Maths Questions By Topic:

Trigonometry

A-Level Edexcel

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1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$2\sin(x - 60°) = \cos(x - 30°)$$

show that

$$\tan x = 3\sqrt{3}$$

(4)

(b) Hence or otherwise solve, for $0 \leqslant \theta < 180°$

$$2\sin 2\theta = \cos(2\theta + 30°)$$

giving your answers to one decimal place.

(4)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2.

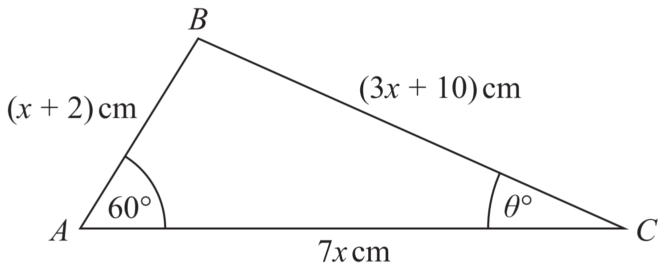

Figure 1

Figure 1 shows a sketch of triangle ABC with $AB = (x + 2)$ cm, $BC = (3x + 10)$ cm, $AC = 7x$ cm, angle $BAC = 60°$ and angle $ACB = \theta°$

(a) (i) Show that $17x^2 - 35x - 48 = 0$

(3)

(ii) Hence find the value of x.

(1)

(b) Hence find the value of θ giving your answer to one decimal place.

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 6 marks)

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos\theta} + \tan\theta \equiv \frac{\cos\theta}{1 - \sin\theta} \qquad \theta \neq (2n+1)90° \quad n \in \mathbb{Z}$$

(3)

Given that $\cos 2x \neq 0$

(b) solve for $0 < x < 90°$

$$\frac{1}{\cos 2x} + \tan 2x = 3\cos 2x$$

giving your answers to one decimal place.

(5)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 8 marks)

4. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that $1 + \cos 2\theta + \sin 2\theta \neq 0$ prove that

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan\theta$$

(4)

(b) Hence solve, for $0 < x < 180°$

$$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3\sin 2x$$

giving your answers to one decimal place where appropriate.

(4)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

5. A parallelogram *PQRS* has area $50\,\text{cm}^2$

Given

- *PQ* has length 14 cm
- *QR* has length 7 cm
- angle *SPQ* is obtuse

find

(a) the size of angle *SPQ*, in degrees, to 2 decimal places,

(3)

(b) the length of the diagonal *SQ*, in cm, to one decimal place.

(2)

Question 5 continued

(Total for Question 5 is 5 marks)

6. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta \leqslant 450°$, the equation

$$5\cos^2\theta = 6\sin\theta$$

giving your answers to one decimal place.

(5)

(ii) (a) A student's attempt to solve the question

"Solve, for $-90° < x < 90°$, the equation $3\tan x - 5\sin x = 0$"

is set out below.

$$3\tan x - 5\sin x = 0$$
$$3\frac{\sin x}{\cos x} - 5\sin x = 0$$
$$3\sin x - 5\sin x\cos x = 0$$
$$3 - 5\cos x = 0$$
$$\cos x = \frac{3}{5}$$
$$x = 53.1°$$

Identify two errors or omissions made by this student, giving a brief explanation of each.

(2)

The first four positive solutions, in order of size, of the equation

$$\cos(5\alpha + 40°) = \frac{3}{5}$$

are α_1, α_2, α_3 and α_4

(b) Find, to the nearest degree, the value of α_4

(2)

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Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. (a) Express $\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$ where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The temperature, θ°C, inside a room on a given day is modelled by the equation

$$\theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2\cos\left(\frac{\pi t}{12} - 3\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (a),

(b) deduce the maximum temperature of the room during this day,

(1)

(c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 7 marks)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta \qquad \theta \neq (180n)° \qquad n \in \mathbb{Z}$$

(3)

(b) Hence, or otherwise, solve for $0 < x < 180°$

$$\operatorname{cosec}x - \sin x = \cos x\cot(3x - 50°)$$

(5)

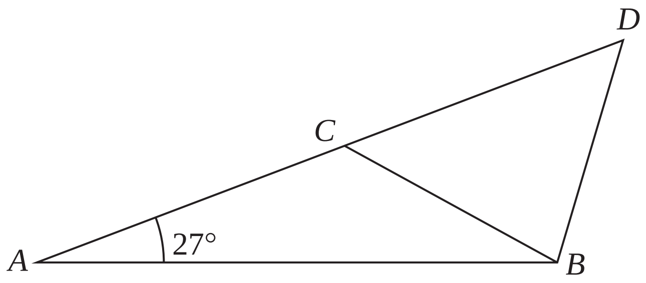
Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 8 marks)

9.



