EXPERT TUITION

# Maths Questions By Topic:

# Trigonometry
# Mark Scheme

# A-Level Edexcel

0207 060 4494
www.expert-tuition.co.uk
online.expert-tuition.co.uk
enquiries@expert-tuition.co.uk
The Foundry, 77 Fulham Palace Road, W6 8JA

# Table Of Contents

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 1(a) | Attempts to use both $\sin(x-60°)=\pm\sin x\cos 60°\pm\cos x\sin 60°$ <br> $\cos(x-30°)=\pm\cos x\cos 30°\pm\sin x\sin 30°$ | M1 | 2.1 |
| | Correct equation <br> $2\sin x\cos 60°-2\cos x\sin 60°=\cos x\cos 30°+\sin x\sin 30°$ | A1 | 1.1b |
| | Either uses $\frac{\sin x}{\cos x}=\tan x$ and attempts to make $\tan x$ the subject <br> E.g. $(2\cos 60°-\sin 30°)\tan x=\cos 30°+2\sin 60°$ <br><br> Or attempts $\sin 30°$ etc with at least two correct and collects terms in $\sin x$ and $\cos x$ <br> E.g. $\left(2\times\frac{1}{2}-\frac{1}{2}\right)\sin x=\left(2\times\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)\cos x$ <br> , | M1 | 2.1 |
| | Proceeds to given answer showing all key steps <br> E.g. $\frac{1}{2}\tan x=\frac{3\sqrt{3}}{2}\Rightarrow\tan x=3\sqrt{3}$ * | A1* | 1.1b |
| | | **(4)** | |
| **(b)** | Deduces that $x=2\theta+60°$ | B1 | 2.2a |
| | $\tan(2\theta+60°)=3\sqrt{3}\Rightarrow 2\theta+60°=79.1°,259.1°,.....$ | M1 | 1.1b |
| | Correct method to find one value of $\theta$ E.g $\theta=\frac{79.1°-60°}{2}$ | dM1 | 1.1b |
| | $\theta=$ awrt $9.6°, 99.6°$ (See note) | A1 | 2.1 |
| | | **(4)** | |
| | | | **(8 marks)** |
| **Notes:** | | | |

**(a)**

**M1:** Attempts to use both compound angle expansions to set up an equation in sin $x$ and cos $x$
The terms must be correct but condone sign errors and a slip on the multiplication of 2

**A1:** Correct equation $2\sin x\cos 60°-2\cos x\sin 60°=\cos x\cos 30°+\sin x\sin 30°$ o.e.
Note that $\cos 60°=\sin 30°$ and $\cos 30°=\sin 60°$
Also allow this mark for candidates who substitute in their trigonometric values "early"

$$2\sin x\times\frac{1}{2}-2\cos x\times\frac{\sqrt{3}}{2}=\cos x\times\frac{\sqrt{3}}{2}+\sin x\times\frac{1}{2}\quad \text{o.e.}$$

**M1:** Shows the necessary progress towards showing the given result.
There are three key moves, two of which must be shown for this mark.

- uses $\frac{\sin x}{\cos x}=\tan x$ to form an equation in just tan $x$.
- uses exact numerical values for $\sin 30°, \sin 60°, \cos 30°, \cos 60°$ with at least two correct
- collects terms in $\sin x$ and $\cos x$ or alternatively in $\tan x$

**A1*:** Proceeds to the given answer with accurate work showing all necessary lines.

Examples of two proofs showing all necessary lines

E.g. I $\quad 2\sin x\cos 60° - 2\cos x\sin 60° = \cos x\cos 30° + \sin x\sin 30°$

$$\sin x\left(2\cos 60° - \sin 30°\right) = \cos x\left(\cos 30° + 2\sin 60°\right)$$ 1. collect terms

$$\left(2\cos 60° - \sin 30°\right)\tan x = \cos 30° + 2\sin 60°$$ 2. $\frac{\sin x}{\cos x} = \tan x$ so M1

$$\tan x = \frac{\cos 30° + 2\sin 60°}{2\cos 60° - \sin 30°} = \frac{\frac{\sqrt{3}}{2} + \sqrt{3}}{1 - \frac{1}{2}} = 3\sqrt{3}$$ 3..uses values and completes proof A1*

E.g II

$$2\sin x \times \frac{1}{2} - 2\cos x \times \frac{\sqrt{3}}{2} = \cos x \times \frac{\sqrt{3}}{2} + \sin x \times \frac{1}{2}$$ 1.uses values

$$\Rightarrow \frac{1}{2}\sin x = \frac{3\sqrt{3}}{2}\cos x$$ 2.collects terms so M1

$$\Rightarrow \tan x = 3\sqrt{3}$$

3. $\frac{\sin x}{\cos x} = \tan x$ completes proof A1*

**(b) Hence**

**B1:** Deduces that $x = 2\theta + 60°$ o.e such as $\theta = \frac{x - 60°}{2}$

This is implied for sight of the equation $\tan\left(2\theta + 60°\right) = 3\sqrt{3}$

**M1:** Proceeds from $\tan\left(2\theta \pm \alpha°\right) = 3\sqrt{3} \Rightarrow 2\theta \pm \alpha° =$ one of $79.1°, 259.1°, .....$ where $\alpha \neq 0$

One angle for $\arctan\left(3\sqrt{3}\right)$ **must** be correct in degrees or radians(3sf). FYI radian answers 1.38, 4.52

**dM1:** Correct method to find one value of $\theta$ from their $2\theta \pm \alpha° = 79.1°$ to $\theta = \frac{79.1° \mp \alpha°}{2}$

This is dependent upon one angle being correct, which must be in degrees, for $\arctan\left(3\sqrt{3}\right)$

$\tan\left(2\theta + 60°\right) = 3\sqrt{3} \Rightarrow \theta = 9.6°$ would imply B1 M1 dM1

**A1:** $\theta =$ awrt $9.6°, 99.6°$ with no other values given in the range

**Otherwise: Via the use of** $\cos\left(2\theta + 30°\right) = \cos 2\theta\cos 30° - \sin 2\theta\sin 30°$.

$$2\sin 2\theta = \cos\left(2\theta + 30°\right) \Rightarrow \tan 2\theta = \frac{\sqrt{3}}{5} \Rightarrow \theta = 9.6°, 99.6°$$

**The order of the marks needs to match up to the main scheme so 0110 is possible.**

B1: For achieving $\tan 2\theta = \frac{\sqrt{3}}{5}$ o.e so allow $\tan 2\theta =$ awrt 0.346 or $\tan 2\theta = \frac{\cos 30°}{2 + \sin 30°}$

Or via double angle identities $\sqrt{3}\tan^2\theta + 10\tan\theta - \sqrt{3} = 0$ o.e.

M1: Attempts to use the compound angle identities to reach a form $\tan 2\theta = k$ where $k$ is a constant not $3\sqrt{3}$ (or expression in trig terms such as cos 30 etc as seen above)

Or via double angle identities reaches a 3TQ in $\tan\theta$

dM1: Correct order of operations from $\tan 2\theta = k$ leading to $\theta = ...$

Correctly solves their $\sqrt{3}\tan^2\theta + 10\tan\theta - \sqrt{3} = 0$ leading to $\theta = ...$

A1: $\theta =$ awrt $9.6°, 99.6°$ with no other values given in the range.

Note that $\tan\left(2\theta + 60°\right) = 3\sqrt{3} \Rightarrow \theta = 9.6°, 99.6°$ is acceptable for full marks

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **2(a)(i)** | $(3x+10)^2 = (x+2)^2 + (7x)^2 - 2(x+2)(7x)\cos 60°$ oe | M1 | 3.1a |
| | Uses cos 60° = ½, expands the brackets and proceeds to a 3 term quadratic equation | dM1 | 1.1b |
| | $17x^2 - 35x - 48 = 0$ * | A1* | 2.1 |
| | | **(3)** | |
| **(ii)** | $x = 3$ | B1 | 3.2a |
| | | **(1)** | |
| **(b)** | $\dfrac{5}{\sin ACB} = \dfrac{19}{\sin 60°} \Rightarrow \sin ACB = ...\left(\dfrac{5\sqrt{3}}{38}\right)$<br>or e.g.<br>$5^2 = 21^2 + 19^2 - 2\times19\times21\cos ACB \Rightarrow \cos ACB = ...\left(\dfrac{37}{38}\right)$ | M1 | 1.1b |
| | $\theta$ = awrt 13.2 | A1 | 1.1b |
| | | **(2)** | |
| | | | **(6 marks)** |

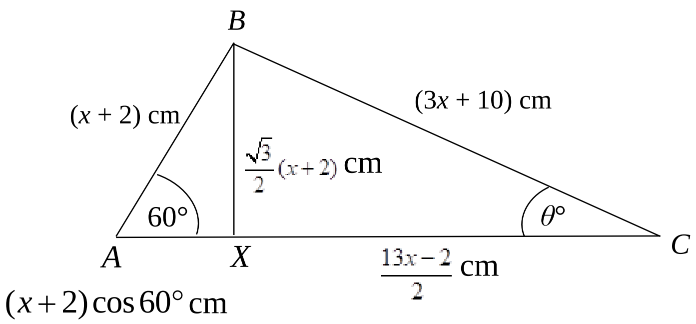
**Notes**

(a)(i) Mark (a) and (b) together

M1: Recognises the need to apply the cosine rule and attempts to use it with the sides in the correct positions and the formula applied correctly. Condone invisible brackets and slips on $3x+10$ as $3x-10$.
Alternatively, uses trigonometry to find *AX* and then equates two expressions for the length *BX*. You may see variations of this if they use Pythagoras or trigonometry to find *BX* and then apply Pythagoras to the triangle *BXC*. See the diagram below to help you.
The angles and lengths must be in the correct positions. Cos 60 may be $\frac{1}{2}$ from the start

dM1: Uses cos 60° = ½, expands the brackets and proceeds to a 3TQ. You may see the use of cos 60° = ½ in earlier work, but they must proceed to a 3TQ as well to score this mark. It is dependent on the first method mark.

A1*: Obtains the correct quadratic equation with the = 0 with no errors seen in the main body of their solution. Condone the recovery of invisible brackets as long as the intention is clear. You do not need to explicitly see cos 60 to score full marks.




(a)(ii)

B1: Selects the appropriate value i.e. $x = 3$ only. The other root must **either** be rejected if found **or** $x = 3$ must be the only root used in part (b). Can be implied by awrt 13.2 in (b)

(b)

M1: Using their value for $x$ this mark is for either:

- applying the sine rule correctly (or considers 2 right angled triangles) and proceeding to obtain a value for sin *ACB* or
- applying the cosine rule correctly and proceeding to obtain a value for cos *ACB*.

Condone slips calculating the lengths *AB*, *BC* and *AC*. At least one of them should be found correctly for their value for $x$

(Also allow if the sine rule or cosine rule is applied correctly to find a value for sin *ABC* $\left(=\frac{21\sqrt{3}}{38}\right)$ or cos *ACB* $\left(=-\frac{11}{38}\right)$)

A1: awrt 13.2 (answers with little working eg just lengths on the diagram can score M1A1)

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| 3(a) | $\frac{1}{\cos\theta}+\tan\theta=\frac{1+\sin\theta}{\cos\theta}$ or $\frac{(1+\sin\theta)\cos\theta}{\cos^2\theta}$ | | M1 | 1.1b |
| | $=\frac{1+\sin\theta}{\cos\theta}\times\frac{1-\sin\theta}{1-\sin\theta}=\frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}=\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$ <br> or <br> $\frac{(1+\sin\theta)\cos\theta}{\cos^2\theta}=\frac{(1+\sin\theta)\cos\theta}{1-\sin^2\theta}=\frac{(1+\sin\theta)\cos\theta}{(1+\sin\theta)(1-\sin\theta)}$ | | dM1 | 2.1 |
| | $=\frac{\cos\theta}{1-\sin\theta}$ * | | A1* | 1.1b |
| | | | **(3)** | |
| (b) | $\frac{1}{\cos 2x}+\tan 2x=3\cos 2x$ <br> $\Rightarrow 1+\sin 2x=3\cos^2 2x=3\left(1-\sin^2 2x\right)$ | $\frac{\cos 2x}{1-\sin 2x}=3\cos 2x$ <br> $\Rightarrow \cos 2x=3\cos 2x(1-\sin 2x)$ | M1 | 2.1 |
| | $\Rightarrow 3\sin^2 2x+\sin 2x-2=0$ | $\Rightarrow \cos 2x(2-3\sin 2x)=0$ | A1 | 1.1b |
| | $\sin 2x=\frac{2}{3},\ (-1)\Rightarrow 2x=...\Rightarrow x=...$ | | M1 | 1.1b |
| | $x=20.9°,\ \ 69.1°$ | | A1 <br> A1 | 1.1b <br> 1.1b |
| | | | **(5)** | |
| | | | | **(8 marks)** |

**Notes**

(a) If starting with the LHS: Condone if another variable for $\theta$ is used except for the final mark

M1: Combines terms with a common denominator. The numerator must be correct for their common denominator.

dM1: Either:

- $\frac{1+\sin\theta}{\cos\theta}$ : Multiplies numerator and denominator by 1 – sin $\theta$, uses the difference of two squares and applies $\cos^2\theta=1-\sin^2\theta$
- $\frac{(1+\sin\theta)\cos\theta}{\cos^2\theta}$ : Uses $\cos^2\theta=1-\sin^2\theta$ on the denominator, applies the difference of two squares

It is dependent on the previous method mark.

A1*: Fully correct proof with correct notation and no errors in the main body of their work. Withhold this mark for writing eg sin instead of $\sin\theta$ anywhere in the solution and for eg $\sin\theta^2$ instead of $\sin^2\theta$

Alt(a) If starting with the RHS: Condone if another variable is used for $\theta$ except for the final mark

M1: Multiplies by $\dfrac{1+\sin\theta}{1+\sin\theta}$ leading to $\dfrac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$ or

Multiplies by $\dfrac{\cos\theta}{\cos\theta}$ leading to $\dfrac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$

dM1: Applies $\cos^2\theta = 1-\sin^2\theta$ and cancels the $\cos\theta$ factor from the numerator and denominator leading to $\dfrac{1+\sin\theta}{\cos\theta}$ or

Applies $\cos^2\theta = 1-\sin^2\theta$ and uses the difference of two squares leading to $\dfrac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$

It is dependent on the previous method mark.

A1*: Fully correct proof with correct notation and no errors in the main body of their work. If they work from both the LHS and the RHS and meet in the middle with both sides the same then they need to conclude at the end by stating the original equation.

(b) ***Be aware that this can be done entirely on their calculator which is not acceptable***

M1: Either multiplies through by cos $2x$ and applies $\cos^2 2x = 1-\sin^2 2x$ to obtain an equation in sin $2x$ only or alternatively sets $\dfrac{\cos 2x}{1-\sin 2x} = 3\cos 2x$ and multiplies by $1-\sin 2x$

A1: Correct equation or equivalent. The = 0 may be implied by their later work (Condone notational slips in their working)

M1: Solves for sin $2x$, uses arcsin to obtain at least one value for $2x$ and divides by 2 to obtain at least one value for $x$. The roots of the quadratic can be found using a calculator. They cannot just write down values for $x$ from their quadratic in sin$2x$

A1: For 1 of the required angles. Accept awrt 21 or awrt 69. Also accept awrt 0.36 rad or awrt 1.21 rad

A1: For both angles (awrt 20.9 and awrt 69.1) and no others inside the range. If they find $x = 45$ it must be rejected. (Condone notational slips in their working)

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 4(a) | $\dfrac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\dfrac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\cos 2\theta+\sin 2\theta}$ <br> or <br> $\dfrac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\dfrac{1-\cos 2\theta+\sin 2\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$ | M1 | 2.1 |
| | $\dfrac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\dfrac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$ | A1 | 1.1b |
| | $=\dfrac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}=\dfrac{2\sin\theta\left(\sin\theta+\cos\theta\right)}{2\cos\theta\left(\cos\theta+\sin\theta\right)}$ | dM1 | 2.1 |
| | $=\dfrac{\sin\theta}{\cos\theta}=\tan\theta$ * | A1* | 1.1b |
| | | **(4)** | |
| **(b)** | $\dfrac{1-\cos 4x+\sin 4x}{1+\cos 4x+\sin 4x}=3\sin 2x\Rightarrow\tan 2x=3\sin 2x$ o.e | M1 | 3.1a |
| | $\Rightarrow\sin 2x-3\sin 2x\cos 2x=0$ <br> $\Rightarrow\sin 2x\left(1-3\cos 2x\right)=0$ <br> $\Rightarrow\left(\sin 2x=0,\right)\ \cos 2x=\dfrac{1}{3}$ | A1 | 1.1b |
| | $x=90°$, awrt $35.3°$, awrt $144.7°$ | A1 <br> A1 | 1.1b <br> 2.1 |
| | | **(4)** | |
| | | | **(8 marks)** |

**Notes**

(a)

M1: Attempts to use a correct double angle formulae for both $\sin 2\theta$ and $\cos 2\theta$ (seen once).
The application of the formula for $\cos 2\theta$ must be the one that cancels out the "1"
So look for $\cos 2\theta = 1 - 2\sin^2\theta$ in the numerator or $\cos 2\theta = 2\cos^2\theta - 1$ in the denominator
Note that $\cos 2\theta = \cos^2\theta - \sin^2\theta$ may be used as well as using $\cos^2\theta + \sin^2\theta = 1$

A1: $\dfrac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$ or $\dfrac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}$

dM1: Factorises numerator and denominator in order to demonstrate cancelling of $(\sin\theta + \cos\theta)$

A1*: Fully correct proof with no errors.

You must see an intermediate line of $\dfrac{2\sin\theta\cancel{\left(\sin\theta+\cos\theta\right)}}{2\cos\theta\cancel{\left(\cos\theta+\sin\theta\right)}}$ or $\dfrac{\sin\theta}{\cos\theta}$ or even $\dfrac{2\sin\theta}{2\cos\theta}$

Withhold this mark if you see, within the body of the proof,

- notational errors. E.g. $\cos 2\theta = 1 - 2\sin^2$ or $\cos\theta^2$ for $\cos^2\theta$
- mixed variables. E.g. $\cos 2\theta = 2\cos^2 x - 1$

(b)

M1: Makes the connection with part (a) and writes the lhs as $\tan 2x$ . Condone $x \leftrightarrow \theta$ $\quad \tan 2\theta = 3\sin 2\theta$

A1: Obtains $\cos 2x = \dfrac{1}{3}$ o.e. with $x \leftrightarrow \theta$ . You may see $\sin^2 x = \dfrac{1}{3}$ or $\cos^2 x = \dfrac{2}{3}$ after use of double angle formulae.

A1: Two "correct" values. Condone accuracy of awrt $90°,\ 35°,\ 145°$
Also condone radian values here. Look for 2 of awrt 0.62, 1.57, 2.53

A1: All correct (allow awrt) and no other values in range. Condone $x \leftrightarrow \theta$ if used consistently

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Answers without working in (b): Just answers and no working score 0 marks.
If the first line is written out, i.e. $\tan 2x = 3\sin 2x$ followed by all three correct answers score 1100 .

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **5 (a)** | Sets $50 = 7\times14\sin(SPQ)$ oe | B1 | 1.2 |
| | Finds $180° - \arcsin\left("\frac{50}{98}"\right)$ | M1 | 1.1b |
| | $= 149.32°$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)** | Method of finding $SQ$<br>$SQ^2 = 14^2 + 7^2 - 2\times14\times7\cos"149.32"$ | M1 | 1.1b |
| | $= 20.3$ cm | A1 | 1.1b |
| | | **(2)** | |
| | | | **(5 marks)** |
| **Alt(a)** | States or uses $14h = 50$ or $7h_1 = 50$ | B1 | 1.2 |
| | Full method to find obtuse $\angle SPQ$.<br>In this case it is $90° + \arccos\left(\frac{h}{7}\right)$ or $90° + \arccos\left(\frac{h_1}{14}\right)$ | M1 | 1.1b |
| | awrt $149.32°$ | A1 | 1.1b |

**Notes**

**(a)**

**B1:** Sets $50 = 7\times14\sin(SPQ)$ oe

**M1:** Attempts the correct method of finding obtuse $\angle SPQ$. See scheme.

**A1:** awrt $149.32°$

**(b)**

**M1:** A correct method of finding $SQ$ using their $\angle SPQ$.

$SQ^2 = 14^2 + 7^2 - 2\times14\times7\cos"149.32"$ scores this mark.

**A1:** awrt 20.3 cm (condone lack of units)

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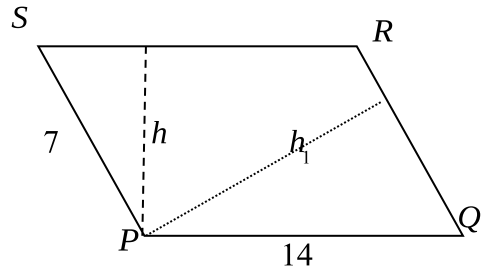
**Alt(a)**




**B1:** States or uses $14h = 50$ or $7h_1 = 50$

**M1:** Full method to find obtuse $\angle SPQ$.

In this case it is $90° + \arccos\left(\frac{h}{7}\right)$ or $90° + \arccos\left(\frac{h_1}{14}\right)$

**A1:** awrt $149.32°$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 6 (i) | Uses $\cos^2\theta = 1-\sin^2\theta$<br>$5\cos^2\theta = 6\sin\theta \Rightarrow 5\sin^2\theta + 6\sin\theta - 5 = 0$ | M1<br>A1 | 1.2<br>1.1b |
| | $\Rightarrow \sin\theta = \dfrac{-3+\sqrt{34}}{5} \Rightarrow \theta = ...$ | dM1 | 3.1a |
| | $\Rightarrow \theta = 34.5°, 145.5°, 394.5°$ | A1<br>A1 | 1.1b<br>1.1b |
| | | **(5)** | |
| (ii) (a) | One of<br>• They cancel by $\sin x$ (and hence they miss the solution $\sin x = 0 \Rightarrow x = 0$)<br>• They do not find all the solutions of $\cos x = \frac{3}{5}$ (in the given range) or they missed the solution $x = -53.1°$ | B1 | 2.3 |
| | Both of the above | B1 | 2.3 |
| | | **(2)** | |
| (ii) (b) | Sets $5\alpha + 40° = 720° - 53.1°$ | M1 | 3.1a |
| | $\alpha = 125°$ | A1 | 1.1b |
| | | **(2)** | |
| | | | **(9 marks)** |

**Notes**

**(i)**

**M1:** Uses $\cos^2\theta = 1-\sin^2\theta$ to form a 3TQ in $\sin\theta$

**A1:** Correct 3TQ=0 $5\sin^2\theta + 6\sin\theta - 5 = 0$

**dM1:** Solves their 3TQ in $\sin\theta$ to produce one value for $\theta$. It is dependent upon having used $\cos^2\theta = \pm 1 \pm \sin^2\theta$

**A1:** Two of awrt $\theta = 34.5°, 145.5°, 394.5°$ (or if in radians two of awrt 0.60, 2.54, 6.89)

**A1:** All three of awrt $\theta = 34.5°, 145.5°, 394.5°$ and no other values

**(i) (a)**

**See scheme**

**(ii)(b)**

**M1:** Sets $5\alpha + 40° = 666.9°$ o.e.

**A1:** awrt $\alpha = 125°$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **7(a)** | $R=\sqrt{5}$ | B1 | 1.1b |
| | $\tan\alpha = 2 \Rightarrow \alpha = ...$ | M1 | 1.1b |
| | $\alpha = 1.107$ | A1 | 1.1b |
| | | **(3)** | |
| | $\theta = 5+\sqrt{5}\sin\left(\frac{\pi t}{12}+1.107-3\right)$ | | |
| **(b)** | $\left(5+\sqrt{5}\right)$ °C or awrt 7.24°C | B1ft | 2.2a |
| | | **(1)** | |
| **(c)** | $\frac{\pi t}{12}+1.107-3=\frac{\pi}{2} \Rightarrow t=$ | M1 | 3.1b |
| | $t =$ awrt 13.2 | A1 | 1.1b |
| | Either 13:14 or 1:14 pm or 13 hours 14 minutes after midnight. | A1 | 3.2a |
| | | **(3)** | |
| | | | **(7 marks)** |
| **Notes:** | | | |

**(a)**

**B1:** $R=\sqrt{5}$ only.

**M1:** Proceeds to a value of $\alpha$ from $\tan\alpha = \pm 2, \tan\alpha = \pm\frac{1}{2}, \sin\alpha = \pm\frac{2}{"R"}$ OR $\cos\alpha = \pm\frac{1}{"R"}$

It is implied by either awrt 1.11 (radians) or 63.4 (degrees)

**A1:** $\alpha =$ awrt 1.107

**(b)**

**B1ft:** Deduces that the maximum temperature is $\left(5+\sqrt{5}\right)$ °C or awrt 7.24°C Remember to isw

Condone a lack of units. Follow through on their value of $R$ so allow $(5+"R")$ °C

**(c)**

**M1:** An complete strategy to find $t$ from $\frac{\pi t}{12}\pm 1.107-3=\frac{\pi}{2}$.

Follow through on their 1.107 but the angle must be in radians.

It is possible via degrees but only using $15t \pm 63.4-171.9=90$

**A1:** awrt $t=13.2$

**A1:** The question asks for the time of day so accept either 13:14, 1:14 pm, 13 hours 14 minutes after midnight, 13h 14, or 1 hour 14 minutes after midday. If in doubt use review

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It is possible to attempt parts (b) and (c) via differentiation but it is unlikely to yield correct results.

$$\frac{d\theta}{dt}=\frac{\pi}{12}\cos\left(\frac{\pi t}{12}-3\right)-\frac{2\pi}{12}\sin\left(\frac{\pi t}{12}-3\right)=0 \Rightarrow \tan\left(\frac{\pi t}{12}-3\right)=\frac{1}{2} \Rightarrow t=13.23=13:14 \text{ scores M1 A1 A1}$$

$$\frac{d\theta}{dt}=\cos\left(\frac{\pi t}{12}-3\right)-2\sin\left(\frac{\pi t}{12}-3\right)=0 \Rightarrow \tan\left(\frac{\pi t}{12}-3\right)=\frac{1}{2} \Rightarrow t=13.23=13:14 \text{ they can score M1 A0 A1 (SC)}$$

A value of $t=1.23$ implies the minimum value has been found and therefore incorrect method M0.

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| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 8(a) | States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ | B1 | 1.2 |
| | $\operatorname{cosec}\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta}$ | M1 | 2.1 |
| | $= \dfrac{\cos^2\theta}{\sin\theta} = \cos\theta \times \dfrac{\cos\theta}{\sin\theta} = \cos\theta\cot\theta$ * | A1* | 2.1 |
| | | **(3)** | |
| **(b)** | $\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50°)$ <br> $\Rightarrow \cos x \cot x = \cos x \cot(3x-50°)$ | | |
| | $\cot x = \cot(3x-50°) \Rightarrow x = 3x - 50°$ | M1 | 3.1a |
| | $x = 25°$ | A1 | 1.1b |
| | Also $\cot x = \cot(3x-50°) \Rightarrow x + 180° = 3x - 50°$ | M1 | 2.1 |
| | $x = 115°$ | A1 | 1.1b |
| | Deduces $x = 90°$ | B1 | 2.2a |
| | | **(5)** | |
| | | | **(8 marks)** |
| **Notes:** | | | |

**(a) Condone a full proof in *x* (or other variable) instead of $\theta$'s here**

**B1:** States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ Do not accept $\operatorname{cosec}\theta = \dfrac{1}{\sin}$ with the $\theta$ missing

**M1:** For the key step in forming a single fraction/common denominator

E.g. $\operatorname{cosec}\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta}$. Allow if written separately $\dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1}{\sin\theta} - \dfrac{\sin^2\theta}{\sin\theta}$

Condone missing variables for this M mark

**A1*:** Shows careful work with all necessary steps shown leading to given answer. See scheme for necessary steps. There should not be any notational or bracketing errors.

**(b) Condone $\theta$'s instead of x's here**

**M1:** Uses part (a), cancels or factorises out the $\cos x$ term, to establish that one solution is found when $x = 3x - 50°$.

You may see solutions where $\cot A - \cot B = 0 \Rightarrow \cot(A-B) = 0$ or $\tan A - \tan B = 0 \Rightarrow \tan(A-B) = 0$.

As long as they don't state $\cot A - \cot B = \cot(A-B)$ or $\tan A - \tan B = \tan(A-B)$ this is acceptable

**A1:** $x = 25°$

**M1:** For the key step in realising that $\cot x$ has a period of $180°$ and a second solution can be found by solving $x + 180° = 3x - 50°$. The sight of $x = 115°$ can imply this mark provided the step $x = 3x - 50°$ has been seen. Using reciprocal functions it is for realising that $\tan x$ has a period of $180°$

**A1:** $x = 115°$ Withhold this mark if there are additional values in the range (0,180) but ignore values outside.

**B1:** Deduces that a solution can be found from $\cos x = 0 \Rightarrow x = 90°$. Ignore additional values here.

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Solutions with limited working. The question demands that candidates show all stages of working.

SC: $\cos x \cot x = \cos x \cot(3x-50°) \Rightarrow \cot x = \cot(3x-50°) \Rightarrow x = 25°, 115°$

They have shown some working so can score B1, B1 marked on epen as 11000

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **9(a)** | States $\frac{\sin\theta}{12}=\frac{\sin 27}{7}$ | M1 | 1.1b |
| | Finds $\theta =$ awrt $51°$ or awrt $129°$ | A1 | 1.1b |
| | $=$ awrt $128.9°$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)** | Attempts to find part or all of $AD$<br>Eg $AD^2=7^2+12^2-2\times12\times7\cos101.9=(AD=15.09)$<br><br>Eg $(AC)^2=7^2+12^2-2\times12\times7\cos(180-"128.9"-27)$<br><br>Eg $12\cos27$ or $7\cos"51"$ | M1 | 1.1b |
| | Full method for the total length $=12+7+7+"15.09"=$ | dM1 | 3.1a |
| | $= 42$ m | A1 | 3.2a |
| | | **(3)** | |
| | | | **(6 marks)** |

**Notes**

(a)

**M1:** States $\frac{\sin\theta}{12}=\frac{\sin 27}{7}$ oe with the sides and angles in the correct positions

Alternatively they may use the cosine rule on $\angle ACB$ and then solve the subsequent quadratic to find $AC$ and then use the cosine rule again

**A1:** awrt $51°$ or awrt $129°$

**A1:** Awrt $128.9°$ only (must be seen in part a))

(b)

**M1:** Attempts a "correct" method of finding either $AD$ or a part of $AD$ eg ($AC$ or $CD$ or forming a perpendicular to split the triangle into two right angled triangles to find $AX$ or $XD$) which may be seen in (a).
You should condone incorrect labelling of the side.
Look for attempted application of the cosine rule

$$(AD)^2=7^2+12^2-2\times12\times7\cos("128.9"-27)$$
$$\text{or } (AC)^2=7^2+12^2-2\times12\times7\cos(180-"128.9"-27)$$

Or an attempted application of the sine rule $\frac{(AD)}{\sin("128.9"-27)}=\frac{7}{\sin27}$

Or $\frac{(AC)}{\sin(180-"128.9"-27)}=\frac{7}{\sin27}$

Or an attempt using trigonometry on a right-angled triangle to find part of $AD$

$12\cos 27$ or $7\cos "51"$

This method can be implied by sight of awrt 15.1 or awrt 6.3 or awrt 8.8 or awrt 10.7 or awrt 4.4

**dM1:** A complete method of finding the TOTAL length.
There must have been an attempt to use the correct combination of angles and sides.
Expect to see $7+7+12+"AD"$ found using a correct method.
This is scored by either $7+7+12+"AD"$ if $\angle ACB = 128.9°$ in a) or
$7+7+12+\text{awrt}15.1$ by candidates who may have assumed $\angle ACB = 51.1°$ in a)

**A1:** Rounds correct 41.09 m (or correct expression) up to 42 m to find steel **bought**

Candidates who assumed $\angle ACB = 51.1°$ (acute) in (a):
Full marks can still be achieved as candidates may have restarted in (b) or not used the acute angle in their calculation which is often unclear. We are condoning any reference to $AC = 15.1$ so ignore any labelling of the lengths they are finding.

Diagram of the correct triangle with lengths and angles:

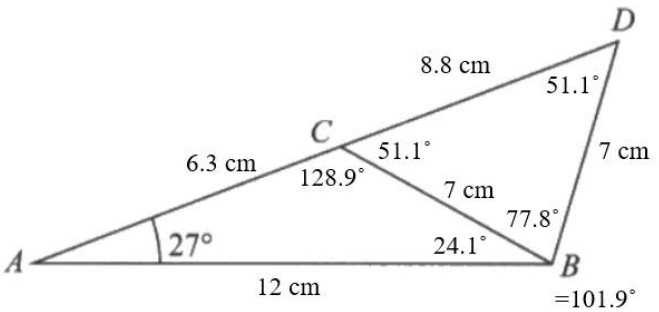
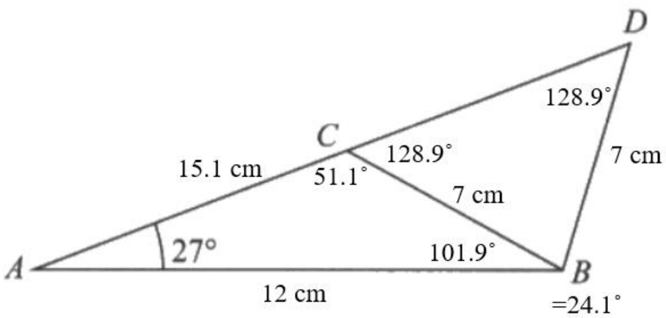



Diagram using the incorrect acute angle:



| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 10 (a) | $(-180°,-3)$ | B1 | 1.1b |
| | | (1) | |
| (b) | (i) $(-720°,-3)$ | B1ft | 2.2a |
| | (ii) $(-144°,-3)$ | B1 ft | 2.2a |
| | | (2) | |
| (c) | Attempts to use both $\tan\theta=\frac{\sin\theta}{\cos\theta}$, $\sin^2\theta+\cos^2\theta=1$ and solves a quadratic equation in $\sin\theta$ to find at least one value of $\theta$ | M1 | 3.1a |
| | $3\cos\theta=8\tan\theta\Rightarrow3\cos^2\theta=8\sin\theta$ | B1 | 1.1b |
| | $3\sin^2\theta+8\sin\theta-3=0$ <br> $(3\sin\theta-1)(\sin\theta+3)=0$ | M1 | 1.1b |
| | $\sin\theta=\frac{1}{3}$ | A1 | 2.2a |
| | awrt $520.5°$ only | A1 | 2.1 |
| | | (5) | |
| | | | **(8 marks)** |

(a)

**B1:** Deduces that $P(-180°,-3)$ or $c=-180^{(o)}, d=-3$

(b)(i)

**B1ft:** Deduces that $P'(-720°,-3)$ Follow through on their $(c,d)\to(4c,d)$ where $d$ is negative

(b)(ii)

**B1ft:** Deduces that $P'(-144°,-3)$ Follow through on their $(c,d)\to(c+36°,d)$ where $d$ is negative

(c)

**M1:** An overall problem solving mark, condoning slips, for an attempt to

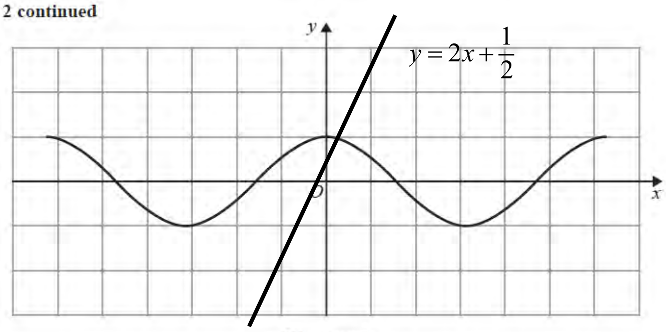
- use $\tan\theta=\frac{\sin\theta}{\cos\theta}$,
- use $\pm\sin^2\theta\pm\cos^2\theta=\pm1$
- find at least one value of $\theta$ from a quadratic equation in $\sin\theta$

**B1:** Uses the correct identity and multiplies across to give $3\cos\theta=8\tan\theta\Rightarrow3\cos^2\theta=8\sin\theta$ oe

**M1:** Uses the correct identity $\sin^2\theta+\cos^2\theta=1$ to form a 3TQ in $\sin\theta$ which they attempt to solve using an appropriate method. It is OK to use a calculator to solve this

**A1:** $\sin\theta=\frac{1}{3}$ Accept sight of $\frac{1}{3}$. Ignore any reference to the other root even if it is "used"

**A1:** Full method with all identities correct leading to the answer of awrt $520.5°$ and no other values.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **11(a)** | 2 continued   Diagram 1 | B1 | 3.1a |
| | For an allowable linear graph and explaining that there is only one intersection | B1 | 2.4 |
| | | **(2)** | |
| **(b)** | $\cos x - 2x - \frac{1}{2} = 0 \Rightarrow 1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$ | M1 | 1.1b |
| | Solves their $x^2 + 4x - 1 = 0$ | dM1 | 1.1b |
| | Allow awrt 0.236 but accept $-2+\sqrt{5}$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(5 marks)** |

**(a)**

**B1:** Draws $y = 2x + \frac{1}{2}$ on Figure 1 or Diagram 1 with an attempt at the correct gradient and the correct intercept. Look for a straight line with an intercept at $\approx \frac{1}{2}$ and a further point at $\approx \left(\frac{1}{2}, 1\frac{1}{2}\right)$ Allow a tolerance of 0.25 of a square in either direction on these two points. It must appear in quadrants 1, 2 and 3.

**B1:** There must be an allowable linear graph on Figure 1 or Diagram1 for this to be awarded
Explains that as there is only one intersection so there is just one root.
This requires a reason and a minimal conclusion.
The question asks candidates to explain but as a bare minimum allow one "intersection"

Note: An allowable linear graph is one with intercept of $\pm\frac{1}{2}$ with one intersection with $\cos x$ **OR** gradient of $\pm 2$ with one intersection with $\cos x$

**(b)**

**M1:** Attempts to use the small angle approximation $\cos x = 1 - \frac{x^2}{2}$ in the given equation.
The equation must be in a single variable but may be recovered later in the question.

**dM1:** Proceeds to a 3TQ in a single variable and attempts to solve. See General Principles
The previous M must have been scored. Allow completion of square or formula or calculator. Do not allow attempts via factorisation unless their equation does factorise. You may have to use your calculator to check if a calculator is used.

**A1:** Allow $-2+\sqrt{5}$ or awrt 0.236.

Do not allow this where there is another root given and it is not obvious that 0.236 has been chosen.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 12(a) | $5\sin 2\theta = 9\tan\theta \Rightarrow 10\sin\theta\cos\theta = 9\times\frac{\sin\theta}{\cos\theta}$ <br> $A\cos^2\theta = B$ or $C\sin^2\theta = D$ or $P\cos^2\theta\sin\theta = Q\sin\theta$ | M1 | 3.1a |
| | For a correct simplified equation in one trigonometric function <br> Eg $10\cos^2\theta = 9$ $10\sin^2\theta = 1$ oe | A1 | 1.1b |
| | Correct order of operations For example <br> $10\cos^2\theta = 9 \Rightarrow \theta = \arccos(\pm)\sqrt{\frac{9}{10}}$ | dM1 | 2.1 |
| | Any one of the four values awrt $\theta = \pm 18.4°, \pm 161.6°$ | A1 | 1.1b |
| | All four values $\theta =$ awrt $\pm 18.4°, \pm 161.6°$ | A1 | 1.1b |
| | $\theta = 0°, \pm 180°$ | B1 | 1.1b |
| | | **(6)** | |
| **(b)** | Attempts to solve $x - 25° = -18.4°$ | M1 | 1.1b |
| | $x = 6.6°$ | A1ft | 2.2a |
| | | **(2)** | |
| | | | **(8 marks)** |

**(a)**

**M1:** Scored for the whole strategy of attempting to form an equation in one function of the form given in the scheme. For this to be awarded there must be an attempt at using $\sin 2\theta = ...\sin\theta\cos\theta$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and possibly $\pm 1 \pm \sin^2\theta = \pm\cos^2\theta$ to form an equation in one "function" usually $\sin^2\theta$ or $\cos^2\theta$

Allow for this mark equations of the form $P\cos^2\theta\sin\theta = Q\sin\theta$ oe

**A1:** Uses the correct identities $\sin 2\theta = 2\sin\theta\cos\theta$ **and** $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to form a correct simplified equation in one trigonometric function. It is usually one of the equations given in the scheme, but you may see equivalent correct equations such as $10 = 9\sec^2\theta$ which is acceptable, but in almost all cases it is for a correct equation in $\sin\theta$ or $\cos\theta$

**dM1:** Uses the correct order of operations for their equation, usually in terms of just $\sin\theta$ or $\cos\theta$, to find at least one value for $\theta$ (Eg. square root before invcos). It is dependent upon the previous M.

Note that some candidates will use $\cos^2\theta = \frac{\pm\cos 2\theta \pm 1}{2}$ and the same rules apply.

Look for correct order of operations.

**A1:** Any one of the four values awrt $\pm 18.4°, \pm 161.6°$. Allow awrt 0.32 (rad) or 2.82 (rad)

**A1:** All four values awrt $\pm 18.4°, \pm 161.6°$ and no other values apart from $0°, \pm 180°$

**B1:** $\theta = 0°, \pm 180°$ This can be scored independent of method.

**(b)**

**M1:** Attempts to solve $x - 25° = "\theta"$ where $\theta$ is a solution of their part (a)

**A1ft**: For awrt $x = 6.6°$ but you may ft on their $\theta + 25°$ where $-25 < \theta < 0$

If multiple answers are given, the correct value for their $\theta$ must be chosen

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **13 (a)** | Attempts to differentiate $x=4\sin 2y$ and inverts $\dfrac{dx}{dy}=8\cos 2y \Rightarrow \dfrac{dy}{dx}=\dfrac{1}{8\cos 2y}$ | M1 | 1.1b |
| | At (0,0) $\dfrac{dy}{dx}=\dfrac{1}{8}$ | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | (i) Uses $\sin 2y \approx 2y$ when $y$ is small to obtain $x \approx 8y$ | B1 | 1.1b |
| | (ii) The value found in (a) is the gradient of the line found in (b)(i) | B1 | 2.4 |
| | | **(2)** | |
| **(c)** | Uses their $\dfrac{dy}{dx}$ as a function of $y$ and, using both $\sin^2 2y+\cos^2 2y=1$ and $x=4\sin 2y$ in an attempt to write $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ as a function of $x$ Allow for $\dfrac{dy}{dx}=k\dfrac{1}{\cos 2y}=..\dfrac{1}{\sqrt{1-(..x)^2}}$ | M1 | 2.1 |
| | A correct answer $\dfrac{dy}{dx}=\dfrac{1}{8\sqrt{1-\left(\dfrac{x}{4}\right)^2}}$ or $\dfrac{dx}{dy}=8\sqrt{1-\left(\dfrac{x}{4}\right)^2}$ | A1 | 1.1b |
| | and in the correct form $\dfrac{dy}{dx}=\dfrac{1}{2\sqrt{16-x^2}}$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(7 marks)** |

**(a)**

**M1:** Attempts to differentiate $x=4\sin 2y$ and inverts.

Allow for $\dfrac{dx}{dy}=k\cos 2y \Rightarrow \dfrac{dy}{dx}=\dfrac{1}{k\cos 2y}$ or $1=k\cos 2y\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx}=\dfrac{1}{k\cos 2y}$

Alternatively, changes the subject and differentiates $x=4\sin 2y \to y=\ldots\arcsin\left(\dfrac{x}{4}\right) \to \dfrac{dy}{dx}=\dfrac{\ldots}{\sqrt{1-\left(\dfrac{x}{4}\right)^2}}$

It is possible to approach this from $x=8\sin y\cos y \Rightarrow \dfrac{dx}{dy}=\pm 8\sin^2 y \pm 8\cos^2 y$ before inverting

**A1:** $\dfrac{dy}{dx}=\dfrac{1}{8}$ Allow both marks for sight of this answer as long as no incorrect working is seen (See below)

Watch for candidates who reach this answer via $\dfrac{dx}{dy}=8\cos 2x \Rightarrow \dfrac{dy}{dx}=\dfrac{1}{8\cos 2x}$ This is M0 A0

**(b)(i)**

**B1:** Uses $\sin 2y \approx 2y$ when $y$ is small to obtain $x = 8y$ oe such as $x = 4(2y)$ .

Do not allow $\sin 2y \approx 2\theta$ to get $x = 8\theta$ but allow recovery in (b)(i) or (b)(ii)

Double angle formula is B0 as it does not satisfy the demands of the question.

**(b)(ii)**

**B1:** Explains the relationship between the answers to (a) and (b) (i).

For this to be scored the first three marks, in almost all cases, must have been awarded and the statement must refer to both answers

Allow for example "The gradients are the same $\left(=\frac{1}{8}\right)$" 'both have $m = \frac{1}{8}$'

Do not accept the statement that 8 and $\frac{1}{8}$ are reciprocals of each other unless further correct work explains the relationship in terms of $\frac{dx}{dy}$ and $\frac{dy}{dx}$

**(c)**

**M1:** Uses their $\frac{dy}{dx}$ as a function of $y$ and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4\sin 2y$, attempts to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of $x$. The $\frac{dy}{dx}$ may not be seen and may be implied by their calculation.

**A1:** A correct (un-simplified) answer for $\frac{dy}{dx}$ or $\frac{dx}{dy}$ Eg. $\frac{dy}{dx} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$

**A1:** $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$ The $\frac{dy}{dx}$ must be seen at least once in part (c) of this solution

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**Alt to (c)** using arcsin

**M1:** Alternatively, changes the subject and differentiates $x = 4\sin 2y \to y = ...\arcsin\left(\frac{x}{4}\right) \to \frac{dy}{dx} = \frac{...}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

Condone a lack of bracketing on the $\frac{x}{4}$ which may appear as $\frac{x^2}{4}$

**A1:** $\frac{dy}{dx} = \frac{1/8}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$ oe

**A1:** $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 14 (a) | Uses $18\sqrt{3}=\frac{1}{2}\times 2x\times 3x\times \sin 60°$ | M1 | 1.1a |
| | Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ and proceeds to $x^2=k$ oe | M1 | 1.1b |
| | $x=\sqrt{12}=2\sqrt{3}$ * | A1* | 2.1 |
| | | (3) | |
| (b) | Uses $BC^2=\left(6\sqrt{3}\right)^2+\left(4\sqrt{3}\right)^2-2\times 6\sqrt{3}\times 4\sqrt{3}\times \cos 60°$ | M1 | 1.1b |
| | $BC^2=84$ | A1 | 1.1b |
| | $BC=2\sqrt{21}$ (cm) | A1 | 1.1b |
| | | **(3)** | |
| | | | **(6 marks)** |

**Notes**

**(a)**

**M1:** Attempts to use the formula $A=\frac{1}{2}ab\sin C$.

If the candidate writes $18\sqrt{3}=\frac{1}{2}\times 5x\times \sin 60°$ **without** sight of a previous correct line then this would be M0

**M1:** Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ or awrt 0.866 and proceeds to $x^2=k$ oe such as $px^2=q$

This may be awarded from the correct formula or $A=ab\sin C$

**A1*:** Look for $x^2=12\Rightarrow x=2\sqrt{3}$, $x^2=4\times 3\Rightarrow x=2\sqrt{3}$ or $x=\sqrt{12}=2\sqrt{3}$

This is a given answer and all aspects must be correct including one of the above intermediate lines. It cannot be scored by using decimal equivalents to $\sqrt{3}$

Alternative using the given answer of $x=2\sqrt{3}$

**M1:** Attempts to use the formula $A=\frac{1}{2}\times 4\sqrt{3}\times 6\sqrt{3}\sin 60°$ oe

**M1:** Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ and proceeds to $A=18\sqrt{3}$

**A1*:** Concludes that $x=2\sqrt{3}$

**(b)**

**M1:** Attempts the cosine rule with the sides in the correct position.

This can be scored from $BC^2=(3x)^2+(2x)^2-2\times 3x\times 2x\times \cos 60°$ as long as there is some attempt to substitute $x$ in later. Condone slips on the squaring

**A1:** $BC^2=84$ Accept $BC^2=7\times 12$, $BC=\sqrt{84}$ or $BC=2\sqrt{21}$

If they replace the surds with decimals they can score the A1 for $BC^2=$ awrt 84.0

**A1:** $BC=2\sqrt{21}$

Condone other variables, say $x=2\sqrt{21}$, but it cannot be scored via decimals.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 15(a) | $\dfrac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\dfrac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{3+2\cos\theta}$ | M1 | 1.1b |
| | $\equiv\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$ | A1 | 1.1b |
| | $\equiv\dfrac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$ | M1 | 1.1b |
| | $\equiv 4-5\cos\theta$ * | A1* | 2.1 |
| | | **(4)** | |
| **(b)** | $4+3\sin x=4-5\cos x\Rightarrow\tan x=-\dfrac{5}{3}$ | M1 | 2.1 |
| | $x=$ awrt $121°, 301°$ | A1 A1 | 1.1b<br>1.1b |
| | | **(3)** | |
| | | | **(7 marks)** |

**Notes**

**(a)**

**M1:** Uses the identity $\sin^2\theta=1-\cos^2\theta$ within the fraction

**A1:** Correct (simplified) expression in just $\cos\theta$ $\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$ or exact equivalent such as $\dfrac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$ Allow for $\dfrac{12-7u-10u^2}{3+2u}$ where they introduce $u=\cos\theta$

We would condone mixed variables here.

