EXPERT TUITION

Maths Questions By Topic:

Sequences & Series

A-Level Edexcel

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1. (i) In an arithmetic series, the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

(3)

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

Given that James takes n weeks to save exactly £64

(a) show that

$$n^2 - 26n + 160 = 0$$

(2)

(b) Solve the equation

$$n^2 - 26n + 160 = 0$$

(1)

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.

(1)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 7 marks)

2. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{2x}{9}\right)^8$$

giving each term in simplest form.

(4)

$$f(x) = \left(\frac{x-1}{2x}\right)\left(3 - \frac{2x}{9}\right)^8$$

(b) Find the coefficient of x^2 in the series expansion of $f(x)$, giving your answer as a simplified fraction.

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 6 marks)

3. The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = k - \frac{24}{u_n} \qquad u_1 = 2$$

where k is an integer.

Given that $u_1 + 2u_2 + u_3 = 0$

(a) show that

$$3k^2 - 58k + 240 = 0$$

(3)

(b) Find the value of k, giving a reason for your answer.

(2)

(c) Find the value of u_3

(1)

Question 3 continued

(Total for Question 3 is 6 marks)

4. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

(a) show that the profit for Year 3 will be £23 328

(1)

(b) find the first year when the yearly profit will exceed £65 000

(3)

(c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000

(2)

Question 4 continued

(Total for Question 4 is 6 marks)

5.

$$f(x) = \frac{50x^2 + 38x + 9}{(5x+2)^2(1-2x)} \qquad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that f(x) can be expressed in the form

$$\frac{A}{5x+2} + \frac{B}{(5x+2)^2} + \frac{C}{1-2x}$$

where A, B and C are constants

(a) (i) find the value of B and the value of C

(ii) show that $A = 0$

(4)

(b) (i) Use binomial expansions to show that, in ascending powers of x

$$f(x) = p + qx + rx^2 + \ldots$$

where p, q and r are simplified fractions to be found.

(ii) Find the range of values of x for which this expansion is valid.

(7)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 11 marks)

6. $$g(x) = (2 + ax)^8 \qquad \text{where } a \text{ is a constant}$$

Given that one of the terms in the binomial expansion of $g(x)$ is $3402x^5$

(a) find the value of a.

(4)

Using this value of a,

(b) find the constant term in the expansion of

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8$$

(3)

Question 6 continued

(Total for Question 6 is 7 marks)

7. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$(1+8x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

(b) Explain how you could use $x = \frac{1}{32}$ in the expansion to find an approximation for $\sqrt{5}$

There is no need to carry out the calculation.

(2)

Question 7 continued

(Total for Question 7 is 5 marks)

8. A car has six forward gears.

The fastest speed of the car

- in 1^{st} gear is 28 km h^{-1}
- in 6^{th} gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

(a) find the fastest speed of the car in 3^{rd} gear.

(3)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5^{th} gear.

(3)

Question 8 continued

(Total for Question 8 is 6 marks)

9. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \qquad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

(3)

(b) For this sequence explain why $k \neq 1$

(1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

(3)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 7 marks)

10. (a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$(1 + kx)^{10}$$

where k is a non-zero constant. Write each coefficient as simply as possible.

(3)

Given that in the expansion of $(1 + kx)^{10}$ the coefficient x^3 is 3 times the coefficient of x,

(b) find the possible values of k.

(3)

Question 10 continued

(Total for Question 10 is 6 marks)

11. (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

giving each coefficient in its simplest form.

(4)

The expansion can be used to find an approximation to $\sqrt{2}$
Possible values of x that could be substituted into this expansion are:

- $x = -14$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{18}} = \dfrac{\sqrt{2}}{6}$
- $x = 2$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$
- $x = -\dfrac{1}{2}$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{\dfrac{9}{2}}} = \dfrac{\sqrt{2}}{3}$

(b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of x should not be used

(1)

(ii) state, giving a reason, which of the three values of x would lead to the most accurate approximation to $\sqrt{2}$

(1)

Question 11 continued

(Total for Question 11 is 6 marks)

12. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds,
(2)

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leqslant r \leqslant 20$, is

$$6 \times 1.05^{r-4}$$

(1)

(c) estimate the total time, in minutes and seconds, that she will take to complete the race.
(4)

Question 12 continued

Question 12 continued

Question 12 continued

(Total for Question 12 is 7 marks)

13. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 + \frac{3x}{4}\right)^6$$

giving each term in its simplest form.

(4)

(b) Explain how you could use your expansion to estimate the value of 1.925^6
You do not need to perform the calculation.

(1)

Question 13 continued

(Total for Question 13 is 5 marks)

14. (a) Use binomial expansions to show that $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$

(6)

A student substitutes $x = \frac{1}{2}$ into both sides of the approximation shown in part (a) in an attempt to find an approximation to $\sqrt{6}$

(b) Give a reason why the student **should not** use $x = \frac{1}{2}$

(1)

(c) Substitute $x = \frac{1}{11}$ into

$$\sqrt{\frac{1+4x}{1-x}} = 1 + \frac{5}{2}x - \frac{5}{8}x^2$$

to obtain an approximation to $\sqrt{6}$. Give your answer as a fraction in its simplest form.

(3)

Question 14 continued

Question 14 continued

Question 14 continued

(Total for Question 14 is 10 marks)

15. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{16}\right)^9$$

giving each term in its simplest form.

(4)

$$f(x) = (a + bx)\left(2 - \frac{x}{16}\right)^9, \text{ where } a \text{ and } b \text{ are constants}$$

Given that the first two terms, in ascending powers of x, in the series expansion of $f(x)$ are 128 and $36x$,

(b) find the value of a,

(2)

(c) find the value of b.

(2)

Question 15 continued

Question 15 continued

Question 15 continued

(Total for Question 15 is 8 marks)

16. (a) Expand $\left(1+\dfrac{3}{x}\right)^2$ simplifying each term.

(2)

(b) Use the binomial expansion to find, in ascending powers of x, the first four terms in the expansion of

$$\left(1+\frac{3}{4}x\right)^6$$

simplifying each term.