Not to scale

Figure 1

Figure 1 shows the design for a structure used to support a roof.

The structure consists of four steel beams, AB, BD, BC and AD.

Given $AB = 12\,\text{m}$, $BC = BD = 7\text{m}$ and angle $BAC = 27°$

(a) find, to one decimal place, the size of angle ACB.

(3)

The steel beams can only be bought in whole metre lengths.

(b) Find the minimum length of steel that needs to be bought to make the complete structure.

(3)

Question 9 continued

(Total for Question 9 is 6 marks)

10.

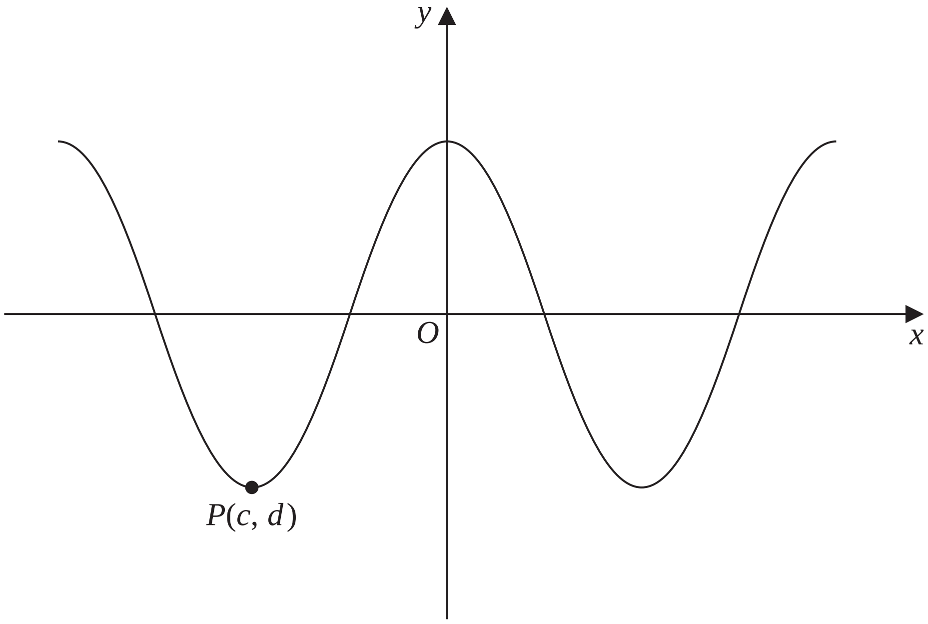


Figure 3

Figure 3 shows part of the curve with equation $y = 3\cos x°$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which a minimum occurs.

(a) State the value of c and the value of d.

(1)

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3\cos x°$ to the curve with equation

(i) $y = 3\cos\left(\dfrac{x°}{4}\right)$

(ii) $y = 3\cos(x - 36)°$

(2)

(c) Solve, for $450° \leqslant \theta < 720°$,

$$3\cos\theta = 8\tan\theta$$

giving your solution to one decimal place.

In part (c) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(5)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 8 marks)

11.