**M1:** A correct attempt to factorise the numerator, usual rules. Allow candidates to use $u=\cos\theta$ oe

**A1*:** A fully correct proof with correct notation and no errors.

Only withhold the last mark for (1) Mixed variable e.g. $\theta$ and $x$'s (2) Poor notation $\cos\theta^2\leftrightarrow\cos^2\theta$ or $\sin^2=1-\cos^2$ within the solution.

Don't penalise incomplete lines if it is obvious that it is just part of their working

E.g. $\dfrac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\dfrac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{}=\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$

**(b)**

**M1:** Attempts to use part (a) and proceeds to an equation of the form $\tan x=k,\quad k\neq 0$

Condone $\theta\leftrightarrow x$ Do not condone $a\tan x=0\Rightarrow\tan x=b\Rightarrow x=...$

Alternatively squares $3\sin x=-5\cos x$ and uses $\sin^2 x=1-\cos^2 x$ oe to reach $\sin x=A, -1<A<1$ or $\cos x=B, -1<B<1$

**A1:** Either $x=$ awrt $121°$ **or** $301°$. Condone awrt 2.11 or 5.25 which are the radian solutions

**A1:** Both $x=$ awrt $121°$ **and** $301°$ and no other solutions.

Answers without working, or with no incorrect working in (b).

Question states hence or otherwise so allow

For 3 marks both $x=$ awrt $121°$ **and** $301°$ and no other solutions.

For 1 marks scored SC 100 for either $x=$ awrt $121°$ **or** $301°$

| Notes on Question 15 continue |
| --- |
| Alternative proof in part (a):<br>**M1:** Multiplies across and form 3TQ in $\cos\theta$ on rhs<br>$10\sin^2\theta - 7\cos\theta + 2 = (4-5\cos\theta)(3+2\cos\theta) \Rightarrow 10\sin^2\theta - 7\cos\theta + 2 = A\cos^2\theta + B\cos\theta + C$<br>**A1:** Correct identity formed $10\sin^2\theta - 7\cos\theta + 2 = -10\cos^2\theta - 7\cos\theta + 12$<br>**dM1:** Uses $\cos^2\theta = 1-\sin^2\theta$ on the rhs or $\sin^2\theta = 1-\cos^2\theta$ on the lhs<br>Alternatively proceeds to $10\sin^2\theta + 10\cos^2\theta = 10$ and makes a statement about $\sin^2\theta + \cos^2\theta = 1$ oe<br>**A1*:** Shows that $(4-5\cos\theta)(3+2\cos\theta) \equiv 10\sin^2\theta - 7\cos\theta + 2$ oe AND makes a minimal statement "hence true" |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 16 | Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in $\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$ | M1 | 1.1b |
| | Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2} \rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify | M1 | 2.1 |
| | $=\frac{4}{3}$ oe | A1 | 1.1b |
| | | **(3)** | |
| | | | **(3 marks)** |

**M1:** Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in the given expression.

See below for description of marking of $\cos 4\theta$

**M1:** Attempts to substitute both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$

$$\rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta} \text{ and attempts to simplify.}$$

Condone missing bracket on the $4\theta$ so $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$ would score the method

Expect to see it simplified to a single term which could be in terms of $\theta$

Look for an answer of $k$ but condone $k\theta$ following a slip

**A1:** Uses both identities and simplifies to $\frac{4}{3}$ or exact equivalent with no incorrect lines BUT allow recovery on missing bracket for $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$.

Eg. $\frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta} = \frac{8\theta^2}{6\theta} = \frac{4}{3}$ is M1 M1 A0

Condone awrt 1.33.

.......................................................................................................................................................

Alt: $\frac{1-\cos 4\theta}{2\theta \sin 3\theta} = \frac{1-\left(1-2\sin^2 2\theta\right)}{2\theta \sin 3\theta} = \frac{2\sin^2 2\theta}{2\theta \sin 3\theta} = \frac{2\times(2\theta)^2}{2\theta \times 3\theta} = \frac{4}{3}$

M1 For an attempt at $\sin 3\theta \approx 3\theta$ or the identity $\cos 4\theta = 1-2\sin^2 2\theta$ with $\sin 2\theta \approx 2\theta$

M1 For both of the above and attempts to simplify to a single term.

A1 $\frac{4}{3}$ oe

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **17** | States or uses $\frac{1}{2}r^2\theta = 11$ | B1 | 1.1b |
| | States or uses $2r + r\theta = 4r\theta$ | B1 | 1.1b |
| | Attempts to solve, full method $r = ...$ | M1 | 3.1a |
| | $r = \sqrt{33}$ | A1 | 1.1b |
| | | | **[4]** |
| | | | **(4 marks)** |

**Notes:**

**B1:** States or uses $\frac{1}{2}r^2\theta = 11$ This may be implied with an embedded found value for $\theta$

**B1:** States or uses $2r + r\theta = 4r\theta$ or equivalent

**M1:** Full method to find $r = ...$ This involves combining the equations to eliminate $\theta$ or find $\theta$

The initial equations must be of the same "form" (see **) but condone slips when attempting to solve.

It cannot be scored from impossible values for $\theta$ Hence only score if $0 < \theta < 2\pi$ FYI $\theta = \frac{2}{3}$ radians

Allow this to be scored from equations such as $...r^2\theta = 11$ and ones that simplify to $...r = ...r\theta$ **

Allow their $2r + r\theta = 4r\theta \Rightarrow \theta = ..$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $2r + r\theta = 4r\theta \Rightarrow r\theta = ..$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $\frac{1}{2}r^2\theta = 11 \Rightarrow \theta = \frac{..}{r^2}$ then substitute into their $2r + r\theta = 4r\theta \Rightarrow r = ..$

**A1:** $r = \sqrt{33}$ only but isw after a correct answer.

..............................................................................................................................................

The whole question can be attempted using $\theta$ in degrees.

B1: States or uses $\frac{\theta}{360} \times \pi r^2 = 11$

B1: States or uses $2r + \frac{\theta}{360} \times 2\pi r = 4 \times \frac{\theta}{360} \times 2\pi r$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **18 (a)** | $D = 5 + 2\sin(30\times 6.5)° = \text{awrt } 4.48\text{ m}$ with units | B1 | 3.4 |
| | | **(1)** | |
| **(b)** | $3.8 = 5 + 2\sin(30t)° \Rightarrow \sin(30t)° = -0.6$ | M1<br>A1 | 1.1b<br>1.1b |
| | $t = 10.77$ | dM1 | 3.1a |
| | 10:46 a.m. or 10:47 a.m. | A1 | 3.2a |
| | | **(4)** | |
| | | | **(5 marks)** |

**Notes:**

**(a)**

**B1:** Scored for using the model ie. substituting $t = 6.5$ into $D = 5 + 2\sin(30t)°$ and stating $D = awrt\ 4.48\text{m}$. The units must be seen somewhere in (a) . So allow when $D = 4.482.. = 4.5\text{ m}$

Allow the mark for a correct answer without any working.

**(b)**

**M1:** For using $D = 3.8$ and proceeding to $\sin(30t)° = k, \quad |k| \le 1$

**A1:** $\sin(30t)° = -0.6$ This may be implied by any correct answer for *t* such as $t = 7.2$

If the A1 implied, the calculation must be performed in degrees.

**dM1:** For finding **the first value** of *t* for their $\sin(30t)° = k$ after $t = 8.5$.

You may well see other values as well which is not an issue for this dM mark

(Note that $\sin(30t)° = -0.6 \Rightarrow 30t = 216.9°$ as well but this gives $t = 7.2$)

For the correct $\sin(30t)° = -0.6 \Rightarrow 30t = 323.1 \Rightarrow t = \text{awrt } 10.8$

For the incorrect $\sin(30t)° = +0.6 \Rightarrow 30t = 396.9 \Rightarrow t = \text{awrt } 13.2$

So award this mark if you see $30t = \text{invsin } their -0.6$ to give the first value of *t* where 30 *t* > 255

**A1:** Allow 10:46 a.m. (12 hour clock notation) or 10:46 (24 hour clock notation ) oe

Allow 10:47 a.m. (12 hour clock notation) or 10:47 (24 hour clock notation ) oe

DO NOT allow 646 minutes or 10 hours 46 minutes.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **19 (a)** | Uses $15=\frac{1}{2}\times5\times10\times\sin\theta$ | M1 | 1.1b |
| | $\sin\theta=\frac{3}{5}$ oe | A1 | 1.1b |
| | Uses $\cos^2\theta=1-\sin^2\theta$ | M1 | 2.1 |
| | $\cos\theta=\pm\frac{4}{5}$ | A1 | 1.1b |
| | | **(4)** | |
| **(b)** | Uses $BC^2=10^2+5^2-2\times10\times5\times"-\frac{4}{5}"$ | M1 | 3.1a |
| | $BC=\sqrt{205}$ | A1 | 1.1b |
| | | **(2)** | |
| | | | **(6 marks)** |

**Notes**

**(a)**

**M1:** Uses the formula $\text{Area}=\frac{1}{2}ab\sin C$ in an attempt to find the value of $\sin\theta$ or $\theta$

**A1:** $\sin\theta=\frac{3}{5}$ oe This may be implied by $\theta=$ awrt 36.9° or awrt 0.644 (radians)

**M1:** Uses their value of $\sin\theta$ to find two values of $\cos\theta$ This may be scored via the formula $\cos^2\theta=1-\sin^2\theta$ or by a triangle method. Also allow the use of a graphical calculator or candidates may just write down the **two values**. The values must be symmetrical $\pm k$

**A1:** $\cos\theta=\pm\frac{4}{5}$ or $\pm0.8$ Condone these values appearing from $\pm0.79....$

**(b)**

**M1:** Uses a suitable method of finding the longest side. For example chooses the negative value (or the obtuse angle) and proceeds to find *BC* using the cosine rule. Alternatively works out *BC* using both values and chooses the larger value. If stated the cosine rule should be correct (with a minus sign). Note if the sign is +ve and the acute angle is chosen the correct value will be seen. It is however M0 A0

**A1:** $BC=\sqrt{205}$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **20 (a)** | $4\cos\theta - 1 = 2\sin\theta\tan\theta \Rightarrow 4\cos\theta - 1 = 2\sin\theta \times \dfrac{\sin\theta}{\cos\theta}$ | M1 | 1.2 |
| | $\Rightarrow 4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe | A1 | 1.1b |
| | $\Rightarrow 4\cos^2\theta - \cos\theta = 2\left(1-\cos^2\theta\right)$ | M1 | 1.1b |
| | $6\cos^2\theta - \cos\theta - 2 = 0$ * | A1* | 2.1 |
| | | **(4)** | |
| **(b)** | For attempting to solve given quadratic | M1 | 1.1b |
| | $(\cos 3x) = \dfrac{2}{3}, -\dfrac{1}{2}$ | B1 | 1.1b |
| | $x = \dfrac{1}{3}\arccos\left(\dfrac{2}{3}\right)$ or $\dfrac{1}{3}\arccos\left(-\dfrac{1}{2}\right)$ | M1 | 1.1b |
| | $x = 40°, 80°$, awrt $16.1°$ | A1 | 2.2a |
| | | **(4)** | |
| | | | **(8 marks)** |

**Notes**

**(a)**

**M1:** Recall and use the identity $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ **Note that it cannot just be stated.**

**A1:** $4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe.

This is scored for a correct line that does not contain any fractional terms.

It may be awarded later in the solution after the identity $1-\cos^2\theta = \sin^2\theta$ has been used Eg for $\cos\theta(4\cos\theta - 1) = 2\left(1-\cos^2\theta\right)$ or equivalent

**M1:** Attempts to use the correct identity $1-\cos^2\theta = \sin^2\theta$ to form an equation in just $\cos\theta$

**A1*:** Proceeds to correct answer through rigorous and clear reasoning. No errors in notation or bracketing. For example $\sin^2\theta = \sin\theta^2$ is an error in notation

**(b)**

**M1:** For attempting to solve the given quadratic "$6y^2 - y - 2 = 0$ " where $y$ could be $\cos 3x$, $\cos x$, or even just $y$. When factorising look for $(ay+b)(cy+d)$ where $ac = \pm 6$ and $bd = \pm 2$

This may be implied by the correct roots (even award for $\left(y \pm \dfrac{2}{3}\right)\left(y \pm \dfrac{1}{2}\right)$), an attempt at factorising, an attempt at the quadratic formula, an attempt at completing the square and even $\pm$ the correct roots.

**B1:** For the roots $\dfrac{2}{3}, -\dfrac{1}{2}$ oe

**M1:** Finds at least one solution for $x$ from $\cos 3x$ **within the given range** for their $\dfrac{2}{3}, -\dfrac{1}{2}$

**A1:** $x = 40°, 80°$, awrt $16.1°$ **only** Withhold this mark if there are **any** other values even if they are outside the range. Condone 40 and 80 appearing as 40.0 and 80.0

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **21(i)** | $\left(2\theta+10°\right)=\arcsin\left(-0.6\right)$ | M1 | 1.1b |
| | $\left(2\theta+10°\right)=-143.13°,-36.87°,216.87°,323.13°$ (Any two) | A1 | 1.1b |
| | Correct order to find $\theta=...$ | dM1 | 1.1b |
| | Two of $\theta=-76.6°,-23.4°,103.4°,156.6°$. | A1 | 1.1b |
| | $\theta=-76.6°,-23.4°,103.4°,156.6°$, only | A1 | 2.1 |
| | | **(5)** | |
| **(ii)** | (a) Explains that the student has not considered the negative value of $x\left(=-29.0°\right)$ when solving $\cos x=\frac{7}{8}$ | B1 | 2.3 |
| | Explains that the student should check if any solutions of $\sin x=0$ (the cancelled term) are solutions of the given equation. $x=0°$ should have been included as a solution | B1 | 2.3 |
| | (b) Attempts to solve $4\alpha+199°=\left(360-29.0\right)°$ | M1 | 2.2a |
| | $\alpha=33.0°$ | A1 | 1.1b |
| | | **(4)** | |
| | | **(9 marks)** | |

Notes:

(i)

**M1:** Attempts $\arcsin\left(-0.6\right)$ implied by any correct answer

**A1:** Any two of $-143.13°,-36.87°,216.87°,323.13°$

**dM1:** Correct method to find any value of $\theta$

**A1:** Any two of $\theta=-76.6°,-23.4°,103.4°,156.6°$.

**A1:** A full solution leading to all four answers and no extras
$\theta=-76.6°,-23.4°,103.4°,156.6°$, only

(ii)(a)

**B1:** See scheme

**B1:** See scheme

(ii)(b)

**M1:** For deducing the smallest positive solution occurs when $4\alpha+199°=\left(360-29.0\right)°$

**A1:** $\alpha=33°$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **22 (a)** | $R = 2.5$ | B1 | 1.1b |
| | $\tan\alpha = \dfrac{1.5}{2}$ o.e. | M1 | 1.1b |
| | $\alpha = 0.6435$, so $2.5\sin(\theta - 0.6435)$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)** | e.g. $D = 6 + 2\sin\left(\dfrac{4\pi(0)}{25}\right) - 1.5\cos\left(\dfrac{4\pi(0)}{25}\right) = 4.5\text{m}$<br>or $D = 6 + 2.5\sin\left(\dfrac{4\pi(0)}{25} - 0.6435\right) = 4.5\text{m}$ | B1 | 3.4 |
| | | **(1)** | |
| **(c)** | $D_{max} = 6 + 2.5 = 8.5$ m | B1ft | 3.4 |
| | | **(1)** | |
| **(d)** | Sets $\dfrac{4\pi t}{25} - \text{"}0.6435\text{"} = \dfrac{5\pi}{2}$ or $\dfrac{\pi}{2}$ | M1 | 1.1b |
| | Afternoon solution $\Rightarrow \dfrac{4\pi t}{25} - \text{"}0.6435\text{"} = \dfrac{5\pi}{2} \Rightarrow t = \dfrac{25}{4\pi}\left(\dfrac{5\pi}{2} + \text{"}0.6435\text{"}\right)$ | M1 | 3.1b |
| | $\Rightarrow t = 16.9052... \Rightarrow$ Time $= 16:54$ or 4:54 pm | A1 | 3.2a |
| | | **(3)** | |
| **(e)(i)** | • An attempt to find the depth of water at 00:00 on 19th October 2017 for at least one of either Tom's model or Jolene's model. | M1 | 3.4 |
| | • At 00:00 on 19th October 2017,<br>Tom: $D = 3.72...$ m and Jolene: $H = 4.5$ m<br>and e.g.<br>• As $4.5 \neq 3.72$ then Jolene's model is not true<br>• Jolene's model is not continuous at 00:00 on 19th October 2017<br>• Jolene's model does not continue on from where Tom's model has ended | A1 | 3.5a |
| **(ii)** | To make the model continuous, e.g.<br>• $H = 5.22 + 2\sin\left(\dfrac{4\pi x}{25}\right) - 1.5\cos\left(\dfrac{4\pi x}{25}\right), \quad 0 \leqslant x < 24$<br>• $H = 6 + 2\sin\left(\dfrac{4\pi(x+24)}{25}\right) - 1.5\cos\left(\dfrac{4\pi(x+24)}{25}\right), \quad 0 \leqslant x < 24$ | B1 | 3.3 |
| | | **(3)** | |
| | | | **(11 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **22 (d) Alt 1** | Sets $\frac{4\pi t}{25} - \text{"0.6435"} = \frac{\pi}{2}$ | M1 | 1.1b |
| | Period $= 2\pi \div \left(\frac{4\pi}{25}\right) = 12.5$<br>Afternoon solution $\Rightarrow t = 12.5 + \frac{25}{4\pi}\left(\frac{\pi}{2} + \text{"0.6435"}\right)$ | M1 | 3.1b |
| | $\Rightarrow t = 16.9052... \Rightarrow \text{Time} = 16:54$ or 4:54 pm | A1 | 3.2a |
| | | **(3)** | |

Question **22** Notes:

| | |
|---|---|
| **(a)** | |
| **B1:** | $R = 2.5$ Condone $R = \sqrt{6.25}$ |
| **M1:** | For either $\tan\alpha = \frac{1.5}{2}$ or $\tan\alpha = -\frac{1.5}{2}$ or $\tan\alpha = \frac{2}{1.5}$ or $\tan\alpha = -\frac{2}{1.5}$ |
| **A1:** | $\alpha =$ awrt 0.6435 |
| **(b)** | |
| **B1:** | Uses Tom's model to find $D = 4.5$ (m) at 00:00 on 18th October 2017 |
| **(c)** | |
| **B1ft:** | Either 8.5 or follow through "6 + their $R$" (by using their $R$ found in part (a)) |
| **(d)** | |
| **M1:** | Realises that $D = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right) = 6 + \text{"2.5"}\sin\left(\frac{4\pi t}{25} - \text{"0.6435"}\right)$ and<br>so maximum depth occurs when $\sin\left(\frac{4\pi t}{25} - \text{"0.6435"}\right) = 1 \Rightarrow \frac{4\pi t}{25} - \text{"0.6435"} = \frac{\pi}{2}$ or $\frac{5\pi}{2}$ |
| **M1:** | Uses the model to deduce that a p.m. solution occurs when $\frac{4\pi t}{25} - \text{"0.6435"} = \frac{5\pi}{2}$ and rearranges this equation to make $t = ...$ |
| **A1:** | Finds that maximum depth occurs in the afternoon at 16:54 or 4:54 pm |
| **(d) Alt 1** | |
| **M1:** | Maximum depth occurs when $\sin\left(\frac{4\pi t}{25} - \text{"0.6435"}\right) = 1 \Rightarrow \frac{4\pi t}{25} - \text{"0.6435"} = \frac{\pi}{2}$ |
| **M1:** | Rearranges to make $t = ...$ and adds on the period, where period $= 2\pi \div \left(\frac{4\pi}{25}\right) \{= 12.5\}$ |
| **A1:** | Finds that maximum depth occurs in the afternoon at 16:54 or 4:54 pm |

| Question **22** Notes Continued: | |
|---|---|
| **(e)(i)** | |
| **M1:** | See scheme |
| **A1:** | See scheme |
| | **Note:** Allow Special Case M1 for a candidate who just states that Jolene's model is not continuous at 00:00 on 19th October 2017 o.e. |
| **(e)(ii)** | |
| **B1:** | Uses the information to set up a new model for $H$. (See scheme) |

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| **23(a)** | **<u>Way 1</u>**<br>Finds third angle of triangle and uses or states<br>$\frac{x}{\sin 60^\circ} = \frac{30}{\sin \text{"}50^\circ\text{"}}$ | **<u>Way 2</u>**<br>Finds third angle of triangle and uses or states<br>$\frac{y}{\sin 70^\circ} = \frac{30}{\sin \text{"}50^\circ\text{"}}$ | M1 | 2.1 |
| | So $x = \frac{30\sin 60^\circ}{\sin 50^\circ}$ $(= 33.9)$ | So $y = \frac{30\sin 70^\circ}{\sin 50^\circ}$ $(= 36.8)$ | A1 | 1.1b |
| | Area = $\frac{1}{2} \times 30 \times x \times \sin 70^\circ$ or $\frac{1}{2} \times 30 \times y \times \sin 60$ | | M1 | 3.1a |
| | $= 478 \text{ m}^2$ | | A1ft | 1.1b |
| | | | **(4)** | |
| **(b)** | Plausible reason e.g. Because the angles and the side length are not given to four significant figures<br>Or e.g. The lawn may not be flat | | B1 | 3.2b |
| | | | **(1)** | |
| | | | | **(5 marks)** |

**Notes:**

**(a)**

**M1:** Uses sine rule with their third angle to find one of the unknown side lengths

**A1:** Finds expression for, or value of either side length

**M1:** Completes method to find area of triangle

**A1ft:** Obtains a correct answer for their value of $x$ or their value of $y$

**(b)**

**B1:** As information given in the question may not be accurate to 4sf or the lawn may not be flat so modelling by a plane figure may not be accurate

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **24** | Uses $\sin^2 x = 1 - \cos^2 x \Rightarrow 12(1-\cos^2 x) + 7\cos x - 13 = 0$ | M1 | 3.1a |
| | $\Rightarrow 12\cos^2 x - 7\cos x + 1 = 0$ | A1 | 1.1b |
| | Uses solution of quadratic to give $\cos x =$ | M1 | 1.1b |
| | Uses inverse cosine on their values, giving two correct follow through values (see note) | M1 | 1.1b |
| | $\Rightarrow x =\ 430.5°,\ 435.5°$ | A1 | 1.1b |
| | | | **(5 marks)** |

**Notes:**

**M1:** Uses correct identity

**A1:** Correct three term quadratic

**M1:** Solves their three term quadratic to give values for cos $x$. (The correct answers are $\cos x = \frac{1}{3}$ or $\frac{1}{4}$ but this is not necessary for this method mark)

**M1:** Uses inverse cosine on their values, giving two correct follow through values - may be outside the given domain

**A1:** Two correct answers in the given domain

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **25(a)** | Uses $s = r\theta \Rightarrow 3 = r \times 0.4$ | M1 | 1.2 |
| | $\Rightarrow OD = 7.5\text{ cm}$ | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | Uses angle $AOB = (\pi - 0.4)$ or uses radius is (12 – '7.5') cm | M1 | 3.1a |
| | Uses area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2}\times(12-7.5)^2\times(\pi-0.4)$ | M1 | 1.1b |
| | $= 27.8\text{cm}^2$ | A1ft | 1.1b |
| | | **(3)** | |
| | | | **(5 marks)** |

**Notes:**

**(a)**

**M1:** Attempts to use the correct formula $s = r\theta$ with $s = 3$ and $\theta = 0.4$

**A1:** $OD$ = 7.5 cm (An answer of 7.5cm implies the use of a correct formula and scores both marks)

**(b)**

**M1:** $AOB = \pi - 0.4$ may be implied by the use of $AOB$ = awrt 2.74 or uses radius is (12 – their '7.5')

**M1:** Follow through on their radius (12 – their $OD$) and their angle

**A1ft:** Allow awrt 27.8 cm$^2$. (Answer 27.75862562). Follow through on their (12 – their '7.5')
Note: Do not follow through on a radius that is negative.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **26(a)** | Deduces that $A=\pm 50$ **or** $b=\frac{1}{4}$ | B1 | 3.4 |
| | Deduces that $A=\pm 50$ **and** $b=\frac{1}{4}$ | B1 | 3.4 |
| | Uses $t=0, H=1 \Rightarrow \alpha = ...$ E.g. $1 = "50"\sin(\alpha)° \Rightarrow \alpha = ...$ | M1 | 3.4 |
| | $H=\left\lvert \pm 50\sin\left(\frac{1}{4}t+1.15\right)^{\circ} \right\rvert$ | A1 | 3.3 |
| | | **(4)** | |
| **(b)** | E.g. the minimum height above the ground of the passenger on the original model was 0 m<br>or<br>Adding "*d*" means the passenger does not touch the ground. | B1 | 3.5b |
| | | **(1)** | |
| | | | **(5 marks)** |
| **Notes:** | | | |

(a) Note that B0B1 is not possible

**B1**: Uses the equation of the given model to deduce that $A=\pm 50$ **or** $b=\frac{1}{4}$ o.e.

May be seen embedded within their equation.

**B1**: Uses the equation of the given model to deduce that $A=\pm 50$ **and** $b=\frac{1}{4}$ o.e.

May be seen embedded within their equation.

**M1**: Uses $t = 0$ and $H = 1$ in the equation of the model to find a value for $\alpha$ .

Follow through on their value for $A$. Allow for $\pm 1 = "50"\sin(\alpha)° \Rightarrow \alpha = ...$ where α is in degrees or radians.

Note that in radians $\sin^{-1}\left(\frac{1}{50}\right) \approx \frac{1}{50}$ (0.0200…) which may appear incorrect but is in fact ok.

Also in degrees a value of e.g. 1.14 (truncated) would indicate the method.

**A1**: Writes down the correct full equation of the model: $H=\left\lvert "\pm" 50\sin\left(\frac{1}{4}t+1.15\right)^{\circ} \right\rvert$ o.e.

Condone omission of degrees symbol and allow awrt 1.15 for $\alpha$.
Allow if a correct equation is seen anywhere in their solution.

(b)

**B1**: Gives a suitable explanation with no contradictory statements.

Condone "so that pod/capsule/seat/passenger/ferris wheel/it etc. will not hit/touch the ground"

Responses that focus on the starting point of the model are likely to score B0

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **27(a)** | Uses the common ratio $\dfrac{5+2\sin\theta}{12\cos\theta}=\dfrac{6\tan\theta}{5+2\sin\theta}$ o.e. | M1 | 3.1a |
| | Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$ <br> $(5+2\sin\theta)^2=6\times12\sin\theta$ | dM1 | 1.1b |
| | Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ <br> $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ * | A1* | 2.1 |
| | | **(3)** | |
| **(a) Alt** | **(a) Alternative example:** | | |
| | Uses the common ratio $12r\cos\theta=5+2\sin\theta,\ 12r^2\cos\theta=6\tan\theta$ <br> $\Rightarrow 12\cos\theta\left(\dfrac{5+2\sin\theta}{12\cos\theta}\right)^2=6\tan\theta$ | M1 | 3.1a |
| | Multiplies up and uses $\tan\theta\times\cos\theta=\sin\theta$ <br> $(5+2\sin\theta)^2=6\tan\theta\times12\cos\theta=72\sin\theta$ | dM1 | 1.1b |
| | Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ <br> $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ * | A1* | 2.1 |
| | | **(3)** | |
| **(b)** | $4\sin^2\theta-52\sin\theta+25=0\Rightarrow\sin\theta=\dfrac{1}{2}\left(,\dfrac{25}{2}\right)$ | M1 | 1.1b |
| | $\theta=\dfrac{5\pi}{6}$ | A1 | 1.2 |
| | | **(2)** | |
| **(c)** | Attempts a value for either $a$ or $r$ <br> e.g. $a=12\cos\theta=12\times-\dfrac{\sqrt{3}}{2}$ **or** $r=\dfrac{5+2\sin\theta}{12\cos\theta}=\dfrac{5+2\times\frac{1}{2}}{12\times-\frac{\sqrt{3}}{2}}$ | M1 | 3.1a |
| | $"a"=-6\sqrt{3}$ **and** $"r"=-\dfrac{1}{\sqrt{3}}$ o.e. | A1 | 1.1b |
| | Uses $S_\infty=\dfrac{a}{1-r}=\dfrac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$ | dM1 | 2.1 |
| | Rationalises denominator <br> $S_\infty=\dfrac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}=\dfrac{-18}{\sqrt{3}+1}\times\dfrac{\sqrt{3}-1}{\sqrt{3}-1}$ | ddM1 | 1.1b |
| | $(S_\infty=)9(1-\sqrt{3})$ | A1 | 2.1 |
| | | **(5)** | |
| | | | **(10 marks)** |
| **Notes:** | | | |

(a) **M1**: For the key step in using the ratio of $\dfrac{a_2}{a_1}=\dfrac{a_3}{a_2}$

**dM1**: Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$

**A1***: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

**Alternative:**

**M1**: Expresses the 2nd and 3rd terms in terms of the first term and the common ratio and eliminates "r"

**dM1**: Multiplies up and uses $\tan\theta \times \cos\theta = \sin\theta$

**A1***: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

| |
|---|
| Other approaches may be seen in (a) and can be marked in a similar way e.g. M1 for correctly obtaining an equation in $\theta$ using the GP, M1 for applying $\tan\theta \times \cos\theta = \sin\theta$ or equivalent and eliminating fractions, A1 as above |
| Example: $u_2 = \dfrac{u_1 \times u_3}{u_2} \Rightarrow 5 + 2\sin\theta = \dfrac{12\cos\theta \times 6\tan\theta}{5 + 2\sin\theta}$ **M1** |
| $\Rightarrow (5 + 2\sin\theta)^2 = 72\sin\theta$ **dM1** |
| $25 + 20\sin\theta + 4\sin^2\theta = 72\sin\theta$ |
| $\Rightarrow 4\sin^2\theta - 52\sin\theta + 25 = 0$ * **A1** |

(b)

**M1**: Attempts to solve $4\sin^2\theta - 52\sin\theta + 25 = 0$. Must be clear they have found $\sin\theta$ and not e.g. just $x$ from $4x^2 - 52x + 25 = 0$. Working does not need to be seen but see general guidance for solving a 3TQ if necessary. Note that the $\dfrac{25}{2}$ does not need to be seen.

**A1**: $\theta = \dfrac{5\pi}{6}$ and no other values unless they are rejected or the $\dfrac{5\pi}{6}$ clearly selected here and not in (c)

A minimum requirement in (b) is e.g. $\sin\theta = \dfrac{1}{2}, \quad \theta = \dfrac{5\pi}{6}$

Do **not** allow 150° for $\dfrac{5\pi}{6}$

**PTO for the notes to part (c)**

(c) Allow full marks in (c) if e.g. $\theta = \frac{\pi}{6}$ is their answer to (b) but $\theta = \frac{5\pi}{6}$ is used here.

or if e.g. $\theta = \frac{5\pi}{6}$ is their answer to (b) but $\theta = \frac{\pi}{6}$ is used here allow the M marks only.

**M1**: For attempting a value (exact or decimal) for either $a$ or $r$ using **their** $\theta$

E.g. $a = 12\cos\theta = \left(12 \times -\frac{\sqrt{3}}{2}\right)$ or $r = \frac{5+2\sin\theta}{12\cos\theta} = \left(\frac{5+2\times\frac{1}{2}}{12\times -\frac{\sqrt{3}}{2}}\right)$ oe e.g. $r = \frac{6\tan\theta}{5+2\sin\theta} = \left(\frac{6\times -\frac{1}{\sqrt{3}}}{5+2\times\frac{1}{2}}\right)$

**A1**: Finds both $a = -6\sqrt{3}$ and $r = -\frac{1}{\sqrt{3}}$ which can be left unsimplified but $\sin\theta = \frac{1}{2}, \cos\theta = -\frac{\sqrt{3}}{2}$

and $\tan\theta = -\frac{\sqrt{3}}{3}$ (if required) must have been used.

**dM1**: Uses both **values** of "$a$" and "$r$" with the equation $S_\infty = \frac{a}{1-r} = \frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$ to create an expression

involving surds where $a$ and $r$ have come from appropriate work and $|r| < 1$

Depends on the first method mark.

**ddM1**: Rationalises denominator. The denominator must be of the form $p \pm q\sqrt{3}$ oe e.g. $p + \frac{q}{\sqrt{3}}$

Depends on both previous method marks.

Note that stating e.g. $\frac{k}{p+q\sqrt{3}} \times \frac{p-q\sqrt{3}}{p-q\sqrt{3}}$ or $\frac{k}{p+\frac{q}{\sqrt{3}}} \times \frac{p-\frac{q}{\sqrt{3}}}{p-\frac{q}{\sqrt{3}}}$ is sufficient.

**A1**: Obtains $(S_\infty =) 9\left(1-\sqrt{3}\right)$

Note that full marks are available in (c) for the use of $\theta = 150°$

Note also that marks may be implied in (c) by e.g.

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\theta}{1-\frac{5+2\sin\theta}{12\cos\theta}} = \frac{144\cos^2\theta}{12\cos\theta - 5 - 2\sin\theta} = \frac{144\cos^2\frac{5\pi}{6}}{12\cos\frac{5\pi}{6} - 5 - 2\sin\frac{5\pi}{6}}$$

$$= \frac{108}{-6-6\sqrt{3}} = \frac{108}{-6-6\sqrt{3}} \times \frac{-6+6\sqrt{3}}{-6+6\sqrt{3}} = \frac{-648+648\sqrt{3}}{-72} = 9\left(1-\sqrt{3}\right)$$

Scores M1A1 implied dM1 ddM1 A1

**See next page for some other cases in (c) and how to mark them:**

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{5\pi}{6}}{1-\dfrac{5+2\sin\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}}} \quad \text{or e.g.} \quad S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{\pi}{6}}{1-\dfrac{5+2\sin\frac{\pi}{6}}{12\cos\frac{\pi}{6}}}$$

And nothing else

scores M1A0dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{5\pi}{6}}{1-\dfrac{5+2\sin\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}}} = 9\left(1-\sqrt{3}\right)$$

Scores M1A1dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{\pi}{6}}{1-\dfrac{5+2\sin\frac{\pi}{6}}{12\cos\frac{\pi}{6}}} = 9\left(1+\sqrt{3}\right)$$

Scores M1A0dM1ddM0A0

$S_\infty = 9\left(1-\sqrt{3}\right)$ with no working scores no marks

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **28** | **Examples:**<br>$4\sin\frac{\theta}{2}\approx 4\left(\frac{\theta}{2}\right),\ 3\cos^2\theta\approx 3\left(1-\frac{\theta^2}{2}\right)^2$<br>$3\cos^2\theta = 3\left(1-\sin^2\theta\right)\approx 3\left(1-\theta^2\right)$<br>$3\cos^2\theta = 3\frac{\left(\cos 2\theta+1\right)}{2}\approx\frac{3}{2}\left(1-\frac{4\theta^2}{2}+1\right)$ | M1 | 1.1a |
| | **Examples:**<br>$4\sin\frac{\theta}{2}+3\cos^2\theta\approx 4\left(\frac{\theta}{2}\right)+3\left(1-\frac{\theta^2}{2}\right)^2$<br>$4\sin\frac{\theta}{2}+3\cos^2\theta = 4\left(\frac{\theta}{2}\right)+3\left(1-\sin^2\theta\right)\approx 2\theta+3\left(1-\theta^2\right)$<br>$4\sin\frac{\theta}{2}+3\cos^2\theta = 4\sin\frac{\theta}{2}+3\frac{\left(\cos 2\theta+1\right)}{2}\approx 4\left(\frac{\theta}{2}\right)+\frac{3}{2}\left(1-\frac{4\theta^2}{2}+1\right)$ | dM1 | 1.1b |
| | $=2\theta+3\left(1-\theta^2+...\right)=3+2\theta-3\theta^2$ | A1 | 2.1 |
| | | **(3)** | |
| | | | **(3 marks)** |

**Notes**

M1: Attempts to use at least one correct approximation **within the given expression**.
Either $\sin\frac{\theta}{2}\approx\frac{\theta}{2}$ or $\cos\theta\approx 1-\frac{\theta^2}{2}$ or e.g. $\sin\theta\approx\theta$ if they write $\cos^2\theta$ as $1-\sin^2\theta$ or e.g. $\cos 2\theta\approx 1-\frac{(2\theta)^2}{2}$ (condone missing brackets) if they write $\cos^2\theta$ as $\frac{1+\cos 2\theta}{2}$.
Allow sign slips only with any identities used but the appropriate approximations must be applied.

dM1: Attempts to use correct approximations **with the given expression** to obtain an expression in terms of $\theta$ only. **Depends on the first method mark.**

A1: Correct terms following correct work. Allow the terms in any order and ignore any extra terms if given correct or incorrect.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **29(a)** | Angle $AOB = \frac{\pi - \theta}{2}$ | B1 | 2.2a |
| | | **(1)** | |
| **(b)** | Area = $2\times\frac{1}{2}r^2\left(\frac{\pi-\theta}{2}\right)+\frac{1}{2}(2r)^2\theta$ | M1 | 2.1 |
| | $=\frac{1}{2}r^2\pi-\frac{1}{2}r^2\theta+2r^2\theta=\frac{3}{2}r^2\theta+\frac{1}{2}r^2\pi=\frac{1}{2}r^2(3\theta+\pi)$ * | A1* | 1.1b |
| | | **(2)** | |
| **(c)** | Perimeter = $4r+2r\left(\frac{\pi-\theta}{2}\right)+2r\theta$ | M1 | 3.1a |
| | $=4r+r\pi+r\theta$ or e.g. $r(4+\pi+\theta)$ | A1 | 1.1b |
| | | **(2)** | |
| | | | **(5 marks)** |

**Notes**

(a)
B1: Deduces the correct expression for angle $AOB$
Note that $\frac{180-\theta}{2}$ scores B0

(b)
M1: Fully correct strategy for the area using their angle from (a) appropriately.
Need to see $2\times\frac{1}{2}r^2\alpha$ or just $r^2\alpha$ where $\alpha$ is their angle in terms of $\theta$ from part (a) + $\frac{1}{2}(2r)^2\theta$ with or without the brackets.

A1*: Correct proof. For this mark you can condone the omission of the brackets in $\frac{1}{2}(2r)^2\theta$ as long as they are recovered in subsequent work e.g. when this term becomes $2r^2\theta$
The first term must be seen expanded as e.g. $\frac{1}{2}r^2\pi-\frac{1}{2}r^2\theta$ or equivalent

(c)
M1: Fully correct strategy for the perimeter using their angle from (a) appropriately
Need to see $4r+2r\alpha+2r\theta$ where $\alpha$ is their angle from part (a) in terms of $\theta$
A1: Correct simplified expression

Note that some candidates may change the angle to degrees at the start and all marks are available e.g.

(a) $\dfrac{180-\frac{180\theta}{\pi}}{2}$

(b) $2\left(\dfrac{180-\frac{180\theta}{\pi}}{2}\right)\times\frac{1}{360}\times\pi r^2+\frac{\theta}{360}\times\frac{180}{\pi}\times\pi(2r)^2=\frac{1}{2}\pi r^2-\frac{1}{2}r^2\theta+2r^2\theta=\frac{1}{2}r^2(3\theta+\pi)$

(c) $4r+2\left(\dfrac{180-\frac{180\theta}{\pi}}{2}\right)\times\frac{1}{360}\times 2\pi r+\frac{180\theta}{\pi}\times\frac{1}{360}\times 2\pi(2r)=4r+\pi r+r\theta$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **30(a)** | $R=\sqrt{5}$ | B1 | 1.1b |
| | $\tan\alpha=\frac{1}{2}$ or $\sin\alpha=\frac{1}{\sqrt{5}}$ or $\cos\alpha=\frac{2}{\sqrt{5}}\Rightarrow\alpha=...$ | M1 | 1.1b |
| | $\alpha=0.464$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)(i)** | $3+2\sqrt{5}$ | B1ft | 3.4 |
| **(ii)** | $\cos(0.5t+0.464)=1\Rightarrow 0.5t+0.464=2\pi$<br>$\Rightarrow t=...$ | M1 | 3.4 |
| | $t$ = 11.6 | A1 | 1.1b |
| | | **(3)** | |
| **(c)** | $3+2\sqrt{5}\cos(0.5t+0.464)=0$<br>$\cos(0.5t+0.464)=-\frac{3}{2\sqrt{5}}$ | M1 | 3.4 |
| | $\cos(0.5t+0.464)=-\frac{3}{2\sqrt{5}}\Rightarrow 0.5t+0.464=\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)$<br>$\Rightarrow t=2\left(\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)-0.464\right)$ | dM1 | 1.1b |
| | So the time required is e.g.:<br>$2(3.977...-0.464)-2(2.306...-0.464)$ | dM1 | 3.1b |
| | = 3.34 | A1 | 1.1b |
| | | **(4)** | |
| **(d)** | e.g. the "3" would need to vary | B1 | 3.5c |
| | | **(1)** | |
| | | | **(11 marks)** |

**Notes**

(a)

B1: $R=\sqrt{5}$ only.

M1: Proceeds to a value for $\alpha$ from $\tan\alpha=\pm\frac{1}{2}$ or $\sin\alpha=\pm\frac{1}{"R"}$ or $\cos\alpha=\pm\frac{2}{"R"}$

It is implied by either awrt 0.464 (radians) or awrt 26.6 (degrees)

A1: $\alpha=$awrt 0.464

(b)(i)

B1ft: For $(3+2\sqrt{5})$m or awrt 7.47 m and remember to isw. Condone lack of units.

Follow through on their $R$ value so allow 3 + 2×Their $R$. (Allow in decimals with at least 3sf accuracy)

(b)(ii)

M1: Uses $0.5t\pm"0.464"=2\pi$ to obtain a value for $t$

Follow through on their 0.464 but this angle must be in radians.

It is possible in degrees but only using $0.5t\pm"26.6"=360$

A1: Awrt 11.6

**Alternative for (b):**

$$H = 3+4\cos(0.5t)-2\sin(0.5t) \Rightarrow \frac{\mathrm{d}H}{\mathrm{d}t} = -2\sin(0.5t)-\cos(0.5t)=0$$

$$\Rightarrow \tan(0.5t) = -\frac{1}{2} \Rightarrow 0.5t = 2.677...,5.819... \Rightarrow t = 5.36,\ 11.6$$

$$t = 11.6 \Rightarrow H = 7.47$$

Score as follows:

M1: For a complete method:

Attempts $\frac{\mathrm{d}H}{\mathrm{d}t}$ and attempts to solve $\frac{\mathrm{d}H}{\mathrm{d}t}=0$ for $t$

A1: For $t$ = awrt 11.6

B1ft: For awrt 7.47 or 3 + 2×Their $R$

(c)

M1: Uses the model and sets $3+2"\sqrt{5}"\cos(...)=0$ and proceeds to $\cos(...)=k$ where $|k|<1$.

Allow e.g. $3+2"\sqrt{5}"\cos(...)<0$

dM1: Solves $\cos(0.5t \pm "0.464")=k$ where $|k|<1$ to obtain at least one value for $t$

This requires e.g. $2\left(\pi+\cos^{-1}(k)\pm\tan^{-1}\left(\frac{1}{2}\right)\right)$ or e.g. $2\left(\pi-\cos^{-1}(k)\pm\tan^{-1}\left(\frac{1}{2}\right)\right)$

**Depends on the previous method mark.**

dM1: A fully correct strategy to find the required duration. E.g. finds 2 consecutive values of $t$ when $H$ = 0 and subtracts. Alternatively finds $t$ when $H$ is minimum and uses the times found correctly to find the required duration.

**Depends on the previous method mark.**

**<u>Examples:</u>**

Second time at water level – first time at water level:

$$2\left(\pi+\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)-\tan^{-1}\left(\frac{1}{2}\right)\right)-2\left(\pi-\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)-\tan^{-1}\left(\frac{1}{2}\right)\right)=7.02685...-3.68492...$$

2×(first time at minimum point – first time at water level):

$$2\left(2\left(\pi-\tan^{-1}\left(\frac{1}{2}\right)\right)-2\left(\pi-\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)-\tan^{-1}\left(\frac{1}{2}\right)\right)\right)=2(5.35589...-3.68492...)$$

**Note that both of these examples equate to $4\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)$ which is not immediately obvious but may be seen as an overall method.**

**There may be other methods – if you are not sure if they deserve credit send to review.**

A1: Correct value. Must be 3.34 (**not** awrt).

**<u>Special Cases in (c):</u>**

Note that if candidates have an incorrect $\alpha$ and have e.g. $3+2\sqrt{5}\cos(0.5t-0.464)$, this has no impact on the final answer. So for candidates using $3+2\sqrt{5}\cos(0.5t\pm\alpha)$ in (c) allow all the marks including the A mark as a correct method should always lead to 3.34

**<u>Some values to look for:</u>**

$$0.5t \pm "0.464" = \pm 2.306,\ \pm 3.977,\ \pm 8.598,\ \pm 10.26$$

(d)

B1: Correct refinement e.g. As in scheme. If they suggest a specific function to replace the “3” then it must be sensible e.g. a trigonometric function rather than e.g. a quadratic/linear one.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 31 (a) | $\cos 3A = \cos(2A + A) = \cos 2A\cos A - \sin 2A\sin A$ | M1 | 3.1a |
| | $= \left(2\cos^2 A - 1\right)\cos A - \left(2\sin A\cos A\right)\sin A$ | dM1 | 1.1b |
| | $= \left(2\cos^2 A - 1\right)\cos A - 2\cos A\left(1 - \cos^2 A\right)$ | ddM1 | 2.1 |
| | $= 4\cos^3 A - 3\cos A$ * | A1* | 1.1b |
| | | **(4)** | |
| **(b)** | $1 - \cos 3x = \sin^2 x \Rightarrow \cos^2 x + 3\cos x - 4\cos^3 x = 0$ | M1 | 1.1b |
| | $\Rightarrow \cos x\left(4\cos^2 x - \cos x - 3\right) = 0$ <br> $\Rightarrow \cos x\left(4\cos x + 3\right)\left(\cos x - 1\right) = 0$ <br> $\Rightarrow \cos x = \ldots$ | dM1 | 3.1a |
| | Two of $-90°, 0, 90°$, awrt $139°$ | A1 | 1.1b |
| | All four of $-90°, 0, 90°$, awrt $139°$ | A1 | 2.1 |
| | | **(4)** | |
| | | | **(8 marks)** |

**Notes:**

**(a)**

**Allow a proof in terms of *x* rather than A**

**M1:** Attempts to use the compound angle formula for cos(2$A$ + $A$) or cos($A$ + 2$A$)
Condone a slip in sign

**dM1:** Uses correct double angle identities for cos 2$A$ and sin 2$A$
$\cos 2A = 2\cos^2 A - 1$ must be used. If either of the other two versions are used expect to see an attempt to replace $\sin^2 A$ by $1 - \cos^2 A$ at a later stage.
**Depends on previous mark**.

**ddM1:** Attempts to get all terms in terms of cos $A$ using correct and appropriate identities.
**Depends on both previous marks**.

**A1*:** A completely correct and rigorous proof including correct notation, no mixed variables, missing brackets etc.

**Alternative right to left is possible:**

$4\cos^3 A - 3\cos A = \cos A\left(4\cos^2 A - 3\right) = \cos A\left(2\cos^2 A - 1 + 2\left(1 - \sin^2 A\right) - 2\right) = \cos A\left(\cos 2A - 2\sin^2 A\right)$

$= \cos A\cos 2A - 2\sin A\cos A\sin A = \cos A\cos 2A - \sin 2A\sin A = \cos(2A + A) = \cos 3A$

Score M1: For $4\cos^3 A - 3\cos A = \cos A\left(4\cos^2 A - 3\right)$

dM1: For $\cos A\left(2\cos^2 A - 1 + 2\left(1 - \sin^2 A\right) - 2\right)$ (Replaces $4\cos^2 A - 1$ by $2\cos^2 A - 1$ and $2\left(1 - \sin^2 A\right)$)

ddM1: Reaches $\cos A\cos 2A - \sin 2A\sin A$

A1: $\cos(2A + A) = \cos 3A$

(b)

**M1:** For an attempt to produce an equation just in cos $x$ using both part (a) and the identity $\sin^2 x = 1 - \cos^2 x$
Allow one slip in sign or coefficient when copying the result from part (a)

**dM1: Dependent upon the preceding mark**. It is for taking the cubic equation in cos $x$ and making a valid attempt to solve. This could include factorisation or division of a cos $x$ term followed by an attempt to solve the 3 term quadratic equation in cos $x$ to reach at least one non zero value for cos $x$.
May also be scored for solving the cubic equation in cos $x$ to reach at least one non zero value for cos $x$.

