Mark (a)(i) and (a)(ii) together.

B1: States the values of at least $a_2 = 5$ and $a_3 = 3$. This is sufficient but if more terms are given they must be correct. There is no need to see e.g. $a_2 = ...,\ a_3 = ...$ just look for values.

Allow an algebraic approach e.g. $a_{n+1} = 8 - a_n,\ a_{n+2} = 8 - (8 - a_n) = a_n$

A conclusion is **not** needed.

(a)(ii)

B1: States that the order of the periodic sequence is 2

Allow "second order", "it repeats every 2 numbers" or equivalent statements that convey the idea of the period being 2.

Note that ±2 is B0

(b)

M1: Attempts a <u>**correct**</u> method to find $\sum_{n=1}^{85} a_n$

For example $\sum_{n=1}^{85} a_n = 42\times(3+5)+3$, $\sum_{n=1}^{85} a_n = \frac{84}{2}\times 3 + 42\times 5 + 3$ or $\sum_{n=1}^{85} a_n = 43\times(3+5)-5$

or $\sum_{n=1}^{85} a_n = 43\times 3 + 42\times 5$ or $\sum_{n=1}^{85} a_n = \frac{85}{2}\times 8 - 1$

There may be other methods e.g. "Chunking": 5×(3 + 5) = 40, 40 × 8 = 320, 320 + 3×3 +2×5 = 339

A1: 339. Correct answer only scores both marks.

Attempts to use an AP formula score M0

Question	Scheme	Marks	AOs
22(a)	$\sqrt{4-9x}=2(1\pm...)^{\frac{1}{2}}$	B1	1.1b
	$\left(1-\text{"}\frac{9x}{4}\text{"}\right)^{\frac{1}{2}}=...+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\left(\text{"}-\frac{9x}{4}\text{"}\right)^2}{2!}$ or $...+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\left(\text{"}-\frac{9x}{4}\text{"}\right)^3}{3!}$	M1	1.1b
	$1+\frac{1}{2}\times\left(-\frac{9x}{4}\right)+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\left(-\frac{9x}{4}\right)^2}{2!}+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\left(-\frac{9x}{4}\right)^3}{3!}$	A1	1.1b
	$\sqrt{4-9x}=2-\frac{9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$	A1	1.1b
		(4)	
(b)	States that the approximation will be an **overestimate** since all terms (after the first one) in the expansion are negative (since $x>0$)	B1	3.2b
		(1)	
			(5 marks)
Notes:			

(a)

B1: Takes out a factor of 4 and writes $\sqrt{4-9x}=2(1\pm...)^{\frac{1}{2}}$ or $\sqrt{4}(1\pm...)^{\frac{1}{2}}$ or $4^{\frac{1}{2}}(1\pm...)^{\frac{1}{2}}$

M1: For an attempt at the binomial expansion of $(1+ax)^{\frac{1}{2}}$ $a\neq 1$ to form term 3 or term 4 with the correct structure. Look for the correct binomial coefficient multiplied by the corresponding power of x e.g.

$\frac{(\frac{1}{2})(\frac{1}{2}-1)}{2!}(...x)^2$ **or** $\frac{(\frac{1}{2})(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(...x)^3$ where$...\neq 1$

Condone missing or incorrect brackets around the x terms but the binomial coefficients must be correct. Allow 2! and/or 3! or 2 and/or 6. Ignore attempts to find more terms.

Do not allow notation such as $\begin{pmatrix}\frac{1}{2}\\1\end{pmatrix}$, $\begin{pmatrix}\frac{1}{2}\\2\end{pmatrix}$ unless these are interpreted correctly.

A1: Correct expression for the expansion of $\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$ e.g.

$$1+\frac{1}{2}\times\left(-\frac{9x}{4}\right)+\frac{\frac{1}{2}\times\left(\frac{1}{2}-1\right)\left(\pm\frac{9x}{4}\right)^2}{2!}+\frac{\frac{1}{2}\times\left(\frac{1}{2}-1\right)\times\left(\frac{1}{2}-2\right)\left(-\frac{9x}{4}\right)^3}{3!}$$

which may be left unsimplified as shown but the bracketing must be correct unless any missing brackets are implied by subsequent work. If the 2 outside this expansion is only partially applied to this expansion then score A0 but if it is applied to all terms this A1 can be implied.

OR at least 2 correct simplified terms **for the final expansion** from, $-\frac{9x}{4}$, $-\frac{81x^2}{64}$, $-\frac{729x^3}{512}$

A1: $\sqrt{4-9x}=2-\frac{9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$ oe and condone e.g. $2+\frac{-9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$

Allow equivalent mixed numbers and/or decimals for the coefficients e.g.:

$$\left(\frac{9}{4}, 2\frac{1}{4}, 2.25\right), \left(\frac{81}{64}, 1\frac{17}{64}, 1.265625\right), \left(\frac{729}{512}, 1\frac{217}{512}, 1.423828125\right)$$

Ignore any extra terms if found. Allow terms to be "listed" and apply isw once a correct expansion is seen. Allow recovery if applicable e.g. if an "x" is lost then "reappears".

Direct expansion in (a) can be marked in a similar way:

$$\sqrt{4-9x} = (4-9x)^{\frac{1}{2}} = 4^{\frac{1}{2}} + \left(\frac{1}{2}\right)4^{-\frac{1}{2}} \times (-9x)^1 + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)4^{-\frac{3}{2}} \times \frac{(-9x)^2}{2!} + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)4^{-\frac{5}{2}} \times \frac{(-9x)^3}{3!}$$

B1: For 2 or $\sqrt{4}$ or $4^{\frac{1}{2}}$ as the constant term in the expansion.

M1: Correct form for term 3 or term 4.

E.g. $\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) \times \frac{(...x)^2}{2!}$ or $\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right) \times \frac{(...x)^3}{3!}$ where $... \neq 1$

Condone missing brackets around the x terms but the binomial coefficients must be correct. Allow 2! and/or 3! or 2 and/or 6. Ignore attempts to find more terms.

Do not allow notation such as $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ unless these are interpreted correctly.

A1: Correct expansion (unsimplified as above)

OR at least 2 correct simplified terms from, $-\frac{9x}{4}$, $-\frac{81x^2}{64}$, $-\frac{729x^3}{512}$

A1: $\sqrt{4-9x} = 2 - \frac{9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$ oe and condone e.g. $2 + \frac{-9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$

Allow equivalent mixed numbers and/or decimals for the coefficients e.g.:

$$\left(\frac{9}{4}, 2\frac{1}{4}, 2.25\right), \left(\frac{81}{64}, 1\frac{17}{64}, 1.265625\right), \left(\frac{729}{512}, 1\frac{217}{512}, 1.423828125\right)$$

Ignore any extra terms if found. Allow terms to be "listed" and apply isw once a correct expansion is seen. Allow recovery if applicable e.g. if an "x" is lost then "reappears".

(b)

B1: States that the approximation will be an **overestimate** due to the fact that all terms (after the first one) in the expansion are negative or equivalent statements e.g.

- Overestimate because the terms are negative
- Overestimate as the terms are being taken away (from 2)

Condone "overestimate as every term is negative"

If you think a response is worthy of credit but are unsure then use Review.

This mark depends on having obtained an expansion in (a) of the form

$k - px - qx^2 - rx^3 \quad k, p, q, r > 0$ but note that if e.g. one of the algebraic terms is zero or was "lost" or there are extra negative terms this mark is still available.

....

Question	Scheme	Marks	AOs
23(a)	$16+(21-1)\times d=24 \Rightarrow d=...$	M1	1.1b
	$d=0.4$	A1	1.1b
	Answer only scores both marks.		
		(2)	
(b)	$S_n=\frac{1}{2}n\{2a+(n-1)d\} \Rightarrow S_{500}=\frac{1}{2}\times 500\{2\times 16+499\times "0.4"\}$	M1	1.1b
	$=57900$	A1	1.1b
	Answer only scores both marks		
		(2)	
	(b) Alternative using $S_n=\frac{1}{2}n\{a+l\}$		
	$l=16+(500-1)\times "0.4"=215.6 \Rightarrow S_{500}=\frac{1}{2}\times 500\{16+"215.6"\}$	M1	1.1b
	$=57900$	A1	1.1b
		(4 marks)	

Notes

(a)

M1: Correct strategy to find the common difference – must be a correct method using a = 16, and n = 21 and the 24. The method may be implied by their working.
If the AP term formula is quoted it must be correct, so use of e.g. $u_n=a+nd$ scores M0

A1: Correct value. Accept equivalents e.g. $\frac{8}{20}, \frac{4}{10}, \frac{2}{5}$ etc.

(b)

M1: Attempts to use a correct sum formula with a = 16, n = 500 and their numerical d from part (a)
If a formula is quoted it must be correct (it is in the formula book)

A1: Correct value

Alternative:

M1: Correct method for the 500th term and then uses $S_n=\frac{1}{2}n\{a+l\}$ with their l

A1: Correct value

Note that some candidates are showing implied use of $u_n=a+nd$ by showing the following:

$$\text{(a) } d=\frac{24-16}{21}=\frac{8}{21} \quad \text{(b) } S_{500}=\frac{1}{2}\times 500\left\{2\times 16+499\times\frac{8}{21}\right\}=55523.80952...$$

This scores (a) M0A0 (b) M1A0

Question	Scheme	Marks	AOs
24	$a=\left(\frac{3}{4}\right)^2$ or $a=\frac{9}{16}$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{\frac{9}{16}}{1-\left(-\frac{3}{4}\right)}=...$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
		(3)	
	Alternative 1:		
	$\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{-\frac{3}{4}}{1-\left(-\frac{3}{4}\right)}=...$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=-\frac{3}{7}-\left(-\frac{3}{4}\right)$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
	Alternative 2:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-...$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+...-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-...$ $\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+...=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ or $-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-...=-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ $\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
	Alternative 3:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=S=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-...$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2\left(1-\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2-...\right)=\left(\frac{3}{4}\right)^2\left(\frac{1}{4}+S\right)\Rightarrow\frac{7}{16}S=\frac{9}{64}\Rightarrow S=...$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
		(3 marks)	

Notes

B1: Deduces the correct value of the **first** term or the common ratio. The correct first term can be

seen as part of them writing down the sequence but must be the **first** term.
M1: Recognises that the series is infinite geometric and applies the sum to infinity GP formula with $a = \frac{9}{16}$ and $r = \pm\frac{3}{4}$
A1*: Correct proof
Alternative 1:
B1: Deduces the correct value for the sum to infinity (starting at n = 1) or the common ratio
M1: Calculates the required value by subtracting the first term from their sum to infinity
A1*: Correct proof
Alternative 2:
B1: Deduces the correct value of the **first** term or the common ratio.
M1: Splits the series into "odds" and "evens", attempts the sum of both parts and calculates the required value by adding both sums
A1*: Correct proof
Alternative 3:
B1: Deduces the correct value of the **first** term
M1: A complete method by taking out the first term, expresses the rhs in terms of the original sum and rearranges for "S"
A1*: Correct proof

Question	Scheme	Marks	AOs
25	${}^7C_4 a^3 (2x)^4$	M1	1.1b
	$\frac{7!}{4!3!}a^3 \times 2^4 = 15120 \Rightarrow a = ...$	dM1	2.1
	$a = 3$	A1	1.1b
		(3)	
			(3 marks)

Notes:

M1: For an attempt at the correct coefficient of x^4.

The coefficient must have

- the correct binomial coefficient
- the correct power of a
- 2 or 2^4 (may be implied)

May be seen within a full or partial expansion.

Accept ${}^7C_4 a^3 (2x)^4$, $\frac{7!}{4!3!}a^3(2x)^4$, $\binom{7}{4}a^3(2x)^4$, $35a^3(2x)^4$, $560a^3x^4$, $\binom{7}{4}a^3 16x^4$ etc.

or ${}^7C_4 a^3 2^4$, $\frac{7!}{4!3!}a^3 2^4$, $\binom{7}{4}a^3 2^4$, $35a^3 2^4$, $560a^3$ etc.

or ${}^7C_3 a^3 (2x)^4$, $\frac{7!}{4!3!}a^3(2x)^4$, $\binom{7}{3}a^3(2x)^4$, $35a^3(2x)^4$, $560a^3x^4$, $\binom{7}{3}a^3 16x^4$ etc.

or ${}^7C_3 a^3 2^4$, $\frac{7!}{4!3!}a^3 2^4$, $\binom{7}{3}a^3 2^4$, $35a^3 2^4$, $560a^3$

You can condone missing brackets around the "2x" so allow e.g. $\frac{7!}{4!3!}a^3 2x^4$

An alternative is to attempt to expand $a^7\left(1+\frac{2x}{a}\right)^7$ to give $a^7\left(...\frac{7\times6\times5\times4}{4!}\left(\frac{2x}{a}\right)^4...\right)$

Allow M1 for e.g. $a^7\left(...\frac{7\times6\times5\times4}{4!}\left(\frac{2x}{a}\right)^4...\right)$, $a^7\left(...\binom{7}{4}\left(\frac{2x}{a}\right)^4...\right)$, $a^7\left(...35\left(\frac{2x}{a}\right)^4...\right)$ etc.

but condone missing brackets around the $\frac{2x}{a}$

Note that ${}^7C_3$, $\binom{7}{3}$ etc. are equivalent to ${}^7C_4$, $\binom{7}{4}$ etc. and are equally acceptable.

If the candidate attempts $(a + 2x)(a + 2x)(a + 2x)$… etc. then it must be a complete method to reach the required term. Send to review if necessary.

dM1: For $"560"a^3 = 15120 \Rightarrow a = ...$ Condone slips on copying the 15120 but their "560" must be an attempt at ${}^7C_4\times 2$ or ${}^7C_4\times 2^4$ and must be attempting the <u>cube root</u> of $\frac{15120}{"560"}$. **Depends on the first mark**.

A1: $a = 3$ and no other values i.e. ±3 scores A0

Note that this is fairly common:

$${}^7C_4 a^3 2x^4 = 70a^3x^4 \Rightarrow 70a^3 = 15120 \Rightarrow a^3 = 216 \Rightarrow a = 6$$

and scores M1 dM1 A0

Question	Scheme	Marks	AOs
26(a)	$S_n = a + ar + ar^2 + \dots\dots + ar^{n-1}$	B1	1.2
	$rS_n = ar + ar^2 + ar^3 + \dots\dots + ar^n \Rightarrow S_n - rS_n = \dots$	M1	2.1
	$S_n - rS_n = a - ar^n$ $S_n(1-r) = a(1-r^n) \Rightarrow S_n = \dfrac{a(1-r^n)}{(1-r)}$ *	A1 A1*	1.1b 2.1
		(4)	
(b)	$\dfrac{a(1-r^{10})}{1-r} = 4\times\dfrac{a(1-r^5)}{1-r}$ or $4\times\dfrac{a(1-r^{10})}{1-r} = \dfrac{a(1-r^5)}{1-r}$ Equation in r^{10} and r^5 (and possibly $1 - r$)	M1	3.1a
	$1 - r^{10} = 4(1-r^5)$	A1	1.1b
	$r^{10} - 4r^5 + 3 = 0 \Rightarrow (r^5-1)(r^5-3) = 0 \Rightarrow r^5 = \dots$ or e.g. $1 - r^{10} = 4(1-r^5) \Rightarrow (1-r^5)(1+r^5) = 4(1-r^5) \Rightarrow r^5 = \dots$	dM1	2.1
	$r = \sqrt[5]{3}$ oe only	A1	1.1b
		(4)	
			(8 marks)

Notes:

(a)

B1: Writes out the sum or lists terms. Condone the omission of *S*.
The sum must include the first and last terms and (at least) two other correct terms and no incorrect terms e.g. ar^n
Note that the sum may be seen embedded within their working.

M1: For the key step in attempting to multiply the first series by *r* and subtracting.

A1: $S_n - rS_n = a - ar^n$ either way around but condone one side being prematurely factorised (but not both) **following correct work but this could follow B0 if insufficient terms were shown.**

A1*: Depends on all previous marks. Proceeds to given result showing all steps including seeing both sides unfactorised at some point in their working.

Note: If terms are <u>listed</u> rather than <u>added</u> then allow the first 3 marks if otherwise correct but withhold the final mark.

(b)

M1: For the correct strategy of producing an equation in just r^{10} and r^5 (and possibly $(1 - r)$) with the "4" on either side using the result from part (a) and makes progress to at least cancel through by *a*
Some candidates retain the "$1 - r$" and start multiplying out e.g. $(1 - r)(1 - r^{10})$ and this mark is still available as long as there is progress in cancelling the "*a*".

A1: Correct equation with the *a* and the $1 - r$ cancelled. Allow any correct equation in just r^5 and r^{10}

dM1: Depends on the first M. Solves as far as $r^5 = \ldots$ by solving a 3 term quadratic in r^5 by a valid method – see general guidance or by difference of 2 squares – see above

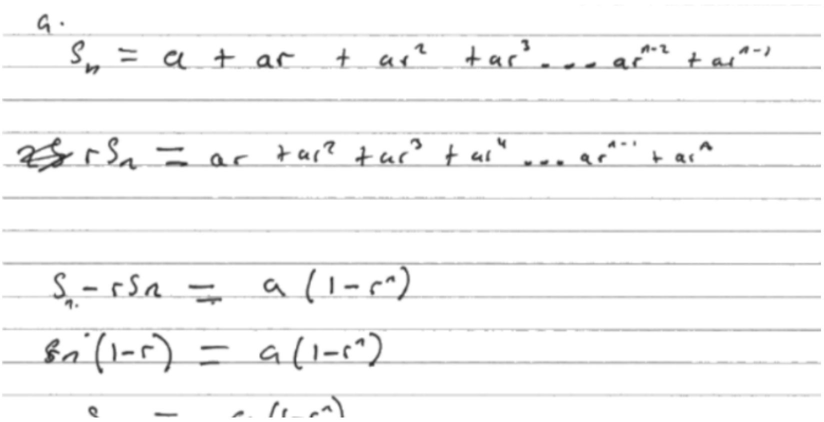
A1: $r = \sqrt[5]{3}$ oe only. The solution $r = 1$ if found must be rejected here.

(b) Note: For candidates who use $S_5 = 4S_{10}$ expect to see:

$$4\times\frac{a\left(1-r^{10}\right)}{1-r}=\frac{a\left(1-r^{5}\right)}{1-r}\Rightarrow 4\left(1-r^{10}\right)=\left(1-r^{5}\right) \text{ M1A0}$$

$$4r^{10}-r^5-3=0\Rightarrow\left(4r^5+3\right)\left(r^5-1\right)=0\Rightarrow r^5=\ldots \text{ or } 4\left(1-r^5\right)\left(1+r^5\right)=\left(1-r^5\right)\Rightarrow r^5=\ldots \text{dM1A0}$$

<u>Example for (a):</u>



This scores B1M1A1A0:

B1: Writes down the sum including first and last terms and at least 2 other correct terms and no incorrect terms

M1: Multiplies by *r* and subtracts

A1: Correct equation (we allow one side to be prematurely factorised)

A0: One side was prematurely factorised

Question	Scheme	Marks	AOs
27 (i) Way 1	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = 20\left(\frac{1}{2}\right)^4 + 20\left(\frac{1}{2}\right)^5 + 20\left(\frac{1}{2}\right)^6 + \ldots$		
	$= \dfrac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= (1.25)(2)\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 2	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=1}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=1}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \dfrac{10}{1-\frac{1}{2}} - (10+5+2.5)$ or $= \dfrac{10}{1-\frac{1}{2}} - \dfrac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 20 - 17.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 3	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=0}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=0}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \dfrac{20}{1-\frac{1}{2}} - (20+10+5+2.5)$ or $= \dfrac{20}{1-\frac{1}{2}} - \dfrac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 40 - 37.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(ii) Way 1	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\}$		
	$= \log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \ldots\ldots + \log_5\left(\frac{50}{49}\right) = \log_5\left(\frac{3}{2}\times\frac{4}{3}\times\ldots\times\frac{50}{49}\right)$	M1	1.1b
		M1	3.1a
	$= \log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$ *	A1*	2.1
		(3)	
(ii) Way 2	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\} \sum_{n=1}^{48} (\log_5(n+2) - \log_5(n+1))$	M1	1.1b
	$= (\log_5 3 + \log_5 4 + \ldots\ldots + \log_5 50) - (\log_5 2 + \log_5 3 + \ldots\ldots + \log_5 49)$	M1	3.1a
	$= \log_5 50 - \log_5 2$ or $\log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$*	A1*	2.1
		(3)	
			(6 marks)

Notes for Question **27**	
(i)	**Way 1**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a complete strategy of applying $\frac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$
A1:	2.5 o.e.
(i)	**Way 2**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\frac{10}{1-\frac{1}{2}} - (10+5+2.5)$ or $\frac{10}{1-\frac{1}{2}} - \frac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
(i)	**Way 3**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\frac{20}{1-\frac{1}{2}} - (20+10+5+2.5)$ or $\frac{20}{1-\frac{1}{2}} - \frac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
Note:	Give M1 M1 A1 for a correct answer of 2.5 from no working in (i)
(ii)	**Way 1**
M1:	Some evidence of applying the addition law of logarithms as part of a valid proof
M1:	Begins to solve the problem by just writing (or by combining) at least three terms including • **either** the first two terms and the last term • **or** the first term and the last two terms
Note:	The 2nd mark can be gained by writing any of • listing $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right), \log_5\left(\frac{50}{49}\right)$ or $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{49}{48}\right), \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + ... + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + ... + \log_5\left(\frac{49}{48}\right) + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2} \times \frac{4}{3} \times ... \times \frac{50}{49}\right)$ ***{this will also gain the 1st M1 mark}*** • $\log_5\left(\frac{3}{2} \times ... \times \frac{49}{48} \times \frac{50}{49}\right)$ ***{this will also gain the 1st M1 mark}***
A1*:	Correct proof leading to a correct answer of 2
Note:	Do not allow the 2nd M1 if $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right)$ are listed and $\log_5\left(\frac{50}{49}\right)$ is used for the first time in their applying the formula $S_{48} = \frac{48}{2}\left(\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{50}{49}\right)\right)$
Note:	<u>**Listing all 48 terms**</u> Give M0 M1 A0 for $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \log_5\left(\frac{5}{4}\right) + ... + \log_5\left(\frac{50}{49}\right) = 2$ {lists all terms} Give M0 M0 A0 for $0.2519... + 0.1787... + 0.1386... + ... + 0.0125... = 2$ {all terms in decimals}

Notes for Question **27**	
(ii)	**Way 2**
M1:	Uses the subtraction law of logarithms to give $\log_5\left(\frac{n+2}{n+1}\right) \to \log_5(n+2) - \log_5(n+1)$
M1:	Begins to solve the problem by writing at least three terms for each of $\log_5(n+2)$ and $\log_5(n+1)$ including • **either** the first two terms and the last term for both $\log_5(n+2)$ and $\log_5(n+1)$ • **or** the first term and the last two terms for both $\log_5(n+2)$ and $\log_5(n+1)$
Note:	This mark can be gained by writing any of • $(\log_5 3 + \log_5 4 + ... + \log_5 50) - (\log_5 2 + \log_5 3 + ... + \log_5 49)$ • $(\log_5 3 + ... + \log_5 49 + \log_5 50) - (\log_5 2 + ... + \log_5 48 + \log_5 49)$ • $(\log_5 3 + \log_5 4 + ... + \log_5 50) - (\log_5 2 + \log_5 3 + ... + \log_5 49)$ • $(\log_5 3 - \log_5 2) + (\log_5 4 - \log_5 3) + ... + (\log_5 50 - \log_5 49)$ • $\log_5 3 - \log_5 2,\ ......, \log_5 49 - \log_5 48,\ \log_5 50 - \log_5 49$
A1*:	Correct proof leading to a correct answer of 2
Note:	The base of 5 can be omitted for the M marks in part (ii), but the base of 5 must be included in the final line (as shown on the mark scheme) of their solution.
Note:	If a student uses a mixture of a Way 1 or Way 2 method, then award the best Way 1 mark only or the best Way 2 mark only.
Note:	Give M1 M0 A0 (1st M for implied use of subtraction law of logarithms) for $\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 91.8237... - 89.8237... = 2$
Note:	Give M1 M1 A1 for $\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = \sum_{n=1}^{48} (\log_5(n+2) - \log_5(n+1))$ $= \log_5(3 \times 4 \times .... \times 50) - \log_5(2 \times 3 \times .... \times 49)$ $= \log_5\left(\frac{50!}{2}\right) - \log_5(49!)$ or $= \log_5(25 \times 49!) - \log_5(49!)$ $= \log_5 25 = 2$

Question	Scheme	Marks	AOs
28	(i) $\sum_{r=1}^{16}\left(3+5r+2^r\right)=131\,798$; (ii) $u_1, u_2, u_3, ...,: u_{n+1}=\frac{1}{u_n}, u_1=\frac{2}{3}$		
(i) Way 1	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\}\sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=\frac{16}{2}(2(8)+15(5))\ +\ \frac{2(2^{16}-1)}{2-1}$	M1	1.1b
		M1	1.1b
	$=728+131\,070\ =\ 131\,798$ *	A1*	2.1
		(4)	
(i) Way 2	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\}\sum_{r=1}^{16}3\ +\ \sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=(3\times16)+\frac{16}{2}(2(5)+15(5))\ +\ \frac{2(2^{16}-1)}{2-1}$	M1	1.1b
		M1	1.1b
	$=48+680+131\,070\ =\ 131\,798$ *	A1*	2.1
		(4)	
(i) Way 3	Sum = $10+17+26+39+60+97+166+299+560+1077+2106$ $+4159+8260+16457+32846+65619=131\,798$ *	M1	3.1a
		M1	1.1b
		M1	1.1b
		A1*	2.1
		(4)	
(ii)	$\left\{u_1=\frac{2}{3}\right\}, u_2=\frac{3}{2}, u_3=\frac{2}{3}, ...$ (***can be implied by later working***)	M1	1.1b
	$\left\{\sum_{r=1}^{100}u_r=\right\}\ 50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$	M1	2.2a
	$=\frac{325}{3}\quad\left(\text{or } 108\frac{1}{3} \text{ or } 108.\dot{3} \text{ or } \frac{1300}{12} \text{ or } \frac{650}{6}\right)$	A1	1.1b
		(3)	
			(7 marks)

Notes for Question **28**	
(i)	
M1:	Uses a correct methodical strategy to enable the given sum, $\sum_{r=1}^{16}\left(3+5r+2^r\right)$ to be found Allow M1 for any of the following: • expressing the given sum as either $\sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}(2^r),\quad \sum_{r=1}^{16}3+\sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}(2^r)$ or $\sum_{r=1}^{16}3+5\sum_{r=1}^{16}r+\sum_{r=1}^{16}(2^r)$ • attempting to find both $\sum_{r=1}^{16}(3+5r)$ and $\sum_{r=1}^{16}(2^r)$ separately • (3×16) and attempting to find both $\sum_{r=1}^{16}(5r)$ and $\sum_{r=1}^{16}(2^r)$ separately
M1:	**Way 1:** Correct method for finding the sum of an AP with $a=8,\ d=5,\ n=16$ **Way 2:** (3×16) and a correct method for finding the sum of an AP
M1:	Correct method for finding the sum of a GP with $a=2,\ r=2,\ n=16$
A1*:	For all steps fully shown (with correct formulae used) leading to 131 798
Note:	**Way 1:** Give 2nd M1 for writing $\sum_{r=1}^{16}(3+5r)$ as $\frac{16}{2}(8+83)$
Note:	**Way 2:** Give 2nd M1 for writing $\sum_{r=1}^{16}3+\sum_{r=1}^{16}(5r)$ as $48+\frac{16}{2}(5+80)$ or $48+680$
Note:	Give 3rd M1 for writing $\sum_{r=1}^{16}(2^r)$ as $\frac{2(1-2^{16})}{1-2}$ or $2(2^{16}-1)$ or $(2^{17}-2)$
(i)	
Way 3	
M1:	At least 6 correct terms **and** 16 terms shown
M1:	At least 10 correct terms (may not be 16 terms)
M1:	At least 15 correct terms (may not be 16 terms)
A1*:	All 16 terms correct **and** an indication that the sum is 131 798
(ii)	
M1:	For some indication that the next two terms of this sequence are $\frac{3}{2},\frac{2}{3}$
M1:	For deducing that the sum can be found by applying $50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$, o.e.
A1:	Obtains $\frac{325}{3}$ or $108\frac{1}{3}$ or $108.\dot{3}$ or an exact equivalent
Note:	Allow 1st M1 for $u_2=\frac{3}{2}$ (or equivalent) **and** $u_3=\frac{2}{3}$ (or equivalent)
Note:	Allow 1st M1 for the first 3 terms written as $\frac{2}{3},\frac{3}{2},\frac{2}{3},...$
Note:	Allow 1st M1 for the 2nd and 3rd terms written as $\frac{3}{2},\frac{2}{3},...$ in the correct order
Note:	Condone $\frac{2}{3}$ written as 0.66 or awrt 0.67 for the 1st M1 mark
Note:	Give A0 for 108.3 or 108.333... without reference to $\frac{325}{3}$ or $108\frac{1}{3}$ or $108.\dot{3}$

Question	Scheme	Marks	AOs
29	Arithmetic sequence, $T_2 = 2k$, $T_3 = 5k - 10$, $T_4 = 7k - 14$		
	$(5k - 10) - (2k) = (7k - 14) - (5k - 10) \Rightarrow k = ...$	M1	2.1
	$\{3k - 10 = 2k - 4 \Rightarrow\}$ $k = 6$	A1	1.1b
	$\{k = 6 \Rightarrow\}$ $T_2 = 12$, $T_3 = 20$, $T_4 = 28$. So $d = 8$, $a = 4$	M1	2.2a
	$S_n = \frac{n}{2}\left(2(4) + (n-1)(8)\right)$	M1	1.1b
	$= \frac{n}{2}\left(8 + 8n - 8\right) = 4n^2 = (2n)^2$ which is a square number	A1	2.1
		(5)	
			(5 marks)

Question **29** Notes:

M1:	Complete method to find the value of *k*
A1:	Uses a correct method to find $k = 6$
M1:	Uses their value of *k* to deduce the common difference and the first term ($\neq T_2$) of the arithmetic series.
M1:	Applies $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$ with their $a \neq T_2$ and their *d*.
A1:	Correctly shows that the sum of the series is $(2n)^2$ and makes an appropriate conclusion.

Question	Scheme	Marks	AOs
30(a)	$\sqrt{(4-x)} = 2\left(1-\frac{1}{4}x\right)^{\frac{1}{2}}$	M1	2.1
	$\left(1-\frac{1}{4}x\right)^{\frac{1}{2}} = 1+\frac{1}{2}\left(-\frac{1}{4}x\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{1}{4}x\right)^2+...$	M1	1.1b
	$\sqrt{(4-x)} = 2\left(1-\frac{1}{8}x-\frac{1}{128}x^2+..\right)$	A1	1.1b
	$\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^2+...$ and $k=-\frac{1}{64}$	A1	1.1b
		(4)	
(b)	The expansion is valid for $\lvert x\rvert < 4$, so $x=1$ can be used	B1	2.4
		(1)	
		(5 marks)	

Notes:

(a)

M1: Takes out a factor of 4 and writes $\sqrt{(4-x)} = 2(1\pm...)^{\frac{1}{2}}$

M1: For an attempt at the binomial expansion with $n=\frac{1}{2}$

Eg. $(1+ax)^{\frac{1}{2}} = 1+\frac{1}{2}(ax)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(ax)^2+...$

A1: Correct expression inside the bracket $1-\frac{1}{8}x-\frac{1}{128}x^2+$ which may be left unsimplified

A1: $\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^2+...$ and $k=-\frac{1}{64}$

(b)

B1: The expansion is valid for $\lvert x\rvert < 4$, so $x=1$ can be used

Question	Scheme	Marks	AOs
31	Attempts $S_\infty = \frac{8}{7} \times S_6 \Rightarrow \frac{a}{1-r} = \frac{8}{7} \times \frac{a(1-r^6)}{1-r}$	M1	2.1
	$\Rightarrow 1 = \frac{8}{7} \times (1-r^6)$	M1	2.1
	$\Rightarrow r^6 = \frac{1}{8} \Rightarrow r = ..$	M1	1.1b
	$\Rightarrow r = \pm \frac{1}{\sqrt{2}}$ (so $k = 2$)	A1	1.1b
			(4 marks)

Notes:

M1: Substitutes the correct formulae for S_∞ and S_6 into the given equation $S_\infty = \frac{8}{7} \times S_6$

M1: Proceeds to an equation just in r

M1: Solves using a correct method

A1: Proceeds to $r = \pm \frac{1}{\sqrt{2}}$ giving $k = 2$

Question Number	Scheme		Marks
32 (a)	$a+(n-1)d=600+9\times120$	This mark is for: $600+9\times120$ or $600+8\times120$	M1
	$=(£)1680$	1680 with or without the "£"	A1
	Answer only scores both marks		
	<u>Listing</u> M1: Lists ten terms starting £600, £720, £840, £960, … A1: Identifies the 10th term as $(£)1680$		
			(2)
(b)	**Allow the use of *n* instead of *N* throughout in (b)**		
	$d=80$ for Kim	Identifies or uses $d=80$ for Kim	B1
	$\frac{N}{2}\left\{2\times600+(N-1)\times120\right\}$ OR $\frac{N}{2}\left\{2\times130+(N-1)\times80\right\}$	Attempts a sum formula for Andy or Kim. A correct formula must be seen or implied with: *a* = 600, *d* = 120 for Andy **or** *a* = 130, *d* = 80 for Kim. If B0 was scored, allow M1 here if Kim's incorrect "*d*" is used.	M1
	$\frac{N}{2}\left\{2\times600+(N-1)\times120\right\}=2\times\frac{N}{2}\left\{2\times130+(N-1)\times80\right\}$ A **<u>correct</u>** equation in any form		A1
	$20N=360\Rightarrow N=...$	Proceeds to find a value for *N*. (Allow if it leads to $N<0$) **Dependent on the first method mark and must be an equation that uses Andy's and Kim's sum.**	dM1
	$(N=)18$	Ignore *N*/*n* = 0 and if a correct value of *N* is seen, isw any further reference to years etc.	A1
	See below for listing approach		
	If you see *N* = 18 with no working send to Review		
			(5)
			(7 marks)

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Andy	600	1320	2160	3120	4200	5400	6720	8160	9720	11400	13200	15120	17160	19320	21600	24000	26520	29160
Kim	130	340	630	1000	1450	1980	2590	3280	4050	4900	5830	6840	7930	9100	10350	11680	13090	14580
Kimx2	260	680	1260	2000	2900	3960	5180	6560	8100	9800	11660	13680	15860	18200	20700	23360	26180	29160

B1: States or uses $d=80$ for Kim
M1: Attempts to find the total savings for Andy or Kim – must see the correct pattern for Andy (600, 1320, 2160,…) or Kim (130, 340, 630,…) (or Kimx2)
A1: Correct totals for Andy and Kim (or Kimx2) at least as far as *n* = 18
M1: Identifies when Andy's total = 2xKim's total
A1: *N* = 18

Question Number	Scheme		Marks
33 (a)	$a_1 = 4 \Rightarrow a_2 = \frac{4}{4+1}$	Attempts to use the given recurrence relation correctly at least once e.g. $a_2 = \frac{4}{4+1}$ or $a_3 = \frac{\text{their } a_2}{(\text{their } a_2)+1}$ or $a_4 = \frac{\text{their } a_3}{(\text{their } a_3)+1}$. May be implied by their term(s).	M1
	$\frac{4}{5}, \frac{4}{9}, \frac{4}{13}$	A1: Two of $\frac{4}{5}, \frac{4}{9}, \frac{4}{13}$ which may be un-simplified. Accept for example $0.8, \frac{0.8}{1.8}, \ldots$ or $\frac{4}{5}, \frac{\frac{4}{5}}{1+\frac{4}{5}}, \ldots$	A1A1
		A1: $\frac{4}{5}, \frac{4}{9}, \frac{4}{13}$ (Allow 0.8 for $\frac{4}{5}$)	
			(3)
(b)	$p = 4$ or e.g. $4 = \frac{4}{p+q}$, $"\frac{4}{5}" = \frac{4}{2p+q}$ $\Rightarrow p = \ldots$ *or* $q = \ldots$	$a_n = \frac{4}{4n \pm ..}$ or $p = 4$ **OR** Uses 2 terms to set up and solve two **<u>correct equations</u> for their fractions** in p and q to obtain a value for p or a value for q.	M1
	$a_n = \frac{4}{4n-3} \Rightarrow$ $p = 4$ and $q = -3$	Either $a_n = \frac{4}{4n-3}$ OR $p = 4$ and $q = -3$	A1
	Correct answer only scores both marks.		
			(2)
(c)	$\frac{4}{4N-3} = \frac{4}{321} \Rightarrow N = \ldots$	Solves their $\frac{4}{pN+q} = \frac{4}{321}$ to obtain a value for N or n.	M1
	$(N =)81$	Cao (ignore what they use for N)	A1
	Allow trial and improvement if 81 is clearly identified and then award both marks following a correct answer in (b) but just trying random values is M0		
			(2)
			(7 marks)