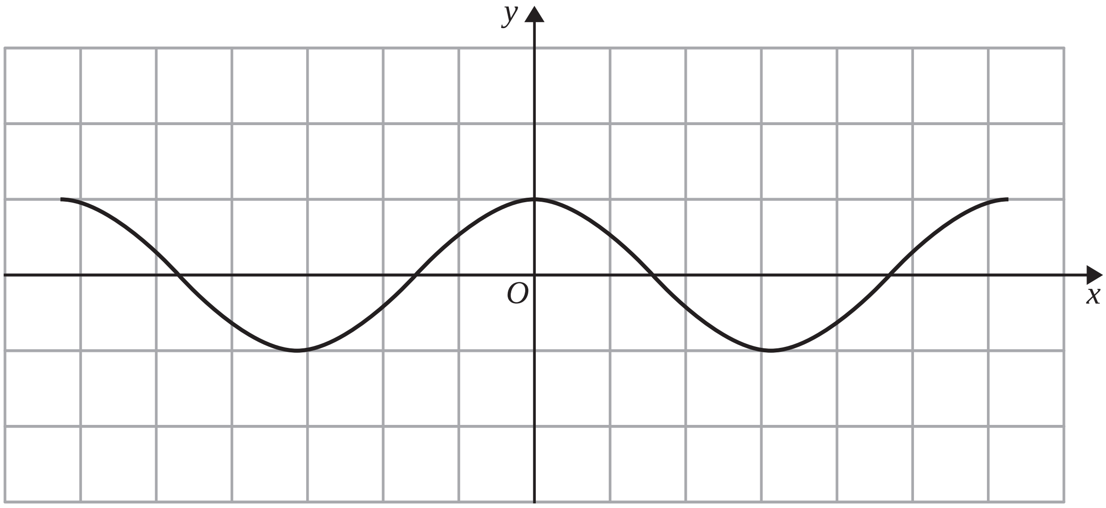


Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians. Diagram 1, on the opposite page, is a copy of Figure 1.

(a) Use Diagram 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

(2)

Given that the root of the equation is α, and that α is small,

(b) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

Question 11 continued

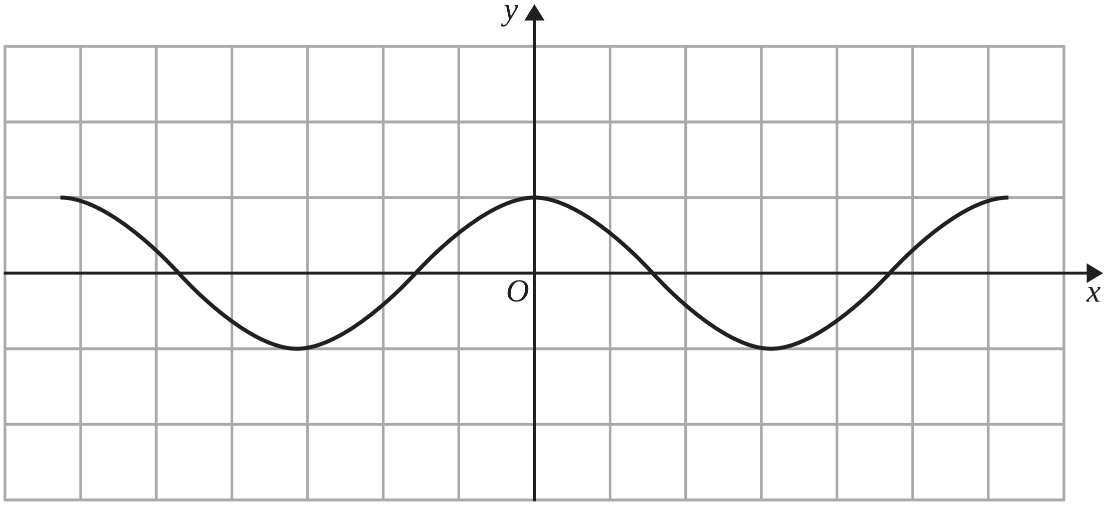


Diagram 1

(Total for Question 11 is 5 marks)

12. (a) Solve, for $-180° \leqslant \theta \leqslant 180°$, the equation

$$5\sin 2\theta = 9\tan\theta$$

giving your answers, where necessary, to one decimal place.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

(b) Deduce the smallest positive solution to the equation

$$5\sin(2x - 50°) = 9\tan(x - 25°)$$

(2)

Question 12 continued

Question 12 continued

Question 12 continued

(Total for Question 12 is 8 marks)

13. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

Question 13 continued

Question 13 continued

(Total for Question 13 is 7 marks)

14.

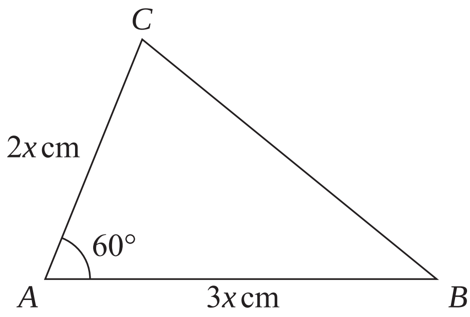


Figure 1

Figure 1 shows a sketch of a triangle *ABC* with $AB = 3x$ cm, $AC = 2x$ cm and angle $CAB = 60°$

Given that the area of triangle *ABC* is $18\sqrt{3}$ cm^2

(a) show that $x = 2\sqrt{3}$

(3)

(b) Hence find the exact length of *BC*, giving your answer as a simplified surd.