(4)

(c) Hence find the coefficient of x in the expansion of

$$\left(1+\frac{3}{x}\right)^2\left(1+\frac{3}{4}x\right)^6$$

(2)

Question 16 continued

(Total for Question 16 is 8 marks)

17. (a) Show that the binomial expansion of

$$(4 + 5x)^{\frac{1}{2}}$$

in ascending powers of x, up to and including the term in x^2 is

$$2 + \frac{5}{4}x + kx^2$$

giving the value of the constant k as a simplified fraction.

(4)

(b) (i) Use the expansion from part (a), with $x = \frac{1}{10}$, to find an approximate value for $\sqrt{2}$

Give your answer in the form $\frac{p}{q}$ where p and q are integers.

(ii) Explain why substituting $x = \frac{1}{10}$ into this binomial expansion leads to a valid approximation.

(4)

Question 17 continued

(Total for Question 17 is 8 marks)

18. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3$$

$$a_{n+1} = \frac{a_n - 3}{a_n - 2}, \qquad n \in \mathbb{N}$$

(a) Find $\sum_{r=1}^{100} a_r$

(3)

(b) Hence find $\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r$

(1)

Question 18 continued

(Total for Question 18 is 4 marks)

19. There were 2100 tonnes of wheat harvested on a farm during 2017.

The mass of wheat harvested during each subsequent year is expected to increase by 1.2% per year.

(a) Find the total mass of wheat expected to be harvested from 2017 to 2030 inclusive, giving your answer to 3 significant figures.

(2)

Each year it costs

- £5.15 per tonne to harvest the first 2000 tonnes of wheat
- £6.45 per tonne to harvest wheat in excess of 2000 tonnes

(b) Use this information to find the expected cost of harvesting the wheat from 2017 to 2030 inclusive. Give your answer to the nearest £1000

(3)

Question 19 continued

(Total for Question 19 is 5 marks)

20. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of $\left(2 - \frac{x}{2}\right)^7$, giving each term in its simplest form.

(4)

(b) Explain how you would use your expansion to give an estimate for the value of 1.995^7

(1)

Question 20 continued

(Total for Question 20 is 5 marks)

21. A sequence of terms $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3$$
$$a_{n+1} = 8 - a_n$$

(a) (i) Show that this sequence is periodic.

(ii) State the order of this periodic sequence.

(2)

(b) Find the value of

$$\sum_{n=1}^{85} a_n$$

(2)

Question 21 continued

(Total for Question 21 is 4 marks)

22. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\sqrt{4 - 9x}$$

writing each term in simplest form.

(4)

A student uses this expansion with $x = \frac{1}{9}$ to find an approximation for $\sqrt{3}$

Using the answer to part (a) and without doing any calculations,

(b) state whether this approximation will be an overestimate or an underestimate of $\sqrt{3}$ giving a brief reason for your answer.

(1)

Question 22 continued

(Total for Question 22 is 5 marks)

23. In an arithmetic series

- the first term is 16
- the 21st term is 24

(a) Find the common difference of the series.

(2)

(b) Hence find the sum of the first 500 terms of the series.

(2)

Question 23 continued

(Total for Question 23 is 4 marks)

24. Show that

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{9}{28}$$

(3)

Question 24 continued

(Total for Question 24 is 3 marks)

25. In the binomial expansion of

$$(a + 2x)^7 \qquad \text{where } a \text{ is a constant}$$

the coefficient of x^4 is 15 120

Find the value of a.

(3)

Question 25 continued

(Total for Question 25 is 3 marks)

26. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a.

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r}$$

(4)

Given also that S_{10} is four times S_5

(b) find the exact value of r.

(4)

Question 26 continued

Question 26 continued

Question 26 continued

(Total for Question 26 is 8 marks)

27. (i) Find the value of

$$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$$

(3)

(ii) Show that

$$\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 2$$

(3)

Question 27 continued

Question 27 continued

Question 27 continued

(Total for Question 27 is 6 marks)

28. (i) Show that $\sum_{r=1}^{16}\left(3 + 5r + 2^r\right) = 131\,798$

(4)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$

(3)

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Question 28 continued

(Total for Question 28 is 7 marks)

29. The second, third and fourth terms of an arithmetic sequence are $2k$, $5k - 10$ and $7k - 14$ respectively, where k is a constant.

Show that the sum of the first n terms of the sequence is a square number.

(5)

Question 29 continued

(Total for Question 29 is 5 marks)

30. (a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \dots$$

where k is a rational constant to be found.

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x.

(1)

Question 30 continued

(Total for Question 30 is 5 marks)

31. In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm\frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

(Total for Question 31 is 4 marks)

Leave blank

32. Each year, Andy pays into a savings scheme. In year one he pays in £600. His payments increase by £120 each year so that he pays £720 in year two, £840 in year three and so on, so that his payments form an arithmetic sequence.

(a) Find out how much Andy pays into the savings scheme in year ten.

(2)

Kim starts paying money into a different savings scheme at the same time as Andy. In year one she pays in £130. Her payments increase each year so that she pays £210 in year two, £290 in year three and so on, so that her payments form a different arithmetic sequence.

At the end of year N, Andy has paid, in total, twice as much money into his savings scheme as Kim has paid, in total, into her savings scheme.

(b) Find the value of N.

(5)

Leave blank

Question 32 continued

Leave blank

Question 32 continued

Leave blank

Question 32 continued

(Total 7 marks)

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Leave blank

33. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4$$
$$a_{n+1} = \frac{a_n}{a_n + 1}, \qquad n \geqslant 1, n \in \mathbb{N}$$

(a) Find the values of a_2, a_3 and a_4

Write your answers as simplified fractions.

(3)

Given that

$$a_n = \frac{4}{pn + q}, \text{ where } p \text{ and } q \text{ are constants}$$

(b) state the value of p and the value of q.

(2)

(c) Hence calculate the value of N such that $a_N = \dfrac{4}{321}$

(2)

Leave blank

Question 33 continued

(Total 7 marks)

Leave blank

34. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 1$$
$$a_{n+1} = \frac{k(a_n + 1)}{a_n}, \qquad n \geqslant 1$$

where k is a positive constant.

(a) Write down expressions for a_2 and a_3 in terms of k, giving your answers in their simplest form.