**A1:** Two of $-90°, 0, 90°$, awrt $139°$ **Depends on the first method mark**.

**A1:** All four of $-90°, 0, 90°$, awrt $139°$ with no extra solutions offered within the range.

**Note that this is an alternative approach for obtaining the cubic equation in (b):**

$$1-\cos 3x=\sin^2 x \Rightarrow 1-\cos 3x=\frac{1}{2}(1-\cos 2x)$$
$$\Rightarrow 2-2\cos 3x=1-\cos 2x$$
$$\Rightarrow 1=2\cos 3x-\cos 2x$$
$$\Rightarrow 1=2\left(4\cos^3 x-3\cos x\right)-\left(2\cos^2 x-1\right)$$
$$\Rightarrow 0=4\cos^3 x-3\cos x-\cos^2 x$$

The M1 will be scored on the penultimate line when they use part (a) and use the correct identity for cos $2x$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **32(a)** | Allow explanations such as<br>• student should have worked in radians<br>• they did not convert degrees to radians<br>• 40 should be in radians<br>• $\theta$ should be in radians<br>• angle (or $\theta$) should be $\frac{40\pi}{180}$ or $\frac{2\pi}{9}$<br>• correct formula is $\pi r^2\left(\frac{\theta}{360}\right)$ {where $\theta$ is in degrees}<br>• correct formula is $\pi r^2\left(\frac{40}{360}\right)$ | B1 | 2.3 |
| | | **(1)** | |
| **(b) Way 1** | {Area of sector = } $\frac{1}{2}(5^2)\left(\frac{2\pi}{9}\right)$ | M1 | 1.1b |
| | $= \frac{25}{9}\pi$ {cm$^2$} or awrt 8.73 {cm$^2$} | A1 | 1.1b |
| | | **(2)** | |
| **(b) Way 2** | {Area of sector = } $\pi(5^2)\left(\frac{40}{360}\right)$ | M1 | 1.1b |
| | $= \frac{25}{9}\pi$ {cm$^2$} or awrt 8.73 {cm$^2$} | A1 | 1.1b |
| | | **(2)** | |
| | | | **(3 marks)** |

**Notes for Question 32**

| | |
|---|---|
| **(a)** | |
| **B1:** | Explains that the formula use is only valid when angle *AOB* is applied in radians.<br>See scheme for examples of suitable explanations. |
| **(b)** | **Way 1** |
| **M1:** | Correct application of the sector formula using a correct value for $\theta$ in radians |
| **Note:** | Allow exact equivalents for $\theta$ e.g. $\theta = \frac{40\pi}{180}$ or $\theta$ in the range [0.68, 0.71] |
| **A1*:** | Accept $\frac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units |
| **(b)** | **Way 2** |
| **M1:** | Correct application of the sector formula in degrees |
| **A1:** | Accept $\frac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units. |
| **Note:** | Allow exact equivalents such as $\frac{50}{18}\pi$ |
| **Note:** | Allow M1 A1 for $500\left(\frac{\pi}{180}\right) = \frac{25}{9}\pi$ {cm$^2$} or awrt 8.73 {cm$^2$} |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **33** | $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta$ | | |
| **(a) Way 1** | $\{\text{LHS}=\}\ \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$ | M1 | 3.1a |
| | $=\dfrac{\cos(3\theta-\theta)}{\sin\theta\cos\theta}\quad\left\{=\dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$ | A1 | 2.1 |
| | $=\dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}\ =\ 2\cot 2\theta$ * | dM1 | 1.1b |
| | | A1 * | 2.1 |
| | | **(4)** | |
| **(a) Way 2** | $\{\text{LHS}=\}\ \dfrac{\cos 2\theta\cos\theta-\sin 2\theta\sin\theta}{\sin\theta}+\dfrac{\sin 2\theta\cos\theta+\cos 2\theta\sin\theta}{\cos\theta}$ | | |
| | $=\dfrac{\cos 2\theta\cos^2\theta-\sin 2\theta\sin\theta\cos\theta+\sin 2\theta\cos\theta\sin\theta+\cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$ | M1 | 3.1a |
| | $=\dfrac{\cos 2\theta(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta}\quad\left\{=\dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$ | A1 | 2.1 |
| | $=\dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}\ =\ 2\cot 2\theta$ * | dM1 | 1.1b |
| | | A1 * | 2.1 |
| | | **(4)** | |
| **(a) Way 3** | $\{\text{RHS}=\}\ \dfrac{2\cos 2\theta}{\sin 2\theta}=\dfrac{2\cos(3\theta-\theta)}{\sin 2\theta}=\dfrac{2(\cos 3\theta\cos\theta+\sin 3\theta\sin\theta)}{\sin 2\theta}$ | M1 | 3.1a |
| | | A1 | 2.1 |
| | $=\dfrac{2(\cos 3\theta\cos\theta+\sin 3\theta\sin\theta)}{2\sin\theta\cos\theta}$ | dM1 | 1.1b |
| | $=\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}$ * | A1 * | 2.1 |
| | | **(4)** | |
| **(b) Way 1** | $\left\{\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=4\Rightarrow\right\}\quad 2\cot 2\theta=4\Rightarrow 2\left(\dfrac{1}{\tan 2\theta}\right)=4$ | M1 | 1.1b |
| | Rearranges to give $\tan 2\theta=k;\ k\neq 0$ and applies $\arctan k$ | dM1 | 1.1b |
| | $\left\{90°<\theta<180°,\ \tan 2\theta=\dfrac{1}{2}\Rightarrow\right\}$ | | |
| | ***Only one solution*** of $\theta=103.3°$ (1 dp) or awrt $103.3°$ | A1 | 2.2a |
| | | **(3)** | |
| **(b) Way 2** | $\left\{\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=4\Rightarrow\right\}\quad 2\cot 2\theta=4\Rightarrow\dfrac{2}{\tan 2\theta}=4$ | M1 | 1.1b |
| | $\dfrac{2}{\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)}=4\Rightarrow 2(1-\tan^2\theta)=8\tan\theta$<br>$\Rightarrow\ \tan^2\theta+4\tan\theta-1=0\Rightarrow\tan\theta=\dfrac{-4\pm\sqrt{(4)^2-4(1)(-1)}}{2(1)}$<br>$\{\Rightarrow\tan\theta=-2\pm\sqrt{5}\}\Rightarrow\tan\theta=k;\ k\neq 0\ \Rightarrow$ applies $\arctan k$ | dM1 | 1.1b |
| | $\{90°<\theta<180°,\ \tan\theta=-2-\sqrt{5}\ \Rightarrow\}$ | | |
| | ***Only one solution*** of $\theta=103.3°$ (1 dp) or awrt $103.3°$ | A1 | 2.2a |
| | | **(3)** | |
| | | **(7 marks)** | |

| Notes for Question **33** | |
|---|---|
| **(a)** | **Way 1 and Way 2** |
| **M1:** | Correct valid method forming a common denominator of $\sin\theta\cos\theta$ <br> i.e. correct process of $\dfrac{(...)\cos\theta+(...)\sin\theta}{\cos\theta\sin\theta}$ |
| **A1:** | Proceeds to show that the numerator of their resulting fraction simplifies to $\cos(3\theta-\theta)$ or $\cos 2\theta$ |
| **dM1:** | **dependent on the previous M mark** <br> Applies a correct $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to the common denominator $\sin\theta\cos\theta$ |
| **A1*** | Correct proof |
| **Note:** | Writing $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos 3\theta\cos\theta}{\sin\theta\cos\theta}+\dfrac{\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$ is considered a correct valid method of forming a common denominator of $\sin\theta\cos\theta$ for the 1st M1 mark |
| **Note:** | Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$ <br> but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta}=\dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$ |
| **Note:** | Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$ <br> but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta}=\dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$ |
| **Note:** | Allow 2nd M1 for stating a correct $\sin 2\theta=2\sin\theta\cos\theta$ and for attempting to apply it to the common denominator $\sin\theta\cos\theta$ |
| **(a)** | **Way 3** |
| **M1:** | Starts from RHS and proceeds to expand $\cos 2\theta$ in the form $\cos 3\theta\cos\theta \pm \sin 3\theta\sin\theta$ |
| **A1:** | Shows, as part of their proof, that $\cos 2\theta = \cos 3\theta\cos\theta + \sin 3\theta\sin\theta$ |
| **dM1:** | **dependent on the previous M mark** <br> Applies $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to their denominator |
| **A1*:** | Correct proof |
| **Note:** | Allow 1st M1 1st A1 (together) for any of LHS $\to \dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ or LHS $\to \dfrac{\cos 2\theta(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta}$ <br> or LHS $\to \cos 2\theta(\cot\theta+\tan\theta)$ or LHS $\to \cos 2\theta\left(\dfrac{1+\tan^2\theta}{\tan\theta}\right)$ <br> (i.e. where $\cos 2\theta$ has been factorised out) |
| **Note:** | Allow 1st M1 1st A1 for progressing as far as LHS $= ... = \cot x - \tan x$ |
| **Note:** | The following is a correct alternative solution <br> $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta}=\dfrac{\frac{1}{2}(\cos 4\theta+\cos 2\theta)-\frac{1}{2}(\cos 4\theta-\cos 2\theta)}{\sin\theta\cos\theta}$ <br> $=\dfrac{\cos 2\theta}{\sin\theta\cos\theta}=\dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}=2\cot 2\theta$ * |
| **Note:** | E.g. going from $\dfrac{\cos 2\theta\cos^2\theta-\sin 2\theta\sin\theta\cos\theta+\sin 2\theta\cos\theta\sin\theta+\cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$ to $\dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ <br> with no intermediate working is 1st A0 |

| Notes for Question **33** Continued | |
|---|---|
| **(b)** | **Way 1** |
| **M1:** | Evidence of applying $\cot 2\theta = \dfrac{1}{\tan 2\theta}$ |
| **dM1:** | **dependent on the previous M mark**<br>Rearranges to give $\tan 2\theta = k,\ k \neq 0,$ and applies $\arctan k$ |
| **A1:** | Uses $90° < \theta < 180°$ to deduce the only solution $\theta =$ awrt $103.3°$ |
| **Note:** | Give M0M0A0 for writing, for example, $\tan 2\theta = 2$ with no evidence of applying $\cot 2\theta = \dfrac{1}{\tan 2\theta}$ |
| **Note:** | 1st M1 can be implied by seeing $\tan 2\theta = \dfrac{1}{2}$ |
| **Note:** | Condone 2nd M1 for applying $\dfrac{1}{2}\arctan\left(\dfrac{1}{2}\right)$ $\{= 13.28...\}$ |
| **(b)** | **Way 2** |
| **M1:** | Evidence of applying $\cot 2\theta = \dfrac{1}{\tan 2\theta}$ |
| **dM1:** | **dependent on the previous M mark**<br>Applies $\tan 2\theta \equiv \dfrac{2\tan\theta}{1-\tan^2\theta}$, forms and uses a correct method for solving a 3TQ to give $\tan\theta = k,\ k \neq 0,$ and applies $\arctan k$ |
| **A1:** | Uses $90° < \theta < 180°$ to deduce the only solution $\theta =$ awrt $103.3°$ |
| **Note:** | Give M1 dM1 A1 for no working leading to $\theta =$ awrt $103.3°$ and no other solutions |
| **Note:** | Give M1 dM1 A0 for no working leading to $\theta =$ awrt $103.3°$ and other solutions which can be either outside or inside the range $90° < \theta < 180°$ |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **34** | (i) $4\sin x = \sec x,\ 0 \le x < \frac{\pi}{2}$; (ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$ | | |
| **(i)**<br>**Way 1** | For $\sec x = \frac{1}{\cos x}$ | B1 | 1.2 |
| | $\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 2\sin 2x = 1 \Rightarrow \sin 2x = \frac{1}{2}$ | M1 | 3.1a |
| | $x = \frac{1}{2}\arcsin\left(\frac{1}{2}\right)$ or $\frac{1}{2}\left(\pi - \arcsin\left(\frac{1}{2}\right)\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$ | dM1 | 1.1b |
| | | A1 | 1.1b |
| | | **(4)** | |
| **(i)**<br>**Way 2** | For $\sec x = \frac{1}{\cos x}$ | B1 | 1.2 |
| | $\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 16\sin^2 x\cos^2 x = 1$<br>$16\sin^2 x(1-\sin^2 x) = 1$ \| $16(1-\cos^2 x)\cos^2 x = 1$<br>$16\sin^4 x - 16\sin^2 x + 1 = 0$ \| $16\cos^4 x - 16\cos^2 x + 1 = 0$<br>$\sin^2 x$ or $\cos^2 x = \frac{16 \pm \sqrt{192}}{32}$ $\left\{= \frac{2 \pm \sqrt{3}}{4}\ \text{or}\ 0.933..., 0.066...\right\}$ | M1 | 3.1a |
| | $x = \arcsin\left(\sqrt{\frac{2 \pm \sqrt{3}}{4}}\right)$ or $x = \arccos\left(\sqrt{\frac{2 \pm \sqrt{3}}{4}}\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$ | dM1 | 1.1b |
| | | A1 | 1.1b |
| | | **(4)** | |
| **(ii)** | Complete strategy, i.e.<br>• Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\sin(\theta - \alpha) = 2$, finds both $R$ and $\alpha$, and proceeds to $\sin(\theta - \alpha) = k,\ \lvert k\rvert < 1,\ k \ne 0$<br>• Applies $(5\sin\theta - 5\cos\theta)^2 = 2^2$, followed by applying both $\cos^2\theta + \sin^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to proceed to $\sin 2\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$ | M1 | 3.1a |
| | $R = \sqrt{50}$<br>$\tan\alpha = 1 \Rightarrow \alpha = 45°$ \| $(5\sin\theta - 5\cos\theta)^2 = 2^2 \Rightarrow$<br>$25\sin^2\theta + 25\cos^2\theta - 50\sin\theta\cos\theta = 4$<br>$\Rightarrow 25 - 25\sin 2\theta = 4$ | M1 | 1.1b |
| | $\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$ \| $\sin 2\theta = \frac{21}{25}$ | A1 | 1.1b |
| | **dependent on the first M mark**<br>e.g. $\theta = \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$ \| e.g. $\theta = \frac{1}{2}\left(\arcsin\left(\frac{21}{25}\right)\right)$ | dM1 | 1.1b |
| | $\theta = \text{awrt } 61.4°,\ \text{awrt } 208.6°$ | A1 | 2.1 |
| | **Note:** Working in radians does not affect any of the first 4 marks | | |
| | | **(5)** | |
| | | | **(9 marks)** |

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>34</td><td colspan="2">(ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$</td><td></td><td></td></tr>
<tr><td rowspan="8">(ii)<br>Alt 1</td><td colspan="2">Complete strategy, i.e.<br>• Attempts to apply $(5\sin\theta)^2 = (2+5\cos\theta)^2$ or $(5\sin\theta-2)^2 = (5\cos\theta)^2$ followed by applying $\cos^2\theta+\sin^2\theta=1$ and solving a quadratic equation in either $\sin\theta$ or $\cos\theta$ to give at least one of $\sin\theta = k$ or $\cos\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$</td><td>M1</td><td>3.1a</td></tr>
<tr><td colspan="2">e.g. $25\sin^2\theta = 4 + 20\cos\theta + 25\cos^2\theta$<br>$\Rightarrow 25(1-\cos^2\theta) = 4 + 20\cos\theta + 25\cos^2\theta$</td><td rowspan="2">M1</td><td rowspan="2">1.1b</td></tr>
<tr><td colspan="2">or e.g. $25\sin^2\theta - 20\sin\theta + 4 = 25\cos^2\theta$<br>$\Rightarrow 25\sin^2\theta - 20\sin\theta + 4 = 25(1-\sin^2\theta)$</td></tr>
<tr><td>$50\cos^2\theta + 20\cos\theta - 21 = 0$</td><td>$50\sin^2\theta - 20\sin\theta - 21 = 0$</td><td></td><td></td></tr>
<tr><td>$\cos\theta = \dfrac{-20 \pm \sqrt{4600}}{100}$, o.e.</td><td>$\sin\theta = \dfrac{20 \pm \sqrt{4600}}{100}$, o.e.</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2">dependent on the first M mark<br>e.g. $\theta = \arccos\left(\dfrac{-2+\sqrt{46}}{10}\right)$     e.g. $\theta = \arcsin\left(\dfrac{2+\sqrt{46}}{10}\right)$</td><td>dM1</td><td>1.1b</td></tr>
<tr><td colspan="2">$\theta = $ awrt $61.4°$, awrt $208.6°$</td><td>A1</td><td>2.1</td></tr>
<tr><td colspan="2"></td><td>(5)</td><td></td></tr>
</table>

| Notes for Question **34** | |
|---|---|
| **(i)** | |
| **B1:** | For recalling that $\sec x = \dfrac{1}{\cos x}$ |
| **M1:** | Correct strategy of<br>• **Way 1:** applying $\sin 2x = 2\sin x\cos x$ and proceeding to $\sin 2x = k,\ \lvert k\rvert \le 1,\ k \ne 0$<br>• **Way 2:** squaring both sides, applying $\cos^2 x + \sin^2 x = 1$ and solving a quadratic equation in either $\sin^2 x$ or $\cos^2 x$ to give $\sin^2 x = k$ or $\cos^2 x = k,\ \lvert k\rvert \le 1,\ k \ne 0$ |
| **dM1:** | Uses the correct order of operations to find at least one value for $x$ in either radians or degrees |
| **A1:** | Clear reasoning to achieve both $x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$ and no other values in the range $0 \le x < \dfrac{\pi}{2}$ |
| **Note:** | Give dM1 for $\sin 2x = \dfrac{1}{2} \Rightarrow$ any of $\dfrac{\pi}{12}, \dfrac{5\pi}{12}, 15°, 75°$, awrt 0.26 or awrt 1.3 |
| **Note:** | Give special case, SC B1M0M0A0 for writing down any of $\dfrac{\pi}{12}, \dfrac{5\pi}{12}, 15°$ or $75°$ with no working |

| Notes for Question **34** Continued | |
|---|---|
| **(ii)** | |
| **M1:** | See scheme |
| **Note:** | **Alternative strategy:** Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\cos(\theta+\alpha) = -2$, finds both $R$ and $\alpha$, and proceeds to $\cos(\theta+\alpha) = k$, $\lvert k\rvert < 1$, $k \neq 0$ |
| **M1:** | Either<br>• uses $R\sin(\theta-\alpha)$ to find the values of both $R$ and $\alpha$<br>• attempts to apply $(5\sin\theta - 5\cos\theta)^2 = 2^2$, uses $\cos^2\theta + \sin^2\theta = 1$ and proceeds to find an equation of the form $\pm\lambda \pm \mu\sin 2\theta = \pm\beta$ or $\pm\mu\sin 2\theta = \pm\beta$; $\mu \neq 0$<br>• attempts to apply $(5\sin\theta)^2 = (2+5\cos\theta)^2$ or $(5\sin\theta - 2)^2 = (5\cos\theta)^2$ and uses $\cos^2\theta + \sin^2\theta = 1$ to form an equation in $\cos\theta$ only or $\sin\theta$ only |
| **A1:** | For $\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$, o.e., $\cos(\theta + 45°) = -\frac{2}{\sqrt{50}}$, o.e. or $\sin 2\theta = \frac{21}{25}$, o.e.<br>or $\cos\theta = \frac{-20 \pm \sqrt{4600}}{100}$, o.e. or $\cos\theta =$ awrt 0.48, awrt $-0.88$<br>or $\sin\theta = \frac{20 \pm \sqrt{4600}}{100}$, o.e., or $\sin\theta =$ awrt 0.88, awrt $-0.48$ |
| **Note:** | $\sin(\theta - 45°)$, $\cos(\theta + 45°)$, $\sin 2\theta$ must be made the subject for A1 |
| **dM1:** | **dependent on the first M mark**<br>Uses the correct order of operations to find at least one value for $x$ in either degrees or radians |
| **Note:** | dM1 can also be given for $\theta = 180° - \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$ or $\theta = \frac{1}{2}\left(180° - \arcsin\left(\frac{21}{25}\right)\right)$ |
| **A1:** | Clear reasoning to achieve both $\theta =$ awrt 61.4°, awrt 208.6° and no other values in the range $0 \leq \theta < 360°$ |
| **Note:** | Give M0M0A0M0A0 for writing down any of $\theta =$ awrt 61.4°, awrt 208.6° with no working |
| **Note:** | <u>**Alternative solutions:**</u> (to be marked in the same way as Alt 1):<br>• $5\sin\theta - 5\cos\theta = 2 \Rightarrow 5\tan\theta - 5 = 2\sec\theta \Rightarrow (5\tan\theta - 5)^2 = (2\sec\theta)^2$<br>$\Rightarrow 25\tan^2\theta - 50\tan\theta + 25 = 4\sec^2\theta \Rightarrow 25\tan^2\theta - 50\tan\theta + 25 = 4(1+\tan^2\theta)$<br>$\Rightarrow 21\tan^2\theta - 50\tan\theta + 21 = 0 \Rightarrow \tan\theta = \frac{50 \pm \sqrt{736}}{42} = \frac{25 \pm 2\sqrt{46}}{21} = 1.8364..., 0.5445...$<br>$\Rightarrow \theta =$ awrt 61.4°, awrt 208.6° only<br>• $5\sin\theta - 5\cos\theta = 2 \Rightarrow 5 - 5\cot\theta = 2\operatorname{cosec}\theta \Rightarrow (5 - 5\cot\theta)^2 = (2\operatorname{cosec}\theta)^2$<br>$\Rightarrow 25 - 50\cot\theta + 25\cot^2\theta = 4\operatorname{cosec}^2\theta \Rightarrow 25 - 50\cot\theta + 25\cot^2\theta = 4(1+\cot^2\theta)$<br>$\Rightarrow 21\cot^2\theta - 50\cot\theta + 21 = 0 \Rightarrow \cot\theta = \frac{50 \pm \sqrt{736}}{42} = \frac{25 \pm 2\sqrt{46}}{21} = 1.8364..., 0.5445...$<br>$\Rightarrow \theta =$ awrt 61.4°, awrt 208.6° only |

| Question | Scheme | Marks | AOs |
| --- | --- | --- | --- |
| **35** | $1-\cos 2\theta \equiv \tan\theta \sin 2\theta,\ \theta \neq \frac{(2n+1)\pi}{2},\ n \in \mathbb{Z}$ | | |
| **(a) Way 1** | $\tan\theta \sin 2\theta = \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta)$ | M1 | 1.1b |
| | $= \left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)(2\sin\theta\cancel{\cos\theta}) = 2\sin^2\theta = 1-\cos 2\theta$ * | M1 | 1.1b |
| | | A1* | 2.1 |
| | | **(3)** | |
| **(a) Way 2** | $1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta$ | M1 | 1.1b |
| | $= \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta) = \tan\theta\sin 2\theta$ * | M1 | 1.1b |
| | | A1* | 2.1 |
| | | **(3)** | |
| | $(\sec^2 x - 5)(1-\cos 2x) = 3\tan^2 x \sin 2x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$ | | |
| **(b) Way 1** | $(\sec^2 x - 5)\tan x \sin 2x = 3\tan^2 x \sin 2x$ **or** $(\sec^2 x - 5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$ | | |
| | Deduces $x = 0$ | B1 | 2.2a |
| | Uses $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ e.g. $(1+\tan^2 x - 3\tan x - 5)\tan x = 0$ **or** $(1+\tan^2 x - 3\tan x - 5)(1-\cos 2x) = 0$ **or** $1+\tan^2 x - 5 = 3\tan x$ | M1 | 2.1 |
| | $\tan^2 x - 3\tan x - 4 = 0$ | A1 | 1.1b |
| | $(\tan x - 4)(\tan x + 1) = 0 \Rightarrow \tan x = ...$ | M1 | 1.1b |
| | $x = -\frac{\pi}{4}, 1.326$ | A1 | 1.1b |
| | | A1 | 1.1b |
| | | **(6)** | |
| | | | **(9 marks)** |

### Notes for Question **35**

| | |
| --- | --- |
| **(a)** | **Way 1** |
| **M1:** | Applies $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to $\tan\theta\sin 2\theta$ |
| **M1:** | Cancels as scheme (may be implied) and attempts to use $\cos 2\theta = 1-2\sin^2\theta$ |
| **A1*:** | For a correct proof showing all steps of the argument |
| **(a) Way 2** | |
| **M1:** | For using $\cos 2\theta = 1-2\sin^2\theta$ |
| **Note:** | If the form $\cos 2\theta = \cos^2\theta - \sin^2\theta$ or $\cos 2\theta = 2\cos^2\theta - 1$ is used, the mark cannot be awarded until $\cos^2\theta$ has been replaced by $1-\sin^2\theta$ |
| **M1:** | Attempts to write their $2\sin^2\theta$ in terms of $\tan\theta$ and $\sin 2\theta$ using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and |
| | $\sin 2\theta = 2\sin\theta\cos\theta$ within the given expression |
| **A1*:** | For a correct proof showing all steps of the argument |
| **Note:** | If a proof meets in the middle; e.g. they show LHS = $2\sin^2\theta$ and RHS = $2\sin^2\theta$; then some indication must be given that the proof is complete. E.g. $1-\cos 2\theta \equiv \tan\theta\sin 2\theta$, QED, box |

| Notes for Question **35** Continued | |
|---|---|
| **(b)** | |
| **B1:** | Deduces that the given equation yields a solution $x=0$ |
| **M1:** | For using the key step of $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ or $\sin 2x$ to produce a quadratic factor or quadratic equation in just $\tan x$ |
| **Note:** | Allow the use of $\pm\sec^2 x = \pm 1 \pm \tan^2 x$ for M1 |
| **A1:** | Correct 3TQ in $\tan x$. E.g. $\tan^2 x - 3\tan x - 4 = 0$ |
| **Note:** | E.g. $\tan^2 x - 4 = 3\tan x$ or $\tan^2 x - 3\tan x = 4$ are acceptable for A1 |
| **M1:** | For a correct method of solving their 3TQ in $\tan x$ |
| **A1:** | Any one of $-\frac{\pi}{4}$, awrt $-0.785$, awrt $1.326$, $-45°$, awrt $75.964°$ |
| **A1:** | Only $x = -\frac{\pi}{4}, 1.326$ **cao** stated in the range $-\frac{\pi}{2} < x < \frac{\pi}{2}$ |

**Note:** <u>**Alternative Method (Alt 1)**</u>

| | | | |
|---|---|---|---|
| $(\sec^2 x - 5)\tan x \sin 2x = 3\tan^2 x \sin 2x$<br>**or** $(\sec^2 x - 5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$ | | | |
| Deduces $x=0$ | | B1 | 2.2a |
| $\sec^2 x - 5 = 3\tan x \Rightarrow \frac{1}{\cos^2 x} - 5 = 3\left(\frac{\sin x}{\cos x}\right)$<br>$1 - 5\cos^2 x = 3\sin x \cos x$<br>$1 - 5\left(\frac{1+\cos 2x}{2}\right) = \frac{3}{2}\sin 2x$ | Complete process (as shown) of using the identities for $\sin 2x$ and $\cos 2x$ to proceed as far as $\pm A \pm B\cos 2x = \pm C\sin 2x$ | M1 | 2.1 |
| $-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$<br>$\{3\sin 2x + 5\cos 2x = -3\}$ | $-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$<br>o.e. | A1 | 1.1b |
| $\sqrt{34}\sin(2x + 1.03) = -3$ | Expresses their answer in the form $R\sin(2x+\alpha) = k$; $k \neq 0$ with values for $R$ and $\alpha$ | M1 | 1.1b |
| $\sin(2x + 1.03) = -\frac{3}{\sqrt{34}}$ | | | |
| $x = -\frac{\pi}{4}, 1.326$ | | A1 | 1.1b |
| | | A1 | 1.1b |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **36** | $\frac{1}{2}r^2(4.8)$ | M1 | 1.1a |
| | $\frac{1}{2}r^2(4.8) = 135 \Rightarrow r^2 = \frac{225}{4} \Rightarrow r = 7.5$ o.e. | A1 | 1.1b |
| | length of minor arc $= 7.5(2\pi - 4.8)$ | dM1 | 3.1a |
| | $= 15\pi - 36 \quad \{a = 15, b = -36\}$ | A1 | 1.1b |
| | | **(4)** | |
| **36 Alt** | $\frac{1}{2}r^2(4.8)$ | M1 | 1.1a |
| | $\frac{1}{2}r^2(4.8) = 135 \Rightarrow r^2 = \frac{225}{4} \Rightarrow r = 7.5$ o.e. | A1 | 1.1b |
| | length of major arc $= 7.5(4.8)\ \{= 36\}$ | | |
| | length of minor arc $= 2\pi(7.5) - 36$ | dM1 | 3.1a |
| | $= 15\pi - 36 \quad \{a = 15, b = -36\}$ | A1 | 1.1b |
| | | **(4)** | |
| | | | **(4 marks)** |

Question **36** Notes:

| | |
|---|---|
| **M1:** | Applies formula for the area of a sector with $\theta = 4.8$; i.e. $\frac{1}{2}r^2\theta$ with $\theta = 4.8$<br>**Note:** Allow M1 for considering ratios. E.g. $\frac{135}{\pi r^2} = \frac{4.8}{2\pi}$ |
| **A1:** | Uses a correct equation $\left(\text{e.g. } \frac{1}{2}r^2(4.8) = 135\right)$ to obtain a radius of 7.5 |
| **dM1:** | Depends on the previous M mark.<br>A complete process for finding the length of the minor arc *AB*, by either<br>• (their $r$)$\times(2\pi - 4.8)$<br>• $2\pi$(their $r$) $-$ (their $r$)(4.8) |
| **A1:** | Correct exact answer in its simplest form, e.g. $15\pi - 36$ or $-36 + 15\pi$ |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **37(a)** | Attempts to substitute $\cos\theta \approx 1 - \frac{1}{2}\theta^2$ into either $1 + 4\cos\theta$ or $3\cos^2\theta$ | M1 | 1.1b |
| | $1 + 4\cos\theta + 3\cos^2\theta \approx 1 + 4\left(1 - \frac{1}{2}\theta^2\right) + 3\left(1 - \frac{1}{2}\theta^2\right)^2$ | | |
| | $= 1 + 4\left(1 - \frac{1}{2}\theta^2\right) + 3\left(1 - \theta^2 + \frac{1}{4}\theta^4\right)$ | M1 | 1.1b |
| | $= 1 + 4 - 2\theta^2 + 3 - 3\theta^2 + \frac{3}{4}\theta^4$ | | |
| | $= 8 - 5\theta^2$ * | A1* | 2.1 |
| | | **(3)** | |
| **(b)(i)** | E.g.<br>• Adele is working in degrees and not radians<br>• Adele should substitute $\theta = \frac{5\pi}{180}$ and not $\theta = 5$ into the approximation | B1 | 2.3 |
| **(b)(ii)** | $8 - 5\left(\frac{5\pi}{180}\right)^2$ = awrt 7.962, so $\theta = 5°$ gives a good approximation. | B1 | 2.4 |
| | | **(2)** | |
| | | **(5 marks)** | |

Question **37** Notes:

| | |
|---|---|
| **(a)(i)** | |
| **M1:** | See scheme |
| **M1:** | Substitutes $\cos\theta \approx 1 - \frac{1}{2}\theta^2$ into $1 + 4\cos\theta + 3\cos^2\theta$ and attempts to apply $\left(1 - \frac{1}{2}\theta^2\right)^2$ |
| | **Note:** It is not a requirement for this mark to write or refer to the term in $\theta^4$ |
| **A1*:** | Correct proof with no errors seen in working. |
| | **Note:** It is not a requirement for this mark to write or refer to the term in $\theta^4$ |
| **(a)(ii)** | |
| **B1:** | See scheme |
| **(b)(i)** | |
| **B1:** | See scheme |
| **(b)(ii)** | |
| **B1:** | Substitutes $\theta = \frac{5\pi}{180}$ or $\frac{\pi}{36}$ into $8 - 5\theta^2$ to give awrt 7.962 ***and*** an appropriate conclusion. |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **38(a)** | $\operatorname{cosec}2x + \cot 2x \equiv \cot x,\ \ x \neq 90n°,\ \ n \in \mathbb{Z}$ | | |
| | $\operatorname{cosec}2x + \cot 2x = \dfrac{1}{\sin 2x} + \dfrac{\cos 2x}{\sin 2x}$ | M1 | 1.2 |
| | $= \dfrac{1 + \cos 2x}{\sin 2x}$ | M1 | 1.1b |
| | $= \dfrac{1 + 2\cos^2 x - 1}{2\sin x \cos x} = \dfrac{2\cos^2 x}{2\sin x \cos x}$ | M1 | 2.1 |
| | | A1 | 1.1b |
| | $= \dfrac{\cos x}{\sin x} = \cot x$ * | A1* | 2.1 |
| | | **(5)** | |
| **(b)** | $\operatorname{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3};\ \ 0 \leqslant \theta < 180°,$ | | |
| | $\cot(2\theta \pm ...°) = \sqrt{3}$ | M1 | 2.2a |
| | $2\theta \pm ... = 30° \Rightarrow \theta = 12.5°$ | M1 | 1.1b |
| | | A1 | 1.1b |
| | $2\theta + 5° = 180° + PV° \Rightarrow \theta = ...°$ | M1 | 2.1 |
| | $\theta = 102.5°$ | A1 | 1.1b |
| | | **(5)** | |
| | | | **(10 marks)** |

| Question **38** Notes: | |
|---|---|
| **(a)** | |
| **M1:** | Writes $\text{cosec}2x = \dfrac{1}{\sin 2x}$ and $\cot 2x = \dfrac{\cos 2x}{\sin 2x}$ |
| **M1:** | Combines into a single fraction with a common denominator |
| **M1:** | Applies $\sin 2x = 2\sin x\cos x$ to the denominator **and** applies either<br>• $\cos 2x = 2\cos^2 x - 1$<br>• $\cos 2x = 1 - 2\sin^2 x$ and $\sin^2 x + \cos^2 x = 1$<br>• $\cos 2x = \cos^2 x - \sin^2 x$ and $\sin^2 x + \cos^2 x = 1$<br>to the numerator and manipulates to give a one term numerator expression |
| **A1:** | Correct algebra leading to $\dfrac{2\cos^2 x}{2\sin x\cos x}$ or equivalent. |
| **A1*:** | Correct proof with correct notation and no errors seen in working |
| **(b)** | |
| **M1:** | Uses the result in part (a) in an attempt to deduce either $2x = 4\theta + 10$ or $x = 2\theta + \ldots$ and uses $x = 2\theta + \ldots$ to write down or imply $\cot(2q \pm \ldots°) = \sqrt{3}$ |
| **M1:** | Applies $\text{arccot}\left(\sqrt{3}\right) = 30°$ or $\arctan\left(\dfrac{1}{\sqrt{3}}\right) = 30°$<br>and attempts to solve $2\theta \pm \ldots = 30°$ to give $\theta = \ldots$ |
| **A1:** | Uses a correct method to obtain $\theta = 12.5°$ |
| **M1:** | Uses $2\theta + 5 = 180 +$ their $PV°$ in a complete method to find the second solution, $\theta = \ldots$ |
| **A1:** | Uses a correct method to obtain $\theta = 102.5°$, with no extra solutions given either inside or outside the required range $0 \leqslant \theta < 180°$ |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **39(a)** | Identifies an error for student A: They use $\frac{\cos\theta}{\sin\theta}=\tan\theta$ It should be $\frac{\sin\theta}{\cos\theta}=\tan\theta$ | B1 | 2.3 |
| | | **(1)** | |
| **(b)** | (i) Shows $\cos(-26.6°)\neq 2\sin(-26.6°)$, so cannot be a solution | B1 | 2.4 |
| | (ii) Explains that the incorrect answer was introduced by squaring | B1 | 2.4 |
| | | **(2)** | |
| | | | **(3 marks)** |

**Notes:**

**(a)**

**B1:** Accept a response of the type 'They use $\frac{\cos\theta}{\sin\theta}=\tan\theta$. This is incorrect as $\frac{\sin\theta}{\cos\theta}=\tan\theta$'

It can be implied by a response such as 'They should get $\tan\theta=\frac{1}{2}$ not $\tan\theta=2$'

Accept also statements such as 'it should be $\cot\theta=2$'

**(b)**

**B1:** Accept a response where the candidate shows that $-26.6°$ is not a solution of $\cos\theta=2\sin\theta$. This can be shown by, for example, finding both $\cos(-26.6°)$ and $2\sin(-26.6°)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6°)=+ve$ and $2\sin(-26.6°)=-ve$ and stating that they therefore cannot be equal.

**B1:** Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x=5$ squared gives $x^2=25$ which has answers $\pm5$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **40(a)** | Uses $\cos^2 x = 1 - \sin^2 x \Rightarrow 3\sin^2 x + \sin x + 8 = 9\left(1 - \sin^2 x\right)$ | M1 | 3.1a |
| | $\Rightarrow 12\sin^2 x + \sin x - 1 = 0$ | A1 | 1.1b |
| | $\Rightarrow (4\sin x - 1)(3\sin x + 1) = 0$ | M1 | 1.1b |
| | $\Rightarrow \sin x = \frac{1}{4}, -\frac{1}{3}$ | A1 | 1.1b |
| | Uses arcsin to obtain two correct values | M1 | 1.1b |
| | All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$ | A1 | 1.1b |
| | | **(6)** | |
| **(b)** | Attempts $2\theta - 30° = -19.47°$ | M1 | 3.1a |
| | $\Rightarrow \theta = 5.26°$ | A1ft | 1.1b |
| | | **(2)** | |
| | | | **(8 marks)** |

**Notes:**

**(a)**

**M1**: Substitutes $\cos^2 x = 1 - \sin^2 x$ into $3\sin^2 x + \sin x + 8 = 9\cos^2 x$ to create a quadratic equation in just $\sin x$

**A1**: $12\sin^2 x + \sin x - 1 = 0$ or exact equivalent

**M1:** Attempts to solve their quadratic equation in $\sin x$ by a suitable method. These could include factorisation, formula or completing the square.

**A1**: $\sin x = \frac{1}{4}, -\frac{1}{3}$

**M1**: Obtains two correct values for their $\sin x = k$

**A1:** All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$

**(b)**

**M1:** For setting $2\theta - 30° = \text{their}\ '-19.47°'$

**A1ft**: $\theta = 5.26°$ but allow a follow through on their $'-19.47°'$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **41(a)** | $R=\sqrt{109}$ | B1 | 1.1b |
| | $\tan\alpha=\frac{3}{10}$ | M1 | 1.1b |
| | $\alpha=16.70°$ so $\sqrt{109}\cos\left(\theta+16.70°\right)$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)** | (i) e.g $H=11-10\cos(80t)°+3\sin(80t)°$ or $H=11-\sqrt{109}\cos\left(80t+16.70\right)°$ | B1 | 3.3 |
| | (ii) $11+\sqrt{109}$ or 21.44 m | B1ft | 3.4 |
| | | **(2)** | |
| **(c)** | Sets $80t+"16.70"=540$ | M1 | 3.4 |
| | $t=\frac{540-"16.70"}{80}=\left(6.54\right)$ | M1 | 1.1b |
| | $t$ = 6 mins 32 seconds | A1 | 1.1b |
| | | **(3)** | |
| **(d)** | Increase the '80' in the formula<br>For example use $H=11-10\cos(90t)°+3\sin(90t)°$ | | 3.3 |
| | | **(1)** | |
| | | | **(9 marks)** |

**Notes:**

**(a)**

**B1:** $R=\sqrt{109}$ Do not allow decimal equivalents

**M1:** Allow for $\tan\alpha=\pm\frac{3}{10}$

**A1:** $\alpha=16.70°$

**(b)(i)**

**B1:** see scheme

**(b)(ii)**

**B1ft:** their $11+$their $\sqrt{109}$ Allow decimals here.

**(c)**

**M1:** Sets $80t+"16.70"=540$. Follow through on their 16.70

**M1:** Solves their $80t+"16.70"=540$ correctly to find $t$

**A1:** $t$ = 6 mins 32 seconds

**(d)**

B1: States that to increase the speed of the wheel the 80's in the equation would need to be increased.

| Question Number | Scheme | Marks |
|---|---|---|
| **42.** (a) | **Usually answered in radians:**<br>**Uses** Area $ZYW = \frac{1}{2}\times 5^2 \times (angle)$, $=12.5\times 0.7 = 8.75$ o.e.$(\text{cm}^2)$ | M1 A1<br>**(2)** |
| (b) | Area of triangle $XYZ = \dfrac{1}{2}\times 7\times 5\times \sin Y = (11.273)$ $(\text{cm}^2)$<br>Area of whole flag = "8.75" + "11.273", = 20.02 $(\text{cm}^2)$ | M1<br>M1, A1<br>**(3)** |
| (c) | $(XZ^2) = 7^2 + 5^2 - 2\times 7\times 5\cos(\pi - 0.7)$, \| Or $(XZ^2) = (7+5\cos 0.7)^2 + (5\sin 0.7)^2$<br>Use of arc length formula $s = 5\theta$ $( = 3.5)$<br>Total perimeter = $12 + "3.5" + "11.293"$<br>= 26.79 cm | M1,<br>M1<br>ddM1<br>A1<br>**(4)**<br><br>**(9 marks)** |

**Notes**

(a)
M1: uses $A = 12.5\times\theta$ with $\theta$ in radians or completely correct work in degrees.
(If the angle is given as $0.7\pi$ and the formula has not been quoted correctly do not give this mark)
A1: 8.75 or $\dfrac{35}{4}$ or equivalent (do not need to see units)
(b)
M1 for use of $A = \dfrac{1}{2}\times 7\times 5\times \sin Y$ (where $Y = 0.7$ or attempt at $(\pi - 0.7)$ they give the same answer) Do not need to see 11.273 (Do not allow use of 0.7 or $\pi - 0.7$ instead of their respective sines )
This may arise from use of $A = \dfrac{1}{2}\times a\times b\times \sin C$ formula or from $A = \dfrac{1}{2}\times b\times h$ with $h$ found by a correct method so either $A = \dfrac{1}{2}\times 7\times (5\sin Y)$ or $A = \dfrac{1}{2}\times 5\times (7\sin Y)$
This may follow a long method finding all the angles and side lengths of triangle $XYZ$. If their answer rounds to 11.3 credit should be given. E.g. $A = \dfrac{1}{2}\times 11.293\times 1.996$
M1 for adding two numerical areas – triangle and sector (not dependent on previous M marks)
A1 for 20.02 (do not need to see units) (Allow answers which round to 20.02 e.g. do not allow 20.05)
(c)
M1: Uses cosine rule with correct angle (allow 2.4) or uses right angle triangle with correct sides.
(do not need to see $XZ$ = 11.293) This may be calculated in part (b)
M1: Uses arc length with correct radius (may use wrong angle)
ddM1: (Needs to have earned both previous M marks) Adds 7 + 5 + their arc length + their $XZ$
This mark should not be awarded if they use their answer for $XZ^2$ instead of $XZ$.
A1: 26.79 – allow awrt

| Question Number | Scheme | Marks |
|---|---|---|
| **43.** (i) | $4\cos(x+70°)=3$ | |
| | $\cos(x+70°)=0.75$, so $x+70°=41.4(1)°$ | M1A1 |
| | $x=248.6°$ or $331.4°$ | M1 A1 |
| | | **(4)** |
| (ii) | $6\cos^2\theta-5=6\sin^2\theta+\sin\theta$ so $6(1-\sin^2\theta)-5=6\sin^2\theta+\sin\theta$ | M1 |
| | $12\sin^2\theta+\sin\theta-1=0$ | A1 |
| | $(4\sin\theta-1)(3\sin\theta+1)=0$ *so* $\sin\theta=$ | M1 |
| | $\theta=0.253,\ 2.89,\ 3.48,\ 5.94$ | A1 A1 |
| | | **(5)** |
| | | **[9]** |

**Notes**

(i)

M1: Divides by 4 and then uses inverse cosine

A1: Any Correct answer for $x+70°$ or for $x$ (not necessarily in the range) Accept awrt 41.4

Or ( $x=$ ) $-28.6$. If an intermediate answer here is not seen the final correct answers imply this mark.

M1: One correct answer (awrt) so awrt 331.4 or 248.6

A1: Both answers – accept awrt (Lose this mark for extra answers in the range) Ignore extra answers outside the range.

4.3 radians and 5.8 radians is special case: M1A0M1A0

(ii)

M1: Uses $\cos^2\theta=1-\sin^2\theta$

A1: correct three term quadratic – any equivalent - so $12\sin^2\theta+\sin\theta=1$ is acceptable

M1: Solves their quadratic to give values for sin $\theta$ (implied if arcsin is used on their answer(s))

1st A1: Need two correct angles (accept awrt)

A1: All four solutions correct accept awrt 3 sf and ignore subsequent rounding or copying errors. (Extra solutions in range lose this A mark, but outside range - ignore)

Special case: All four angles correct but in degrees ( awrt 14.5, 166, 199, 341) gets A1 A0

| Question Number | Scheme | Marks |
|---|---|---|
| **44.** | $\dfrac{\sin x}{16} = \dfrac{\sin 50^0}{13}$ | M1 |
| | $(\sin x) = \dfrac{16\times\sin 50}{13}$ $(= 0.943 \text{ but accept } 0.94)$ | A1 |
| | $x$ = awrt 70.5(3) **and** 109.5 **or** 70.6 and 109.4 | dM1 A1<br>**(4)**<br>**[4]** |

**Notes**

M1: use sine formula correctly **in any form.** Allow awrt 0.77 for $\sin 50°$

A1: give the correct value or correct expression for sin $x$ (this implies the M1 mark).

If it is given as expression they do not need degrees symbol. $\dfrac{\sin 50\times 16}{13}$ is fine,

If this is given as a decimal allow answers which round to 0.94.

Allow awrt −0.323 (radians) here but no further marks are available.

If they give this as $x$ (not sin$x$) and do not recover this is A0

dM1: Correct work leading to $x$=… (via inverse sin) expression or value for sin$x$

If the previous A mark has been awarded for a correct expression then this is for getting to awrt 70.5 or 109.5 (allow for 70.6 or 109.4).

If the previous A mark was not gained, e.g. rounding errors were made in rearranging the correct sine formula then award dM1 for evidence of use of **inverse sin in degrees** on their value for sin$x$ (may need to check on calculator).

**NB** 70.5 following a correct sine formula will gain M1A1M1.

A1: deduce and state **both of the answers** $x$ = 70.5 and 109.5 (do not need degrees) Accept awrt these. Also accept 70.6 and 109.4.

(Second answer is sometimes obtained by a long indirect route but still scores A1)

If working in radians throughout, answers are 1.23 and 1.91 and this can be awarded M1 A1 M1 A0

(Working with 50 radians gives probable answers of -0.3288 and 3.47 giving M1A1M0A0)

**Special case:** Wrong labelling of triangle. This simplifies the problem as there is only one solution for angle $x$. So it is not treated as a misread. If they find the missing side as awrt 12.6 then proceed to find an angle or its sine or cosine then give M1A0M0A0

**Alternative Method using cosine rule**

Let $BC = a$.

M1: uses the cosine rule to form to form a three term quadratic equation in $a$ (e.g. $a^2 - 32a\cos 50^\circ + 87 = 0$ or $a^2 - \text{awrt}20.6a + 87 = 0$ though allow slips in signs rearranging)

A1: Solves and obtains a correct value for $a$ of awrt 14.6 **or** awrt 5.95.

dM1: A **correct full** method to find (at least) one of the two angles. May use cosine rule again, or find angle $BAC$ and then use sine rule. As in the main scheme, if the previous A mark has been awarded then they should obtain one of the correct angles for this mark.

A1: deduces both correct answer as in main scheme.