(3)

Question 14 continued

(Total for Question 14 is 6 marks)

15. (a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leqslant x < 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

(3)

Question 15 continued

Question 15 continued

Question 15 continued

(Total for Question 15 is 7 marks)

16. Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1 - \cos 4\theta}{2\theta \sin 3\theta}$$

(3)

Question 16 continued

(Total for Question 16 is 3 marks)

17.

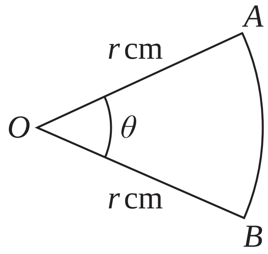


Figure 1

Figure 1 shows a sector *AOB* of a circle with centre *O* and radius *r* cm.

The angle *AOB* is θ radians.
The area of the sector *AOB* is 11 cm^2

Given that the perimeter of the sector is 4 times the length of the arc *AB*, find the exact value of *r*.

(4)

Question 17 continued

(Total for Question 17 is 4 marks)

18. The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)° \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

Question 18 continued

(Total for Question 18 is 5 marks)

19. In a triangle ABC, side AB has length 10 cm, side AC has length 5 cm, and angle $BAC = \theta$ where θ is measured in degrees. The area of triangle ABC is 15 cm^2

(a) Find the two possible values of $\cos\theta$

(4)

Given that BC is the longest side of the triangle,

(b) find the exact length of BC.

(2)

Question 19 continued

(Total for Question 19 is 6 marks)

20. (a) Show that the equation

$$4\cos\theta - 1 = 2\sin\theta\tan\theta$$

can be written in the form

$$6\cos^2\theta - \cos\theta - 2 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < 90°$

$$4\cos 3x - 1 = 2\sin 3x\tan 3x$$

giving your answers, where appropriate, to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

Question 20 continued

Question 20 continued

Question 20 continued

(Total for Question 20 is 8 marks)

21. (i) Solve, for $-90° \leqslant \theta < 270°$, the equation,

$$\sin(2\theta + 10°) = -0.6$$

giving your answers to one decimal place.

(5)

(ii) (a) A student's attempt at the question

"Solve, for $-90° < x < 90°$, the equation $7\tan x = 8\sin x$"

is set out below.

$$7\tan x = 8\sin x$$

$$7 \times \frac{\sin x}{\cos x} = 8\sin x$$

$$7\sin x = 8\sin x \cos x$$

$$7 = 8\cos x$$

$$\cos x = \frac{7}{8}$$

$$x = 29.0° \text{ (to 3 sf)}$$

Identify two mistakes made by this student, giving a brief explanation of each mistake.

(2)

(b) Find the smallest positive solution to the equation

$$7\tan(4\alpha + 199°) = 8\sin(4\alpha + 199°)$$

(2)

Question 21 continued

(Total for Question 21 is 9 marks)

22. (a) Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$

State the value of R and give the value of α to 4 decimal places.

(3)

Tom models the depth of water, D metres, at Southview harbour on 18th October 2017 by the formula

$$D = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), \qquad 0 \leqslant t \leqslant 24$$

where t is the time, in hours, after 00:00 hours on 18th October 2017.

Use Tom's model to

(b) find the depth of water at 00:00 hours on 18th October 2017,

(1)

(c) find the maximum depth of water,

(1)

(d) find the time, in the afternoon, when the maximum depth of water occurs. Give your answer to the nearest minute.

(3)

Tom's model is supported by measurements of D taken at regular intervals on 18th October 2017. Jolene attempts to use a similar model in order to model the depth of water at Southview harbour on 19th October 2017.

Jolene models the depth of water, H metres, at Southview harbour on 19th October 2017 by the formula

$$H = 6 + 2\sin\left(\frac{4\pi x}{25}\right) - 1.5\cos\left(\frac{4\pi x}{25}\right), \qquad 0 \leqslant x \leqslant 24$$

where x is the time, in hours, after 00:00 hours on 19th October 2017.

By considering the depth of water at 00:00 hours on 19th October 2017 for both models,

(e) (i) explain why Jolene's model is not correct,

(ii) hence find a suitable model for H in terms of x.

(3)

Question 22 continued

Question 22 continued

Question 22 continued

(Total for Question 22 is 11 marks)

23.