(3)

Given that $\sum_{r=1}^{3} a_r = 10$

(b) find an exact value for k.

(3)

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Leave blank

Question 34 continued

(Total 6 marks)

Leave blank

35. A company, which is making 140 bicycles each week, plans to increase its production. The number of bicycles produced is to be increased by d each week, starting from 140 in week 1, to $140 + d$ in week 2, to $140 + 2d$ in week 3 and so on, until the company is producing 206 in week 12.

(a) Find the value of d.

(2)

After week 12 the company plans to continue making 206 bicycles each week.

(b) Find the total number of bicycles that would be made in the first 52 weeks starting from and including week 1.

(5)

Leave blank

Question 35 continued

(Total 7 marks)

Leave blank

36. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4,$$

$$a_{n+1} = 5 - ka_n, \quad n \geqslant 1$$

where k is a constant.

(a) Write down expressions for a_2 and a_3 in terms of k.

(2)

Find

(b) $\sum_{r=1}^{3}(1 + a_r)$ in terms of k, giving your answer in its simplest form,

(3)

(c) $\sum_{r=1}^{100}(a_{r+1} + ka_r)$

(1)

(Total 6 marks)

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Leave blank

37. On John's 10th birthday he received the first of an annual birthday gift of money from his uncle. This first gift was £60 and on each subsequent birthday the gift was £15 more than the year before. The amounts of these gifts form an arithmetic sequence.

(a) Show that, immediately after his 12th birthday, the total of these gifts was £225

(1)

(b) Find the amount that John received from his uncle as a birthday gift on his 18th birthday.

(2)

(c) Find the total of these birthday gifts that John had received from his uncle up to and including his 21st birthday.

(3)

When John had received n of these birthday gifts, the total money that he had received from these gifts was £3375

(d) Show that $n^2 + 7n = 25 \times 18$

(3)

(e) Find the value of n, when he had received £3375 in total, and so determine John's age at this time.

(2)

Leave blank

Question 37 continued

(Total 11 marks)

Leave blank

38. (i) A sequence $U_1, U_2, U_3, \ldots$ is defined by

$$U_{n+2} = 2U_{n+1} - U_n, \quad n \geqslant 1$$

$$U_1 = 4 \text{ and } U_2 = 4$$

Find the value of

(a) U_3

(1)

(b) $\sum_{n=1}^{20} U_n$

(2)

(ii) Another sequence $V_1, V_2, V_3, \ldots$ is defined by

$$V_{n+2} = 2V_{n+1} - V_n, \quad n \geqslant 1$$

$$V_1 = k \text{ and } V_2 = 2k, \text{ where } k \text{ is a constant}$$

(a) Find V_3 and V_4 in terms of k.

(2)

Given that $\sum_{n=1}^{5} V_n = 165$,

(b) find the value of k.

(3)

Leave blank

Question 38 continued

(Total 8 marks)

Leave blank

39. Jess started work 20 years ago. In year 1 her annual salary was £17 000. Her annual salary increased by £1 500 each year, so that her annual salary in year 2 was £18 500, in year 3 it was £20 000 and so on, forming an arithmetic sequence. This continued until she reached her maximum annual salary of £32 000 in year k. Her annual salary then remained at £32 000.

(a) Find the value of the constant k.

(2)

(b) Calculate the total amount that Jess has earned in the 20 years.

(5)

(Total 7 marks)

Leave blank

40. A sequence of numbers $a_1, a_2, a_3 \ldots$ is defined by

$$a_{n+1} = 5a_n - 3, \qquad n \geqslant 1$$

Given that $a_2 = 7$,

(a) find the value of a_1

(2)

(b) Find the value of $\sum_{r=1}^{4} a_r$

(3)

(Total 5 marks)

Leave blank

41. In the year 2000 a shop sold 150 computers. Each year the shop sold 10 more computers than the year before, so that the shop sold 160 computers in 2001, 170 computers in 2002, and so on forming an arithmetic sequence.

(a) Show that the shop sold 220 computers in 2007.

(2)

(b) Calculate the total number of computers the shop sold from 2000 to 2013 inclusive.

(3)

In the year 2000, the selling price of each computer was £900. The selling price fell by £20 each year, so that in 2001 the selling price was £880, in 2002 the selling price was £860, and so on forming an arithmetic sequence.

(c) In a particular year, the selling price of each computer in £s was equal to three times the number of computers the shop sold in that year. By forming and solving an equation, find the year in which this occurred.

(4)

Leave blank

Question 41 continued

(Total 9 marks)

Leave blank

42. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 4a_n - 3, \qquad n \geqslant 1$$

$$a_1 = k, \qquad \text{where } k \text{ is a positive integer.}$$

(a) Write down an expression for a_2 in terms of k.

(1)

Given that $\sum_{r=1}^{3} a_r = 66$

(b) find the value of k.

(4)

Leave blank

Question 42 continued

(Total 5 marks)

Leave blank

43. Xin has been given a 14 day training schedule by her coach.

Xin will run for A minutes on day 1, where A is a constant.

She will then increase her running time by $(d + 1)$ minutes each day, where d is a constant.

(a) Show that on day 14, Xin will run for

$$(A + 13d + 13) \text{ minutes.}$$

(2)

Yi has also been given a 14 day training schedule by her coach.

Yi will run for $(A - 13)$ minutes on day 1.

She will then increase her running time by $(2d - 1)$ minutes each day.

Given that Yi and Xin will run for the same length of time on day 14,

(b) find the value of d.

(3)

Given that Xin runs for a total time of 784 minutes over the 14 days,

(c) find the value of A.

(3)

Leave blank

Question 43 continued

(Total 8 marks)

Leave blank

44. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4$$
$$a_{n+1} = k(a_n + 2), \qquad \text{for } n \geqslant 1$$

where k is a constant.

(a) Find an expression for a_2 in terms of k.

(1)

Given that $\sum_{i=1}^{3} a_i = 2$,

(b) find the two possible values of k.

(6)

(Total 7 marks)

Leave blank

45. A company, which is making 200 mobile phones each week, plans to increase its production.

The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, to 240 in week 3 and so on, until it is producing 600 in week N.

(a) Find the value of N.

(2)

The company then plans to continue to make 600 mobile phones each week.