**NB** obtaining only one correct angle will usually score M1A1M1A0 in any method.

| Question Number | Scheme | Marks |
| --- | --- | --- |
| **45.** (a) | **Usually answered in radians: Uses** $BCD = 3.5 \times (angle), = 3.5 \times 1.77 = 6.195$ (m) (accept awrt 6.20) | M1 A1<br>**(2)** |
| (b) | Area = $\frac{1}{2}(3.5)^2 \times 1.77 = 10.84$ $(\text{m}^2)$ | M1 A1<br><br>**(2)** |
| (c) | Area of triangle = $\frac{1}{2} \times 3.7 \times 3.5 \times \sin(\text{angle})$ , = $\frac{1}{2} \times 3.7 \times 3.5 \times \sin(\frac{\pi}{2} - \frac{1.77}{2})$ (=awrt 4.1)<br>Total area = $"10.84" + 2 \times "4.101"$<br>= 19.04 | M1, A1<br>M1<br>A1cao<br><br>**(4)**<br>**[8]** |
| | **Notes** | |
| (a)<br><br>(b)<br><br><br>(c) | M1: uses $s = 3.5 \times \theta$ with $\theta$ in radians or completely correct work in degrees<br>A1: awrt 6.20 or just 6.2 (do not need to see units) Correct answer can imply the method.<br>M1 for attempt to use $A = \frac{1}{2} \times 3.5^2 \times \theta$ (Accept $\theta$ in degrees.)<br>A1 for awrt 10.84 (do not need to see units) isw if correct answer is followed by 10.8. Correct answer can imply the method.<br>M1: Uses area of triangle $\frac{1}{2} \times 3.7 \times 3.5 \times \sin(\text{angle})$ Must be correct method for area of triangle but may be less direct.<br>A1: Correct expression using correct angle i.e. $\frac{\pi}{2} - \frac{1.77}{2}$ or awrt 0.69 or awrt 39 degrees (need at least 2 sf if no other work seen, but may be implied by correct final answer) If correct expression is given then isw (so e.g. isw an answer of 0.0775 implying angle set to degrees on calculator)<br>M1: Adds **twice** their **second calculated area** (even if rectangle or segment) to their sector area (may have been slips or errors in one or both formulae – such as missing ½ or mixture of degrees and radians or weak attempt at triangle area) so M0A0M1A0 is a possible mark distribution<br>A1: 19.04 cao (common answer through insufficient accuracy is 19.08 which loses final mark)<br>**Special Case**. The mark profile M1A0M1A0M1A0M1A0 can be given if the angle is misunderstood as $1.77\pi$ or as *AFB* for example<br>If $"10.84" + 3.5 \times 3.7 \sin(\text{angle})$ is used then this can gain both M marks and the A marks if correct.<br>But use of $3.5 \times 3.7 \sin(\text{angle})$ and later doubled and added to "10.84" is 1st M0, 2nd M1. | |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **46.** (a) | Way 1 | Way 2 | |
| | $1-\sin^2 x = 8\sin^2 x-6\sin x$ | $2=(3\sin x-1)^2$ gives $9\sin^2 x-6\sin x+1=2$<br>so $\sin^2 x+8\sin^2 x-6\sin x+1=2$ | B1 |
| | E.g. $9\sin^2 x-6\sin x=1$ or<br>$9\sin^2 x-6\sin x-1=0$ or<br>$9\sin^2 x-6\sin x+1=2$ | so $8\sin^2 x-6\sin x=1-\sin^2 x$ | M1 |
| | So $9\sin^2 x-6\sin x+1=2$ or<br>$(3\sin x-1)^2-2=0$<br>so $(3\sin x-1)^2=2$ or<br>$2=(3\sin x-1)^2$ * | $8\sin^2 x-6\sin x=\cos^2 x$ * | A1cso* |
| | | | **(3)** |
| (b) | Way 1: $(3\sin x-1)=(\pm)\sqrt{2}$ | Way 2: Expands $(3\sin x-1)^2=2$ and uses quadratic formula on 3TQ | M1 |
| | $\sin x=\dfrac{1\pm\sqrt{2}}{3}$ or awrt 0.8047 **and** awrt $-$ 0.1381 | | A1 |
| | $x$ = 53.58 , 126.42 (or 126.41), 352.06, 187.94 | | dM1A1<br>A1 |
| | | | **(5)**<br>**[8]** |

| | **Notes** |
|---|---|
| (a) | **Way 1**<br>B1: Uses $\cos^2 x=1-\sin^2 x$<br>M1: Collects $\sin^2 x$ terms to form a three term quadratic or into a suitable completed square format. May be sign slips in the collection of terms.<br>A1*: cso This needs an intermediate step from 3 term quadratic and no errors in answer and printed answer stated but allow $2=(3\sin x-1)^2$. If sin is used throughout instead of sin*x* it is A0.<br>**Way 2**<br>B1: **Needs correct expansion** and **split**<br>M1: Collects $1-\sin^2 x$ together<br>A1*: Conclusion and no errors seen |
| (b) | M1: Square roots both sides(Way 1), or expands and uses quadratic formula (Way 2) Attempts at factorization after expanding are M0.<br>A1: **Both** correct answers for sin*x* (need plus and minus). Need not be simplified.<br>dM1: Uses inverse sin to give one of the given correct answers<br>1st A1: Need two correct angles (allow awrt) Note that the scheme allows 126.41 in place of 126.42 though 126.42 is preferred<br>A1: All four solutions correct (Extra solutions in range lose this A mark, but outside range - ignore)<br>(**Premature approximation**:– in the final three marks lose first A1 then ft other angles for second A mark)<br>Do **not** require degrees symbol for the marks<br>**Special case: Working in radians**<br>M1A1A0 for the *correct* 0.94, 2.21, 6.14, 3.28 |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **47.** | $1-2\cos\left(\theta-\frac{\pi}{5}\right)=0$ ; $-\pi<\theta\leqslant\pi$ | | |
| (i) | $\cos\left(\theta-\frac{\pi}{5}\right)=\frac{1}{2}$ | Rearranges to give $\cos\left(\theta-\frac{\pi}{5}\right)=\frac{1}{2}$ or $-\frac{1}{2}$ | M1 |
| | $\theta=\left\{-\frac{2\pi}{15},\ \frac{8\pi}{15}\right\}$ | At least one of $-\frac{2\pi}{15}$ or $\frac{8\pi}{15}$ or $-24^{\circ}$ or $96^{\circ}$ or awrt 1.68 or awrt -0.419 | A1 |
| | | Both $-\frac{2\pi}{15}$ and $\frac{8\pi}{15}$ | A1 |
| | | | **[3]** |
| **NB Misread** | **Misreading** $\frac{\pi}{5}$ as $\frac{\pi}{6}$ or $\frac{\pi}{3}$ (or anything else)– treat as misread so M1 A0 A0 is maximum mark | | |
| | $4\cos^2 x+7\sin x-2=0$, $0\leqslant x<360^{\circ}$ | | |
| (ii) | $4(1-\sin^2 x)+7\sin x-2=0$ | Applies $\cos^2 x=1-\sin^2 x$ | M1 |
| | $4-4\sin^2 x+7\sin x-2=0$ | | |
| | $4\sin^2 x-7\sin x-2=0$ | Correct 3 term, $4\sin^2 x-7\sin x-2\ \{=0\}$ | A1 oe |
| | $(4\sin x+1)(\sin x-2)\ \{=0\}$ , $\sin x=...$ | Valid attempt at solving and $\sin x=...$ | M1 |
| | $\sin x=-\frac{1}{4}$, $\{\sin x=2\}$ | $\sin x=-\frac{1}{4}$ (See notes.) | A1 cso |
| | $x=\text{awrt}\{194.5,\ 345.5\}$ | At least one of awrt 194.5 **or** awrt 345.5 or awrt 3.4 or awrt 6.0 | A1ft |
| | | awrt 194.5 **and** awrt 345.5 | A1 |
| | | | **[6]** |
| | | | **9** |
| **NB Misread** | **Writing equation as** $4\cos^2 x-7\sin x-2=0$ with a sign error should be marked by applying the scheme as **it simplifies** the solution (do not treat as misread) Max mark is 3/6 | | |
| | $4(1-\sin^2 x)-7\sin x-2=0$ | | **M1** |
| | $4\sin^2 x+7\sin x-2=0$ | | **A0** |
| | $(4\sin x-1)(\sin x+2)\ \{=0\}$ , $\sin x=...$ | Valid attempt at solving and $\sin x=...$ | **M1** |
| | $\sin x=+\frac{1}{4}$, $\{\sin x=-2\}$ | $\sin x=\frac{1}{4}$ (See notes.) | **A0** |
| | $x=\text{awrt}165.5$ | | **A1ft** |
| | Incorrect answers | | **A0** |

| Question 47 Notes | | |
|---|---|---|
| (i) | **M1** | Rearranges to give $\cos\left(\theta - \frac{\pi}{5}\right) = \pm\frac{1}{2}$ |
| | **Note** | M1 can be implied by seeing either $\frac{\pi}{3}$ or $60^\circ$ as a result of taking $\cos^{-1}(...)$. |
| | **A1** | Answers **may be in degrees or radians** for this mark and may have just one correct answer<br>Ignore mixed units in working if correct answers follow (recovery) |
| | **A1** | Both answers correct and in radians as multiples of π $-\frac{2\pi}{15}$ and $\frac{8\pi}{15}$<br>Ignore EXTRA solutions outside the range $-\pi < \theta \leq \pi$ but lose this mark for extra solutions in this range. |
| (ii) | **1st M1** | Using $\cos^2 x = 1 - \sin^2 x$ on the given equation. [Applying $\cos^2 x = \sin^2 x - 1$, scores M0.] |
| | **1st A1** | Obtaining a correct three term equation eg. either $4\sin^2 x - 7\sin x - 2 \{= 0\}$<br>or $-4\sin^2 x + 7\sin x + 2 \{= 0\}$ or $4\sin^2 x - 7\sin x = 2$ or $4\sin^2 x = 7\sin x + 2$, etc. |
| | **2nd M1** | For a valid attempt at solving a 3TQ quadratic in sine. Methods include factorization, quadratic formula, completion of the square (unlikely here) and calculator. (See notes on page 6 for general principles on awarding this mark) Can use any variable here, *s*, *y*, *x* or $\sin x$, and an attempt to find at least one of the solutions for sin*x*. This solution may be outside the range for sin*x* |
| | **2nd A1** | $\sin x = -\frac{1}{4}$ BY A CORRECT SOLUTION ONLY UP TO THIS POINT**.** Ignore extra answer of $\sin x = 2$, but penalise if candidate states an incorrect result. e.g. $\sin x = -2$. |
| | **Note** | $\sin x = -\frac{1}{4}$ can be implied by later correct working if no errors are seen. |
| | **3rd A1ft** | At least one of awrt 194.5 **or** awrt 345.5 or awrt 3.4 or awrt 6.0. This is a limited follow through.<br>Only follow through on the error $\sin x = \frac{1}{4}$ and allow for 165.5 special case (as this is equivalent work) This error is likely to earn M1A1M1A0A1A0 so 4/6 or M1A0M1A0A1A0 if the quadratic had a sign slip. |
| | **4thA1**<br>**Note** | awrt 194.5 **and** awrt 345.5<br>If there are any EXTRA solutions inside the range $0 \leqslant x < 360^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final A1 mark.<br>Ignore EXTRA solutions outside the range $0 \leqslant x < 360^\circ$. |
| | Special Cases | Rounding error Allow M1A1M1A1A1A0 for those who give two correct answers but wrong accuracy e.g. awrt 194, 346 (Remove final A1 for this error)<br>Answers in radians:– **lose final** mark so either or both of 3.4, 6.0 gets A1ftA0<br>It is possible to earn M1A0A1A1 on the final 4 marks if an error results fortuitously in sin*x* = -1/4 then correct work follows. |



EXPERT TUITION

| Question Number | Scheme | Marks |
|---|---|---|
| **48.(a)** | In triangle $OCD$ **complete method** used to find angle $COD$ so: | |
| | Either $\cos COD = \dfrac{8^2+8^2-7^2}{2\times8\times8}$ or uses $\angle COD = 2\times\arcsin\frac{3.5}{8}$ oe **so** $\angle COD =$ | M1 |
| | ( $\angle COD = 0.9056(331894)$ ) $= 0.906$ (3sf) * accept awrt 0.906 | A1 * **(2)** |
| (b) | Uses $s = 8\theta$ for any $\theta$ in radians or $\dfrac{\theta}{360}\times 2\pi\times 8$ for any $\theta$ in degrees | M1 |
| | $\theta = \dfrac{\pi - "COD"}{2}$ $(= awrt\ 1.12)$ or $2\theta(= awrt\ 2.24)$ and Perimeter = 23+( 16 $\times\theta$ ) | M1 |
| | accept awrt 40.9 (cm) | A1 **(3)** |
| (c) | Either Way 1: (Use of Area of two sectors + area of triangle)<br>Area of triangle = $\frac{1}{2}\times8\times8\times\sin 0.906$ (or 25.1781155 accept awrt 25.2)or<br>$\frac{1}{2}\times8\times7\times\sin 1.118$ or $\frac{1}{2}\times7\times h$ after $h$ calculated from correct Pythagoras or trig. | M1 |
| | Area of sector = $\frac{1}{2}8^2\times"1.117979732"$ (or 35.77535142 accept awrt 35.8 ) | M1 |
| | Total Area = Area of two sectors + area of triangle =awrt 96.7 or 96.8 or 96.9 ( $\text{cm}^2$ ) | A1 **(3)** |
| | Or Way 2: (Use of area of semicircle – area of segment)<br>Area of semi-circle = $\frac{1}{2}\times\pi\times8\times8$ ( or 100.5) | M1 |
| | Area of segment = $\frac{1}{2}8^2\times("0.906"-\sin"0.906")$ ( or 3.807) | M1 |
| | So area required = awrt 96.7 or 96.8 or 96.9 ( $\text{cm}^2$ ) | A1 **(3)** **[8]** |

Notes

(a) **M1**: Either use correctly quoted cosine rule – may quote as $7^2 = 8^2+8^2-2\times8\times8\cos\alpha \Rightarrow \alpha = .....$
Or split isosceles triangle into two right angled triangles and use arcsin or longer methods using Pythagoras and arcos (i.e. $\pi - 2\times\arccos\frac{3.5}{8}$ ). **There are many ways of showing this result.**
Must conclude that $\angle COD =$

**A1***: (NB this is a given answer) If any errors or over-approximation is seen this is A0. It needs correct work **leading to stated answer** of 0.906 or awrt 0.906 for A1. The cosine of $COD$ is equal to 79/128 or awrt 0.617. Use of 0.62 (2sf) does not lead to printed answer. They may give 51.9 in degrees then convert to radians. This is fine.

The minimal solution $7^2 = 8^2+8^2-2\times8\times8\cos\alpha \Rightarrow \alpha = .....0.906$ (with no errors seen) can have M1A1 but errors rearranging result in M1A0

(b) **M1**: Uses formula for arc length with $r = 8$ and any angle i.e. $s = 8\theta$ if working in rads or $s = \dfrac{\theta}{360}\times2\pi\times8$ in degrees
(If the formula is quoted with $r$ the 8 may be implied by the value of their $r\theta$ )

**M1**: Uses angles on straight line (or other geometry) to find angle $BOC$ or $AOD$ and uses Perimeter = 23 + arc lengths $BC$ and $AD$ (may make a slip – in calculation or miscopying)

**A1:** correct work leading to awrt 40.9 not 40.8 (do not need to see cm) This answer implies M1M1A1

(c) Way 1: **M1**: Mark is given for **correct** statement of area of triangle $\frac{1}{2}\times8\times8\times\sin 0.906$ (must use correct angle) or for correct answer (awrt 25.2) Accept alternative correct methods using Pythagoras and ½ base×height

**M1**: Mark is given for formula for area of sector $\frac{1}{2}8^2\times"1.117979732"$ with $r = 8$ and their angle $BOC$ or $AOD$ or ($BOC + AOD$) **not** $COD$. May use $A = \dfrac{\theta}{360}\times\pi\times8^2$ **if working in degrees**

**A1**: Correct work leading to awrt 96.7, 96.8 or 96.9 (This answer implies M1M1A1)

NB. Solution may combine the two sectors for part (b) and (c) and so might use $2\times\angle BOC$ rather than $\angle BOC$

Way 2: **M1**: Mark is given for **correct** statement of area of semicircle $\frac{1}{2}\times\pi\times8\times8$ or for correct answer 100.5

**M1**: Mark is given for formula for area of segment $\frac{1}{2}8^2\times("0.906"-\sin"0.906")$ with $r = 8$ or 3.81 **A1**: As in Way 1

| Question Number | Scheme | Marks |
|---|---|---|
| **49.** (i) | Way 1: Divides by $\cos 3\theta$ to give $\tan 3\theta = \sqrt{3}$ so $(3\theta) = \frac{\pi}{3}$ <br> Or Way 2: Squares both sides, uses $\cos^2 3\theta + \sin^2 3\theta = 1$, obtains $\cos 3\theta = \pm\frac{1}{2}$ or $\sin 3\theta = \pm\frac{\sqrt{3}}{2}$ so $(3\theta) = \frac{\pi}{3}$ | M1 |
| | Adds $\pi$ or $2\pi$ to previous value of angle ( to give $\frac{4\pi}{3}$ or $\frac{7\pi}{3}$) | M1 |
| | So $\theta = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}$ (all three, no extra in range) | A1 **(3)** |
| (ii)(a) | $4(1-\cos^2 x) + \cos x = 4 - k$ Applies $\sin^2 x = 1 - \cos^2 x$ | M1 |
| | Attempts to solve $4\cos^2 x - \cos x - k = 0$, to give $\cos x =$ | dM1 |
| | $\cos x = \frac{1 \pm \sqrt{1+16k}}{8}$ or $\cos x = \frac{1}{8} \pm \sqrt{\frac{1}{64} + \frac{k}{4}}$ or other correct equivalent | A1 **(3)** |
| (b) | $\cos x = \frac{1 \pm \sqrt{49}}{8} = 1$ and $-\frac{3}{4}$ (see the note below if errors are made) | M1 |
| | Obtains two solutions from 0 , 139 , 221 (0 or 2.42 or 3.86 in radians) | dM1 |
| | $x = 0$ and 139 and 221 (allow awrt 139 and 221) must be in degrees | A1 <br> **(3)** <br> **[9]** |

Notes

(i) **M1**: Obtains $\frac{\pi}{3}$. Allow $x = \frac{\pi}{3}$ or even $\theta = \frac{\pi}{3}$. Need not see working here. May be implied by $\theta = \frac{\pi}{9}$ in final answer ( allow $(3\theta) = 1.05$ or $\theta = 0.349$ as decimals or $(3\theta) = 60$ or $\theta = 20$ as degrees for this mark)

Do not allow $\tan 3\theta = -\sqrt{3}$ nor $\tan 3\theta = \pm\frac{1}{\sqrt{3}}$

**M1**: Adding $\pi$ or $2\pi$ to a previous value however obtained. It is not dependent on the previous mark.

(May be implied by final answer of $\theta = \frac{4\pi}{9}$ **or** $\frac{7\pi}{9}$). This mark may also be given for answers as decimals [4.19 or 7.33], or degrees ( 240 or 420).

**A1**: Need all three correct answers in terms of $\pi$ and **no extras in range**.

**Three correct answers implies M1M1A1**

NB : $\theta = 20°, 80°, 140°$ earns M1M1A0 and 0.349, 1.40 and 2.44 earns M1M1A0

(ii) (a) **M1:** Applies $\sin^2 x = 1 - \cos^2 x$ (allow even if brackets are missing e.g. $4 \times 1 - \cos^2 x$ ).

This must be awarded in (ii) (a) for an expression with $k$ not after $k = 3$ is substituted.

**dM1**: Uses formula or completion of square to obtain $\cos x =$ expression in $k$

(Factorisation attempt is M0) **A1:** cao - award for their final simplified expression

(b) **M1**: **Either** attempts to substitute $k = 3$ into their answer to obtain two values for $\cos x$

**Or** restarts with $k = 3$ to find two values for $\cos x$ (They cannot earn marks in ii(a) for this)

**In both cases** they need to have applied $\sin^2 x = 1 - \cos^2 x$ (brackets may be missing) **and** correct method for solving their quadratic (usual rules – see notes) The values for $\cos x$ may be >1 or < -1

**dM1:** Obtains **two correct** values for $x$

**A1**: Obtains **all three correct values** in degrees (allow awrt 139 and 221) including 0. Ignore excess answers outside range (including 360 degrees) Lose this mark for excess answers in the range or radian answers.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **50.(a)** | Area $BDE$ $=\frac{1}{2}(5)^2(1.4)$ | M1: Use of the correct formula or method for the area of the sector | M1A1 |
| | $=17.5\ (\text{cm}^2)$ | A1: 17.5 oe | |
| | | | **[2]** |
| **(b)** | **Parts (b) and (c) can be marked together** | | |
| | $6.1^2=5^2+7.5^2-\left(2\times5\times7.5\cos DBC\right)$ or $\cos DBC=\dfrac{5^2+7.5^2-6.1^2}{2\times5\times7.5}$ (or equivalent) | | M1 |
| | M1: A correct statement involving the angle $DBC$ | | |
| | Angle $DBC=0.943201...$ | awrt 0.943 | A1 |
| | **Note that work for (b) may be seen on the diagram or in part (c)** | | |
| | | | **[2]** |
| **(c)** | **Note that candidates may work in degrees in (c) (**Angle $DBC=54.04....\text{degrees}$) | | |
| | Area $CBD=\frac{1}{2}5(7.5)\sin(0.943)$ | | |
| | Angle $EBA=\pi-1.4-\text{"}0.943\text{"}$ **(Maybe seen on the diagram)** | Area $CBD=\frac{1}{2}5(7.5)\sin(\text{their } 0.943)$ or awrt 15.2. (Note area of $CBD=15.177...$) A correct method for the area of triangle $CBD$ which can be implied by awrt 15.2 | M1 |
| | $\pi-1.4-\text{"their }0.943\text{"}$ A value for angle $EBA$ of awrt 0.8 (from 0.7985926536... or 0.7983916536...) or value for angle $EBA$ of $\left(1.74159...-\text{their angle } DBC\right)$ would imply this mark. | | M1 |
| | $AB=5\cos(\pi-1.4-\text{"}0.943\text{"})$ or $AE=5\sin(\pi-1.4-\text{"}0.943\text{"})$ | | |
| | | $AB=5\cos(\pi-1.4-\text{their } 0.943)$ $AB=5\cos(0.79859...)=3.488577938...$ Allow M1 for $AB=$ awrt 3.49 Or $AE=5\sin(\pi-1.4-\text{their } 0.943)$ $AE=5\sin(0.79859...)=3.581874365688...$ Allow M1 for $AE=$ awrt 3.58 **It must be clear that** $\pi-1.4-\text{"}0.943\text{"}$ **is being used for angle EBA. Note that some candidates use the sin rule here but it must be used correctly – do not allow mixing of degrees and radians.** | M1 |
| | Area $EAB=\frac{1}{2}5\cos(\pi-1.4-\text{"}0.943\text{"})\times5\sin(\pi-1.4-\text{"}0.943\text{"})$ | | |
| | **This is dependent on the previous M1 and there must be no other errors in finding the area of triangle EAB** | | **d**M1 |
| | Allow M1 for area $EAB$ = awrt 6.2 | | |
| | Area $ABCDE=15.17...+17.5+6.24...=38.92...$ | | |
| | | awrt 38.9 | A1**cso** |
| | | | **[5]** |
| | **Note that a sign error in (b) can give the obtuse angle (2.198....) and could lead to the correct answer in (c) – this would lose the final mark in (c)** | | **Total 9** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **51.** | (i) $9\sin(\theta+60^{\circ})=4$; $0\le\theta<360^{\circ}$<br>(ii) $2\tan x-3\sin x=0$; $-\pi\le x<\pi$ | | |
| **(i)** | $\sin(\theta+60^{\circ})=\frac{4}{9}$, so $(\theta+60^{\circ})=26.3877...$<br>$(\alpha=26.3877...)$ | Sight of $\sin^{-1}\left(\frac{4}{9}\right)$ or awrt $26.4^{\circ}$ or $0.461^{c}$<br>Can also be implied for $\theta=$ awrt $-33.6$ (i.e. $26.4-60$) | M1 |
| | So, $\theta+60^{\circ}=\{153.6122...,\ 386.3877...\}$ | $\theta+60^{\circ}$ = either "$180-$ their $\alpha$" or "$360^{\circ}+$ their $\alpha$" and not for $\theta=$ either "$180-$ their $\alpha$" or "$360^{\circ}+$ their $\alpha$". This can be implied by later working. The candidate's $\alpha$ could also be in radians but do not allow mixing of degrees and radians. | M1 |
| | and $\theta=\{93.6122...,\ 326.3877...\}$ | A1: At least one of awrt $93.6^{\circ}$ or awrt $326.4^{\circ}$ | A1 A1 |
| | | A1: Both awrt $93.6^{\circ}$ and awrt $326.4^{\circ}$ | |
| | **Both answers are cso and must come from correct work** | | |
| | Ignore extra solutions outside the range.<br>In an otherwise fully correct solution deduct the final A1for any extra solutions in range | | |
| | | | **[4]** |
| **(ii)** | $2\left(\frac{\sin x}{\cos x}\right)-3\sin x=0$ | Applies $\tan x=\frac{\sin x}{\cos x}$ | M1 |
| | Note: Applies $\tan x=\frac{\sin x}{\cos x}$ can be implied by $2\tan x-3\sin x=0\Rightarrow\tan x(2-3\cos x)$ | | |
| | $2\sin x-3\sin x\cos x=0$ | | |
| | $\sin x(2-3\cos x)=0$ | | |
| | $\cos x=\frac{2}{3}$ | $\cos x=\frac{2}{3}$ | A1 |
| | $x=$ awrt $\{0.84,\ -0.84\}$ | A1: One of either awrt $0.84$ or awrt $-0.84$ | A1A1ft |
| | | A1ft: You can apply ft for $x=\pm\alpha$, where $\alpha=\cos^{-1}k$ and $-1\le k\le1$ | |
| | **In this part of the solution, if there are any extra answers in range in an otherwise correct solution withhold the A1ft.** | | |
| | $\{\sin x=0\ \Rightarrow\}\ x=0$ **and** $-\pi$ | **Both** $x=0$ **and** $-\pi$ or awrt $-3.14$ from $\sin x=0$<br>**In this part of the solution**, **ignore extra solutions in range.** | B1 |
| | Note solutions are: $x=\{-3.1415...,\ -0.8410...,\ 0,\ 0.8410...\}$<br>Ignore extra solutions outside the range | | |
| | For **all** answers in degrees in (ii) M1A1A0A1ftB0 is possible | | |
| | **Allow the use of θ in place of $x$ in (ii)** | | |
| | | | **[5]** |
| | | | **Total 9** |
| | | | |



EXPERT TUITION

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **52(a)** | Length *DEA* $= 7(2.1) = 14.7$ | M1: $7 \times 2.1$ only<br>A1: 14.7 | M1A1 |
| | | | **[2]** |
| **(b)** | Angle *CBD* $= \pi - 2.1$ | **May be seen on the diagram** (allow awrt 1.0 and allow 180 – 120). Could score for sight of **Angle *CBD* = awrt 60 degrees.** | M1 |
| | **Both** $7\cos(\pi - 2.1)$ **and** $7\sin(\pi - 2.1)$<br>or<br>**Both** $7\cos(\pi - 2.1)$ **and** $\sqrt{7^2 - (7\cos(\pi - 2.1))^2}$<br>or<br>**Both** $7\sin(\pi - 2.1)$ **and** $\sqrt{7^2 - (7\sin(\pi - 2.1))^2}$<br>**Or equivalents to these** | A correct attempt to find BC and BD. You can ignore how the candidate assigns *BC* and *CD*. $7\cos(\pi - 2.1)$ can be implied by awrt 3.5 and $7\sin(\pi - 2.1)$ can be implied by awrt 6. Note if the sin rule is used, do not allow mixing of degrees and radians unless their answer implies a correct interpretation. **Dependent on the previous method mark**. | dM1 |
| | **Note that 2.1 radians is 120 degrees (to 3sf) which if used gives angle CBD as 60 degrees. If used this gives a correct perimeter of 31.3 and could score full marks.** | | |
| | $P = 7\cos(\pi - 2.1) + 7\sin(\pi - 2.1) + 7 + 14.7$ | their BC + their CD + 7 + their DEA<br>**Dependent on <u>both</u> previous method marks** | ddM1 |
| | $= 31.2764...$ | Awrt 31.3 | A1 |
| | | | **[4]** |
| | | | **Total 6** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 53.(i) | $\frac{\sin 2\theta}{(4\sin 2\theta - 1)} = 1;\ \ 0 \leqslant \theta < 180^\circ$ | | |
| | $\sin 2\theta = \frac{1}{3}$ | $\sin 2\theta = k$ where $-1 < k < 1$ **Must be 2θ and not θ.** | M1 |
| | $\{2\theta = \{19.4712..., 160.5288...\}\}$ | | |
| | $\theta = \{9.7356..., 80.2644...\}$ | A1: Either awrt 9.7 or awrt 80.3<br>A1: Both awrt 9.7 and awrt 80.3 | A1 A1 |
| | **Do not penalise poor accuracy more than once e.g. 9.8 and 80.2 from correct work could score M1A1A0** | | |
| | **If both answers are correct in radians award A1A0 otherwise A0A0**<br>**Correct answers are 0.2 and 1.4** | | |
| | **Extra solutions in range in an otherwise fully correct solution deduct the last A1** | | |
| | | | **[3]** |
| (ii) | $5\sin^2 x - 2\cos x - 5 = 0,\ \ 0 \le x < 2\pi.$ | | |
| | $5(1-\cos^2 x) - 2\cos x - 5 = 0$ | Applies $\sin^2 x = 1 - \cos^2 x$ | M1 |
| | $5\cos^2 x + 2\cos x = 0$<br>$\cos x(5\cos x + 2) = 0$<br>$\Rightarrow \cos x = ....$ | Cancelling out $\cos x$ or a valid attempt at solving the quadratic in cos$x$ **and** giving cos$x$ = … **Dependent on the previous method mark.** | dM1 |
| | awrt 1.98 or awrt 4.3(0) | Degrees: 113.58, 246.42 | A1 |
| | Both 1.98 and 4.3(0) | or their $\alpha$ and their $2\pi - \alpha$, where $\alpha \neq \frac{\pi}{2}$.<br>If working in degrees allow 360 – their α | A1ft |
| | awrt 1.57 or $\frac{\pi}{2}$ **and** 4.71 or $\frac{3\pi}{2}$<br>**or**<br>$90^\circ$ and $270^\circ$ | **These answers only but ignore other answers outside the range** | B1 |
| | | | **[5]** |
| | NB: $x$ = awrt $\left\{1.98,\ 4.3(0),\ 1.57 \text{ or } \frac{\pi}{2},\ 4.71 \text{ or } \frac{3\pi}{2}\right\}$ | | **8** |
| | **Answers in degrees: 113.58, 246.42, 90, 270**<br>**Could score M1M1A0A1ftB1 (4/5)** | | |

| Question Number | Scheme | Marks |
|---|---|---|
| 54. (a) | Mark (a) and (b) together. | |
| | **Usually answered in radians: Uses** either $\frac{1}{2}ab\sin(\text{angle})$ **or** $\frac{1}{2}(12)^2(\text{angle})$ or both | M1 |
| | Area $= \frac{1}{2}(23)(12)\sin 0.64$ **or** $\frac{1}{2}(12)^2(\pi - 0.64)$ $\{= 82.41297091...$ **or** $180.1146711...\}$ | A1 |
| | Area $= \frac{1}{2}(23)(12)\sin 0.64 + \frac{1}{2}(12)^2(\pi - 0.64)$ $\{= 82.41297091... + 180.1146711...\}$ | A1 |
| | $\{\text{Area} = 262.527642...\}$ = awrt 262.5 ($\text{m}^2$) or 262.4($\text{m}^2$) or 262.6 ($\text{m}^2$) | A1 |
| | | **(4)** |
| (b) | $CDE = 12\times(angle),\ = 12(\pi - 0.64)\{\Rightarrow CDE = 30.01911...\}$ | M1, A1 |
| | $AE^2 = 23^2 + 12^2 - 2(23)(12)\cos(0.64) \Rightarrow AE^2 =$ or $AE =$ $\{AE = 15.17376...\}$ | M1 |
| | Perimeter $= 23 + 12 + 15.17376... + 30.01911...$ | M1 |
| | $= 80.19287... =$ awrt 80.2 (m) | A1 |
| | | **(5)** |
| | | **[9]** |

| | Notes for Question 54 |
|---|---|
| (a) | M1: uses either area of triangle formula as given in mark scheme, or area of sector or both (may be implied by answer) |
| | A1: one correct area expression (with **correct angle** used) $\frac{1}{2}(23)(12)\sin 0.64$ or $\frac{1}{2}(12)^2(\pi - 0.64)$ or see awrt 82.4 **or** awrt 180 (180 may be split as 226.2(semicircle) minus 46.1(small sector)) |
| | A1: two correct area expressions (with correct angles) **added together (**allow 2.5 as implying $\pi - 0.64$) or see awrt 82.4 **+** awrt 180 ( or 226 - 46 ) |
| | A1: answers which round to 262.5 or 262.4 or 262.6 |
| (b) | 1st M1 for attempt to use $s = r\,\theta$ (any angle) |
| | 1st A1 for $\pi - 0.64$ in the formula (or 2.5) |
| | 2nd M1: Uses correct cosine rule to obtain $AE$ or $AE^2$ (this may appear in part (a)) |
| | 3rd M1(**independent)**: Perimeter = $23 + 12 +$ their $AE$ + their $CDE$ |
| | 2nd A1: awrt 80.2 (ignore units – even incorrect units) |
| **Degrees (a)** | Uses either $\frac{1}{2}ab\sin(\text{angle})$ **or** $\frac{\text{angle in degrees}}{360}\times\pi(12)^2$ or both for M1 |
| | Area $= \frac{1}{2}(23)(12)\sin 36.7$ **or** $\frac{(180\text{-}36.7)}{360}\times\pi(12)^2\{=$ *awrt* $82.4...$ **or** $180\}$ A1 |
| | Area $= \frac{1}{2}(23)(12)\sin 36.7 + \frac{(180\text{-}36.7)}{360}\times\pi(12)^2$ $\{=$ *awrt* $82.4... + 180\}$ A1 |
| | Final mark as before |
| (b) | $CDE = \frac{\text{Angle in degrees}}{360}\times 24\pi,\ = \frac{180-36.7}{360}\times 24\pi\{\Rightarrow CDE = 30.01268...\}$ M1, A1 |
| | Final three marks as before |

| Question Number | Scheme | Marks |
|---|---|---|
| **55. (i)** | $(\lvert\alpha\rvert = 56.3099...)$ | |
| | $x = \{\alpha + 40 = 96.309993...\} =$ **awrt 96.3** | B1 |
| | $x - 40^{\circ} = -180 + "56.3099"...$ or $x - 40^{\circ} = -\pi + "0.983"...$ | M1 |
| | $x = \{-180 + 56.3099... + 40 = -83.6901...\} =$ **awrt -83.7** | A1 |
| | | **(3)** |
| **(ii)(a)** | $\sin\theta\left(\dfrac{\sin\theta}{\cos\theta}\right) = 3\cos\theta + 2$ | M1 |
| | $\left(\dfrac{1-\cos^2\theta}{\cos\theta}\right) = 3\cos\theta + 2$ | dM1 |
| | $1-\cos^2\theta = 3\cos^2\theta + 2\cos\theta \quad \Rightarrow 0 = 4\cos^2\theta + 2\cos\theta - 1$ * | A1 cso * |
| | | **(3)** |
| **(b)** | $\cos\theta = \dfrac{-2 \pm \sqrt{4-4(4)(-1)}}{8}$ | M1 |
| | **or** $4(\cos\theta \pm \frac{1}{4})^2 \pm q \pm 1 = 0,$ or $(2\cos\theta \pm \frac{1}{2})^2 \pm q \pm 1 = 0,\ q \neq 0$ so $\cos\theta = ...$ | |
| | One solution is $72^{\circ}$ or $144^{\circ}$, Two solutions are $72^{\circ}$ and $144^{\circ}$ | A1, A1 |
| | $\theta = \{72, 144, 216, 288\}$ | M1 A1 |
| | | **(5)** |
| | | **[11]** |

| | **Notes for Question 55** |
|---|---|
| **(i)** | B1: 96.3 by any or no method<br>M1: Takes 180 degrees from arctan (1.5) or from their "96.3" May be implied by A1. (Could be obtained by adding 180, then subtracting 360).<br>A1: awrt –83.7<br>Extra answers: ignore extra answers outside range. Any extra answers in range lose final A mark (if earned)<br>Working in radians – could earn M1 for $x - 40^{\circ} = -\pi + "0.983"...$ so B0M1A0 |
| **(ii) (a)** | M1: uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ or equivalent in equation (not just $\tan = \dfrac{\sin}{\cos}$, with no argument)<br>dM1: uses $\sin^2\theta = 1-\cos^2\theta$ (quoted correctly) in equation<br>A1: completes proof correctly, with no errors to give printed answer*. Need at least three steps in proof and need to achieve the correct quadratic with all terms on one side and "=0" |
| **(b)** | M1: Attempts to solve quadratic by correct quadratic formula, or completion of the square . Factorisation attempts score M0.<br>1st A1: Either 72 **or** 144, 2nd A1: both 72 **and** 144 (allow 72.0 etc.)<br>M1: 360 – "a previous solution" (provided that cos was being used) (not dependent on previous M)<br>A1: All four solutions correct (Extra solutions in range lose this A mark, but outside range - ignore)<br>(**Premature approximation**: e.g. 71.9, 144.1, 288.1 and 215.9 – lose first A1 then ft other angles)<br>Do **not** require degrees symbol for the marks<br>**Special case: Working in radians**<br>M1: as before, A1 for either $\theta = \frac{2}{5}\pi$ or $\theta = \frac{4}{5}\pi$ or decimal equivalents, and 2nd A1: both<br>M1: $2\pi - \alpha_1$ or $2\pi - \alpha_2$ then A0 so 4/5 |

| Question Number | Scheme | Marks |
|---|---|---|
| 56.(a) | **Way 1**: $10^2 = 7^2 + 13^2 - 2\times 7\times 13\cos\theta$ or $\cos\theta = \frac{7^2+13^2-10^2}{2\times 7\times 13}$ | M1 |
| | $\cos\theta = \frac{59}{91}$ or $\cos\theta = \frac{7^2+13^2-10^2}{2\times 7\times 13}$ or $\cos\theta = 0.6483$ or $0.8644$ | A1 o.e |
| | ($\theta$ = 0.8653789549… ) = 0.865 * (to 3 dp) | A1* cso (3) |
| | **Way 2**: Uses $\cos\theta = \frac{x}{7}$, where $7^2 - x^2 = 10^2 - (13-x)^2$ and finds $x$ ( = 59/13) | M1 |
| | $\cos\theta = \frac{59}{91}$ and ($\theta$ = 0.8653789549… ) = 0.865 * (to 3 dp) – as in Way 1 | A1, A1 (3) |
| (b) | Area triangle $ABC = \frac{1}{2}\times 13\times 7\sin 0.865$ or $\frac{1}{2}\times 13\times 7\sin 49.6$ or $20\sqrt{3}$ | M1 |
| | Area sector $ABD = \frac{1}{2}\times 7^2\times 0.865$ or $\frac{49.6}{360}\times\pi\times 7^2$ | M1 |
| | =34.6 (triangle) **or** 21.2 (Sector) | A1 |
| | Area of $S = \frac{1}{2}\times 13\times 7\sin 0.865 - \frac{1}{2}\times 7^2\times 0.865$ $(=13.4)$ | M1 A1 |
| | (Amount of seed = ) 13.4 x 50 = 670g or 680g (need one of these two answers) | M1 A1 (7) |
| | | **Total 10** |

| | **Notes for Question 56** |
|---|---|
| **(a)** | M1: use correct cosine **formula in any form** A1: give a value for $\cos\theta$<br>NB $\cos\theta = \frac{7^2+13^2-10^2}{2\times 7\times 13}$ earns M1A1<br>A1: deduce and state the printed answer θ = 0.865 |
| **(b)** | M1: Uses Correct method for area of the correct triangle i.e. *ABC*<br>M1: Uses Correct method for the area of the sector<br>A1: This is earned for one of the correct answers. May be implied if these answers are not calculated but the final answer is correct with no errors (or shaded area is 13.4 or13.5)<br>M1: Their area of Triangle *ABC*– **Area** of Sector (may have $kr^2\theta$ but not $k\theta$ )<br>A1: **Correct expression** or awrt 13.4 or 13.5 (may be implied by final answer)<br>M1: Multiply their previous answer by 50<br>A1: 670g or 680 g (There is an argument for rounding answer up to provide enough seed) |

N.B. $(\frac{1}{2}\times 13\times 7\sin 0.865 - \frac{1}{2}\times 7^2\times 0.865)\times 50$ =670 or 680 earns full marks

$(\frac{1}{2}\times 13\times 7\sin 0.865 - \frac{1}{2}\times 7^2\times 0.865)\times 50$ = **awrt** 670 or 680 just loses last mark

$(\frac{1}{2}\times 13\times 7\sin 0.865 - \frac{1}{2}\times 7^2\times 0.865)\times 50$ = wrong answer M1M1A0M1A1M1A0

| Question Number | Scheme | Marks |
| --- | --- | --- |
| **57.(a)** | $\sin(2\theta-30)=-0.6$ or $2\theta-30=-36.9$ or implied by 216.9 | B1 |
| | $2\theta-30=216.87$ = (180 + 36.9) | M1 |
| | $\theta=\dfrac{216.87+30}{2}=123.4$ or 123.5 | A1 |
| | $2\theta-30=360-36.9$ or 323.1 | M1 |
| | $\theta=\dfrac{323.1+30}{2}=176.6$ | A1 (5) |
| **(b)** | $9\cos^2 x-11\cos x+3(1-\cos^2 x)=0$ or $6\cos^2 x-11\cos x+3(\sin^2 x+\cos^2 x)=0$ | M1 |
| | $6\cos^2 x-11\cos x+3=0$ {as $(\sin^2 x+\cos^2 x)=1$} | A1 |
| | $(3\cos x-1)(2\cos x-3)=0$ implies $\cos x=$ | M1 |
| | $\cos x=\dfrac{1}{3}, \left(\dfrac{3}{2}\right)$ | A1 |
| | $x=70.5$ | B1 |
| | $x=360-"70.5"$<br>$x=289.5$ | M1<br>A1cao<br>(7)<br>**Total 12** |
| | **Notes for Question 57** | |
| **(a)** | B1: This statement seen and must contain no errors or may implied by – 36.9<br>M1: Uses 180 – arcsin (-0.6) i.e. 180 + 36.9 ( or $\pi+\arcsin(0.6)$ in radians) (in 3rd quadrant)<br>A1: allow answers which round to 123.4 or 123.5 must be in degrees<br>M1: Uses 360 + arcsin (-0.6) i.e. 360 – 36.9 ( or $2\pi+\arcsin(-0.6)$ in radians) ( in 4th quadrant)<br>A1: allow answers which round to 176.6 must be in degrees (A1 implies M1)<br>Ignore extra answers outside range but lose final A1 for extra answers in the range if both B and A marks have been earned)<br>Working in radians may earn B1M1A0M1A0 | |
| **(b)** | M1: **Use** of $\sin^2 x=(1-\cos^2 x)$ or $(\sin^2 x+\cos^2 x)=1$ in the given equation<br>A1: Correct three term quadratic in any equivalent form<br>M1: Uses standard method to solve quadratic and obtains cos*x* =<br>A1: A1 for $\frac{1}{3}$ with $\frac{3}{2}$ ignored but A0 if $\frac{3}{2}$ is incorrect<br>B1: 70.5 or answers which round to this value<br>M1: 360 –arcos(*their1*/3) ( or $2\pi-\arccos(their1/3)$ in radians)<br>A1: Second answer<br>Working in radians in (b) may earn M1A1M1A1B0M1A0<br>Extra values in the range coming from arcos (1/3) – deduct final A mark - so A0 | |

| | | | |
|---|---|---|---|
| | | | |
| **58.** | | | |
| | $\cos^{-1}\left(-0.4\right)\ =113.58\ \ (\alpha)$ | Awrt 114 | B1 |
| | $3x-10=\alpha \Rightarrow x=\dfrac{\alpha+10}{3}$ | Uses their α to find $x$. Allow $x=\dfrac{\alpha\pm10}{3}$ **not** $\dfrac{\alpha}{3}\pm10$ | M1 |
| | | | |
| | $x=41.2$ | Awrt | A1 |
| | | | |
| | $\left(3x-10=\right)360-\alpha\ \ (246.4....)$ | $360-\alpha$ (can be implied by 246.4...) | M1 |
| | $x$ = 85.5 | Awrt | A1 |
| | $\left(3x-10=\right)360+\alpha\ \left(=473.57....\right)$ | $360+\alpha$ (Can be implied by 473.57...) | M1 |
| | $x$ = 161.2 | Awrt | A1 |

| | | | |
|---|---|---|---|
| **59.** | | | |
| **(a)** | $9^2 = 4^2 + 6^2 - 2\times4\times6\cos\alpha \Rightarrow \cos\alpha = .....$ | Correct use of cosine rule leading to a value for cos α | M1 |
| | $\cos\alpha = \dfrac{4^2+6^2-9^2}{2\times4\times6}\left(= -\dfrac{29}{48} = -0.604..\right)$ | | |
| | $\alpha = 2.22$ * | Cso (2.22 must be seen here) | A1 |
| | (NB $\alpha = 2.219516005$) | | **(2)** |
| **(a) Way 2** | $XY^2 = 4^2 + 6^2 - 2\times4\times6\cos 2.22 \Rightarrow XY^2 = ..$ | Correct use of cosine rule leading to a value for $XY^2$ | M1 |
| | $XY^2 = 81.01....$ | | |
| | $XY = 9.00....$ | | A1 |
| | | | **(2)** |
| **(b)** | $2\pi - 2.22(= 4.06366......)$ | $2\pi - 2.22$ or awrt 4.06 | B1 |
| | $\frac{1}{2}\times4^2\times"4.06"$ | Correct method for major sector area. | M1 |
| | 32.5 | Awrt 32.5 | A1 |
| | | | **(3)** |
| **(b) Way2** | **Circle – Minor sector** | | |
| | $\pi\times4^2$ | Correct expression for circle area | B1 |
| | $\pi\times4^2 - \dfrac{1}{2}\times4^2\times2.22 = 32.5$ | Correct method for circle - minor sector area | M1 |
| | $= 32.5$ | Awrt 32.5 | A1 |
| | | | **(3)** |
| **(c)** | Area of triangle = $\frac{1}{2}\times4\times6\times\sin 2.22(= 9.56)$ | Correct expression for the area of triangle XYZ | B1 |
| | **So area required =** "9.56" + "32.5" | Their Triangle XYZ + (part (b) answer or correct attempt at major sector) | M1 |
| | = 42.1 cm$^2$ or 42.0 cm$^2$ | Awrt 42.1 or 42.0 (Or just 42) | A1 |
| | | | **(3)** |
| | | | |
| **(d)** | Arc length $= 4\times4.06(= 16.24)$ Or $8\pi - 4\times2.22$ | M1: $4\times their\,(2\pi - 2.22)$ Or circumference – minor arc<br>A1: Correct ft expression | M1A1ft |
| | Perimeter = $ZY$ + $WY$ + Arc Length | 9 + 2 + Any Arc | M1 |
| | Perimeter = 27.2 or 27.3 | Awrt 27.2 or awrt 27.3 | A1 |
| | | | |
| | | | **(4)** |
| | | | **[12]** |

| Question number | Scheme | | Marks |
|---|---|---|---|
| 60(a) | States or uses $\tan 2x = \dfrac{\sin 2x}{\cos 2x}$ | | M1 |
| | $\dfrac{\sin 2x}{\cos 2x} = 5\sin 2x \Rightarrow \sin 2x - 5\sin 2x\cos 2x = 0 \Rightarrow \sin 2x(1-5\cos 2x) = 0$ * | | A1 (2) |
| (b) | sin $2x$ = 0 gives $2x$ = 0, 180, 360 so $x$ = 0, 90, 180 | B1 for two correct answers, second B1 for all three correct. Excess in range – lose last B1 | B1, B1 |
| | $\cos 2x = \frac{1}{5}$ gives $2x$ = 78.46 (or 78.5 or 78.4) **or** $2x$ = 281.54 (or 281.6) | | M1 |
| | $x$ = 39.2 (or 39.3), 140.8 (or 141) | | A1, A1 (5) |
| | | | **7 marks** |
| Notes | (a) **M1**: Statement that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ or Replacement of tan (wherever it appears). Must be a correct statement but may involve $\theta$ instead of 2$x$.<br>**A1**: the answer is given so all steps should be given.<br>N.B. $\sin 2x - 5\sin 2x\cos 2x = 0$ or $-5\sin 2x\cos 2x + \sin 2x = 0$ or $\sin 2x(\dfrac{1}{\cos 2x} - 5) = 0$ o.e. **must be seen** and be followed by printed answer for A1 mark<br>$\sin 2x = 5\sin 2x\cos 2x$ is not sufficient.<br>(b) Statement of 0 and 180 with no working gets B1 B0 (bod) as it is two solutions<br>**M1**: This mark for one of the two statements given (must relate to 2$x$ not just to $x$)<br>**A1, A1**: first A1 for 39.2, second for 140.8<br>*Special case* solving $\cos 2x = -1/5$ giving 2$x$ = 101.5 or 258.5 is awarded M1A0A0<br>140.8 omitted would give M1A1A0<br>Allow answers which round to 39.2 or 39.3 and which round to 140.8 and allow 141<br>Answers in radians lose last A1 awarded (These are 0, 0.68, 1.57, 2.46 and 3.14)<br>Excess answers **in range** lose last A1 Ignore excess answers outside range.<br>All 5 correct answers with no extras and no working gets **full marks** in part (b). The answers imply the method here | | |
| | | | |



EXPERT TUITION

| Question number | Scheme | Marks |
|---|---|---|
| **61 (a)** | $r\theta = 6\times 0.95, = 5.7$ (cm) | M1, A1 (2) |
| **(b)** | $\frac{1}{2}r^2\theta = \frac{1}{2}\times 6^2 \times 0.95, = 17.1\ \left(\text{cm}^2\right)$ | M1, A1 (2) |
| **(c)** | Let $AD = x$ then $\dfrac{x}{\sin 0.95} = \dfrac{6}{\sin 1.24}$ so $x = 5.16$ * <br> OR $x = 3 / \cos 0.95$ OR so $x = 3/ \sin 0.62$ so $x = 5.16$ * <br> OR $x^2 = 6^2 + x^2 - 12x\cos 0.95$ leading to $x =$ , so $x = 5.16$ * | M1 A1 (2) |
| **(d)** | Perimeter = '5.7'+5.16 +6 – 5.16= "11.7" **or 6 + their 5.7** | M1A1 ft (2) |
| **(e)** | Area of triangle $ABD$ = $\frac{1}{2}\times 6\times 5.16\times \sin 0.95$ = 12.6 or <br> $\frac{1}{2}\times 6\times 3\times \tan 0.95$ = 12.6 (½ base x height) or $\frac{1}{2}\times 5.16\times 5.16\times \sin 1.24$ =12.6 <br> So Area of $R$ = '17.1' – '12.6' = 4.5 | M1 A1 <br> M1 A1 (4) <br> **12** |
| Notes | (a) **M1:** Needs $\theta$ in radians for this formula. Could convert to degrees and use degrees formula. <br> **A1:** Does not need units <br> (b**) M1:** Needs $\theta$ in radians for this formula. Could convert to degrees and use degrees formula. <br> **A1:** Does not need units <br> (c) **M1:** Needs complete correct trig method to achieve $x$ = <br> May have worked in degrees, using 54.4 degrees and 71.1 degrees <br> Using angles of triangle sum to 360degrees is not correct method so is M0 <br> **A1:** accept answers which round to 5.16 (NB This is given answer) <br> If the answer 5.16 is **assumed** and **verified** award M1A0 for correct work <br> (d) **M1:** Accept answer only as implying method, or just 6 + 5.7 <br> **A1 :** can be scored even following wrong answer to part (c) <br> (e) **M1:** needs complete method for area of triangle $ABD$ not $ABC$ <br> **A1:** Accept awrt 12.6 (If area of triangle is not evaluated or is given as 12.5 (truncated) this mark may be implied by 4.5 later) <br> **M1:** Uses area of $R$ = area of sector – area of triangle $ABD$ (not $ABC$) <br> **A1:** Answers wrt 4.5 | |
| Alternative For part (e) | Finds area of segment **and** area of triangle $BDC$ by correct methods **M1** <br> Obtains 2.4585 **and** 2.0498 – accept answers wrt 2.5, 2.1 **A1** <br> Uses area of segment + area of triangle $BDC$ ,to obtain 4.5 (not 4.6) **M1, A1** <br> **NB Just finding area of segment is M0** | |

| Question number | Scheme | Marks |
|---|---|---|
| **62 (i)** | $\sin(3x-15)=\frac{1}{2}$ so $3x-15=30$ $(\alpha)$ and $x$ = 15 | M1 A1 |
| | Need $3x-15=180-\alpha$ or $3x-15=540-\alpha$ | M1 |
| | Need $3x-15=180-\alpha$ and $3x-15=360+\alpha$ and $3x-15=540-\alpha$ | M1 |
| | $x$ = 55 or 175 | A1 |
| | $x$ = 55 , 135 , 175 | A1 (6) |
| Notes | **M1** Correct order of operation: inverse sine then linear algebra - not just 3$x$-15 = 30 (slips in linear algebra lose Accuracy mark)<br>**A1** Obtains first solution 15<br>**M1** Uses either 180 – α or 540 – α,<br>**M1** uses all three 180 – α and 360 + α and 540 – α<br>**A1**, for one further correct solution 55 or 175, (depends only on second M1)<br>**A1** – all 3 further correct solutions<br>If more than 4 solutions in range, lose last **A1**<br>Common slips: Just obtains 15 and 55, or 15 and 175 – usually M1A1M1M0A1A0<br>Just obtains 15 and 135 is usually M1A1M0M0A0A0 (It is easy to get this erroneously)<br>Obtains 5, 45, 125 and 165 – usually M1A0M1M1A0A0<br>Obtains 25, 65, 145, (185) usually M1A0M1M1A0A0<br>Working in radians – lose last A1 earned for $\frac{\pi}{12}$, $\frac{11\pi}{36}$, $\frac{3\pi}{4}$ and $\frac{35\pi}{36}$ or numerical equivalents<br>Mixed radians and degrees is usually Method marks only<br>Methods involving no working should be sent to Review | |
| **62 (ii)** | At least one of $(\frac{a\pi}{10}-b)=$ 0 (or $n\pi$)<br>$(\frac{a3\pi}{5}-b)=$ $\pi$ {or $(n+1)\pi$} or in degrees<br>or $(\frac{a11\pi}{10}-b)=$ $2\pi$ {or $(n+2)\pi$} | M1 |
| | If two of **above equations** used eliminates $a$ or $b$ to find one or both of these or uses period property of curve to find $a$<br>or uses other valid method to find either $a$ or $b$ ( May see $\frac{5\pi}{10}a=\pi$ so $a$ = ) | M1 |
| | Obtains $a$ = 2 | A1 |
| | Obtains $b=\frac{\pi}{5}$ (must be in radians) | A1 (4) |

| | |
|---|---|
| Notes | M1: Award for $(\frac{a\pi}{10} - b) = 0$ or $\frac{a\pi}{10} = b$ BUT $\sin(\frac{a\pi}{10} - b) = 0$ is M0<br>M1: As described above but solving $(\frac{a\pi}{10} - b) = 0$ with $(\frac{a3\pi}{5} - b) = 0$ is M0 (It gives $a = b = 0$)<br>Special cases:<br>Can obtain full marks here for both correct answers with no working M1M1A1A1<br>For $a$ = 2 only, with no working, award M0M1A1A0 For $b = \frac{\pi}{5}$ only with no working M1M0A0A1 |
| Alternative | Some use translations and stretches to give answers.<br>If they achieve $a=2$ they earn second method and first accuracy. If they achieve correct value for $b$ they earn first method and second accuracy.<br>Common error is $a = 2$ and $b = \frac{\pi}{10}$. This is usually M0M1A1A0 unless they have stated $(\frac{a\pi}{10} - b) = 0$ earlier in which case they earn first M1. |

| Question Number | Scheme | Marks |
|---|---|---|
| **63.** (a) | $\frac{1}{2}r^2\theta = \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right) = 6\pi$ or 18.85 or awrt 18.8 (cm)$^2$ — Using $\frac{1}{2}r^2\theta$ (See notes) | M1 |
| | $6\pi$ or 18.85 or awrt 18.8 | A1 |
| | | **[2]** |
| (b) | $\sin\left(\frac{\pi}{6}\right) = \frac{r}{6-r}$ — $\sin\left(\frac{\pi}{6}\right)$ or $\sin 30^\circ = \frac{r}{6-r}$ | M1 |
| | $\frac{1}{2} = \frac{r}{6-r}$ — Replaces sin by numeric value | dM1 |
| | $6 - r = 2r \Rightarrow r = 2$ — $r = 2$ | A1 **cso** |
| | | **[3]** |
| (c) | Area $= 6\pi - \pi(2)^2 = 2\pi$ or awrt 6.3 (cm)$^2$ — their area of sector $- \pi r^2$ | M1 |
| | $2\pi$ or awrt 6.3 | A1 **cao** |
| | | **[2]** |
| | | **7** |

**(a)** M1: Needs $\theta$ in radians for this formula.
Candidate could convert to degrees and use the degrees formula.
A1: Does not need units. Answer should be either $6\pi$ or 18.85 or awrt 18.8
Correct answer with no working is M1A1.
This M1A1 can only be awarded in part (a).