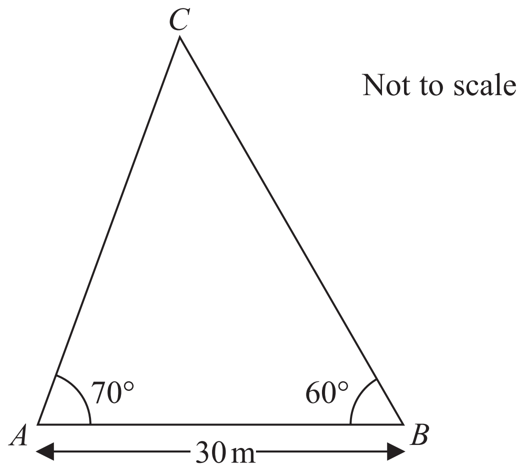


Figure 1

A triangular lawn is modelled by the triangle *ABC*, shown in Figure 1. The length *AB* is to be 30 m long.

Given that angle $BAC = 70°$ and angle $ABC = 60°$,

(a) calculate the area of the lawn to 3 significant figures.

(4)

(b) Why is your answer unlikely to be accurate to the nearest square metre?

(1)

Question 23 continued

(Total for Question 23 is 5 marks)

24. Solve, for $360° \leqslant x < 540°$,

$$12\sin^2 x + 7\cos x - 13 = 0$$

Give your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Question 24 continued

(Total for Question 24 is 5 marks)

25.

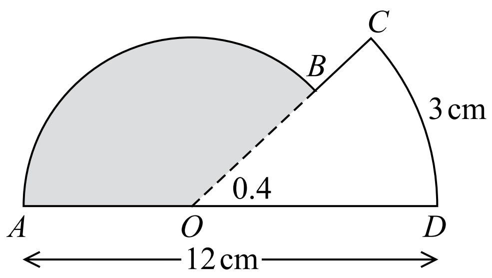


Figure 1

The shape $ABCDOA$, as shown in Figure 1, consists of a sector COD of a circle centre O joined to a sector AOB of a different circle, also centre O.

Given that arc length $CD = 3$ cm, $\angle COD = 0.4$ radians and AOD is a straight line of length 12 cm,

(a) find the length of OD,

(2)

(b) find the area of the shaded sector AOB.

(3)

(Total for Question 25 is 5 marks)

26.

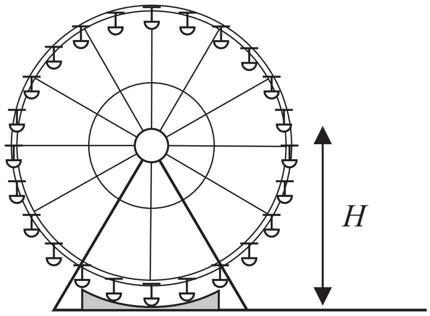


Figure 4

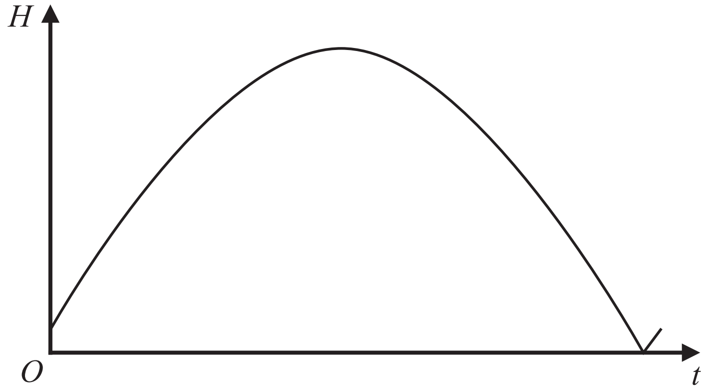


Figure 5

Figure 4 shows a sketch of a Ferris wheel.

The height above the ground, H m, of a passenger on the Ferris wheel, t seconds after the wheel starts turning, is modelled by the equation

$$H = |A\sin(bt + \alpha)°|$$

where A, b and α are constants.

Figure 5 shows a sketch of the graph of H against t, for one revolution of the wheel.

Given that

- the maximum height of the passenger above the ground is 50 m
- the passenger is 1 m above the ground when the wheel starts turning
- the wheel takes 720 seconds to complete one revolution

(a) find a complete equation for the model, giving the exact value of A, the exact value of b and the value of α to 3 significant figures.