Question Number	Scheme		Marks
34.(a)	$(a_2 =)2k$	$2k$ only	B1
	$(a_3 =)\dfrac{k("2k"+1)}{"2k"}$	For substituting their a_2 into $a_3 = \dfrac{k(a_2+1)}{a_2}$ to find a_3 in terms of just k	M1
	$(a_3 =)\dfrac{2k+1}{2}$	$(a_3 =)\dfrac{2k+1}{2}$ or exact simplified equivalent such as $(a_3 =)k+\dfrac{1}{2}$ or $\dfrac{1}{2}(2k+1)$ but not $k+\dfrac{k}{2k}$ **Must be seen in (a)** but isw once a correct simplified answer is seen.	A1
			(3)
	Note that there are <u>no</u> marks in (b) for using an AP (or GP) sum formula unless their terms do form an AP (or GP).		
(b)	$\sum_{r=1}^{3} a_r = 10 \Rightarrow 1+"2k"+"\dfrac{2k+1}{2}"=10$	Writes 1 + their a_2 + their a_3 = 10. E.g. $1+2k+\dfrac{2k^2+k}{2k}=10$. Must be a correct follow through equation in terms of k only.	M1
	$\Rightarrow 2+4k+2k+1=20 \Rightarrow k=...$ or e.g. $\Rightarrow 6k^2-17k=0 \Rightarrow k=...$	Solves their equation in k which has come from the sum of 3 terms = 10, and reaches $k=\ldots$ Condone poor algebra but if a quadratic is obtained then the usual rules apply for solving – see General Principles. (Note that it does not need to be a 3-term quadratic in this case)	M1
	$(k =)\dfrac{17}{6}$	$k=\dfrac{17}{6}$ or exact equivalent e.g. $2\dfrac{5}{6}$ Do **not** allow $k=\dfrac{8.5}{3}$ or $k=\dfrac{17/2}{3}$ Ignore any reference to $k = 0$. Allow 2.83 recurring as long as the recurring is clearly indicated e.g. a dot over the 3.	A1
			(3)
			(6 marks)

Question Number	Scheme		Marks
35. (a)	$206=140+(12-1)\times d \Rightarrow d=...$	Uses $206=140+(12-1)\times d$ and proceeds as far as $d=...$	M1
	$(d=)6$	Correct answer only can score both marks.	A1
			(2)
(b)	$S_{12}=\frac{12}{2}(140+206)$ or $S_{12}=\frac{12}{2}(2\times140+(12-1)\times"6")$ or $S_{11}=\frac{11}{2}(140+206-"6")$ or $S_{11}=\frac{11}{2}(2\times140+(11-1)\times"6")$	Attempts $S_n=\frac{n}{2}(a+l)$ or $S_n=\frac{n}{2}(2a+(n-1)d)$ with $n=12$, $a=140, l=206, d='6'$ **WAY 1** Or Attempts $S_n=\frac{n}{2}(a+l)$ or $S_n=\frac{n}{2}(2a+(n-1)d)$ with $n=11$, $a=140, l=206-'6', d='6'$ **WAY2** If they are using $S_n=\frac{n}{2}(2a+(n-1)d)$, the n must be used consistently.	M1
	$S=2076$ **WAY1** or $S=1870$ **WAY 2**	Correct sum (may be implied)	A1
	$(52-12)\times206=...$ or $(52-11)\times206=...$	Attempts to find $(52-12)\times206$ or $(52-11)\times206$. Does **not** have to be consistent with their n used for the first Method mark.	M1
	Total $="2076"+"8240"=...$ **(WAY 1)** or Total $="1870"+"8446"=...$ **(WAY 2)**	Attempts to find the total by adding the sum to 12 terms with (52 - 12) lots of 206 or attempts to find the total by adding the sum to 11 terms with (52 - 11) lots of 206. I.e. consistency is now required for this mark. **Dependent on both previous method marks.**	**dd**M1
	10316	cao	A1
			(5)
			(7 marks)

Listing in (b):

Week	1	2	3	4	5	6	7
Bicycles	140	146	152	158	164	170	176
Total	140	286	438	596	760	930	1106

8	9	10	11	12	13	…	52
182	188	194	200	206	206	…	206
1288	1476	1670	1870	2076	2282	…	10316

M1: Attempts the sum of either 12 or 11 terms of a series with first term 140 and their d up to 140 +11d or 140 + 10d.
A1: S = 2076 or 1870
Then follow the scheme

Special case in (b) – Treats as single AP with *n* = 52

$$S_n = \frac{52}{2}\left(2\times 140 + (52-1)\times "6"\right) = 15236$$ Scores 11000

M1: $S_n = \frac{n}{2}\left(2a+(n-1)d\right)$ with n = 52, a = 140, d = “6” **A1:** 15236

EXPERT TUITION

Question Number	Scheme	Notes	Marks
	$a_1 = 4,\ a_{n+1} = 5 - ka_n\ ,\ n \ldots 1$		
36. (a)	$a_2 = 5 - ka_1 = 5 - 4k$ $a_3 = 5 - ka_2 = 5 - k(5 - 4k)$	M1: Uses the recurrence relation correctly at least once. This may be implied by $a_2 = 5 - 4k$ or by the use of $a_3 = 5 - k\left(\text{their } a_2\right)$ A1: Two correct expressions – need not be simplified but must be seen in (a). Allow $a_2 = 5 - k4$ and $a_3 = 5 - 5k + k^2 4$ Isw if necessary for a_3.	M1A1
			[2]
(b)	$\sum_{r=1}^{3}(1) = 1+1+1$	Finds 1+1+1 or 3 somewhere in their solution (may be implied by e.g. $5 + 6 - 4k + 6 - 5k + 4k^2$). Note that $5 + 6 - 4k + 6 - 5k + 4k^2$ would score B1 and the M1 below.	B1
	$\sum_{r=1}^{3} a_r = 4 + "5 - 4k" + "5 - 5k + 4k^2"$	Adds 4 to their a_2 and their a_3 where a_2 and a_3 are functions of k. The statement as shown is sufficient.	M1
	$\sum_{r=1}^{3}(1 + a_r) = 17 - 9k + 4k^2$	Cao but condone '= 0' after the expression	A1
	Allow full marks in (b) for correct answer only		
			[3]
(c)	500	cao	B1
			[1]
			6 marks

Question Number	Scheme	Notes	Marks
37.(a)	John; arithmetic series, $a = 60$, $d = 15$.		
	$60 + 75 + 90 = 225$* or $S_3 = \frac{3}{2}\left(120 + (3-1)(15)\right) = 225$*	Finds and adds the first 3 terms or uses sum of 3 terms of an AP and obtains the printed answer, with no errors.	B1 *
	Beware: **The 12th term of the sequence is 225 also so look out for** $60 + (12-1)\times 15 = 225$. **This is B0.**		
			[1]
(b)	$t_9 = 60 + (n-1)15 = (£)180$	M1: Uses $60 + (n - 1)15$ with $n = 8$ or 9 A1: (£)180	M1 A1
	Listing: M1: Uses $a = 60$ and $d = 15$ to select the 8th or 9th term (allow arithmetic slips) A1: (£)180 (Special case (£)165 **only** scores M1A0)		
			[2]
(c)	$S_n = \frac{n}{2}\left(120 + (n-1)(15)\right)$ or $S_n = \frac{n}{2}\left(60 + 60 + (n-1)(15)\right)$	Uses correct formula for sum of n terms with $a = 60$ and $d = 15$ (must be a correct formula but ignore the value they use for n or could be in terms of n)	M1
	$S_n = \frac{12}{2}\left(120 + (12-1)(15)\right)$	Correct numerical expression	A1
	$= (£)1710$	cao	A1
	Listing: M1: Uses $a = 60$ and $d = 15$ and finds the sum of at least 12 terms (allow arithmetic slips) A2: (£)1710		
			[3]
(d)	$3375 = \frac{n}{2}\left(120 + (n-1)(15)\right)$	Uses correct formula for sum of n terms with $a = 60$, $d = 15$ and puts $= 3375$	M1
	$6750 = 15n\left(8 + (n-1)\right) \Rightarrow 15n^2 + 105n = 6750$	**Correct three term** quadratic. E.g. $6750 = 105n + 15n^2$, $3375 = \frac{15}{2}n^2 + \frac{105}{2}n$ This may be implied by equations such as $6750 = 15n(n+7)$ or $3375 = \frac{15}{2}\left(n^2 + 7n\right)$	A1
	$n^2 + 7n = 25\times 18$ *	Achieves the printed answer with no errors but must see the 450 or 450 in factorised form or e.g. 6750, 3375 in factorised form i.e. an intermediate step.	A1*
			[3]
(e)	$n = 18 \Rightarrow$ Aged 27	M1: Attempts to solve the given quadratic or states $n = 18$ A1: Age = 27 or just 27	M1 A1
	Age = 27 only scores both marks (i.e. $n = 18$ need not be seen)		
	Note that (e) is not hence so allow valid attempts to solve the given equation for M1		
			[2]
			11 marks

n	1	2	3	4	5	6	7	8	9
u_n	60	75	90	105	120	135	150	165	180
S_n	60	135	225	330	450	585	735	900	1080
Age	10	11	12	13	14	15	16	17	18

n	10	11	12	13	14	15	16	17	18
u_n	195	210	225	240	255	270	285	300	315
S_n	1275	1485	1710	1950	2205	2475	2760	3060	3375
Age	19	20	21	22	23	24	25	26	27

Question Number	Scheme		Marks
38(i).(a)	$U_3 = 4$	cao	B1
			(1)
(b)	$\sum_{n=1}^{n=20} U_n = 4+4+4..........+4$ or 20×4	For realising that all 20 terms are 4 and that the sum is required. Possible ways are 4+4+4…..+4 or 20×4 or $\frac{1}{2}\times 20(2\times 4+19\times 0)$ or $\frac{1}{2}\times 20(4+4)$ (Use of a correct sum formula with $n = 20$, $a = 4$ and $d = 0$ or $n = 20$, $a = 4$ and $l = 4$)	M1
	$= 80$	cao	A1
	Correct answer with no working scores M1A1		
			(2)
(ii)(a)	$V_3 = 3k, \quad V_4 = 4k$	May score in (b) if clearly identified as V_3 and V_4	B1, B1
			(2)
(b)	$\sum_{n=1}^{n=5} V_n = k+2k+3k+4k+5k = 165$ or $\frac{1}{2}\times 5(2\times k+4\times k) = 165$ or $\frac{1}{2}\times 5(k+5k) = 165$	Attempts V_5, adds their V_1, V_2, V_3, V_4, V_5 AND sets equal to 165 or Use of a correct sum formula with $a = k$, $d = k$ and $n = 5$ or $a = k$, $l = 5k$ and $n = 5$ AND sets equal to 165	M1
	$15k = 165 \Rightarrow k = ..$	Attempts to solve their linear equation in k **having set the sum of their first 5 terms equal to 165.** Solving $V_5 =$ 165 scores no marks.	M1
	$k = 11$	cao and cso	A1
			(3)
			(8 marks)

Question Number	Scheme		Marks
39.(a)	$32000 = 17000 + (k-1)\times 1500 \Rightarrow k = ...$	Use of 32000 with a **correct** formula in an attempt to find *k*. A correct formula could be implied by a correct answer.	M1
	(k =) 11	Cso (Allow n = 11)	A1
	Accept correct answer only.		
	32000 = 17000 + 1500k $\Rightarrow k = 10$ is M0A0 (wrong formula) $\frac{32000-17000}{1500} = 10 \therefore k = 11$ is M1A1 (correct formula implied)		
	Listing: All terms must be listed up to 32000 and 11 correctly identified. A solution that scores 2 if fully correct and 0 otherwise.		
			(2)
(b)	M1: $S = \frac{k}{2}(2\times 17000 + (k-1)\times 1500)$ or $\frac{k}{2}(17000+32000)$ $S = \frac{k-1}{2}(2\times 17000 + (k-2)\times 1500)$ or $\frac{k-1}{2}(17000+30500)$	M1: Use of correct sum formula with their **integer** $n = k$ or $k - 1$ from part (a) where $3 < k < 20$ and a = 17000 and d = 1500. **See below for special case for using n = 20.**	M1A1
	A1: $S = \frac{11}{2}(2\times 17000 + 10\times 1500)$ or $\frac{11}{2}(17000+32000)$ $S = \frac{10}{2}(2\times 17000 + 9\times 1500)$ or $\frac{10}{2}(17000+30500)$ (= 269 500 or 237 500)	A1: Any correct un-simplified numerical expression with n = 11 or n = 10	
	$32000\times\alpha$	$32000\times\alpha$ where α is an integer and $3 < \alpha < 18$	M1
	288 000 + 269 500 = 557 500 or 320 000 + 237 500 = 557 500	M1: Attempts to add their two values. It is dependent upon the two previous M's being scored and must be the sum of 20 terms i.e. $\alpha + k = 20$ A1: 557 500	ddM1A1
	Special Case: If they just find S_{20} (£625 000) in (b) score the first M1 otherwise apply the scheme.		
			(5)
			(7 marks)

Listing:

n	1	2	3	4	5	6	7	8	9	10
u_n	17000	18500	20000	21500	23000	24500	26000	27500	29000	30500
n	11	12	13	14	15	16	17	18	19	20
u_n	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000

Look for a sum before awarding marks. Award the M's as above then A2 for 557 500
If they sum the 'parts' separately then apply the scheme.

Question Number	Scheme	Marks
40.	(a) $7 = 5a_1 - 3 \quad \Rightarrow a_1 = ..$	M1
	$a_1 = 2$	A1
		(2)
	(b) $a_3 = "32"$ and $a_4 = "157"$	M1
	$\sum_{r=1}^{r=4} a_r = a_1 + a_2 + a_3 + a_4$	
	= "2"+ "7"+ "32"+ "157"	dM1
	= 198	A1
		(3)
		(5 marks)

Notes

(a) M1 Writes $7 = 5a_1 - 3$ and attempts to solve leading to an answer for a_1 . If they rearrange wrongly before any substitution this is M0

A1 Cao $a_1 = 2$

Special case: Substitutes n = 1 into 5n – 3 and obtains answer 2. This is fortuitous and gets M0A0 but full marks are available on (b).

(b) M1 Attempts to find either their a_3 or their a_4 using $a_{n+1} = 5a_n - 3,\ a_2 = 7$
Needs clear attempt to use formula or is implied by correct answers or correct follow through of their a_3

dM1 (Depends on previous M mark) Sum of their four adjacent terms from the given sequence.
n.b May be given for 9 + a_3 + a_4 as they may add 2 + 7 to give 9
(dM0 for sum of an Arithmetic series)

A1 cao 198

Special case
(a) $a_1 = 32$ is M0 A0
(b) Adds for example 7+32+157 + 782 = or 32+157 + 782 + 3907 is M1 M1 A0
Total mark possible is 2 / 5
(This is not treated as a misread – as it changes the question)

Question Number	Scheme	Marks
41.	(a) Use $n^{th}\text{term} = a+(n-1)d$ with d = 10; a = 150 and n = 8, or a = 160 and n = 7, or a = 170 and n = 6 : = 150+7×10 or 160 +6 ×10 or 170 + 5 ×10 = 220* (Or gives clear list – see note)	M1 A1* (2)
Or	If answer 220 is assumed and 150 + (n – 1) 10 =220 or variation is solved for n= Then n = 8, so 2007 is the year (must conclude the year)	M1 A1* (2)
	(b) Use $S_n = \frac{n}{2}\{2a+(n-1)10\}$ Or $S_n = \frac{n}{2}\{a+l\}$ **and** l= a + (n – 1)10 = 7(300+13×10) or 7(150 + 280) = 7×430 = 3010	M1 A1 A1 (3)
	(c) Cost in year n = 900+(n–1)×-20 Sales in year n = 150+(n–1)×10 Cost =3×Sales ⇒ 900+(n–1)×-20 = 3×(150+(n–1)×10) 900–20n+20 = 450+30n–30 500 = 50n n =10 Year is 2009 As n is not defined they may work correctly from another base year to get the answer 2009 and their n may not equal 10. If doubtful – send to review.	M1 M1 M1 A1 (4) (9 marks)

Notes

(a) M1 Attempt to use $n^{th}\text{term} = a+(n-1)d$ with d = 10, and correct combination of a and n i.e. a = 150 and n = 8 or a = 160 and n = 7, or a = 170 and n = 6

A1 * Shows that 220 computers are sold in 2007 with no errors

Note that this is a given solution, so needed 150+7×10 or 160+6×10 or 170+5×10 or equivalent.

Accept a correct list showing all values and years for both marks Just 150,160,170,180,190,200,210,220 is M1A0

Need some reference to years as well as the list of numbers of computers for A1.

(b) M1 Attempts to use $S_n = \frac{n}{2}\{2a+(n-1)d\}$ with d = 10, and correct combination of a and n i.e. a = 150 and n = 14, or a = 160 and n = 13, or a = 170 and n =12

A1 Uses $S_n = \frac{n}{2}\{2a+(n-1)d\}$ with a = 150, d = 10 and n = 14 [N.B. $S_n = \frac{n}{2}\{a+l\}$ needs l= a + (n - 1)d as well

NB A0 for a = 160 and n = 13 or a = 170 and n = 12 unless they then add the first, or first two terms respectively.

A1 Cao 3010 . This answer (with no working) implies correct method M1A1A1.

Special case: If a complete list 150+160+170+180+190+200+210+220+230+240+250+260+270+280 is seen, then there is an error finding the sum then score M1A1A0, but incomplete or wrong lists score M0A0A0

(c) M1 Writes down an expression for the cost = 900+(n–1)×–20 or writes 900 + (n – 1) d and states d = -20 Allow 900 + n ×–20. Allow recovery from invisible brackets.

M1 **Attempts** to write down an equation in n for statement 'cost =3×sales' 900+(n-1)×-20 = 3×(150+(n-1)×10). Accept the 3 on the wrong side and allow use of 20 instead of -20 and allow n (consistently) instead of n – 1 for this mark. Ignore £ signs in equation.

M1 Solves the correct linear equation in n to achieve n = 10 (for those using n – 1) or n = 9 (for those using n). Ignore £ signs.

A1 Cso Year 2009 (A0 for the answer Year 10 if 2009 is not given)

Special case. **Just answer or trial and improvement** with no equation leading to answer scores SC 0,0,1,1

Equations satisfying the method mark descriptors followed by trial and improvement could get all four marks

Question Number	Scheme	Marks
42.(a)	$(a_2 =)\quad 4k-3$	B1 (1)
(b)	$a_3 = 4(4k-3)-3$	M1
	$\sum_{r=1}^{3} a_r = k+4k-3+4(4k-3)-3 = ..k \pm ...$	M1
	$21k-18=66 \Rightarrow k = ...$	dM1
	$k=4$	A1 (4) (5 marks)

(a) B1 $4k-3$ cao

(b) M1 An attempt to find a_3 from iterative formula $a_3 = 4a_2 - 3$. Condone bracketing errors for the M mark

M1 Attempt to sum their a_1, a_2 and a_3 to get a linear expression in *k* (Sum of Arithmetic series is M0)

dM1 Sets their linear expression to 66 and solves to find a value for *k*. It is dependent upon the previous M mark

A1 cao $k=4$

Question Number	Scheme	Marks
43(a).	Attempts to use $a+(n-1)"d"$ with a=A and "d"=d+1 and n = 14 $A+13(d+1)=A+13d+13$ *	M1 A1* (2)
(b)	Calculates time for Yi on Day 14= $(A-13)+13(2d-1)$ Sets **times** equal $A+13d+13=\ (A-13)+13(2d-1)\Rightarrow d=...$ $d=3$	M1 M1 A1 cso (3)
(c)	Uses $\frac{n}{2}\{2A+(n-1)(D)\}$ with n =14, and with D=d or d + 1 Attempts to solve $\frac{14}{2}\{2A+13\times'(d+1)'\}=784\Rightarrow A=...$ $A=30$	M1 dM1 A1 (3) (8 marks)

(a) M1 Attempts to use $a+(n-1)d$ with a=A and d =d+1 AND n =14

A1* cao This is a given answer and there is an expectation that the intermediate answer is seen and that **all work is correct** with correct brackets.

The expressions $A+13(d+1)$ and $A+13d+13$ should be seen

N.B. If **brackets are missing and formula is not stated**

e.g. $A+13d+1\Rightarrow A+13d+13$ or $A+(13)d+1\Rightarrow A+13d+13$ then this is **M0A0**

If **formula is quoted and a = A and d = d + 1 is quoted or implied** , then M1 A0 may be given

So $a+(n-1)d$ followed by $A+(13)d+1=A+13d+13$ achieves **M1A0**

(b) M1 States a time for Yi on Day 14 = $(A-13)+13(2d-1)$

M1 Sets their **time** for Yi, equal to $A+13d+13$ and uses this equation to proceed to d =

A1 cso d =3 Needs both M marks and must be simplified to 3 (not 39/13)

[NB Setting **each** of the times separately equal to 0 leads to d = 3 – this will gain M0A0]

(c) M1 Uses the sum formula $\frac{n}{2}\{2A+(n-1)(D)\}$ with n = 14 and D = d +1 or allow D = d (usually 4 or 3)

NB May use $\frac{n}{2}\{A+(A+13D)\}$ with n = 14 and and D = d +1 or allow D = d (usually 4 or 3)

dM1 Attempts to solve $\frac{14}{2}\{2A+13\times'4'\}="784"\Rightarrow A=...$ (Must use their d + 1 this time)

Allow miscopy of 784

A1 cao $A=30$

Question Number	Scheme		Marks
	For this question, mark (a) and (b) together and ignore labelling.		
44(a)	$(a_2 =)\ k(4+2)\ \ (= 6k)$	Any correct (possibly un-simplified) expression	B1
			(1)
(b)	$a_3 = k(\text{their } a_2 + 2)\ (= 6k^2 + 2k)$	An attempt at a_3. Can follow through their answer to (a) but a_2 must be an expression in k.	M1
	$a_1 + a_2 + a_3 = 4 + (6k) + (6k^2 + 2k)$	An attempt to find their $a_1 + a_2 + a_3$	M1
	$4 + (6k) + (6k^2 + 2k) = 2$	A **<u>correct</u>** equation in any form.	A1
	Solves $6k^2 + 8k + 2 = 0$ to obtain $k =$ $(6k^2 + 8k + 2 = 2(3k+1)(k+1))$	Solves their 3TQ as far as $k =$ .... according to the general principles. (An independent mark for solving their three term quadratic)	M1
	$k = -1/3$	Any equivalent fraction	A1
	$k = -1$	Must be from a correct equation. (Do not accept un-simplified)	B1
	Note that it is quite common to think the sequence is an AP. Unless they find a_3, this is likely only to score the M1 for solving their quadratic.		
			(6)
			[7]

Question Number	Scheme		Marks
45(a)	600 = 200 + $(N-1)20 \Rightarrow N = ...$	Use of 600 with a **correct** formula in an attempt to find N. A correct formula could be implied by a correct answer.	M1
	$N = 21$	cso	A1
	Accept correct answer only.		
	$600 = 200 + 20N \Rightarrow N = 20$ is M0A0 (wrong formula) $\frac{600-200}{20} = 20 \therefore N = 21$ is M1A1 (correct formula implied)		
	Listing: All terms must be listed up to 600 and 21 correctly identified. A solution that scores 2 if fully correct and 0 otherwise.		
			(2)
(b)	**Look for an AP first:**		
	$S = \frac{21}{2}(2\times200+20\times20)$ or $\frac{21}{2}(200+600)$ or $S = \frac{20}{2}(2\times200+19\times20)$ or $\frac{20}{2}(200+580)$ (= 8400 or 7800)	M1: Use of correct sum formula with their **integer** $n = N$ or $N-1$ from part (a) where $3 < N < 52$ and $a = 200$ and $d = 20$. A1: Any correct un-simplified numerical expression with $n = 20$ or $n = 21$ (No follow through here)	M1A1
	Then for the constant terms:		
	$600\times(52-\text{"}N\text{"})\ (= 18600)$	M1: $600\times k$ where k is an integer and $3 < k < 52$ A1: A correct un-simplified follow through expression with their k consistent with n so that $n + k = 52$	M1A1ft
	So total is 27000	Cao	A1
	Note that for the constant terms, they may correctly use an AP sum with $d = 0$.		
	There are no marks in (b) for just finding S_{52}		
			(5)
			[7]
	If they obtain $N = 20$ in (a) (0/2) and then in (b) proceed with, $S = \frac{20}{2}(2\times200+19\times20)+32\times600 = 7800+19\ 200 = 27\ 000$ allow them to 'recover' and score full marks in (b) Similarly If they obtain $N = 22$ in (a) (0/2) and then in (b) proceed with, $S = \frac{21}{2}(2\times200+20\times20)+31\times600 = 8400+18\ 600 = 27\ 000$ allow them to 'recover' and score full marks in (b)		

Question Number	Scheme	Notes	Marks
46.(a)	$x_2 = 1 - k$	Accept un-simplified e.g. 1^2 - $1k$	B1
			(1)
(b)	$x_3 = (1-k)^2 - k(1-k)$	Attempt to substitute their x_2 into $x_3 = (x_2)^2 - kx_2$ with their x_2 in terms of k.	M1
	$= 1 - 3k + 2k^2$ *	Answer given	A1*
			(2)
(c)	$1 - 3k + 2k^2 = 1$	Setting $1 - 3k + 2k^2 = 1$	M1
	$\left(2k^2 - 3k = 0\right)$		
	$k(2k-3) = 0 \Rightarrow k = ..$	Solving their quadratic to obtain a value for k. **Dependent on the previous M1.**	dM1
	$k = \frac{3}{2}$	Cao and cso (ignore any reference to $k = 0$)	A1
			(3)
(d)	$\sum_{n=1}^{100} x_n = 1 + \left(-\frac{1}{2}\right) + 1 + ...$ Or $= 1 + \left(1 - 'k'\right) + 1 + ...$		M1
	Writing out at least 3 terms with the third term equal to the first term. Allow in terms of k as well as numerical values. Evidence that the sequence is oscillating between 1 and 1 – k. This may be implied by a correct sum.		
	$50 \times \frac{1}{2}$ or $50 \times 1 - 50 \times \frac{1}{2}$ or $\frac{1}{2} \times 50 \times (1 - \frac{1}{2})$	An attempt to combine the terms correctly. Can be in terms of k here e.g 100 – 50k	M1
	= 25	Allow an equivalent fraction, e.g. 50/2 or 100/4	A1
	Note that the use of $\frac{1}{2}n(a+l)$ is acceptable here but $\frac{1}{2}n(2a+(n-1)d)$ is not.		
			(3)
	Allow correct answer only		
			[9]

Question Number	Scheme	Notes	Marks
47.(a)	$U_{10} = 500 + (10-1)\times 200$	Uses $a+(n-1)d$ with a=500, d=200 and n = 9,10 or 11	M1
	=(£)2300		A1
	If the term formula is not quoted and the numerical expression is incorrect score M0. A correct answer with no working scores full marks.		**(2)**
(b)	**Mark parts (b) and (c) together**		
	$\frac{n}{2}\{2\times 500+(n-1)\times 200\} = 67200$	M1: Attempt to use $S = \frac{n}{2}\{2a+(n-1)d\}$ with , $S_n = 67200$, $a = 500$ *and* $d = 200$	M1A1
		A1: Correct equation	
	If the sum formula is not quoted and the equation is incorrect score M0.		
	$n^2+4n-672=0$	M1: An attempt to remove brackets and collect terms. **Dependent on the previous M1**	dM1A1
		A1: A correct **three term** equation in any form	
	E.g. allow $n^2+4n=672$, $n^2=672-4n$, $672-4n-n^2=0$, $200n^2+800n=134400$ etc.		
	$n^2+4n-24\times 28=0$ *	Replaces 672 with 24×28 with the equation **as printed** (including = 0) with no errors. (= 0 may not appear on the last line but must be seen at some point)	A1
			(5)
(c)	$(n-24)(n+28)=0 \Rightarrow n = ..$ or $n(n+4)=24\times 28 \Rightarrow n = ..$	Solves the **given** quadratic in an attempt to find n. They may use the quadratic formula.	M1
	24	States that n = 24, or the number of years is 24	A1
	Allow correct answer only in (c)		
			(2)
			[9]

Question Number	Scheme		Marks
48.	$u_2 = 9,\ u_{n+1} = 2u_n - 1,\ \ n \geqslant 1$		
(a)	$u_3 = 2u_2 - 1 = 2(9) - 1 \quad (= 17)$	$u_3 = 2(9) - 1.$ Can be implied by $u_3 = 17$	M1
	$u_4 = 2u_3 - 1 = 2(17) - 1 = 33$	Both $u_3 = 17$ and $u_4 = 33$	A1
			[2]
(b)	$\sum_{r=1}^{4} u_r = u_1 + u_2 + u_3 + u_4$		
	$(u_1) = 5$	$(u_1) = 5$	B1
	$\sum_{r=1}^{4} u_r = "5" + 9 + "17" + "33" = 64$	Adds their first four terms obtained legitimately (see notes below)	M1
		64	A1
			[3]
			5 marks

	Notes
(a)	**M1**: Substitutes 9 into RHS of iteration formula **A1**: Needs **both** 17 and 33 (but **allow if either or both seen in part (b)**)
(b)	**B1: for** u_1=5 (however obtained – may appear in (a)) May be called *a=5* **M1:** Uses their u_1 found from $u_2 = 2u_1 - 1$ stated explicitly, or uses $u_1 = 4$ *or* $5\frac{1}{2}$, and adds it to u_2, their u_3 and their u_4 only. (See special cases below). There should be no fifth term included. Use of sum of AP is irrelevant and scores M0 **A1**: 64

Question Number	Scheme	Marks
49.	Lewis; arithmetic series, $a = 140$, $d = 20$.	
(a)	$T_{20} = 140 + (20-1)(20); = 520$ Or lists 20 terms to get to 520	M1; A1
	OR 120 + (20)(20)	**[2]**
	Method 1 / Method 2	
(b)	**Either:** Uses $\frac{1}{2}n(2a+(n-1)d)$ / **Or:** Uses $\frac{1}{2}n(a+l)$	M1
	$\frac{20}{2}(2\times 140 + (20-1)(20))$ / $\frac{20}{2}(140 + \text{"520"})$ ft 520	A1
	6600	A1
		[3]
(c)	Sian; arithmetic series, $a = 300$, $l = 700$, $S_n = 8500$	
	Either: Attempt to use $8500 = \frac{n}{2}(a+l)$ / **Or:** May use both $8500 = \frac{1}{2}n(2a+(n-1)d)$ and $l = a+(n-1)d$ **and eliminate** d	M1
	$8500 = \frac{n}{2}(300 + 700)$ / $8500 = \frac{n}{2}(600 + 400)$	A1
	$\Rightarrow n = 17$	A1
		[3]
		8 marks
	Notes	
(a)	**M1:** Attempt to use formula for **20th term** of Arithmetic series with first term **140** and d = **20. Normal formula rules** apply – see General principles at the start of the mark scheme re "Method Marks" **Or:** uses 120 + 20n with n = 20 **Or: Listing method :** Lists 140, 160, 180, 200, 220, 240, 260, 280, … 520. M1A1 if correct M0A0 if wrong. (So 2 marks or zero) **A1:** For 520	
(b)	**M1:** An attempt to apply $\frac{1}{2}n(2a+(n-1)d)$ **or** $\frac{1}{2}n(a+l)$ with their values for a, n, d and l **A1:** Uses a = 140, d = 20, n = 20 in their formula (two alternatives given above) but **ft on their value of l** from (a) if they use Method 2. **A1:** 6600 cao **Or: Listing method :** Lists 140, 160, 180, 200, 220, 240, 260, 280, … 520 and adds **6600** gets M1A1A1- any other answer gets M1 A0A0 provided there are 20 numbers, the first is 140 and the last is 520.	
(c) **First method**	**M1:** Attempt to use $S_n = \frac{n}{2}(a+l)$ with their values for a, and l and S =8500 **A1:** Uses formula with correct values **A1:** Finds exact value 17	
Alternative method	**M1:** If both formulae $8500 = \frac{1}{2}n(2a+(n-1)d)$ and $l = a+(n-1)d$ are used, then d **must be eliminated** before this mark is awarded by valid work. Should not be using d = 400. This would be **M0**. **A1:** Correct equation in n only then **A1** for 17 exactly **Trial and error methods:** Finds d = 25 and n = 17 and list from 300 to 700 with total checked – 3/3	

Question Number	Scheme	Marks
	$a_1 = 3,\ a_{n+1} = 2a_n - c\ ,\ \ n \geq 1\ ,\ c$ is a constant	
50. (a)	$\{a_2 =\}\ 2\times 3 - c$ or $2(3) - c$ or $6 - c$	B1
		[1]
(b)	$\{a_3 =\}\ 2\times(\text{"}6-c\text{"}) - c$	M1
	$= 12 - 3c$ **(*)**	A1 cso
		[2]
(c)	$a_4 = 2\times(\text{"}12-3c\text{"}) - c \qquad \{= 24 - 7c\}$	M1
	$\left\{\sum_{i=1}^{4} a_i =\right\}\ 3 + (6 - c) + (12 - 3c) + (24 - 7c)$	M1
	$\text{"}45 - 11c\text{"} \geq 23$ or $\text{"}45 - 11c\text{"} = 23$	M1
	$c \leq 2$ or $2 \geq c$	A1 cso
		[4]
		7
	Notes	
(a)	The answer to part (a) cannot be recovered from candidate's working in part (b) or part (c). Once the candidate has achieved the correct result you can ignore subsequent working in this part.	
(b)	**M1:** For a correct substitution of ***their*** a_2 ***which must include term(s) in c*** into $2a_2 - c$ giving a result for a_3 in terms of only *c*. Candidates must use correct bracketing for this mark. **A1:** for correct solution only. No incorrect working/statements seen. (**Note**: the answer is given!)	
(c)	**1st M1:** For a correct substitution of a_3 ***which must include term(s) in c*** into $2a_3 - c$ giving a result for a_4 in terms of only *c*. Candidates must use correct bracketing (can be implied) for this mark. **2nd M1:** for an attempt to sum their a_1, a_2, a_3 and a_4 only. **3rd M1:** for their sum (of 3 or 4 or 5 consecutive terms) $=$ or $\geq$ or $>$ 23 to form a linear inequality or equation in *c*. **A1:** for $c \leq 2$ or $2 \geq c$ from a correct solution only. **Beware:** $-11c \geq -22 \Rightarrow c \geq 2$ is A0. **Note:** $45 - 11c \geq 23 \Rightarrow -11c \leq -22 \Rightarrow c \leq 2$ would be A0 cso. **Note:** Applying either $S_n = \frac{n}{2}(2a + (n-1)d)$ or $S_n = \frac{n}{2}(a + l)$ is 2nd M0, 3rd M0. **Note:** If a candidate gives a numerical answer in part (a); they will then get M0A0 in part (b); but if they use the printed result of $a_3 = 12 - 3c$ they could potentially get M0M1M1A0 in part (c) **Note:** If a candidate only adds numerical values (not in terms of *c*) in part (c) then they could potentially get only M0M0M1A0. **Note:** For the 3rd M1 candidates will usually sum a_1, a_2, a_3 and a_4 or a_2, a_3 and a_4 or a_2, a_3, a_4 and a_5 or a_1, a_2, a_3, a_4 and a_5	

Question Number	Scheme	Marks
	Boy's Sequence: 10, 15, 20, 25, ...	
51 (a)	$\{a = 10,\ d = 5 \Rightarrow T_{15} =\}\ a + 14d = 10 + 14(5);= 80$ or $0.1 + 14(0.05);= £0.80$	M1; A1
		[2]
(b)	$\{S_{60} =\}\ \frac{60}{2}\left[2(10) + 59(5)\right]$	M1 A1
	$= 30(315) = 9450$ or $£94.50$	A1
		[3]
	Boy's Sister's Sequence: 10, 20, 30, 40, ...	
(c)	$\{a = 10,\ d = 10 \Rightarrow S_m =\}\ \frac{m}{2}\left(2(10) + (m-1)(10)\right)$ $\left(\text{or } \frac{m}{2}\times 10(m+1) \text{ or } 5m(m+1)\right)$	M1 A1
	63 or $6300 = \frac{m}{2}\left(2(10) + (m-1)(10)\right)$	dM1
	$6300 = \frac{m}{2}(10)(m+1)$ or $12600 = 10m(m+1)$	
	$1260 = m(m+1)$	
	$35 \times 36 = m(m+1)$ **(*)**	A1 **cso**
		[4]
(d)	$\{m =\}\ 35$	B1
		[1]
		10
	Notes	
(a)	**M1:** for using the formula $a + 14d$ with either a or d correct. **A1:** for 80 or 80p or £0.80 or £0.80p and apply ISW. Otherwise, £80 or 0.80 or 0.80p would be A0. Award M0 if candidate applies $a + 59d$. Listing the first 15 terms and highlighting that the 15th term is 80 or listing 15 terms with the final 15th term aligned with 80 will then be awarded all two marks of M1A1. Writing down 80 with no working is M1A1.	
(b)	**M1**: for use of correct $\frac{60}{2}\left[2(10) + 59(5)\right]$ or $\frac{15}{2}\left(2(10) + 14(5)\right)$ with $a = 10,\ d = 5$ and $n = 60$ or $a = 10,\ d = 5$ and $n = 15$. If a candidate uses $\frac{n}{2}(a + l)$ with $n = 60$ or 15, there must be a full method of finding or stating l as either $a + 59d\ (= 305)$ or $a + 14d\ (= 80)$, respectively. **1st A1:** for a correct expression for S_{60}. ie. $\frac{60}{2}\left[2(10) + 59(5)\right]$ or $\frac{60}{2}\left[2(0.1) + 59(0.05)\right]$ or $\frac{60}{2}\left[10 + 305\right]$ or $\frac{60}{2}\left[0.10 + 3.05\right]$. This mark can be implied by later working. **2nd A1:** for 9450 or 9450p or £94.50 and apply ISW. Otherwise, £9450 or 94.50 without £ sign is A0. **Note**: the bracketing error of $\frac{60}{2}\,2(10) + 59(5)$ is A0 unless recovered from later working. Adding together the first 60 terms to obtain 9450 will then be awarded all three marks of M1A1A1.	