**(b)** M1: Also allow $\cos\left(\frac{\pi}{3}\right)$ or $\cos 60^\circ = \frac{r}{6-r}$.

1st M1: Needs correct trigonometry method. Candidates could state $\sin\left(\frac{\pi}{6}\right) = \frac{r}{x}$ and $x + r = 6$ or equivalent in their working to gain this method mark.

dM1: Replaces sin by numerical value. $0.009... = \frac{r}{6-r}$ from working "incorrectly" in degrees is fine here for dM1.
A1: For $r = 2$ from correct solution only.
<u>**Alternative:**</u> 1st M1 for $\frac{r}{OC} = \sin 30$ or $\frac{r}{OC} = \cos 60$. 2nd M1 for $OC = 2r$ and then A1 for $r = 2$.
<u>**Note**</u> seeing $OC = 2r$ is M1M1.
<u>**Special Case:**</u> If a candidate states an answer of $r = 2$ (must be in part (b)) as a guess or from an incorrect method then award SC: M0M0B1. Such a candidate could then go on to score M1A1 in part (c).

**(c)** M1: For "their area of sector – their area of circle", where $r > 0$ is ft from their answer to part (b).
Allow the method mark if "their area of sector" < "their area of circle". The candidate must show somewhere in their working that they are subtracting the correct way round, even if their answer is negative.
Some candidates in part (c) will either use their value of $r$ from part (b) or even introduce a value of $r$ in part (c). You can apply the scheme to award either M0A0 or M1A0 or M1A1 to these candidates.
<u>**Note:**</u> Candidates can get M1 by writing "their part (a) answer $- \pi r^2$", where the radius of the circle is not substituted.
A1: cao – accept exact answer or awrt 6.3
Correct answer only with no working in (c) gets M1A1
<u>**Beware: The answer in (c) is the same as the arc length of the pendant**</u>

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **64.** | (a) $3\sin(x+45^\circ)=2$; $0\le x<360^\circ$ (b) $2\sin^2 x+2=7\cos x$; $0\le x<2\pi$ | | |
| (a) | $\sin(x+45^\circ)=\frac{2}{3}$, so $(x+45^\circ)=41.8103...$ $(\alpha=41.8103...)$ | $\sin^{-1}\left(\frac{2}{3}\right)$ or awrt 41.8 or awrt $0.73^c$ | M1 |
| | So, $x+45^\circ=\{138.1897...,\ 401.8103...\}$ | $x+45^\circ$ = either "180 − their $\alpha$" or "$360^\circ$ + their $\alpha$" ($\alpha$ could be in radians). | M1 |
| | and $x=\{93.1897...,\ 356.8103...\}$ | Either awrt $93.2^\circ$ or awrt $356.8^\circ$ | A1 |
| | | Both awrt $93.2^\circ$ and awrt $356.8^\circ$ | A1 |
| | | | **[4]** |
| (b) | $2(1-\cos^2 x)+2=7\cos x$ | Applies $\sin^2 x=1-\cos^2 x$ | M1 |
| | $2\cos^2 x+7\cos x-4=0$ | Correct 3 term, $2\cos^2 x+7\cos x-4\ \{=0\}$ | A1 **oe** |
| | $(2\cos x-1)(\cos x+4)\ \{=0\}$, $\cos x=...$ | Valid attempt at solving and $\cos x=...$ | M1 |
| | $\cos x=\frac{1}{2}$, $\{\cos x=-4\}$ | $\cos x=\frac{1}{2}$ (See notes.) | A1 cso |
| | $\left(\beta=\frac{\pi}{3}\right)$ | | |
| | $x=\frac{\pi}{3}$ or $1.04719...^c$ | Either $\frac{\pi}{3}$ or awrt $1.05^c$ | B1 |
| | $x=\frac{5\pi}{3}$ or $5.23598...^c$ | Either $\frac{5\pi}{3}$ or awrt $5.24^c$ or $2\pi$ − their $\beta$ (See notes.) | B1 ft |
| | | | **[6]** |
| | | | **10** |



EXPERT TUITION

| Question Number | Notes | Marks |
|---|---|---|
| (a) | 1st M1: can also be implied for $x = \text{awrt } -3.2$<br>2nd M1: for $x + 45^{\circ} =$ either $"180 - \text{their } \alpha"$ or $"360^{\circ} + \text{their } \alpha"$. This can be implied by later working. The candidate's $\alpha$ could also be in radians.<br>**Note that this mark is not** for $x =$ either $"180 - \text{their } \alpha"$ or $"360^{\circ} + \text{their } \alpha"$.<br>**Note:** Imply the first two method marks or award M1M1A1 for either awrt $93.2^{\circ}$ or awrt $356.8^{\circ}$.<br>**Note:** Candidates who apply the following incorrect working of $3\sin(x + 45^{\circ}) = 2$<br>$\Rightarrow 3(\sin x + \sin 45) = 2$, etc will usually score M0M0A0A0.<br>If there are any EXTRA solutions inside the range $0 \le x < 360$ and the candidate would otherwise score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question).<br>Also ignore EXTRA solutions outside the range $0 \le x < 360$.<br>**Working in Radians:** Note the answers in radians are $x = \text{awrt } 1.6, \text{ awrt } 6.2$<br>If a candidate works in radians then mark part (a) as above awarding the A marks in the same way.<br>If the candidate would then score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question.)<br>**No working:** Award M1M1A1A0 for one of awrt $93.2^{\circ}$ or awrt $356.8^{\circ}$ seen without any working.<br>Award M1M1A1A1 for both awrt $93.2^{\circ}$ and awrt $356.8^{\circ}$ seen without any working.<br>Allow benefit of the doubt (FULL MARKS) for final answer of<br>$\sin x \{\text{and not } x\} = \{\text{awrt } 93.2, \text{ awrt } 356.8\}$ | |

| Question Number | Notes | Marks |
|---|---|---|
| (b) | 1st M1: for a correct method to use $\sin^2 x = 1 - \cos^2 x$ on the given equation.<br>Give bod if the candidate omits the bracket when substituting for $\sin^2 x$, but $2 - \cos^2 x + 2 = 7\cos x$, without supporting working, (eg. seeing "$\sin^2 x = 1 - \cos^2 x$") would score 1st M0.<br>Note that applying $\sin^2 x = \cos^2 x - 1$, scores M0.<br>1st A1: for obtaining either $2\cos^2 x + 7\cos x - 4$ or $-2\cos^2 x - 7\cos x + 4$.<br>1st A1: can also awarded for a correct three term equation eg. $2\cos^2 x + 7\cos x = 4$ or $2\cos^2 x = 4 - 7\cos x$ etc.<br>2nd M1: for a valid attempt at factorisation of a quadratic (either 2TQ or 3TQ) in cos, can use any variable here, $c$, $y$, $x$ or $\cos x$, and an attempt to find at least one of the solutions. See introduction to the Mark Scheme. *Alternatively*, using a correct formula for solving the quadratic. Either the formula must be stated correctly or the correct form must be implied by the substitution.<br>2nd A1: for $\cos x = \frac{1}{2}$, BY A CORRECT SOLUTION ONLY UP TO THIS POINT. Ignore extra answer of $\cos x = -4$, but penalise if candidate states an incorrect result e.g. $\cos x = 4$. If they have used a substitution, a correct value of their $c$ or their $y$ or their $x$.<br>**Note:** 2nd A1 for $\cos x = \frac{1}{2}$ can be implied by later working.<br>1st B1: for either $\frac{\pi}{3}$ or awrt $1.05^c$<br>2nd B1: for either $\frac{5\pi}{3}$ or awrt $5.24^c$ or can be ft from $2\pi$ – their $\beta$ or $360^\circ$ – their $\beta$ where $\beta = \cos^{-1}(k)$, such that $0 < k < 1$ or $-1 < k < 0$, but $k \neq 0$, $k \neq 1$ or $k \neq -1$.<br>If there are any EXTRA solutions inside the range $0 \leq x < 2\pi$ and the candidate would otherwise score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question).<br>Also ignore EXTRA solutions outside the range $0 \leq x < 2\pi$.<br>**Working in Degrees:** Note the answers in degrees are $x = 60,\ 300$<br>If a candidate works in degrees then mark part (b) as above awarding the B marks in the same way. If the candidate would then score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question.)<br>**Answers from no working:**<br>$x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$ scores M0A0M0A0B1B1,<br>$x = 60$ and $x = 300$ scores M0A0M0A0B1B0,<br>$x = \frac{\pi}{3}$ ONLY or $x = 60$ ONLY scores M0A0M0A0B1B0,<br>$x = \frac{5\pi}{3}$ ONLY or $x = 120$ ONLY scores M0A0M0A0B0B1.<br>**No working:** You cannot apply the ft in the B1ft if the answers are given with NO working.<br>Eg: $x = \frac{\pi}{5}$ and $x = \frac{9\pi}{3}$ FROM NO WORKING scores M0A0M0A0B0B0.<br>**For candidates using trial & improvement, please forward these to your Team Leader.** | |

| Question Number | Scheme | Marks |
|---|---|---|
| **65.** **(a)** | $11^2 = 8^2 + 7^2 - \left(2 \times 8 \times 7\cos C\right)$<br>$\cos C = \dfrac{8^2 + 7^2 - 11^2}{2 \times 8 \times 7}$ (or equivalent)<br>$\left\{\hat{C} = 1.64228...\right\} \Rightarrow \hat{C} = \text{awrt } 1.64$ | M1<br>A1<br>A1 **cso**<br>(3) |
| **(b)** | Use of Area $\Delta ABC = \dfrac{1}{2}ab\sin(\text{their } C)$, where $a, b$ are any of 7, 8 or 11.<br>$= \dfrac{1}{2}\left(7 \times 8\right)\sin C$ using the value of their $C$ from part (a).<br>$\{= 27.92848... \text{ or } 27.93297...\} = \text{awrt } 27.9$ (from angle of either $1.64^c$ or $94.1^\circ$) | M1<br>A1 ft<br>A1 **cso**<br>(3)<br>**[6]** |
| | **<u>Notes</u>** | |
| **(a)** | M1 is also scored for $8^2 = 7^2 + 11^2 - \left(2 \times 7 \times 11\cos C\right)$ or $7^2 = 8^2 + 11^2 - \left(2 \times 8 \times 11\cos C\right)$<br>or $\cos C = \dfrac{7^2 + 11^2 - 8^2}{2 \times 7 \times 11}$ or $\cos C = \dfrac{8^2 + 11^2 - 7^2}{2 \times 8 \times 11}$<br>1st A1: Rearranged correctly to make $\cos C = ...$ and numerically correct (possibly unsimplified). Award A1 for any of $\cos C = \dfrac{8^2 + 7^2 - 11^2}{2 \times 8 \times 7}$ or $\cos C = \dfrac{-8}{112}$ or $\cos C = -\dfrac{1}{14}$ or $\cos C = \text{awrt } -0.071$.<br>SC: Also allow 1st A1 for $112\cos C = -8$ or equivalent.<br>Also note that the 1st A1 can be implied for $\hat{C} = \text{awrt } 1.64$ or $\hat{C} = \text{awrt } 94.1^\circ$.<br>**Special Case:** $\cos C = \dfrac{1}{14}$ or $\cos C = \dfrac{11^2 - 8^2 - 7^2}{2 \times 8 \times 7}$ scores a SC: M1A0A0.<br>2nd A1: for awrt 1.64 **cao**<br>**Note that** $A = 0.6876...^c \left(\text{or } 39.401...^\circ\right)$, $B = 0.8116...^c \left(\text{or } 46.503...^\circ\right)$ | |
| **(b)** | M1: alternative methods must be fully correct to score the M1.<br>For any (or both) of the M1 or the 1st A1; their $C$ can either be in degrees or radians.<br>Candidates who use $\cos C = \dfrac{1}{14}$ to give $C = 1.499...$, can achieve the correct answer of awrt 27.9 in part (b). These candidates will score M1A1A0cso, in part (b).<br>Finding $C = 1.499...$ in part (a) and achieving awrt 27.9 with no working scores M1A1A0.<br>Otherwise with no working in part (b), awrt 27.9 scores M1A1A1.<br>**Special Case: If the candidate gives awrt 27.9 from any of the below then award M1A1A1.**<br>$\dfrac{1}{2}\left(7 \times 11\right)\sin(0.8116^c \text{ or } 46.503^\circ) = \text{awrt } 27.9$, $\dfrac{1}{2}\left(8 \times 11\right)\sin(0.6876...^c \text{ or } 39.401...^\circ) = \text{awrt } 27.9$.<br>**<u>Alternative: Hero's Formula:</u>** $A = \sqrt{13(13-11)(13-8)(13-7)} = \text{awrt } 27.9$, where M1 is attempt to apply $A = \sqrt{s(s-11)(s-8)(s-7)}$ and the first A1 is for the correct application of the formula. | |

| Question Number | Scheme | | Marks |
| --- | --- | --- | --- |
| **66.** (a) | $3\sin^2 x + 7\sin x = \cos^2 x - 4$; $0 \le x < 360^\circ$ | | |
| | $3\sin^2 x + 7\sin x = (1 - \sin^2 x) - 4$ | | M1 |
| | $4\sin^2 x + 7\sin x + 3 = 0$ **AG** | | A1 * **cso** (2) |
| (b) | $(4\sin x + 3)(\sin x + 1)\{= 0\}$ | Valid attempt at factorisation and $\sin x = ...$ | M1 |
| | $\sin x = -\frac{3}{4}$, $\sin x = -1$ | Both $\sin x = -\frac{3}{4}$ and $\sin x = -1$. | A1 |
| | $(\lvert\alpha\rvert = 48.59...)$ | | |
| | $x = 180 + 48.59$ or $x = 360 - 48.59$ | Either $(180 + \lvert\alpha\rvert)$ or $(360 - \lvert\alpha\rvert)$ | dM1 |
| | $x = 228.59...$, $x = 311.41...$ | Both awrt 228.6 and awrt 311.4 | A1 |
| | $\{\sin x = -1\} \Rightarrow x = 270$ | 270 | B1 (5) **[7]** |
| | **Notes** | | |
| (a) | M1 for a correct method to change $\cos^2 x$ into $\sin^2 x$ (must use $\cos^2 x = 1 - \sin^2 x$).<br>Note that applying $\cos^2 x = \sin^2 x - 1$, scores M0.<br>A1 for obtaining the printed answer without error (except for implied use of zero.), although the equation at the end of the proof **must be = 0.** Solution **just** written only as above would score M1A1. | | |
| (b) | 1st M1 for a valid attempt at factorisation, can use any variable here, *s*, *y*, *x* or $\sin x$, and an attempt to find at least one of the solutions.<br>*Alternatively*, using a correct formula for solving the quadratic. Either the formula must be stated correctly or the correct form must be implied by the substitution.<br>1st A1 for the two correct values of $\sin x$. If they have used a substitution, a correct value of their *s* or their *y* or their *x*.<br>2nd M1 for solving $\sin x = -k$, $0 < k < 1$ and realising a solution is either of the form $(180 + \lvert\alpha\rvert)$ or $(360 - \lvert\alpha\rvert)$ where $\alpha = \sin^{-1}(k)$. Note that you **cannot** access this mark from $\sin x = -1 \Rightarrow x = 270$. Note that this mark is dependent upon the 1st M1 mark awarded.<br>2nd A1 for both awrt 228.6 and awrt 311.4<br>B1 for 270.<br>If there are any EXTRA solutions inside the range $0 \le x < 360^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final bA2 mark (the fourth mark in this part of the question).<br>Also ignore EXTRA solutions outside the range $0 \le x < 360^\circ$.<br>**Working in Radians:** Note the answers in radians are $x = 3.9896...,\ 5.4351...,\ 4.7123...$<br>If a candidate works in radians then mark part (b) as above awarding the 2nd A1 for both awrt 4.0 and awrt 5.4 and the B1 for awrt 4.7 or $\frac{3\pi}{2}$. If the candidate would then score FULL MARKS then withhold the final bA2 mark (the fourth mark in this part of the question.)<br>**No working:** Award B1 for 270 seen without any working.<br>Award M0A0M1A1 for awrt 228.6 and awrt 311.4 seen without any working.<br>Award M0A0M1A0 for any one of awrt 228.6 or awrt 311.4 seen without any working. | | |

| Question Number | Scheme | Marks |
|---|---|---|
| **67** | (a) $\tan\theta = \frac{2}{5}$ (or 0.4) (i.s.w. if a value of $\theta$ is subsequently found)<br>Requires the correct value with no incorrect working seen. | B1 (1) |
| | (b) awrt $21.8$ ($\alpha$)<br>(Also allow awrt 68.2, ft from $\tan\theta = \frac{5}{2}$ in (a), but no other ft)<br>(This value must be seen in part (b). It may be implied by a correct solution, e.g. 10.9) | B1 |
| | $180+\alpha$ $(=201.8)$, or $90+(\alpha/2)$ (if division by 2 has already occurred)<br>($\alpha$ found from $\tan 2x = ...$ or $\tan x = ...$ or $\sin 2x = \pm...$ or $\cos 2x = \pm...$) | M1 |
| | $360+\alpha$ $(=381.8)$, or $180+(\alpha/2)$<br>($\alpha$ found from $\tan 2x = ...$ or $\sin 2x = ...$ or $\cos 2x = ...$)<br>OR $540+\alpha$ $(=561.8)$, or $270+(\alpha/2)$<br>($\alpha$ found from $\tan 2x = ...$) | M1 |
| | Dividing at least one of the angles by 2<br>($\alpha$ found from $\tan 2x = ...$ or $\sin 2x = ...$ or $\cos 2x = ...$) | M1 |
| | $x = 10.9,\ 100.9,\ 190.9,\ 280.9$ (Allow awrt) | A1 (5)<br>**6** |

(b) Extra solution(s) in range: Loses the final A mark.

Extra solutions outside range: Ignore (whether correct or not).

Common answers:

10.9 and 100.9 would score B1 M1 M0 M1 A0 (Ensure that these M marks are awarded)

10.9 and 190.9 would score B1 M0 M1 M1 A0 (Ensure that these M marks are awarded)

Alternatives:

(i) $2\cos 2x - 5\sin 2x = 0$ $\quad R\cos(2x+\lambda) = 0$ $\quad \lambda = 68.2 \Rightarrow 2x + 68.2 = 90$ B1

$2x+\lambda = 270$ M1

$2x+\lambda = 450$ or $2x+\lambda = 630$ M1

Subtracting $\lambda$ and dividing by 2 (at least once) M1

(ii) $25\sin^2 2x = 4\cos^2 2x = 4(1-\sin^2 2x)$

$29\sin^2 2x = 4$ $\quad 2x = 21.8$ B1

The M marks are scored as in the main scheme, but extra solutions will be likely, losing the A mark.

Using radians:

B1: Can be given for awrt 0.38 ($\beta$)

M1: For $\pi+\beta$ or $180+\beta$

M1: For $2\pi+\beta$ or $3\pi+\beta$ (Must now be consistently radians)

M1: For dividing at least one of the angles by 2

A1: For this mark, the answers must be in degrees.

(Correct) answers only (or by graphical methods):

B and M marks can be awarded by implication, e.g.

10.9 scores B1 M0 M0 M1 A0

10.9, 100.9 scores B1 M1 M0 M1 A0

10.9, 100.9, 190.9, 280.9 scores full marks.

Using 11, etc. instead of 10.9 can still score the M marks by implication.

| Question Number | Scheme | Marks |
|---|---|---|
| **68** | (a) $r\theta = 9\times 0.7 = 6.3$ (Also allow 6.30, or awrt 6.30) | M1 A1<br>(2) |
| | (b) $\frac{1}{2}r^2\theta = \frac{1}{2}\times 81\times 0.7 = 28.35$<br>(Also allow 28.3 or 28.4, or awrt 28.3 or 28.4)<br>(Condone $28.35^2$ written instead of $28.35\,\text{cm}^2$) | M1 A1<br>(2) |
| | (c) $\tan 0.7 = \frac{AC}{9}$<br>$AC = 7.58$ (Allow awrt) <u>NOT</u> 7.59 (see below) | M1<br>A1<br>(2) |
| | (d) Area of triangle $AOC$ = $\frac{1}{2}(9\times \text{their } AC)$ (or other complete method)<br>Area of $R$ = "34.11" – "28.35" (triangle – sector) or (sector – triangle)<br>(needs a <u>value</u> for each)<br>= 5.76 (Allow awrt) | M1<br>M1<br><br>A1<br>(3)<br>**9** |
| | (a) M: Use of $r\theta$ (with $\theta$ in radians), or equivalent (could be working in degrees with a correct degrees formula).<br><br>(b) M: Use of $\frac{1}{2}r^2\theta$ (with $\theta$ in radians), or equivalent (could be working in degrees with a correct degrees formula).<br><br>(c) M: Other methods must be fully correct,<br>e.g. $\frac{AC}{\sin 0.7} = \frac{9}{\sin\left(\frac{\pi}{2} - 0.7\right)}$<br>$(\pi - 0.7)$ instead of $\left(\frac{\pi}{2} - 0.7\right)$ here is <u>not</u> a fully correct method.<br><br><u>Premature approximation (e.g. taking angle $C$ as 0.87 radians)</u>:<br>This will often result in loss of A marks, e.g. $AC$ = 7.59 in (c) is A0. | |

| Question Number | Scheme | Marks |
|---|---|---|
| 69 (a) | $5\sin x = 1 + 2\left(1 - \sin^2 x\right)$ | M1 |
| | $2\sin^2 x + 5\sin x - 3 = 0$ **(*)** | A1cso (2) |
| (b) | $(2s-1)(s+3) = 0$ **giving** $s$ = | M1 |
| | $[\sin x = -3 \text{ has no solution}]$ **so** $\sin x = \frac{1}{2}$ | A1 |
| | $\therefore \quad x = 30,\ 150$ | B1, B1ft (4) **[6]** |

(a)

M1 for a correct method to change $\cos^2 x$ into $\sin^2 x$ (must use $\cos^2 x = 1 - \sin^2 x$)

A1 need 3 term quadratic printed in any order with =0 included

(b)

M1 for attempt to solve given quadratic (usual rules for solving quadratics) (can use any variable here, *s*, *y*, *x*, or sin*x* )

A1 requires no incorrect work seen and is for $\sin x = \frac{1}{2}$ **or** $x = \sin^{-1}\frac{1}{2}$

$y = \frac{1}{2}$ is A0 (unless followed by *x* = 30)

B1 for 30 ($\alpha$) not dependent on method

2nd B1 for 180 - $\alpha$ provided in required range (otherwise 540 - $\alpha$)

Extra solutions outside required range: Ignore

Extra solutions inside required range: Lose final B1

Answers in radians: Lose final B1

S.C. Merely writes down two correct answers is M0A0B1B1

Or $\sin x = \frac{1}{2} \quad \therefore \quad x = 30,\ 150$ **is M1A1B1B1**

Just gives one answer : 30 only is M0A0B1B0 or 150 only is M0A0B0B1

**NB** Common error is to factorise wrongly giving $(2\sin x + 1)(\sin x - 3) = 0$

$[\sin x = 3 \text{ gives no solution}]\ \sin x = -\frac{1}{2} \quad \Rightarrow \quad x = 210,\ 330$

This earns M1 A0 B0 B1ft

Another common error is to factorise correctly $(2\sin x - 1)(\sin x + 3) = 0$ and follow this with $\sin x = \frac{1}{2}$, $\sin x = 3$ then $x = 30^\circ, 150^\circ$

This would be M1 A0 B1 B1

| Question Number | Scheme | Marks |
|---|---|---|
| 70 (a) | **Either** $\frac{\sin(A\hat{C}B)}{5} = \frac{\sin 0.6}{4}$<br>$\therefore\ A\hat{C}B = \arcsin(0.7058...)$<br>$= [0.7835..\ \text{ or }\ 2.358]$<br>Use angles of triangle<br>$A\hat{B}C = \pi - 0.6 - A\hat{C}B$<br>(But as $AC$ is the longest side so)<br>$A\hat{B}C = 1.76$ **(*)**(3sf) [Allow $100.7^\circ \to 1.76$]<br>In degrees $0.6 = 34.377^\circ$, $\text{A}\hat{\text{C}}\text{B} = 44.9^\circ$<br><br>**or** $4^2 = b^2 + 5^2 - 2\times b\times 5\cos 0.6$<br>$\therefore b = \frac{10\cos 0.6 \pm \sqrt{(100\cos^2 0.6 - 36)}}{2}$<br>$= [6.96\ \text{ or }\ 1.29]$<br>Use sine / cosine rule with value for $b$<br>$\sin B = \frac{\sin 0.6}{4}\times b$ or $\cos B = \frac{25+16-b^2}{40}$<br>(But as $AC$ is the longest side so)<br>$A\hat{B}C = 1.76$ **(*)**(3sf) | M1<br>M1<br>M1,<br>A1<br>(4) |
| (b) | $\left\|C\hat{B}D = \pi - 1.76 = 1.38\right\|$ Sector area = $\frac{1}{2}\times 4^2\times(\pi - 1.76) = [11.0 \sim 11.1]$ $\frac{1}{2}\times 4^2\times 79.3$ is M0<br>Area of $\Delta ABC = \frac{1}{2}\times 5\times 4\times\sin(1.76) = [9.8]$ or $\frac{1}{2}\times 5\times 4\times\sin 101$<br>Required area = awrt 20.8 or 20.9 or 21.0 or gives 21 (2sf) after correct work. | M1<br>M1<br>A1 (3)<br>**[7]** |
| (a) | 1st M1 for correct use of sine rule to find $ACB$ or cosine rule to find $b$ (M0 for ABC here or for use of sin x where $x$ could be $ABC$)<br>2nd M1 for a correct expression for angle $ACB$ (This mark may be implied by .7835 or by arcsin (.7058)) and needs accuracy. In second method this M1 is for correct expression for $b$ – may be implied by 6.96. [Note $10\cos 0.6 \approx 8.3$ ] (do not need two answers)<br>3rd M1 for a correct method to get angle $ABC$ in method (i) or sin$ABC$ or cos$ABC$ , in method (ii) (If sin $B$ >1, can have M1A0)<br>A1cso for correct work leading to 1.76 3sf . Do not need to see angle 0.1835 considered and rejected. | |
| (b) | 1st M1 for a correct expression for sector area or a value in the range 11.0 – 11.1<br>2nd M1 for a correct expression for the area of the triangle or a value of 9.8<br>Ignore 0.31 (working in degrees) as subsequent work.<br>A1 for answers which round to 20.8 or 20.9 or 21.0. No need to see units. | |
| (a) | <u>Special case</u> If answer 1.76 is assumed then usual mark is M0 M0 M0 A0. A Fully checked method may be worth M1 M1 M0 A0. A maximum of 2 marks. The mark is either 2 or 0.<br><u>Either</u> **M1** for $A\hat{C}B$ is found to be 0,7816 (angles of triangle) then<br>**M1** for checking $\frac{\sin(A\hat{C}B)}{5} = \frac{\sin 0.6}{4}$ with conclusion giving numerical answers<br>**This gives a maximum mark of 2/4**<br><u>OR</u> **M1** for $b$ is found to be 6.97 (cosine rule)<br>**M1** for checking $\frac{\sin(ABC)}{b} = \frac{\sin 0.6}{4}$ with conclusion giving numerical answers<br>**This gives a maximum mark of 2/4**<br>Candidates making this assumption need a complete method. They cannot earn M1M0.<br>So the score will be 0 or 2 for part (a). Circular arguments earn 0/4. | |


EXPERT TUITION

| Question Number | Scheme | Marks |
| --- | --- | --- |
| 71 (i) | $\tan\theta = -1 \Rightarrow \quad \theta = -45, \quad 135$ | B1, B1ft |
| | $\sin\theta = \frac{2}{5} \Rightarrow \quad \theta = 23.6, \quad 156.4$ (AWRT: 24, 156) | B1, B1ft (4) |
| (ii) | $4\sin x = \dfrac{3\sin x}{\cos x}$ | M1 |
| | $4\sin x\cos x = 3\sin x \quad \Rightarrow \quad \sin x(4\cos x - 3) = 0$<br>Other possibilities (after squaring): $\sin^2 x(16\sin^2 x - 7) = 0$,<br>$(16\cos^2 x - 9)(\cos^2 x - 1) = 0$ | M1 |
| | $x = 0$, 180 <u>seen</u> | B1, B1 |
| | $x = 41.4$, 318.6 (AWRT: 41, 319) | B1, B1ft (6) |
| | | **[10]** |
| (i) | 1st B1 for $-45$ seen ($\alpha$, where $\lvert\alpha\rvert < 90$)<br>2nd B1 for 135 seen, <u>or ft</u> (180 + $\alpha$) if $\alpha$ is negative, or ($\alpha$ – 180) if $\alpha$ is positive.<br>If $\tan\theta = k$ is obtained from <u>wrong working</u>, 2nd B1ft is still available.<br>3rd B1 for awrt 24 ($\beta$, where $\lvert\beta\rvert < 90$)<br>4th B1 for awrt 156, <u>or ft</u> (180 – $\beta$) if $\beta$ is positive, or – (180 + $\beta$) if $\beta$ is negative.<br>If $\sin\theta = k$ is obtained from <u>wrong working</u>, 4th B1ft is still available. | |
| (ii) | 1st M1 for use of $\tan x = \dfrac{\sin x}{\cos x}$. Condone $\dfrac{3\sin x}{3\cos x}$.<br>2nd M1 for correct work leading to 2 factors (may be implied).<br>1st B1 for 0, 2nd B1 for 180.<br>3rd B1 for awrt 41 ($\gamma$, where $\lvert\gamma\rvert < 180$)<br>4th B1 for awrt 319, <u>or ft</u> (360 – $\gamma$).<br>If $\cos\theta = k$ is obtained from <u>wrong working</u>, 4th B1ft is still available.<br>N.B. Losing $\sin x = 0$ usually gives a maximum of 3 marks M1M0B0B0B1B1<br><u>Alternative:</u> (squaring both sides)<br>1st M1 for squaring both sides and using a 'quadratic' identity.<br>e.g. $16\sin^2\theta = 9(\sec^2\theta - 1)$<br>2nd M1 for reaching a factorised form.<br>e.g. $(16\cos^2\theta - 9)(\cos^2\theta - 1) = 0$<br>Then marks are equivalent to the main scheme. Extra solutions, if not rejected, are penalised as in the main scheme.<br><br><u>For both parts of the question</u>:<br><u>Extra solutions outside required range</u>: Ignore<br><u>Extra solutions inside required range</u>:<br>For each <u>pair</u> of B marks, the 2nd B mark is lost if there are two correct values and one or more extra solution(s), e.g. $\tan\theta = -1 \Rightarrow \theta = 45, -45, 135$ is B1 B0<br><u>Answers in radians</u>:<br>Loses a maximum of 2 B marks in the whole question (to be deducted at the first and second occurrence). | |

| Question Number | Scheme | Marks |
|---|---|---|
| **72** (a) | $\frac{1}{2}r^2\theta = \frac{1}{2}\times 6^2 \times 2.2 = 39.6 \quad (\text{cm}^2)$ | M1 A1 (2) |
| (b) | $\left(\frac{2\pi - 2.2}{2} =\right) \pi - 1.1 = 2.04 \quad (\text{rad})$ | M1 A1 (2) |
| | (c) $\Delta DAC = \frac{1}{2}\times 6 \times 4\sin 2.04 \qquad (\approx 10.7)$ | M1 A1ft |
| | Total area = sector + 2 triangles = 61 $\quad (\text{cm}^2)$ | M1 A1 (4) **[8]** |

| | |
|---|---|
| (a) | **M1:** Needs $\theta$ in radians for this formula. Could convert to degrees and use degrees formula.<br>**A1:** Does not need units. Answer should be 39.6 exactly.<br>Answer with no working is **M1 A1**.<br>This **M1A1** can only be awarded in part (a). |
| (b) | **M1:** Needs full method to give angle in radians<br>**A1:** Allow answers which round to 2.04 (Just writes 2.04 – no working is 2/2) |
| (c) | **M1:** Use $\frac{1}{2}\times 6\times 4\sin A$ (if any other triangle formula e.g. $\frac{1}{2}b\times h$ is used the method must be complete for this mark) (No value needed for $A$, but should not be using 2.2)<br>**A1:** ft the value obtained in part (b) – need not be evaluated- could be in degrees<br>**M1:** Uses Total area = sector + 2 triangles or other complete method<br>**A1:** Allow answers which round to 61. (Do not need units)<br><br>Special case degrees: Could get M0A0, M0A0, M1A1M1A0<br>Special case: Use $\Delta$ *BDC* – $\Delta$ *BAC* Both areas needed for first **M1**<br>Total area = sector + area found is second **M1**<br>**NB** Just finding lengths BD, DC, and angle BDC then assuming area BDC is a sector to find area BDC is 0/4 |

| Question Number | Scheme | Marks |
|---|---|---|
| **73** (a) | $4(1-\cos^2 x)+9\cos x-6=0$ $\quad$ $4\cos^2 x-9\cos x+2=0$ (*) | M1 A1 (2) |
| (b) | $(4\cos x-1)(\cos x-2)=0$ $\quad$ $\cos x=...,$ $\quad \dfrac{1}{4}$ | M1 A1 |
| | $x=75.5$ $\quad (\alpha)$ | B1 |
| | $360-\alpha,$ $\quad$ $360+\alpha$ or $720$ - $\alpha$ | M1, M1 |
| | 284.5, 435.5, 644.5 | A1 (6) **[8]** |
| (a) | **M1:** Uses $\sin^2 x=1-\cos^2 x$ (may omit bracket) **not** $\sin^2 x=\cos^2 x-1$<br>**A1:** Obtains the printed answer without error – **must have = 0** | |
| (b) | **M1:** Solves the quadratic with usual conventions<br>**A1:** Obtains ¼ accurately- ignore extra answer 2 but penalise e.g. -2.<br>**B1:** allow answers which round to 75.5<br>**M1:** $360-\alpha$ ft their value, **M1:** $360+\alpha$ ft their value or 720 - $\alpha$ ft<br>**A1:** Three **and only three** correct exact answers in the range achieves the mark | |
| Special cases | In part (b) Error in solving quadratic (4cosx-1)(cosx+2)<br>Could yield, **M1A0B1M1M1A1** losing one mark for the error<br><br>Works in radians:<br>Complete work in radians :Obtains 1.3 **B0**. Then allow **M1 M1** for $2\pi-\alpha$, $2\pi+\alpha$ or $4\pi-\alpha$ Then gets 5.0, 7.6, 11.3 **A0 so 2/4**<br>Mixed answer 1.3, 360 – 1.3, 360 + 1.3, 720 – 1.3 still gets **B0M1M1A0** | |

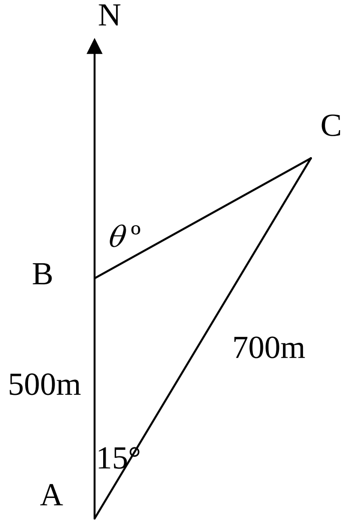
| Question number | Scheme | Marks |
|---|---|---|
| **74.** | (a) $r\theta = 7 \times 0.8 = 5.6$ (cm)<br>(b) $\frac{1}{2}r^2\theta = \frac{1}{2} \times 7^2 \times 0.8 = 19.6$ $(\text{cm}^2)$<br>(c) $BD^2 = 7^2 + (\text{their } AD)^2 - \left(2 \times 7 \times (\text{their } AD) \times \cos 0.8\right)$<br>$BD^2 = 7^2 + 3.5^2 - \left(2 \times 7 \times 3.5 \times \cos 0.8\right)$ (or awrt 46° for the angle)<br>($BD$ = 5.21)<br>Perimeter = (their $DC$) + "5.6" + "5.21" = 14.3 (cm) (Accept awrt)<br>(d) $\Delta ABD = \frac{1}{2} \times 7 \times (\text{their } AD) \times \sin 0.8$ (or awrt 46° for the angle) (ft their $AD$)<br>(= 8.78…)<br>(If the correct formula $\frac{1}{2}ab\sin C$ is <u>quoted</u> the use of any two of the sides of $\Delta ABD$ as $a$ and $b$ scores the M mark).<br>Area = "19.6" – "8.78…" = 10.8 $(\text{cm}^2)$ (Accept awrt) | M1 A1 (2)<br>M1 A1 (2)<br>M1<br>A1<br><br>M1 A1 (4)<br>M1 A1ft<br><br><br>M1 A1 (4)<br>**12** |
| | Units (cm or $\text{cm}^2$) are not required in any of the answers.<br>(a) and (b): Correct answers without working score both marks.<br>(a) M: Use of $r\theta$ (with $\theta$ in radians), or equivalent (could be working in degrees with a correct degrees formula).<br>(b) M: Use of $\frac{1}{2}r^2\theta$ (with $\theta$ in radians), or equivalent (could be working in degrees with a correct degrees formula).<br>(c) 1st M: Use of the (correct) cosine rule formula to find $BD^2$ or $BD$.<br>Any other methods need to be complete methods to find $BD^2$ or $BD$.<br>2nd M: Adding their $DC$ to their arc $BC$ and their $BD$.<br><u>Beware</u>: If 0.8 is used, but calculator is in degree mode, this can still earn M1 A1 (for the required expression), but this gives $BD$ = 3.50… so the perimeter may appear as 3.5 + 5.6 + 3.5 (earning M1 A0).<br>(d) 1st M: Use of the (correct) area formula to find $\Delta ABD$.<br>Any other methods need to be complete methods to find $\Delta ABD$.<br>2nd M: Subtracting their $\Delta ABD$ from their sector $ABC$.<br>Using segment formula $\frac{1}{2}r^2(\theta - \sin\theta)$ scores no marks in part (d). | |

| Question number | Scheme | Marks |
|---|---|---|
| **75.** | (a) 45 ($\alpha$) (This mark can be implied by an answer 65)<br>$180-\alpha$, Add 20 (for at least one angle)<br>65 155 | B1<br>M1, M1<br>A1 (4) |
| | (b) 120 or 240 ($\beta$): (This mark can be implied by an answer 40 or 80)<br>(Could be achieved by working with 60, using $180-60$ and/or $180+60$)<br>$360-\beta$, $360+\beta$ (or 120 + an angle that has been divided by 3)<br>Dividing by 3 (for at least one angle)<br>40 80 160 200 280 320 First A1: at least 3 correct | B1<br><br>M1, M1<br>M1<br>A1 A1 (6)<br>**10** |
| | (a) Extra solution(s) in range: Loses the A mark.<br>Extra solutions outside range: Ignore (whether correct or not).<br>Common solutions:<br>65 (only correct solution) will score B1 M0 M1 A0 (2 marks)<br>65 and 115 will score B1 M0 M1 A0 (2 marks)<br>44.99 (or similar) for $\alpha$ is B0, and 64.99, 155.01 (or similar) is A0.<br>(b) Extra solution(s) in range: Loses the final A mark.<br>Extra solutions outside range: Ignore (whether correct or not).<br>Common solutions:<br>40 (only correct solution) will score B1 M0 M0 M1 A0 A0 (2 marks)<br>40 and 80 (only correct solutions) B1 M1 M0 M1 A0 A0 (3 marks)<br>40 and 320 (only correct solutions) B1 M0 M0 M1 A0 A0 (2 marks)<br>Answers without working:<br>Full marks can be given (in both parts), B and M marks by implication.<br>Answers given in radians:<br>Deduct a maximum of 2 marks (misread) from B and A marks. (Deduct these at first and second occurrence.)<br>Answers that begin with statements such as $\sin(x-20)=\sin x-\sin 20$ or $\cos x=-\frac{1}{6}$, then go on to find a value of '$\alpha$' or '$\beta$', however badly, can continue to earn the first M mark in either part, but will score no further marks.<br>Possible misread: $\cos 3x=\frac{1}{2}$, giving 20, 100, 140, 220, 260, 340<br>Could score up to 4 marks B0 M1 M1 M1 A0 A1 for the above answers. | |

| | | | |
|---|---|---|---|
| 76. (a) | $3\sin^2\theta - 2\cos^2\theta = 1$ | | |
| | $3\sin^2\theta - 2(1 - \sin^2\theta) = 1$ (M1: Use of $\sin^2\theta + \cos^2\theta = 1$) | M1 | |
| | $3\sin^2\theta - 2 + 2\sin^2\theta = 1$ | | |
| | $5\sin^2\theta = 3$ cso **AG** | A1 | (2) |
| (b) | $\sin^2\theta = \frac{3}{5}$, so $\sin\theta = (\pm)\sqrt{0.6}$ | M1 | |
| | Attempt to solve both $\sin\theta = +..$ and $\sin\theta = -$ (may be implied by later work) | M1 | |
| | $\theta = 50.7685°$ awrt $\theta = 50.8°$ (dependent on first M1 only) | A1 | |
| | $\theta$ $(= 180° - 50.7685_c°)$; $= 129.23...°$ awrt $129.2°$ | M1; A1 √ | |
| | [f.t. dependent on first M and 3rd M] | | |
| | $\sin\theta = -\sqrt{0.6}$ | | |
| | $\theta = 230.785°$ and $309.23152°$ awrt $230.8°$, $309.2°$ (both) | M1A1 | (7) |
| | | | **[9]** |

| | |
|---|---|
| Notes: | (a) N.B: **AG**; need to see at least one line of working after substituting $\cos^2\theta$. |
| | (b) First M1: Using $5\sin^2\theta = 3$ to find value for $\sin\theta$ or $\theta$ |
| | Second M1: Considering the – value for $\sin\theta$. (usually later) |
| | First A1: Given for awrt 50.8°. Not dependent on second M. |
| | Third M1: For $(180 - 50.8_c)°$, need not see written down |
| | Final M1: **Dependent** on second M (but may be implied by answers) |
| | For (180 + candidate's 50.8)° **or** $(360 - 50.8_c)°$ **or equiv.** |
| | Final A1: Requires both values. (no follow through) |
| | [ Finds $\cos^2\theta = k$ ($k$ = 2/5) and so $\cos\theta = (\pm)$...M1, then mark equivalently] |

77.




| | | |
|---|---|---|
| (a) | $BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700 \cos 15°$ <br> $( = 63851.92\ldots)$ <br> $BC = 253$ awrt | M1 A1 <br><br> A1 (3) |
| (b) | $\dfrac{\sin B}{700} = \dfrac{\sin 15}{\text{candidate's } BC}$ | M1 |
| | $\sin B = \sin 15 \times 700 / 253_c = 0.716..$ and giving an **obtuse** $B$ (134.2°) dep | M1 |
| | $\theta = 180° -$ candidate's angle $B$ (Dep. on first M only, B can be acute) | M1 |
| | $\theta = 180 - 134.2 = (0)45.8$ (allow 46 or awrt 45.7, 45.8, 45.9) <br> [46 needs to be from correct working] | A1 (4) **[7]** |

Notes:

(a) If use cos 15° = ....., then A1 not scored until written as $BC^2 = \ldots$ correctly

*Splitting into 2 triangles BAX and CAX, where X is foot of perp. from B to AC*
Finding value for $BX$ and $CX$ and using Pythagoras M1
$BC^2 = (500\sin 15°)^2 + (700 - 500\cos 15°)^2$ A1
$BC = 253$ awrt A1

(b) Several alternative methods: (Showing the M marks, 3rd M dep. on first M))

(i) $\cos B = \dfrac{500^2 + \text{candidate's}BC^2 - 700^2}{2\text{x}500\text{xcandidate's}BC}$ or $700^2 = 500^2 + BC_c^{\ 2} - 2x500xBC_c$ M1
Finding angle $B$ M1, then M1 as above

(ii) 2 triangle approach, as defined in notes for (a)
$\tan CBX = \dfrac{700 - valueforAX}{valueforBX}$ M1
Finding value for $\angle CBX$ ($\approx 59°$) M1
$\theta = [180° - (75° + candidate's\ \angle CBX)]$ M1

(iii) Using sine rule (or cos rule) to find $C$ first:
Correct use of sine or cos rule for C M1, Finding value for C M1
Either $B = 180° - (15° + \text{candidate's } C)$ or $\theta = (15° + \text{candidate's } C)$ M1

(iv) $700\cos 15° = 500 + BC\cos\theta$ M2 {first two Ms earned in this case}
Solving for $\theta$; $\theta = 45.8$ (allow 46 or5.7, 45.8, 45.9 M1;A1

| Question Number | Scheme | Marks |
|---|---|---|
| **78(i)** | $\frac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5\Rightarrow \tan(2x+32°)=5$ | B1 |
| | $\Rightarrow x=\frac{\arctan 5-32°}{2}$ | M1 |
| | $\Rightarrow x=\text{awrt } 23.35°,-66.65°$ | A1A1 |
| | | **(4)** |
| **(ii)(a)** | $\tan(3\theta-45°)=\frac{\tan 3\theta-\tan 45°}{1+\tan 45°\tan 3\theta}=\frac{\tan 3\theta-1}{1+\tan 3\theta}$ | M1A1* |
| | | **(2)** |
| **(b)** | $(1+\tan 3\theta)\tan(\theta+28°)=\tan 3\theta-1$ | |
| | $\Rightarrow \tan(\theta+28°)=\tan(3\theta-45°)$ | B1 |
| | $\theta+28°=3\theta-45°\Rightarrow\theta=36.5°$ | M1A1 |
| | $\theta+28°+180°=3\theta-45°\Rightarrow\theta=126.5°$ | dM1A1 |
| | | **(5)** |
| | | **(11 marks)** |
| **78(i) ALT 1** | $\frac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5\Rightarrow \tan 2x=\frac{5-\tan 32°}{1+5\tan 32°}=awrt 1.06$ | B1 |
| | $\Rightarrow x=\frac{\arctan\left(\frac{5-\tan 32°}{1+5\tan 32°}\right)}{2}$ | M1 |
| | $\Rightarrow x=23.35°,-66.65°$ | A1A1 |
| | | **(4)** |
| **78(ii) ALT 2** | $\frac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5\Rightarrow\frac{2\tan x}{1-\tan^2 x}+\tan 32°=5-5\times\frac{2\tan x}{1-\tan^2 x}\tan 32°$ | |
| | $\Rightarrow(5-\tan 32°)\tan^2 x+(2+10\tan 32°)\tan x+\tan 32°-5=0$ | |
| | OR $\Rightarrow awrt\ 4.38\tan^2 x+8.25\tan x-4.38=0$ | B1 |
| | Quadratic formula $\Rightarrow\tan x=0.4316,-2.3169\Rightarrow x=..$ | M1 |
| | $\Rightarrow x=23.35°,-66.65°$ | A1 A1 |
| | | **(4)** |

(i)

**B1:** Stating or implying by subsequent work $\tan(2x+32°)=5$

**M1:** Scored for the correct order of operations from $\tan(2x\pm 32°)=5$ to $x=..$ $\quad x=\frac{\arctan 5\pm 32°}{2}$

This may be implied by one correct answer

**A1:** One of awrt $x=23.3/23.4°,-66.6/-66.7°$ **One dp accuracy required for this penultimate mark.**

**A1:** Both of $x=\text{awrt } 23.35°,-66.65°$ and no other solutions in the range $-90°<x<90°$

Using Alt I
B1: $\tan 2x = \text{awrt}1.06$
M1: For attempting to make $\tan 2x$ the subject followed by correct inverse operations to find a value for $x$ from their $\tan 2x = k$
If they write down $\tan(2x+32°) = 5$ and then the answers that is fine for all 4 marks.
Answers mixing degrees and radians can only score the first B1

(ii)(a)
**M1:** States or implies (just rhs) $\tan(3\theta - 45°) = \dfrac{\tan 3\theta \pm \tan 45°}{1 \pm \tan 45° \tan 3\theta}$

**A1*:** Complete proof with the lhs, the correct identity $\dfrac{\tan 3\theta - \tan 45°}{1 + \tan 45° \tan 3\theta}$ and either stating that $\tan 45° = 1$ or substituting $\tan 45° = 1$ (which may only be seen on the numerator) and proceeding to given answer.
It is possible to work backwards here $\dfrac{\tan 3\theta - 1}{1 + \tan 3\theta} = \dfrac{\tan 3\theta - \tan 45°}{1 + \tan 45° \tan 3\theta} = \tan(3\theta - 45°)$ with M1 A1 scored at the end. Do not allow the final A1* if there are errors.