(4)

(b) Explain why an equation of the form

$$H = |A\sin(bt + \alpha)°| + d$$

where d is a positive constant, would be a more appropriate model.

(1)

Question 26 continued

(Total for Question 26 is 5 marks)

27. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given that the first three terms of a geometric series are

$$12\cos\theta \qquad 5 + 2\sin\theta \qquad \text{and} \qquad 6\tan\theta$$

(a) show that

$$4\sin^2\theta - 52\sin\theta + 25 = 0$$

(3)

Given that θ is an obtuse angle measured in radians,

(b) solve the equation in part (a) to find the exact value of θ

(2)

(c) show that the sum to infinity of the series can be expressed in the form

$$k\left(1 - \sqrt{3}\right)$$

where k is a constant to be found.

(5)

Question 27 continued

Question 27 continued

Question 27 continued

(Total for Question 27 is 10 marks)

28. Given that θ is small and measured in radians, use the small angle approximations to show that

$$4\sin\frac{\theta}{2} + 3\cos^2\theta \approx a + b\theta + c\theta^2$$

where a, b and c are integers to be found.

(3)

Question 28 continued

(Total for Question 28 is 3 marks)

29.

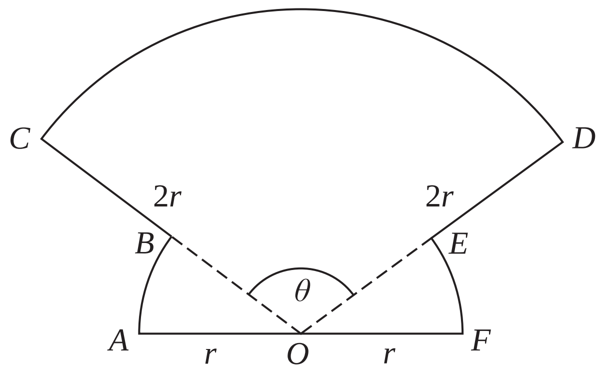


Figure 1

The shape *OABCDEFO* shown in Figure 1 is a design for a logo.

In the design

- *OAB* is a sector of a circle centre *O* and radius r
- sector *OFE* is congruent to sector *OAB*
- *ODC* is a sector of a circle centre *O* and radius $2r$
- *AOF* is a straight line

Given that the size of angle *COD* is θ radians,

(a) write down, in terms of θ, the size of angle *AOB*

(1)

(b) Show that the area of the logo is

$$\frac{1}{2}r^2(3\theta + \pi)$$

(2)

(c) Find the perimeter of the logo, giving your answer in simplest form in terms of r, θ and π.

(2)

Question 29 continued

Question 29 continued

Question 29 continued

(Total for Question 29 is 5 marks)

30. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α in radians to 3 decimal places.

(3)

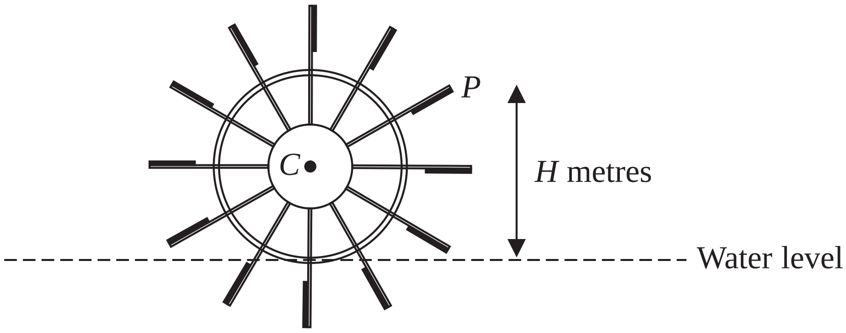


Figure 6

Figure 6 shows the cross-section of a water wheel.

The wheel is free to rotate about a fixed axis through the point C.

The point P is at the end of one of the paddles of the wheel, as shown in Figure 6.

The water level is assumed to be horizontal and of constant height.

The vertical height, H metres, of P above the water level is modelled by the equation

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t)$$

where t is the time in seconds after the wheel starts rotating.

Using the model, find

(b) (i) the maximum height of P above the water level,

(ii) the value of t when this maximum height first occurs, giving your answer to one decimal place.

(3)

In a single revolution of the wheel, P is below the water level for a total of T seconds.

According to the model,

(c) find the value of T giving your answer to 3 significant figures.