(c)	**1st M1:** for correct use of S_m formula with one of *a* or *d* correct. **1st A1:** for a correct expression for S_m. Eg: $\frac{m}{2}\left(2(10)+(m-1)(10)\right)$ or $\frac{m}{2}\times 10(m+1)$ or $5m(m+1)$ **2nd M1:** for forming a suitable equation using 63 or 6300 and their S_m. **Dependent on 1st M1.** **2nd A1cso:** for ***reaching the printed result*** with no incorrect working seen. Long multiplication is not necessary for the final accuracy mark. **Note:** $\frac{m}{2}\left(2(10)+(m-1)(10)\right)=630$ and not either 6300 or 63 is dM0. **<u>Beware</u>:** Some candidates will try and fudge the result given on the question paper. **<u>Notes for awarding 2nd A1</u>** Going from $m(m+1)=1260$ straight to $m(m+1)=35\times 36$ is 2nd A1. Going from $m(m+1)=$ some factor decomposition of 6300 straight to $m(m+1)=35\times 36$ is 2nd A1. Going from $10m(m+1)=12600$ straight to $m(m+1)=35\times 36$ is 2nd A0. Going from $m(m+1)=\frac{6300}{5}$ straight to $m(m+1)=35\times 36$ is 2nd A0. ***<u>Alternative: working in an different letter, say n or p.</u>*** **M1A1:** for $\frac{n}{2}\left(2(10)+(n-1)(10)\right)$ (although mixing letters eg. $\frac{n}{2}\left(2(10)+(m-1)(10)\right)$ is M0A0). **dM1:** for 63 or 6300 = $\frac{n}{2}\left(2(10)+(n-1)(10)\right)$ Leading to $6300=\frac{n}{2}(10)(n+1)\Rightarrow 1260=n(n+1)\Rightarrow 35\times 36=n(n+1)$ The candidate then needs to write either $35\times 36=m(m+1)$ or $m\equiv n$ or $m=n$ to gain the final A1.
(d)	**B1:** for 35 only.

Question	Scheme	Marks
52. **(a)**	$(x_2 =)\ a + 5$	B1 (1)
(b)	$(x_3) = a"(a+5)"+5$	M1
	$= a^2 + 5a + 5$ **(*)**	A1cso (2)
(c)	$41 = a^2 + 5a + 5 \Rightarrow a^2 + 5a - 36 (= 0)$ or $36 = a^2 + 5a$	M1
	$(a + 9)(a - 4) = 0$	M1
	$a = 4$ or -9	A1 (3)
		6 marks
	Notes	
(a)	B1 accept $a1 + 5$ or $1 \times a + 5$ (etc)	
(b)	M1 must see a(their x_2) + 5 A1cso must have seen $a(a[1] + 5) + 5$ (etc or better) Must have both brackets (....) and no incorrect working seen	
(c)	1st M1 for forming a suitable equation using x_3 and 41 and an attempt to collect like terms and reduce to 3TQ (o.e). Allow one error in sign. Accept for example $a^2 + 5a + 46 (= 0)$ If completing the square should get to $\left(a \pm \frac{5}{2}\right)^2 = 36 + \frac{25}{4}$ 2nd M1 Attempting to solve their relevant 3TQ (**see General Principles**) A1 for both 4 and -9 seen. If $a = 4$ and -9 is followed by $-9 < a < 4$ apply ISW. No working or trial and improvement leading to both answers scores 3/3 but no marks for only one answer. Allow use of other letters instead of a	

Question	Scheme	Marks
53. (a)	$S_{10} = \frac{10}{2}\left[2P + 9\times 2T\right]$ <u>or</u> $\frac{10}{2}\left(P + [P + 18T]\right)$	M1
	e.g. $5[2P + 18T]$ = (£) (10P + 90T) <u>or</u> (£) 10P + 90T **(*)**	A1cso (2)
(b)	Scheme 2: $S_{10} = \frac{10}{2}\left[2(P + 1800) + 9T\right] = \left\{10P + 18000 + 45T\right\}$	M1A1
	10P + 90T = 10P + 18000 + 45T	M1
	90T = 18000 + 45T	
	T = 400 (only)	A1 (4)
(c)	Scheme 2, Year 10 salary: $\left[a + (n-1)d =\right](P + 1800) + 9T$	B1ft
	P + 1800 + "3600" = 29850	M1
	P = (£) <u>24450</u>	A1 (3)
		9 marks
	Notes	
(a)	M1 for identifying $a = P$ or $d = 2T$ and attempt at S_{10}. Using n = 10 and one of a or d correct. Must see evidence for M mark, at least one line before the answer.	
	A1cso for simplifying to given answer. No incorrect working seen. Do not penalise missing end bracket in working eg 5(2P + 18T	
List	M1A1 for a full list seen (with + signs or written in columns) and no incorrect working seen. Any missing terms is M0A0	
(b)	1st M1 for attempting S_{10} for scheme 2 (allow missing (…) brackets e.g. 2P + 1800 + 9T) Using n = 10 and at least one of a or d correct.	
	1st A1 for a correct expression for S_{10} using scheme 2 (needn't be multiplied out)	
List	Allow M1A1 if they reach 10P + 18000 + 45T with no incorrect working seen 10P + 18000 + 45T with no working is M1A1	
	2nd M1 for forming an equation using the two sums that would enable P to be eliminated. Follow through their expressions provided P would disappear.	
	2nd A1 for T = 400 Answer only (4/4)	
(c)	B1 for using u_{10} for scheme 2 . Can be 9T or follow through their <u>value</u> of T	
	M1 for forming an equation based on u_{10} for scheme 2 and using 29850 and their <u>value</u> of T	
	A1 for 24450 seen Answer only (3/3)	
MR	If they misread scheme 2 as scheme 1 in part (c) apply MR rule and award B0M1A0 max for an equation based on u_{10} for scheme 1 and using 29850 and their <u>value</u> of T	

Question Number	Scheme	Marks
54. **(a)**	$(a_2 =)\ 5k+3$	B1 (1)
(b)	$(a_3 =)\ 5(5k+3)+3$	M1
	$= 25k+18$ (*)	A1 cso (2)
(c) **(i)**	$a_4 = 5(25k+18)+3 \quad (=125k+93)$	M1
	$\sum_{r=1}^{4} a_r = k+(5k+3)+(25k+18)+(125k+93)$	M1
	$= 156k+114$	A1 cao
(ii)	$= 6(26k+19)$ (or explain each term is divisible by 6)	A1 ft
		(4) **7**

Notes

(a) $5k+3$ must be seen in (a) to gain the mark

(b) 1st M: Substitutes their a_2 into $5a_2+3$ - note the answer is given so working must be seen.

(c) 1st M1: Substitutes their a_3 into $5a_3+3$ or uses $125k+93$

2nd M1: for **their** sum $k+a_2+a_3+a_4$ - must see evidence of **four terms with plus signs and must not be sum of AP**

1st A1: All correct so far

2nd A1ft: Limited ft – previous answer must be divisible by 6 (eg $156k+42$) . This is dependent on second M mark in (c)

Allow $\frac{156k+114}{6} = 26k+19$ without explanation. No conclusion is needed.

Question Number	Scheme	Marks
55. **(a)**	Series has 50 terms $S=\frac{1}{2}(50)(2+100)=2550$ or $S=\frac{1}{2}(50)(4+49\times 2)=2550$	B1 M1 A1 (3)
(b) **(i)**	$\frac{100}{k}$	B1
(ii)	Sum: $\frac{1}{2}\left(\frac{100}{k}\right)(k+100)$ or $\frac{1}{2}\left(\frac{100}{k}\right)\left(2k+\left(\frac{100}{k}-1\right)k\right)$ $=50+\frac{5000}{k}$ (*)	M1 A1 A1 cso (4)
(c)	50th term = $a+(n-1)d$ $=(2k+1)+49\text{"}(2k+3)\text{"}$ $=100k+148$ Or $2k+49(2k)+1+49(3)$ $=100k+148$	M1 A1 (2) **9**
	Notes (a) B for seeing attempt to use n = 50 or n = 50 stated M for attempt to use $\frac{1}{2}n(a+l)$ or $\frac{1}{2}n(2a+(n-1)d)$ with a = 2 and values for other variables (Using n = 100 may earn B0 M1A0) (b) M for use of $a=k$ and $d=k$ or l = 100 with their value for n , could be numerical or even letter n in correct formula for sum. A1: Correct formula with n = 100/k A1: NB Answer is printed – so no slips should have appeared in working (c) M for use of formula $a+49d$ with a = 2k +1 and with d obtained from difference of terms A1: Requires this simplified answer	

Question Number	Scheme	Marks
56 **(a)**	$(a_2 =)\ 6 - c$	B1 (1)
(b)	$a_3 = 3(\text{their } a_2) - c \quad (= 18 - 4c)$ $a_1 + a_2 + a_3 = 2 + "(6-c)" + "(18-4c)"$ $"26 - 5c" = 0$ So $c = 5.2$	M1 M1 A1ft A1 o.a.e (4) **5**
	Notes	
(b)	1st M1 for attempting a_3. Can follow through their answer to (a) but it must be an expression in *c*. 2nd M1 for an attempt to find the sum $a_1 + a_2 + a_3$ must see evidence of sum 1st A1ft for their sum put equal to 0. Follow through their values but answer must be in the form $p + qc = 0$ A1 – accept any correct equivalent answer	

Question Number	Scheme	Marks
57. (a)	$S_{10} = \frac{10}{2}\left[2a + 9d\right]$ or $S_{10} = a + a + d + a + 2d + a + 3d + a + 4d + a + 5da + 6d + a + 7d + a + 8d + a + 9d$ $162 = 10a + 45d$ *	M1 A1cso (2)
(b)	$\left(u_n = a + (n-1)d \quad \Rightarrow \quad\right) 17 = a + 5d$	B1 (1)
	$10 \times (b)$ gives $10a + 50d = 170$ (a) is $10a + 45d = 162$ Subtract $5d = 8$ so $d = \underline{1.6}$ o.e. Solving for a $a = 17 - 5d$ so $a = \underline{9}$	M1 A1 M1 A1 (4) **7**
	Notes	
(a) (b)	M1 for use of S_n with $n = 10$ 1st M1 for an attempt to eliminate a or d from their two linear equations 2nd M1 for using their value of a or d to find the other value.	

Question Number	Scheme	Marks
58.		
(a)	$a_2 = \left(\sqrt{4+3}\right) = \sqrt{7}$	B1
	$a_3 = \sqrt{\text{"their 7"}+3} = \sqrt{10}$	B1ft (2)
(b)	$a_4 = \sqrt{10+3}\left(=\sqrt{13}\right)$	M1
	$a_5 = \sqrt{13+3} = 4$ *	A1 cso (2)
		4
	Notes	
(a)	1st B1 for $\sqrt{7}$ only 2nd B1ft follow through their "7" in correct formula provided they have $\sqrt{n}$,where n is an integer.	
(b)	M1 for an attempt to find a_4. Should see $\sqrt{\text{"their"}\left(a_3\right)^2+3}$. Must see evidence for M1. $a_4 = \sqrt{13}$ provided this follows from their a_3 working or answer is sufficient A1cso for a correct solution (M1 explicit) must include the = 4. Ending at $\sqrt{16}$ only is A0 and ending with $\pm$ 4 is A0. Ignore any incorrect statements that are not used e.g. common difference = $\sqrt{3}$ Listing: A full list: $2\left(=\sqrt{4}\right),\ \sqrt{7},\ \sqrt{10},\ \sqrt{13},\ \sqrt{16}=4$ is fine for M1A1	
ALT	Formula: Some may state (or use) $a_n = \sqrt{3n+1}$ leading to $a_5 = \sqrt{3\times 5+1} = 4$. This will get marks in (a) [if correct values are seen] and can score the M1 in (b) if $a_n = \sqrt{3n+1}$ or $a_4 = \sqrt{13}$ are seen.	
$\pm\sqrt{\ }$	If $\pm\sqrt{\ }$ appear any where ignore in part (a) and withhold the final A mark only	

Question Number	Scheme	Marks
59. **(a)**	$a+29d=40.75$ or $a=40.75-29d$ or $29d=40.75-a$	M1 A1 (2)
(b)	$(S_{30})=\frac{30}{2}(a+l)$ or $\frac{30}{2}(a+40.75)$ or $\frac{30}{2}(2a+(30-1)d)$ or $15(2a+29d)$	M1
	So $1005=15[a+40.75]$ *	A1 cso (2)
(c)	$67 = a + 40.75$ so a = (£) 26.25 or 2625p or $26\frac{1}{4}$ NOT $\frac{105}{4}$	M1 A1
	$29d = 40.75 - 26.25$	M1
	$= 14.5$ so d = (£)0.50 or 0.5 or 50p or $\frac{1}{2}$	A1 (4)
		8

	Notes
(a)	M1 for attempt to use $a+(n-1)d$ with $n=30$ to form an equation . So $a+(30-1)d$ = any number is OK A1 as written. Must see $29d$ not just $(30-1)d$. Ignore any floating £ signs e.g. $a+29d=£40.75$ is OK for M1A1 These two marks must be scored in (a). Some may omit (a) but get correct equation in (c) [or (b)] but we do not give the marks retrospectively.
	Parts (b) and (c) may run together
(b)	M1 for an attempt to use an S_n formula with $n=30$. **Must see one of the printed forms.** ($S_{30}=$ is not required) A1cso for forming an equation with 1005 and S_n and simplifying to printed answer. Condone £ signs e.g. 15[a+ £40.75]=1005 is OK for A1
(c)	1st M1 for an attempt to simplify the given linear equation for a. Correct processes. Must get to $ka = \dots$ or $k = a + m$ i.e. one step (division or subtraction) from $a = \dots$ Commonly: $15a = 1005 - 611.25$ (= 393.75) 1st A1 For $a = 26.25$ or 2625p or $26\frac{1}{4}$ NOT $\frac{105}{4}$ or any other fraction 2nd M1 for correct attempt at a linear equation for d, follow through their a or equation in (a) Equation just has to be linear in d, they don't have to simplify to $d = \dots$ 2nd A1 depends upon 2nd M1 and use of correct a. Do not penalise a second time if there were minor arithmetic errors in finding a provided $a = 26.25$ (o.e.) is used. Do not accept other fractions other than $\frac{1}{2}$ **If answer is in pence a "p" must be seen.**
Sim Equ	Use this scheme: 1st M1A1 for a and 2nd M1A1 for d. Typically solving: $1005=30a+435d$ and $40.75=a+29d$. If they find d first then follow through use of their d when finding a.

Question number	Scheme	Marks
60	(a) $a+9d=150+9\times 10=240$	M1 A1 (2)
	(b) $\frac{1}{2}n\{2a+(n-1)d\}=\frac{20}{2}\{2\times 150+19\times 10\},\ =4900$	M1 A1, A1 (3)
	(c) Kevin: $\frac{1}{2}n\{2a+(n-1)d\}=\frac{20}{2}\{2A+19\times 30\}$ Kevin's total = $2\times$"4900" (or "4900" = $2\times$ Kevin's total) $\frac{20}{2}\{2A+19\times 30\}=2\times$"4900" $A=205$	B1 M1 A1ft A1 (4) **[9]**
	(a) M: Using $a+9d$ with at least one of a = 150 and d = 10. Being 'one off' (e.g. equivalent to a + 10d), scores M0. Correct answer with no working scores both marks. (b) M: Attempting to use the correct sum formula to obtain S_{20}, with at least one of a = 150 and d = 10. If the wrong value of n or a or d is used, the M mark is only scored if the correct sum formula has been quoted. 1st A: Any fully correct numerical version. (c) B: A correct expression, in terms of A, for Kevin's total. M: Equating Kevin's total to twice Jill's total, or Jill's total to twice Kevin's. For this M mark, the expression for Kevin's total need not be correct, but must be a linear function of A (or a). 1st A: (Kevin's total, correct, possibly unsimplified) = 2(Jill's total), ft Jill's total from part (b). 'Listing' and other methods (a) M: Listing terms (found by a correct method with at least one of a = 150 and d = 10), and picking the 10th term. (There may be numerical slips). (b) M: Listing sums, or listing and adding terms (found by a correct method with at least one of a = 150 and d = 10), far enough to establish the required sum. (There may be numerical slips). Note: 20th term is 340. A2 (scored as A1 A1) for 4900 (clearly selected as the answer). If no working (or no legitimate working) is seen, but the answer 4900 is given, allow one mark (scored as M1 A0 A0). (c) By trial and improvement: Obtaining a value of A for which Kevin's total is twice Jill's total, or Jill's total is twice Kevin's (using Jill's total from (b)): M1 Obtaining a value of A for which Kevin's total is twice Jill's total (using Jill's total from (b)): A1ft Fully correct solutions then score the B1 and final A1. The answer 205 with no working (or no legitimate working) scores no marks.	

Question Number	Scheme	Marks
61 (a)	$a+9d=2400 \qquad a+39d=600$	M1
	$d=\frac{-1800}{30} \qquad d=-60$ (accept $\pm$ 60 for A1)	M1 A1 (3)
(b)	$a-540=2400 \qquad a=2940$	M1 A1 (2)
(c)	Total = $\frac{1}{2}n\{2a+(n-1)d\}=\frac{1}{2}\times 40\times(5880+39\times -60)$ (ft values of a and d)	M1 A1ft
	$=\underline{70\ 800}$	A1cao (3) **[8]**
	Note: If the sequence is considered 'backwards', an equivalent solution may be given using $d = 60$ with $a = 600$ and $l = 2940$ for part (b). This can still score full marks. **Ignore labelling of (a) and (b)**	
(a)	1st M1 for an attempt to use 2400 and 600 in $a+(n-1)d$ formula. Must use both values i.e. need $a + pd = 2400$ <u>and</u> $a + qd = 600$ where $p = 8$ or 9 and $q = 38$ or 39 (any combination) 2nd M1 for an attempt to solve <u>their</u> 2 linear equations in a and d as far as d =… A1 for $d = \pm$ 60. Condone correct equations leading to $d = 60$ or $a + 8d = 2400$ and $a + 38d = 600$ leading to $d = -60$. They should get penalised in (b) and (c). **NB This is a "one off" ruling for A1. Usually an A mark must follow from their work.** ALT 1st M1 for $(30d) = \pm (2400 - 600)$ 2nd M1 for $(d=)\pm\frac{(2400-600)}{30}$ A1 for $d = \pm 60$ $a + 9d = 600$, $a + 39d = 2400$ **only** scores M0 BUT if they solve to find $d = \pm 60$ then use ALT scheme above.	
(b)	M1 for use of <u>their</u> d in a **correct** linear equation to find a leading to a = … A1 their a must be compatible with their d so $d = 60$ must have $a = 600$ and $d = -60$, $a = 2940$ So for example they can have $2400 = a + 9(60)$ leading to a = … for M1 but it scores A0 Any approach using a list scores M1A1 for a correct a but M0A0 otherwise	
(c)	M1 for use of a correct S_n formula with $n = 40$ and at least one of a, d or l correct or correct ft. 1st A1ft for use of a correct S_{40} formula and both a, d or a, l correct or correct follow through ALT Total = $\frac{1}{2}n\{a+l\}=\frac{1}{2}\times 40\times(2940+600)$ (ft value of a) M1 A1ft 2nd A1 for 70800 only	

Question Number	Scheme	Marks
62 (a)	$(a_2 =)2k-7$	B1 (1)
(b)	$(a_3 =)2(2k-7)-7$ or $4k-14-7, = 4k-21$ (*)	M1, A1cso (2)
(c)	$(a_4 =)2(4k-21)-7 \quad (= 8k-49)$	M1
	$\sum_{r=1}^{4} a_r = k + "(2k-7)" + (4k-21) + "(8k-49)"$	M1
	$k+(2k-7)+(4k-21)+(8k-49) = 15k-77 = 43 \qquad k = 8$	M1 A1 (4) **[7]**
(b) (c)	M1 must see 2(their a_2) - 7 or $2(2k-7)-7$ or $4k - 14 - 7$. Their a_2 must be a function of *k*. A1cso must see the $2(2k-7)-7$ or $4k - 14 - 7$ expression and the $4k$ - 21 with no incorrect working 1st M1 for an attempt to find a_4 using the given rule. Can be awarded for $8k$ - 49 seen. **Use of formulae for the sum of an arithmetic series scores M0M0A0 for the next 3 marks.** 2nd M1 for attempting the sum of the 1st 4 terms. Must have "+" not just , or clear attempt to sum. Follow through their a_2 and a_4 provided they are linear functions of *k*. Must lead to linear expression in *k*. Condone use of their linear $a_3 \neq 4k-21$ here too. 3rd M1 for forming a linear equation in *k* using their sum and the 43 and attempt to solve for *k* as far as $pk = q$ A1 for $k = 8$ only so $k = \frac{120}{15}$ is A0 <u>Answer Only </u>(e.g. trial improvement) Accept $k = 8$ <u>only if </u> 8 + 9 + 11 + 15 = 43 is seen as well <u>Sum $a_2 + a_3 + a_4 + a_5$</u> or <u>$a_2 + a_3 + a_4$</u> Allow: M1 if $8k$ - 49 is seen, M0 for the sum (since they are not adding the 1st 4 terms) then M1 if they use their sum along with the 43 to form a linear equation and attempt to solve but A0	

Question Number	Scheme	Marks
63 (a)	$a+17d=25$ or equiv. (for 1st B1), $a+20d=32.5$ or equiv. (for 2nd B1),	B1, B1 (2)
(b)	Solving (Subtract) $3d = 7.5$ so **d = 2.5**	M1
	$a = 32.5 - 20\times 2.5$ so **a = -17.5** (*)	A1cso (2)
(c)	$2750 = \frac{n}{2}\left[-35 + \frac{5}{2}(n-1)\right]$	M1A1ft
	{ $4\times 2750 = n(5n-75)$ }	
	$4\times 550 = n(n-15)$	M1
	$n^2 - 15n = 55\times 40$ (*)	A1cso (4)
(d)	$n^2 - 15n - 55\times 40 = 0$ or $n^2 - 15n - 2200 = 0$	M1
	$(n-55)(n+40) = 0$ $n = \ldots$	M1
	n = 55 (ignore - 40)	A1 (3) **[11]**

Mark parts (a) and (b) as 'one part', ignoring labelling.

(a) Alternative:

1st B1: $d = 2.5$ or equiv. or $d = \frac{32.5-25}{3}$. No method required, but $a = -17.5$ must not be assumed.

2nd B1: Either $a+17d=25$ or $a+20d=32.5$ seen, or used with a value of d…
or for 'listing terms' or similar methods, 'counting back' 17 (or 20) terms.

(b) M1: In main scheme: for a full method (allow numerical or sign slips) leading to solution for d or a without assuming $a = -17.5$
In alternative scheme: for using a d value to find a value for a.

A1: Finding correct values for both a and d (allowing equiv. fractions such as $d = \frac{15}{6}$), with no incorrect working seen.

In the main scheme, if the given a is used to find d from one of the equations, then allow M1A1 if both values are checked in the 2nd equation.

(c) 1st M1 for attempt to form equation with correct S_n formula and 2750, with values of a and d.
1st A1ft for a correct equation following through their d.
2nd M1 for expanding and simplifying to a 3 term quadratic.
2nd A1 for correct working leading to printed result (no incorrect working seen).

(d) 1st M1 forming the correct 3TQ = 0 . Can condone missing "= 0" but all terms must be on one side.
First M1 can be implied (perhaps seen in (c), but there must be an attempt at (d) for it to be scored).
2nd M1 for attempt to solve 3TQ, by factorisation, formula or completing the square (see general marking principles at end of scheme). If this mark is earned for the 'completing the square' method or if the factors are written down directly, the 1st M1 is given by implication.
A1 for n = 55 dependent on both Ms. Ignore – 40 if seen.
No working or 'trial and improvement' methods in (d) score all 3 marks for the answer 55, otherwise no marks.

Question number	Scheme	Marks	
64(a)	$[x_2 =]\ a-3$	B1	(1)
(b)	$[x_3 =]\ ax_2 - 3$ or $a(a-3)-3$	M1	
	$= a(a-3)-3$ } both lines needed for A1		
	$= a^2 - 3a - 3$ (*)	A1cso	(2)
(c)	$a^2 - 3a - 3 = 7$		
	$a^2 - 3a - 10 = 0$ or $a^2 - 3a = 10$	M1	
	$(a-5)(a+2) = 0$	dM1	
	$\underline{a = 5 \text{ or } -2}$	A1	(3)
			6
(a)	B1 for $a \times 1 - 3$ or better. Give for $a-3$ in part (a) or if it appears in (b) they must state $x_2 = a-3$ This must be seen in (a) or before the $a(a-3)-3$ step.		
(b)	M1 for clear show that. Usually for $a(a-3)-3$ but can follow through their x_2 and even allow $ax_2 - 3$ A1 for correct processing leading to printed answer. Both lines needed and no incorrect working seen.		
(c)	1st M1 for attempt to form a correct equation and start to collect terms. It must be a quadratic but need not lead to a 3TQ=0 2nd dM1 This mark is dependent upon the first M1. for attempt to factorize their 3TQ=0 or to solve their 3TQ=0. The "=0" can be implied. $(x \pm p)(x \pm q) = 0$, where $pq = 10$ or $(x \pm \frac{3}{2})^2 \pm \frac{9}{4} - 10 = 0$ or correct use of quadratic formula with $\underline{\pm}$ They must have a form that leads directly to 2 values for *a*. Trial and Improvement that leads to only one answer gets M0 here. A1 for both correct answers. Allow $x = \ldots$ Give 3/3 for correct answers with no working or trial and improvement that gives <u>both</u> values for *a*		

Question number		Scheme	Marks	
65	(a)	5, 7, 9, 11 or 5+2+2+2=11 or 5+6=11 use $a = 5$, $d = 2$, $n = 4$ and $t_4 = 5 + 3\times 2 = 11$	B1	(1)
	(b)	$t_n = a + (n-1)d$ with one of $a = 5$ or $d = 2$ correct (can have a letter for the other)	M1	
		$= 5 + 2(n - 1)$ or $2n + 3$ or $1 + 2(n + 1)$	A1	(2)
	(c)	$S_n = \frac{n}{2}[2\times 5 + 2(n-1)]$ or use of $\frac{n}{2}(5 + \text{"their } 2n+3\text{"})$ (may also be scored in (b))	M1A1	
		$= \{n(5 + n - 1)\} = n(n + 4)$ **(*)**	A1cso	(3)
	(d)	$43 = 2n + 3$	M1	
		$[n] = 20$	A1	(2)
	(e)	$S_{20} = 20\times 24$, $= \underline{480}$ (km)	M1A1	(2)
			10	

(a)	B1	Any other sum must have a convincing argument
(b)	M1	for an attempt to use $a + (n - 1)d$ with one of a or d correct (the other can be a letter) Allow any answer of the form $2n + p$ ($p \neq 5$) to score M1.
	A1	for a correct expression (needn't be simplified) [**Beware** $5 + (2n-1)$ scores A0] Expression must be in n not x. Correct answers with no working scores 2/2.
(c)	M1	for an attempt to use S_n formula with $a = 5$ or $d = 2$ or $a = 5$ and their "$2n + 3$"
	1st A1	for a fully correct expression
	2nd A1	for correctly simplifying to given answer. No incorrect working seen. Must see S_n used.
		Do not give credit for part (b) if the equivalent work is given in part (d)
(d)	M1	for forming a suitable equation in n (ft their (b)) and attempting to solve leading to $n =$…
	A1	for 20 Correct answer only scores 2/2 . Allow 20 following a restart but check working. eg $43 = 2n + 5$ that leads to $40 = 2n$ and $n = 20$ should score M1A0.
(e)	M1	for using their answer for n in $n(n + 4)$ or S_n formula, their n must be a value.
	A1	for 480 (ignore units but accept 480 000 m etc)[no matter where their 20 comes from]
		NB "attempting to solve" eg part (d) means we will allow sign slips and slips in arithmetic but not in processes. So dividing when they should subtract etc would lead to M0. Listing in parts (d) and (e) can score 2 (if correct) or 0 otherwise in each part. Poor labelling may occur (especially in (b) and (c)) . If you see work to get $n(n + 4)$ mark as (c)

Question number	Scheme	Marks	
66.	(a) $1(p+1)$ or $p+1$	B1	(1)
	(b) $((a))(p+(a))$ [(a) must be a function of p]. $[(p+1)(p+p+1)]$	M1	
	$=1+3p+2p^2$ (*)	A1cso	(2)
	(c) $1+3p+2p^2=1$	M1	
	$p(2p+3)=0$ $p=...$	M1	
	$p=-\frac{3}{2}$ (ignore $p = 0$, if seen, even if 'chosen' as the answer)	A1	(3)
	(d) Noting that even terms are the same.	M1	
	This M mark can be implied by listing at least 4 terms, e.g. $1, -\frac{1}{2}, 1, -\frac{1}{2},...$		
	$x_{2008}=-\frac{1}{2}$	A1	(2)
			8
	(b) M: Valid attempt to use the given recurrence relation to find x_3. Missing brackets, e.g. $p+1(p+p+1)$ Condone for the M1, then if all terms in the expansion are correct, with working fully shown, M1 A1 is still allowed. Beware 'working back from the answer', e.g. $1+3p+2p^2=(1+p)(1+2p)$ scores no marks unless the recurrence relation is justified. (c) 2nd M: Attempt to solve a quadratic equation in p (e.g. quadratic formula or completing the square). The equation must be based on $x_3=1$. The attempt must lead to a non-zero solution, so just stating the zero solution $p = 0$ is M0. A: The A mark is dependent on both M marks. (d) M: Can be implied by a correct answer for their p (answer is $p + 1$), and can also be implied if the working is 'obscure'). Trivialising, e.g. $p = 0$, so every term = 1, is M0. If the additional answer $x_{2008}=1$ (from $p = 0$) is seen, ignore this (isw).		

Question number	Scheme	Marks
67.	(a) $u_{25} = a + 24d = 30 + 24 \times (-1.5)$	M1
	$= -6$	A1 (2)
	(b) $a + (n-1)d = 30 - 1.5(r-1) = 0$	M1
	$r = 21$	A1 (2)
	(c) $S_{20} = \frac{20}{2}\{60 + 19(-1.5)\}$ or $S_{21} = \frac{21}{2}\{60 + 20(-1.5)\}$ or $S_{21} = \frac{21}{2}\{30 + 0\}$	M1 A1ft
	$= 315$	A1 (3)
		7
	(a) M: Substitution of $a = 30$ and $d = \pm 1.5$ into $(a + 24d)$. Use of $a + 25d$ (or any other variations on 24) scores M0. (b) M: Attempting to use the term formula, equated to 0, to form an equation in r (with no other unknowns). Allow this to be called n instead of r. Here, being 'one off' (e.g. equivalent to $a + nd$), scores M1. (c) M: Attempting to use the correct sum formula to obtain S_{20}, S_{21}, or, with their r from part (b), S_{r-1} or S_r. 1st A(ft): A correct numerical expression for S_{20}, S_{21}, or, with their r from part (b), S_{r-1} or S_r …. but the ft is dependent on an <u>integer</u> value of r. Methods such as calculus to find a maximum only begin to score marks <u>after</u> establishing a value of r at which the maximum sum occurs. This value of r can be used for the M1 A1ft, but must be a positive integer to score A marks, so evaluation with, say, $n = 20.5$ would score M1 A0 A0. <u>'Listing' and other methods</u> (a) M: Listing terms (found by a correct method), and picking the <u>25th</u> term. (There may be numerical slips). (b) M: Listing terms (found by a correct method), until the zero term is seen. (There may be numerical slips). 'Trial and error' approaches (or where working is unclear or non-existent) score M1 A1 for 21, M1 A0 for 20 or 22, and M0 A0 otherwise. (c) M: Listing sums, or listing and adding terms (found by a correct method), at least as far as the 20th term. (There may be numerical slips). A2 (scored as A1 A1) for 315 (clearly selected as the answer). 'Trial and error' approaches essentially follow the main scheme, beginning to score marks when trying S_{20}, S_{21}, or, with their r from part (b), S_{r-1} or S_r. If no working (or no legitimate working) is seen, but the answer 315 is given, allow one mark (scored as M1 A0 A0). <u>For reference</u>: Sums: 30, 58.5, 85.5, 111, 135, 157.5, 178.5, 198, 216, 232.5, 247.5, 261, 273, 283.5, 292.5, 300, 306, 310.5, 313.5, 315, ……..	

Question Number	Scheme	Marks
68. (a)	$\left(2+kx\right)^7$ $2^7 + {}^7C_1 2^6(kx) + {}^7C_2 2^5(kx)^2 + {}^7C_3 2^4(kx)^3 ...$ First term of 128 $\left({}^7C_1 \times ... \times x\right) + \left({}^7C_2 \times ... \times x^2\right) + \left({}^7C_3 \times ... \times x^3\right) ...$ $=(128 ..) + 448kx + 672k^2x^2 + 560k^3x^3 ..$	 B1 M1 A1, A1 **(4)**
(b)	$560k^3 = 1890$ $k^3 = \frac{1890}{560}$ so $k =$ $k = 1.5$ o.e.	M1 dM1 A1 **(3)** **(7marks)**
Alternative method For (a)	$\left(2+kx\right)^7 = 2^7(1+\frac{kx}{2})^7$ $2^7(1 + {}^7C_1(\frac{k}{2}x) + {}^7C_2(\frac{k}{2}x)^2 + {}^7C_3(\frac{k}{2}x)^3$ ...) Scheme is applied exactly as before	

Notes

(a)

B1: The constant term should be 128 in their expansion (should not be followed by other constant terms)

M1: Two of the three binomial coefficients must be correct and must be with the correct power of x. Accept ${}^7C_1$ or $\binom{7}{1}$ or 7 **as a coefficient, and** ${}^7C_2$ or $\binom{7}{2}$ or 21 **as another and** ${}^7C_3$ or $\binom{7}{3}$ or 35 **as another……..**

Pascal's triangle may be used to establish coefficients.

A1: Two of the final three terms correct (i.e. two of $448kx + 672k^2x^2 + 560k^3x^3..$).

A1: All three final terms correct. (Accept answers without + signs, can be listed with commas or appear on separate lines)

e.g. The common error $=(128 ..) + 448kx + 672kx^2 + 560kx^3..$ would earn B1, M1, A0, A0, so 2/4 Then would gain a maximum of 1/3 in part (b)

If extra terms are given then isw

If the **final** answer is given as $=(128 ..) + 448kx + 672(kx)^2 + 560(kx)^3..$ with correct brackets and no errors are seen, this may be given full marks. If they continue and remove the brackets wrongly then they lose the accuracy marks.

Special case using Alternative Method: Uses $2\ (1+\frac{kx}{2})^7$ is likely to result in a maximum mark of B0M1A0A0 then M1M1A0

If the correct expansion is seen award the marks and isw

(b)

M1: Sets their **Coefficient** of x^3 equal to 1890. They should have an equation which does not include a power of x. This mark may be recovered if they continue on to get $k = 1.5$

dM1: This mark depends upon the previous M mark. Divides then attempts a cube root of their answer to give k – the intention must be clear. (You may need to check on a calculator) The correct answer implies this mark.