(ii)(b)
**B1:** Uses (ii)(a) to state or imply that $\tan(\theta + 28°) = \tan(3\theta - 45°)$
Allow this mark for $(1 + \tan 3\theta)\tan(\theta + 28°) = (1 + \tan 3\theta)\tan(3\theta - 45°)$

**M1:** $\theta + 28° = 3\theta - 45° \Rightarrow \theta = ..$
**We have seen two incorrect methods that should be given M0.**
$\tan(\theta + 28°) = \tan(3\theta - 45°) \Rightarrow \tan(3\theta - 45°) - \tan(\theta + 28°) = 0 \Rightarrow (3\theta - 45°) - (\theta + 28°) = 0 \Rightarrow \theta = ...$
and $\tan 3\theta - \tan 45° = \tan \theta + \tan 28° \Rightarrow 3\theta - 45° = \theta + 28° \Rightarrow \theta = ...$

**A1:** $\theta = 36.5°$ oe such as $\dfrac{73}{2}$

**dM1:** A correct method of finding a 2nd solution $\theta + 28° + 180° = 3\theta - 45° \Rightarrow \theta = ..$ The previous M must have been awarded. The method may be implied by their $\theta_1 + 90°$ but only if the previous M was scored.
It is an incorrect method to substitute the acute angle into one side of $\tan(\theta + 28°) = \tan(3\theta - 45°)$
Eg. $\tan(36.5 + 28°) = \tan(3\theta - 45°)$ and use trig to find another solution.

**A1:** $\theta = 36.5°, 126.5°$ oe and no other solutions in the range.

The questions states 'hence' so the minimum expected working is $\tan(\theta + 28°) = \tan(3\theta - 45°)$. Full marks can be awarded when this point is reached.
(ii) (b) Alternative solution using compound angles.

……………………………………………………………………………………………………..

(ii) (b) Alternative solution using compound angles.
From the B1 mark, $\tan(\theta + 28°) = \tan(3\theta - 45°)$ they proceed to

$$\frac{\sin(\theta + 28°)}{\cos(\theta + 28°)} = \frac{\sin(3\theta - 45°)}{\cos(3\theta - 45°)} \Rightarrow \sin((3\theta - 45°) - (\theta + 28°)) = 0 \text{ via the compound angle identity}$$

So, M1 is gained for an attempt at one value for $\sin(2\theta - 73°) = 0$, condoning slips andA1 for $\theta = 36.5°$

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| Question Number | Scheme | Marks |
|---|---|---|
| **79.(a)** | $R=\sqrt{5}$ | B1 |
| | $\tan\alpha=2\Rightarrow\alpha=\text{awrt }1.107$ | M1A1 |
| | | (3) |
| **(b)(i)** | $'40+9R^2'=85$ | M1A1 |
| **(ii)** | $\theta=\frac{\pi}{2}+1.107\Rightarrow\theta=\text{awrt }2.68$ | B1ft |
| | | (3) |
| **(c )(i)** | 6 | B1 |
| **(ii)** | $2\theta-'1.107'=3\pi\Rightarrow\theta=\text{awrt }5.27$ | M1A1 |
| | | (3) |
| | | **( 9 marks)** |

(a)
**B1:** Accept $R=\sqrt{5}$ **Do not accept** $R=\pm\sqrt{5}$
**M1:** For sight of $\tan\alpha=\pm 2, \tan\alpha=\pm\frac{1}{2}$. Condone $\sin\alpha=2,\cos\alpha=1\Rightarrow\tan\alpha=\frac{2}{1}$

If $R$ is found first, accept $\sin\alpha=\pm\frac{2}{R}, \cos\alpha=\pm\frac{1}{R}$

**A1:** $\alpha=\text{awrt }1.107$. The degrees equivalent $63.4°$ is A0.

(b)(i)
**M1:** Attempts $'40+9R^2'$ OR $'40+3R^2'$ using their $R$.
Can be scored for sight of the statement $'40+9R^2'$

It can be done via calculus. The M mark will probably be awarded when $\left("\alpha"-\frac{\pi}{2}\right)=-0.464$ is substituted into $M(\theta)$

**A1:** 85 exactly. Without any method this scores both marks. Do not accept awrt 85.

(b)(ii)
**B1ft:** For awrt 2.68 or $\left(\frac{\pi}{2}+"\alpha"\right)$ A simple way would be to add 1.57 to their $\alpha$ to 2dp

Accept awrt $153.4°$ for candidates who work in degrees. Follow through in degrees on $90°+'\alpha'$

(c)(i)
**B1:** 6
(c)(ii)
**M1:** Using $2\theta\pm'1.107'=n\pi$ where $n$ is a positive integer leading to a value for $\theta$
In degrees for $2\theta\pm\text{their}'63.43'=180n$ where $n$ is a positive integer leading to a value for $\theta$

Another alternative is to solve $\tan 2\theta=2$ so score for $\frac{180n+\arctan 2}{2}$ or $\frac{\pi n+\arctan 2}{2}$

**A1:** $\theta=\text{awrt }5.27$ or if candidate works in degrees awrt $301.7°$

| Question Number | Scheme | Marks |
|---|---|---|
| **80.(a)** | $R=\sqrt{29}$ | B1 |
| | $\tan\alpha=\dfrac{2}{5}\Rightarrow\alpha=\text{awrt }0.381$ | M1A1 |
| | | (3) |
| **(b)** | $5\cot 2x-3\operatorname{cosec}2x=2\Rightarrow 5\dfrac{\cos 2x}{\sin 2x}-\dfrac{3}{\sin 2x}=2$ | M1 |
| | $\Rightarrow 5\cos 2x-2\sin 2x=3$ | A1 |
| | | (2) |
| **(c)** | $5\cos 2x-2\sin 2x=3\Rightarrow\cos(2x+0.381)=\dfrac{3}{\sqrt{29}}$ | M1 |
| | $2x+0.381=\arccos\left(\dfrac{3}{\sqrt{29}}\right)\Rightarrow x=...$ | dM1 |
| | $x=\text{awrt }0.30,\ 2.46$ | A1A1 |
| | | (4) |
| | | **(9 marks)** |
| **Alt I (c)** | $5\cos 2x-2\sin 2x=3\Rightarrow 10\cos^2 x-5-4\sin x\cos x=3$ | |
| | $\Rightarrow 4\tan^2 x+2\tan x-1=0$ | M1 |
| | $\Rightarrow\tan x=\dfrac{-1\pm\sqrt{5}}{4}\Rightarrow x=..$ | dM1 |
| | $x=\text{awrt }0.30,\ 2.46$ | A1A1 |
| | | **(4)** |
| **Alt II (c)** | $5\cos 2x-2\sin 2x=3\Rightarrow(5\cos 2x)^2=(3+2\sin 2x)^2$ **&** $\cos^2 2x=1-\sin^2 2x$ | |
| | $\Rightarrow 29\sin^2 2x+12\sin 2x-16=0$ | M1 |
| | $\Rightarrow\sin 2x=\dfrac{-12\pm\sqrt{2000}}{58}\Rightarrow 2x=..\Rightarrow x=..$ | dM1 |
| | $x=\text{awrt }0.30,\ 2.46$ | A1A1 |
| | | **(4)** |

(a)

B1 $R=\sqrt{29}$

Condone $R=\pm\sqrt{29}$ (Do not allow decimals for this mark Eg 5.39 but remember to isw after $\sqrt{29}$ )

M1 $\tan\alpha=\pm\dfrac{2}{5},\ \tan\alpha=\pm\dfrac{5}{2}\Rightarrow\alpha=...$

If $R$ is used to find $\alpha$ accept $\sin\alpha=\pm\dfrac{2}{R}$ or $\cos\alpha=\pm\dfrac{5}{R}\Rightarrow\alpha=...$

A1 $\alpha = \text{awrt } 0.381$ Note that the degree equivalent $\alpha = \text{awrt } 21.8°$ is A0

(b)

M1 Replaces $\cot 2x$ by $\frac{\cos 2x}{\sin 2x}$ **and** $\operatorname{cosec} 2x$ by $\frac{1}{\sin 2x}$ in the lhs

Do not be concerned by the coefficients 5 and -3.

Replacing $\cot 2x$ by $\frac{1}{\tan 2x}$ does not score marks until the $\tan 2x$ has been replaced by $\frac{\sin 2x}{\cos 2x}$

They may state $\times \sin 2x \Rightarrow 5\cos 2x - 3 = 2\sin 2x$ which implies this mark

A1 cso $5\cos 2x - 2\sin 2x = 3$ There is no need to state the value of '$c$'

The notation must be correct. They cannot mix variables within their equation

Do not accept for the final A1 $\tan 2x = \frac{\sin}{\cos} 2x$ within their equations

(c)

M1 Attempts to use part (a) and (b). They must be using their $R$ and $\alpha$ from part (a) and their $c$ from part (b)

Accept $\cos(2x \pm '\alpha') = \frac{'c'}{'R'}$ Condone $\cos(\theta \pm '\alpha') = \frac{'c'}{'R'}$ or even $\cos(x \pm '\alpha') = \frac{'c'}{'R'}$ for the first M

dM1 Score for dealing with the cos, the $\alpha$ and the 2 **correctly** and in that order to reach $x = ..$

Don't be concerned if they change the variable in the question and solve for $\theta =$ (as long as all operations have been undone).You may not see any working. It is implied by one correct answer.

You may need to check with a calculator.

Eg for an incorrect $\alpha$ $\cos(2x + 1.19) = \frac{3}{\sqrt{29}} \Rightarrow x = -0.105$ would score M1 dM1 A0 A0

A1 One solution correct, usually $x = 0.3/0.30$ or $x = 2.46$ or in degrees $17.2°$ **or** $141.(0)°$

A1 Both solutions correct awrt $x = \text{awrt } 0.30, 2.46$ and no extra values in the range.

Condone candidates who write 0.3 and 2.46 without any (more accurate) answers

In degrees accept awrt 1 dp $17.2°, 141.(0)°$ and no extra values in the range.

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Special case: For candidates who are misreading the question and using their part (a) with 2 on the rhs.

They will be allowed to score a maximum of SC M1 dM1 A0 A0

M1 Attempts to use part (a) with 2. They must be using their $R$ and $\alpha$ from part (a)

Accept $\cos(2x \pm '\alpha') = \frac{2}{'R'}$ Condone $\cos(\theta \pm '\alpha') = \frac{2}{'R'}$ or even $\cos(x \pm '\alpha') = \frac{2}{'R'}$ for the first M

dM1 Score for dealing with the cos, the $\alpha$ and the 2 **correctly** and in that order to reach $x = ..$

You may not see any working. It is implied by one correct answer. You may need to check with a calculator.

Eg for an correct $\alpha$ and $R$ $\cos(2x + 0.381) = \frac{2}{\sqrt{29}} \Rightarrow x = 0.405$

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Alt to part (c)

M1 Attempts both double angle formulae condoning sign slips on $\cos 2x$, divides by $\cos^2 x$ and forms a quadratic in tan by using the identity $\pm 1 \pm \tan^2 x = \sec^2 x$

dM1 Attempts to solve their quadratic in $\tan x$ leading to a solution for $x$.

A1 A1 As above

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| Question Number | Scheme | Marks |
| --- | --- | --- |
| **81(a)** | $\sin 2x - \tan x = 2\sin x\cos x - \tan x$ | M1 |
| | $= \dfrac{2\sin x\cos^2 x}{\cos x} - \dfrac{\sin x}{\cos x}$ | M1 |
| | $= \dfrac{\sin x}{\cos x} \times (2\cos^2 x - 1)$ | |
| | $= \tan x \cos 2x$ | dM1 A1* |
| | | **(4)** |
| **(b)** | $\tan x\cos 2x = 3\tan x\sin x \Rightarrow \tan x(\cos 2x - 3\sin x) = 0$ | |
| | $\cos 2x - 3\sin x = 0$ | M1 |
| | $\Rightarrow 1 - 2\sin^2 x - 3\sin x = 0$ | M1 |
| | $\Rightarrow 2\sin^2 x + 3\sin x - 1 = 0 \Rightarrow \sin x = \dfrac{-3 \pm \sqrt{17}}{4} \Rightarrow x = ...$ | M1 |
| | Two of $x = 16.3°, 163.7°, 0, 180°$ | A1 |
| | All four of $x = 16.3°, 163.7°, 0, 180°$ | A1 |
| | | **(5)** |
| | | **(9 marks)** |

(a)

M1 Uses a correct double angle identity involving $\sin 2x$ Accept $\sin(x+x) = \sin x\cos x + \cos x\sin x$

M1 **Uses** $\tan x = \dfrac{\sin x}{\cos x}$ with $\sin 2x = 2\sin x\cos x$ and attempts to combine the two terms using a common denominator. This can be awarded on two separate terms with a common denominator.
Alternatively uses $\sin x = \tan x\cos x$ and attempts to combine two terms using factorisation of tan $x$

dM1 Both M's must have been scored. Uses a correct double angle identity involving $\cos 2x$ .

A1* A fully correct solution with no errors or omissions. All notation must be correct and variables must be consistent
Withhold this mark if for instance they write $\tan x = \dfrac{\sin}{\cos}$

If the candidate $\times \cos x$ on line 1 and/or $\div \sin x$ they cannot score any more than one mark unless they are working with both sides of the equation or it is fully explained.

(b)

M1 The $\tan x$ must be cancelled or factorised out to produce $\cos 2x - 3\sin x = 0$ or $\dfrac{\cos 2x}{\sin x} = 3$ oe Condone slips

M1 Uses $\cos 2x = 1 - 2\sin^2 x$ to form a 3TQ=0 in $\sin x$ The = 0 may be implied by later work

M1 Uses the formula/completion of square or GC with invsin to produce at least one value for $x$
It may be implied by one correct value.
This mark **can** be scored from factorisation of their 3TQ in $\sin x$ **but only if** their quadratic factorises.

A1 Two of $x = 0, 180°, \text{awrt } 16.3°, \text{awrt } 163.7°$ or in radians two of awrt 0.28, 2.86, 0 and $\pi$ or 3.14
This mark can be awarded as a SC for those students who just produce $0, 180°$ ( or 0 and $\pi$) from $\tan x = 0$ or $\sin x = 0$.

A1 All four values in degrees $x = 0, 180°, \text{awrt } 16.3°, \text{awrt } 163.7°$ and no extra's inside the range $0 \leqslant x < 360°$.
Condone $0 = 0.0$ and $180° = 180.0°$ Ignore any roots outside range.

Alternatives to parts (a) and (b)

| | | |
|---|---|---|
| **(a) Alt 1** | $\tan x\cos 2x = \tan x\left(2\cos^2 x - 1\right)$ | M1 |
| | $= 2\tan x\cos^2 x - \tan x$ | |
| | $= 2\dfrac{\sin x}{\cos x}\cos^2 x - \tan x$ | M1 |
| | $= 2\sin x\cos x - \tan x$ | |
| | $= \sin 2x - \tan x$ | dM1 A1 |
| | | **(4)** |

a) Alt 1 Starting from the rhs

M1 Uses a correct double angle identity for $\cos 2x$. Accept any correct version including $\cos(x+x) = \cos x\cos x - \sin x\sin x$

M1 **Uses** $\tan x = \dfrac{\sin x}{\cos x}$ with $\cos 2x = 2\cos^2 x - 1$ and attempts to multiply out the bracket

dM1 Both M's must have been scored. It is for using $2\sin x\cos x =\ \sin 2x$

A1* A fully correct solution with no errors or omissions. All notation must be correct and variables must be consistent. See Main scheme for how to deal with candidates who $\div\tan x$

| | | |
|---|---|---|
| **(a) Alt 2** | $\sin 2x - \tan x \equiv \tan x\cos 2x$ | |
| | $2\sin x\cos x - \tan x \equiv \tan x(2\cos^2 x - 1)$ | M1 |
| | $2\sin x\cos x - \cancel{\tan x} \equiv 2\tan x\cos^2 x - \cancel{\tan x}$ | |
| | $2\sin x\cos x \equiv 2\dfrac{\sin x}{\cos x}\cos^2 x$ | M1 |
| | $2\sin x\cos x \equiv 2\sin x\cos x$ | dM1 |
| | +statement that it must be true | A1* |

a) Alt 2 Candidates who use both sides

M1 Uses a correct double angle identity involving $\sin 2x$ **or** $\cos 2x$. Can be scored from either side
Accept $\sin(x+x) = \sin x\cos x + \cos x\sin x$ or $\cos(x+x) = \cos x\cos x - \sin x\sin x$

M1 **Uses** $\tan x = \dfrac{\sin x}{\cos x}$ with $\cos 2x = 2\cos^2 x - 1$ and cancels the $\tan x$ term from both sides

dM1 Uses a correct double angle identity involving $\sin 2x$ Both previous M's must have been scored

A1* A fully correct solution with no errors or omissions AND statement "hence true", "a tick", "QED". W[5]
All notation must be correct and variables must be consistent

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It is possible to solve part (b) without using the given identity. There are various ways of doing this, one of which is shown below.

$$\sin 2x - \tan x = 3\tan x\sin x \Rightarrow\ 2\sin x\cos x - \frac{\sin x}{\cos x} = 3\frac{\sin x}{\cos x}\sin x$$

$2\sin x\cos^2 x - \sin x = 3\sin^2 x$ — M1 Equation in $\sin x$ and $\cos x$

$2\sin x\left(1-\sin^2 x\right) - \sin x = 3\sin^2 x$ — M1 Equation in $\sin x$ only

$\left(2\sin^2 x + 3\sin x - 1\right)\sin x = 0$

$x = ..$ — M1 Solving equation to find at least one $x$

Two of $x = 16.3°, 163.7°, 0, 180°$ — A1

All four of $x = 16.3°, 163.7°, 0, 180°$ and no extras A1

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| Question | Scheme | Marks |
|---|---|---|
| **82.(a)** | $R=\sqrt{5}$ | B1 |
| | $\tan\alpha=\dfrac{1}{2}\Rightarrow\alpha=26.57°$ | M1A1 |
| | | **(3)** |
| **(b)** | $\dfrac{2}{2\cos\theta-\sin\theta-1}=15\Rightarrow\dfrac{2}{\sqrt{5}\cos(\theta+26.6°)-1}=15$ | |
| | $\Rightarrow\cos(\theta+26.6°)=\dfrac{17}{15\sqrt{5}}=(awrt\ 0.507)$ | M1A1 |
| | $\theta+26.57°=59.54°$ | |
| | $\Rightarrow\theta=awrt\ 33.0°$ or $awrt\ 273.9°$ | A1 |
| | $\theta+26.6°=360°-\text{their }'59.5°'$ | dM1 |
| | $\Rightarrow\theta=awrt\ 273.9°$ and $awrt\ 33.0°$ | A1 |
| | | **(5)** |
| **(c)** | $\theta-\text{their }26.57°=\text{their }59.54°\Rightarrow\theta=...$ | M1 |
| | $\theta=\text{awrt }86.1°$ | A1 |
| | | **(2)** |
| | | **(10 marks)** |

(a)

B1 $R=\sqrt{5}$. Condone $R=\pm\sqrt{5}$ Ignore decimals

M1 $\tan\alpha=\pm\dfrac{1}{2},\ \tan\alpha=\pm\dfrac{2}{1}\Rightarrow\alpha=...$

If their value of $R$ is used to find the value of $\alpha$ only accept $\cos\alpha=\pm\dfrac{2}{R}$ OR $\sin\alpha=\pm\dfrac{1}{R}\Rightarrow\alpha=...$

A1 $\alpha=\text{awrt }26.57°$

(b)

M1 Attempts to use part (a) $\Rightarrow\cos(\theta\pm\text{their }26.6°)=K,\ |K|\leq 1$

A1 $\cos(\theta\pm\text{their }26.6°)=\dfrac{17}{15\sqrt{5}}=(\text{awrt }0.507)$. Can be implied by $(\theta\pm\text{their }26.6°)=\text{awrt }59.5°/59.6°$

A1 One solution correct, $\theta=awrt\ 33.0°$ or $\theta=awrt\ 273.9°$ Do not accept 33 for 33.0.

dM1 Obtains a second solution in the range. It is dependent upon having scored the previous M.
Usually for $\theta\pm\text{their }26.6°=360°-\text{their }59.5°\Rightarrow\theta=...$

A1 Both solutions $\theta=awrt\ 33.0°$ and $awrt\ 273.9°$. Do not accept 33 for 33.0.
Extra solutions inside the range withhold this A1. Ignore solutions outside the range $0\leq\theta<360°$

(c)

M1 $\theta-\text{their }26.57°=\text{their }59.54°\Rightarrow\theta=...$
Alternatively $-\theta+\text{their }26.6°=-\text{their }59.5°\Rightarrow\theta=...$
If the candidate has an incorrect sign for $\alpha$, for example they used $\cos(\theta-26.57°)$ in part (b) it would be scored for $\theta+\text{their }26.57°=\text{their }59.54°\Rightarrow\theta=...$

A1 awrt $86.1°$ ONLY. Allow both marks following a correct (a) and (b)
They can restart the question to achieve this result. Do not award if 86.1 was their smallest answer in (b). This occurs when they have $\cos(\theta-26.57°)$ instead of $\cos(\theta+26.57°)$ in (b)

Answers in radians: Withhold only one A mark, the first time a solution in radians appears

FYI (a) $\alpha=0.46$ (b) $\theta_1=\text{awrt }0.58$ and $\theta_2=\text{awrt }4.78$ (c) $\theta_3=\text{awrt }1.50$. Require 2 dp accuracy

| Question | Scheme | Marks |
|---|---|---|
| **83 (a)** | $2\cot 2x+\tan x\equiv\dfrac{2}{\tan 2x}+\tan x$ | B1 |
| | $\equiv\dfrac{(1-\tan^2 x)}{\tan x}+\dfrac{\tan^2 x}{\tan x}$ | M1 |
| | $\equiv\dfrac{1}{\tan x}$ | M1 |
| | $\equiv\cot x$ | A1* |
| | | **(4)** |
| **(b)** | $6\cot 2x+3\tan x=\operatorname{cosec}^2 x-2\Rightarrow 3\cot x=\operatorname{cosec}^2 x-2$ | |
| | $\Rightarrow 3\cot x=1+\cot^2 x-2$ | M1 |
| | $\Rightarrow 0=\cot^2 x-3\cot x-1$ | A1 |
| | $\Rightarrow \cot x=\dfrac{3\pm\sqrt{13}}{2}$ | M1 |
| | $\Rightarrow \tan x=\dfrac{2}{3\pm\sqrt{13}}\Rightarrow x=..$ | M1 |
| | $\Rightarrow x=0.294,\,-2.848,\,-1.277\,,1.865$ | A2,1,0 |
| | | **(6)** |
| | | **(10 marks)** |
| **83 (a)alt 1** | $2\cot 2x+\tan x\equiv\dfrac{2\cos 2x}{\sin 2x}+\tan x$ | B1 |
| | $\equiv 2\dfrac{\cos^2 x-\sin^2 x}{2\sin x\cos x}+\dfrac{\sin x}{\cos x}$ | M1 |
| | $\equiv\dfrac{\cos^2 x-\sin^2 x}{\sin x\cos x}+\dfrac{\sin^2 x}{\sin x\cos x}\equiv\dfrac{\cos^2 x}{\sin x\cos x}$ | M1 |
| | $\equiv\dfrac{\cos x}{\sin x}$ | |
| | $\equiv\cot x$ | A1* |
| **83 (a)alt 2** | $2\cot 2x+\tan x\equiv 2\dfrac{(1-\tan^2 x)}{2\tan x}+\tan x$ | B1M1 |
| | $\equiv\dfrac{2}{2\tan x}-\dfrac{2\tan^2 x}{2\tan x}+\tan x$ or $\dfrac{(1-\tan^2 x)+\tan^2 x}{\tan x}$ | |
| | $\equiv\dfrac{2}{2\tan x}=\cot x$ | M1A1* |
| **Alt (b)** | $6\cot 2x+3\tan x=\operatorname{cosec}^2 x-2\Rightarrow\dfrac{3\cos x}{\sin x}=\dfrac{1}{\sin^2 x}-2$ | |
| | $\left(\times\sin^2 x\right)\Rightarrow 3\sin x\cos x=1-2\sin^2 x$ | M1 |
| | $\Rightarrow\dfrac{3}{2}\sin 2x=\cos 2x$ | M1A1 |
| | $\Rightarrow\tan 2x=\dfrac{2}{3}\Rightarrow x=..$ | M1 |
| | $\Rightarrow x=0.294,\,-2.848,\,-1.277\,,1.865$ | A2,1,0 |
| | | **(6)** |

(a)

B1 States or uses the identity $2\cot 2x = \dfrac{2}{\tan 2x}$ or alternatively $2\cot 2x = \dfrac{2\cos 2x}{\sin 2x}$

This may be implied by $2\cot 2x = \dfrac{1-\tan^2 x}{\tan x}$. Note $2\cot 2x = \dfrac{1}{2\tan 2x}$ is B0

M1 Uses the correct double angle identity $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

Alternatively uses $\sin 2x = 2\sin x\cos x$, $\cos 2x = \cos^2 x - \sin^2 x$ oe and $\tan x = \dfrac{\sin x}{\cos x}$

M1 Writes their two terms with a single common denominator and simplifies to a form $\dfrac{ab}{cd}$ .

For this to be scored the expression must be in either sin *x* and cos *x* or just tan *x*.

In alternative 2 it is for splitting the complex fraction into parts and simplifying to a form $\dfrac{ab}{cd}$ .

You are awarding this for a correct method to proceed to terms like $\dfrac{\cos^2 x}{\sin x\cos x}$, $\dfrac{2\cos^3 x}{2\sin x\cos^2 x}$, $\dfrac{2}{2\tan x}$

A1* cso. For proceeding to the correct answer. This is a given answer and all aspects must be correct including the consistent use of variables. If the candidate approaches from both sides there must be a conclusion for this mark to be awarded. Occasionally you may see a candidate attempting to prove $\cot x - \tan x \equiv 2\cot 2x$. This is fine but again there needs to be a conclusion for the A1*
If you are unsure of how some items should be marked then please use review

(b)

M1 For using part (a) and writing $6\cot 2x + 3\tan x$ as $k\cot x$, $k \neq 0$ in their equation (or equivalent)

WITH an attempt at using $\text{cosec}^2 x = \pm 1 \pm \cot^2 x$ to produce a quadratic equation in just $\cot x / \tan x$

A1 $\cot^2 x - 3\cot x - 1 = 0$ The = 0 may be implied by subsequent working

Alternatively accept $\tan^2 x + 3\tan x - 1 = 0$

M1 Solves a 3TQ=0 in $\cot x$ (or tan) using the formula or any suitable method for their quadratic to find at least one solution. Accept answers written down from a calculator. You may have to check these from an incorrect quadratic. FYI answers are $\cot x = \text{awrt } 3.30, \quad -0.30$

Be aware that $\cot x = \dfrac{3 \pm \sqrt{13}}{2} \Rightarrow \tan x = \dfrac{-3 \pm \sqrt{13}}{2}$

M1 For $\tan x = \dfrac{1}{\cot x}$ and using arctan producing at least one answer for $x$ in degrees or radians.

You may have to check these with your calculator.

A1 Two of $x = 0.294, -2.848, -1.277, 1.865$ (awrt 3dp) in radians or degrees.

In degrees the answers you would accept are (awrt 2dp) $x = 16.8°, 106.8°, -73.2°, -163.2°$

A1 All four of $x = 0.294, -2.848, -1.277, 1.865$ (awrt 3 dp) with no extra solutions in the range
$-\pi$ „ xx $\pi$

See main scheme for Alt to (b) using Double Angle formulae still entered M A M M A A in epen

1st M1 For using part (a) and writing $6\cot 2x + 3\tan x$ as $k\cot x$, $k \neq 0$ in their equation (or equivalent)

then using $\cot x = \dfrac{\cos x}{\sin x}$, $\text{cosec}^2 x = \dfrac{1}{\sin^2 x}$ and $\times \sin^2 x$ to form an equation sin and cos

1st A1 For $\dfrac{3}{2}\sin 2x = \cos 2x$ or equivalent. **Attached to the next M**

2nd M1 For using both correct double angle formula

3rd M1 For moving from $\tan 2x = C$ to *x*=..using the correct order of operations.

| Question Number | Scheme | Marks |
|---|---|---|
| **84.(a)** | $\tan 2\theta° = \dfrac{2\tan\theta°}{1-\tan^2\theta°} = \dfrac{2p}{1-p^2}$ Final answer | M1A1 |
| | | **(2)** |
| **(b)** | $\cos\theta° = \dfrac{1}{\sec\theta°} = \dfrac{1}{\sqrt{1+\tan^2\theta°}} = \dfrac{1}{\sqrt{1+p^2}}$ Final answer | M1A1 |
| | | **(2)** |
| **(c)** | $\cot(\theta-45)° = \dfrac{1}{\tan(\theta-45)°} = \dfrac{1+\tan\theta°\tan 45°}{\tan\theta°-\tan 45°} = \dfrac{1+p}{p-1}$ Final answer | M1A1 |
| | | **(2)** **(6 marks)** |

(a)

M1 Attempt to use the double angle formula for tangent followed by the substitution $\tan\theta = p$.

For example accept $\tan 2\theta° = \dfrac{2\tan\theta°}{1\pm\tan^2\theta°} = \dfrac{2p}{1\pm p^2}$

Condone unconventional notation such as $\tan 2\theta° = \dfrac{2\tan\theta°}{1\pm\tan\theta^{2}{}°}$ followed by an attempt to substitute $\tan\theta = p$ for the M mark. Recovery from this notation is allowed for the A1.

Alternatively use $\tan(A+B) = \dfrac{\tan A+\tan B}{1\pm\tan A\tan B}$ with an attempt at substituting $\tan A = \tan B = p$ . The unsimplified answer $\dfrac{p+p}{1-p\times p}$ is evidence

It is possible to use $\tan 2\theta° = \dfrac{\sin 2\theta°}{\cos 2\theta°} = \dfrac{2\sin\theta°\cos\theta°}{2\cos^2\theta°-1} = \dfrac{2\times\dfrac{p}{\sqrt{1\pm p^2}}\times\dfrac{1}{\sqrt{1\pm p^2}}}{2\times\dfrac{1}{1\pm p^2}-1}$ but it is unlikely to succeed.

A1 Correct **simplified** answer of $\tan 2\theta° = \dfrac{2p}{1-p^2}$ or $\dfrac{2p}{(1-p)(1+p)}$.

Do not allow if they "simplify" to $\dfrac{2}{1-p}$

Allow the correct answer for both marks as long as no incorrect working is seen.

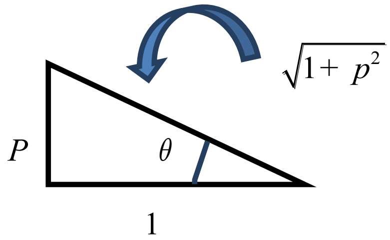
(b)

M1 Attempt to use **both** $\cos\theta = \dfrac{1}{\sec\theta}$ **and** $1 + \tan^2\theta = \sec^2\theta$ with $\tan\theta = p$ in an attempt to obtain an expression for $\cos\theta$ in terms of $p$. Condone a slip in the sign of the second identity.

Evidence would be $\cos^2\theta = \dfrac{1}{\pm 1 \pm p^2}$

Alternatively use a triangle method, attempt Pythagoras' theorem and use $\cos\theta = \dfrac{adj}{hyp}$

The attempt to use Pythagoras must attempt to use the squares of the lengths.




A1 $\cos\theta° = \dfrac{1}{\sqrt{1+p^2}}$ Accept versions such as $\cos\theta° = \sqrt{\dfrac{1}{1+p^2}}$, $\cos\theta° = \pm\dfrac{1}{\sqrt{1+p^2}}$

Withhold this mark if the candidate goes on to write $\cos\theta° = \dfrac{1}{1+p}$

(c)

M1 Use the correct identity $\cot(\theta - 45) = \dfrac{1}{\tan(\theta - 45)}$ and an attempt to use the $\tan(A-B)$ formula with $A$=$\theta$ , $B$=45 and $\tan\theta = p$ .

For example accept an unsimplified answer such as $\dfrac{1}{\dfrac{\tan\theta \pm \tan 45}{1 \pm \tan\theta\tan 45}} = \dfrac{1}{\dfrac{p \pm \tan 45}{1 \pm p\tan 45}}$

It is possible to use $\cot(\theta - 45) = \dfrac{\cos(\theta - 45)}{\sin(\theta - 45)}$ and an attempt to use the formulae for $\sin(A-B)$ and $\cos(A-B)$ with $A$=$\theta$ , $B$=45 . $\sin\theta = \dfrac{p}{\sqrt{1\pm p^2}}$ and $\cos\theta = \dfrac{1}{\sqrt{1\pm p^2}}$

Sight of an expression $\dfrac{\dfrac{1}{\sqrt{1\pm p^2}}\cos 45 \pm \dfrac{p}{\sqrt{1\pm p^2}}\sin 45}{\dfrac{p}{\sqrt{1\pm p^2}}\cos 45 \pm \dfrac{1}{\sqrt{1\pm p^2}}\sin 45}$ is evidence.

A1 Uses $\tan 45 = 1$ or $\sin 45 = \cos 45 = \dfrac{\sqrt{2}}{2}$ $oe$ and simplifies answer.

Accept $-\dfrac{1+p}{1-p}$ or $1 + \dfrac{2}{p-1}$

**Note that there is no isw in any parts of this question.**



EXPERT TUITION

| Question Number | Scheme | Marks |
|---|---|---|
| **85(a)** | $4\cos 2\theta + 2\sin 2\theta = R\cos(2\theta - \alpha)$ | |
| | $R = \sqrt{4^2 + 2^2} = \sqrt{20} = \left(2\sqrt{5}\right)$ | B1 |
| | $\alpha = \arctan\left(\frac{1}{2}\right) = 26.565°... = \text{awrt } 26.57°$ | M1A1 |
| | | **(3)** |
| **(b)** | $\sqrt{20}\cos(2\theta -\ 26.6) = 1 \text{Þ } \cos(2\theta -\ 26.57) = \frac{1}{\sqrt{20}}$ | M1 |
| | $\text{Þ } (2\theta -\ 26.57) = +77.1.. \text{Þ } \theta = ...$ | dM1 |
| | $\theta = \text{ awrt } 51.8°$ | A1 |
| | $2\theta -\ 26.57 = \text{'}-\ 77.1...\text{' Þ } \theta = -\ \text{awrt } 25.3°$ | ddM1A1 |
| | | **(5)** |
| **(c)** | $k < -\ \sqrt{20}, k > \sqrt{20}$ | B1ft either<br>B1ft both |
| | | **(2)** |
| | | **(10 marks)** |

You can marks parts (a) and (b) together as one.

(a)

B1 For $R = \sqrt{20} = 2\sqrt{5}$ . Condone $R = \pm\sqrt{20}$

M1 For $\alpha = \arctan\left(\pm\frac{1}{2}\right)$ or $\alpha = \arctan(\pm 2)$ leading to a solution of $\alpha$

Condone any solutions coming from $\cos\alpha = 4, \sin\alpha = 2$

Condone for this mark $2\alpha = \arctan\left(\pm\frac{1}{2}\right) \Rightarrow \alpha = ..$

If $R$ has been used to find $\alpha$ award for only $\alpha = \text{arcos}\left(\pm\frac{4}{'R'}\right)$ $\alpha = \arcsin\left(\pm\frac{2}{'R'}\right)$

A1 $\alpha = \text{awrt } 26.57°$

(b)

M1 Using part (a) and proceeding as far as $\cos(2\theta \pm \text{their } 26.57) = \dfrac{1}{\text{their } R}$.

This may be implied by $(2\theta \pm \text{their } 26.57) = \arccos\left(\dfrac{1}{\text{their} R}\right)$

Allow this mark for $\cos(\theta \pm \text{their } 26.57) = \dfrac{1}{\text{their } R}$

dM1 Dependent upon the first M1- it is for a correct method to find $\theta$ from their principal value
Look for the correct order of operations, that is dealing with the "26.57" before the "2".
Condone subtracting 26.57 instead of adding.

$\cos(2\theta \pm \text{their } 26.57) = \ldots \Rightarrow 2\theta \pm \text{their } 26.57 = \beta \Rightarrow \theta = \dfrac{\beta \pm \text{their } 26.57}{2}$

A1 awrt $\theta = 51.8°$

ddM1 For a correct method to find a secondary value of $\theta$ in the range
Either $2\theta \pm 26.57 = '-\beta' \Rightarrow \theta =$ OR $2\theta \pm 26.57 = 360 - '\beta' \Rightarrow \theta =$ THEN MINUS 180

A1 awrt $\theta = -25.3°$
Withhold this mark if there are extra solutions in the range.

Radian solution: Only lose the first time it occurs.

FYI. In radians desired accuracy is awrt 2 dp (a) $\alpha = 0.46$ and (b) $\theta_1 = 0.90, \theta_2 = -0.44$

Mixing degrees and radians only scores the first M

(c)

B1ft Follow through on their $R$. Accept decimals here including $\sqrt{20} \approx$ awrt 4.5 .

Score for one of the ends $k > \sqrt{20}$, $k < -\sqrt{20}$

Condone versions such as $g(\theta) > \sqrt{20}$, $y > \sqrt{20}$

or both ends including the boundaries $k \geqslant \sqrt{20}$, $k \leqslant -\sqrt{20}$

B1 ft For both intervals in terms of $k$.

Accept $k > \sqrt{20}$ or $k < -\sqrt{20}$. Accept $|k| > \sqrt{20}$ Accept $k \in (\sqrt{20}, \infty) \cup (-\infty, -\sqrt{20})$

Condone $k > \sqrt{20}, k < -\sqrt{20}$ $k > \sqrt{20}$ and $k < -\sqrt{20}$ for both marks

but $-\sqrt{20} > k > \sqrt{20}$ is B1 B0

| Question Number | Scheme | Marks |
|---|---|---|
| **86(a)** | $\sec 2A + \tan 2A = \dfrac{1}{\cos 2A} + \dfrac{\sin 2A}{\cos 2A}$ | B1 |
| | | M1 |
| | $= \dfrac{1 + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$ | M1 |
| | $= \dfrac{(\cos A + \sin A)(\cos A + \sin A)}{(\cos A + \sin A)(\cos A - \sin A)}$ | M1 |
| | $= \dfrac{\cos A + \sin A}{\cos A - \sin A}$ | A1* |
| | | **(5)** |
| **(b)** | $\sec 2\theta + \tan 2\theta = \dfrac{1}{2} \Rightarrow \dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \dfrac{1}{2}$ | |
| | $\Rightarrow 2\cos\theta + 2\sin\theta = \cos\theta - \sin\theta$ | |
| | $\Rightarrow \tan\theta = -\dfrac{1}{3}$ | M1 A1 |
| | $\Rightarrow \theta = awrt\ 2.820, 5.961$ | dM1A1 |
| | | **(4)** |
| | | **(9 marks)** |

(a)

B1 A correct identity for $\sec 2A = \dfrac{1}{\cos 2A}$ **OR** $\tan 2A = \dfrac{\sin 2A}{\cos 2A}$.

It need not be in the proof and it could be implied by the sight of $\sec 2A = \dfrac{1}{\cos^2 A - \sin^2 A}$

M1 For setting their expression as a single fraction. The denominator must be correct for their fractions and at least two terms on the numerator.

This is usually scored for $\dfrac{1 + \cos 2A \tan 2A}{\cos 2A}$ or $\dfrac{1 + \sin 2A}{\cos 2A}$

M1 For getting an expression in just sin $A$ and cos $A$ by using the double angle identities

$\sin 2A = 2\sin A\cos A$ and $\cos 2A = \cos^2 A - \sin^2 A$, $2\cos^2 A - 1$ or $1 - 2\sin^2 A$.

Alternatively for getting an expression in just sin $A$ and cos $A$ by using the double angle identities $\sin 2A = 2\sin A\cos A$ and $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$ with $\tan A = \dfrac{\sin A}{\cos A}$.

For example $= \dfrac{1}{\cos^2 A - \sin^2 A} + \dfrac{2\sin A / \cos A}{1 - \sin^2 A / \cos^2 A}$ is B1M0M1 so far

M1 In the main scheme it is for replacing 1 by $\cos^2 A + \sin^2 A$ **and** factorising both numerator and denominator

A1* Cancelling to produce given answer with no errors.
Allow a consistent use of another variable such as $\theta$, but mixing up variables will lose the A1*.

(b)

M1 For using part (a), cross multiplying, dividing by $\cos\theta$ to reach $\tan\theta = k$
Condone $\tan 2\theta = k$ for this mark only

A1 $\tan\theta = -\frac{1}{3}$

dM1 Scored for $\tan\theta = k$ leading to at least one value (with 1 dp accuracy) for $\theta$ between 0 and $2\pi$. You may have to use a calculator to check. Allow answers in degrees for this mark.

A1 $\theta =$ awrt $2.820, 5.961$ with no extra solutions within the range. Condone 2.82 for 2.820.