(b) Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1.

(5)

(Total 7 marks)

Leave blank

46. A sequence $x_1, x_2, x_3 \ldots$ is defined by

$$x_1 = 1$$

$$x_{n+1} = (x_n)^2 - kx_n, \quad n \geqslant 1$$

where k is a constant, $k \neq 0$

(a) Find an expression for x_2 in terms of k.

(1)

(b) Show that $x_3 = 1 - 3k + 2k^2$

(2)

Given also that $x_3 = 1$,

(c) calculate the value of k.

(3)

(d) Hence find the value of $\sum_{n=1}^{100} x_n$

(3)

Leave blank

Question 46 continued

(Total 9 marks)

Leave blank

47. Each year, Abbie pays into a savings scheme. In the first year she pays in £500. Her payments then increase by £200 each year so that she pays £700 in the second year, £900 in the third year and so on.

(a) Find out how much Abbie pays into the savings scheme in the tenth year.

(2)

Abbie pays into the scheme for n years until she has paid in a total of £67200.

(b) Show that $n^2 + 4n - 24 \times 28 = 0$

(5)

(c) Hence find the number of years that Abbie pays into the savings scheme.

(2)

Leave blank

Question 47 continued

(Total 9 marks)

Leave blank

48. A sequence $u_1, u_2, u_3, \ldots$ satisfies

$$u_{n+1} = 2u_n - 1, \quad n \geqslant 1$$

Given that $u_2 = 9$,

(a) find the value of u_3 and the value of u_4,

(2)

(b) evaluate $\sum_{r=1}^{4} u_r$.

(3)

(Total 5 marks)

Leave blank

49. Lewis played a game of space invaders. He scored points for each spaceship that he captured.

Lewis scored 140 points for capturing his first spaceship.

He scored 160 points for capturing his second spaceship, 180 points for capturing his third spaceship, and so on.

The number of points scored for capturing each successive spaceship formed an arithmetic sequence.

(a) Find the number of points that Lewis scored for capturing his 20th spaceship.
(2)

(b) Find the total number of points Lewis scored for capturing his first 20 spaceships.
(3)

Sian played an adventure game. She scored points for each dragon that she captured. The number of points that Sian scored for capturing each successive dragon formed an arithmetic sequence.

Sian captured n dragons and the total number of points that she scored for capturing all n dragons was 8500.

Given that Sian scored 300 points for capturing her first dragon and then 700 points for capturing her nth dragon,

(c) find the value of n.
(3)

Leave blank

Question 49 continued

(Total 8 marks)

Leave blank

50. A sequence of numbers a_1, a_2, a_3 ... is defined by

$$a_1 = 3$$

$$a_{n+1} = 2a_n - c \qquad (n \geqslant 1)$$

where c is a constant.

(a) Write down an expression, in terms of c, for a_2

(1)

(b) Show that $a_3 = 12 - 3c$

(2)

Given that $\sum_{i=1}^{4} a_i \geqslant 23$

(c) find the range of values of c.

(4)

(Total 7 marks)

EXPERT TUITION

Leave blank

51. A boy saves some money over a period of 60 weeks. He saves 10p in week 1, 15p in week 2, 20p in week 3 and so on until week 60. His weekly savings form an arithmetic sequence.

(a) Find how much he saves in week 15

(2)

(b) Calculate the total amount he saves over the 60 week period.

(3)

The boy's sister also saves some money each week over a period of m weeks. She saves 10p in week 1, 20p in week 2, 30p in week 3 and so on so that her weekly savings form an arithmetic sequence. She saves a total of £63 in the m weeks.

(c) Show that

$$m(m + 1) = 35 \times 36$$

(4)

(d) Hence write down the value of m.

(1)

Leave blank

Question 51 continued

(Total 10 marks)

Leave blank

52. A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1$$

$$x_{n+1} = ax_n + 5, \qquad n \geqslant 1$$

where a is a constant.

(a) Write down an expression for x_2 in terms of a.

(1)

(b) Show that $x_3 = a^2 + 5a + 5$

(2)

Given that $x_3 = 41$

(c) find the possible values of a.

(3)

(Total 6 marks)

Leave blank

53. A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is £P.
Salary increases by £($2T$) each year, forming an arithmetic sequence.

Scheme 2: Salary in Year 1 is £(P + 1800).
Salary increases by £T each year, forming an arithmetic sequence.

(a) Show that the **total** earned under Salary Scheme 1 for the 10-year period is

$$£(10P + 90T)$$

(2)

For the 10-year period, the **total** earned is the same for both salary schemes.

(b) Find the value of T.

(4)

For this value of T, the salary in Year 10 under Salary Scheme 2 is £29 850

(c) Find the value of P.

(3)

Leave blank

Question 53 continued

(Total 9 marks)

Leave blank

54. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$
$$a_{n+1} = 5a_n + 3, \qquad n \geqslant 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k.

(1)

(b) Show that $a_3 = 25k + 18$.

(2)

(c) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k, in its simplest form.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 6.

(4)

(Total 7 marks)

Leave blank

55. (a) Calculate the sum of all the even numbers from 2 to 100 inclusive,

$$2 + 4 + 6 + \ldots\ldots + 100$$

(3)

(b) In the arithmetic series

$$k + 2k + 3k + \ldots\ldots + 100$$

k is a positive integer and k is a factor of 100.

(i) Find, in terms of k, an expression for the number of terms in this series.

(ii) Show that the sum of this series is

$$50 + \frac{5000}{k}$$

(4)

(c) Find, in terms of k, the 50th term of the arithmetic sequence

$$(2k + 1), (4k + 4), (6k + 7), \ldots\ldots,$$

giving your answer in its simplest form.

(2)

Leave blank

Question 55 continued

(Total 9 marks)

Leave blank

56. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 2$$
$$a_{n+1} = 3a_n - c$$

where c is a constant.

(a) Find an expression for a_2 in terms of c.

(1)

Given that $\sum_{i=1}^{3} a_i = 0$

(b) find the value of c.

(4)

(Total 5 marks)

Leave blank

57. An arithmetic sequence has first term a and common difference d. The sum of the first 10 terms of the sequence is 162.