A1: Any equivalent to 1.5 If they give – 1.5 as a second answer this is A0

Question Number	Scheme	Marks
69.(a)	$10000 = \dfrac{a}{1-(-0.9)}$	M1
	$a = 19000$	A1
		(2)
(b)	Use ar^4	M1
	$19000 \times (-0.9)^4 = 12465.9$ (accept awrt 12466)	A1
		(2)
(c)	$S = \dfrac{a(1-r^{12})}{1-r}$ or lists and adds their first twelve terms with their a	M1
	$S = \dfrac{\text{"19000"}(1-(-0.9)^{12})}{1-(-0.9)}$ or $S = 10000(1-(-0.9)^{12})$	A1ft
	= 7176 only	A1cso
		(3)
		[7]

Notes

(a) M1: Correct use of formula for sum to infinity as above, or states correct formula and makes small slip such as replacing r with 0.9 instead of – 0.9
A1: Correct answer

(b) M1: Correct use of formula with $n - 1 = 4$, allow 0.9 instead of -0.9 here. Condone invisible brackets.
A1: accept awrt 12466 (even following use of 0.9) Correct answer implies M1A1 even with no method shown. Accept correct equivalents such as mixed or improper fractions

(c) M1: Correct use of formula with power 12 (or adds 12 terms) with their a (not 10000) and $r = +0.9$ or -0.9
A1ft: Correct unsimplified with their a and with $r = +0.9$ or -0.9 or for listing method as follows
19000 + -17100 + 15390 + -13851 + 12465.9 + -11219.31 + 10097.379 + -9087.6411 + 8178.87699 + -7360.989291 + 6624.890362 + -5962.401326 = (Do not follow through for listing method)
A1cso: 7176 only

Special case: $S = \dfrac{a(1-r^n)}{1-r}$ so $S = \dfrac{\text{"19000"}(1+(0.9)^{12})}{1+(0.9)}$ is M1A0A0

Whereas $S = \dfrac{\text{"19000"}(1+(0.9)^{12})}{1+(0.9)}$ on its own with no formula quoted is M0A0A0

$S = \dfrac{\text{"19000"}(1--0.9^{12})}{1--0.9}$ should have M1 (bod) then final two A marks depend on whether answer is correct so if this is followed by 7176 the A1A1 should be awarded. If it is followed by 12824 then A0A0 is implied.

Question Number	Scheme	Marks
70.	$\left(3-\frac{1}{3}x\right)^5$ - $3^5 + {}^5C_1 3^4(-\frac{1}{3}x) + {}^5C_2 3^3(-\frac{1}{3}x)^2 + {}^5C_3 3^2(-\frac{1}{3}x)^3 ...$ First term of 243 $\left({}^5C_1 \times ... \times x\right) + \left({}^5C_2 \times ... \times x^2\right) + \left({}^5C_3 \times ... \times x^3\right)...$ $=(243\ ..\ ) - \frac{405}{3}x + \frac{270}{9}x^2 - \frac{90}{27}x^3 ...$ $=(243\ ..\ ) - 135x + 30x^2 - \frac{10}{3}x^3 ..$	 B1 M1 A1 A1 (4) **[4]**
Alternative method	$\left(3-\frac{1}{3}x\right)^5 = 3^5(1-\frac{x}{9})^5$ $3^5(1 + {}^5C_1(-\frac{1}{9}x) + {}^5C_2(-\frac{1}{9}x)^2 + {}^5C_3(-\frac{1}{9}x)^3\ ...)$ Scheme is applied exactly as before	
	Notes B1: The constant term should be 243 in their expansion M1: Two of the three binomial coefficients must be correct and must be with the correct power of x. Accept ${}^5C_1$ or $\binom{5}{1}$ or 5 **as a coefficient, and** ${}^5C_2$ or $\binom{5}{2}$ or 10 **as another and** ${}^5C_3$ or $\binom{5}{3}$ or 10 **as another……..** Pascal's triangle may be used to establish coefficients. NB: If they only include the first two of these terms then the M1 may be awarded. A1: Two of the final three terms correct – may be unsimplified i.e. two of $-135x + 30x^2 - \frac{10}{3}x^3$ correct, or two of $-\frac{405}{3}x + \frac{270}{9}x^2 - \frac{90}{27}x^3$ (may be just two terms) A1: All three final terms correct and simplified. (Can be listed with commas or appear on separate lines. Accept in reverse order.) Accept correct alternatives to $-\frac{10}{3}$ e.g. $-3\frac{1}{3}$ or $-3.\dot{3}$ the recurring must be clear. 3.3 is not acceptable. Allow e.g. $+-135x$	
	e.g. The common error $3^5 + {}^5C_1 3^4(-\frac{1}{3})x + {}^5C_2 3^3(-\frac{1}{3})x^2 + {}^5C_3 3^2(-\frac{1}{3})x^3 = (243) - 135x - 90x^2 - 30x^3$ would earn B1, M1, A0, A0, so 2/4 If extra terms are given then isw No negative signs in answer also earns B1, M1, A0, A0 If the series is divided through by 3 at the final stage after an error or omission resulting in all multiple of three coefficients then apply scheme to series before this division and ignore subsequent work (isw) **Special Case**: Only gives first three terms $=(243\ ..\ ) - 135x + 30x^2\ ...$ or $243 - \frac{405}{3}x + \frac{270}{9}x^2$ Follow the scheme to give B1 M1 A1 A0 special case. (Do not treat as misread.) Answers such as $243\ + 405 - \frac{1}{3}x + 270 - \frac{1}{9}x^2 + 90 - \frac{1}{27}x^3 ..$ gain no credit as the binomial coefficients are not linked to the x terms.	

Question Number	Scheme	Marks
71(a)	$a = 7k-5,\ ar = 5k-7$ and $ar^2 = 2k+10$	B1
	(So $r =$) $\dfrac{5k-7}{7k-5} = \dfrac{2k+10}{5k-7}$ or $(7k-5)(2k+10) = (5k-7)^2$ or equivalent	M1
	See $(5k-7)^2 = 25k^2 - 70k + 49$	M1
	$14k^2 + 60k - 50 = 25k^2 - 70k + 49 \rightarrow\ 11k^2 - 130k + 99 = 0$ *	A1cso * **(4)**
(b)	$(k-11)(11k-9)$ so $k =$	M1
	$k = 9/11$ only* (after rejecting 11)	A1*
	<u>**N.B. Special case k = 9/11 can be verified in (b) (1 mark only)**</u>	
	$11\times\left(\dfrac{9}{11}\right)^2 - 130\times\left(\dfrac{9}{11}\right) + 99 = \dfrac{81}{11} - \dfrac{1170}{11} + \dfrac{1089}{11} = 0$ M1A0	**(2)**
(c)	$a = \dfrac{8}{11}$	B1
	$\dfrac{5\times\frac{9}{11}-7}{7\times\frac{9}{11}-5}$ *or* $\dfrac{2\times\frac{9}{11}+10}{5\times\frac{9}{11}-7}$ so $r = -4$	B1
	(i) Fourth term = $ar^3 = -\dfrac{512}{11}$	M1A1
	(ii) $S_{10} = \dfrac{a(1-r^{10})}{(1-r)} = \dfrac{\frac{8}{11}(1-(-4)^{10})}{(1-(-4))} = -152520$	M1A1
		(6) **[12]**

Notes

(a) Mark parts (a) and (b) together

B1: Correct statement (needs all three terms)**– this may be omitted and implied** by correct statement in k only, as candidates may use geometric mean, or may use ratio of terms being equal and give a correct line 2 without line 1. (This would earn the B1M1 immediately)

M1: Valid Attempt to eliminate a and r and to obtain equation in k only

M1: Correct expansion of $(5k-7)^2 = 25k^2 - 70k + 49$ - may have four terms $(5k-7)^2 = 25k^2 - 35k - 35k + 49$

A1cso: No incorrect work seen. The printed answer is obtained including "=0".

(b) M1: Attempt to solve quadratic by usual methods (factorisation, completion of square or formula – see notes at start of mark scheme) **or** see 9/11 substituted and given as "=0" for M1A0

A1*: 9/11 **only and** 11 should be seen and rejected. Accept 9/11 underlined or k= 9/11 written on following line. Alternatively (k − 11) may be seen in the factorisation and a statement 'k not integer' given with k=9/11 stated.

(c) Mark parts (i) and (ii) together

B1: $a = \dfrac{8}{11}$ or any equivalent (If not stated explicitly or used in formula may be implied by correct answer to (ii))

B1:Substitutes k = 9/11 completely and obtain $r = -4$ (If not stated explicitly, may be implied by correct answer to (i) or (ii))

(i) M1: Use of correct formula with n = 4 a and/or r may still be in terms of k or uses $(2k+10)\times r$. May assume $r = k$.
A1: Correct exact answer

(ii) M1: Use of correct formula with n = 10 a and/or r may still be in terms of k May assume $r = k$ A1 : −152520 cao

NB Correct formula **with negative sign** in numerator followed by the incorrect $(8/11)(1+4^{10})/(1-(-4))$ usually found equal to 152520.2909 with no negative sign can be allowed M1A0 but if the incorrect numerical expression appears on its own with no formula then M0A0

Listing terms can get: B1 (first term) B1 M1A1 (implied by correct 4th term) M1A1 (implied by −152520)

Question Number	Scheme		Marks
72.	$r=\frac{3}{4},\ S_4=175$		
(a) **Way 1**	$\frac{a\left(1-\left(\frac{3}{4}\right)^4\right)}{1-\frac{3}{4}}$ or $\frac{a\left(1-\frac{3}{4}^4\right)}{1-\frac{3}{4}}$ or $\frac{a\left(1-0.75^4\right)}{1-0.75}$	Substituting $r=\frac{3}{4}$ or 0.75 and $n=4$ into the formula for S_n	M1
	$175=\frac{a\left(1-\left(\frac{3}{4}\right)^4\right)}{1-\frac{3}{4}} \Rightarrow a=\frac{175\left(1-\frac{3}{4}\right)}{\left(1-\left(\frac{3}{4}\right)^4\right)}\ \left\{\Rightarrow a=\frac{\left(\frac{175}{4}\right)}{\left(\frac{175}{256}\right)} \Rightarrow\right\}\ \underline{a=64}\ *$	Correct proof	A1*
			[2]
(a) **Way 2**	$a+a\left(\frac{3}{4}\right)+a\left(\frac{3}{4}\right)^2+a\left(\frac{3}{4}\right)^3$	$a+a\left(\frac{3}{4}\right)+a\left(\frac{3}{4}\right)^2+a\left(\frac{3}{4}\right)^3$	M1
	$\frac{175}{64}a=175\ \left(\Rightarrow a=\frac{175}{\left(\frac{175}{64}\right)}\right) \Rightarrow \underline{a=64}\ *$ or $2.734375a=175 \Rightarrow \underline{a=64}$	Correct proof	A1*
			[2]
(a) **Way 3**	$\{S_4=\}\ \frac{64\left(1-\left(\frac{3}{4}\right)^4\right)}{1-\frac{3}{4}}$ or $\frac{64\left(1-\frac{3}{4}^4\right)}{1-\frac{3}{4}}$ or $\frac{64\left(1-0.75^4\right)}{1-0.75}$	Applying the formula for S_n with $r=\frac{3}{4}$, $n=4$ and a as 64.	M1
	$=175$ so $a=64*$	Obtains 175 with no errors seen and concludes $a=64*$.	A1*
			[2]
(b)	$\{S_\infty\}=\frac{64}{\left(1-\frac{3}{4}\right)}$; $=256$	$S_\infty=\frac{(\text{their } a)}{1-\frac{3}{4}}$ or $\frac{64}{1-\frac{3}{4}}$	M1;
		256	A1cao
			[2]
(c)	$\{D=T_9-T_{10}=\}\ 64\left(\frac{3}{4}\right)^8-64\left(\frac{3}{4}\right)^9$	Writes down either "64"$\left(\frac{3}{4}\right)^8$ or awrt 6.4 or "64"$\left(\frac{3}{4}\right)^9$ or awrt 4.8, using $a=64$ or their a	M1
		A correct expression for the difference (i.e. $\pm(T_9-T_{10})$) using $a=64$ or their a.	dM1
	$\left\{=64\left(\frac{3}{4}\right)^8\left(\frac{1}{4}\right)=1.6018066...\right\}=\underline{1.602}$ (3dp)	1.602 or -1.602	A1 cao
			[3]
			7

		Question 72 Notes
72. (a)		**Allow invisible brackets around fractions throughout all parts of this question**.
	M1	There are three possible methods as described above.
	A1	Note that this is a "show that" question with a printed answer. In **Way 1** this mark **usually** requires $a = p/q$ where p and q may be unsimplified brackets from the formula (or could be 11200/175 for example) as an intermediate step before the conclusion $a = 64$. Exceptions include $a = 175/4 * 256/175$ i.e. multiplication by reciprocal rather than division or $175 = 175a/64$ followed by the obvious $a = 64$ These also get A1 In "reverse" methods such as **Way 3** we need a conclusion "so $a = 64$" or some implication that their argument is reversible. Also a conclusion can be implied from a preamble, eg: "If I assume $a = 64$ then find S= 175 as given this implies $a = 64$ as required" This is a show that question and there should be no loss of accuracy. In all the methods **if** decimals are used there should **not be rounding**. If 0.68359375 appears this is correct. If it is rounded it would not give the exact answer. $64(1-0.31640625)$ or 43.75 are each correct – if they are rounded then treat this as incorrect e.g. **Way 3: "**43.75/0.25 = 175 so $a = 64$ is A1" but **"**43/0.25 = 175 so $a = 64$ is A0" and "44/0.25 = 175 so a = 64 is A0" Yet another **variant on Way 3**: take a=64 then find the next 3 terms as 48, 36, 27 then add 64+48+36+27 to get 175. Again need conclusion that $a = 64$ or some implication that their argument is reversible. Otherwise M1 A0
(b)	**M1**	$S_\infty = \dfrac{64}{1-\frac{3}{4}}$ or $\dfrac{(\text{their } a \text{ found in part } (a))}{1-\frac{3}{4}}$
	A1	256 cao
(c)	**NB**	Using **Sum of 10 terms** minus **Sum of 9 terms** is NOT a misread Scores **M0M0A0**
	M1	Can be **implied.** Writes down either $64\left(\frac{3}{4}\right)^8$ **or** $64\left(\frac{3}{4}\right)^9$, using $a = 64$ (or their a found in part (a)).
	Note	Ignore candidate's labelling of terms.
	Note	$64\left(\frac{3}{4}\right)^8 = 6.407226563...$ and $64\left(\frac{3}{4}\right)^9 = 4.805419922\ldots$
	dM1	**This is dependent on previous M mark and can be implied.** Either $64\left(\frac{3}{4}\right)^8 - 64\left(\frac{3}{4}\right)^9$ or $64\left(\frac{3}{4}\right)^9 - 64\left(\frac{3}{4}\right)^8$ or awrt $6.4 -$ awrt 4.8, using $a = 64$ (or their a from part (a)) .
	Note	1st M1 and 2nd M1 can be implied by the value of their $\text{difference} = \text{"their } a \text{ found in part (a)"} \times \dfrac{3^8}{4^9} \approx \dfrac{\text{"their } a \text{ found in part (a)"}}{40}$
	Note	Either $64\left(\frac{3}{4}\right)^9 - 64\left(\frac{3}{4}\right)^{10}$ or $64\left(\frac{3}{4}\right)^{10} - 64\left(\frac{3}{4}\right)^9$ is 1st M1, 2nd M0.
	A1	1.602 or -1.602 **cao (This answer with no working is M1M1A1) But 1.6 with no working is M0M0A0**
	Note	$\left\{D = \frac{1}{4}T_9 \Rightarrow\right\}$ $D = \frac{1}{4}(64)\left(\frac{3}{4}\right)^8$ is 1st M1, 2nd M1
	Special case	Obtains awrt 6.4, then obtains awrt 4.8 but rounds to 6 – 5 when subtracting – award M1M1A0

Question Number	Scheme		Marks
73.	(a) $(2-9x)^4 = 2^4 + {}^4C_1 2^3(-9x) + {}^4C_2 2^2(-9x)^2$, (b) $f(x) = (1+kx)(2-9x)^4 = A - 232x + Bx^2$		
(a) **Way 1**	First term of 16 in their final series		B1
	At least one of $({}^4C_1 \times ... \times x)$ or $({}^4C_2 \times ... \times x^2)$		M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
(a) **Way 2**	$(2-9x)^4 = (4-36x+81x^2)(4-36x+81x^2)$		
		First term of 16 in their final series	B1
	$= 16 - 144x + 324x^2 - 144x + 1296x^2 + 324x^2$	Attempts to multiply a 3 term quadratic by the same 3 term quadratic to achieve either 2 terms in x or at least 2 terms in x^2.	M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
(a) **Way 3**	$\{(2-9x)^4 =\}\ 2^4\left(1 - \frac{9}{2}x\right)^4$	First term of 16 in final series	B1
	$= 2^4\left(1 + \underline{4\left(-\frac{9}{2}x\right) + \frac{4(3)}{2}\left(-\frac{9}{2}x\right)^2} + ...\right)$	At least one of $\underline{(4 \times ... \times x)\text{ or }\left(\frac{4(3)}{2} \times ... \times x^2\right)}$	M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
	Parts (b), (c) and (d) may be marked together		
(b)	$A = "16"$	Follow through their value from (a)	B1ft
			[1]
(c)	$\{(1+kx)(2-9x)^4\} = (1+kx)(16 - 288x + \{1944x^2 + ...\})$	*May be seen in part (b) or (d) and can be implied by work in parts (c) or (d).*	M1
	x terms: $-288x + 16kx = -232x$		
	giving, $16k = 56 \Rightarrow \underline{k = \frac{7}{2}}$	$\underline{k = \frac{7}{2}}$	A1
			[2]
(d)	x^2 terms: $1944x^2 - 288kx^2$		
	So, $B = 1944 - 288\left(\frac{7}{2}\right)$; $= 1944 - 1008 = 936$	See notes	M1
		936	A1
			[2]
			9

	Question 73 Notes	
(a) Ways 1 and 3	**B1 cao**	16
	M1	Correct binomial coefficient associated with correct power of *x i.e* $\left({}^4C_1 \times ... \times x\right)$ or $\left({}^4C_2 \times ... \times x^2\right)$ They may have 4 and 6 or 4 and $\frac{4(3)}{2}$ or even $\begin{pmatrix}4\\1\end{pmatrix}$ and $\begin{pmatrix}4\\2\end{pmatrix}$ as their coefficients. Allow missing signs and brackets for the M marks.
	1st A1	At least one of $-288x$ or $+1944x^2$ (allow +- 288*x*)
	2nd A1	Both $-288x$ **and** $+1944x^2$ (May list terms separated by commas) Also full marks for correct answer with no working here. Again allow +- 288*x*
	Note	If the candidate then divides their final correct answer through by 8 or any other common factor then isw and mark correct series when first seen. So (a) B1M1A1A1 .It is likely that this approach will be followed by (b) B0, (c) M1A0, (d) M1A0 if they continue with their new series e.g. $2 - 36x + 283x^2 + ...$ (Do not ft the value 2 as a mark was awarded for 16)
Way 2b	**Special Case**	**Slight Variation on the solution given in the scheme** $(2-9x)^4 = (2-9x)(2-9x)(4-36x+81x^2)$ $= (2-9x)(8-108x+486x^2+...)$
		$= 16 - 216x + 972x^2 - 72x + 972x^2$ — First term of 16 **B1**; Multiplies out to give either 2 terms in *x* or 2 terms in x^2. **M1**
		$= (16) - 288x + 1944x^2 + ...$ — At least one of $-288x$ or $+1944x^2$ **A1**; Both $-288x$ **and** $+1944x^2$ **A1**
		Parts (b), (c) and (d) may be marked together.
(b)	**B1ft**	Must **identify** $A = 16$ or $A =$ *their* constant term found in part (a). Or may write just 16 if this is clearly their answer to part (b). If they expand their series and have 16 as first term of a series it is not sufficient for this mark.
(c)	**M1**	Candidate shows intention to multiply (1+*kx*) by part of their series from (a) e.g. Just $(1+kx)\left(16-288x+...\right)$ or $(1+kx)\left(16-288x+1944x^2+...\right)$ are fine for M1.
	Note	This mark can also be implied by candidate multiplying out to find **two terms** (or coefficients) in *x*. i.e. f.t. their $-288x + 16kx$ N.B. $-288kx = -232x$ with no evidence of brackets is M0 – allow copying slips, or use of factored series, as this is a method mark
	A1	$k = \frac{7}{2}$ o.e. so 3.5 is acceptable
(d)	**M1**	Multiplies out their $(1+kx)\left(16-288x+1944x^2+...\right)$ to give **exactly** two terms (or coefficients) in x^2 and attempts to find *B* using **these two** terms and a numerical value of *k*.
	A1	936
	Note	Award A0 for $B = 936x^2$ But allow A1 for $B = 936x^2$ **followed by** $B = 936$ and treat this as a correction Correct answers in parts (c) and (d) with no method shown may be awarded full credit.

Question Number	Scheme		Marks
74.	$\left(2-\frac{x}{4}\right)^{10}$		
Way 1	$2^{10}+\binom{10}{1}2^9\left(-\frac{1}{4}x\right)+\binom{10}{2}2^8\left(-\frac{1}{4}x\right)^2+\ldots$	For **either** the x term **or** the x^2 term including a correct binomial coefficient with a correct power of x	M1
		First term of 1024	B1
	$=1024-1280x+720x^2$	**Either** $-1280x$ **or** $720x^2$ (Allow +-1280x here)	A1
		Both $-1280x$ and $720x^2$ (Do not allow +-1280x here)	A1 **[4]**
Way 2	$\left(2-\frac{x}{4}\right)^{10}=2^k\left(1-10\times\frac{x}{8}+\frac{10\times 9}{2}\left(-\frac{x}{8}\right)^2\right)$		M1
	$1024(1\pm.......)$		
	$=1024-1280x+720x^2$		B1A1 A1 **[4]**

Notes

M1: For either the x term or the x^2 term having correct structure i.e. a correct binomial coefficient in any form with the correct power of x. Condone sign errors and condone missing brackets and allow alternative forms for binomial coefficients e.g. ${}^{10}C_1$ or $\binom{10}{1}$ or even $\left(\frac{10}{1}\right)$ or 10. The powers of 2 or of ¼ may be wrong or missing.

B1: Award this for 1024 when first seen as a distinct constant term (not $1024x^0$) and not 1 + 1024

A1: For one correct term in x with coefficient simplified. Either -1280x or $720x^2$ (allow +-1280x here)

Allow $720x^2$ to come from $\left(\frac{x}{4}\right)^2$ with no negative sign. So use of + sign throughout could give M1 B1 A1 A0

A1: For both correct simplified terms i.e. -1280x and $720x^2$ (**Do not** allow +-1280x here)

Allow terms to be listed for full marks e.g. 1024, $-1280x$, $+720x^2$

N.B. If they follow a correct answer by a factor such as $512-640x+360x^2$ then isw

Terms may be listed. Ignore any extra terms.

Notes for Way 2

M1: Correct structure for at least one of the underlined terms. i.e. a correct binomial coefficient in any form with the correct power of x. Condone sign errors and condone missing brackets and allow alternative forms for binomial coefficients e.g. ${}^{10}C_1$ or $\binom{10}{1}$ or even $\left(\frac{10}{1}\right)$ or 10. k may even be 0 or 2^k may not be seen. Just consider the bracket for this mark.

B1: Needs 1024(1…. To become 1024

A1, A1: as before

Question Number	Scheme	Marks
75.(i)	**Mark (a) and (b) together**	
(a)	$a+ar=34$ or $\frac{a(1-r^2)}{(1-r)}=34$ or $\frac{a(r^2-1)}{(r-1)}=34$; $\frac{a}{1-r}=162$	B1; B1
(Way 1)	Eliminate a to give $(1+r)(1-r)=\frac{17}{81}$ or $1-r^2=\frac{34}{162}$ .. (not a cubic)	aM1
	(and so $r^2=\frac{64}{81}$ and) $r=\frac{8}{9}$ only	aA1 **(4)**
(b)	Substitute their $r=\frac{8}{9}$ ($0<r<1$) to give $a=$ $a=18$	bM1 bA1 **(2)**
(Way 2) Part (b) first	Eliminate r to give $\frac{34-a}{a}=1-\frac{a}{162}$	bM1
	gives $a=18$ or 306 and rejects 306 to give $a=18$	bA1
Then part (a) again	Substitute a = 18 to give $r=$	aM1
	$r=\frac{8}{9}$	aA1
(ii)	$\frac{42(1-\frac{6}{7}^n)}{1-\frac{6}{7}}>290$ (For trial and improvement approach see notes below)	M1
	to obtain So $(\frac{6}{7})^n<(\frac{4}{294})$ or equivalent e.g. $(\frac{7}{6})^n>(\frac{294}{4})$ or $(\frac{6}{7})^n<(\frac{2}{147})$	A1
	So $n>\frac{\log"(\frac{4}{294})"}{\log(\frac{6}{7})}$ or $\log_{\frac{6}{7}}"(\frac{4}{294})"$ or equivalent but must be log of positive quantity	M1
	(i.e. $n>$ 27.9) so $n=28$	A1 **(4)**

Notes

(i) (a) **B1**: Writes **a correct** equation connecting a and r and 34 (allow equivalent equations – may be implied)
B1: Writes a **correct** equation connecting a and r and 162 (allow equivalent equation – may be implied)

Way 1: a**M1**: Eliminates a correctly for these two equations to give $(1+r)(1-r)=\frac{17}{81}$ or $(1+r)(1-r)=\frac{34}{162}$ or equivalent –
not a cubic – should have factorized (1 - r) to give a correct quadratic
a**A1:** Correct value for r. Accept 0.8 recurring or 8/9 (not 0.889) Must only have positive value.

b**M1**: Substitutes their r ($0<r<1$) into a correct formula to give value for a. Can be implied by $a=18$
b**A1**: must be 18 (not answers which round to 18)

Way 2: Finds *a* first - B1, B1: As before then award the (b) M and A marks before the (a) M and A marks

b**M1**: Eliminates r correctly to give $\frac{34-a}{a}=1-\frac{a}{162}$ or $a^2-324a+5508=0$ or equivalent
b**A1:** Correct value for a so $a=18$ only. (Only award after 306 has been rejected)
a**M1**: Substitutes their 18 to give $r=$
a**A1**: $r=\frac{8}{9}$ only

(ii) **M1:** Allow n **or** $n-1$ and any symbols from ">", "<", or "=" etc **A1 :** Must be power n (**not** $n-1$) with any symbol
M1: Uses logs correctly on $(\frac{6}{7})^n$ or $(\frac{7}{6})^n$ **not on** $(36)^n$ to get as far as n Allow any symbol
A1: $n=28$ cso (any errors with inequalities earlier e.g. failure to reverse the inequality when dividing by the negative $\log(\frac{6}{7})$ or any contradictory statements must be penalised here) Those with equals throughout may gain this mark if they follow 27.9 by n=28. Just $n=28$ without mention of 27.9 is only allowed following correct inequality work.

Special case: Trial and improvement: Gives $n=28$ as S = awrt 290.1 (M1A1)and when $n=27$ S = (awrt) 289 so $n=28$ (M1A1) – $n=28$ with no working is M1A0M0A0 and insufficient accuracy is M1A0M1A0
Uses nth term instead of sum of n terms – over simplified – do not treat as misread – award 0/4

Question Number	Scheme		Marks
76. (a)	$(2-3x)^6 = 64 + ...$	64 seen as the only constant term in their expansion.	B1
	$\{(2-3x)^6\} = (2)^6 + \underline{{}^6C_1}(2)^5(-3\underline{x}) + \underline{{}^6C_2}(2)^4(-3\underline{x})^2 + ...$		M1
	M1: $({}^6C_1 \times ... \times x)$ or $({}^6C_2 \times ... \times x^2)$. For either the x term or the x^2 term. Requires correct binomial coefficient in any form with the correct power of x, but the other part of the coefficient (perhaps including powers of 2 and/or -3) may be wrong or missing. The terms can be "listed" rather than added. Ignore any extra terms.		
	${}^6C_1 2^5 -3x + {}^6C_2 2^4 -3x^2 + ...$ Scores M0 unless later work implies a correct method		
	$= 64 - 576x + 2160x^2 + ...$	A1: Either $-576x$ or $2160x^2$ (Allow $+ -576x$ here) A1: Both $-576x$ and $2160x^2$ (Do not allow $+ -576x$ here)	A1A1
			[4]
(a) Way 2	$(2-3x)^6 = 64 + ...$	64 seen as the only constant term in their expansion.	B1
	$\left(1-\frac{3}{2}x\right)^6 = 1 + \underline{{}^6C_1}\left(\frac{-3}{2}\underline{x}\right) + \underline{{}^6C_2}\left(\frac{-3}{2}\underline{x}\right)^2 + ...$	M1: $({}^6C_1 \times ... \times x)$ or $({}^6C_2 \times ... \times x^2)$. For either the x term or the x^2 term. Requires correct binomial coefficient in any form with the correct power of x, but the other part of the coefficient (perhaps including powers of 2 and/or -3) may be wrong or missing. The terms can be "listed" rather than added. Ignore any extra terms.	M1
	$= 64 - 576x + 2160x^2 + ...$	A1: Either $-576x$ or $2160x^2$ (Allow $+ -576x$ here) A1: Both $-576x$ and $2160x^2$ (Do not allow $+ -576x$ here)	A1A1
(b)	Candidate writes down $\left(1+\frac{x}{2}\right) \times$ (their part (a) answer, at least up to the term in x). (Condone missing brackets) $\left(1+\frac{x}{2}\right)(64-576x+...)$ or $\left(1+\frac{x}{2}\right)(64-576x+2160x^2+...)$ or $\left(1+\frac{x}{2}\right)64 - \left(1+\frac{x}{2}\right)576x$ or $\left(1+\frac{x}{2}\right)64 - \left(1+\frac{x}{2}\right)576x + \left(1+\frac{x}{2}\right)2160x^2$ or $64+32x, -576x-288x^2,\ 2160x^2+1080x^3$ are fine.		M1
	$= 64 - 544x + 1872x^2 + ...$	A1: At least 2 terms correct as shown. (Allow $+ -544x$ here) A1: $64 - 544x + 1872x^2$ The terms can be "listed" rather than added. Ignore any extra terms.	A1A1
			[3]
			Total 7
	SC: If a candidate expands in descending powers of x, only the M marks are available		
	e.g. $\{(2-3x)^6\} = (-3x)^6 + \underline{{}^6C_1}(2)^2(-3\underline{x})^5 + \underline{{}^6C_2}(2)^2(-3\underline{x})^4 + ...$		

Question Number	Scheme		Marks
77(a)	$S_\infty = \dfrac{20}{1-\frac{7}{8}}$; $=160$	M1: Use of a correct S_∞ formula A1: 160	M1A1
	Accept correct answer only (160)		
			[2]
(b)	$S_{12} = \dfrac{20\left(1-(\frac{7}{8})^{12}\right)}{1-\frac{7}{8}}$; $=127.77324...$	M1: Use of a correct S_n formula with $n = 12$ (condone missing brackets around 7/8) A1: **awrt** 127.8	M1A1
	T & I in (b) requires all 12 terms to be calculated correctly for M1 and A1 for awrt 127.8		
			[2]
(c)	$160 - \dfrac{20(1-(\frac{7}{8})^N)}{1-\frac{7}{8}} < 0.5$	Applies S_N (**GP only**) with $a = 20$, $r = \frac{7}{8}$ and "uses" 0.5 and their S_∞ at any point in their working. (condone missing brackets around 7/8)(Allow =, <, >, ≥, ≤) but see note below.	M1
	$160\left(\dfrac{7}{8}\right)^N < (0.5)$ or $\left(\dfrac{7}{8}\right)^N < \left(\dfrac{0.5}{160}\right)$	Attempt to isolate $+160\left(\frac{7}{8}\right)^N$ or $+\left(\frac{7}{8}\right)^N$ oe (Allow =, <, >, ≥, ≤) but see note below. **Dependent on the previous M1**	**d**M1
	$N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{160}\right)$	Uses the power law of logarithms or takes logs base 0.875 correctly to obtain an equation or an inequality of the form $N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{\text{their } S_\infty}\right)$ or $N > \log_{0.875}\left(\dfrac{0.5}{\text{their } S_\infty}\right)$ (Allow =, <, >, ≥, ≤) but see note below.	M1
	$N > \dfrac{\log\left(\frac{0.5}{160}\right)}{\log\left(\frac{7}{8}\right)} = 43.19823... \Rightarrow N = 44$	$N = 44$ (Allow $N \geq 44$ but not $N > 44$	A1 **cso**
	An incorrect **inequality** statement at any stage in a candidate's working loses the final mark. Some candidates do not realise that the direction of the inequality is reversed in the final line of their solution. **BUT** it is possible to gain full marks for using =, as long as no incorrect working seen.		
			[4]
			Total 8
	Trial & Improvement Method in (c):		
	1st M1: Attempts $160 - S_N$ or S_N with at least one value for $N > 40$		
	2nd M1: Attempts $160 - S_N$ or S_N with $N = 43$ or $N = 44$		
	3rd M1: For evidence of examining $160 - S_N$ or S_N for **both** $N = 43$ **and** $N = 44$ with **both** values correct to 2 DP Eg: $160 - S_{43} =$ awrt 0.51 and $160 - S_{44} =$ awrt 0.45 or $S_{43} =$ awrt 159.49 and $S_{44} =$ awrt 159.55		
	A1: $N = 44$ **cso**		
	Answer of $N = 44$ only with no working scores no marks		

Question Number	Scheme		Marks
	$\left(1+\frac{3x}{2}\right)^8$		
	$1+12x$	Both terms correct as printed (allow $12x^1$ but not 1^8)	B1
78.	$... + \frac{8(7)}{2!}\left(\frac{3x}{2}\right)^2 + \frac{8(7)(6)}{3!}\left(\frac{3x}{2}\right)^3 + ...$ $... + {}^8C_2\left(\frac{3x}{2}\right)^2 + {}^8C_3\left(\frac{3x}{2}\right)^3 + ...$	$\left(\frac{8(7)}{2!}\times ...\times x^2\right)$ or $\left(\frac{8(7)(6)}{3!}\times ...\times x^3\right)$ or $\left({}^8C_2\times ...\times x^2\right)$ or $\left({}^8C_3\times ...\times x^3\right)$ M1: For <u>either</u> the x^2 term <u>or</u> the x^3 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or 3 or signs) may be wrong or missing.	M1
	<u>Special Case:</u> Allow this M1 <u>only</u> for an attempt at a descending expansion provided the equivalent conditions are met for any term <u>other than the first</u> e.g. $... + 8\left(\frac{3x}{2}\right)^7(1) + \frac{8(7)}{2!}\left(\frac{3x}{2}\right)^6(1)^2 + ...$ $... + {}^8C_1\left(\frac{3x}{2}\right)^7 + {}^8C_2\left(\frac{3x}{2}\right)^6 + ...$		
	$... + 63x^2 + 189x^3 + ...$	A1: **Either** $63x^2$ **or** $189x^3$ A1: **Both** $63x^2$ **and** $189x^3$	A1A1
	Terms may be listed but must be positive		
			[4]
			Total 4
	Note it is common not to square the 2 in the denominator of $\left(\frac{3x}{2}\right)$ and this gives $1+12x+126x^2+756x^3$. This could score B1M1A0A0.		
	Note $... + {}^8C_2\left(1^4+\frac{3x}{2}\right)^2 + {}^8C_3\left(1^3+\frac{3x}{2}\right)^3 + ...$ would score M0 unless a correct method was implied by later work		