You may condone different/ mixed variables in part (b)

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There are some long winded methods. Eg. M1, dM1 applied as in main scheme

$$\Rightarrow (2\cos\theta + 2\sin\theta)^2 = (\cos\theta - \sin\theta)^2 \Rightarrow 4 + 4\sin 2\theta = 1 - \sin 2\theta$$

$$\Rightarrow \sin 2\theta = -\frac{3}{5}$$ is M1 ( for $\sin 2\theta = k$ ) A1

$$\Rightarrow \theta = 2.820, 5.961$$ for dM1 (for $\theta = \frac{\arcsin k}{2}$ ) A1

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$$\cos\theta + 3\sin\theta = 0 \Rightarrow (\sqrt{10})\cos(\theta - 1.25) = 0$$ M1 for..$\cos(\theta - \alpha) = 0, \alpha = \arctan\left(\pm\frac{3}{1} \text{ or } \pm\frac{1}{3}\right)$ A1

$$\Rightarrow \theta = 2.820, 5.961$$ dM1 A1

$$\cos\theta + 3\sin\theta = 0 \Rightarrow (\sqrt{10})\sin(\theta + 0.32) = 0$$ M1 A1

$$\Rightarrow \theta = 2.820, 5.961$$ dM1 A1

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$$\cos\theta = -3\sin\theta \Rightarrow \cos^2\theta = 9\sin^2\theta \Rightarrow \sin^2\theta = \frac{1}{10} \Rightarrow \sin\theta = (\pm)\sqrt{\frac{1}{10}}$$ M1 A1

$$\Rightarrow \theta = 2.820, 5.961$$ dM1 A1

$$\cos\theta = -3\sin\theta \Rightarrow \cos^2\theta = 9\sin^2\theta \Rightarrow \cos^2\theta = \frac{9}{10} \Rightarrow \cos\theta = (\pm)\sqrt{\frac{9}{10}}$$ M1 A1

$$\Rightarrow \theta = 2.820, 5.961$$ dM1 A1

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| Question Number | Scheme | Marks |
|---|---|---|
| **Alt I** <br> **From RHS** | $\frac{\cos A+\sin A}{\cos A-\sin A} = \frac{\cos A+\sin A}{\cos A-\sin A}\times\frac{\cos A+\sin A}{\cos A+\sin A}$ <br> $=\frac{\cos^2 A+\sin^2 A+2\sin A\cos A}{\cos^2 A-\sin^2 A}$ <br> $=\frac{1+\sin 2A}{\cos 2A}$ <br> $=\frac{1}{\cos 2A}+\frac{\sin 2A}{\cos 2A}$ <br> $=\sec 2A+\tan 2A$ | (Pythagoras) **M1** <br> (Double Angle) **M1** <br> (Single Fraction) **M1** <br> **B1**(Identity), **A1*** |
| **Alt II** <br> **Both sides** | Assume true $\sec 2A+\tan 2A=\frac{\cos A+\sin A}{\cos A-\sin A}$ <br> $\frac{1}{\cos 2A}+\frac{\sin 2A}{\cos 2A}=\frac{\cos A+\sin A}{\cos A-\sin A}$ <br> $\frac{1+\sin 2A}{\cos 2A}=\frac{\cos A+\sin A}{\cos A-\sin A}$ <br> $\frac{1+2\sin A\cos A}{\cos^2 A-\sin^2 A}=\frac{\cos A+\sin A}{\cos A-\sin A}$ <br> $\times(\cos A-\sin A)\Rightarrow\frac{1+2\sin A\cos A}{\cos A+\sin A}=\cos A+\sin A$ <br> $1+2\sin A\cos A=\cos^2 A+2\sin A\cos A+\sin^2 A=1+2\sin A\cos A$ **True** | B1 (identity) <br> M1 (single fraction) <br> M1(double angles) <br> M1(Pythagoras)A1* |
| **Alt 111** <br> **Very difficult** | $\sec 2A+\tan 2A=\frac{1}{\cos 2A}+\tan 2A$ <br> $=\frac{1}{\cos 2A}+\frac{2\tan A}{1-\tan^2 A}$ <br> $=\frac{1-\tan^2 A+2\tan A\cos 2A}{\cos 2A(1-\tan^2 A)}$ <br> $=\frac{1-\tan^2 A+2\tan A(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)(1-\tan^2 A)}$ <br> $=\frac{1-\frac{\sin^2 A}{\cos^2 A}+2\frac{\sin A}{\cos A}(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)\left(1-\frac{\sin^2 A}{\cos^2 A}\right)}$ <br> $\times\cos^2 A=\frac{\cos^2 A-\sin^2 A+2\sin A\cos A(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)(\cos^2 A-\sin^2 A)}$ <br> $=\frac{\cancel{(\cos^2 A-\sin^2 A)}(1+2\sin A\cos A)}{\cancel{(\cos^2 A-\sin^2 A)}(\cos^2 A-\sin^2 A)}$ <br> Final two marks as in main scheme | (Identity) **B1** <br> (Single fraction) **M1** <br> (Double Angle and in just sin and cos ) **M1** <br> **M1A1*** |



EXPERT TUITION

| Question Number | Scheme | Marks |
| --- | --- | --- |
| **87.(a)** | $\text{cosec}2x+\cot 2x=\dfrac{1}{\sin 2x}+\dfrac{\cos 2x}{\sin 2x}$ | M1 |
| | $=\dfrac{1+\cos 2x}{\sin 2x}$ | M1 |
| | $=\dfrac{1+2\cos^2 x-1}{2\sin x\cos x}$ | |
| | $=\dfrac{2\cos^2 x}{2\sin x\cos x}$ | M1 A1 |
| | $=\dfrac{\cos x}{\sin x}=\cot x$ | A1* |
| | | (5) |
| **(b)** | $\text{cosec}(4\theta+10°)+\cot(4\theta+10°)=\sqrt{3}$ | |
| | $\cot(2\theta\pm..°)=\sqrt{3}$ | M1 |
| | $2\theta\pm...=30°\Rightarrow\theta=12.5°$ | dM1, A1 |
| | $2\theta\pm...=180+PV°\Rightarrow\theta=..°$ | dM1 |
| | $\theta=102.5°$ | A1 |
| | | (5) |
| | | **(10 marks)** |

(a)

M1 Writing $\operatorname{cosec} 2x = \frac{1}{\sin 2x}$ **and** $\cot 2x = \frac{\cos 2x}{\sin 2x}$ *or* $\frac{1}{\tan 2x}$

M1 Writing the lhs as a single fraction $\frac{a+b}{c}$. The denominator must be correct for their terms.

M1 Uses the appropriate double angle formulae/trig identities to produce a fraction in a form containing no addition or subtraction signs. A form $\frac{p \times q}{s \times t}$ or similar

A1 A correct intermediate line. Accept $\frac{2\cos^2 x}{2\sin x \cos x}$ or $\frac{2\sin x \cos x}{2\sin x \cos x \tan x}$ or similar

This cannot be scored if errors have been made

A1* Completes the proof by cancelling and using either $\frac{\cos x}{\sin x} = \cot x$ or $\frac{1}{\tan x} = \cot x$

The cancelling could be implied by seeing $\frac{2}{2}\frac{\cos x}{\sin x}\frac{\cos x}{\cos x} = \cot x$

The proof cannot rely on expressions like $\cot = \frac{\cos}{\sin}$ (with missing $x$'s) for the final A1

(b)

M1 Attempt to use the solution to part (a) with $2x = 4\theta + 10 \Rightarrow$ to write or imply $\cot(2\theta \pm ...°) = \sqrt{3}$

Watch for attempts which start $\cot \alpha = \sqrt{3}$. The method mark here is not scored until the $\alpha$ has been replaced by $2\theta \pm ...°$

Accept a solution from $\cot(2x \pm ...°) = \sqrt{3}$ where $\theta$ has been replaced by another variable.

dM1 Proceeds from the previous method and uses $\tan .. = \frac{1}{\cot ..}$ and

$\arctan\left(\frac{1}{\sqrt{3}}\right) = 30°$ to solve $2\theta \pm ...° = 30° \Rightarrow \theta = ..$

A1 $\theta = 12.5°$ or exact equivalent. Condone answers such as $x = 12.5°$

dM1 This mark is for the correct method to find a second solution to θ. It is dependent upon the first M only.

Accept $2\theta \pm ... = 180 + PV° \Rightarrow \theta = ..°$

A1 $\theta = 102.5°$ or exact equivalent. Condone answers such as $x = 102.5°$

Ignore any solutions outside the range. This mark is withheld for any extra solutions within the range.

If radians appear they could just lose the answer marks. So for example

$2\theta \pm ... = \frac{\pi}{6}(0.524) \Rightarrow \theta = ..$is M1dM1A0 followed by

$2\theta \pm ... = \pi + '\frac{\pi}{6}' \Rightarrow \theta = ..$dM1A0

Special case 1: For candidates in (b) who solve $\cot(4\theta \pm ...°) = \sqrt{3}$ the mark scheme is severe, so we are awarding a special case solution, scoring 00011.

$\cot(4\theta + \beta°) = \sqrt{3} \Rightarrow 4\theta + \beta = 30° \Rightarrow \theta = ..$ is M0M0A0 where $\beta = 5°$ or $10°$

$\Rightarrow 4\theta + \beta = 210° \Rightarrow \theta = ..$ can score M1A1 Special case.

If $\beta = 5°$, $\theta = 51.25$ If $\beta = 10°$, $\theta = 50$

Special case 2: Just answers in (b) **with no working** scores 1 1 0 0 0 for 12.5 and 102.5

BUT $\cot(2\theta \pm 5°) = \sqrt{3} \Rightarrow \theta = 12.5°, 102.5°$ scores all available marks.

| Question Number | Scheme | Marks |
|---|---|---|
| **87.(a)Alt 1** | $\operatorname{cosec}2x + \cot 2x = \frac{1}{\sin 2x} + \frac{1}{\tan 2x}$ | 1ST M1 |
| | $= \frac{1}{2\sin x\cos x} + \frac{1-\tan^2 x}{2\tan x}$ | |
| | $= \frac{\tan x + (1-\tan^2 x)\sin x\cos x}{2\sin x\cos x\tan x}$ or $= \frac{2\tan x + 2(1-\tan^2 x)\sin x\cos x}{4\sin x\cos x\tan x}$ | 2nd M1 |
| | $= \frac{\tan x + \sin x\cos x - \tan^2 x\sin x\cos x}{2\sin x\cos x\tan x}$ | |
| | $= \frac{\tan x + \sin x\cos x - \tan x\sin^2 x}{2\sin x\cos x\tan x}$ | |
| | $= \frac{\tan x(1-\sin^2 x) + \sin x\cos x}{2\sin x\cos x\tan x}$ | |
| | $= \frac{\tan x\cos^2 x + \sin x\cos x}{2\sin x\cos x\tan x}$ | |
| | $= \frac{\sin x\cos x + \sin x\cos x}{2\sin x\cos x\tan x}$ | |
| | $= \frac{2\sin x\cos x}{2\sin x\cos x\tan x}$ oe | 3rd M1A1 |
| | $= \frac{1}{\tan x} = \cot x$ | A1* (5) |
| **87.(a)Alt 2** | **Example of how main scheme could work in a roundabout route** | |
| | $\operatorname{cosec}2x + \cot 2x = \cot x \Leftrightarrow \frac{1}{\sin 2x} + \frac{1}{\tan 2x} = \frac{1}{\tan x}$ | 1st M1 |
| | $\Leftrightarrow \tan 2x\tan x + \sin 2x\tan x = \sin 2x\tan 2x$ | 2nd M1 |
| | $\Leftrightarrow \frac{2\tan x}{1-\tan^2 x} \times \tan x + 2\sin x\cancel{\cos x} \times \frac{\sin x}{\cancel{\cos x}} = 2\sin x\cos x \times \frac{2\tan x}{1-\tan^2 x}$ | |

| Question Number | Scheme | Marks |
| --- | --- | --- |
| | $\Leftrightarrow \dfrac{2\tan^2 x}{1-\tan^2 x} + 2\sin^2 x = \dfrac{4\sin^2 x}{1-\tan^2 x}$ | |
| | $\times(1-\tan^2 x) \Leftrightarrow 2\tan^2 x + 2\sin^2 x(1-\tan^2 x) = 4\sin^2 x$ | |
| | $\Leftrightarrow 2\tan^2 x - 2\sin^2 x\tan^2 x = 2\sin^2 x$ | |
| | $\Leftrightarrow 2\tan^2 x(1-\sin^2 x) = 2\sin^2 x$ | 3rd M1 |
| | $\div 2\tan^2 x \Leftrightarrow 1-\sin^2 x = \cos^2 x$ | A1 |
| | As this is true, initial statement is true | A1* |
| | | (5) |

| Question Number | Scheme | Marks |
|---|---|---|
| **88.(a)** | $R=\sqrt{20}$ | B1 |
| | $\tan\alpha=\frac{4}{2}\Rightarrow\alpha=\text{awrt }1.107$ | M1A1 |
| | | (3) |
| **(b)(i)** | $'4+5R^2'=104$ | B1ft |
| **(ii)** | $3\theta-'1.107'=\frac{\pi}{2}\Rightarrow\theta=\text{awrt }0.89$ | M1A1 |
| | | (3) |
| **(c )(i)** | 4 | B1 |
| **(ii)** | $3\theta-'1.107'=2\pi\Rightarrow\theta=\text{awrt }2.46$ | M1A1 |
| | | (3) |
| | | **( 9 marks)** |

(a)

B1 Accept $R = \sqrt{20}$ or $2\sqrt{5}$ or awrt 4.47

Do not accept $R = \pm\sqrt{20}$

This could be scored in parts (b) or (c) as long as you are certain it is *R*

M1 for sight of $\tan\alpha = \pm\frac{4}{2}, \tan\alpha = \pm\frac{2}{4}$. Condone $\sin\alpha = 4, \cos\alpha = 2 \Rightarrow \tan\alpha = \frac{4}{2}$

If *R* is found first only accept $\sin\alpha = \pm\frac{4}{R}, \cos\alpha = \pm\frac{2}{R}$

A1 $\alpha =$ awrt $1.107$. The degrees equivalent $63.4°$ is A0.

If a candidate does all the question in degrees they will lose just this mark.

(b)(i)

B1ft Either 104 or if *R* was incorrect allow for the numerical value of their $'4+5R^2'$.

Allow a tolerance of 1 dp on decimal *R*'s.

(b)(ii)

M1 Using $3\theta \pm \text{their}'1.107' = \frac{\pi}{2} \Rightarrow \theta = ..$

Accept $3\theta \pm \text{their}'1.107' = (2n+1)\frac{\pi}{2} \Rightarrow \theta = ..$ where *n* is an integer

Allow slips on the lhs with an extra bracket such as

$3(\theta \pm \text{their}'1.107') = \frac{\pi}{2} \Rightarrow \theta = ..$

The degree equivalent is acceptable $3\theta - \text{their}'63.4°' = 90° \Rightarrow \theta =$

Do not allow mixed units in this question

A1 awrt $0.89$ radians or $51.1°$. Do not allow multiple solutions for this mark.

(c)(i)

B1 4

(c)(ii)

M1 Using $3\theta \pm \text{their}'1.107' = 2\pi \Rightarrow \theta = ...$

Accept $3\theta \pm \text{their}'1.107' = n\pi \Rightarrow \theta = ..$ where *n* is an integer, including 0

Allow slips on the lhs with an extra bracket such as

$3(\theta \pm \text{their}'1.107') = 2\pi \Rightarrow \theta = ..$

The degree equivalent is acceptable $3\theta - \text{their}'63.4°' = 360° \Rightarrow \theta =$ but

Do not allow mixed units in this question

A1 $\theta =$ awrt $2.46$ radians or $141.1°$ Do not allow multiple solutions for this mark.

| Question Number | Scheme | Marks |
|---|---|---|
| **89. (i) (a)** | $2\dfrac{\sin x}{\cos x}-\dfrac{\cos x}{\sin x}=\dfrac{5}{\sin x}$ | B1 |
| | Uses common denominator to give $2\sin^2 x-\cos^2 x=5\cos x$ | M1 |
| | Replaces $\sin^2 x$ by $(1-\cos^2 x)$ to give $2(1-\cos^2 x)-\cos^2 x=5\cos x$ | M1 |
| | Obtains $3\cos^2 x+5\cos x-2=0$ ( $a$ =3, $b$ = 5, $c$ = -2 ) | A1<br>(4) |
| **(b)** | Solves $3\cos^2 x+5\cos x-2=0$ to give cos $x$ = | M1 |
| | cos $x$ = $\frac{1}{3}$ only (rejects cos $x$ = -2) | A1 |
| | So $x$ = 1.23 or 5.05 | dM1A1<br>(4) |
| **(ii)** | Either | |
| | $\tan\theta+\cot\theta\equiv\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}$ | B1 |
| | $\equiv\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$ | M1 |
| | $\equiv\dfrac{2}{\sin 2\theta}$ | M1 |
| | $\equiv 2\operatorname{cosec}2\theta$ (so $\lambda=2$) | A1 |
| | Or | |
| | $\tan\theta+\cot\theta\equiv\tan\theta+\dfrac{1}{\tan\theta}$ | B1 |
| | $\equiv\dfrac{\tan^2\theta+1}{\tan\theta}$ | M1 |
| | $\equiv\dfrac{1}{\cos^2\theta\times\frac{\sin\theta}{\cos\theta}}\equiv\dfrac{2}{\sin 2\theta}$ | M1 |
| | $\equiv 2\operatorname{cosec}2\theta$ (so $\lambda=2$) | A1<br>(4)<br>**12 marks** |
| | **Alternatives to Main Scheme** | |
| **89. (i) (a)** | $2\tan x-\dfrac{1}{\tan x}=\dfrac{5}{\sin x}$ does not score any marks until | |
| | $\times\tan x\Rightarrow 2\tan^2 x+1=5\sec x$ | B1, M1 |
| | Replaces $\tan^2 x$ by $(\sec^2 x-1)$ to give $2(\sec x^2-1)+1=5\sec x$ | M1 |
| | Obtains $3\cos^2 x+5\cos x-2=0$ ( $a$ =3, $b$ = 5, $c$ = -2 ) | A1<br>(4) |
| **(b)** | Solves $3\cos^2 x+5\cos x-2=0$ to give cos $x$ =<br>or $2\sec^2 x-5\sec x-3=0\Rightarrow\sec x=..$ | M1 |
| | cos $x$ = $\frac{1}{3}$ only (rejects cos $x$ = -2) | A1 |
| | So $x$ = 1.23 or 5.05 | dM1A1<br>(4) |
| **89. (ii)** | $\tan\theta+\cot\theta=\lambda\text{cosec}2\theta\Rightarrow\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}=\dfrac{\lambda}{\sin 2\theta}=\left(\cancel{\dfrac{\lambda}{2\sin\theta\cos\theta}}\right)$ | B1 |
| | $\times 2\sin\theta\cos\theta\Rightarrow 2\sin^2\theta+2\cos^2\theta=\lambda$ | M1 |
| | Factorises $2(\sin^2\theta+\cos^2\theta)=\lambda\Rightarrow 2=\lambda$ | M1 |
| | All above correct + a statement like ‘hence true’, ‘QED’ | A1<br>(4) |

**(i)(a)**

B1 Uses definitions $\tan x = \dfrac{\sin x}{\cos x}$, $\cot x = \dfrac{\cos x}{\sin x}$ and $\operatorname{cosec} x = \dfrac{1}{\sin x}$ to write the equation in terms of $\cos x$ and $\sin x$. Condone $5\operatorname{cosec} x = \dfrac{1}{5\sin x}$ as the intention is clear.

Alternatively uses $\cot x = \dfrac{1}{\tan x}$ and $\operatorname{cosec} x = \dfrac{1}{\sin x}$ to write the equation in terms of $\tan x$ and $\sin x$

This may be implied by later work that achieves $A\tan^2 x \pm B = C\sec x$

M1 Either uses common denominator and cross multiples, or multiplies each term by $\sin x \cos x$ to achieve an equation of the form equivalent to $A\sin^2 x \pm B\cos^2 x = C\cos x$. It may be seen on the numerator of a fraction

Alternatively multiplies by $\tan x$ to achieve $A\tan^2 x \pm B = C\sec x$

M1 Uses a correct Pythagorean relationship, usually $\sin^2 x = 1 - \cos^2 x$ to form a quadratic equation in terms of $\cos x$. In the alternative uses $\tan^2 x = \sec^2 x - 1$ to form a quadratic in sec $x$, followed by $\sec x = \dfrac{1}{\cos x}$ to form a quadratic equation in terms of $\cos x$

A1 Obtains $\pm K\left(3\cos^2 x + 5\cos x - 2\right) = 0$ ( $a$ =3, $b$ = 5, $c$ = -2 )

**(i)(b)**

M1 Uses a standard method to solve their quadratic equation in $\cos x$ from **(i)(a) OR** sec$x$ from an earlier line in (a)
See General Principles for Core Mathematics on how to solve quadratics

A1 $\cos x = \frac{1}{3}$ only Do not need to see –2 rejected

dM1 Uses arcos on their value to obtain at least one answer. It is dependent upon the previous M.
It may be implied by one correct answer

A1 Both values correct awrt 3sf $x$ = 1.23 and 5.05.
Ignore any solutions outside the range. Any extra solutions in the range will score A0.
Answers in degrees will score A0.

**(ii)**

B1 Uses a definition of cot with matching expression for tan. Acceptable answers are

$\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$, $\dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\frac{\sin\theta}{\cos\theta}}$, $\tan\theta + \dfrac{1}{\tan\theta}$. Condone a miscopy on the sign. Eg Allow $\tan\theta - \dfrac{1}{\tan\theta}$

M1 Uses common denominator, writing their expression as a single fraction. In the examples given above, example 2 would need to be inverted. The denominator has to be correct and one of the terms must be adapted.

M1 Uses identities $\sin^2\theta + \cos^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$ specifically to achieve an expression of the form $\dfrac{\lambda}{\sin 2\theta}$

Alternatively uses $1 + \tan^2\theta = \sec^2\theta = \dfrac{1}{\cos^2\theta}$, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ specifically to achieve an expression of the form $\dfrac{\lambda}{\sin 2\theta}$. A line of $\dfrac{1}{\sin\theta\cos\theta}$ achieved on the lhs followed by $\lambda = \dfrac{1}{2}$ or $2$ would imply this mark

A1 Achieves printed answer with no errors.

Allow for a different variable as long as it is used consistently.

| Question Number | Scheme | Marks |
|---|---|---|
| **90.(a)** | $R=\sqrt{\left(6^2+2.5^2\right)}=6.5$ | B1 |
| | $\tan\alpha=\dfrac{2.5}{6}, \quad \Rightarrow \quad \alpha=\text{awrt }0.395$ | M1A1 |
| | | (3) |
| **(b)** | $(0,6)$, | B1 |
| | awrt $(1.97,0) \quad (5.11,0)$ | M1A1 |
| | | (3) |
| **(c)** | $H_{\max}=18.5, H_{\min}=5.5$ | M1A1A1 |
| | | (3) |
| **(d)** | Sub $H=16$ and proceed to $'6.5'\cos\left(\dfrac{2\pi t}{52}\pm'0.395'\right)=4$ | M1 |
| | $\left(\dfrac{2\pi t}{52}-'0.395'\right)=\text{awrt }0.91$ | A1 |
| | $t=\left(awrt\,0.908\pm'0.395'\right)\times\dfrac{52}{2\pi}=11\ (10.78)$ | dM1A1 |
| | $\left(\dfrac{2\pi t}{52}\pm'0.395'\right)=awrt\,2\pi-0.908\Rightarrow t=48\ (47.75)$ | ddM1A1 |
| | | (6) |
| | | **(15 marks)** |

(a)

B1 $R=6.50,\dfrac{13}{2}$. Accept $R=\text{awrt }6.50$. Do not accept $R=\pm6.50$

M1 For reaching $\tan\alpha=\pm\dfrac{2.5}{6}$ *or* $\tan\alpha=\pm\dfrac{6}{2.5}$.

If R has been attempted first then only accept $\sin\alpha=\pm\dfrac{2.5}{'R'}$ *or* $\cos\alpha=\pm\dfrac{6}{'R'}$

A1 Correct value $\alpha=awrt\ 0.395$. The answer in degrees $22.6°$ is A0

(b)

B1 The correct y intercept. Accept $y=6, (0,6)$, awrt y = 6.00, $f(0)=6$ or it marked on the curve.

Do not accept $(6,0)$

M1 Attempt to find either $x$ intercept from $\dfrac{\pi}{2}+\text{their }0.395$, or $\dfrac{3\pi}{2}+\text{their }0.395$

If the candidate is working in degrees accept $90+\text{their }22.6$ or $270+\text{their }22.6$

One answer correct will imply this.

A1 Both answers correct. Accept awrt $(1.97,0)$ and $(5.11,0)$, Accept $x=1.97$ and $x=5.11$ or both being marked on the curve. Do not accept $(0,1.97)$ and $(0,5.11)$ for both marks

In degrees accept $(112.6,0)$ and $(292.6,0)$

(c)

M1 Attempts either $12+'R'$ OR $12-'R'$

A1 Either of 18.5 or 5.5. Accept one of these for two marks

A1 Both 18.5 and 5.5.

Accept for 3 marks answers just written down with limited or no working.

Attempted answers via differentiation will be few and far between but can score 3 marks.

M1 Differentiates to $H'=\pm A\sin\left(\frac{2\pi t}{52}\right)\pm B\cos\left(\frac{2\pi t}{52}\right)$, followed by $H'=0$

$\Rightarrow \tan\left(\frac{2\pi t}{52}\right)=\left(\frac{5}{12}\right)\Rightarrow t=\text{awrt } 3.2..,29.2..$.

For the M to be scored they need to sub one value of t (which may not be correct ) into $H=$

A1 Either of 18.5 or 5.5. A 1 Both 18.5 and 5.5.

(d)

M1 Substitutes $H=16$ into the equation for $H$ and proceeds to $'6.5'\cos\left(\frac{2\pi t}{52}\pm'0.395'\right)=4$

Accept for this mark $'6.5'\cos(x\pm'0.395')=4$

A1 A correct intermediate line, which may be implied by a correct final answer. Follow through on their numerical value of $\alpha$

Accept in terms of '*t*' $\left(\frac{2\pi t}{52}-'0.395'\right)=awrt\,0.91$ or in terms of '*x*' $(x-'0.395')=awrt\,0.91$

Accept in terms of '*t*' $\left(\frac{2\pi t}{52}-'0.395'\right)=\text{invcos}\frac{4}{6.5}$

dM1 A full method to find one value of $t$. It is dependent upon the previous M mark having been awarded.

Accept $t=(their\,0.908\pm'0.395')\times\frac{52}{2\pi}$.

Don't be overly concerned with the mechanics of this but the '0.395' the $2\pi$ and the 52 must have been used to find *t*.

A1 One correct value of *t* with a correct solution. Both M's must have been scored.

Accept awrt 10.7/10.8 or 11 or 47.7/47.8 or 48.

ddM1 A full method to find a secondary value of $t$. It is dependent upon both previous M's.

$$\left(\frac{2\pi t}{52}\pm their\,'0.395'\right)=awrt\,2\pi-their\,0.91\Rightarrow t=..$$

Don't be overly concerned with the mechanics of this but the '0.395' the $2\pi$ and the 52 must have been used to find *t*.

A1 Accept 11 and 48 coming from awrt 10.8/10.7 and 47.7/47.8. Both values of *t* need to be correct and have been rounded from *t* values that were correct to 1 dp. The intermediate values can be implied by seeing the whole calculation as written out in the mark scheme

Answers obtained by graphical or numerical means are not acceptable.

Answers obtained from degrees are perfectly acceptable only if degrees were used throughout (d) with $\pi$, being replaced by $180°$ in the formula and the answers in degrees converted back to radians at the end.

Mixed units can only score the first M1A1

$$6.5\cos\left(\frac{2\pi t}{52}-'22.6'\right)=4\Rightarrow\left(\frac{2\pi t}{52}-'22.6'\right)=awrt\,52.0$$

| Question Number | Scheme | Marks |
|---|---|---|
| **91(a)** | $2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$ | M1 |
| | $\sin x(\cos 40 + 2\sin 50) = \cos x(2\cos 50 - \sin 40)$ | |
| | $\div\cos x \Rightarrow \tan x(\cos 40 + 2\sin 50) = 2\cos 50 - \sin 40$ | M1 |
| | $\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50}$, (or numerical answer awrt 0.28) | A1 |
| | States or uses cos50 = sin40 and cos40 =sin50 and so $\tan x^{\circ} = \frac{1}{3}\tan 40^{\circ}$ * cao | A1 * **(4)** |
| **(b)** | Deduces $\tan 2\theta = \frac{1}{3}\tan 40$ | M1 |
| | $2\theta =$ 15.6 so $\theta =$ awrt 7.8(1) One answer | A1 |
| | Also $2\theta =$ 195.6, 375.6 , 555.6 $\Rightarrow \theta = ..$ | M1 |
| | $\theta =$ awrt 7.8 , 97.8, 187.8, 277.8 All 4 answers | A1 |
| | | **(4)** |
| | | **[8 marks ]** |
| **Alt 1** **91(a)** | $2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$ | M1 |
| | $2\cos x\sin 40 - 2\sin x\cos 40 = \sin x\cos 40 + \cos x\sin 40$ | |
| | $\div\cos x \Rightarrow 2\sin 40 - 2\tan x\cos 40 = \tan x\cos 40 + \sin 40$ | M1 |
| | $\tan x = \dfrac{\sin 40}{3\cos 40}$( or numerical answer awrt 0.28), $\Rightarrow \tan x = \dfrac{1}{3}\tan 40$ | A1,A1 |
| **Alt 2** **91(a)** | $2\cos(x+50) = \sin(x+40) \Rightarrow 2\sin(40-x) = \sin(x+40)$ | |
| | $2\cos x\sin 40 - 2\sin x\cos 40 = \sin x\cos 40 + \cos x\sin 40$ | M1 |
| | $\div\cos x \Rightarrow 2\sin 40 - 2\tan x\cos 40 = \tan x\cos 40 + \sin 40$ | M1 |
| | $\tan x = \dfrac{\sin 40}{3\cos 40}$( or numerical answer awrt 0.28), $\Rightarrow \tan x = \dfrac{1}{3}\tan 40$ | A1,A1 |

**Notes for Question 91**

(a)

M1 Expand both expressions using $\cos(x+50) = \cos x\cos 50 - \sin x\sin 50$ and $\sin(x+40) = \sin x\cos 40 + \cos x\sin 40$. Condone a missing bracket on the lhs.

The terms of the expansions must be correct as these are given identities. You may condone a sign error on one of the expressions.

Allow if written separately and not in a connected equation.

M1 Divide by $\cos x$ to reach an equation in $\tan x$.

Below is an example of M1M1 with incorrect sign on left hand side

$2\cos x\cos 50 + 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$

$\Rightarrow 2\cos 50 + 2\tan x\sin 50 = \tan x\cos 40 + \sin 40$

This is independent of the first mark.

A1 $\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50}$

Accept for this mark $\tan x = \text{awrt } 0.28...$ as long as M1M1 has been achieved.

A1* States or uses $\cos 50 = \sin 40$ and $\cos 40 = \sin 50$ leading to showing

$$\tan x = \frac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50} = \frac{\sin 40}{3\cos 40} = \frac{1}{3}\tan 40$$

This is a given answer and all steps above must be shown. The line above is acceptable.

Do not allow from $\tan x = \text{awrt } 0.28...$

(b)

M1 For linking part (a) with (b). Award for writing $\tan 2\theta = \frac{1}{3}\tan 40$

A1 Solves to find one solution of $\theta$ which is usually (awrt) 7.8

M1 Uses the correct method to find at least another value of $\theta$. It must be a full method but can be implied by any correct answer.

Accept $\theta = \dfrac{180 + their\,\alpha}{2}, (or)\dfrac{360 + their\,\alpha}{2}, (or)\dfrac{540 + their\,\alpha}{2}$

A1 Obtains all four answers awrt 1dp. $\theta = 7.8, 97.8, 187.8, 277.8$.

Ignore any extra solutions outside the range.

Withhold this mark for extras inside the range.

Condone a different variable. Accept $x$= 7.8, 97.8, 187.8, 277.8

Answers fully given in radians, loses the first A mark.

Acceptable answers in rads are awrt 0.136, 1.71, 3.28, 4.85

Mixed units can only score the first M 1

| Question Number | Scheme | Marks |
|---|---|---|
| **92(a)** | $R=\sqrt{\left(7^2+24^2\right)}=25$ | B1 |
| | $\tan\alpha=\frac{24}{7},\quad\Rightarrow\quad\alpha=\text{awrt}\,73.74°$ | M1A1 |
| | | **(3)** |
| **(b)** | maximum value of 24sin$x$ + 7cos$x$ = 25 so $V_{\min}=\frac{21}{25}=(0.84)$ | M1A1 |
| | | **(2)** |
| **(c)** | Distance $AB=\frac{7}{\sin\theta}$, with $\theta=\alpha$ | M1, B1 |
| | So distance = 7.29m $=\frac{175}{24}\text{m}$ | A1 |
| | | **(3)** |
| **(d)** | $R\cos(\theta-\alpha)=\frac{21}{1.68}\Rightarrow\cos(\theta-\alpha)=0.5$ | M1, A1 |
| | $\theta-\alpha=60\Rightarrow\theta=..,\theta-\alpha=-60\Rightarrow\theta=..$ | dM1, dM1 |
| | $\theta=\text{awrt}\,133.7, 13.7$ | A1, A1 |
| | | **(6)** |
| | | **(14 marks)** |

**Notes for Question 92**

(a)

B1 25. Accept 25.0 but not $\sqrt{625}$ or answers that are not exactly 25. Eg 25.0001

M1 For $\tan\alpha=\pm\frac{24}{7}, \tan\alpha=\pm\frac{7}{24}$.

If the value of R is used only accept $\sin\alpha=\pm\frac{24}{R}, \cos\alpha=\pm\frac{7}{R}$

A1 Accept answers which round to 73.74 – must be in degrees for this mark

(b)

M1 Calculates $V=\frac{21}{their\,'R'}$ NOT - R

A1 Obtains correct answer. $V=\frac{21}{25}$ Accept 0.84

Do not accept if you see incorrect working- ie from $\cos(\theta-\alpha)=-1$ or the minus just disappearing from a previous line.

Questions involving differentiation are acceptable. To score M1 the candidate would have to differentiate $V$ by the quotient rule (or similar) , set $V'$=0 to find $\theta$ and then sub this back into V to find its value.

## Notes for Question 92 Continued

(c)

M1 Uses the trig equation $\sin\theta = \dfrac{7}{AB}$ with a numerical $\theta$ to find $AB = ...$

B1 Uses $\theta =$ their value of $\alpha$ in a trig calculation involving sin. ($\sin\alpha = \dfrac{AB}{7}$ is condoned)

A1 Obtains answer $\dfrac{175}{24}$ or awrt 7.29

(d)

M1 Substitutes $V = 1.68$ and their answer to part (a) in $V = \dfrac{21}{24\sin\theta + 7\cos\theta}$ to get an equation

of the form $R\cos(\theta \pm \alpha) = \dfrac{21}{1.68}$ or $1.68R\cos(\theta \pm \alpha) = 21$ or $\cos(\theta \pm \alpha) = \dfrac{21}{1.68R}$ .

Follow through on their $R$ and $\alpha$

A1 Obtains $\cos(\theta \pm \alpha) = 0.5$ oe. Follow through on their $\alpha$. It may be implied by later working.

dM1 Obtains one value of $\theta$ **in the range** $0 < \theta < 150$ from inverse cos +their α

It is dependent upon the first M being scored.

dM1 Obtains second angle of $\theta$ **in the range** $0 < \theta < 150$ from inverse cos +their α

It is dependent upon the first M being scored.

A1 one correct answer awrt $\theta = 133.7\ or\ 13.7$ 1dp

A1 both correct answers awrt $\theta = 133.7\ and\ 13.7$ 1dp.

Extra solutions in the range loses the last A1.

Answers in radians, lose the first time it occurs. Answers must be to 3dp

For your info $\alpha = 1.287, \theta_1 = 2.334, \theta_2 = 0.240$

| Question Number | Scheme | Marks |
| --- | --- | --- |
| **93.** | (a) $\cot 40^0 = \dfrac{1}{\tan 40^0} = \dfrac{1}{p}$ | B1 |
| | | **(1)** |
| | (b) Attempts to use $1 + \tan^2 40^0 = \sec^2 40^0$ | M1 |
| | $\Rightarrow \sec 40^0 = \sqrt{(1 + p^2)}$ | A1 |
| | | **(2)** |
| | (c ) Attempts to use $\tan 85^0 = \tan(45^0 + 40^0) = \dfrac{\tan 45^0 + \tan 40^0}{1 - \tan 45^0 \tan 40^0}$ | M1 |
| | $\Rightarrow \tan 85^0 = \dfrac{1+p}{1-p}$ | A1 |
| | | **(2)** |
| | | **(5 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **94.** | (a) $\dfrac{\cos x}{1-\sin x}+\dfrac{1-\sin x}{\cos x}=\dfrac{\cos x\cos x+(1-\sin x)(1-\sin x)}{(1-\sin x)\cos x}$ | M1A1 |
| | $=\dfrac{\cos^2 x+\sin^2 x+1-2\sin x}{(1-\sin x)\cos x}$ | |
| | $=\dfrac{2-2\sin x}{(1-\sin x)\cos x}$ | M1 |
| | $=\dfrac{2\cancel{(1-\sin x)}}{\cancel{(1-\sin x)}\cos x}$ | |
| | $=\dfrac{2}{\cos x}=2\sec x$ | A1* |
| | | **(4)** |
| | (b) $\dfrac{\cos x}{1-\sin x}+\dfrac{1-\sin x}{\cos x}=8\sin x$ | |
| | $2\sec x=8\sin x$ | |
| | $1=4\sin x\cos x$ $\quad$ $a\sin x\cos x=b$ | M1 |
| | $\sin 2x=\dfrac{1}{2}$ $\quad$ $\sin 2x=2\sin x\cos x$ | M1 |
| | $x=\dfrac{1}{2}\arcsin(\dfrac{1}{2})=\dfrac{\pi}{12}$ | M1,A1 |
| | | **(4)** |
| | | **(8 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **95.** | (a) $9\cos\theta - 2\sin\theta = R\cos(\theta+\alpha)$ | |
| | $R = \sqrt{(9^2+2^2} = \sqrt{85}$ | B1 |
| | $\alpha = \arctan(\frac{2}{9}) = 0.21866... = \text{awrt } 0.2187$ | M1A1 |
| | | **(3)** |
| | (b) (i) $\sqrt{85}$ | B1√ |
| | (ii) $\theta+\alpha = 2\pi \Rightarrow \theta = \text{awrt } 6.06 \text{ 2dp}$ | M1A1 |
| | | **(3)** |
| | (c ) Seeing (or implied by their working) | |
| | $H = 10 - R\cos(\frac{\pi t}{5}+\alpha)$ for their $R$ and α | M1 |
| | $H_{\max} = 10 + \textit{their } R = 10+\sqrt{85}$ $(=19.22\text{m})$ | A1√ |
| | Maximum occurs when $\cos(\frac{\pi t}{5}+\alpha) = -1$ or $(\frac{\pi t}{5}+\alpha) = \pi$ | M1 |
| | $t$= awrt 4.65 | A1 |
| | | **(4)** |
| | (d) Setting and solving $\frac{\pi t}{5} = 2\pi$ (for 1 cycle) or $\frac{\pi t}{5} = 4\pi$ (for 2 cycles) | M1 |
| | Two revolutions = 20 minutes | A1 |
| | | **(2)** |
| | | **(12 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **96.(a)** | $7\cos x + \sin x = R\cos(x-\alpha)$ | |
| | $R = \sqrt{(7^2+1^2)} = \sqrt{50} = (5\sqrt{2})$ | B1 |
| | $\alpha = \arctan\left(\frac{1}{7}\right) = 8.13... = \text{awrt } 8.1^0$ | M1A1 |
| | | **(3)** |
| **(b)** | $\sqrt{50}\cos(x-8.1) = 5 \Rightarrow \cos(x-8.1) = \frac{5}{\sqrt{50}}$ | M1 |
| | $x - 8.1 = 45 \Rightarrow x = 53.1^0$ | M1,A1 |
| | AND $x - 8.1 = 315 \Rightarrow x = 323.1^0$ | M1A1 |
| | | **(5)** |
| **(c )** | One solution if $\frac{k}{\sqrt{50}} = \pm 1, \Rightarrow k = \pm\sqrt{50}$ ft on $R$ | M1A1ft |
| | | **(2)** |
| | | **(10 marks)** |

**Notes for Question 96**

(a)

B1 $R=\sqrt{50}$ . Accept $5\sqrt{2}$ Accept $R=\pm\sqrt{50}$

Do not accept $R=\sqrt{(7^2+1^2)}$ or the decimal equivalent 7.07…unless you see $\sqrt{50}$ or $5\sqrt{2}$ as well

M1 For $\tan\alpha=\pm\frac{1}{7}$ or $\tan\alpha=\pm\frac{7}{1}$. Condone if this comes from $\cos\alpha=7, \sin\alpha=1$

If $R$ is used then only accept $\sin\alpha=\pm\frac{1}{R}$ or $\cos\alpha=\pm\frac{7}{R}$

A1 $\alpha=$ awrt 8.1.

Be aware that $\tan\alpha=7\Rightarrow\alpha=81.9$ can easily be mistaken for the correct answer

Note that the radian answer awrt 0.1418… is A0

(b)

M1 For using their answers to part (a) and moving from $R\cos(x\pm\alpha)=5\Rightarrow\cos(x\pm\alpha)=\frac{5}{R}$ using their numerical values of $R$ and $\alpha$

This may be implied for sight of 53.1 if $R$ and $\alpha$ were correct

M1 For achieving $x\pm\alpha=$ awrt $45^0$ or 315, leading to one value of $x$ **in the range**

Note that for this to be scored $R$ has to be correct (to 2sf) as awrt 45, 315 must be achieved

This may be implied for achieving an answer of either $45+\textit{their }\alpha$ or $315+\textit{their }\alpha$

A1 One correct answer, either awrt 53.1° or 323.1°

M1 For an attempt at finding a secondary value of $x$ **in the range**.

Usually this is an attempt at solving $x-\textit{their } 8.1^0=360^0-\textit{their } 45^0\Rightarrow x=..$

A1 Both values correct awrt 53.1° and 323.1°.

Withhold this mark if there are extra values in the range.

Ignore extra values outside the range

(c )

M1 For stating that $\frac{k}{\textit{their } R}=1$ OR $\frac{k}{\textit{their } R}=-1$

This may be implied by seeing $k=(\pm)\textit{their } R$

A1ft Both values $k=\pm\sqrt{50}$ oe . Follow through on their numerical R

Answers all in radians. Lose the first time that it appears but demand an accuracy of 2dp.

Part (a) $R=\sqrt{50}$ $\alpha=\textit{awrt } 0.14$

Part (b) $x=\textit{awrt } 0.927,\ 5.64$ . Accuracy must be to 3 sf.

With correct working this would score (a) B1M1A0 (b) M1A1A1M1A1

Mixed degrees and radians refer to the main scheme

| Question Number | Scheme | Marks |
|---|---|---|
| **97.** | (a) $R^2 = 6^2 + 8^2 \Rightarrow R = 10$ | M1A1 |
| | $\tan\alpha = \frac{8}{6} \Rightarrow \alpha = \text{awrt } 0.927$ | M1A1 |
| | | (4) |
| | (b)(i) $p(x) = \frac{4}{12 + 10\cos(\theta - 0.927)}$ | |
| | $p(x) = \frac{4}{12 - 10}$ | M1 |
| | Maximum = 2 | A1 |
| | | (2) |
| | (b)(ii) $\theta - \text{'}their\ \alpha\text{'} = \pi$ | M1 |
| | $\theta = \text{awrt } 4.07$ | A1 |
| | | (2) |
| | | **(8 marks)** |

(a) M1 Using Pythagoras' Theorem with 6 and 8 to find *R*. Accept $R^2 = 6^2 + 8^2$

If α has been found first accept $R = \pm\frac{8}{\sin\text{'}\alpha\text{'}}$ or $R = \pm\frac{6}{\cos\text{'}\alpha\text{'}}$

A1 $R = 10$. Many candidates will just write this down which is fine for the 2 marks. Accept $\pm 10$ but not -10

M1 For $\tan\alpha = \pm\frac{8}{6}$ or $\tan\alpha = \pm\frac{6}{8}$

If *R* is used then only accept $\sin\alpha = \pm\frac{8}{R}$ or $\cos\alpha = \pm\frac{6}{R}$

A1 $\alpha = \text{awrt } 0.927$. Note that $53.1^0$ is A0

(b) Note that (b)(i) and (b)(ii) can be marked together

(i) M1 Award for $p(x) = \frac{4}{12 - \text{'}R\text{'}}$.

A1 Cao $p(x)_{\max} = 2$.

The answer is acceptable for both marks as long as no incorrect working is seen

(ii) M1 For setting $\theta - \text{'}their\ \alpha\text{'} = \pi$ and proceeding to $\theta$=..

If working exclusively in degrees accept $\theta - \text{'}their\ \alpha\text{'} = 180$

Do not accept mixed units

A1 $\theta = \text{awrt } 4.07$. If the final A mark in part (a) is lost for 53.1, then accept awrt 233.1

| Question Number | Scheme | Marks |
|---|---|---|
| **98.** | (i) $(\sin 22.5+\cos 22.5)^2=\sin^2 22.5+\cos^2 22.5+......$ | M1 |
| | $=\sin^2 22.5+\cos^2 22.5+2\sin 22.5\cos 22.5$ | |
| | States or uses $\sin^2 22.5+\cos^2 22.5=1$ | B1 |
| | **Uses** $2\sin x\cos x=\sin 2x \Rightarrow 2\sin 22.5\cos 22.5=\sin 45$ | M1 |
| | $(\sin 22.5+\cos 22.5)^2=1+\sin 45$ | A1 |
| | $=1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$ cso | A1 |
| | | (5) |
| | (ii) (a) $\cos 2\theta+\sin\theta=1 \Rightarrow 1-2\sin^2\theta+\sin\theta=1$ | M1 |
| | $\sin\theta-2\sin^2\theta=0$ | |
| | $2\sin^2\theta-\sin\theta=0$ or $k=2$ | A1* |
| | | (2) |
| | (b) $\sin\theta(2\sin\theta-1)=0$ | M1 |
| | $\sin\theta=0,\quad \sin\theta=\frac{1}{2}$ | A1 |
| | Any two of 0,30,150,180 | B1 |
| | All four answers 0,30,150,180 | A1 |
| | | (4) |
| | | **(11 marks)** |

(i) M1 Attempts to expand $(\sin 22.5+\cos 22.5)^2$. Award if you see $\sin^2 22.5+\cos^2 22.5+......$

There must be > two terms. Condone missing brackets ie $\sin 22.5^2+\cos 22.5^2+......$

B1 Stating or using $\sin^2 22.5+\cos^2 22.5=1$. Accept $\sin 22.5^2+\cos 22.5^2=1$ as the intention is clear. Note that this may also come from using the double angle formula

$\sin^2 22.5+\cos^2 22.5=(\frac{1-\cos 45}{2})+(\frac{1+\cos 45}{2})=1$

M1 Uses $2\sin x\cos x=\sin 2x$ to write $2\sin 22.5\cos 22.5$ as sin 45 or sin(2×22.5)

A1 Reaching the intermediate answer $1+\sin 45$

A1 Cso $1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$. Be aware that both 1.707 and $\frac{2+\sqrt{2}}{2}$ can be found by using a calculator for 1+sin45. Neither can be accepted on their own without firstly seeing one of the two answers given above. **Each stage should be shown as required by the mark scheme.**

Note that if the candidates use $(\sin\theta+\cos\theta)^2$ they can pick up the first M and B marks, but no others until they use $\theta=22.5$. All other marks then become available.

(iia) M1 Substitutes $\cos 2\theta=1-2\sin^2\theta$ in $\cos 2\theta+\sin\theta=1$ to produce an equation in $\sin\theta$ only.

It is acceptable to use $\cos 2\theta=2\cos^2\theta-1$ *or* $\cos^2\theta-\sin^2\theta$ as long as the $\cos^2\theta$ is subsequently replaced by $1-\sin^2\theta$

A1* Obtains the correct simplified equation in $\sin\theta$.

$\sin\theta-2\sin^2\theta=0$ or $\sin\theta=2\sin^2\theta$ must be written in the form $2\sin^2\theta-\sin\theta=0$ as required by the question. Also accept $k=2$ as long as no incorrect working is seen.

(iib) M1 Factorises or divides by $\sin\theta$. For this mark $1='k'\sin\theta$ is acceptable. If they have a 3 TQ in $\sin\theta$ this can be scored for correct factorisation

A1 **Both** $\sin\theta=0$, and $\sin\theta=\frac{1}{2}$

B1 Any two answers from 0, 30, 150, 180.