(a) Show that $10a + 45d = 162$

(2)

Given also that the sixth term of the sequence is 17,

(b) write down a second equation in a and d,

(1)

(c) find the value of a and the value of d.

(4)

(Total 7 marks)

Leave blank

58. A sequence of positive numbers is defined by

$$a_{n+1} = \sqrt{(a_n^2 + 3)}, \qquad n \geqslant 1,$$
$$a_1 = 2$$

(a) Find a_2 and a_3, leaving your answers in surd form.

(2)

(b) Show that $a_5 = 4$

(2)

(Total 4 marks)

Leave blank

59. A farmer has a pay scheme to keep fruit pickers working throughout the 30 day season. He pays £a for their first day, £$(a + d)$ for their second day, £$(a + 2d)$ for their third day, and so on, thus increasing the daily payment by £d for each extra day they work.

A picker who works for all 30 days will earn £40.75 on the final day.

(a) Use this information to form an equation in a and d.

(2)

A picker who works for all 30 days will earn a total of £1005

(b) Show that $15(a+40.75) = 1005$

(2)

(c) Hence find the value of a and the value of d.

(4)

(Total 8 marks)

Leave blank

60. Jill gave money to a charity over a 20-year period, from Year 1 to Year 20 inclusive. She gave £150 in Year 1, £160 in Year 2, £170 in Year 3, and so on, so that the amounts of money she gave each year formed an arithmetic sequence.

(a) Find the amount of money she gave in Year 10.

(2)

(b) Calculate the total amount of money she gave over the 20-year period.

(3)

Kevin also gave money to the charity over the same 20-year period.

He gave £A in Year 1 and the amounts of money he gave each year increased, forming an arithmetic sequence with common difference £30.
The total amount of money that Kevin gave over the 20-year period was **twice** the total amount of money that Jill gave.

(c) Calculate the value of A.

(4)

Leave blank

Question 60 continued

(Total 9 marks)

Leave blank

61. A 40-year building programme for new houses began in Oldtown in the year 1951 (Year 1) and finished in 1990 (Year 40).

The numbers of houses built each year form an arithmetic sequence with first term a and common difference d.

Given that 2400 new houses were built in 1960 and 600 new houses were built in 1990, find

(a) the value of d,
(3)

(b) the value of a,
(2)

(c) the total number of houses built in Oldtown over the 40-year period.
(3)

(Total 8 marks)

Leave blank

62. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$

$$a_{n+1} = 2a_n - 7, \qquad n \geqslant 1,$$

where k is a constant.

(a) Write down an expression for a_2 in terms of k.

(1)

(b) Show that $a_3 = 4k - 21$.

(2)

Given that $\sum_{r=1}^{4} a_r = 43$,

(c) find the value of k.

(4)

(Total 7 marks)

Leave blank

63. The first term of an arithmetic series is a and the common difference is d.

The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

(a) Use this information to write down two equations for a and d.

(2)

(b) Show that $a = -17.5$ and find the value of d.

(2)

The sum of the first n terms of the series is 2750.

(c) Show that n is given by

$$n^2 - 15n = 55 \times 40.$$

(4)

(d) Hence find the value of n.

(3)

Leave blank

Question 63 continued

(Total 11 marks)

Leave blank

64. A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1,$$

$$x_{n+1} = ax_n - 3, \quad n \geqslant 1,$$

where a is a constant.

(a) Find an expression for x_2 in terms of a.

(1)

(b) Show that $x_3 = a^2 - 3a - 3$.

(2)

Given that $x_3 = 7$,

(c) find the possible values of a.

(3)

(Total 6 marks)

Leave blank

65. Sue is training for a marathon. Her training includes a run every Saturday starting with a run of 5 km on the first Saturday. Each Saturday she increases the length of her run from the previous Saturday by 2 km.

(a) Show that on the 4th Saturday of training she runs 11 km.
(1)

(b) Find an expression, in terms of n, for the length of her training run on the nth Saturday.
(2)

(c) Show that the total distance she runs on Saturdays in n weeks of training is $n(n + 4)$ km.
(3)

On the nth Saturday Sue runs 43 km.

(d) Find the value of n.
(2)

(e) Find the total distance, in km, Sue runs on Saturdays in n weeks of training.
(2)

Leave blank

Question 65 continued

(Total 10 marks)

Leave blank

66. A sequence is given by:

$$x_1 = 1,$$
$$x_{n+1} = x_n(p + x_n),$$

where p is a constant ($p \neq 0$).

(a) Find x_2 in terms of p.

(1)

(b) Show that $x_3 = 1 + 3p + 2p^2$.

(2)

Given that $x_3 = 1$,

(c) find the value of p,

(3)

(d) write down the value of x_{2008}.

(2)

Leave blank

Question 66 continued

(Total 8 marks)

Leave blank

67. The first term of an arithmetic sequence is 30 and the common difference is –1.5

(a) Find the value of the 25th term.

(2)

The rth term of the sequence is 0.

(b) Find the value of r.

(2)

The sum of the first n terms of the sequence is S_n.

(c) Find the largest positive value of S_n.

(3)

Leave blank

Question 67 continued

(Total 7 marks)

Leave blank

68. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(2 + kx)^7$$

where k is a non-zero constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^3 in this expansion is 1890

(b) find the value of k.

(3)

Leave blank

Question 68 continued

(Total 7 marks)

EXPERT TUITION

Leave blank

69. A geometric series with common ratio $r = -0.9$ has sum to infinity 10 000

For this series,

(a) find the first term,

(2)

(b) find the fifth term,

(2)

(c) find the sum of the first twelve terms, giving this answer to the nearest integer.

(3)

Leave blank

Question 69 continued

(Total 7 marks)

Leave blank

70. Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{1}{3}x\right)^5$$

giving each term in its simplest form.

(4)

Leave blank

Question 70 continued

(Total 4 marks)

Leave blank

71. The first three terms of a geometric sequence are

$$7k - 5, \ 5k - 7, \ 2k + 10$$

where k is a constant.

(a) Show that $11k^2 - 130k + 99 = 0$

(4)

Given that k is not an integer,

(b) show that $k = \dfrac{9}{11}$

(2)

For this value of k,

(c) (i) evaluate the fourth term of the sequence, giving your answer as an exact fraction,

(ii) evaluate the sum of the first ten terms of the sequence.