Question Number	Scheme		Marks
79. (a)	$S_\infty = 6a$		
	$\dfrac{a}{1-r} = 6a$	Either $\dfrac{a}{1-r} = 6a$ or $\dfrac{6a}{1-r} = a$ or $\dfrac{6}{1-r} = 1$	M1
	$\{\Rightarrow 1 = 6(1-r) \Rightarrow\}\ r = \dfrac{5}{6}$*	cso	A1*
	Allow verification e.g. $\dfrac{a}{1-r} = 6a \Rightarrow \dfrac{a}{1-\frac{5}{6}} = 6a \Rightarrow \dfrac{a}{\frac{1}{6}} = 6a \Rightarrow 6a = 6a$		
			[2]
(b)	$\{T_4 = ar^3 = 62.5 \Rightarrow\}\ a\left(\dfrac{5}{6}\right)^3 = 62.5$	$a\left(\dfrac{5}{6}\right)^3 = 62.5$ (Correct statement using the 4th term. Do not accept $a\left(\dfrac{5}{6}\right)^4 = 62.5$)	M1
	$\Rightarrow a = 108$	108	A1
			[2]
(c)	$S_\infty = 6(\text{their } a)$ or $\dfrac{\text{their } a}{1-\frac{5}{6}}\{= 648\}$	Correct method to find S_∞	M1
	$\{S_{30} =\}\ \dfrac{108\left(1-(\frac{5}{6})^{30}\right)}{1-\frac{5}{6}}\ \{= 645.2701573...\}$	M1: $S_{30} = \dfrac{(\text{their } a)\left(1-\left(\frac{5}{6}\right)^{30}\right)}{1-\left(\frac{5}{6}\right)}$ **(Condone invisible brackets around 5/6)**	M1 A1ft
		A1ft: Correct follow through expression (follow through their a). **Do not condone invisible brackets around 5/6 unless <u>their</u> evaluation or final answer implies they were intended.**	
	$\{S_\infty - S_{30}\} = 2.72984...$	**awrt** 2.73	A1
			[4]
			Total 8
(c)	**<u>Alternative:</u>** Difference $= \dfrac{ar^{30}}{1-r} = \dfrac{108\left(\frac{5}{6}\right)^{30}}{1-\frac{5}{6}} = 2.72984...$ M1M1: For an attempt to apply $\dfrac{ar^{30}}{1-r}$. A1ft: $\dfrac{(\textit{their } a)\times r^{30}}{1-r}$ with their ft a. A1: awrt 2.73		

Question Number	Scheme	Marks
80. (a)	$\{r =\}\ \dfrac{2}{3}$	B1
		(1)
(b)	$\{p =\}\ 8$	B1 cao
		(1)
(c)	$\{S_{15} =\}\ \dfrac{18\left(1-\left(\frac{2}{3}\right)^{15}\right)}{1-\frac{2}{3}}$	M1
	$\{S_{15} = 53.87668...\} \Rightarrow S_{15} = \text{awrt } 53.877$	A1
		(2)
		[4]

	Notes for Question 80
(a)	B1: Accept $\dfrac{12}{18}$, $0.\dot{6}$ or 0.6 recurring, or even 0.667 (3sf) but not 0.6 or 0.67
(b)	B1: accept 8 only
(c)	M1: Applies this formula $S_{15} = \dfrac{18\left(1-(\text{their } r)^{15}\right)}{1-(\text{their } r)}$, can be implied by their answer. For this mark they may use any value for r except $r = 1$ or $r = 0$ (even 3/2 or -6 may be used) A1: Answers which round to 53.877
Alternative method for (c)	M1: (Adding terms is an unlikely method for this question) Need to see 15 terms listed as 18+12+……0.06165877 or can be implied by correct answer A1: awrt 53.877 **Answer only** : 53.9 is M0A0 with no working, but 53.877 with no working is M1A1

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>
81. (a)

(b)</td><td>$(2+3x)^4$ - Mark (a) and (b) together
$2^4 + {}^4C_1 2^3(3x) + {}^4C_2 2^2(3x)^2 + {}^4C_3 2^1(3x)^3 + (3x)^4$
First term of 16
$\left({}^4C_1 \times \ldots \times x\right) + \left({}^4C_2 \times \ldots \times x^2\right) + \left({}^4C_3 \times \ldots \times x^3\right) + \left({}^4C_4 \times \ldots \times x^4\right)$
$=(16+)\ 96x + 216x^2 + 216x^3 + 81x^4$ Must use Binomial – otherwise A0, A0

$(2-3x)^4 = 16 - 96x + 216x^2 - 216x^3 + 81x^4$</td><td>

B1
M1
A1 A1

(4)
B1ft
(1)
5</td></tr>
<tr><td>**Alternative method (a)**</td><td>$(2+3x)^4 = 2^4(1+\frac{3x}{2})^4$
$2^4(1 + {}^4C_1(\frac{3x}{2}) + {}^4C_2(\frac{3x}{2})^2 + {}^4C_3(\frac{3x}{2})^3 + (\frac{3x}{2})^4)$
Scheme is applied exactly as before</td><td></td></tr>
<tr><td></td><td colspan="2">**Notes for Question 81**</td></tr>
<tr><td>(a)

(b)</td><td colspan="2">B1: The constant term should be 16 in their expansion
M1: Two binomial coefficients must be correct and must be with the correct power of x. Accept ${}^4C_1$ or $\binom{4}{1}$ or 4 **as a coefficient, and** ${}^4C_2$ or $\binom{4}{2}$ or 6 **as another……..** Pascal's triangle may be used to establish coefficients.
A1: Any two of the final four terms correct (i.e. two of $96x + 216x^2 + 216x^3 + 81x^4$) in expansion following Binomial Method.
A1: All four of the final four terms correct in expansion. (Accept answers without + signs, can be listed with commas or appear on separate lines)
B1ft: Award for correct answer as printed above or **ft their previous answer** provided it has **five terms** ft and must be **subtracting the x and x^3 terms**
Allow terms in (b) to be in descending order and allow +-96x and +-216x^3 in the series. (Accept answers without + signs, can be listed with commas or appear on separate lines)</td></tr>
<tr><td></td><td colspan="2">e.g. The common error $2^4 + {}^4C_1 2^3 3x + {}^4C_2 2^2 3x^2 + {}^4C_3 2^1 3x^3 + 3x^4 = (16) + 96x + 72x^2 + 24x^3 + 3x^4$ would earn B1, M1, A0, A0, and if followed by $=(16) - 96x + 72x^2 - 24x^3 + 3x^4$ gets B1ft so 3/5
Fully correct answer with no working can score B1 in part (a) and B1 in part (b). The question stated use the Binomial theorem and if there is no evidence of its use then M mark and hence A marks cannot be earned.
Squaring the bracket and squaring again may also earn **B1 M0 A0 A0 B1 if correct**
Omitting the final term but otherwise correct is B1 M1 A1 A0 B0ft so 3/5
If the series is divided through by 2 or a power of 2 at the final stage after an error or omission resulting in all even coefficients then apply scheme to series before this division and ignore subsequent work (isw)</td></tr>
</table>

Question Number	Scheme	Marks
82. Way 1	$\left(2-\frac{1}{2}x\right)^8 = 2^8 + \binom{8}{1}.2^7\left(-\frac{1}{2}x\right) + \binom{8}{2}2^6\left(-\frac{1}{2}x\right)^2 + \binom{8}{3}2^5\left(-\frac{1}{2}x\right)^3$	
	First term of 256	B1
	$\left({}^8C_1 \times \ldots \times x\right) + \left({}^8C_2 \times \ldots \times x^2\right) + \left({}^8C_3 \times \ldots \times x^3\right)$	M1
	$= (256) - 512x + 448x^2 - 224x^3$	A1, A1
		(4)
		Total 4
Way 2	$\left(2-\frac{1}{2}x\right)^8 = 2^8\left(1-\frac{1}{4}x\right)^8 = 2^8\left(1+\binom{8}{1}.\left(-\frac{1}{4}x\right)+\binom{8}{2}\left(-\frac{1}{4}x\right)^2+\binom{8}{3}\left(-\frac{1}{4}x\right)^3\right)$ Scheme is applied exactly as before except in special case below*	
	Notes for Question 82	
	B1: The first term should be 256 in their expansion M1: Two binomial coefficients must be correct and must be with the correct power of x. Accept ${}^8C_1$ or $\binom{8}{1}$ or 8 **as a coefficient, and** ${}^8C_2$ or $\binom{8}{2}$ or 28 **as another……..** Pascal's triangle may be used to establish coefficients. A1: Any two of the final three terms correct (but allow +- instead of -) A1: All three of the final three terms correct and simplified. (Deduct last mark for +-512x *and* +-224x^3 in the series). Also deduct last mark for the three terms correct but unsimplified. (Accept answers without + signs, can be listed with commas or appear on separate lines) The common error $\left(2-\frac{1}{2}x\right)^8 = 256 + \binom{8}{1}.2^7\left(-\frac{1}{2}x\right) + \binom{8}{2}2^6\left(-\frac{1}{2}x^2\right) + \binom{8}{3}2^5\left(-\frac{1}{2}x^3\right)$ would earn B1, M1, A0, A0 **Ignore extra terms** involving higher powers. Condone terms in reverse order i.e. $= -224x^3 + 448x^2 - 512x + (256)$ ***In Way 2 the error** $= 2\left(1+\binom{8}{1}.\left(-\frac{1}{4}x\right)+\binom{8}{2}\left(-\frac{1}{4}x\right)^2+\binom{8}{3}\left(-\frac{1}{4}x\right)^3\right)$ giving $= 2 - 4x + \frac{7}{2}x^2 - \frac{7}{4}x^3$ is a **special case** B0, M1, A1, A0 i.e. 2/4	

Question Number	Scheme	Marks
83.(a)	$a = 4p,\ ar = (3p+15)$ and $ar^2 = 5p+20$	B1
	(So $r =$) $\dfrac{5p+20}{3p+15} = \dfrac{3p+15}{4p}$ or $4p(5p+20) = (3p+15)^2$ or equivalent	M1
	See $(3p+15)^2 = 9p^2 + 90p + 225$	M1
	$20p^2 + 80p = 9p^2 + 90p + 225 \rightarrow\ 11p^2 - 10p - 225 = 0$ *	A1 *
		(4)
(b)	$(p-5)(11p+45)$ so $p =$	M1
	$p = 5$ only (after rejecting - 45/11)	A1
	<u>N.B. Special case p = 5 can be verified in (b) (1 mark only)</u>	
	$11\times5^2 - 10\times5 - 225 \quad = 275 - 50 - 225 = 0$ M1A0	
		(2)
(c)	$\dfrac{3\times5+15}{4\times5}$ or $\dfrac{5\times5+20}{3\times5+15}$	M1
	$r = \dfrac{3}{2}$	A1
		(2)
(d)	$S_{10} = \dfrac{20\left(1-\left(\text{"}\frac{3}{2}\text{"}\right)^{10}\right)}{\left(1-\text{"}\frac{3}{2}\text{"}\right)}$	M1A1ft
	(= 2266.601568…) = 2267	A1
		(3) **Total 11**
	Notes for Question 83	
(a)	B1: Correct statement (needs all three terms)**– this may be omitted and implied** by correct statement in *p* only as candidates may use geometric mean, or may use ratio of terms being equal and give a correct line 2 without line 1. (This would earn the B1M1 immediately) M1: Valid Attempt to eliminate *a* and *r* and to obtain equation in *p* only M1: Correct expansion of $(3p+15)^2 = 9p^2 + 90p + 225$ A1cso: No incorrect work seen. The printed answer is obtained. **NB Those who show *p* = 5 in part (a) obtain no credit for this**	
(b)	M1: Attempt to solve quadratic by usual methods (factorisation, completion of square or formula) Must appear in part (b) – not part (a) A1: 5 **only and** -45/11 should be seen and rejected or (11*p* + 45) seen and statement $p > 0$	
(c)	M1: Substitutes *p* = 5 completely and attempt ratio (correct way up) A1: 1.5 or any equivalent	
(d)	M1: Use of correct formula with *n* = 10 *a* and/or *r* may still be in terms of *p* A1ft: Correct expression ft on their *r* only – must have *a* = 20 and power = 10 here A1 2267 (accept awrt 2267)	

Question Number	Scheme		Marks
84.	$(2-5x)^6$		
	$(2^6 =)\ 64$	Award this when first seen (not $64x^0$)	B1
	$+6\times(2)^5(-5x)+\frac{6\times5}{2}(2)^4(-5x)^2$	Attempt binomial expansion with correct structure for at least one of these terms. E.g. a term of the form: $\binom{6}{p}\times(2)^{6-p}(-5x)^p$ with p = 1 or p = 2 consistently. Condone sign errors. Condone missing brackets if later work implies correct structure and allow alternative forms for binomial coefficients e.g. ${}^6C_1$ or $\binom{6}{1}$ or even $\left(\frac{6}{1}\right)$	M1
	$-960x$	**Not** $+-960x$	A1 (first)
	$(+)6000x^2$		A1 (Second)
			(4)
Way 2	$64(1\pm............)$	64 **and** (1 ± ..... – Award when first seen.	B1
	$\left(1-\frac{5x}{2}\right)^6=1\ \underline{-6\times\frac{5x}{2}}+\underline{\frac{6\times5}{2}\left(-\frac{5x}{2}\right)^2}$	Correct structure for at least one of the underlined terms. E.g. a term of the form: $\binom{6}{p}\times(kx)^p$ with $p=1$ or $p=2$ consistently and $k\neq\pm5$ Condone sign errors. Condoned missing brackets if later work implies correct structure but it must be an expansion of $(1-kx)^6$ where $k\neq\pm5$	M1
	$-960x$	**Not** $+-960x$	A1
	$(+)6000x^2$		A1
			(4)

85.			
(a)	$120000\times(1.05)^3=138915\,*$	Or $120000\times1.05\times1.05\times1.05=138915$ Or 120000, 126000, 132300, 138915 Or $a=120000$ and $a\times(1.05)^3=138915$	B1
			(1)
(b)	$120000\times(1.05)^{n-1}>200000$	Allow n or $n-1$ and ">", "<", or "=" etc.	M1
	$\log1.05^{n-1}>\log\left(\frac{5}{3}\right)$	Takes logs correctly Allow n or $n-1$ and ">", "<", or "=" etc.	M1
	$(n-1>)\frac{\log\left(\frac{5}{3}\right)}{\log1.05}$ or equivalent e.g $(n>)\frac{\log\left(\frac{7}{4}\right)}{\log1.05}$	Allow n or $n-1$ and ">", "<", or "=" etc. Allow $1.\dot{6}$ or awrt 1.67 for 5/3.	A1
	2024	M1: Identifies a calendar year using their value of n or n - 1 A1: 2024	M1A1
			(5)
(c)	$\frac{a(1-r^n)}{1-r}=\frac{120000\left(1-1.05^{11}\right)}{1-1.05}$	M1: Correct sum formula with n = 10, 11 or 12 A1: Correct numerical expression with $n=11$	M1 A1
	1704814	Cao (Allow 1704814.00)	A1
			(3)
			[9]

	Listing or trial/improvement in (b)	
	U_{10} = 186 159.39, U_{11} = 195 467.36, U_{12} = 205 240.72	
	Attempt to find at least the 10th or 11th or 12th terms correctly using a common ratio of 1.05 (all the terms need **not** be listed)	M1
	Forms the geometric progression correctly to reach a term > 200 000	M1
	Obtains an "11th" term of awrt 195 500 **and** a "12th" term of awrt 205 200	A1
	Uses their number of terms to identify a calendar year	M1
	2024	A1
		(5)

Question number	Scheme	Marks
86	$\left[(2-3x)^5\right] = \ldots \quad +\binom{5}{1}2^4(-3x)+\binom{5}{2}2^3(-3x)^2+\ldots,\ \ \ldots\ldots$ $=32,\ -240x,\ +720x^2$	M1 B1, A1 , A1 **Total 4**
Notes	**M1**: The **method** mark is awarded for an attempt at Binomial to get the second and/or third term – need **correct** binomial coefficient combined **with correct power of *x*.** Ignore errors (or omissions) in powers of 2 or 3 or sign or bracket errors. Accept any notation for ${}^5C_1$ and ${}^5C_2$, e.g. $\binom{5}{1}$ and $\binom{5}{2}$ (unsimplified) or 5 and 10 from Pascal's triangle This mark may be given if no working is shown, but either or both of the terms including *x* is correct. **B1:** must be simplified to 32 (writing just 2^5 is **B0). 32** must be the only constant term in the final answer- so 32 +80 - 3*x*+80 +9 x^2 is B0 but may be eligible for M1A0A0 . **A1: is cao and is for –**240 *x*. (not +-240x) The *x* is required for this mark **A1:** is c.a.o and is for $720x^2$ (can follow omission of negative sign in working) A list of correct terms may be given credit i.e. series appearing on different lines Ignore extra terms in x^3 and/or x^4 (isw)	
Special Case	Special Case*: Descending powers* of *x* would be $(-3x)^5+2\times5\times(-3x)^4+2^2\times\binom{5}{3}\times(-3x)^3+\ldots$ i.e. $-243x^5+810x^4-1080x^3+\ldots$ This is a misread but award as s.c. M1B1A0A0 if completely "correct" or M1 B0A0A0 for <u>correct</u> binomial coefficient in any form with the correct power of *x*	
Alternative Method	**Method 1:** $\left[(2-3x)^5\right]=2^5(1+\binom{5}{1}(-\frac{3x}{2})+\binom{5}{2}(\frac{-3x}{2})^2+\ldots\ )$ is **M1B0A0A0 {** The M1 is for the expression **in the bracket** and as in first method– need **correct** binomial coefficient combined with correct power of *x*. Ignore bracket errors or errors (or omissions) in powers of 2 or 3 or sign or bracket errors} – answers must be simplified to $=32, -240x, +720x^2$ for full marks (awarded as before) $\left[(2-3x)^5\right]=2(1+\binom{5}{1}(-\frac{3x}{2})+\binom{5}{2}(\frac{-3x}{2})^2+\ldots\ )$ would also be awarded **M1B0A0A0** **Method 2: Multiplying out** : B1 for 32 and M1A1A1 for other terms with M1 awarded if *x* or *x*^2 term is correct. Completely correct is 4/4	

Question	Scheme	Marks
87 (a)	$(S_n =)\ a + ar + (ar^2) + \ldots \quad + ar^{n-1}$ **and** $rS_n = ar + ar^2 + (ar^3)\ldots \quad + ar^n$	M1
	$S_n - rS_n = a - ar^n$	M1
	$S_n(1-r) = a(1-r^n)$	dM1
	And so result $S_n = \dfrac{a(1-r^n)}{(1-r)}$ *	A1 (4)
(b)	Divides one term by other (either way) to give $r^2 = \ldots$ then square roots to give r = **Or:** (*Method 2)* Finds geometric mean i.e 3.24 and divides one term by 3.24 or 3.24 by one term	M1
	$r^2 = \dfrac{1.944}{5.4}$, r = 0.6 (ignore – 0.6) r = 0.6 (ignore – 0.6)	A1 (2)
(c)	Uses $5.4 \div r^2$ or $1.944 \div r^4$, to give a = a = 15	M1, A1ft (2)
(d)	Uses $S = \dfrac{15}{1-0.6}$, to obtain 37.5	M1A1 ,A1 (3)
		11 marks
Notes Special Case	**(a) M1:** Lists **both** of these sums (S_n =) may be omitted, rS_n (or rS) must be stated 1st two terms must be correct in each series. Last term must be ar^{n-1} or ar^n in first series and the corresponding ar^n or ar^{n+1} in second series. Must be n and not a number. Reference made to other terms e.g. space or dots to indicate missing terms **M1:** Subtracts series for rS from series for S (or other way round) to give RHS $= \pm(a - ar^n)$. This may have been obtained by following a pattern**. If wrong power stated on line 1 M0 here**. (Ignore LHS)M0M0M0A0 d**M1**: Factorises both sides correctly– must follow from a previous M1 (It is possible to obtain M0M1M1A0 or M1M0M1A0) **A1**: completes the proof with **no errors** seen No errors seen: **First line absolutely correct**, omission of second line, third and fourth lines correct: M1M0M1A1 See next sheet of common errors. Refer any attempts involving **sigma notation**, or any **proofs by induction** to team leader. Also attempts which **begin with the answer and work backwards**. **(b) M1:** Deduces r^2 by dividing either term by other and attempts square root **A1:** any correct equivalent for r e.g. 3/5 Answer only is 2/2 (*Method 2*) Those who find fourth term must use $\sqrt{ab}$ and **not** $\frac{1}{2}(a+b)$ then must use it in a division with given term to obtain r = **(c) M1:** May be done in two steps or more e.g. $5.4 \div r$ then divided by r again **A1ft:** follow through their value of r . Just a = 15 **with no wrong working** implies M1A1 (d) **M1**: States sum to infinity formula with values of a and r found earlier, provided $\lvert r \rvert$ <1 **A1** : uses 15 and 0.6 (or 3/5) (This is **not** a ft mark) **A1**: 37.5 or exact equivalent	
Common errors	(i) Fraction inverted in (b) $r^2 = \dfrac{5.4}{1.944}$ and $r = 1\frac{2}{3}$, then correct ft gives M1A0 M1 A1ft M0A0A0 i.e. 3/7 (ii) Uses r = 0.36: (b)M0A0 (c)M1A1ft (d) M1A0A0 i.e. 3/7 (iii) Uses ar^3 = 5.4, ar^5 = 1.944 Likely to have (b)M1A1 (c)M0A0 (d) M1A0A0 i.e.3/7	

Question number	Scheme	Marks
88 (a)	Uses $360\times\left(\frac{7}{8}\right)^{19}$, to obtain 28.5	M1, A1 (2)
(b)	Uses $S=\dfrac{360(1-(\frac{7}{8})^{20})}{1-\frac{7}{8}}$, or $S=\dfrac{360((\frac{7}{8})^{20}-1)}{\frac{7}{8}-1}$ to obtain 2680	M1, A1 (2)
(c)	Uses $S=\dfrac{360}{1-\frac{7}{8}}$, to obtain 2880	M1, A1cao (2) **6**
Notes	**(a) M1:** Correct use of formula with power = 19 **A1:** Accept 28.47, or 28.474 or indeed 28.47446075 **(b) M1:** Correct use of formula with n = 20 **A1:** Accept 2681, 2680.7, 2680.68 or 2680.679 or indeed 2680.678775 (N.B. 2680.67 or 2680.0 is **A0**) **(c) M1:** Correct use of formula **A1:** Accept 2880 only	
Alternative method	**Alternative to (a)** Gives **all 20 terms** 315, 275.6(25), 241.17(1875), … (1st 3 accurate)	M1
	All correct and last term as above **A1:** Accept 28.5, 28.47, or 28.474 or indeed 28.47446075	A1
	Alternative to (b) Gives **all 20 terms** 315, 275.6(25), 241.17(1875), … (1st 3 accurate) and adds	M1
	Sum correct **A1:** Accept 2680, 2681, 2680.7, 2680.68 or 2680.679 or indeed 2680.678775	A1

Question number	Scheme	Marks
89 (a).	$(1+\frac{x}{4})^8 = 1+2x+..,\ \ ......$	B1
	$+\frac{8\times7}{2}(\frac{x}{4})^2+\frac{8\times7\times6}{2\times3}(\frac{x}{4})^3,$	M1 A1
	$= \quad +\frac{7}{4}x^2+\frac{7}{8}x^3$ or $= \quad +1.75x^2+0.875x^3$	A1 (4)
(b)	States or implies that $x = 0.1$	B1
	Substitutes their value of x (provided it is <1) into series obtained in (a)	M1
	i.e. 1 + 0.2 + 0.0175 + 0.000875, = 1.2184	A1 cao (3) **7**
Alternative for (b) **Special case**	Starts again and expands $(1+0.025)^8$ to $1+\ 8\times0.025\ +\frac{8\times7}{2}(0.025)^2+\frac{8\times7\times6}{2\times3}(0.025)^3,\ = 1.2184$ (Or 1 + 1/5 + 7/400 + 7/8000 = 1.2184)	B1,M1,A1
Notes	**(a) B1** must be simplified The **method** mark **(M1)** is awarded for an attempt at Binomial to get the third and/or fourth term – need correct binomial coefficient combined with correct power of x. Ignore bracket errors or errors in powers of 4. Accept any notation for $^8C_2$ and $^8C_3$, e.g. $\binom{8}{2}$ and $\binom{8}{3}$ (unsimplified) or 28 and 56 from Pascal's triangle. (The terms may be listed without + signs) First **A1** is for two completely correct unsimplified terms **A1** needs the fully simplified $\frac{7}{4}x^2$ *and* $\frac{7}{8}x^3$. **(b) B1 –** states or uses x =0.1 or $\frac{x}{4}=\frac{1}{40}$ **M1** for substituting their value of x (0 < x <1) into expansion (e.g. 0.1 (correct) or 0.01, 0.00625 or even 0.025 but **not** 1 nor 1.025 which would earn **M0**) **A1** Should be answer printed cao (not answers which round to) and should follow correct work. **Answer with no working at all** is B0, M0, A0 States 0.1 then just writes down answer is B1 M0A0	

<table>
<tr><th>Question Number</th><th>Scheme</th><th></th><th>Marks</th></tr>
<tr><td>90.
(a)</td><td>$\{(3+bx)^5\} = (3)^5 + \underline{{}^5C_1(3)^4(bx)} + \underline{{}^5C_2(3)^3(bx)^2} + \ldots$
$= 243 + 405bx + 270b^2x^2 + \ldots$</td><td>243 as a constant term seen.
$405bx$
$\left({}^5C_1 \times \ldots \times x\right)$ or $\left({}^5C_2 \times \ldots \times x^2\right)$
$270b^2x^2$ or $270(bx)^2$</td><td>B1
B1
<u>M1</u>
A1 **[4]**</td></tr>
<tr><td>(b)</td><td>$\{2(\text{coeff } x) = \text{coeff } x^2\} \Rightarrow 2(405b) = 270b^2$
So, $\left\{b = \frac{810}{270} \Rightarrow\right\} b = 3$</td><td>Establishes an equation from their coefficients. Condone 2 on the wrong side of the equation.
$b = 3$ (Ignore $b = 0$, if seen.)</td><td>M1
A1
[2]
6</td></tr>
<tr><td>(a)

(b)</td><td colspan="3">The terms can be “listed” rather than added. Ignore any extra terms.
1st B1: A constant term of 243 seen. Just writing $(3)^5$ is B0.
2nd B1: Term must be simplified to $405bx$ for B1. The x is required for this mark. Note $405 + bx$ is B0.
M1: For <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 3 and/or b) may be wrong or missing.
<u>Allow</u> binomial coefficients such as $\binom{5}{2}, \left(\frac{5}{2}\right), \binom{5}{1}, \left(\frac{5}{1}\right), {}^5C_2, {}^5C_1$.
A1: For either $270b^2x^2$ or $270(bx)^2$. (If $270bx^2$ follows $270(bx)^2$, isw and allow A1.)
<u>Alternative:</u>
Note that a factor of 3^5 can be taken out first: $3^5\left(1+\frac{bx}{3}\right)^5$, but the mark scheme still applies.
<u>Ignore subsequent working (isw)</u>: Isw if necessary after correct working:
e.g. $243 + 405bx + 270b^2x^2 + \ldots$ leading to $9 + 15bx + 10b^2x^2 + \ldots$ scores B1B1M1A1 isw.
Also note that full marks could also be available in part (b), here.
<u>Special Case</u>: Candidate writing down the first three terms in ***descending*** powers of x usually get $(bx)^5 + {}^5C_4(3)^1(bx)^4 + {}^5C_3(3)^2(bx)^3 + \ldots = b^5x^5 + 15b^4x^4 + 90b^3x^3 + \ldots$
So award SC: B0B0M1A0 for either $\left({}^5C_4 \times \ldots \times x^4\right)$ or $\left({}^5C_3 \times \ldots \times x^3\right)$
M1 for equating 2 times their coefficient of x to the coefficient of x^2 to get an equation in b,
<u>or</u> equating their coefficient of x to 2 times that of x^2, to get an equation in b.
Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, eg: $2(405b) = 270b$, but beware $b = 3$ from this, which is A0.
<u>An equation in b alone</u> is required:
e.g. $2(405b)x = 270b^2x^2 \Rightarrow b = 3$ or similar will be **Special Case** SC: M1A0 (as equation in coefficients only is not seen here).
e.g. $2(405b)x = 270b^2x^2 \Rightarrow 2(405b) = 270b^2 \Rightarrow b = 3$ will get M1A1 (as coefficients rather than terms have now been considered).
Note: Answer of 3 from no working scores M1A0.
Note: The mistake $k\left(1+\frac{bx}{3}\right)^5, k \neq 243$ would give a maximum of 3 marks: B0B0M1A0, M1A1
Note: For $270bx^2$ in part (a), followed by $2(405b) = 270b^2 \Rightarrow b = 3$, in part (b), allow recovery M1A1.</td></tr>
</table>

Question Number	Scheme		Marks
91. (a)	$\{ ar = 192$ and $ar^2 = 144 \}$		
	$r = \dfrac{144}{192}$	Attempt to eliminate a. (See notes.)	M1
	$r = \frac{3}{4}$ or 0.75	$\frac{3}{4}$ or 0.75	A1
			[2]
(b)	$a(0.75) = 192$		M1
	$a\left\{= \dfrac{192}{0.75}\right\} = 256$	256	A1
			[2]
(c)	$S_\infty = \dfrac{256}{1-0.75}$	Applies $\dfrac{a}{1-r}$ correctly using both their a **and** their $\lvert r \rvert < 1$.	M1
	So, $\{S_\infty =\}$ 1024	1024	A1 cao
			[2]
(d)	$\dfrac{256(1-(0.75)^n)}{1-0.75} > 1000$	Applies S_n with their a and r and "uses" 1000 at any point in their working. (Allow with = or <).	M1
	$(0.75)^n < 1 - \dfrac{1000(0.25)}{256} \left\{= \dfrac{6}{256}\right\}$	Attempt to isolate $+(r)^n$ from S_n formula. (Allow with = or >).	M1
	$n\log(0.75) < \log\left(\dfrac{6}{256}\right)$	Uses the power law of logarithms correctly. (Allow with = or >). (See notes.)	M1
	$n > \dfrac{\log\left(\frac{6}{256}\right)}{\log(0.75)} = 13.0471042... \Rightarrow n = 14$	See notes **and** $n = 14$	A1 **cso**
			[4] **10**
(a)	M1: **for eliminating *a*** by eg. $192r = 144$ or by either dividing $ar^2 = 144$ by $ar = 192$ or dividing $ar = 192$ by $ar^2 = 144$, to achieve an equation in r or $\dfrac{1}{r}$ Note that $r^2 - r = \dfrac{144}{192}$ is M0. Note also that any of $r = \dfrac{144}{192}$ or $r = \dfrac{192}{144}\left\{= \dfrac{4}{3}\right\}$ or $\dfrac{1}{r} = \dfrac{192}{144}$ or $\dfrac{1}{r} = \dfrac{144}{192}$ are fine for the award of M1. **Note:** A candidate just writing $r = \dfrac{144}{192}$ with no reference to a can also get the method mark. **Note**: $ar^2 = 192$ and $ar^3 = 144$ leading to $r = \frac{3}{4}$ scores M1A1. This is because r is the ratio between any two consecutive terms. These candidates, however, will usually be penalised in part (b).		
(b)	M1 for inserting their r into either of the correct equations of either $ar = 192$ or $\{a =\} \dfrac{192}{r}$ or $ar^2 = 144$ or $\{a =\} \dfrac{144}{r^2}$. No slips allowed here for M1. M1: can also be awarded for writing down $144 = a\left(\dfrac{192}{a}\right)^2$ A1 for $a = 256$ only. Note 256 from any working scores M1A1. **Note**: Some candidates incorrectly confuse notation to give $r = \frac{4}{3}$ or 1.33 in part (a) (getting M1A0). In part (b), they recover to write $a = 192 \times \frac{4}{3}$ for M1 and then 256 for A1.		

Question Number	Scheme	Marks
(c)	M1: for applying $\frac{a}{1-r}$ correctly (no slips allowed!) using both their *a* **and** their *r*, where $\|r\| < 1$. A1: for 1024, cao. In parts (a) or (b) or (c), the correct answer with no working scores full marks.	
(d)	1st M1: For applying S_n with their *a* and either "the letter *r*" or their *r* and "uses" 1000. 2nd M1: For isolating $+(r)^n$ and not $(ar)^n$, (eg. $(192)^n$) as the subject of an equation or inequality. $+(r)^n$ must be derived from the S_n formula. 3rd M1: For applying the power law to $\lambda^k = \mu$ to give $k\log\lambda = \log\mu$ oe. where $\lambda, \mu > 0$. or 3rd M1: For solving $\lambda^k = \mu$ to give $k = \log_\lambda \mu$, where $\lambda, \mu > 0$. A1: cso If a candidate uses inequalities, a fully correct method with inequalities is required here. So, an <u>incorrect</u> inequality statement at any stage in a candidate's working for this part loses this mark. **Note:** Some candidates do not realise that the direction of the inequality is reversed in the final line of their solution. Or A1: cso Note a candidate can achieve full marks here if they do not use inequalities. So, if a candidate uses equations rather than inequalities in their working then they need to state in the final line of their working that $n = 13.04$ (truncated) or $n =$ awrt $13.05 \Rightarrow n = 14$ for A1. $n = 14$ from no working gets SC: M0M0M1A1. A method of $T_n > 1000 \Rightarrow 256(0.75)^{n-1} > 1000$ can score M0M0M1A0 for a correct application of the power law of logarithms. **<u>Trial & Improvement Method:</u>** For $a = 256$ and $r = 0.75$, apply the following scheme:	
	$S_{13} = \frac{256(1-(0.75)^{13})}{1-0.75} = 999.6725616...$ — Attempt to find either S_{13} or S_{14}.	M1
	EITHER (1) $S_{13} =$ awrt 999.7 or truncated 999 OR (2) $S_{14} =$ awrt 1005.8 or truncated 1005.	M1
	$S_{14} = \frac{256(1-(0.75)^{14})}{1-0.75} = 1005.754421...$ — Attempt to find both S_{13} and S_{14}.	M1
	So, $n = 14$. — BOTH (1) $S_{13} =$ awrt 999.7 or truncated 999 AND (2) $S_{14} =$ awrt 1005.8 or truncated 1005 AND $n = 14$.	A1

Question Number	Scheme	Marks
	Note: A similar scheme would apply for T&I for candidates using their *a* and their *r*. So,... 1st M1: For attempting to find one of the correct S_n's either side (but next to) 1000. 2nd M1: For one of these S_n's correct for their *a* and their *r*. (You may need to get your calculators out!) 3rd M1: For attempting to find both of the correct S_n's either side (but next to) 1000. A1: Cannot be gained for wrong *a* and/or *r*. **Trial & Improvement Cumulative Approach:** A similar scheme to T&I will be applied here: 1st M1: For getting as far as the cumulative sum of 13 terms. 2nd M1: (1) S_{13} = awrt 999.7 or truncated 999. 3rd M1: For getting as far as the cumulative sum to 14 terms. Also at this stage $S_{13} < 1000$ and $S_{14} > 1000$. A1: BOTH (1) S_{13} = awrt 999.7 or truncated 999 AND (2) S_{14} = awrt 1005.8 or truncated 1005 AND $n = 14$. **Trial & Improvement Method:** for $(0.75)^n < \frac{6}{256} = 0.0234375$ 3rd M1: For evidence of examining both $n = 13$ and $n = 14$. Eg: $(0.75)^{13}\{= 0.023757...\}$ and $(0.75)^{14}\{= 0.0178179...\}$ A1: $n = 14$ **Any misreads,** $S_n > 10000$ etc, please escalate up to your Team Leader.	

Question Number	Scheme	Marks
92.		
(a)	$ar = 750$ and $ar^4 = -6$ (could be implied from later working in either (a) or (b)).	B1
	$r^3 = \frac{-6}{750}$	M1
	$r = -\frac{1}{5}$ Correct answer from no working, except for special case below gains all three marks.	A1
		(3)
(b)	$a(-0.2) = 750$	M1
	$a\left\{= \frac{750}{-0.2}\right\} = -3750$	A1 ft
		(2)
(c)	Applies $\frac{a}{1-r}$ correctly using both their a and their $\lvert r\rvert < 1$. Eg. $\frac{-3750}{1--0.2}$	M1
	So, $S_\infty = -3125$	A1
		(2) **[7]**
	Notes	
(a)	B1: for both $ar = 750$ and $ar^4 = -6$ (may be implied from later working in either (a) or (b)). M1: **for eliminating *a*** by either dividing $ar^4 = -6$ by $ar = 750$ or dividing $ar = 750$ by $ar^4 = -6$, to achieve an equation in r^3 or $\frac{1}{r^3}$ Note that $r^4 - r = -\frac{6}{750}$ is M0. Note also that any of $r^3 = \frac{-6}{750}$ or $r^3 = \frac{750}{-6}\{= -125\}$ or $\frac{1}{r^3} = \frac{-6}{750}$ or $\frac{1}{r^3} = \frac{750}{-6}\{= -125\}$ are fine for the award of M1. SC: $ar^\alpha = 750$ and $ar^\beta = -6$ leading to $r^\delta = \frac{-6}{750}$ or $r^\delta = \frac{750}{-6}\{= -125\}$ or $\frac{1}{r^\delta} = \frac{-6}{750}$ or $\frac{1}{r^\delta} = \frac{750}{-6}\{= -125\}$ where $\delta = \beta - \alpha$ and $\delta \geq 2$ are fine for the award of M1. SC: $ar^2 = 750$ and $ar^5 = -6$ leading to $r = -\frac{1}{5}$ scores B0M1A1.	
(b)	M1 for inserting their r into either of their original correct equations of either $ar = 750$ or $\{a =\}\frac{750}{r}$ or $ar^4 = -6$ or $\{a =\}\frac{-6}{r^4}$ – in ***both a* and *r***. No slips allowed here for M1. A1 for either $a = -3750$ or a equal to the correct follow through result expressed either as an exact integer, or a fraction in the form $\frac{c}{d}$ where both c and d are integers, or correct to awrt 1 dp.	
(c)	M1 for applying $\frac{a}{1-r}$ correctly (only a slip in substituting r is allowed) using both their a and their $\lvert r\rvert < 1$. Eg. $\frac{-3750}{1--0.2}$. A1 for -3125 In parts (a) or (b) or (c), the correct answer with no working scores full marks.	