A1 All four answers 0, 30, 150, 180 with no extra solutions inside the range. Ignore solutions outside the range.

| Question Number | Scheme | Marks |
| --- | --- | --- |
| **98.alt 1** | (i) $(\sin 22.5+\cos 22.5)^2=\sin^2 22.5+\cos^2 22.5+......$ | M1 |
| | $=\sin^2 22.5+\cos^2 22.5+2\sin 22.5\cos 22.5$ | |
| | States or uses $\sin^2 22.5+\cos^2 22.5=1$ | B1 |
| | **Uses** $2\sin x\cos x=2\sqrt{\frac{1-\cos 2x}{2}}\sqrt{\frac{\cos 2x+1}{2}}\Rightarrow\sqrt{1-\cos 45}\sqrt{1+\cos 45}$ | M1 |
| | $=\sqrt{1-\cos^2 45}$ | A1 |
| | Hence $(\sin 22.5+\cos 22.5)^2=1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$ | A1 |
| | | (5) |

| Question Number | Scheme | Marks |
| --- | --- | --- |
| **98.alt 2** | (i) Uses Factor Formula $(\sin 22.5+\sin 67.5)^2=\left(2\sin 45\cos 22.5\right)^2$ | M1,A1 |
| | Reaching the stage $=2\cos^2 22.5$ | B1 |
| | Uses the double angle formula $=2\cos^2 22.5=1+\cos 45$ | M1 |
| | $=1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$ | A1 |
| | | (5) |

| Question Number | Scheme | Marks |
| --- | --- | --- |
| **98.alt 3** | (i) Uses Factor Formula $(\cos 67.5+\cos 22.5)^2=\left(2\cos 45\cos 22.5\right)^2$ | M1,A1 |
| | Reaching the stage $=2\cos^2 22.5$ | B1 |
| | Uses the double angle formula $=2\cos^2 22.5=1+\cos 45$ | M1 |
| | $=1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$ | A1 |
| | | (5) |

| Question Number | Scheme | Marks |
|---|---|---|
| **99.** | (a) $4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = \dfrac{4}{\sin^2 2\theta} - \dfrac{1}{\sin^2\theta}$ | |
| | $= \dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta}$ | B1 B1 |
| | | (2) |
| | (b) $\dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta} = \dfrac{\cancel{4}}{\cancel{4}\sin^2\theta\cos^2\theta} - \dfrac{1}{\sin^2\theta}$ | |
| | $= \dfrac{1}{\sin^2\theta\cos^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta\cos^2\theta}$ | M1 |
| | Using $1-\cos^2\theta = \sin^2\theta$ $\quad = \dfrac{\cancel{\sin^2\theta}}{\cancel{\sin^2\theta}\cos^2\theta}$ | M1 |
| | $= \dfrac{1}{\cos^2\theta} = \sec^2\theta$ | M1A1* |
| | | (4) |
| | (c ) $\sec^2\theta = 4 \Rightarrow \sec\theta = \pm 2 \Rightarrow \cos\theta = \pm\dfrac{1}{2}$ | M1 |
| | $\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ | A1,A1 |
| | | (3) |
| | | **(9 marks)** |

**Note (a) and (b) can be scored together**

(a) B1 One term correct. Eg. writes $4\operatorname{cosec}^2 2\theta$ as $\dfrac{4}{(2\sin\theta\cos\theta)^2}$ **or** $\operatorname{cosec}^2\theta$ as $\dfrac{1}{\sin^2\theta}$. Accept terms like $\operatorname{cosec}^2\theta = 1+\cot^2\theta = 1+\dfrac{\cos^2\theta}{\sin^2\theta}$. The question merely asks for an expression in sin $\theta$ and cos $\theta$

B1 A fully correct expression in sin $\theta$ and cos $\theta$. Eg. $\dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta}$ **Accept equivalents**

Allow a different variable say $x$'s instead of $\theta$'s but do not allow mixed units.

b) M1 Attempts to combine their expression in $\sin\theta$ and $\cos\theta$ using a common denominator. The terms can be separate but the denominator must be correct and one of the numerators must have been adapted

M1 Attempts to form a 'single' term on the numerator by using the identity $1-\cos^2\theta = \sin^2\theta$

M1 Cancels correctly by $\sin^2\theta$ terms and replaces $\dfrac{1}{\cos^2\theta}$ with $\sec^2\theta$

A1* Cso. This is a given answer. All aspects must be correct

**IF IN ANY DOUBT SEND TO REVIEW OR CONSULT YOUR TEAM LEADER**

c ) M1 For $\sec^2\theta = 4$ leading to a solution of $\cos\theta$ by taking the root and inverting in either order . Similarly accept $\tan^2\theta = 3, \sin^2\theta = \dfrac{3}{4}$ leading to solutions of $\tan\theta, \sin\theta$. Also accept $\cos 2\theta = -\dfrac{1}{2}$

A1 Obtains one correct answer usually $\theta = \dfrac{\pi}{3}$ Do not accept decimal answers or degrees

A1 Obtains both correct answers. $\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ Do not award if there are extra solutions inside the range. Ignore solutions outside the range.

| Question Number | Scheme | Marks |
|---|---|---|
| **100.** | (a) $R=25$ | B1 |
| | $\tan\alpha=\frac{24}{7}\Rightarrow\alpha=(\text{awrt})73.7^0$ | M1A1 |
| | | (3) |
| | (b) $\cos(2x+\textit{their }\alpha)=\frac{12.5}{\textit{their } R}$ | M1 |
| | $2x+\textit{their}\,'\alpha'=60^0$ | A1 |
| | $2x+\textit{their}\,'\alpha'=\text{their }300^0\text{ or their }420^0\Rightarrow x=..$ | M1 |
| | $x=\textit{awrt }113.1^0,173.1^0$ | A1A1 |
| | | (5) |
| | (c ) Attempts to use $\cos 2x=2\cos^2 x-1$ **AND** $\sin 2x=2\sin x\cos x$ in the expression | M1 |
| | $14\cos^2 x-48\sin x\cos x=7(\cos 2x+1)-24\sin 2x$ | |
| | $=7\cos 2x-24\sin 2x+7$ | A1 |
| | | (2) |
| | (d) $14\cos^2 x-48\sin x\cos x=R\cos(2x+\alpha)+7$ | |
| | Maximum value =’$R$’+’$c$’ | M1 |
| | = 32 cao | A1 |
| | | (2) |
| | | **(12 marks)** |

(a) B1 Accept 25, awrt 25.0, √625. Condone ±25

M1 For $\tan\alpha=\pm\frac{24}{7}$ $\tan\alpha=\pm\frac{7}{24}$ $\sin\alpha=\pm\frac{24}{\textit{their } R}$, $\cos\alpha=\pm\frac{7}{\textit{their } R}$

A1 $\alpha=(\text{awrt})73.7^0$. The answer 1.287 (radians) is A0

(b) M1 For using part (a) and dividing by their $R$ to reach $\cos(2x+\textit{their }\alpha)=\frac{12.5}{\textit{their } R}$

A1 Achieving $2x+\textit{their }\alpha=60^{(0)}$ . This can be implied by $113.1^{(0)}/113.2^{(0)}$ or $173.1^{(0)}/173.2^{(0)}$ or $-6.8^{(0)}/-6.85^{(0)}/-6.9^{(0)}$

M1 Finding a secondary value of $x$ from their principal value. A correct answer will imply this mark

Look for $\frac{360\pm\text{'their' principal value}\pm\text{'their' }\alpha}{2}$

A1 $x=\textit{awrt }113.1^0/113.2^0$ OR $173.1^0/173.2^0$.

A1 $x=\textit{awrt }113.1^0$ AND $173.1^0$. Ignore solutions outside of range. Penalise this mark for extra solutions inside the range

(c ) M1 Attempts to use $\cos 2x = 2\cos^2 x - 1$ **and** $\sin 2x = 2\sin x \cos x$ in expression.
Allow slips in sign on the $\cos 2x$ term. So accept $2\cos^2 x = \pm\cos 2x \pm 1$
A1 Cao $= 7\cos 2x - 24\sin 2x + 7$. The order of terms is not important. Also accept a=7, b=-24, c=7

(d) M1 This mark is scored for adding their $R$ to their $c$
A1 cao 32

**Radian solutions- they will lose the first time it occurs (usually in a with 1.287 radians) Part b will then be marked as follows**

(b) M1 For using part (a) and dividing by their $R$ to reach $\cos(2x + their\,\alpha) = \dfrac{12.5}{their\ R}$

A1 The correct principal value of $\dfrac{\pi}{3}$ or awrt $1.05$ radians. Accept $60^{(0)}$
This can be implied by awrt – 0.12 radians or awrt or 1.97 radians or awrt 3.02 radians

M1 Finding a secondary value of $x$ from their principal value. A correct answer will imply this mark
Look for $\dfrac{2\pi \pm \text{'their' principal value} \pm \text{'their' } \alpha}{2}$ Do not allow mixed units.

A1 $x = awrt\, 1.97$ OR $3.02$.

A1 $x = awrt\, 1.97$ AND $3.02$. Ignore solutions outside of range. Penalise this mark for extra solutions inside the range

| Question No | Scheme | Marks |
|---|---|---|
| **101.** | Uses the identity $cot^2(3\theta) = cosec^2(3\theta) - 1$ in $2cot^2(3\theta) = 7cosec(3\theta) - 5$ | **M1** |
| | $2cosec^2(3\theta) - 7cosec(3\theta) + 3 = 0$ | **A1** |
| | $(2cosec3\theta - 1)(cosec3\theta - 3) = 0$ | **dM1** |
| | $cosec3\theta = 3$ | **A1** |
| | $\theta = \frac{invsin(\frac{1}{3})}{3}, \ \frac{19.5°}{3} = awrt\ 6.5°$ | **ddM1, A1** |
| | $\theta = \frac{180° - invsin(\frac{1}{3})}{3}, 53.5°$ Correct 2nd value | **ddM1,A1** |
| | $\theta = \frac{360° + invsin(\frac{1}{3})}{3}$ Correct 3rd value | **ddM1** |
| | All 4 correct answers awrt $6.5^0, 53.5^0, 126.5\ ^0$ or $173.5^0$ | **A1** **(10 marks)** |

M1 Uses the substitution $cot^2(3\theta) = \pm 1 \pm cosec^2(3\theta)$ to produce a quadratic equation in $cosec(3\theta)$
Accept 'invisible' brackets in which $2cot^2(3\theta)$ is replaced by $2cosec^2(3\theta) - 1$
A (longer) but acceptable alternative is to convert everything to $sin(3\theta)$.
For this to be scored $cot^2 3\theta$ must be replaced by $\frac{cos^2(3\theta)}{sin^2(3\theta)}$, $cosec(3\theta)$ must be replaced by $\frac{1}{\sin 3\theta}$.
An attempt must be made to multiply by $sin^2(3\theta)$ and finally $cos^2(3\theta)$replaced by $= \pm 1 \pm sin^2(3\theta)$

A1 A correct equation (=0) written or implied by working is obtained. Terms must be collected together on one side of the equation. The usual alternatives are
$2cosec^2(3\theta) - 7cosec(3\theta) + 3 = 0$ or $3sin^2(3\theta) - 7sin(3\theta) + 2 = 0$

dM1 Either an attempt to factorise a 3 term quadratic **in $\boldsymbol{cosec(3\theta)}$ or $\boldsymbol{sin(3\theta)}$** with the usual rules
Or use of a correct formula to produce a solution in $cosec(3\theta)$ or $sin(3\theta)$

A1 Obtaining the correct value of $cosec(3\theta) = 3$ or $\sin(3\theta) = \frac{1}{3}$. Ignore other values

ddM1 Correct method to produce the principal value of θ. It is dependent upon the two M's being scored.
Look for $\theta = \frac{invsin(their\ \frac{1}{3})}{3}$

A1 Awrt 6.5

ddM1 Correct method to produce a secondary value. This is dependent upon the candidate having scored the first 2 M's. Usually you look for $\frac{180 - \text{their } 19.5}{3}$ or $\frac{360 + \text{their } 19.5}{3}$ $or$ $\frac{540 - \text{their } 19.5}{3}$
**Note 180-their 6.5 must be marked correct BUT 360+their 6.5 is incorrect**

A1 Any other correct answer. Awrt 6.5,53.5,126.5 or 173.5

ddM1 Correct method to produce a THIRD value. This is dependent upon the candidate having scored the first 2 M's . See above for alternatives

A1 All 4 correct answers awrt 6.5,53.5,126.5 or 173.5 and no extras inside the range. Ignore any answers outside the range.

Radian answers: awrt 0.11, 0.93, 2.21, 3.03. Accuracy must be to 2dp.
Lose the first mark that could have been scored. Fully correct radian answer scores 1,1,1,1,1,0,1,1,1,1=9 marks
Candidates cannot mix degrees and radians for method marks.

**Special case: Some candidates solve the equation in $\boldsymbol{cosec(\theta\ or\ x)}$, $\boldsymbol{\sin(\theta\ or\ x)}$ to produce $\boldsymbol{cosec(\theta\ or\ x) = 3}$**

$$\boldsymbol{\sin(\theta\ or\ x\ ) = \frac{1}{3}}.$$

M1 Combines the two fractions to form a single fraction with a common denominator. Cubic denominators are fine for this mark. Allow slips on the numerator but one must have been adapted. Allow 'invisible' brackets. Accept two separate fractions with the same denominator. Amongst many possible options are

Correct $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$ ,Invisible bracket $\frac{3x+1-2x-1}{(2x-1)(x+4)}$ ,

Cubic and separate $\frac{3(x+1)(x+4)}{(2x^2+7x-4)(x+4)} - \frac{2x^2+7x-4}{(2x^2+7x-4)(x+4)}$

M1 Simplifies the (now) single fraction to one with a linear numerator divided by a quadratic factorised denominator. Any cubic denominator must have been fully factorised (check first and last terms) and cancelled with terms on a fully factorised numerator (check first and last terms).

A1* Cso. This is a given solution and it must be fully correct. All bracketing/algebra must have been correct. You can however accept $\frac{x+4}{(2x-1)(x+4)}$ going to $\frac{1}{2x-1}$ without the need for 'seeing' the cancelling

**For example** $\frac{3(x+1)-2x-1}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A0. Incorrect line leading to solution.

**Whereas** $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A1

**(b)**

M1 This is awarded for an attempt to make x or a swapped y the subject of the formula. The minimum criteria is that they start by multiplying by (2$x$-1) and finish with x= or swapped y=. Allow 'invisible' brackets.

M1 For applying the order of operations correctly. Allow maximum of one 'slip'. Examples of this are

$y = \frac{1}{2x-1} \to y(2x-1) = 1 \to 2x - 1 = \frac{1}{y} \to x = \frac{\frac{1}{y} \pm 1}{2}$ (allow slip on sign)

$y = \frac{1}{2x-1} \to y(2x-1) = 1 \to 2xy - y = 1 \to 2xy = 1 \pm y \to x = \frac{1 \pm y}{2y}$ (allow slip on sign)

$y = \frac{1}{2x-1} \to 2x - 1 = \frac{1}{y} \to 2x = \frac{1}{y} + 1 \to x = \frac{1}{2y} + 1$ (allow slip on ÷2)

A1 Must be written in terms of x but can be $y = \frac{1+x}{2x}$ or equivalent inc $y = \frac{\frac{1}{x}+1}{2}$, $y = \frac{x^{-1}+1}{2}$, $y = \frac{1}{2x} + \frac{1}{2}$

**( c)**

B1 Accept x>0, (0,∞), domain is all values more than 0. **Do not accept** x$\geq 0$ , y>0, [0, ∞], $f^{-1}(x) > 0$

**(d)**

M1 Attempt to write down fg(x) and set it equal to 1/7.
The order must be correct but accept incorrect or lack of bracketing. Eg $\frac{1}{2\ln x+1-1} = \frac{1}{7}$

A1 Achieving correctly the line $\ln(x+1) = 4$. Accept also $\ln(x+1)^2 = 8$

M1 Moving from $\ln(x \pm A) = c \quad A \neq 0 \text{ to } x =$ The ln work must be correct
Alternatively moving from $\ln(x+1)^2 = c$ to $x = \cdots$
Full solutions to calculate $x$ leading from $gf(x) = \frac{1}{7}$, that is $\ln\left(\frac{1}{2x-1} + 1\right) = \frac{1}{7}$ can score this mark.

A1 Correct answer only $= e^4 - 1$ . Accept $e^4 - e^0$

| Question No | Scheme | Marks |
| --- | --- | --- |
| 102 | (a) $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}$ | M1A1 |
| | $= \frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}$ $(\div\, cosAcosB)$ | M1 |
| | $= \frac{tanA+tanB}{1-tanAtanB}$ | A1 * |
| | | (4) |
| | (b) $\tan\left(\theta+\frac{\pi}{6}\right) = \frac{tan\theta+tan\frac{\pi}{6}}{1-tan\theta tan\frac{\pi}{6}}$ | M1 |
| | $= \frac{tan\theta+\frac{1}{\sqrt{3}}}{1-tan\theta\frac{1}{\sqrt{3}}}$ | M1 |
| | $= \frac{\sqrt{3}tan\theta+1}{\sqrt{3}-tan\theta}$ | A1 * |
| | | (3) |
| | (c ) $\tan\left(\theta+\frac{\pi}{6}\right) = \tan(\pi-\theta)$. | M1 |
| | $\left(\theta+\frac{\pi}{6}\right) = (\pi-\theta)$ | dM1 |
| | $\theta=\frac{5}{12}\pi$ | ddM1 A1 |
| | $\tan\left(\theta+\frac{\pi}{6}\right) = \tan(2\pi-\theta)$ | dddM1 |
| | $\theta=\frac{11}{12}\pi$ | A1 |
| | | (6) |
| | | (13 MARKS) |

**(a)**

**M1** Uses the identity **{** $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}\} = \frac{sinAcosB\pm cosAsinB}{cosAcosB\mp sinAsinB}$. Accept incorrect signs for this.
Just the right hand side is acceptable.

A1 Fully correct statement in terms of cos and sin $\{\tan(A+B)\} = \frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}$

M1 Divide **both** numerator and denominator by cosAcosB. This can be stated or implied by working. If implied you must have seen at least one term modified on both the numerator and denominator.

A1* This is a given solution. The last two principal's reports have highlighted lack of evidence in such questions. Both sides of the identity must be seen or implied. Eg lhs=
The minimum expectation for full marks is

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{sinAcosB + cosAsinB}{cosAcosB - sinAsinB} = \frac{\frac{sinA}{cosA} + \frac{sinB}{cosB}}{1 - \frac{sinAsinB}{cosAcosB}} = \frac{tanA + tanB}{1 - tanAtanB}$$

The solution $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB} = \frac{tanA+tanB}{1-tanAtanB}$ scores M1A1M0A0

The solution $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}\ (\div\ cosAcosB) = \frac{tanA+tanB}{1-tanAtanB}$ scores M1A1M1A0

**(b)**

M1 An attempt to use part (a) with A=θ and B=$\frac{\pi}{6}$. Seeing $\frac{tan\theta+tan\frac{\pi}{6}}{1-tan\theta tan\frac{\pi}{6}}$ is enough evidence. Accept sign slips

M1 Uses the identity $\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}\ or\ \frac{\sqrt{3}}{3}$ in the rhs of the identity on both numerator and denominator

A1* cso. This is a given solution. Both sides of the identity must be seen. All steps must be correct with no unreasonable jumps. Accept

$$\tan\left(\theta+\frac{\pi}{6}\right) = \frac{tan\theta + tan\frac{\pi}{6}}{1 - tan\theta tan\frac{\pi}{6}} = \frac{tan\theta + \frac{1}{\sqrt{3}}}{1 - tan\theta\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}tan\theta + 1}{\sqrt{3} - tan\theta}$$

However the following is only worth 2 out of 3 as the last step is an unreasonable jump without further explanation
.

$$\tan\left(\theta+\frac{\pi}{6}\right) = \frac{tan\theta + tan\frac{\pi}{6}}{1 - tan\theta tan\frac{\pi}{6}} = \frac{tan\theta + \frac{\sqrt{3}}{3}}{1 - tan\theta\frac{\sqrt{3}}{3}} = \frac{\sqrt{3}tan\theta + 1}{\sqrt{3} - tan\theta}$$

**(c )**

M1 Use the given identity in (b) to obtain $\tan\left(\theta+\frac{\pi}{6}\right) = \tan(\pi-\theta)$. Accept sign slips

dM1 Writes down an equation that will give one value of θ , usually $\theta + \frac{\pi}{6} = \pi - \theta$ . This is dependent upon the first M mark. Follow through on slips

ddM1 Attempts to solve their equation in θ. It must end θ= and the first two marks must have been scored.

A1 Cso θ=$\frac{5}{12}\pi$ or $\frac{11}{12}\pi$

dddM1 Writes down an equation that would produce a second value of θ . Usually $\theta + \frac{\pi}{6} = 2\pi - \theta$

A1 cso θ= $\frac{5}{12}\pi\ (accept\ \frac{\pi}{2.4})$and $\frac{11}{12}\pi$ with no extra solutions in the range. Ignore extra solutions outside the range.

Note that under this method one correct solution would score 4 marks. A small number of candidates find the second solution only. They would score 1,1,1,1,0,0

**Alternative to (a) starting from rhs**

M1 Uses correct identities for both *tan*A and *tan*B in the rhs expression. Accept only errors in signs

A1 $\frac{tanA+tanB}{1-tanAtanB} = \frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}$

M1 Multiplies both numerator and denominator by *cos*A*cos*B. This can be stated or implied by working. If implied you must have seen at least one term modified on both the numerator and denominator

A1 This is a given answer. Correctly completes proof. All three expressions must be seen or implied.

$$\frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}=\frac{\sin(A+B)}{\cos(A+B)}=\tan(A+B)$$

**Alternative to (a) starting from both sides**

The usual method can be marked like this

M1 Uses correct identities for both *tan*A and *tan*B in the rhs expression. Accept only errors in signs

A1 $\frac{tanA+tanB}{1-tanAtanB}=\frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}$

M1 Multiplies both numerator and denominator by *cos*A*cos*B. This can be stated or implied by working. If implied you must have seen at least one term modified on both the numerator and denominator

A1 Completes proof. Starting now from the lhs writes $\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}=\frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}$

**And** then states that the lhs is equal to the rhs **Or** hence proven. There must be a statement of closure

**Alternative to (b) from sin and cos**

**M1** Writes $\tan\left(\theta+\frac{\pi}{6}\right)=\frac{\sin(\theta+\frac{\pi}{6})}{\cos(\theta+\frac{\pi}{6})}=\frac{sin\theta cos\frac{\pi}{6}+cos\theta sin\frac{\pi}{6}}{cos\theta cos\frac{\pi}{6}-sin\theta sin\frac{\pi}{6}}$

**M1** Uses the identities $\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$ and $cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$ oe in the rhs of the identity on both numerator and denominator and divides both numerator and denominator by $\cos\theta$ to produce an identity in $\tan\theta$

**A1** As in original scheme

<u>**Alternative solution for c.**</u> Starting with $1+\sqrt{3}tan\theta=(\sqrt{3}-\tan\theta)\,tan(\pi-\theta)$

Let $\tan\theta=t$

$$1+\sqrt{3}t=(\sqrt{3}-t)(-t)$$

$$t^2-2\sqrt{3}t-1=0$$

$$t=\frac{2\sqrt{3}\pm\sqrt{(12+4)}}{2}$$

$$=\sqrt{3}\pm2$$

Must find an exact surd

$$\theta = \frac{5\pi}{12}, \ \frac{11\pi}{12}$$

Accept the use of a calculator for the A marks as long as there is an exact surd for the solution of the quadratic and exact answers are given.

M1 Starting with $1+\sqrt{3}tan\theta = (\sqrt{3} - \tan\theta)\, tan(\pi-\theta)$ expand $\tan(\pi-\theta)$ by the correct compound angle identity (or otherwise) and substitute tan π=0 to produce an equation in $\tan\theta$

dM1 Collect terms and produce a 3 term quadratic in $\tan\theta$

ddM1 Correct use of quadratic formula to produce exact solutions to $\tan\theta$. All previous marks must have been scored.

dddM1 All 3 previous marks must have been scored. This is for producing two exact values for $\theta$

A1 One solution $\frac{5}{12}\pi\ (accept\ \frac{\pi}{2.4})$ or $\frac{11}{12}\pi$

A1 Both solutions $\frac{5}{12}\pi\ \ (accept\ \frac{\pi}{2.4})$ and $\frac{11}{12}\pi$ and no extra solutions inside the range. Ignore extra solutions outside the range.

**Special case**: Watch for candidates who write $tan(\pi-\theta) = tan(\pi) - tan(\theta) = -tan(\theta)$ and proceed correctly. They will lose the first mark but potentially can score the others.

**Solutions in degrees**

Apply as before. Lose the first correct mark that would have been scored-usually $75^0$

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **103.** **(a)** | $7\cos x - 24\sin x = R\cos(x+\alpha)$ | | |
| | $7\cos x - 24\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$ | | |
| | Equate $\cos x$: $7 = R\cos\alpha$<br>Equate $\sin x$: $24 = R\sin\alpha$ | | |
| | $R = \sqrt{7^2 + 24^2}\,; = 25$ | $R = 25$ | B1 |
| | $\tan\alpha = \frac{24}{7} \Rightarrow \alpha = 1.287002218...^c$ | $\tan\alpha = \frac{24}{7}$ or $\tan\alpha = \frac{7}{24}$<br>awrt 1.287 | M1<br>A1 |
| | Hence, $7\cos x - 24\sin x = 25\cos(x + 1.287)$ | | (3) |
| **(b)** | Minimum value $= \underline{-25}$ | $-25$ or $-R$ | B1ft<br>(1) |
| **(c)** | $7\cos x - 24\sin x = 10$ | | |
| | $25\cos(x+1.287) = 10$ | | |
| | $\cos(x+1.287) = \dfrac{10}{25}$ | $\cos(x \pm \text{their } \alpha) = \dfrac{10}{(\text{their } R)}$ | M1 |
| | $\text{PV} = 1.159279481...^c$ or $66.42182152...^\circ$ | For applying $\cos^{-1}\left(\dfrac{10}{\text{their } R}\right)$ | M1 |
| | So, $x + 1.287 = \{1.159279...^c, 5.123906...^c, 7.442465...^c\}$ | either $2\pi$ + or − their $\text{PV}^c$ or<br>$360^\circ$ + or − their $\text{PV}^\circ$ | M1 |
| | gives, $x = \{3.836906..., 6.155465...\}$ | awrt 3.84 OR 6.16<br>awrt 3.84 AND 6.16 | A1<br>A1 |
| | | | (5)<br>**[9]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **104.** | $2\cos 2\theta = 1 - 2\sin\theta$ | | |
| | $2\left(1 - 2\sin^2\theta\right) = 1 - 2\sin\theta$ | Substitutes either $1 - 2\sin^2\theta$ or $2\cos^2\theta - 1$ or $\cos^2\theta - \sin^2\theta$ for $\cos 2\theta$. | M1 |
| | $2 - 4\sin^2\theta = 1 - 2\sin\theta$ | | |
| | $4\sin^2\theta - 2\sin\theta - 1 = 0$ | Forms a "quadratic in sine" = 0 | M1(*) |
| | $\sin\theta = \dfrac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}$ | Applies the quadratic formula<br>See notes for alternative methods. | M1 |
| | PVs: $\alpha_1 = 54^\circ$ or $\alpha_2 = -18^\circ$ | | |
| | $\theta = \{54, 126, 198, 342\}$ | Any one correct answer<br>180-their pv<br>All four solutions correct. | A1<br>dM1(*)<br>A1 |
| | | | **[6]** |

| Question Number | Scheme | Marks |
|---|---|---|
| **105.** (a) | $\dfrac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$ <br> $\dfrac{\cancel{2}\sin\theta\cancel{\cos\theta}}{\cancel{2}\cos\theta\cancel{\cos\theta}} = \tan\theta$ (as required) **AG** | M1 <br> A1 **cso** <br> (2) |
| (b) | $2\tan\theta = 1 \Rightarrow \tan\theta = \dfrac{1}{2}$ <br> $\theta_1$ = awrt $26.6^\circ$ <br> $\theta_2$ = awrt $-153.4^\circ$ | M1 <br> A1 <br> A1 $\sqrt{\ }$ <br> (3) <br> **[5]** |
| | (a) M1: Uses **both** a correct identity for $\sin 2\theta$ **and** a correct identity for $\cos 2\theta$. <br> Also allow a candidate writing $1+\cos 2\theta = 2\cos^2\theta$ on the denominator. <br> Also note that angles **must be consistent** in when candidates apply these identities. <br> A1: Correct proof. No errors seen. <br><br> (b) 1st M1 for either $2\tan\theta = 1$ or $\tan\theta = \dfrac{1}{2}$, seen or implied. <br> A1: awrt 26.6 <br> A1 $\sqrt{\ }$ : awrt $-153.4^\circ$ or $\theta_2 = -180^\circ + \theta_1$ <br><br> **Special Case**: For candidate solving, $\tan\theta = k$, where $k \neq \frac{1}{2}$, to give $\theta_1$ and $\theta_2 = -180^\circ + \theta_1$, then award M0A0B1 in part (b). <br> **Special Case:** Note that those candidates who writes $\tan\theta = 1$, and gives ONLY two answers of $45^\circ$ and $-135^\circ$ that are inside the range will be awarded SC M0A0B1. | |

| Question Number | Scheme | Marks |
|---|---|---|
| **106.**(a) | $R = \sqrt{6.25}$ or $2.5$ | B1 |
| | $\tan\alpha = \frac{1.5}{2} = \frac{3}{4} \Rightarrow \alpha = \text{awrt } 0.6435$ | M1A1 |
| | | (3) |
| (b) (i) | Max Value $= 2.5$ | B1 $\sqrt{\ }$ |
| (ii) | $\underline{\sin(\theta - 0.6435) = 1}$ or $\underline{\theta - \text{their } \alpha = \frac{\pi}{2}}; \Rightarrow \theta = \text{awrt } 2.21$ | <u>M1</u>;A1 $\sqrt{\ }$ |
| | | (3) |
| (c) | $H_{\text{Max}} = 8.5$ (m) | B1 $\sqrt{\ }$ |
| | $\underline{\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1}$ or $\underline{\frac{4\pi t}{25} = \text{their (b) answer}}; \Rightarrow t = \text{awrt } 4.41$ | M1;A1 |
| | | (3) |
| (d) | $\Rightarrow 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7; \Rightarrow \sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5} = 0.4$ | M1;M1 |
| | $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \sin^{-1}(0.4)$ or awrt $0.41$ | A1 |
| | Either $t = \text{awrt } 2.1$ or awrt $6.7$ | A1 |
| | So, $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \{\pi - 0.411517\ldots \text{ or } 2.730076\ldots^c\}$ | ddM1 |
| | Times $= \{14:06,\ 18:43\}$ | A1 (6) |
| | | **[15]** |
| | (a) B1: $R = 2.5$ or $R = \sqrt{6.25}$. For $R = \pm 2.5$, award B0.<br>M1: $\tan\alpha = \pm\frac{1.5}{2}$ or $\tan\alpha = \pm\frac{2}{1.5}$<br>A1: $\alpha = \text{awrt } 0.6435$<br>(b) B1 $\sqrt{\ }$ : 2.5 or follow through the value of $R$ in part (a).<br>M1: For $\sin(\theta - \text{their } \alpha) = 1$<br>A1 $\sqrt{\ }$ : awrt 2.21 or $\frac{\pi}{2} + \text{their } \alpha$ rounding correctly to 3 sf.<br>(c) B1 $\sqrt{\ }$ : 8.5 or 6 + their $R$ found in part (a) as long as the answer is greater than 6.<br>M1: $\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 1$ or $\frac{4\pi t}{25} = \text{their (b) answer}$<br>A1: For $\sin^{-1}(0.4)$ This can be implied by awrt 4.41 or awrt 4.40.<br>(d) M1: $6 + (\text{their } R)\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 7$, M1: $\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = \frac{1}{\text{their } R}$<br>A1: For $\sin^{-1}(0.4)$. This can be implied by awrt 0.41 or awrt 2.73 or other values for different $\alpha$'s. Note this mark can be implied by seeing 1.055.<br>A1: Either $t = \text{awrt } 2.1$ or $t = \text{awrt } 6.7$<br>ddM1: either $\pi$ – their PV$^c$. Note that this mark is dependent upon the two M marks. This mark will usually be awarded for seeing either 2.730… or 3.373…<br>A1: Both $t = 14:06$ and $t = 18:43$ or both 126 (min) and 403 (min) or both 2 hr 6 min and 6 hr 43 min. | |



EXPERT TUITION

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 107 (a) | $5\cos x - 3\sin x = R\cos(x+\alpha)$, $R > 0$, $0 < x < \frac{\pi}{2}$ | | |
| | $5\cos x - 3\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$ | | |
| | Equate $\cos x$: $5 = R\cos\alpha$<br>Equate $\sin x$: $3 = R\sin\alpha$ | | |
| | $R = \sqrt{5^2+3^2}\,; = \sqrt{34}\ \{= 5.83095..\}$ | $R^2 = 5^2 + 3^2$<br>$\sqrt{34}$ or awrt 5.8 | M1;<br>A1 |
| | $\tan\alpha = \frac{3}{5} \Rightarrow \alpha = 0.5404195003...^c$ | $\tan\alpha = \pm\frac{3}{5}$ or $\tan\alpha = \pm\frac{5}{3}$ or<br>$\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{5}{\text{their } R}$ | M1 |
| | | $\alpha =$ awrt 0.54 or<br>$\alpha =$ awrt $0.17\pi$ or $\alpha = \dfrac{\pi}{\text{awrt } 5.8}$ | A1 |
| | Hence, $5\cos x - 3\sin x = \sqrt{34}\cos(x + 0.5404)$ | | (4) |
| (b) | $5\cos x - 3\sin x = 4$ | | |
| | $\sqrt{34}\cos(x+0.5404) = 4$ | | |
| | $\cos(x+0.5404) = \dfrac{4}{\sqrt{34}}\ \{= 0.68599...\}$ | $\cos(x \pm \text{their } \alpha) = \dfrac{4}{\text{their } R}$ | M1 |
| | $(x+0.5404) = 0.814826916...^c$ | For applying $\cos^{-1}\left(\dfrac{4}{\text{their } R}\right)$ | M1 |
| | $x = 0.2744...^c$ | awrt $0.27^c$ | A1 |
| | $(x+0.5404) = 2\pi - 0.814826916...^c\ \{= 5.468358...^c\}$ | $2\pi -$ their 0.8148 | ddM1 |
| | $x = 4.9279...^c$ | awrt $4.93^c$ | A1 |
| | Hence, $x = \{0.27, 4.93\}$ | | (5) |
| | | | **[9]** |

**Part (b)**: If there are any EXTRA solutions inside the range $0 \le x < 2\pi$, then withhold the final accuracy mark if the candidate would otherwise score all 5 marks. Also ignore EXTRA solutions outside the range $0 \le x < 2\pi$.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 108 | $\text{cosec}^2 2x - \cot 2x = 1, \quad (\text{eqn } *) \qquad 0 \le x \le 180^\circ$ | | |
| | Using $\text{cosec}^2 2x = 1 + \cot^2 2x$ gives<br>$1 + \cot^2 2x - \cot 2x = 1$ | Writing down or using $\text{cosec}^2 2x = \pm 1 \pm \cot^2 2x$ or $\text{cosec}^2\theta = \pm 1 \pm \cot^2 \theta$. | M1 |
| | $\underline{\cot^2 2x - \cot 2x} = 0 \quad \text{or} \quad \cot^2 2x = \cot 2x$ | For either $\underline{\cot^2 2x - \cot 2x}\ \{= 0\}$ or $\cot^2 2x = \cot 2x$ | A1 |
| | $\cot 2x(\cot 2x - 1) = 0 \quad \text{or} \quad \cot 2x = 1$ | Attempt to factorise or solve a quadratic (See rules for factorising quadratics) or cancelling out $\cot 2x$ from both sides. | dM1 |
| | $\cot 2x = 0 \quad \text{or} \quad \cot 2x = 1$ | Both $\cot 2x = 0$ and $\cot 2x = 1$. | A1 |
| | $\cot 2x = 0 \Rightarrow (\tan 2x \to \infty) \Rightarrow 2x = 90, 270$<br>$\Rightarrow x = 45, 135$<br>$\cot 2x = 1 \Rightarrow \tan 2x = 1 \Rightarrow 2x = 45, 225$<br>$\Rightarrow x = 22.5, 112.5$ | Candidate attempts to divide at least one of their principal angles by 2. This will be usually implied by seeing $x = 22.5$ resulting from $\cot 2x = 1$. | ddM1 |
| | Overall, $x = \{22.5, 45, 112.5, 135\}$ | ***Both*** $x = 22.5$ and $x = 112.5$<br>***Both*** $x = 45$ and $x = 135$ | A1<br>B1 |
| | | | **[7]** |

If there are any EXTRA solutions inside the range $0 \le x \le 180^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final accuracy mark (the sixth mark in this question).
Also ignore EXTRA solutions outside the range $0 \le x \le 180^\circ$.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 109 (a) | $\cos^2\theta + \sin^2\theta = 1 \quad (\div \cos^2\theta)$ | | |
| | $\underline{\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}}$ | Dividing $\cos^2\theta + \sin^2\theta = 1$ by $\cos^2\theta$ to give <u>underlined</u> equation. | M1 |
| | $1 + \tan^2\theta = \sec^2\theta$ | | |
| | $\tan^2\theta = \sec^2\theta - 1$ (as required) **AG** | Complete proof. No errors seen. | A1 cso |
| | | | (2) |
| (b) | $2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2, \quad (\text{eqn } *) \qquad 0 \le \theta < 360^\circ$ | | |
| | $2(\sec^2\theta - 1) + 4\sec\theta + \sec^2\theta = 2$ | Substituting $\tan^2\theta = \sec^2\theta - 1$ into eqn * to get a quadratic in $\sec\theta$ only | M1 |
| | $2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$ | | |
| | $\underline{3\sec^2\theta + 4\sec\theta - 4} = 0$ | Forming a three term "one sided" quadratic expression in $\sec\theta$. | M1 |
| | $(\sec\theta + 2)(3\sec\theta - 2) = 0$ | Attempt to factorise or solve a quadratic. | M1 |
| | $\sec\theta = -2$ or $\sec\theta = \frac{2}{3}$ | | |
| | $\dfrac{1}{\cos\theta} = -2$ or $\dfrac{1}{\cos\theta} = \dfrac{2}{3}$ | | |
| | $\underline{\cos\theta = -\tfrac{1}{2}}$; or $\cos\theta = \frac{3}{2}$ | $\underline{\cos\theta = -\tfrac{1}{2}}$ | A1; |
| | $\alpha = 120^\circ$ or $\alpha =$ no solutions | | |
| | $\theta_1 = \underline{120^\circ}$ | $\underline{120^\circ}$ | <u>A1</u> |
| | $\theta_2 = 240^\circ$ | $\underline{240^\circ}$ or $\theta_2 = 360^\circ - \theta_1$ when solving using $\cos\theta = \ldots$ | B1 $\sqrt{}$ |
| | $\theta = \{120^\circ, 240^\circ\}$ | Note the final A1 mark has been changed to a B1 mark. | (6) |
| | | | **[8]** |

| Question Number | Scheme | | Marks |
| --- | --- | --- | --- |
| 110 (a) | $A = B \Rightarrow \cos(A+A) = \cos 2A = \underline{\cos A \cos A - \sin A \sin A}$ | Applies $A = B$ to $\cos(A+B)$ to give the underlined equation or $\cos 2A = \underline{\cos^2 A - \sin^2 A}$ | M1 |
| | $\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives | | |
| | $\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required) | Complete proof, with a link between LHS and RHS. No errors seen. | A1 AG |
| | | | (2) |
| (b) | $C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$ | Eliminating $y$ correctly. | M1 |
| | $3\sin 2x = 4\left(\frac{1-\cos 2x}{2}\right) - 2\cos 2x$ | Using result in part (a) to substitute for $\sin^2 x$ as $\frac{\pm 1 \pm \cos 2x}{2}$ or $k\sin^2 x$ as $k\left(\frac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles. | M1 |
| | $3\sin 2x = 2(1-\cos 2x) - 2\cos 2x$ | | |
| | $3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$ | | |
| | $3\sin 2x + 4\cos 2x = 2$ | Rearranges to give correct result | A1 AG |
| | | | (3) |
| (c) | $3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$ | | |
| | $3\sin 2x + 4\cos 2x = R\cos 2x\cos\alpha + R\sin 2x\sin\alpha$ | | |
| | Equate $\sin 2x$: $3 = R\sin\alpha$<br>Equate $\cos 2x$: $4 = R\cos\alpha$ | | |
| | $R = \sqrt{3^2 + 4^2}; = \sqrt{25} = 5$ | $R = 5$ | B1 |
| | $\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765...^\circ$ | $\tan\alpha = \pm\frac{3}{4}$ or $\tan\alpha = \pm\frac{4}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{4}{\text{their } R}$<br>awrt 36.87 | M1<br>A1 |
| | Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$ | | |
| | | | (3) |



EXPERT TUITION

| Question Number | Scheme | | Marks |
| --- | --- | --- | --- |
| (d) | $3\sin 2x + 4\cos 2x = 2$ | | |
| | $5\cos(2x - 36.87) = 2$ | | |
| | $\cos(2x - 36.87) = \frac{2}{5}$ | $\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$ | M1 |
| | $(2x - 36.87) = 66.42182...^\circ$ | awrt 66 | A1 |
| | $(2x - 36.87) = 360 - 66.42182...^\circ$ | | |
| | Hence, $x = 51.64591...^\circ,\ 165.22409...^\circ$ | One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3 | A1 |
| | | Both awrt 51.6 AND awrt 165.2 | A1 |
| | | | (4) |
| | | If there are any EXTRA solutions inside the range $0 \le x < 180^\circ$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 \le x < 180^\circ$. | |
| | | | **[12]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 111 (a) | $\sin 2x = \underline{2\sin x \cos x}$ | $\underline{2\sin x\cos x}$ | B1 aef<br>(1) |
| (b) | $\operatorname{cosec} x - 8\cos x = 0\,,\qquad 0 < x < \pi$ | | |
| | $\dfrac{1}{\sin x} - 8\cos x = 0$ | Using $\operatorname{cosec} x = \dfrac{1}{\sin x}$ | M1 |
| | $\dfrac{1}{\sin x} = 8\cos x$ | | |
| | $1 = 8\sin x\cos x$ | | |
| | $1 = 4(2\sin x\cos x)$ | | |
| | $1 = 4\sin 2x$ | | |
| | $\underline{\sin 2x = \tfrac{1}{4}}$ | $\sin 2x = k$, where $-1 < k < 1$ and $k \neq 0$<br>$\underline{\sin 2x = \tfrac{1}{4}}$ | M1<br>A1 |
| | Radians $2x = \{0.25268\ldots,\ 2.88891\ldots\}$<br>Degrees $2x = \{14.4775\ldots,\ 165.5225\ldots\}$ | | |
| | Radians $x = \{0.12634\ldots,\ 1.44445\ldots\}$<br>Degrees $x = \{7.23875\ldots,\ 82.76124\ldots\}$ | Either arwt 7.24 or 82.76 or 0.13 or 1.44 or 1.45 or awrt $0.04\pi$ or awrt $0.46\pi$.<br>Both $\underline{0.13}$ and $\underline{1.44}$<br>Solutions for the final two A marks must be given in $x$ only. If there are any EXTRA solutions inside the range $0 < x < \pi$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 < x < \pi$. | A1<br>A1 cao<br>(5)<br>**[6]** |

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| **112.** | (a)(i) $\sin 3\theta = \sin(2\theta+\theta)$ | | | |
| | $= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$ | | | |
| | $= 2\sin\theta\cos\theta.\cos\theta + \left(1-2\sin^2\theta\right)\sin\theta$ | | M1 A1 | |
| | $= 2\sin\theta\left(1-\sin^2\theta\right)+\sin\theta-2\sin^3\theta$ | | M1 | |
| | $= 3\sin\theta - 4\sin^3\theta$ $\ast$ | cso | A1 | **(4)** |
| | (ii) $8\sin^3\theta - 6\sin\theta + 1 = 0$ | | | |
| | $-2\sin 3\theta + 1 = 0$ | | M1 A1 | |
| | $\sin 3\theta = \frac{1}{2}$ | | M1 | |
| | $3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ | | | |
| | $\theta = \frac{\pi}{18}, \frac{5\pi}{18}$ | | A1 A1 | **(5)** |
| | (b) $\sin 15° = \sin(60° - 45°) = \sin 60°\cos 45° - \cos 60°\sin 45°$ | | M1 | |
| | $= \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}} - \frac{1}{2}\times\frac{1}{\sqrt{2}}$ | | M1 A1 | |
| | $= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$ $\ast$ | cso | A1 | **(4)** |
| | | | | **[13]** |
| | *Alternatives to* (b) | | | |
| | ① $\sin 15° = \sin(45° - 30°) = \sin 45°\cos 30° - \cos 45°\sin 30°$ | | M1 | |
| | $= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\times\frac{1}{2}$ | | M1 A1 | |
| | $= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$ $\ast$ | cso | A1 | **(4)** |
| | ② Using $\cos 2\theta = 1 - 2\sin^2\theta$, $\cos 30° = 1 - 2\sin^2 15°$ | | | |
| | $2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\sqrt{3}}{2}$ | | | |
| | $\sin^2 15° = \frac{2-\sqrt{3}}{4}$ | | M1 A1 | |
| | $\left(\frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)\right)^2 = \frac{1}{16}\left(6+2-2\sqrt{12}\right) = \frac{2-\sqrt{3}}{4}$ | | M1 | |
| | Hence $\sin 15° = \frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$ $\ast$ | cso | A1 | **(4)** |

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| **113.** | (a) $R^2 = 3^2 + 4^2$ | | M1 | |
| | $R = 5$ | | A1 | |
| | $\tan\alpha = \frac{4}{3}$ | | M1 | |
| | $\alpha = 53\ ...\ °$ | awrt $53°$ | A1 | **(4)** |
| | (b) Maximum value is 5 | ft their $R$ | B1 ft | |
| | At the maximum, $\cos(\theta - \alpha) = 1$ *or* $\theta - \alpha = 0$ | | M1 | |
| | $\theta = \alpha = 53\ ...\ °$ | ft their $\alpha$ | A1 ft | **(3)** |
| | (c) $\mathrm{f}(t) = 10 + 5\cos(15t - \alpha)°$ | | | |
| | Minimum occurs when $\cos(15t - \alpha)° = -1$ | | M1 | |
| | The minimum temperature is $(10-5)° = 5°$ | | A1 ft | **(2)** |
| | (d) $15t - \alpha = 180$ | | M1 | |
| | $t = 15.5$ | awrt 15.5 | M1 A1 | **(3)** |
| | | | | **[12]** |

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| **114.** | (a) $R^2 = 5^2 + 12^2$ | | M1 | |
| | $R = 13$ | | A1 | |
| | $\tan\alpha = \dfrac{12}{5}$ | | M1 | |
| | $\alpha \approx 1.176$ | cao | A1 | (4) |
| | (b) $\cos(x-\alpha) = \dfrac{6}{13}$ | | M1 | |
| | $x - \alpha = \arccos\dfrac{6}{13} = 1.091\ \ldots$ | | A1 | |
| | $x = 1.091\ \ldots + 1.176\ \ldots \approx 2.267\ldots$ | awrt 2.3 | A1 | |
| | $x - \alpha = -1.091\ \ldots$ | accept … = 5.19 … for M | M1 | |
| | $x = -1.091\ \ldots + 1.176\ \ldots \approx 0.0849\ldots$ | awrt 0.084 or 0.085 | A1 | (5) |
| | (c)(i) $R_{\max} = 13$ | ft their $R$ | B1 ft | |
| | (ii) At the maximum, $\cos(x-\alpha) = 1$ *or* $x - \alpha = 0$ | | M1 | |
| | $x = \alpha = 1.176\ \ldots$ | awrt 1.2, ft their $\alpha$ | A1ft | (3) |
| | | | | **[12]** |

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| **115.** | (a) $\sin^2\theta+\cos^2\theta=1$ | | | |
| | $\div\sin^2\theta$ $\quad \dfrac{\sin^2\theta}{\sin^2\theta}+\dfrac{\cos^2\theta}{\sin^2\theta}=\dfrac{1}{\sin^2\theta}$ | | M1 | |
| | $1+\cot^2\theta=\operatorname{cosec}^2\theta$ ∗ | cso | A1 | (2) |
| | *Alternative for* (a) | | | |
| | $1+\cot^2\theta=1+\dfrac{\cos^2\theta}{\sin^2\theta}=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}=\dfrac{1}{\sin^2\theta}$ | | M1 | |
| | $=\operatorname{cosec}^2\theta$ ∗ | cso | A1 | |
| | (b) $2\left(\operatorname{cosec}^2\theta-1\right)-9\operatorname{cosec}\theta=3$ | | M1 | |
| | $2\operatorname{cosec}^2\theta-9\operatorname{cosec}\theta-5=0$ *or* $5\sin^2\theta+9\sin\theta-2=0$ | | M1 | |
| | $(2\operatorname{cosec}\theta+1)(\operatorname{cosec}\theta-5)=0$ *or* $(5\sin\theta-1)(\sin\theta+2)=0$ | | M1 | |
| | $\operatorname{cosec}\theta=5$ *or* $\sin\theta=\dfrac{1}{5}$ | | A1 | |
| | $\theta=11.5°, 168.5°$ | | A1 A1 | (6) |
| | | | | **[8]** |

| Question Number | Scheme | Marks |
|---|---|---|
| **116.** | (a) $\cos(2x+x)=\cos 2x\cos x-\sin 2x\sin x$ | M1 |
| | $=(2\cos^2 x-1)\cos x-(2\sin x\cos x)\sin x$ | M1 |
| | $=(2\cos^2 x-1)\cos x-2(1-\cos^2 x)\cos x$ any correct expression | A1 |
| | $=4\cos^3 x-3\cos x$ | A1 (4) |
| | (b)(i) $\dfrac{\cos x}{1+\sin x}+\dfrac{1+\sin x}{\cos x}=\dfrac{\cos^2 x+(1+\sin x)^2}{(1+\sin x)\cos x}$ | M1 |
| | $=\dfrac{\cos^2 x+1+2\sin x+\sin^2 x}{(1+\sin x)\cos x}$ | A1 |
| | $=\dfrac{2(1+\sin x)}{(1+\sin x)\cos x}$ | M1 |
| | $=\dfrac{2}{\cos x}=2\sec x$ * cso | A1 (4) |
| | (c) $\sec x=2$ *or* $\cos x=\dfrac{1}{2}$ | M1 |
| | $x=\dfrac{\pi}{3},\dfrac{5\pi}{3}$ accept awrt 1.05, 5.24 | A1, A1 (3) |
| | | **[11]** |
| **117.** | (a) $\dfrac{dy}{dx}=6\cos 2x-8\sin 2x$ | M1 A1 |
| | $\left(\dfrac{dy}{dx}\right)_0=6$ | B1 |
| | $y-4=-\dfrac{1}{6}x$ or equivalent | M1 A1 (5) |
| | (b) $R=\sqrt{(3^2+4^2)}=5$ | M1 A1 |
| | $\tan\alpha=\dfrac{4}{3},\ \alpha\approx 0.927$ awrt 0.927 | M1 A1 (4) |
| | (c) $\sin(2x+\text{ their }\alpha)=0$ | M1 |
| | $x=-2.03,-0.46,1.11,2.68$ | A1 A1 A1 (4) |
| | First A1 any correct solution; second A1 a second correct solution; third A1 all four correct and to the specified accuracy or better. Ignore the $y$-coordinate. | **[13]** |



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