(6)

Leave blank

Question 71 continued

Leave blank

Question 71 continued

Leave blank

Question 71 continued

(Total 12 marks)

Leave blank

72. A geometric series has first term a and common ratio $r = \frac{3}{4}$

The sum of the first 4 terms of this series is 175

(a) Show that $a = 64$

(2)

(b) Find the sum to infinity of the series.

(2)

(c) Find the difference between the 9th and 10th terms of the series.
Give your answer to 3 decimal places.

(3)

(Total 7 marks)

Leave blank

73. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 9x)^4$$

giving each term in its simplest form.

(4)

$$f(x) = (1 + kx)(2 - 9x)^4, \text{ where } k \text{ is a constant}$$

The expansion, in ascending powers of x, of $f(x)$ up to and including the term in x^2 is

$$A - 232x + Bx^2$$

where A and B are constants.

(b) Write down the value of A.

(1)

(c) Find the value of k.

(2)

(d) Hence find the value of B.

(2)

Leave blank

Question 73 continued

(Total 9 marks)

Leave blank

74. Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{4}\right)^{10}$$

giving each term in its simplest form.

(4)

(Total 4 marks)

EXPERT TUITION

Leave blank

75. (i) All the terms of a geometric series are positive. The sum of the first two terms is 34 and the sum to infinity is 162

Find

(a) the common ratio,

(4)

(b) the first term.

(2)

(ii) A different geometric series has a first term of 42 and a common ratio of $\frac{6}{7}$.

Find the smallest value of n for which the sum of the first n terms of the series exceeds 290

(4)

Leave blank

Question 75 continued

(Total 10 marks)

Leave blank

76. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 3x)^6$$

giving each term in its simplest form.

(4)

(b) Hence, or otherwise, find the first 3 terms, in ascending powers of x, of the expansion of

$$\left(1 + \frac{x}{2}\right)(2 - 3x)^6$$

(3)

Leave blank

Question 76 continued

(Total 7 marks)

EXPERT TUITION

Leave blank

77. The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$

The sum to infinity of the series is S_∞

(a) Find the value of S_∞

(2)

The sum to N terms of the series is S_N

(b) Find, to 1 decimal place, the value of S_{12}

(2)

(c) Find the smallest value of N, for which

$$S_\infty - S_N < 0.5$$

(4)

Leave blank

Question 77 continued

(Total 8 marks)

Leave blank

78. Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(1 + \frac{3x}{2}\right)^8$$

giving each term in its simplest form.

(4)

(Total 4 marks)

Leave blank

79. A geometric series has first term a, where $a \neq 0$, and common ratio r.
The sum to infinity of this series is 6 times the first term of the series.

(a) Show that $r = \dfrac{5}{6}$

(2)

Given that the fourth term of this series is 62.5

(b) find the value of a,

(2)

(c) find the difference between the sum to infinity and the sum of the first 30 terms, giving your answer to 3 significant figures.

(4)

Leave blank

Question 79 continued

(Total 8 marks)

Leave blank

80. The first three terms of a geometric series are

$$18, 12 \text{ and } p$$

respectively, where p is a constant.

Find

(a) the value of the common ratio of the series,

(1)

(b) the value of p,

(1)

(c) the sum of the first 15 terms of the series, giving your answer to 3 decimal places.

(2)

(Total 4 marks)

Leave blank

81. (a) Use the binomial theorem to find all the terms of the expansion of

$$(2 + 3x)^4$$

Give each term in its simplest form.

(4)

(b) Write down the expansion of

$$(2 - 3x)^4$$

in ascending powers of x, giving each term in its simplest form.

(1)

(Total 5 marks)

Leave blank

82. Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{1}{2}x\right)^8$$

giving each term in its simplest form.

(4)

(Total 4 marks)

Leave blank

83. The first three terms of a geometric series are $4p$, $(3p + 15)$ and $(5p + 20)$ respectively, where p is a **positive** constant.

(a) Show that $11p^2 - 10p - 225 = 0$

(4)

(b) Hence show that $p = 5$

(2)

(c) Find the common ratio of this series.

(2)

(d) Find the sum of the first ten terms of the series, giving your answer to the nearest integer.

(3)

Leave blank

Question 83 continued

(Total 11 marks)

EXPERT TUITION

Leave blank

84. Find the first 3 terms, in ascending powers of x, in the binomial expansion of

$$(2-5x)^6$$

Give each term in its simplest form.

(4)

(Total 4 marks)

Leave blank

85. A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05

(a) Show that the predicted profit in the year 2016 is £138 915

(1)

(b) Find the first year in which the yearly predicted profit exceeds £200 000

(5)

(c) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound.

(3)

(Total 9 marks)

Leave blank

86. Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 3x)^5$$

giving each term in its simplest form.

(4)

(Total 4 marks)

Leave blank

87. A geometric series is $a + ar + ar^2 + \ldots$

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

(4)

The third and fifth terms of a geometric series are 5.4 and 1.944 respectively and all the terms in the series are positive.

For this series find,

(b) the common ratio,

(2)

(c) the first term,

(2)

(d) the sum to infinity.

(3)

Leave blank

Question 87 continued

(Total 11 marks)

Leave blank

88. A geometric series has first term $a = 360$ and common ratio $r = \frac{7}{8}$

Giving your answers to 3 significant figures where appropriate, find

(a) the 20th term of the series,

(2)

(b) the sum of the first 20 terms of the series,

(2)

(c) the sum to infinity of the series.

(2)

(Total 6 marks)

EXPERT TUITION

Leave blank

89. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(1+\frac{x}{4}\right)^{8}$$

giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)

(Total 7 marks)

Leave blank

90. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(3 + bx)^5$$

where b is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in this expansion, the coefficient of x^2 is twice the coefficient of x,

(b) find the value of b.

(2)

(Total 6 marks)

EXPERT TUITION

Leave blank

91. The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

(a) the common ratio,

(2)

(b) the first term,

(2)

(c) the sum to infinity,

(2)

(d) the smallest value of n for which the sum of the first n terms of the series exceeds 1000.