Question Number	Scheme		Marks
93.			
(a)	$\binom{40}{4} = \frac{40!}{4!b!}$; $(1+x)^n$ coefficients of x^4 and x^5 are p and q respectively. $b = 36$ Candidates should usually "identify" two terms as their p and q respectively.		B1 (1)
(b)	Term 1: $\binom{40}{4}$ or ${}^{40}C_4$ or $\frac{40!}{4!36!}$ or $\frac{40(39)(38)(37)}{4!}$ or 91390 Term 2: $\binom{40}{5}$ or ${}^{40}C_5$ or $\frac{40!}{5!35!}$ or $\frac{40(39)(38)(37)(36)}{5!}$ or 658008	Any one of Term 1 or Term 2 correct. (Ignore the label of p and/or q.) Both of them correct. (Ignore the label of p and/or q.)	M1 A1
	Hence, $\frac{q}{p} = \frac{658008}{91390}$ $\left\{= \frac{36}{5} = 7.2\right\}$	for $\frac{658008}{91390}$ oe	A1 oe **cso** (3) **[4]**
	Notes		
(a)	B1: for only $b = 36$.		
(b)	The candidate may expand out their binomial series. At this stage no marks should be awarded until they start to identify either one or both of the terms that they want to focus on. Once they identify their terms then if one out of two of them (ignoring which one is p and which one is q) is correct then award M1. If both of the terms are identified correctly (ignoring which one is p and which one is q) then award the first A1. Term 1 $= \binom{40}{4}x^4$ or ${}^{40}C_4(x^4)$ or $\frac{40!}{4!36!}x^4$ or $\frac{40(39)(38)(37)}{4!}x^4$ or $91390x^4$, Term 2 $= \binom{40}{5}x^5$ or ${}^{40}C_5(x^5)$ or $\frac{40!}{5!35!}x^5$ or $\frac{40(39)(38)(37)(36)}{5!}x^5$ or $658008x^5$ are fine for any (or both) of the first two marks in part (b). 2nd A1 for stating $\frac{q}{p}$ as $\frac{658008}{91390}$ or equivalent. Note that $\frac{q}{p}$ must be independent of x. Also note that $\frac{36}{5}$ or 7.2 or any equivalent fraction is fine for the 2nd A1 mark. SC: If candidate states $\frac{p}{q} = \frac{5}{36}$, then award M1A1A0. Note that either $\frac{4!36!}{5!35!}$ or $\frac{5!35!}{4!36!}$ would be awarded M1A1.		

EXPERT TUITION

Question Number	Scheme	Marks
94	(a) $(1+ax)^7 = 1+7ax....$ or $1+7(ax)....$ (<u>Not</u> unsimplified versions)	B1
	$+\frac{7\times 6}{2}(ax)^2 + \frac{7\times 6\times 5}{6}(ax)^3$ Evidence from <u>one</u> of these terms is enough	M1
	$+21a^2x^2$ or $+21(ax)^2$ or $+21(a^2x^2)$	A1
	$+35a^3x^3$ or $+35(ax)^3$ or $+35(a^3x^3)$	A1 (4)
	(b) $21a^2 = 525$	M1
	$a = \pm 5$ (Both values are required) (The answer $a = 5$ with no working scores M1 A0)	A1 (2) **6**
	(a) The terms can be 'listed' rather than added. M1: Requires correct structure: a correct binomial coefficient in any form (perhaps from Pascal's triangle) with the correct power of x. Allow missing a's and wrong powers of a, e.g. $\frac{7\times 6}{2}ax^2$, $\frac{7\times 6\times 5}{3\times 2}x^3$ However, $21+a^2x^2+35+a^3x^3$ or similar is M0. $1+7ax+21+a^2x^2+35+a^3x^3 = 57+.....$ scores the B1 (isw). $\binom{7}{2}$ and $\binom{7}{3}$ or equivalent such as ${}^7C_2$ and ${}^7C_3$ are acceptable, but <u>not</u> $\left(\frac{7}{2}\right)$ or $\left(\frac{7}{3}\right)$ (unless subsequently corrected). 1st A1: Correct x^2 term. 2nd A1: Correct x^3 term (The binomial coefficients <u>must</u> be simplified). <u>Special case</u>: If $(ax)^2$ and $(ax)^3$ are seen within the working, but then lost… … A1 A0 can be given if $21ax^2$ and $35ax^3$ are <u>both</u> achieved. <u>a's omitted throughout</u>: Note that only the M mark is available in this case. (b) M: Equating their coefficent of x^2 to 525. An equation in a or a^2 alone is required for this M mark, but allow 'recovery' that shows <u>the required coefficient</u>, e.g. $21a^2x^2 = 525 \Rightarrow 21a^2 = 525$ is acceptable, but $21a^2x^2 = 525 \Rightarrow a^2 = 25$ is not acceptable. After $21ax^2$ in the answer for (a), allow 'recovery' of a^2 in (b) so that full marks are available for (b) (but not retrospectively for (a)).	

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>95 (a)

(b)

(c)

(d)</td><td>$18000\times(0.8)^3 \quad = £9216$ * [may see $\frac{4}{5}$ or 80% or equivalent].

$18000\times(0.8)^n < 1000$
$n\log(0.8) < \log\left(\frac{1}{18}\right)$
$n > \frac{\log\left(\frac{1}{18}\right)}{\log(0.8)} = 12.952....$ so $n = 13$.

$u_5 = 200\times(1.12)^4, \quad = £314.70$ or $£314.71$

$S_{15} = \frac{200(1.12^{15}-1)}{1.12-1}$ or $\frac{200(1-1.12^{15})}{1-1.12}$, = 7455.94….. awrt £7460</td><td>B1cso (1)

M1
M1
A1 cso (3)

M1, A1 (2)

M1A1, A1
(3)
[9]</td></tr>
<tr><td>(a)

(b)

(c)

(d)</td><td>B1 NB Answer is printed so need working. May see as above or ×0.8 in three steps giving 14400, 11520, 9216. Do not need to see £ sign but should see 9216 .

1st M1 for an attempt to use nth term and 1000. Allow n or n – 1 and allow > or =
2nd M1 for use of logs to find n Allow n or n – 1 and allow > or =
A1 Need n = 13 This is an accuracy mark and must follow award of both M marks but should not follow incorrect work using n – 1 for example.
Condone slips in inequality signs here.

M1 for use of their a and r in formula for 5th term of GP
A1 cao need one of these answers – answer can imply method here
NB 314.7 – A0

M1 for use of sum to 15 terms of GP using their a and their r (allow if formula stated correctly and one error in substitution, but must use n not n - 1)
1st A1 for a fully correct expression (not evaluated)</td><td></td></tr>
<tr><td>
(b)

(c)

(d)</td><td>Alternative Methods
Trial and Improvement
See 989.56 (or 989 or 990) identified with 12, 13 or 14 years for first M1
See 1236.95 (or 1236 or 1237) identified with 11, 12 or 13 years for second M1
Then n = 13 is A1 (needs both Ms)
Special case $18000\times(0.8)^n < 1000$ so n = 13 as 989.56<1000 is M1M0A0 (not discounted n = 12)

May see the terms 224, 250.88, 280.99, 314.71 with a small slip for M1 A0, or done accurately for M1A1

Adds 15 terms 200 + 224 + 250.88+… + (977.42) M1
Seeing 977… is A1
Obtains answer 7455.94 A1 or awrt £7460 NOT 7450</td><td></td></tr>
</table>

EXPERT TUITION

Question Number	Scheme	Marks
96 (a)	$(7\times\ldots\times x)$ or $(21\times\ldots\times x^2)$ The 7 or 21 can be in 'unsimplified' form.	M1
	$(2+kx)^7 = 2^7 + 2^6\times 7\times kx + 2^5\times\binom{7}{2}k^2x^2$	
	$= 128;\quad +448kx,\quad +672k^2x^2$ [or $672(kx)^2$]	B1; A1, A1
	(If $672kx^2$ follows $672(kx)^2$, isw and allow A1)	(4)
(b)	$6\times 448k = 672k^2$	M1
	$k = 4$ (Ignore $k = 0$, if seen)	A1 (2) **[6]**
(a)	The terms can be 'listed' rather than added. Ignore any extra terms. M1 for <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or k) may be wrong or missing. <u>Allow</u> binomial coefficients such as $\binom{7}{1}, \left(\frac{7}{1}\right), \binom{7}{2}, {}^7C_1, {}^7C_2$. However, $448+kx$ or similar is M0. B1, A1, A1 for the <u>simplified</u> versions seen above. <u>Alternative</u>: Note that a factor 2^7 can be taken out first: $2^7\left(1+\frac{kx}{2}\right)^7$, but the mark scheme still applies. <u>Ignoring subsequent working (isw)</u>: Isw if necessary after correct working: e.g. $128+448kx+672k^2x^2$ M1 B1 A1 A1 $= 4+14kx+21k^2x^2$ isw (Full marks are still available in part (b)).	
(b)	M1 for equating their coefficient of x^2 to 6 times that of x… to get an equation in k, … <u>or</u> equating their coefficient of x to 6 times that of x^2, to get an equation in k. Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, e.g. $6\times 448k = 672k$, but beware $k = 4$ following from this, which is A0. <u>An equation in k alone</u> is required for this M mark, so... e.g. $6\times 448kx = 672k^2x^2 \Rightarrow k = 4$ or similar is M0 A0 (equation in coefficients only is never seen), but ... e.g. $6\times 448kx = 672k^2x^2 \Rightarrow 6\times 448k = 672k^2 \Rightarrow k = 4$ will get M1 A1 (as coefficients rather than terms have now been considered). The mistake $2\left(1+\frac{kx}{2}\right)^7$ would give a maximum of 3 marks: M1B0A0A0, M1A1	

Question Number	Scheme	Marks
97 (a)	$324r^3 = 96$ or $r^3 = \frac{96}{324}$ or $r^3 = \frac{8}{27}$	M1
	$r = \frac{2}{3}$ (*)	A1cso (2)
(b)	$a\left(\frac{2}{3}\right)^2 = 324$ or $a\left(\frac{2}{3}\right)^5 = 96$ $a = \ldots,$ 729	M1, A1 (2)
(c)	$S_{15} = \frac{729\left(1-\left[\frac{2}{3}\right]^{15}\right)}{1-\frac{2}{3}},$ $= 2182.00\ldots$ (AWRT 2180)	M1A1ft, (3)
(d)	$S_\infty = \frac{729}{1-\frac{2}{3}},$ $= 2187$	M1, A1 (2)
		[9]
(a)	M1 for forming an equation for r^3 based on 96 and 324 (e.g. $96r^3 = 324$ scores M1). The equation must involve multiplication/division rather than addition/subtraction. A1 Do not penalise solutions with working in decimals, providing these are correctly rounded or truncated to at least 2dp <u>and</u> the final answer 2/3 is seen. <u>Alternative</u>: (verification) M1 Using $r^3 = \frac{8}{27}$ and multiplying 324 by this (or multiplying by $r = \frac{2}{3}$ three times). A1 Obtaining 96 (cso). (A conclusion is not required). $324 \times \left(\frac{2}{3}\right)^3 = 96$ (no real evidence of calculation) is not quite enough and scores M1 A0.	
(b)	M1 for the use of a correct formula or for 'working back' by dividing by $\frac{2}{3}$ (or by their r) twice from 324 (or 5 times from 96). Exceptionally, allow M1 also for using $ar^3 = 324$ or $ar^6 = 96$ instead of $ar^2 = 324$ or $ar^5 = 96$, or for dividing by r three times from 324 (or 6 times from 96)… but no other exceptions are allowed.	
(c)	M1 for use of sum to 15 terms formula with values of a and r. If the wrong power is used, e.g. 14, the M mark is scored only if the correct sum formula is stated. 1st A1ft for a correct expression or correct ft their a with $r = \frac{2}{3}$. 2nd A1 for awrt 2180, even following 'minor inaccuracies'. Condone missing brackets round the $\frac{2}{3}$ for the marks in part (c). <u>Alternative</u>: M1 for adding 15 terms and 1st A1ft for adding the 15 terms that ft from their a and $r = \frac{2}{3}$.	
(d)	M1 for use of correct sum to infinity formula with their a. For this mark, if a value of r different from the given value is being used, M1 can still be allowed providing $\|r\| < 1$.	

Question Number	Scheme	Marks
98	$(3-2x)^5=243$, $\quad \ldots\ldots+5\times(3)^4(-2x)=-810x \quad \ldots\ldots$ $+\frac{5\times4}{2}(3)^3(-2x)^2= \qquad +1080x^2$	B1, B1 M1 A1 (4) **[4]**
Notes	First term must be 243 for **B1**, writing just 3^5 is B0 (Mark their final answers except in second line of special cases below). Term must be simplified to –810*x* for **B1** The *x* is required for this mark**.** The **method** mark **(M1) is generous** and is awarded for an attempt at Binomial to get the third term. There must be an x^2 (or no *x*- i.e. not wrong power) and attempt at Binomial Coefficient and at dealing with powers of 3 and 2. The power of 3 should not be one, but the power of 2 may be one (regarded as bracketing slip). So allow $\binom{5}{2}$ or $\binom{5}{3}$ or ${}^5C_2$ or ${}^5C_3$ or even $\left(\frac{5}{2}\right)$ or $\left(\frac{5}{3}\right)$ or use of '10' (maybe from Pascal's triangle) May see ${}^5C_2(3)^3(-2x)^2$ or ${}^5C_2(3)^3(-2x^2)$ or ${}^5C_2(3)^5(-\frac{2}{3}x^2)$ or $10(3)^3(2x)^2$ which would each score the **M1** **A1**is c.a.o and needs $1080x^2$ (if $1080x^2$ is written with no working this is awarded both marks i.e. **M1 A1.)**	
Special cases	$243+810x+1080x^2$ is **B1B0M1A1** (condone no negative signs) Follows correct answer with $27-90x+120x^2$ can isw here (sp case)– full marks for correct answer Misreads *ascending* and gives $-32x^5+240x^4-720x^3$ is marked as **B1B0M1A0** special case and must be completely correct. *(If any slips could get B0B0M1A0)* Ignores 3 and expands $(1\pm2x)^5$ is **0/4** 243, -810*x*, 1080x^2 is full marks but 243, -810, 1080 is **B1,B0,M1,A0** NB Alternative method $3^5(1-\frac{2}{3}x)^5=3^5-5\times3^5\times\left(\frac{2}{3}x\right)+\binom{5}{3}3^5\left(-\frac{2}{3}x\right)^2+..$ is **B0B0M1A0** – answers must be simplified to 243 –810*x* +1080x^2 for full marks (awarded as before) Special case $3(1-\frac{2}{3}x)^5=3-5\times3\times\left(\frac{2}{3}x\right)+\binom{5}{3}3\left(-\frac{2}{3}x\right)^2+..$ is **B0, B0, M1, A0** Or $\quad 3(1-2x)^5$ is **B0B0M0A0**	

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>99
(a)

(b)

(c)

(d)</td><td>Initial step: Two of: $a = k+4,\ ar = k,\ ar^2 = 2k-15$
Or one of: $r = \frac{k}{k+4},\quad r = \frac{2k-15}{k},\quad r^2 = \frac{2k-15}{k+4}$,
Or $k = \sqrt{(k+4)(2k-15)}$ or even $k^3 = (k+4)k(2k-15)$
$k^2 = (k+4)(2k-15)$, so $k^2 = 2k^2 + 8k - 15k - 60$
Proceed to $k^2 - 7k - 60 = 0$ (*)

$(k-12)(k+5) = 0 \qquad k = 12$ (*)

Common ratio: $\frac{k}{k+4}$ or $\frac{2k\text{-}15}{k} = \frac{12}{16}\left(= \frac{3}{4} \text{ or } 0.75\right)$

$\frac{a}{1-r} = \frac{16}{\left(\frac{1}{4}\right)} = 64$</td><td>

M1

M1, A1
A1 (4)

M1 A1 (2)

M1 A1 (2)

M1 A1 (2)
[10]</td></tr>
<tr><td>(a)

(b)

(c)

(d)</td><td>M1: The ‘initial step’, scoring the first M mark, may be implied by next line of proof
M1: Eliminates a and r to give valid equation in k only. Can be awarded for equation involving fractions.
A1 : need some correct expansion and working and answer equivalent to required quadratic but with uncollected terms. Equations involving fractions do not get this mark. (No fractions, no brackets – could be a cubic equation)
A1: as answer is printed this mark is for cso (Needs = 0)
All four marks must be scored in part (a)
M1: Attempt to solve quadratic
A1: This is for correct factorisation or solution and $k = 12$. Ignore the extra solution ($k = -5$ or even $k = 5$), if seen.
Substitute and verify is M1 A0
Marks must be scored in part (b)
M1: Complete method to find r Could have answer in terms of k
A1: 0.75 or any correct equivalent
Both Marks must be scored in (c)
M1: Tries to use $\frac{a}{1-r}$, (even with $r>1$). Could have an answer still in terms of k.
A1: This answer is 64 cao.</td><td></td></tr>
</table>

Question number	Scheme	Marks
100.	(a) $(1+ax)^{10} = 1+10ax......$ (<u>Not</u> unsimplified versions)	B1
	$+\frac{10\times 9}{2}(ax)^2 + \frac{10\times 9\times 8}{6}(ax)^3$ Evidence from <u>one</u> of these terms is sufficient	M1
	$+45(ax)^2, +120(ax)^3$ or $+45a^2x^2, +120a^3x^3$	A1, A1 (4)
	(b) $120a^3 = 2\times 45a^2$ $a=\frac{3}{4}$ or equiv. $\left(\text{e.g.} \frac{90}{120}, 0.75\right)$ Ignore $a = 0$, if seen	M1 A1 (2)
		6
	(a) The terms can be 'listed' rather than added. M1: Requires correct structure: 'binomial coefficient' (perhaps from Pascal's triangle) and the correct power of x. (The M mark can also be given for an expansion in <u>descending</u> powers of x). Allow 'slips' such as: $\frac{10\times 9}{2}ax^2$, $\frac{10\times 9}{3\times 2}(ax)^3$, $\frac{10\times 9}{2}x^2$, $\frac{9\times 8\times 7}{3\times 2}a^3x^3$ However, $45+a^2x^2+120+a^3x^3$ or similar is M0. $\binom{10}{2}$ and $\binom{10}{3}$ or equivalent such as ${}^{10}C_2$ and ${}^{10}C_3$ are acceptable, and even $\left(\frac{10}{2}\right)$ and $\left(\frac{10}{3}\right)$ are acceptable for the method mark. 1st A1: Correct x^2 term. 2nd A1: Correct x^3 term (These <u>must</u> be simplified). If simplification is not seen in (a), but correct simplified terms are seen in (b), these marks can be awarded. However, if <u>wrong</u> simplification is seen in (a), this takes precedence. <u>Special case</u>: If $(ax)^2$ and $(ax)^3$ are seen within the working, but then lost… … A1 A0 can be given if $45ax^2$ and $120ax^3$ are <u>both</u> achieved. (b) M: Equating their coefficent of x^3 to twice their coefficient of x^2 … … <u>or</u> equating their coefficent of x^2 to twice their coefficient of x^3. (… or coefficients can be <u>correct</u> coefficients rather than their coefficients). Allow this mark even if the equation is trivial, e.g. $120a = 90a$. An equation in a alone is required for this M mark, although… …condone, e.g. $120a^3x^3 = 90a^2x^2 \Rightarrow (120a^3 = 90a^2 \Rightarrow)\ a=\frac{3}{4}$. <u>Beware</u>: $a=\frac{3}{4}$ following $120a = 90a$, which is A0.	

Question number	Scheme	Marks	
101.	(a) $T_{20} = 5 \times \left(\frac{4}{5}\right)^{19} = 0.072$ (Accept awrt) Allow $5 \times \frac{4}{5}^{19}$ for M1	M1 A1	(2)
	(b) $S_\infty = \frac{5}{1-0.8} = 25$	M1 A1	(2)
	(c) $\frac{5(1-0.8^k)}{1-0.8} > 24.95$ (Allow with = or <)	M1	
	$1-0.8^k > 0.998$ (or equiv., see below) (Allow with = or <)	A1	
	$k \log 0.8 < \log 0.002$ or $k > \log_{0.8} 0.002$ (Allow with = or <)	M1	
	$k > \frac{\log 0.002}{\log 0.8}$ (*)	A1cso	(4)
	(d) $k = 28$ (Must be this integer value) Not $k > 27$, or $k < 28$, or $k > 28$	B1	(1)
			9
	(a) and (b): Correct answer without working scores both marks. (a) M: Requires use of the correct formula ar^{n-1}. (b) M: Requires use of the correct formula $\frac{a}{1-r}$ (c) 1st M: The sum may have already been 'manipulated' (perhaps wrongly), but this mark can still be allowed. 1st A: A 'numerically correct' version that has dealt with $(1-0.8)$ denominator, e.g. $1-\left(\frac{4}{5}\right)^k > 0.998$, $5(1-0.8^k) > 4.99$, $25(1-0.8^k) > 24.95$, $5-5(0.8^k) > 4.99$. In any of these, $\frac{4}{5}$ instead of 0.8 is fine, and condone $\frac{4}{5}^k$ if correctly treated later. 2nd M: Introducing logs and using laws of logs correctly (this must include dealing with the power k so that $p^k = k \log p$). 2nd A: An <u>incorrect</u> statement (including equalities) at any stage in the working loses this mark (this is often identifiable at the stage $k \log 0.8 > \log 0.002$). (So a fully correct method with inequalities is required.)		

Question Number	Scheme	Marks
102.(a)	Complete method, using terms of form ar^k, to find r [e.g. **Dividing** ar^6 = 80 by ar^3 = 10 to find r ; $r^6 - r^3 = 8$ is M0] r = 2	M1 A1 (2)
(b)	Complete method for finding a [e.g. Substituting value for r into equation of form ar^k = 10 or 80 and finding a value for a.]	M1
	(8a = 10) $a = \frac{5}{4} = 1\frac{1}{4}$ (equivalent single fraction or 1.25)	A1 (2)
(c)	Substituting their values of a and r into **correct** formula for sum. $S = \frac{a(r^n - 1)}{r - 1} = \frac{5}{4}(2^{20} - 1)$ (= 1310718.75) 1 310 719 (only this)	M1 A1 (2) **[6]**
Notes:	(a) M1: Condone errors in powers, e.g. ar^4 = 10 and/or ar^7 = 80, A1: For r = 2, allow even if ar^4 = 10 and ar^7 = 80 used (just these) (M mark can be implied from numerical work, if used correctly) (b) M1: Allow for numerical approach: e.g. $\frac{10}{r_c^{\,3}} \leftarrow \frac{10}{r_c^{\,2}} \leftarrow \frac{10}{r_c} \leftarrow 10$ In (a) and (b) correct answer, with no working, allow both marks. (c) Attempt 20 terms of series and add is M1 (correct last term 655360) If formula **not** quoted, errors in applying their a and/or r is M0 Allow full marks for correct answer with no working seen.	
103.(a)	$\left(1+\frac{1}{2}x\right)^{10}$ **=** 1 + $\underline{\binom{10}{1}\left(\frac{1}{2}x\right) + \binom{10}{2}\left(\frac{1}{2}x\right)^2 + \binom{10}{3}\left(\frac{1}{2}x\right)^3}$	M1 A1
	= 1 + 5x ; + $\frac{45}{4}$ (or 11.25)x^2 + $15x^3$ (coeffs need to be these, i.e, simplified) [Allow A1A0, if totally correct with unsimplified, single fraction coefficients)	A1; A1 (4)
(b)	$(1 + \frac{1}{2} \times 0.01)^{10}$ = 1 + 5(0.01) + $(\frac{45}{4}\ or\ 11.25)(0.01)^2$ + $15(0.01)^3$ = 1 + 0.05 + 0.001125 + 0.000015 = 1.05114 cao	M1 A1√ A1 (3) **[7]**
Notes:	(a) For M1 first A1: Consider underlined expression only. M1 Requires correct structure for at least two of the three terms: (i) Must be attempt at binomial coefficients. (ii) Must have increasing powers of x , (iii) May be listed, need not be added; *this applies for all marks*. First A1: Requires all three correct terms but need not be simplified, allow 1^{10} etc, ${}^{10}C_2$ etc, and condone omission of brackets around powers of ½x Second A1: Consider as B1 for 1 + 5x (b) For M1: Substituting their (0.01) into their (a) result First A1 (f.t.): Substitution of (0.01) into their 4 termed expression in (a) Answer with no working scores no marks (calculator gives this answer)	

EXPERT TUITION

Question Number	Scheme	Notes	Marks
104. (a)	$\sqrt{(4-9x)} = (4-9x)^{\frac{1}{2}} = \underline{(4)^{\frac{1}{2}}}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}} = \underline{2}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$	<u>B1</u>
	$=\{2\}\left[1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2+...\right]$	see notes	M1 A1ft
	$=\{2\}\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{9x}{4}\right)^2+...\right]$		
	$=2\left[1-\frac{9}{8}x-\frac{81}{128}x^2+...\right]$	see notes	
	$=2-\frac{9}{4}x;-\frac{81}{64}x^2+...$	isw	A1; A1
			[5]
(b)	$\sqrt{310} = 10\sqrt{3.1} = 10\sqrt{(4-9(0.1))}$, so $x=0.1$	E.g. For $10\sqrt{3.1}$ (can be implied by later working) **and** $x=0.1$ (or uses $x=0.1$) **Note:** $\sqrt{(100)(3.1)}$ by itself is B0	B1
	When $x=0.1$ $\sqrt{(4-9x)} \approx 2-\frac{9}{4}(0.1)-\frac{81}{64}(0.1)^2+...$	Substitutes their x, where $\lvert x\rvert<\frac{4}{9}$ into all three terms of their binomial expansion	M1
	$=2-0.225-0.01265625=1.76234375$		
	So, $\sqrt{310}\approx 17.6234375 = \underline{17.623}$ (3 dp)	17.623 **cao**	A1 **cao**
	Note: the calculator value of $\sqrt{310}$ is 17.60681686... which is 17.607 to 3 decimal places		**[3]**
			8 marks

Question 104 Notes

104. (a)	**B1**	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$ outside brackets or $\underline{2}$ as candidate's constant term in their binomial expansion
	M1	Expands $(...+kx)^{\frac{1}{2}}$ to give any 2 terms out of 3 terms simplified or un-simplified, E.g. $1+\left(\frac{1}{2}\right)(kx)$ or $\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ or $1+...+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ where k is a numerical value and **where** $k\neq 1$
	A1ft	A correct simplified or un-simplified $1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ expansion with **consistent** (kx)
	Note	(kx), $k\neq 1$ must be consistent (on the RHS, not necessarily on the LHS) in their expansion
	Note	Award B1M1A0 for $2\left[1+\left(\frac{1}{2}\right)(-9x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{9x}{4}\right)^2+...\right]$ because (kx) is not consistent
	Note	<u>**Incorrect bracketing:**</u> $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{9x^2}{4}\right)+...\right]$ is B1M1A0 unless recovered
	A1	$2-\frac{9}{4}x$ **(simplified fractions) or** allow $2-2.25x$ or $2-2\frac{1}{4}x$
	A1	Accept only $-\frac{81}{64}x^2$ or $-1\frac{17}{64}x^2$ or $-1.265625x^2$

		Question 104 Notes Continued
104. (a) ctd.	**SC**	If a candidate ***would otherwise score*** 2^{nd} A0, 3^{rd} A0 (i.e. scores A0A0 in the final two marks to (a)) then **allow Special Case 2^{nd} A1 for either** **SC:** $2\left[1-\frac{9}{8}x;...\right]$ or **SC:** $2\left[1+...-\frac{81}{128}x^2+...\right]$ or **SC:** $\lambda\left[1-\frac{9}{8}x-\frac{81}{128}x^2+...\right]$ or **SC:** $\left[\lambda-\frac{9\lambda}{8}x-\frac{81\lambda}{128}x^2+...\right]$ (where λ can be 1 or omitted), where each term in the $[.....]$ is a simplified fraction or a decimal, **OR SC:** for $2-\frac{18}{8}x-\frac{162}{128}x^2+...$ (i.e. for not simplifying their correct coefficients)
	Note	Candidates who write $2\left[1+\left(\frac{1}{2}\right)\left(\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{9x}{4}\right)^2+...\right]$, where $k=\frac{9}{4}$ and not $-\frac{9}{4}$ and achieve $2+\frac{9}{4}x; -\frac{81}{64}x^2+...$ will get B1M1A1A0A1
	Note	**Ignore** extra terms beyond the term in x^2
	Note	You can ignore subsequent working following a correct answer
	Note	Allow B1M1A1 for $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{9x}{4}\right)^2+...\right]$
	Note	Allow B1M1A1A1A1 for $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{9x}{4}\right)^2+...\right]=2-\frac{9}{4}x-\frac{81}{64}x^2+...$
(b)	**Note**	Give B1 M1 for $\sqrt{310}\approx 10\left(2-\frac{9}{4}(0.1)-\frac{81}{64}(0.1)^2\right)$
	Note	**<u>Other alternative suitable values for *x* for</u>** <u>$\sqrt{310}\approx\beta\sqrt{4-9(\text{their }x)}$</u>

b	*x*	Estimate	*b*	*x*	Estimate
7	$-\frac{38}{147}$	17.479	14	$\frac{79}{294}$	18.256
8	$-\frac{3}{32}$	17.599	15	$\frac{118}{405}$	18.555
9	$\frac{14}{729}$	17.607	16	$\frac{119}{384}$	18.899
10	$\frac{1}{10}$	17.623	17	$\frac{94}{289}$	19.283
11	$\frac{58}{363}$	17.690	18	$\frac{493}{1458}$	19.701
12	$\frac{133}{648}$	17.819	19	$\frac{126}{361}$	20.150
13	$\frac{122}{507}$	18.009	20	$\frac{43}{120}$	20.625

	Note	Apply the scheme in the same way for their β and their x E.g. Give B1 M1 A1 for $\sqrt{310}\approx 12\left(2-\frac{9}{4}\left(\frac{133}{648}\right)-\frac{81}{64}\left(\frac{133}{648}\right)^2\right)=17.819$ (3 dp)
	Note	Allow B1 M1 A1 for $\sqrt{310}\approx 100\left(2-\frac{9}{4}(0.441)-\frac{81}{64}(0.441)^2\right)=76.161$ (3 dp)
	Note	Give B1 M1 A0 for $\sqrt{310}\approx 10\left(2-\frac{9}{4}(0.1)-\frac{81}{64}(0.1)^2-\frac{729}{512}(0.1)^3\right)=17.609$ (3 dp)

	Question 104 Notes Continued	
104.(b)	**Note**	***Send to review*** using $\beta = \sqrt{155}$ and $x = \frac{2}{9}$ (which gives 17.897 (3 dp))
	Note	***Send to review*** using $\beta = \sqrt{1000}$ and $x = 0.41$ (which gives 27.346 (3 dp))

104. (a) **Alt 1**	**<u>Alternative method 1:</u>** Candidates can apply an alternative form of the binomial expansion $\left\{(4-9x)^{\frac{1}{2}}\right\} = (4)^{\frac{1}{2}} + \left(\tfrac{1}{2}\right)(4)^{-\frac{1}{2}}(-9x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(4)^{-\frac{3}{2}}(-9x)^2$	
	B1	$(4)^{\frac{1}{2}}$ or 2
	M1	Any two of three (un-simplified) terms correct
	A1	All three (un-simplified) terms correct
	A1	$2 - \frac{9}{4}x$ **(simplified fractions)** or allow $2 - 2.25x$ or $2 - 2\frac{1}{4}x$
	A1	Accept only $-\frac{81}{64}x^2$ or $-1\frac{17}{64}x^2$ or $-1.265625x^2$
	Note	The terms in C need to be evaluated. So ${}^{\frac{1}{2}}C_0(4)^{\frac{1}{2}} + {}^{\frac{1}{2}}C_1(4)^{-\frac{1}{2}}(-9x); + {}^{\frac{1}{2}}C_2(4)^{-\frac{3}{2}}(-9x)^2$ without further working is B0M0A0

104. (a)	**<u>Alternative Method 2: Maclaurin Expansion</u>** $f(x) = (4-9x)^{\frac{1}{2}}$		
	$f''(x) = -\frac{81}{4}(4-9x)^{-\frac{3}{2}}$	Correct $f''(x)$	B1
	$f'(x) = \frac{1}{2}(4-9x)^{-\frac{1}{2}}(-9)$	$\pm a(4-9x)^{-\frac{1}{2}}$; $a \neq \pm 1$	M1
		$\frac{1}{2}(4-9x)^{-\frac{1}{2}}(-9)$	A1 oe
	$\left\{\therefore f(0) = 2,\ f'(0) = -\frac{9}{4} \text{ and } f''(0) = -\frac{81}{32}\right\}$		
	So, $f(x) = 2 - \frac{9}{4}x; - \frac{81}{64}x^2 + \ldots$		A1; A1

Question Number	Scheme	Notes	Marks
105.	$\left\{(2+kx)^{-3} = 2^{-3}\left(1+\frac{kx}{2}\right)^{-3} = \frac{1}{8}\left(1+(-3)\left(\frac{kx}{2}\right)+\frac{(-3)(-3-1)}{2!}\left(\frac{kx}{2}\right)^2+\ldots\right)\right\},\ k>0$		
(a)	$\{A=\}\ \frac{1}{8}$	$\frac{1}{8}$ or 2^{-3} or 0.125, clearly identified as A **or** as their answer to part (a)	B1 **cao**
			[1]
(b)		Uses the x^2 term of the binomial expansion to give	
		either $\frac{(-3)(-4)}{2!}$ **or** $\left(\frac{k}{2}\right)^2$ **or** $\left(\frac{kx}{2}\right)^2$ **or** $\frac{(-3)(-4)}{2}$ **or** 6	M1
	$\left(\frac{1}{8}\right)\frac{(-3)(-4)}{2!}\left(\frac{k}{2}\right)^2$	**either** (their A)$\frac{(-3)(-4)}{2!}\left(\frac{k}{2}\right)^2$ **or** (their A)$\frac{(-3)(-4)}{2!}\left(\frac{kx}{2}\right)^2$, where (their A) $\neq$ 1, **or** $\frac{3}{16}k^2$ **or** $\frac{3}{16}k^2x^2$ **or** $(2^{-5})\frac{(-3)(-4)}{2!}(kx)^2$ **or** $(2^{-5})\frac{(-3)(-4)}{2!}(k)^2$	M1 o.e.
	$\left\{\text{So, } \left(\frac{1}{8}\right)\frac{(-3)(-4)}{2!}\left(\frac{k}{2}\right)^2 = \frac{243}{16} \Rightarrow \frac{3}{16}k^2 = \frac{243}{16} \Rightarrow k^2 = 81\right\}$		
	So, $k=9$	$k=9$ cao	A1 **cso**
	Note: $k=\pm 9$ with no reference to $k=9$ only is A0		**[3]**
(c)	$"\left(\frac{1}{8}\right)"(-3)\left(\frac{k}{2}\right)$	Uses the x term of the binomial expansion to give **either** (their A)$(-3)\left(\frac{k}{2}\right)$ **or** (their A)$(-3)\left(\frac{kx}{2}\right)$; where (their A) $\neq$ 1, **or** $(2)^{-4}(-3)(k)$ **or** $(2)^{-4}(-3)(kx)$ **or** $-\frac{3k}{16}$	M1
	$\left\{\text{So, } B = \left(\frac{1}{8}\right)(-3)\left(\frac{9}{2}\right) \Rightarrow\right\}$ $\underline{B = -\frac{27}{16}}$	$-\frac{27}{16}$ or $-1\frac{11}{16}$ or -1.6875	A1 **cso**
			[2]
			6