(*Solutions based entirely on calculator technology are not acceptable*.)

(4)

In reality, the water level may not be of constant height.

(d) Explain how the equation of the model should be refined to take this into account.

(1)

Question 30 continued

Question 30 continued

Question 30 continued

Question 30 continued

(Total for Question 30 is 11 marks)

31. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos 3A \equiv 4\cos^3 A - 3\cos A$$

(4)

(b) Hence solve, for $-90° \leqslant x \leqslant 180°$, the equation

$$1 - \cos 3x = \sin^2 x$$

(4)

Question 31 continued

Question 31 continued

Question 31 continued

(Total for Question 31 is 8 marks)

32.

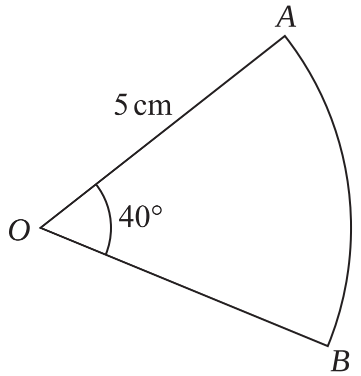


Figure 1

Figure 1 shows a sector *AOB* of a circle with centre *O*, radius 5 cm and angle *AOB* = 40°

The attempt of a student to find the area of the sector is shown below.

$$\text{Area of sector} = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2} \times 5^2 \times 40$$
$$= 500\text{ cm}^2$$

(a) Explain the error made by this student.

(1)

(b) Write out a correct solution.

(2)

Question 32 continued

(Total for Question 32 is 3 marks)

33. (a) Prove

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} \equiv 2\cot 2\theta \qquad \theta \neq (90n)°,\ n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $90° < \theta < 180°$, the equation

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = 4$$

giving any solutions to one decimal place.

(3)

Question 33 continued

Question 33 continued

Question 33 continued

(Total for Question 33 is 7 marks)

34. (i) Solve, for $0 \leqslant x < \frac{\pi}{2}$, the equation

$$4\sin x = \sec x$$

(4)

(ii) Solve, for $0 \leqslant \theta < 360°$, the equation

$$5\sin\theta - 5\cos\theta = 2$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

Question 34 continued

Question 34 continued

Question 34 continued

(Total for Question 34 is 9 marks)

35. (a) Prove that

$$1 - \cos 2\theta \equiv \tan\theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the equation

$$(\sec^2 x - 5)(1 - \cos 2x) = 3\tan^2 x \sin 2x$$

Give any non-exact answer to 3 decimal places where appropriate.

(6)

Question 35 continued

Question 35 continued

Question 35 continued

(Total for Question 35 is 9 marks)

36.

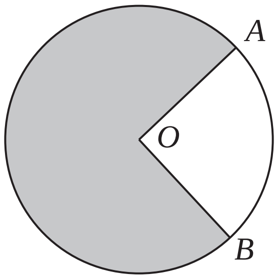


Figure 1

Figure 1 shows a circle with centre O. The points A and B lie on the circumference of the circle.

The area of the major sector, shown shaded in Figure 1, is $135\,\text{cm}^2$.

The reflex angle AOB is 4.8 radians.

Find the exact length, in cm, of the minor arc AB, giving your answer in the form $a\pi + b$, where a and b are integers to be found.

(4)

Question 36 continued

(Total for Question 36 is 4 marks)

37. (a) Given that θ is small, use the small angle approximation for $\cos\theta$ to show that

$$1 + 4\cos\theta + 3\cos^2\theta \approx 8 - 5\theta^2$$

(3)

Adele uses $\theta = 5°$ to test the approximation in part (a).

Adele's working is shown below.

> Using my calculator, $1 + 4\cos(5°) + 3\cos^2(5°) = 7.962$, to 3 decimal places.
>
> Using the approximation $8 - 5\theta^2$ gives $8 - 5(5)^2 = -117$
>
> Therefore, $1 + 4\cos\theta + 3\cos^2\theta \approx 8 - 5\theta^2$ is not true for $\theta = 5°$

(b) (i) Identify the mistake made by Adele in her working.