(4)

Leave blank

Question 91 continued

(Total 10 marks)

Leave blank

92. The second and fifth terms of a geometric series are 750 and –6 respectively.

Find

(a) the common ratio of the series,

(3)

(b) the first term of the series,

(2)

(c) the sum to infinity of the series.

(2)

(Total 7 marks)

Leave blank

93. Given that $\binom{40}{4} = \dfrac{40!}{4!b!}$,

(a) write down the value of b.

(1)

In the binomial expansion of $(1+x)^{40}$, the coefficients of x^4 and x^5 are p and q respectively.

(b) Find the value of $\dfrac{q}{p}$.

(3)

(Total 4 marks)

Leave blank

94.(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1+ax)^7$, where a is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 in this expansion is 525,

(b) find the possible values of a.

(2)

(Total 6 marks)

EXPERT TUITION

Leave blank

95. A car was purchased for £18 000 on 1st January.
On 1st January each following year, the value of the car is 80% of its value on 1st January in the previous year.

(a) Show that the value of the car exactly 3 years after it was purchased is £9216.

(1)

The value of the car falls below £1000 for the first time n years after it was purchased.

(b) Find the value of n.

(3)

An insurance company has a scheme to cover the maintenance of the car.
The cost is £200 for the first year, and for every following year the cost increases by 12% so that for the 3rd year the cost of the scheme is £250.88

(c) Find the cost of the scheme for the 5th year, giving your answer to the nearest penny.

(2)

(d) Find the total cost of the insurance scheme for the first 15 years.

(3)

Leave blank

Question 95 continued

(Total 9 marks)

Leave blank

96. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 + kx)^7$$

where k is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 is 6 times the coefficient of x,

(b) find the value of k.

(2)

(Total 96 marks)

Leave blank

97. The third term of a geometric sequence is 324 and the sixth term is 96

(a) Show that the common ratio of the sequence is $\frac{2}{3}$

(2)

(b) Find the first term of the sequence.

(2)

(c) Find the sum of the first 15 terms of the sequence.

(3)

(d) Find the sum to infinity of the sequence.

(2)

Leave blank

Question 97 continued

(Total 9 marks)

EXPERT TUITION

Leave blank

98. Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(3-2x)^5$, giving each term in its simplest form.

(4)

(Total 4 marks)

EXPERT TUITION

Leave blank

99. The first three terms of a geometric series are $(k + 4)$, k and $(2k - 15)$ respectively, where k is a positive constant.

(a) Show that $k^2 - 7k - 60 = 0$.

(4)

(b) Hence show that $k = 12$.

(2)

(c) Find the common ratio of this series.

(2)

(d) Find the sum to infinity of this series.

(2)

Leave blank

Question 99 continued

(Total 10 marks)

Leave blank

100. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 + ax)^{10}$, where a is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in this expansion, the coefficient of x^3 is double the coefficient of x^2,

(b) find the value of a.

(2)

(Total 6 marks)

EXPERT TUITION

Leave blank

101. A geometric series has first term 5 and common ratio $\frac{4}{5}$.

Calculate

(a) the 20th term of the series, to 3 decimal places,

(2)

(b) the sum to infinity of the series.

(2)

Given that the sum to k terms of the series is greater than 24.95,

(c) show that $k > \dfrac{\log 0.002}{\log 0.8}$,

(4)

(d) find the smallest possible value of k.

(1)

Leave blank

Question 101 continued

(Total 9 marks)

Leave blank

102. The fourth term of a geometric series is 10 and the seventh term of the series is 80.

For this series, find

(a) the common ratio,

(2)

(b) the first term,

(2)

(c) the sum of the first 20 terms, giving your answer to the nearest whole number.

(2)

(Total 6 marks)

Leave blank

103. (a) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)

(Total 7 marks)

Leave blank

104. (a) Find the binomial series expansion of

$$\sqrt{4 - 9x}, \quad |x| < \frac{4}{9}$$

in ascending powers of x, up to and including the term in x^2
Give each coefficient in its simplest form.

(5)

(b) Use the expansion from part (a), with a suitable value of x, to find an approximate value for $\sqrt{310}$
Show all your working and give your answer to 3 decimal places.

(3)

Leave blank

Question 104 continued

(Total 8 marks)

Leave blank

105. $f(x) = (2 + kx)^{-3}$, $|kx| < 2$, where k is a positive constant

The binomial expansion of f(x), in ascending powers of x, up to and including the term in x^2 is

$$A + Bx + \frac{243}{16}x^2$$

where A and B are constants.

(a) Write down the value of A.

(1)

(b) Find the value of k.

(3)

(c) Find the value of B.

(2)

Leave blank

Question 105 continued

(Total 6 marks)

Leave blank

106. Use the binomial series to find the expansion of

$$\frac{1}{(2+5x)^3}, \qquad |x| < \frac{2}{5}$$

in ascending powers of x, up to and including the term in x^3.
Give each coefficient as a fraction in its simplest form.

(6)

(Total 6 marks)

EXPERT TUITION

Leave blank

107. (a) Find the binomial expansion of

$$(4 + 5x)^{\frac{1}{2}}, \quad |x| < \frac{4}{5}$$

in ascending powers of x, up to and including the term in x^2.
Give each coefficient in its simplest form.

(5)

(b) Find the exact value of $(4 + 5x)^{\frac{1}{2}}$ when $x = \frac{1}{10}$

Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined.

(1)

(c) Substitute $x = \frac{1}{10}$ into your binomial expansion from part (a) and hence find an approximate value for $\sqrt{2}$

Give your answer in the form $\frac{p}{q}$ where p and q are integers.

(2)

(Total 8 marks)

Leave blank

108. Given that the binomial expansion of $(1 + kx)^{-4}$, $|kx| < 1$, is

$$1 - 6x + Ax^2 + \dots$$

(a) find the value of the constant k,

(2)

(b) find the value of the constant A, giving your answer in its simplest form.

(3)

Leave blank

Question 108 continued

(Total 5 marks)

Leave blank

109. (a) Find the binomial expansion of

$$\frac{1}{\sqrt{(9-10x)}}, \qquad |x| < \frac{9}{10}$$

in ascending powers of x up to and including the term in x^2.
Give each coefficient as a simplified fraction.