Question 105 Notes

NOTE	**IN THIS QUESTION IGNORE LABELLING AND MARK ALL PARTS TOGETHER.**
Note	$(2+kx)^{-3} = \frac{1}{8}\left(1-\frac{3}{2}kx+\frac{3}{2}k^2x^2+\ldots\right) = \frac{1}{8}-\frac{3}{16}kx+\frac{3}{16}k^2x^2+\ldots$
Note	Writing down $\left\{\left(1+\frac{kx}{2}\right)^{-3}\right\} = 1+(-3)\left(\frac{kx}{2}\right)+\frac{(-3)(-3-1)}{2!}\left(\frac{kx}{2}\right)^2+\ldots$. gets (b) 1st M1
Note	Writing down $\{(2+kx)^{-3}\} = \frac{1}{8}\left(1+(-3)\left(\frac{kx}{2}\right)+\frac{(-3)(-3-1)}{2!}\left(\frac{kx}{2}\right)^2+\ldots\right)$ gets (b) 1st M1 2nd M1 and (c) M1
Note	Writing down $\{(2+kx)^{-3}\} = 2^{-3}+(-3)(2^{-4})(kx)+\frac{(-3)(-4)}{2}(2^{-5})(kx)^2$ gets (b) 1st M1 2nd M1 and (c) M1
Note	Writing down $\{(2+kx)^{-3}\} = (\text{their } A)\left(1+(-3)\left(\frac{kx}{2}\right)+\frac{(-3)(-3-1)}{2!}\left(\frac{kx}{2}\right)^2+\ldots\right)$ where (their A) $\neq$ 1, gets (b) 1st M1 2nd M1 and (c) M1

		Question 105 Notes
105. (b), (c)	**Note**	(their A) is defined as either • their answer to part (a) • their stated $A = ...$ • their "2^{-3}" in their stated $2^{-3}\left(1+\frac{kx}{2}\right)^{-3}$
	Note	Give 2nd M0 in part (b) if (their A) = 1
	Note	Give M0 in part (c) if (their A) = 1
105. (c)	**Note**	Allow M1 for $(\text{their } A)(-3)\left(\frac{\text{their } k \text{ from (b)}}{2}\right)$
	Note	Award A0 for $B = -\frac{27}{16}x$
	Note	Allow A1 for $B = -\frac{27}{16}x$ followed by $B = -\frac{27}{16}$ **or** $-1\frac{11}{16}$ **or** -1.6875
	Note	$k = -9$ leading to $B = \frac{27}{16}$ **or** $1\frac{11}{16}$ **or** 1.6875 is A0
	Note	Give A0 for finding both $B = -\frac{27}{16}$ and $B = \frac{27}{16}$ (without rejecting $B = \frac{27}{16}$) as their final answer.
	Note	The A1 mark in part (c) is for a correct solution only.
	Note	**Be careful!** It is possible to award M0A0 in part (c) for a solution leading to $B = -\frac{27}{16}$. E.g. $f(x) = (2+kx)^{-3} = 2^{-3}(1+kx)^{-3} = \frac{1}{8}\left(1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2+...\right) = \frac{1}{8} - \frac{3k}{8}x + \frac{3k^2}{4}x^2 + ...$ leading to (a) $A = \frac{1}{8}$, (b) $k = \frac{9}{2}$, (c) $B = -\frac{27}{16}$, gets (a) B1, (b) M1M0A0 (c) M0A0
105. (b), (c)	**Note**	${}^{-3}C_0(2)^{-3} + {}^{-3}C_1(2)^{-4}(kx) + {}^{-3}C_2(2)^{-5}(kx)^2$ with the C terms not evaluated gets (b) 1st M0 2nd M0 and (c) M0

Question Number		Notes	Marks
106. Way 1	$\left\{\frac{1}{(2+5x)^3} = \right\} (2+5x)^{-3}$	Writes down $(2+5x)^{-3}$ or uses power of -3	M1
	$= \underline{(2)^{-3}}\left(1+\frac{5x}{2}\right)^{-3} = \underline{\frac{1}{8}}\left(1+\frac{5x}{2}\right)^{-3}$	$\underline{2^{-3}}$ or $\underline{\frac{1}{8}}$	B1
	$= \left\{\frac{1}{8}\right\}\left[1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3+\ldots\right]$	see notes	M1 A1
	$= \left\{\frac{1}{8}\right\}\left[1+(-3)\left(\frac{5x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{5x}{2}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(\frac{5x}{2}\right)^3+\ldots\right]$		
	$= \frac{1}{8}\left[1-\frac{15}{2}x+\frac{75}{2}x^2-\frac{625}{4}x^3+\ldots\right]$		
	$= \frac{1}{8}\left[1-7.5x+37.5x^2-156.25x^3+\ldots\right]$		
	$= \frac{1}{8}-\frac{15}{16}x;+\frac{75}{16}x^2-\frac{625}{32}x^3+\ldots$ **or** $\frac{1}{8}-\frac{15}{16}x;+4\frac{11}{16}x^2-19\frac{17}{32}x^3+\ldots$		A1; A1
			[6]
			6
Way 2	$f(x)=(2+5x)^{-3}$	Writes down $(2+5x)^{-3}$ or uses power of -3	M1
	$f''(x)=300(2+5x)^{-5},\ f'''(x)=-7500(2+5x)^{-6}$	Correct $f''(x)$ and $f'''(x)$	B1
	$f'(x)=-15(2+5x)^{-4}$	$\pm a(2+5x)^{-4},\ a\neq\pm1$	M1
		$-15(2+5x)^{-4}$	A1 oe
	$\left\{\therefore f(0)=\frac{1}{8},\ f'(0)=-\frac{15}{16},\ f''(0)=\frac{75}{8}\ \text{and}\ f'''(0)=-\frac{1875}{16}\right\}$		
	So, $f(x)=\frac{1}{8}-\frac{15}{16}x;+\frac{75}{16}x^2-\frac{625}{32}x^3+\ldots$	Same as in Way 1	A1; A1
			[6]
Way 3	$(2+5x)^{-3}$	Same as in Way 1	M1
	$= \underline{(2)^{-3}}+(-3)(2)^{-4}(5x)+\frac{(-3)(-4)}{2!}(2)^{-5}(5x)^2+\frac{(-3)(-4)(-5)}{3!}(2)^{-6}(5x)^3$	Same as in Way 1	B1
		Any two terms correct	M1
		All four terms correct	A1
	$= \frac{1}{8}-\frac{15}{16}x;+\frac{75}{16}x^2-\frac{625}{32}x^3+\ldots$	Same as in Way 1	A1; A1
	Note: Terms can be simplified or un-simplified for B1 2nd M1 1st A1		**[6]**
	Note: The terms in C need to be evaluated So ${}^{-3}C_0(2)^{-3}+{}^{-3}C_1(2)^{-4}(5x)+{}^{-3}C_2(2)^{-5}(5x)^2+{}^{-3}C_3(2)^{-6}(5x)^3$ without further working is B0 1st M0 1st A0		

106.	**1st M1**	mark can be implied by a constant term of $(2)^{-3}$ or $\frac{1}{8}$.
	B1	$\underline{2^{-3}}$ or $\frac{1}{8}$ outside brackets or $\frac{1}{8}$ as candidate's constant term in their binomial expansion.
	2nd M1	Expands $(...+kx)^{-3}, k=$ a value $\neq 1$, to give any 2 terms out of 4 terms simplified or un-simplified, Eg: $1+(-3)(kx)$ or $\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3$ or $1+\ ...\ +\frac{(-3)(-4)}{2!}(kx)^2$ or $\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3$ are fine for M1.
	1st A1	A correct simplified or un-simplified $1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3$ expansion with consistent (kx). Note that (kx) must be consistent and $k=$ a value $\neq 1$. (on the RHS, not necessarily the LHS) in a candidate's expansion.
	Note	You would award B1M1A0 for $\frac{1}{8}\left[1+(-3)\left(\frac{5x}{2}\right)+\frac{(-3)(-4)}{2!}(5x)^2+\frac{(-3)(-4)(-5)}{3!}\left(\frac{5x}{2}\right)^3+...\right]$ because (kx) is not consistent.
	Note	**Incorrect bracketing:** $=\left\{\frac{1}{8}\right\}\left[1+(-3)\left(\frac{5x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{5x^2}{2}\right)+\frac{(-3)(-4)(-5)}{3!}\left(\frac{5x^3}{2}\right)+...\right]$ is M1A0 unless recovered.
	2nd A1	For $\frac{1}{8}-\frac{15}{16}x$ **(simplified)** or also allow $0.125-0.9375x$.
	3rd A1	Accept only $\frac{75}{16}x^2-\frac{625}{32}x^3$ or $4\frac{11}{16}x^2-19\frac{17}{32}x^3$ or $4.6875x^2-19.53125x^3$
	SC	If a candidate ***would otherwise score*** 2nd A0, 3rd A0 then **allow Special Case 2nd A1 for either**
		SC: $\frac{1}{8}\left[1-\frac{15}{2}x\ ;\ ...\right]$ or **SC:** $\frac{1}{8}\left[1+...+\frac{75}{2}x^2+...\right]$ or **SC:** $\frac{1}{8}\left[1+...-\frac{625}{4}x^3+...\right]$
		SC: $\lambda\left[1-\frac{15}{2}x+\frac{75}{2}x^2-\frac{625}{4}x^3+...\right]$ or **SC:** $\left[\lambda-\frac{15\lambda}{2}x+\frac{75\lambda}{2}x^2-\frac{625\lambda}{4}x^3+...\right]$
		(where λ can be 1 or omitted), where each term in the $[.....]$ is a simplified fraction or a decimal
	SC	<u>**Special case for the 2nd M1 mark**</u> Award Special Case 2nd M1 for a correct simplified or un-simplified $1+n(kx)+\frac{n(n-1)}{2!}(kx)^2+\frac{n(n-1)(n-2)}{3!}(kx)^3$ expansion with their $n\neq -3$, $n \neq$ ***positive integer*** and a consistent (kx). Note that (kx) must be consistent (on the RHS, not necessarily the LHS) in a candidate's expansion. **Note** that $k\neq 1$ **.**
	Note	Ignore extra terms beyond the term in x^3
	Note	You can ignore subsequent working following a correct answer.

Question Number	Scheme	Marks
107. (a)	$(4+5x)^{\frac{1}{2}} = \underline{(4)^{\frac{1}{2}}}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}} = \underline{2}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}$ $\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$	B1
	$=\{2\}\left[1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2+\ldots\right]$ see notes	M1 A1ft
	$=\{2\}\left[1+\left(\frac{1}{2}\right)\left(\frac{5x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{5x}{4}\right)^2+\ldots\right]$ $=2\left[1+\frac{5}{8}x-\frac{25}{128}x^2+\ldots\right]$ See notes below!	
	$=2+\frac{5}{4}x;-\frac{25}{64}x^2+\ldots$ isw	A1; A1
		[5]
(b)	$\left\{x=\frac{1}{10}\Rightarrow(4+5(0.1))^{\frac{1}{2}}=\sqrt{4.5}=\sqrt{\frac{9}{2}}=\frac{3}{\sqrt{2}}=\frac{3}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}\right\}$	
	$=\frac{3}{2}\sqrt{2}$ $\frac{3}{2}\sqrt{2}$ or $k=\frac{3}{2}$ or 1.5 o.e.	B1
		[1]
(c)	$\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\frac{3}{\sqrt{2}}=2+\frac{5}{4}\left(\frac{1}{10}\right)-\frac{25}{64}\left(\frac{1}{10}\right)^2+\ldots\ \{=2.121\ldots\}$ See notes	M1
	So, $\frac{3}{2}\sqrt{2}=\frac{543}{256}$ or $\frac{3}{\sqrt{2}}=\frac{543}{256}$	
	yields, $\sqrt{2}=\frac{181}{128}$ or $\sqrt{2}=\frac{256}{181}$ $\frac{181}{128}$ or $\frac{362}{256}$ or $\frac{543}{384}$ **or** $\frac{256}{181}$ etc.	A1 oe
		[2] **8**

		Question 107 Notes
107. (a)	**B1**	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$ outside brackets or $\underline{2}$ as candidate's constant term in their binomial expansion.
	M1	Expands $(\ldots+kx)^{\frac{1}{2}}$ to give any 2 terms out of 3 terms simplified or un-simplified, Eg: $1+\left(\frac{1}{2}\right)(kx)$ or $\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ or $1+\ldots+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ where k is a numerical value and **where** $k\neq 1$.
	A1	A correct simplified or un-simplified $1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ expansion with **consistent** (kx).
	Note	(kx), $k\neq 1$, must be consistent (on the RHS, not necessarily on the LHS) in a candidate's expansion.

107. (a) ctd	**Note**	Award B1M1A0 for $2\left[1+\left(\frac{1}{2}\right)(5x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{5x}{4}\right)^2+...\right]$ because (kx) is not consistent.
	Note	**<u>Incorrect bracketing:</u>** $2\left[1+\left(\frac{1}{2}\right)\left(\frac{5x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{5x^2}{4}\right)+...\right]$ is B1M1A0 unless recovered.
	A1	$2+\frac{5}{4}x$ **(simplified fractions)** or allow $2+1.25x$ or $2+1\frac{1}{4}x$
	A1	Accept only $-\frac{25}{64}x^2$ or $-0.390625x^2$
	SC	If a candidate ***would otherwise score*** 2nd A0, 3rd A0 then **allow Special Case 2nd A1 for either**
		SC: $2\left[1+\frac{5}{8}x;...\right]$ or **SC:** $2\left[1+...-\frac{25}{128}x^2+...\right]$ or **SC:** $\lambda\left[1+\frac{5}{8}x-\frac{25}{128}x^2+...\right]$
		or **SC:** $\left[\lambda+\frac{5\lambda}{8}x-\frac{25\lambda}{128}x^2+...\right]$ (where λ can be 1 or omitted), where each term in the $[.....]$ is a simplified fraction or a decimal,
		OR SC: for $2+\frac{10}{8}x-\frac{50}{128}x^2+...$ (i.e. for not simplifying their correct coefficients.)
	Note	Candidates who write $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{5x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{5x}{4}\right)^2+...\right]$, where $k=-\frac{5}{4}$ and not $\frac{5}{4}$
		and achieve $2-\frac{5}{4}x-\frac{25}{64}x^2+...$ will get B1M1A1A0A1
	Note	Ignore extra terms beyond the term in x^2.
	Note	You can ignore subsequent working following a correct answer.
(b)	**B1**	$\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $k=\frac{3}{2}$ or 1.5 o.e. (Ignore how $k=\frac{3}{2}$ is found.)
(c)	**M1**	Substitutes $x=\frac{1}{10}$ or 0.1 into their binomial expansion found in part (a) which must contain both an x term and an x^2 term (or even an x^3 term) **and** equates this to either $\frac{3}{\sqrt{2}}$ or their $k\sqrt{2}$ from (b), where k is a numerical value.
	Note	M1 can be implied by $\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\frac{3}{\sqrt{2}}$ = awrt 2.121
	Note	M1 ***can be implied*** by $\frac{1}{k}\left(\text{their } \frac{543}{256}\right)$, with their k found in part (b).
	Note	M1 ***cannot be implied*** by $(k)\left(\text{their } \frac{543}{256}\right)$, with their k found in part (b).
	A1	$\frac{181}{128}$ **or any equivalent fraction**, eg: $\frac{362}{256}$ or $\frac{543}{384}$. Also allow $\frac{256}{181}$ **or any equivalent fraction.**
	Note	Also allow A1 for $p=181, q=128$ or $p=181\lambda, q=128\lambda$ or $p=256, q=181$ or $p=256\lambda, q=181\lambda$, where $\lambda\in\mathbb{Z}^+$
	Note	You can recover work for part (c) in part (b). You cannot recover part (b) work in part (c).
	Note	Candidates are allowed to restart and gain all 2 marks in part (c) from an incorrect part (b).
	Note	Award M1 A1 for the correct answer from no working.

107. (a)	**Alternative methods for part (a)**
	Alternative method 1: Candidates can apply an alternative form of the binomial expansion.
	$\left\{(4+5x)^{\frac{1}{2}}\right\} = (4)^{\frac{1}{2}} + \left(\tfrac{1}{2}\right)(4)^{-\frac{1}{2}}(5x) + \dfrac{(\frac{1}{2})(-\frac{1}{2})}{2!}(4)^{-\frac{3}{2}}(5x)^2$

B1	$(4)^{\frac{1}{2}}$ or 2
M1	Any two of three (un-simplified) terms correct.
A1	All three (un-simplified) terms correct.
A1	$2+\frac{5}{4}x$ **(simplified fractions)** or allow $2+1.25x$ or $2+1\frac{1}{4}x$
A1	Accept only $-\frac{25}{64}x^2$ or $-0.390625x^2$
Note	The terms in C need to be evaluated. So ${}^{\frac{1}{2}}C_0(4)^{\frac{1}{2}} + {}^{\frac{1}{2}}C_1(4)^{-\frac{1}{2}}(5x); + {}^{\frac{1}{2}}C_2(4)^{-\frac{3}{2}}(5x)^2$ without further working is B0M0A0.

Alternative Method 2: Maclaurin Expansion $f(x)=(4+5x)^{\frac{1}{2}}$

$f''(x) = -\frac{25}{4}(4+5x)^{-\frac{3}{2}}$	Correct $f''(x)$	B1
$f'(x) = \frac{1}{2}(4+5x)^{-\frac{1}{2}}(5)$	$\pm a(4+5x)^{-\frac{1}{2}};\ a \neq \pm 1$	M1
	$\frac{1}{2}(4+5x)^{-\frac{1}{2}}(5)$	A1 oe
$\left\{\therefore f(0)=2,\ f'(0)=\frac{5}{4} \text{ and } f''(0)=-\frac{25}{32}\right\}$		
So, $f(x) = 2+\frac{5}{4}x; -\frac{25}{64}x^2+\ldots$		A1; A1

Question Number	Scheme	Marks
108.	$\left\{(1+kx)^{-4} = 1 + (-4)(kx) + \frac{(-4)(-4-1)}{2!}(kx)^2 + ...\right\}$	
(a)	**Either** $(-4)k = -6$ **or** $(1+kx)^{-4} = 1 + (-4)(kx)$ **see notes**	M1
	leading to $k = \frac{3}{2}$ $\quad k = \frac{3}{2}$ or 1.5 or $\frac{6}{4}$	A1
		[2]
(b)	$\frac{(-4)(-5)}{2}(k)^2$ **Either** $\frac{(-4)(-5)}{2!}$ **or** $(k)^2$ **or** $(kx)^2$	M1
	Either $\frac{(-4)(-5)}{2!}(k)^2$ **or** $\frac{(-4)(-5)}{2!}(kx)^2$	M1
	$\left\{A = \frac{(-4)(-5)}{2!}\left(\frac{3}{2}\right)^2\right\} \Rightarrow A = \frac{45}{2}$ $\quad \frac{45}{2}$ or 22.5	A1
		[3] **5**

	Question 108 Notes	
Note	**In this question ignore part labelling and mark part (a) and part (b) together.**	
	Note	Writing down $\left\{(1+kx)^{-4}\right\} = 1 + (-4)(kx) + \frac{(-4)(-4-1)}{2!}(kx)^2 + ...$ gets all the method marks in Q2. i.e. (a) M1 and (b) M1M1
(a)	**M1**	Award M1 for • **either** writing down $(-4)k = -6$ or $4k = 6$ • **or** expanding $(1+kx)^{-4}$ to give $1 + (-4)(kx)$ • **or** writing down $(-4)kx = -6$ or $(-4k) = -6x$ or $-4kx = -6x$
	A1	$k = \frac{3}{2}$ or 1.5 or $\frac{6}{4}$ **from no incorrect sign errors.**
	Note	The M1 mark can be implied by a candidate writing down the correct value of k.
	Note	Award M1 for writing down $4k = 6$ and then A1 for $k = 1.5$ (or equivalent).
	Note	Award M0 for $4k = -6$ (if there is no evidence that $(1+kx)^{-4}$ expands to give $1 + (-4)(kx) + ...$)
	Note	$1 + (-4)(kx)$ leading to $(-4)k = 6$ leading to $k = \frac{3}{2}$ is M1A0.
(b)	**M1**	For **either** $\frac{(-4)(-4-1)}{2!}$ **or** $\frac{(-4)(-5)}{2!}$ **or** 10 **or** $(k)^2$ **or** $(kx)^2$
	M1	**Either** $\frac{(-4)(-4-1)}{2!}(k)^2$ **or** $\frac{(-4)(-5)}{2!}(k)^2$ **or** $\frac{(-4)(-5)}{2!}(kx)^2$ **or** $\frac{(-4)(-5)}{2!}(\text{their } k)^2$ **or** $10k^2$
	Note	Candidates are allowed to use 2 instead of 2!
	A1	Uses $k = 1.5$ to give $A = \frac{45}{2}$ or 22.5
	Note	$A = \frac{90}{4}$ which has not been simplified is A0.
	Note	Award A0 for $A = \frac{45}{2}x^2$.
	Note	Allow A1 for $A = \frac{45}{2}x^2$ followed by $A = \frac{45}{2}$
	Note	$k = -1.5$ leading to $A = \frac{45}{2}$ or 22.5 is A0.

Question Number	Scheme		Marks
109. (a)	$\left\{\dfrac{1}{\sqrt{(9-10x)}}=\right\}(9-10x)^{-\frac{1}{2}}$	$(9-10x)^{-\frac{1}{2}}$ or uses power of $-\dfrac{1}{2}$	B1
	$=\underline{(9)^{-\frac{1}{2}}}\left(1-\dfrac{10x}{9}\right)^{-\frac{1}{2}}=\underline{\dfrac{1}{3}}\left(1-\dfrac{10x}{9}\right)^{-\frac{1}{2}}$	$\underline{(9)^{-\frac{1}{2}}}$ or $\underline{\dfrac{1}{3}}$	<u>B1</u>
	$=\left\{\dfrac{1}{3}\right\}\left[1+\left(-\dfrac{1}{2}\right)(kx)+\dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(kx)^2+\ldots\right]$	At least two correct terms. **See notes**	M1
	$=\left\{\dfrac{1}{3}\right\}\left[1+\left(-\dfrac{1}{2}\right)\left(\dfrac{-10x}{9}\right)+\dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2!}\left(\dfrac{-10x}{9}\right)^2+\ldots\right]$		
	$=\dfrac{1}{3}\left[1+\dfrac{5}{9}x+\dfrac{25}{54}x^2+\ldots\right]$		
	$=\dfrac{1}{3}+\dfrac{5}{27}x;+\dfrac{25}{162}x^2+\ldots$		A1; A1
			[5]
(b)	$\dfrac{3+x}{\sqrt{(9-10x)}}=(3+x)(9-10x)^{-\frac{1}{2}}$		
	$=(3+x)\left(\dfrac{1}{3}+\dfrac{5}{27}x+\left\{\dfrac{25}{162}x^2+\right\}\ldots\right)$	*Can be implied by later work* **See notes**	M1
	$=1+\dfrac{5}{9}x+\dfrac{25}{54}x^2+\dfrac{1}{3}x+\dfrac{5}{27}x^2+\ldots$	Multiplies out to give exactly one constant term, exactly 2 terms in x and exactly 2 terms in x^2. Ignore terms in x^3. Can be implied.	M1
	$=1+\dfrac{8}{9}x+\dfrac{35}{54}x^2+\ldots$		A1
			[3] **8**

Question 109 Notes

(a)	B1	Writes down $(9-10x)^{-\frac{1}{2}}$ or uses power of $-\dfrac{1}{2}$. This mark can be implied by a constant term of $(9)^{-\frac{1}{2}}$ or $\dfrac{1}{3}$.
	<u>B1</u>	$\underline{(9)^{-\frac{1}{2}}}$ or $\underline{\dfrac{1}{3}}$ outside brackets or $\dfrac{1}{3}$ as candidate's constant term in their binomial expansion.
	M1	Expands $(\ldots+kx)^{-\frac{1}{2}}$ to give any 2 terms out of 3 terms simplified or an un-simplified, $1+(-\frac{1}{2})(kx)$ or $(-\frac{1}{2})(kx)+\dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(kx)^2$ or $1+\ldots\ldots+\dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(kx)^2$, **where** $k\neq 1$.
	A1	$\dfrac{1}{3}+\dfrac{5}{27}x$ **(simplified fractions)**
	A1	Accept only $\dfrac{25}{162}x^2$

109. (a) ctd	**Note**	**You cannot recover correct work for part (a) in part (b).** i.e. if the correct answer to (a) appears as part of their solution in part (b), it cannot be credited in part (a).
	SC	If a candidate ***would otherwise score*** A0A0 then **allow Special Case 1st A1 for either**
		SC: $\frac{1}{3}\left[1+\frac{5}{9}x; ...\right]$ or **SC**: $\lambda\left[1+\frac{5}{9}x+\frac{25}{54}x^2+...\right]$ or **SC**: $\left[\lambda+\frac{5\lambda}{9}x+\frac{25\lambda}{54}x^2+...\right]$
		(where λ can be 1 or omitted), with each term in the $[.........]$ is a simplified fraction
	SC	**Special case for the M1 mark**
		Award Special Case M1 for a correct simplified or un-simplified $1+n(kx)+\frac{n(n-1)}{2!}(kx)^2$ expansion with a value of $n \neq -\frac{1}{2}$, $n \neq$ ***positive integer*** and a consistent (kx). Note that (kx) must be consistent (on the RHS, not necessarily the LHS) in a candidate's expansion. **Note** that $k \neq 1$.
	Note	Candidates who write $\left\{\frac{1}{3}\right\}\left[1+\left(-\frac{1}{2}\right)\left(\frac{10x}{9}\right)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}\left(\frac{10x}{9}\right)^2+...\right]$ where $k=\frac{10}{9}$ and not $-\frac{10}{9}$ and achieve $\frac{1}{3}-\frac{5}{27}x;+\frac{25}{162}x^2+...$ will get B1B1M1A0A1.
(b)		
	M1	Writes down $(3+x)($their part (a) answer, at least 2 of the 3 terms.$)$
	Note	$(3+x)\left(\frac{1}{4}+\frac{5}{4}x+...\right)$ or $(3+x)\left(\frac{1}{3}+\frac{5}{27}x+\frac{25}{162}x^2+...\right)$ are fine for M1.
	Note	This mark can also be implied by candidate multiplying out to find two terms (or coefficients) in x.
	M1	Multiplies out to give exactly one constant term, exactly 2 terms in x and exactly 2 terms in x^2.
	Note	This M1 mark can be implied. You can also ignore x^3 terms.
	A1	$1+\frac{8}{9}x+\frac{35}{54}x^2+...$

Alternative Methods for part (a)

Alternative method 1: Candidates can apply an alternative form of the binomial expansion.

$$\left\{\frac{1}{\sqrt{(9-10x)}}=\right\}(9-10x)^{-\frac{1}{2}}=(9)^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)(9)^{-\frac{3}{2}}(-10x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(9)^{-\frac{5}{2}}(-10x)^2$$

B1	Writes down $(9-10x)^{-\frac{1}{2}}$ or uses power of $-\frac{1}{2}$.
B1	$9^{-\frac{1}{2}}$ or $\frac{1}{3}$
M1	Any two of three (un-simplified or simplified) terms correct.
A1	$\frac{1}{3}+\frac{5}{27}x$
A1	$\frac{25}{162}x^2$
Note	The terms in C need to be evaluated, so ${}^{-\frac{1}{2}}C_0(9)^{-\frac{1}{2}}+{}^{-\frac{1}{2}}C_1(9)^{-\frac{3}{2}}(-10x)+{}^{-\frac{1}{2}}C_2(9)^{-\frac{5}{2}}(-10x)^2$ without further working is B1B0M0A0A0.

109. (a)	**Alternative Method 2: *Maclaurin Expansion***		
	Let $f(x) = \dfrac{1}{\sqrt{(9-10x)}}$		
	$\{f(x) =\}\ (9-10x)^{-\frac{1}{2}}$	$(9-10x)^{-\frac{1}{2}}$	B1
	$f''(x) = 75(9-10x)^{-\frac{5}{2}}$	Correct $f''(x)$	B1 oe
	$f'(x) = \left(-\frac{1}{2}\right)(9-10x)^{-\frac{3}{2}}(-10)$	$\pm a(9-10x)^{-\frac{3}{2}};\ a \neq \pm 1$	M1
	$\left\{\therefore f(0) = \dfrac{1}{3},\ f'(0) = \dfrac{5}{27} \text{ and } f''(0) = \dfrac{75}{243} = \dfrac{25}{81}\right\}$		
	$f(x) = \dfrac{1}{3} + \dfrac{5}{27}x;\ + \dfrac{25}{162}x^2 + ...$		A1; A1

Question Number	Scheme		Marks
110. (a)	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)}\right\} = (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	$(1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	B1
	$= \left(1+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2+\ldots\right)\times\left(1+\left(-\frac{1}{2}\right)(-x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2+\ldots\right)$	See notes	M1 A1 A1
	$= \left(1+\frac{1}{2}x-\frac{1}{8}x^2+\ldots\right)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2+\ldots\right)$		
	$= 1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{1}{2}x+\frac{1}{4}x^2-\frac{1}{8}x^2+\ldots$	See notes	M1
	$= 1+x+\frac{1}{2}x^2$	*Answer is given in the question.*	A1 *
			[6]
(b)	$\sqrt{\left(\frac{1+(\frac{1}{26})}{1-(\frac{1}{26})}\right)} = 1+\left(\frac{1}{26}\right)+\frac{1}{2}\left(\frac{1}{26}\right)^2$		M1
	ie: $\frac{3\sqrt{3}}{5} = \frac{1405}{1352}$		B1
	so, $\sqrt{3} = \frac{7025}{4056}$	$\frac{7025}{4056}$	A1 **cao**
			[3] **9**

Notes for Question 110

(a)

B1: $(1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ or $\sqrt{(1+x)}(1-x)^{-\frac{1}{2}}$ seen or implied. (Also allow $\left((1+x)(1-x)^{-1}\right)^{\frac{1}{2}}$).

M1: Expands $(1+x)^{\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified,

Eg: $1+\frac{1}{2}x$ or $+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$ or $1+\ldots.+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$

or expands $(1-x)^{-\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified,

Eg: $1+\left(-\frac{1}{2}\right)(-x)$ or $+\left(-\frac{1}{2}\right)(-x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2$ or $1+\ldots.+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2$

Also allow: $1+\ldots.+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(x)^2$ for M1.

A1: At least one binomial expansion correct (either un-simplified or simplified). (ignore x^3 and x^4 terms)
A1: Two binomial expansions are correct (either un-simplified or simplified). (ignore x^3 and x^4 terms)
Note: Candidates can give decimal equivalents when expanding out their binomial expansions.
M1: Multiplies out to give 1, exactly two terms in x and exactly three terms in x^2.
A1: Candidate achieves the result on the exam paper. Make sure that their working is sound.

Special Case: Award SC FINAL M1A1 for ***a correct*** $\left(1+\frac{1}{2}x-\frac{1}{8}x^2+\ldots\right)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2+\ldots\right)$

multiplied out with no errors to give either $1+x+\frac{3}{8}x^2+\frac{1}{4}x^2-\frac{1}{8}x^2$ or $1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{1}{2}x+\frac{1}{8}x^2$ or

$1+\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{2}x+\frac{1}{4}x^2$ or $1+\frac{1}{2}x+\frac{5}{8}x^2+\frac{1}{2}x-\frac{1}{8}x^2$ leading to the correct answer of $1+x+\frac{1}{2}x^2$.

Notes for Question 110 Continued	
110. (a) ctd	**Note:** If a candidate writes down either $(1+x)^{\frac{1}{2}} = 1+\frac{1}{2}x-\frac{1}{8}x^2+...$ **or** $(1-x)^{-\frac{1}{2}} = 1+\frac{1}{2}x+\frac{3}{8}x^2+...$ with no working then you can award 1st M1, 1st A1. **Note:** If a candidate writes down both correct binomial expansions with no working, then you can award 1st M1, 1st A1, 2nd A1.
(b)	**M1:** Substitutes $x=\frac{1}{26}$ into **both** sides of $\sqrt{\left(\frac{1+x}{1-x}\right)}$ and $1+x+\frac{1}{2}x^2$ **B1:** For sight of $\sqrt{\frac{27}{25}}$ (or better) and $\frac{1405}{1352}$ or equivalent fraction Eg: $\frac{3\sqrt{3}}{5}$ and $\frac{1405}{1352}$ or $0.6\sqrt{3}$ and $\frac{1405}{1352}$ or $\frac{3\sqrt{3}}{5}$ and $1\frac{53}{1352}$ or $\sqrt{3}$ and $\frac{5}{3}\left(\frac{1405}{1352}\right)$ are fine for B1. **A1:** $\frac{7025}{4056}$ **or any equivalent fraction**, eg: $\frac{14050}{8112}$ or $\frac{182650}{105456}$ etc. **Special Case:** Award SC: M1B1A0 for $\sqrt{3} \approx 1.732001972..$ or truncated 1.732001 or awrt 1.732002. **Note** that $\frac{7025}{4056} = 1.732001972...$ and $\sqrt{3} = 1.732050808...$

Aliter 2. (a) Way 2	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)} = \sqrt{\frac{(1+x)(1-x)}{(1+x)(1-x)}} = \sqrt{\frac{(1-x^2)}{(1-x)^2}} = \right\} = (1-x^2)^{\frac{1}{2}}(1-x)^{-1}$	$(1-x^2)^{\frac{1}{2}}(1-x)^{-1}$	B1
	$= \left(1+\left(\frac{1}{2}\right)(-x^2)+...\right)\times\left(1+(-1)(-x)+\frac{(-1)(-2)}{2!}(-x)^2+...\right)$	See notes	M1A1A1
	$= \left(1-\frac{1}{2}x^2+...\right)\times\left(1+x+x^2+...\right)$		
	$= 1+x+x^2-\frac{1}{2}x^2$	See notes	M1
	$= 1+x+\frac{1}{2}x^2$	*Answer is given in the question.*	A1 *
			[6]

Aliter 2. (a) Way 2	**B1**: $(1-x^2)^{\frac{1}{2}}(1-x)^{-1}$ seen or implied. **M1:** Expands $(1-x^2)^{\frac{1}{2}}$ to give both terms simplified or un-simplified, $1+\left(\frac{1}{2}\right)(-x^2)$ **or** expands $(1-x)^{-1}$ to give any 2 out of 3 terms simplified or un-simplified, Eg: $1+(-1)(-x)$ or $...+(-1)(-x)+\frac{(-1)(-2)}{2!}(-x)^2$ or $1+....+\frac{(-1)(-2)}{2!}(-x)^2$ **A1:** At least one binomial expansion correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **A1:** Two binomial expansions are correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **M1:** Multiplies out to give 1, exactly one term in x and exactly two terms in x^2. **A1:** Candidate achieves the result on the exam paper. Make sure that their working is sound.

Notes for Question 110 Continued		
Aliter 110. (a) Way 3	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)} = \sqrt{\frac{(1+x)(1+x)}{(1-x)(1+x)}} = \right\} = (1+x)(1-x^2)^{-\frac{1}{2}}$ $\quad (1+x)(1-x^2)^{-\frac{1}{2}}$	B1
	$= (1+x)\left(1+\frac{1}{2}x^2+...\right)$ Must follow on from above.	M1A1A1
	$= 1+x+\frac{1}{2}x^2$	dM1A1
	Note: The final M1 mark is dependent on the previous method mark for Way 3.	
Aliter 110. (a) Way 4	**Assuming the result on the Question Paper. (You need to be convinced that a candidate is applying this method before you apply the Mark Scheme for Way 4).**	
	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)} = \frac{\sqrt{(1+x)}}{\sqrt{(1-x)}} = 1+x+\frac{1}{2}x^2\right\} \Rightarrow (1+x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}}$	B1
	$(1+x)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2+... \left\{= 1+\frac{1}{2}x-\frac{1}{8}x^2+...\right\}$,	
	$(1-x)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)(-x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2+... \left\{= 1-\frac{1}{2}x-\frac{1}{8}x^2+...\right\}$	M1A1A1
	$\text{RHS} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)\left(1-\frac{1}{2}x-\frac{1}{8}x^2+...\right)$	
	$= 1-\frac{1}{2}x-\frac{1}{8}x^2+x-\frac{1}{2}x^2+\frac{1}{2}x^2$ See notes	M1
	$= 1+\frac{1}{2}x-\frac{1}{8}x^2$	
	So, $\text{LHS} = 1+\frac{1}{2}x-\frac{1}{8}x^2 = \text{RHS}$	A1 *
		[6]
	B1: $(1+x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}}$ seen or implied. **M1: For Way 4, this M1 mark is dependent on the first B1 mark.** Expands $(1+x)^{\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified, Eg: $1+\frac{1}{2}x$ or $+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$ or $1 + ... + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$ **or** expands $(1-x)^{\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified, Eg: $1+\left(\frac{1}{2}\right)(-x)$ or $+\left(\frac{1}{2}\right)(-x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2$ or $1 + ... + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2$ **A1:** At least one binomial expansion correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **A1:** Two binomial expansions are correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **M1: For Way 4, this M1 mark is dependent on the first B1 mark.** Multiplies out RHS to give 1, exactly two terms in x and exactly three terms in x^2. **A1:** Candidate achieves the result on the exam paper. Candidate needs to have correctly processed both the LHS and RHS of $(1+x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}}$.	