(5)

(b) Hence, or otherwise, find the expansion of

$$\frac{3+x}{\sqrt{(9-10x)}}, \qquad |x| < \frac{9}{10}$$

in ascending powers of x, up to and including the term in x^2.
Give each coefficient as a simplified fraction.

(3)

(Total 8 marks)

Leave blank

110. (a) Use the binomial expansion to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1 + x + \frac{1}{2}x^2, \quad |x| < 1$$

(6)

(b) Substitute $x = \frac{1}{26}$ into

$$\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2$$

to obtain an approximation to $\sqrt{3}$

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

(3)

(Total 9 marks)

Leave blank

111. (a) Find the binomial expansion of

$$\sqrt[3]{(8-9x)}, \qquad |x| < \frac{8}{9}$$

in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(6)

(b) Use your expansion to estimate an approximate value for $\sqrt[3]{7100}$, giving your answer to 4 decimal places. State the value of x, which you use in your expansion, and show all your working.

(3)

(Total 9 marks)

Leave blank

112. (a) Find the binomial expansion of

$$\sqrt{(9 + 8x)}, \quad |x| < \frac{9}{8}$$

in ascending powers of x, up to and including the term in x^2.
Give each coefficient as a simplified fraction.

(5)

(b) Use your expansion to estimate the value of $\sqrt{(11)}$, giving your answer as a single fraction.

(3)

(Total 8 marks)

Leave blank

113. Given

$$f(x) = (2 + 3x)^{-3}, \qquad |x| < \frac{2}{3}$$

find the binomial expansion of $f(x)$, in ascending powers of x, up to and including the term in x^3.

Give each coefficient as a simplified fraction.

(5)

(Total 5 marks)

Leave blank

114. $$f(x) = \frac{6}{\sqrt{(9-4x)}}, \quad |x| < \frac{9}{4}$$

(a) Find the binomial expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(6)

Use your answer to part (a) to find the binomial expansion in ascending powers of x, up to and including the term in x^3, of

(b) $g(x) = \dfrac{6}{\sqrt{(9+4x)}}, \quad |x| < \dfrac{9}{4}$

(1)

(c) $h(x) = \dfrac{6}{\sqrt{(9-8x)}}, \quad |x| < \dfrac{9}{8}$

(2)

Leave blank

Question 114 continued

(Total 9 marks)

Leave blank

115. (a) Expand

$$\frac{1}{(2-5x)^2}, \quad |x| < \frac{2}{5}$$

in ascending powers of x, up to and including the term in x^2, giving each term as a simplified fraction.

(5)

Given that the binomial expansion of $\dfrac{2+kx}{(2-5x)^2}$, $|x| < \dfrac{2}{5}$, is

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \dots$$

(b) find the value of the constant k,

(2)

(c) find the value of the constant A.

(2)

Leave blank

Question 115 continued

(Total 9 marks)

Leave blank

116. $$f(x) = \frac{1}{\sqrt{(9+4x^2)}}, \qquad |x| < \frac{3}{2}$$

Find the first three non-zero terms of the binomial expansion of $f(x)$ in ascending powers of x. Give each coefficient as a simplified fraction.

(6)

(Total 6 marks)

Leave blank

117. (a) Use the binomial theorem to expand

$$(2-3x)^{-2}, \quad |x|<\frac{2}{3},$$

in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(5)

$$f(x)=\frac{a+bx}{(2-3x)^2}, \quad |x|<\frac{2}{3}, \quad \text{where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of $f(x)$, in ascending powers of x, the coefficient of x is 0 and the coefficient of x^2 is $\frac{9}{16}$. Find

(b) the value of a and the value of b,

(5)

(c) the coefficient of x^3, giving your answer as a simplified fraction.

(3)

Leave blank

Question 117 continued

(Total 13 marks)

Leave blank

118.

$$\frac{2x^2+5x-10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}$$

(a) Find the values of the constants A, B and C.

(4)

(b) Hence, or otherwise, expand $\dfrac{2x^2+5x-10}{(x-1)(x+2)}$ in ascending powers of x, as far as the term in x^2. Give each coefficient as a simplified fraction.

(7)

(Total 11 marks)

Leave blank

119. (a) Find the binomial expansion of

$$\sqrt{(1-8x)}, \quad |x| < \frac{1}{8},$$

in ascending powers of x up to and including the term in x^3, simplifying each term.

(4)

(b) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt{(1-8x)}$ is $\frac{\sqrt{23}}{5}$.

(2)

(c) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (a) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places.

(3)

(Total 9 marks)

Leave blank

120. $$f(x) = \frac{1}{\sqrt{(4+x)}}, \qquad |x| < 4$$

Find the binomial expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(6)

(Total 6 marks)

Leave blank

121.

$$f(x)=\frac{27x^2+32x+16}{(3x+2)^2(1-x)}, \quad |x|<\frac{2}{3}$$

Given that f(x) can be expressed in the form

$$f(x)=\frac{A}{(3x+2)}+\frac{B}{(3x+2)^2}+\frac{C}{(1-x)},$$

(a) find the values of B and C and show that $A = 0$.

(4)

(b) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^2. Simplify each term.

(6)

(c) Find the percentage error made in using the series expansion in part (b) to estimate the value of f(0.2). Give your answer to 2 significant figures.

(4)

Leave blank

Question 121 continued

(Total 14 marks)

Leave blank

122. (a) Expand $\dfrac{1}{\sqrt{(4-3x)}}$, where $|x| < \frac{4}{3}$, in ascending powers of x up to and including the term in x^2. Simplify each term.

(5)

(b) Hence, or otherwise, find the first 3 terms in the expansion of $\dfrac{x+8}{\sqrt{(4-3x)}}$ as a series in ascending powers of x.

(4)

(Total 9 marks)

Leave blank

123. (a) Use the binomial theorem to expand

$$(8-3x)^{\frac{1}{3}}, \qquad |x| < \tfrac{8}{3},$$

in ascending powers of x, up to and including the term in x^3, giving each term as a simplified fraction.

(5)

(b) Use your expansion, with a suitable value of x, to obtain an approximation to $\sqrt[3]{(7.7)}$. Give your answer to 7 decimal places.

(2)

(Total 7 marks)

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