Question Number	Scheme		Marks
111. (a)	$\left\{\sqrt[3]{(8-9x)}\right\} = (8-9x)^{\frac{1}{3}}$	Power of $\frac{1}{3}$	M1
	$= \underline{(8)^{\frac{1}{3}}}\left(1-\frac{9x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\frac{9x}{8}\right)^{\frac{1}{3}}$	$\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$	<u>B1</u>
	$= \{2\}\left[1+\left(\frac{1}{3}\right)(kx)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3+\ldots\right]$	see notes	M1 A1
	$= \{2\}\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{-9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}\left(\frac{-9x}{8}\right)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{-9x}{8}\right)^3+\ldots}\right]$		
	$= 2\left[1-\frac{3}{8}x;-\frac{9}{64}x^2-\frac{45}{512}x^3+\ldots\right]$	See notes below!	
	$= 2-\frac{3}{4}x;-\frac{9}{32}x^2-\frac{45}{256}x^3+\ldots$		A1; A1
			[6]
(b)	$\left\{\sqrt[3]{7100} = 10\sqrt[3]{7.1} = 10\sqrt[3]{(8-9x)},\right\}$ so $x = 0.1$	Writes down or uses $x = 0.1$	B1
	When $x = 0.1$, $\sqrt[3]{(8-9x)} \approx 2-\frac{3}{4}(0.1)-\frac{9}{32}(0.1)^2-\frac{45}{256}(0.1)^3+\ldots$		M1
	$= 2-0.075-0.0028125-0.00017578125$		
	$= 1.922011719$		
	So, $\sqrt[3]{7100} = 19.220117919\ldots = \underline{19.2201}$ (4 dp)	19.2201 **cso**	A1 **cao**
			[3]
			9

Notes for Question 111

(a)

M1: Writes or uses $\frac{1}{3}$. This mark can be implied by a constant term of $8^{\frac{1}{3}}$ or 2.

<u>B1</u>: $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$ outside brackets or $\underline{2}$ as candidate's constant term in their binomial expansion.

M1: Expands $(\ldots+kx)^{\frac{1}{3}}$ to give any 2 terms out of 4 terms simplified or un-simplified,

Eg: $1+\left(\frac{1}{3}\right)(kx)$ or $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3$ or $1+\ldots\ldots+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2$

or $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3$ **where** $k \neq 1$ are fine for M1.

A1: A correct simplified or un-simplified $1+\left(\frac{1}{3}\right)(kx)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3$

expansion with consistent (kx). Note that (kx) must be consistent (on the RHS, not necessarily the LHS) in a candidate's expansion. **Note** that $k \neq 1$.

You would award B1M1A0 for $2\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{-9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-9x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{-9x}{8}\right)^3+\ldots}\right]$

because (kx) is not consistent.

Notes for Question 111 Continued	
111. (a) ctd	***"Incorrect bracketing"*** $=\{2\}\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{-9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}\left(\frac{-9x^2}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{-9x^3}{8}\right)+...}\right]$ is M1A0 unless recovered. **A1:** For $2-\frac{3}{4}x$ **(simplified please)** or also allow $2-0.75x$. **Allow Special Case A1A0 for either SC:** $=2\left[1-\frac{3}{8}x;\ ...\right]$ **or SC**: $K\left[1-\frac{3}{8}x-\frac{9}{64}x^2-\frac{45}{512}x^3+...\right]$ (where K can be 1 or omitted), with each term in the $[.........]$ either a simplified fraction or a decimal. **A1:** Accept only $-\frac{9}{32}x^2-\frac{45}{256}x^3$ or $-0.28125x^2-0.17578125x^3$ Candidates who write $=2\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}\left(\frac{9x}{8}\right)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{9x}{8}\right)^3+...}\right]$ where $k=\frac{9}{8}$ and not $-\frac{9}{8}$ and achieve $2+\frac{3}{4}x;-\frac{9}{32}x^2+\frac{45}{256}x^3+...$ will get B1M1A1A0A0. <u>**Note for final two marks:**</u> $2\left[1-\frac{3}{8}x;-\frac{9}{64}x^2-\frac{45}{512}x^3+...\right]=2+\frac{3}{4}x-\frac{9}{32}x^2-\frac{45}{256}x^3+...$ scores final A0A1. $2\left[1-\frac{3}{8}x;-\frac{9}{64}x^2-\frac{45}{512}x^3+...\right]=2-\frac{3}{4}-\frac{9}{32}x^2-\frac{45}{256}x^3+...$ scores final A0A1 <u>***Alternative method:***</u> Candidates can apply an alternative form of the binomial expansion. $\left\{\sqrt[3]{(8-9x)}\right\}=(8-9x)^{\frac{1}{3}}=(8)^{\frac{1}{3}}+\left(\frac{1}{3}\right)(8)^{-\frac{2}{3}}(-9x)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(-9x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(-9x)^3$ **B1:** $(8)^{\frac{1}{3}}$ or 2 **M1:** Any two of four (un-simplified or simplified) terms correct. **A1:** All four (un-simplified or simplified) terms correct. **A1:** $2-\frac{3}{4}x$ **A1:** $-\frac{9}{32}x^2-\frac{45}{256}x^3$ **Note:** The terms in C need to be evaluated, so ${}^{\frac{1}{3}}C_0(8)^{\frac{1}{3}}+{}^{\frac{1}{3}}C_1(8)^{-\frac{2}{3}}(-9x)+{}^{\frac{1}{3}}C_2(8)^{-\frac{5}{3}}(-9x)^2+{}^{\frac{1}{3}}C_3(8)^{-\frac{8}{3}}(-9x)^3$ without further working is B0M0A0.
(b)	**B1:** Writes down or uses $x=0.1$ **M1:** Substitutes their x, where $\lvert x\rvert<\frac{8}{9}$ into at least two terms of their binomial expansion. **A1:** 19.2201 **cao** **Be Careful!** The binomial answer is 19.22011719 and the calculated $\sqrt[3]{7100}$ is 19.21997343... which is 19.2200 to 4 decimal places.

Question Number	Scheme		Marks
	** represents a constant (which must be consistent for first accuracy mark)		
112. (a)	$\sqrt{(9+8x)} = (9+8x)^{\frac{1}{2}} = \underline{(9)^{\frac{1}{2}}}\left(1+\frac{8x}{9}\right)^{\frac{1}{2}} = \underline{3}\left(1+\frac{8x}{9}\right)^{\frac{1}{2}}$	$\underline{(9)^{\frac{1}{2}}}$ or $\underline{3}$ outside brackets	$\underline{\text{B1}}$
		Expands $(1+{**}x)^{\frac{1}{2}}$ to give a simplified or an un-simplified $1+(\tfrac{1}{2})({**}x)$;	M1;
	$= 3\left[\underline{1+(\tfrac{1}{2})({**}x);+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}({**}x)^2+\ldots}\right]$ **with** $** \neq 1$	A correct simplified or an un-simplified [..........] expansion with candidate's followed through $({**}x)$	A1 $\sqrt{\ }$
		Award SC M1 if you see $\frac{1}{2}({**}x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}({**}x)^2$ or $1+\ldots+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}({**}x)^2$	
	$= 3\left[\underline{1+\left(\frac{1}{2}\right)\left(\frac{8x}{9}\right);+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{8x}{9}\right)^2+\ldots}\right]$		
	$= 3\left[1+\frac{4}{9}x;-\frac{8}{81}x^2+\ldots\right]$	$3\left[1+\tfrac{4}{9}x;\ldots\right]$ or SC $K\left[1+\tfrac{4}{9}x-\tfrac{8}{81}x^2+\ldots\right]$	A1 oe
	$= 3+\frac{4}{3}x;-\frac{8}{27}x^2+\ldots$	$-\frac{8}{27}x^2$	A1
			[5]
(b)	$\sqrt{11} = \sqrt{(9+8x)} \Rightarrow \underline{x=\frac{1}{4}}$	$\underline{x=\frac{1}{4}}$	B1 oe
	$\sqrt{11} \approx 3+\frac{4}{3}\left(\frac{1}{4}\right)-\frac{8}{27}\left(\frac{1}{4}\right)^2 \quad \left\{=3+\frac{1}{3}-\frac{1}{54}\right\}$	Substitutes their x into their binomial expansion	M1
	$= 3\frac{17}{54} = \frac{179}{54}$	$3\frac{17}{54}$ or $\frac{179}{54}$.	A1
			[3] **8**
	Notes on Question 112		
(b)	**B1:** Writes down or uses $x=\frac{1}{4}$ oe. **M1:** Substitutes their x, where $\lvert x\rvert < \frac{9}{8}$ into at least one of the x or x^2 term of their binomial expansion. **A1:** Either $3\frac{17}{54}$ or $\frac{179}{54}$.		

Question Number	Scheme		Marks
113.	$(2+3x)^{-3} = \underline{(2)^{-3}}\left(1+\frac{3x}{2}\right)^{-3} = \underline{\frac{1}{8}}\left(1+\frac{3x}{2}\right)^{-3}$	$\underline{(2)^{-3}}$ or $\underline{\frac{1}{8}}$	<u>B1</u>
	$= \left\{\frac{1}{8}\right\}\left[1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3+\ldots\right]$	see notes	M1 A1
	$= \left\{\frac{1}{8}\right\}\left[\underline{1+(-3)\left(\frac{3x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{3x}{2}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(\frac{3x}{2}\right)^3+\ldots}\right]$		
	$= \frac{1}{8}\left[1-\frac{9}{2}x;+\frac{27}{2}x^2-\frac{135}{4}x^3+\ldots\right]$	See notes below!	
	$= \frac{1}{8}-\frac{9}{16}x;+\frac{27}{16}x^2-\frac{135}{32}x^3+\ldots$		A1; A1
			[5] **5**

<u>B1</u>: $\underline{(2)^{-3}}$ or $\frac{1}{8}$ outside brackets or $\frac{1}{8}$ as constant term in the binomial expansion.

M1: Expands $(\ldots+kx)^{-3}$ to give any 2 terms out of 4 terms simplified or un-simplified,

Eg: $1+(-3)(kx)$ or $(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2$ or $1+\ldots\ldots+\frac{(-3)(-4)}{2!}(kx)^2$

or $\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3$ **where** $k \neq 1$ are ok for M1.

A1: A correct simplified or un-simplified $1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3$ expansion with consistent (kx) where $k \neq 1$.

"Incorrect bracketing" $\left\{\frac{1}{8}\right\}\left[\underline{1+(-3)\left(\frac{3x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{3x^2}{2}\right)+\frac{(-3)(-4)(-5)}{3!}\left(\frac{3x^3}{2}\right)+\ldots}\right]$ is M1A0 unless recovered.

A1: For $\frac{1}{8}-\frac{9}{16}x$ **(simplified fractions)** or also allow $0.125-0.5625x$.

Allow Special Case A1 for either SC: $\frac{1}{8}\left[1-\frac{9}{2}x;\ldots\right]$ or **SC**: $K\left[1-\frac{9}{2}x+\frac{27}{2}x^2-\frac{135}{4}x^3+\ldots\right]$

(where K can be 1 or omitted), with each term in the $[\ldots\ldots\ldots]$ either a simplified fraction or a decimal.

A1: Accept only $\frac{27}{16}x^2-\frac{135}{32}x^3$ or $1\frac{11}{16}x^2-4\frac{7}{32}x^3$ or $1.6875x^2-4.21875x^3$

113. ctd	Candidates who write $=\frac{1}{8}\left[1+(-3)\left(-\frac{3x}{2}\right)+\frac{(-3)(-4)}{2!}\left(-\frac{3x}{2}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(-\frac{3x}{2}\right)^3+\ldots\right]$ where $k=-\frac{3}{2}$ and not $\frac{3}{2}$ and achieve $\frac{1}{8}+\frac{9}{16}x+\frac{27}{16}x^2+\frac{135}{32}x^3+\ldots$ will get B1M1A1A0A0. ***<u>Alternative method:</u>*** Candidates can apply an alternative form of the binomial expansion. $(2+3x)^{-3} = (2)^{-3}+(-3)(2)^{-4}(3x)+\frac{(-3)(-4)}{2!}(2)^{-5}(3x)^2+\frac{(-3)(-4)(-5)}{3!}(2)^{-6}(3x)^3$ **B1:** $\frac{1}{8}$ or $(2)^{-3}$ **M1:** Any two of four (un-simplified) terms correct. **A1:** All four (un-simplified) terms correct. **A1:** $\frac{1}{8}-\frac{9}{16}x$ **A1:** $+\frac{27}{16}x^2-\frac{135}{32}x^3$ **Note:** The terms in C need to be evaluated, so ${}^{-3}C_0(2)^{-3}+{}^{-3}C_1(2)^{-4}(3x)+{}^{-3}C_2(2)^{-5}(3x)^2+{}^{-3}C_3(2)^{-6}(3x)^3$ without further working is B0M0A0.

Question Number	Scheme	Marks
114.	(a) $\mathrm{f}(x) = \ldots(\ldots - \ldots x)^{-\frac{1}{2}}$	M1
	$= 6 \times 9^{-\frac{1}{2}}(\ldots)$ $\quad\quad \frac{6}{9^{\frac{1}{2}}}, \frac{6}{3}, 2$ or equivalent	B1
	$= \ldots\left(1 + (-\tfrac{1}{2})(kx); + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(kx)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(kx)^3 + \ldots\right)$	M1; A1ft
	$= 2\left(1 + \frac{2}{9}x + \ldots\right)$ $\quad\quad$ or $2 + \frac{4}{9}x$	A1
	$= 2 + \frac{4}{9}x + \frac{4}{27}x^2 + \frac{40}{729}x^3 + \ldots$	A1 **(6)**
	(b) $\mathrm{g}(x) = 2 - \frac{4}{9}x + \frac{4}{27}x^2 - \frac{40}{729}x^3 + \ldots$	B1ft **(1)**
	(c) $\mathrm{h}(x) = 2 + \frac{4}{9}(2x) + \frac{4}{27}(2x)^2 + \frac{40}{729}(2x)^3 + \ldots$	M1 A1 **(2)**
	$\left(= 2 + \frac{8}{9}x + \frac{16}{27}x^2 + \frac{320}{729}x^3 + \ldots\right)$	**[9]**

Question Number	Scheme		Marks
115. (a)	$\frac{1}{(2-5x)^2} = (2-5x)^{-2} = \underline{(2)^{-2}}\left(1-\frac{5x}{2}\right)^{-2} = \underline{\frac{1}{4}}\left(1-\frac{5x}{2}\right)^{-2}$	$\underline{(2)^{-2}}$ or $\underline{\frac{1}{4}}$	$\underline{\text{B1}}$
	$=\left\{\frac{1}{4}\right\}\left[1+(-2)(**x)+\frac{(-2)(-3)}{2!}(**x)^2+\ldots\right]$	see notes	M1 A1ft
	$=\left\{\frac{1}{4}\right\}\left[\underline{1+(-2)\left(-\frac{5x}{2}\right)+\frac{(-2)(-3)}{2!}\left(-\frac{5x}{2}\right)^2+\ldots}\right]$		
	$=\frac{1}{4}\left[1+5x;+\frac{75}{4}x^2+\ldots\right]$	See notes below!	
	$=\frac{1}{4}+\frac{5}{4}x;+\frac{75}{16}x^2+\ldots$		A1; A1
			[5]
(b)	$\left\{\frac{2+kx}{(2-5x)^2}\right\} = (2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\left\{\frac{75}{16}x^2+\ldots\right\}\right)$	*Can be implied by later work even in part (c).*	M1
	x terms: $\frac{2(5x)}{4}+\frac{kx}{4}=\frac{7x}{4}$		
	giving, $10+k=7 \Rightarrow \underline{k=-3}$	$\underline{k=-3}$	A1
			[2]
(c)	x^2 terms: $\frac{150x^2}{16}+\frac{5kx^2}{4}$		M1
	So, $A=\frac{75}{8}+\frac{5(-3)}{4} = \frac{75}{8}-\frac{15}{4} = \underline{\frac{45}{8}}$	$\underline{\frac{45}{8}}$ or $\underline{5\frac{5}{8}}$ or $\underline{5.625}$	A1
			[2] **9**
(a)	$\underline{\textbf{B1}}$: $\underline{(2)^{-2}}$ or $\underline{\frac{1}{4}}$ outside brackets or $\underline{\frac{1}{4}}$ as candidate's constant term in their binomial expansion.		
	M1: Expands to give a simplified or an un-simplified, $1+(-2)(**x)$ or $(-2)(**x)+\frac{(-2)(-3)}{2!}(**x)^2$ or $1+\ldots\ldots+\frac{(-2)(-3)}{2!}(**x)^2$, **where** $**\neq 1$.		
	A1: A correct simplified or an un-simplified $1+(-2)(**x)+\frac{(-2)(-3)}{2!}(**x)^2$ expansion with candidate's follow through $(**x)$. Note that $(**x)$ must be consistent.		
	You would award B1M1A0 for $=\frac{1}{4}\left[\underline{1+(-2)\left(-\frac{5x}{2}\right)+\frac{(-2)(-3)}{2!}(-5x)^2+\ldots}\right]$ because ** is not consistent.		
	Invisible brackets $\left\{\frac{1}{4}\right\}\left[\underline{1+(-2)\left(-\frac{5x}{2}\right)+\frac{(-2)(-3)}{2!}\left(-\frac{5x^2}{2}\right)+\ldots}\right]$ is M1A0 unless recovered.		
	A1: For $\frac{1}{4}+\frac{5}{4}x$ **(simplified fractions)** or Also allow $0.25+1.25x$ or $\frac{1}{4}+1\frac{1}{4}x$.		
	Allow Special Case A1 for either SC: $\frac{1}{4}[1+5x;\ldots]$ or **SC:** $K\left[1+5x+\frac{75}{4}x^2+\ldots\right]$.		
	A1: Accept only $\frac{75}{16}x^2$ or $4\frac{11}{16}x^2$ or $4.6875x^2$		
	$\underline{\textbf{Alternative method:}}$ Candidates can apply an alternative form of the binomial expansion. (See next page).		

115. (b) **M1:** Candidate writes down $(2+kx)$(their part (a) answer, at least up to the term in x.)

$(2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\ ...\right)$ or $(2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\frac{75}{16}x^2+...\right)$ are fine.

This mark can also be implied by candidate multiplying out to find two terms (or coefficients) in x.

A1: $k=-3$

(c) **M1:** Multiplies out their $(2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\frac{75}{16}x^2+...\right)$ to give **exactly** two terms (or coefficients) in x^2 and attempts to find A using a numerical value of k.

A1: Either $\frac{45}{8}$ or $5\frac{5}{8}$ or 5.625 **Note:** $\frac{45}{8}x^2$ is A0.

<u>*Alternative method for part (a)*</u>

$(2-5x)^{-2} = (2)^{-2}+(-2)(2)^{-3}(-5x);+\frac{(-2)(-3)}{2!}(2)^{-4}(-5x)^2$

B1: $\frac{1}{4}$ or $(2)^{-2}$,

M1: Any two of three (un-simplified) terms correct.

A1: All three (un-simplified) terms correct.

A1: $\frac{1}{4}+\frac{5}{4}x$

A1: $\frac{75}{16}x^2$

Note: The terms in C need to be evaluated, so ${}^{-2}C_0(2)^{-2}+{}^{-2}C_1(2)^{-3}(-5x);+{}^{-2}C_2(2)^{-4}(-5x)^2$ without further working is B0M0A0.

<u>*Alternative method for parts (b) and (c)*</u>

$(2+kx) = (2-5x)^2\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+...\right)$

$(2+kx) = (4-20x+25x^2)\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+...\right)$

$(2+kx) = 2+(7x-10x)+\left(4Ax^2-35x^2+\frac{25}{2}x^2\right)$

Equate x terms: $\underline{k=-3}$

Equate x^2 terms: $0=4A-35+\frac{25}{2} \Rightarrow 4A=\frac{45}{2} \Rightarrow \underline{A=\frac{45}{8}}$

(b) **M1:** For $(2+kx) = (4\pm\lambda x+25x^2)\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+...\right)$, where $\lambda\neq 0$

A1: $k=-3$

(c) **M1:** Multiplies out to obtain three x^2 terms/coefficients, equates to 0 and attempts to find A.

A1: Either $\frac{45}{8}$ or $5\frac{5}{8}$ or 5.625 **Note:** $\frac{45}{8}x^2$ is A0.

Question Number	Scheme		Marks	
116.	$\mathrm{f}(x) = (\ \dots\ +\ \dots)^{-\frac{1}{2}}$		M1	
	$= 9^{-\frac{1}{2}}(\ \dots\ +\ \dots)^{\dots}$	3^{-1}, $\frac{1}{3}$ or $\frac{1}{9^{\frac{1}{2}}}$	B1	
	$(1+kx^2)^n = 1 + nkx^2 + \dots$	n not a natural number, $k \neq 1$	M1	
	$(1+kx^2)^{-\frac{1}{2}} = \dots + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(kx^2)^2$	ft their $k \neq 1$	A1 ft	
	$\left(1+\frac{4}{9}x^2\right)^{-\frac{1}{2}} = 1 - \frac{2}{9}x^2 + \frac{2}{27}x^4$		A1	
	$\mathrm{f}(x) = \frac{1}{3} - \frac{2}{27}x^2 + \frac{2}{81}x^4$		A1	**(6)**
				[6]

Question Number	Scheme	Marks
117.		
(a)	$(2-3x)^{-2} = 2^{-2}\left(1-\frac{3}{2}x\right)^{-2}$	B1
	$\left(1-\frac{3}{2}x\right)^{-2} = 1+(-2)\left(-\frac{3}{2}x\right)+\frac{-2.-3}{1.2}\left(-\frac{3}{2}x\right)^2+\frac{-2.-3.-4}{1.2.3}\left(-\frac{3}{2}x\right)^3+\ \ldots$	M1 A1
	$= 1+3x+\frac{27}{4}x^2+\frac{27}{2}x^3+\ \ldots$	
	$(2-3x)^{-2} = \frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\ \ldots$	M1 A1 (5)
(b)	$\text{f}(x) = (a+bx)\left(\frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\ \ldots\right)$	
	Coefficient of x; $\frac{3a}{4}+\frac{b}{4}=0$ $(3a+b=0)$	M1
	Coefficient of x^2; $\frac{27a}{16}+\frac{3b}{4}=\frac{9}{16}$ $(9a+4b=3)$ A1 either correct	M1 A1
	Leading to $a=-1,\ b=3$	M1 A1 (5)
(c)	Coefficient of x^3 is $\frac{27a}{8}+\frac{27b}{16}=\frac{27}{8}\times(-1)+\frac{27}{16}\times 3$	M1 A1ft
	$=\frac{27}{16}$ cao	A1 (3)
		[13]

Question Number	Scheme		Marks
118.	(a) $A = 2$		B1
	$2x^2 + 5x - 10 = A(x-1)(x+2) + B(x+2) + C(x-1)$		
	$x \to 1 \quad -3 = 3B \Rightarrow B = -1$		M1 A1
	$x \to -2 \quad -12 = -3C \Rightarrow C = 4$		A1 **(4)**
	(b) $\dfrac{2x^2+5x-10}{(x-1)(x+2)} = 2 + (1-x)^{-1} + 2\left(1+\dfrac{x}{2}\right)^{-1}$		M1
	$(1-x)^{-1} = 1 + x + x^2 + \ ...$		B1
	$\left(1+\dfrac{x}{2}\right)^{-1} = 1 - \dfrac{x}{2} + \dfrac{x^2}{4} + \ ...$		B1
	$\dfrac{2x^2+5x-10}{(x-1)(x+2)} = (2+1+2) + (1-1)x + \left(1+\dfrac{1}{2}\right)x^2 + \ ...$		M1
	$= 5 + \ ...$	ft their $A - B + \frac{1}{2}C$	A1 ft
	$= \ ... + \dfrac{3}{2}x^2 + \ ...$	$0x$ stated or implied	A1 A1 **(7)**
			[11]

Question Number	Scheme	Marks
119	(a) $(1-8x)^{\frac{1}{2}} = 1+\left(\tfrac{1}{2}\right)(-8x)+\dfrac{\left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)}{2}(-8x)^2+\dfrac{\left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)}{3!}(-8x)^3+\ldots$	M1 A1
	$=1-4x-8x^2;-32x^3-\ \ldots$	A1; A1 (4)
	(b) $\sqrt{(1-8x)}=\sqrt{\left(1-\dfrac{8}{100}\right)}$	M1
	$=\sqrt{\dfrac{92}{100}}=\sqrt{\dfrac{23}{25}}=\dfrac{\sqrt{23}}{5}$ * cso	A1 (2)
	(c) $1-4x-8x^2-32x^3=1-4(0.01)-8(0.01)^2-32(0.01)^3$	
	$=1-0.04-0.0008-0.000\,032=0.959\,168$	M1
	$\sqrt{23}=5\times 0.959\,168$	M1
	$=4.795\,84$ cao	A1 (3)
		[9]

Question Number	Scheme	Marks
120	$f(x) = \frac{1}{\sqrt{(4+x)}} = (4+x)^{-\frac{1}{2}}$	M1
	$= (4)^{-\frac{1}{2}}(1+ \ \dots\ )^{\cdots}$ $\quad \frac{1}{2}(1+\ \dots\ )^{\cdots}$ or $\frac{1}{2\sqrt{(1+\dots)}}$	B1
	$= \ \dots\ \left(1+(-\tfrac{1}{2})\left(\frac{x}{4}\right)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2}\left(\frac{x}{4}\right)^2+\frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}\left(\frac{x}{4}\right)^3+\ \dots\right)$	M1 A1ft
	ft their $\left(\frac{x}{4}\right)$	
	$= \frac{1}{2} - \frac{1}{16}x, + \frac{3}{256}x^2 - \frac{5}{2048}x^3 + \ \dots$	A1, A1 (6)
		[6]
	Alternative	
	$f(x) = \frac{1}{\sqrt{(4+x)}} = (4+x)^{-\frac{1}{2}}$	M1
	$= \underline{4^{-\frac{1}{2}}} + (-\tfrac{1}{2})4^{-\frac{3}{2}}x + \frac{(-\frac{1}{2})(-\frac{3}{2})}{1.2}4^{-\frac{5}{2}}x^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{1.2.3}4^{-\frac{7}{2}}x^3 + \ \dots$	<u>B1</u> M1 A1
	$= \frac{1}{2} - \frac{1}{16}x, + \frac{3}{256}x^2 - \frac{5}{2048}x^3 + \ \dots$	A1, A1 (6)

Question Number	Scheme		Marks
121. (a)	$27x^2+32x+16 \equiv A(3x+2)(1-x)+B(1-x)+C(3x+2)^2$	Forming this identity	M1
	$x=-\frac{2}{3},\quad 12-\frac{64}{3}+16=\left(\frac{5}{3}\right)B \Rightarrow \frac{20}{3}=\left(\frac{5}{3}\right)B \Rightarrow B=4$ $x=1,\quad 27+32+16=25C \Rightarrow 75=25C \Rightarrow C=3$	Substitutes either $x=-\frac{2}{3}$ or $x=1$ into their identity or equates 3 terms or substitutes in values to write down three simultaneous equations.	M1
		Both $B=4$ and $C=3$	A1
		(Note the A1 is dependent on both method marks in this part.)	
	Equate x^2: $27=-3A+9C \Rightarrow 27=-3A+27 \Rightarrow 0=-3A \Rightarrow A=0$ $x=0,\quad 16=2A+B+4C$ $\Rightarrow 16=2A+4+12 \Rightarrow 0=2A \Rightarrow A=0$	Compares coefficients or substitutes in a third x-value or uses simultaneous equations to show $A=0$.	B1
			[4]
(b)	$f(x)=\dfrac{4}{(3x+2)^2}+\dfrac{3}{(1-x)}$		
	$=4(3x+2)^{-2}+3(1-x)^{-1}$	Moving powers to top on any one of the two expressions	M1
	$=4\left[2\left(1+\frac{3}{2}x\right)^{-2}\right]+3(1-x)^{-1}$		
	$=1\left(1+\frac{3}{2}x\right)^{-2}+3(1-x)^{-1}$		
	$=1\left\{\underline{1+(-2)(\frac{3x}{2});+\frac{(-2)(-3)}{2!}(\frac{3x}{2})^2+\ldots}\right\}$	Either $1\pm(-2)(\frac{3x}{2})$ or $1\pm(-1)(-x)$ from either first or second expansions respectively	dM1;
	$+3\left\{\underline{1+(-1)(-x);+\frac{(-1)(-2)}{2!}(-x)^2+\ldots}\right\}$	Ignoring 1 and 3, any one correct $\{\underline{\ldots\ldots\ldots\ldots}\}$ expansion.	A1
		Both $\{\underline{\ldots\ldots\ldots\ldots}\}$ correct.	A1
	$=\left\{1-3x+\frac{27}{4}x^2+\ldots\right\}+3\left\{1+x+x^2+\ldots\right\}$		
	$=4+0x\,;+\frac{39}{4}x^2$	$4+(0x)\,;\ \frac{39}{4}x^2$	A1; A1
			[6]

EXPERT TUITION

Question Number	Scheme		Marks
121. (c)	$\text{Actual} = f(0.2) = \dfrac{1.08+6.4+16}{(6.76)(0.8)}$ $= \dfrac{23.48}{5.408} = 4.341715976... = \dfrac{2935}{676}$	Attempt to find the actual value of f(0.2) or seeing awrt 4.3 and believing it is candidate's actual f(0.2).	
	Or $\text{Actual} = f(0.2) = \dfrac{4}{(3(0.2)+2)^2} + \dfrac{3}{(1-0.2)}$ $= \dfrac{4}{6.76} + 3.75 = 4.341715976... = \dfrac{2935}{676}$	Candidates can also attempt to find the actual value by using $\dfrac{A}{(3x+2)} + \dfrac{B}{(3x+2)^2} + \dfrac{C}{(1-x)}$ with their A, B and C.	M1
	$\text{Estimate} = f(0.2) = 4 + \frac{39}{4}(0.2)^2$ $= 4 + 0.39 = 4.39$	Attempt to find an estimate for f(0.2) using their answer to (b)	M1 $\sqrt{}$
	$\text{\%age error} = \dfrac{\lvert 4.39 - 4.341715976... \rvert}{4.341715976...} \times 100$	$\left\lvert \dfrac{\text{their estimate - actual}}{\text{actual}} \right\rvert \times 100$	M1
	$= 1.112095408... = 1.1\%(2\text{sf})$	1.1%	A1 **cao** **[4]**
			14 marks

EXPERT TUITION

Question Number	Scheme		Marks
	** represents a constant (which must be consistent for first accuracy mark)		
122. (a)	$\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)^{-\frac{1}{2}}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}} = \underline{\frac{1}{2}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$	$\underline{(4)^{-\frac{1}{2}}}$ or $\underline{\frac{1}{2}}$ outside brackets	<u>B1</u>
		Expands $(1+{**}x)^{-\frac{1}{2}}$ to give a simplified or an un-simplified $1+(-\frac{1}{2})({**}x)$;	M1;
	$=\frac{1}{2}\left[\underline{1+(-\frac{1}{2})({**}x);+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}({**}x)^2+\ldots}\right]$ **with** ${**}\neq 1$	A correct simplified or an un-simplified [.........] expansion with candidate's followed through $({**}x)$	A1 $\sqrt{}$
	$=\frac{1}{2}\left[\underline{1+(-\frac{1}{2})(-\frac{3x}{4})+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\frac{3x}{4})^2+\ldots}\right]$	Award SC M1 if you see $(-\frac{1}{2})({**}x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}({**}x)^2$	
	$=\frac{1}{2}\left[1+\frac{3}{8}x;+\frac{27}{128}x^2+\ldots\right]$	$\frac{1}{2}\left[1+\frac{3}{8}x;\ldots\right]$	A1 isw
		SC: $K\left[1+\frac{3}{8}x+\frac{27}{128}x^2+\ldots\right]$	
		$\frac{1}{2}\left[\ldots\ldots;\ \frac{27}{128}x^2\right]$	A1 isw
	$\left\{=\frac{1}{2}+\frac{3}{16}x;+\frac{27}{256}x^2+\ldots\right\}$	*Ignore subsequent working*	
			[5]
(b)	$(x+8)\left(\frac{1}{2}+\frac{3}{16}x+\frac{27}{256}x^2+\ldots\right)$	Writing $(x+8)$ multiplied by candidate's part (a) expansion.	M1
	$= \frac{1}{2}x+\frac{3}{16}x^2+\ldots..$ $+\ 4+\frac{3}{2}x+\frac{27}{32}x^2+\ldots..$	Multiply out brackets to find a constant term, two x terms and two x^2 terms.	M1
	$=\ 4+2x;+\frac{33}{32}x^2+\ldots$	Anything that cancels to $4+2x;\ \frac{33}{32}x^2$	A1; A1
			[4]
			9 marks

Question Number	Scheme		Marks
	** represents a constant (which must be consistent for first accuracy mark)		
123. (a)	$(8-3x)^{\frac{1}{3}} = \underline{(8)^{\frac{1}{3}}}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}}$	Takes 8 outside the bracket to give any of $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$.	B1
	$= 2\left\{1+(\tfrac{1}{3})(**x); + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(**x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(**x)^3 + \ldots\right\}$ **with** $** \neq 1$	Expands $(1+**x)^{\frac{1}{3}}$ to give a simplified or an un-simplified $1+(\frac{1}{3})(**x)$;	M1;
		A correct simplified or an un-simplified $\{\ldots\ldots\ldots\}$ expansion with candidate's followed through $(**x)$	A1 $\sqrt{}$
	$= 2\left\{1+(\tfrac{1}{3})(-\tfrac{3x}{8}) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-\tfrac{3x}{8})^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(-\tfrac{3x}{8})^3 + \ldots\right\}$	Award SC M1 if you see $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(**x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(**x)^3$	
	$= 2\left\{1 - \tfrac{1}{8}x; - \tfrac{1}{64}x^2 - \tfrac{5}{1536}x^3 - \ldots\right\}$	Either $2\{1-\frac{1}{8}x \ldots\ldots\}$ or anything that cancels to $2-\frac{1}{4}x$;	A1;
	$= 2 - \frac{1}{4}x; - \frac{1}{32}x^2 - \frac{5}{768}x^3 - \ldots$	Simplified $-\frac{1}{32}x^2 - \frac{5}{768}x^3$	A1
			[5]
(b)	$(7.7)^{\frac{1}{3}} \approx 2 - \frac{1}{4}(0.1) - \frac{1}{32}(0.1)^2 - \frac{5}{768}(0.1)^3 - \ldots$	***Attempt to substitute*** $x=0.1$ into a candidate's binomial expansion.	M1
	$= 2 - 0.025 - 0.0003125 - 0.0000065104166\ldots$		
	$= 1.97468099\ldots$	awrt 1.9746810	A1
			[2]
			7 marks

You would award B1M1A0 for

$$=2\left\{1+(\tfrac{1}{3})(-\tfrac{3x}{8}) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-\tfrac{3x}{8})^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(-3x)^3 + \ldots\right\}$$

because ** is not consistent.

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822\ldots$

Question Number	Scheme		Marks
Aliter **2.** (a) **Way 2**	$(8-3x)^{\frac{1}{3}}$		
		2 or $(8)^{\frac{1}{3}}$ **(See note ↓)**	B1
	$= \left\{ (8)^{\frac{1}{3}} + (\tfrac{1}{3})(8)^{-\frac{2}{3}}(**x); + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(**x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(**x)^3 + \ldots \right\}$	Expands $(8-3x)^{\frac{1}{3}}$ to give an un-simplified or simplified $(8)^{\frac{1}{3}} + (\frac{1}{3})(8)^{-\frac{2}{3}}(**x)$;	M1;
	with $** \neq 1$	A correct un-simplified or simplified $\{\ldots\ldots\ldots\}$ expansion with candidate's followed through $(**x)$	A1 $\sqrt{}$
	$= \left\{ (8)^{\frac{1}{3}} + (\tfrac{1}{3})(8)^{-\frac{2}{3}}(-3x); + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(-3x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(-3x)^3 + \ldots \right\}$	Award SC M1 if you see $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(**x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(**x)^3$	
	$= \left\{ 2 + (\tfrac{1}{3})(\tfrac{1}{4})(-3x) + (-\tfrac{1}{9})(\tfrac{1}{32})(9x^2) + (\tfrac{5}{81})(\tfrac{1}{256})(-27x^3) + \ldots \right\}$		
	$= 2 - \frac{1}{4}x; - \frac{1}{32}x^2 - \frac{5}{768}x^3 - \ldots$	Anything that cancels to $2 - \frac{1}{4}x$; or $2\{1 - \frac{1}{8}x \ldots\ldots\ldots\}$	A1;
		Simplified $-\frac{1}{32}x^2 - \frac{5}{768}x^3$	A1
			[5]

Attempts using Maclaurin expansion should be escalated up to your team leader.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822\ldots$

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.