(ii) Show that $8 - 5\theta^2$ can be used to give a good approximation to $1 + 4\cos\theta + 3\cos^2\theta$ for an angle of size 5°

(2)

Question 37 continued

(Total for Question 37 is 5 marks)

38. (a) Show that

$$\operatorname{cosec} 2x + \cot 2x \equiv \cot x, \qquad x \neq 90n°, n \in \mathbb{Z}$$

(5)

(b) Hence, or otherwise, solve, for $0 \leqslant \theta < 180°$,

$$\operatorname{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3}$$

You must show your working.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

Question 38 continued

(Total for Question 38 is 10 marks)

39. Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation

$$\cos\theta = 2\sin\theta$$

The attempts of two of the students are shown below.

Student *A*	Student *B*
$\cos\theta = 2\sin\theta$ $\tan\theta = 2$ $\theta = 63.4°$	$\cos\theta = 2\sin\theta$ $\cos^2\theta = 4\sin^2\theta$ $1 - \sin^2\theta = 4\sin^2\theta$ $\sin^2\theta = \frac{1}{5}$ $\sin\theta = \pm\frac{1}{\sqrt{5}}$ $\theta = \pm 26.6°$

(a) Identify an error made by student *A*.

(1)

Student *B* gives $\theta = -26.6°$ as one of the answers to $\cos\theta = 2\sin\theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

(Total for Question 39 is 3 marks)

40. (a) Solve, for $-180° \leqslant x < 180°$, the equation

$$3 \sin^2 x + \sin x + 8 = 9 \cos^2 x$$

giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9 \cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

(2)

Question 40 continued

(Total for Question 40 is 8 marks)

41. (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$
Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

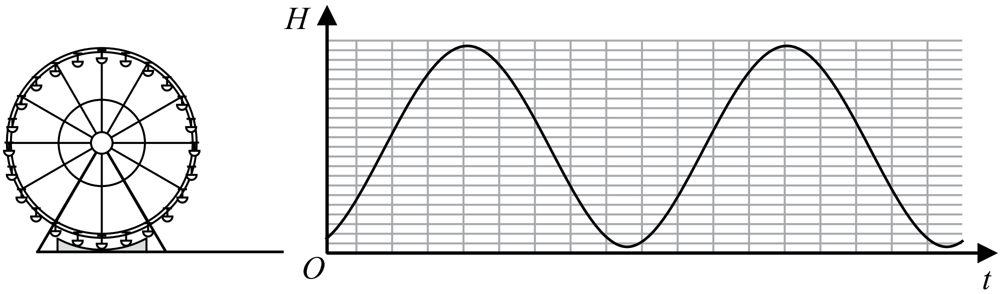


Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10\cos(80t)° + 3\sin(80t)°$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.

(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

(1)

Question 41 continued

(Total for Question 41 is 9 marks)

42.

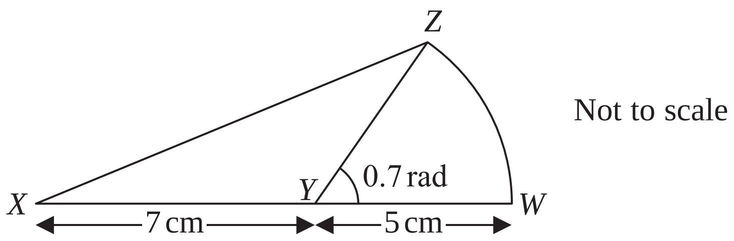


Figure 2

Figure 2 shows a flag *XYWZX*.

The flag consists of a triangle *XYZ* joined to a sector *ZYW* of a circle with radius 5 cm and centre *Y*.

The angle of the sector, angle *ZYW*, is 0.7 radians.

The points *X*, *Y* and *W* lie on a straight line with $XY = 7$ cm and $YW = 5$ cm.

Find

(a) the area of the sector *ZYW* in cm^2,

(2)

(b) the area of the flag, in cm^2, to 2 decimal places,

(3)

(c) the length of the perimeter, *XYWZX*, of the flag, in cm to 2 decimal places.

(4)

Leave blank

Question 42 continued

Leave blank

Question 42 continued

Leave blank

Question 42 continued

(Total 9 marks)

43. *In this question solutions based entirely on graphical or numerical methods are not acceptable.*

(i) Solve for $0 \leqslant x < 360°$,

$$4\cos(x + 70°) = 3$$

giving your answers in degrees to one decimal place.

(4)

(ii) Find, for $0 \leqslant \theta < 2\pi$, all the solutions of

$$6\cos^2\theta - 5 = 6\sin^2\theta + \sin\theta$$

giving your answers in radians to 3 significant figures.

(5)

Leave blank

Question 43 continued

Leave blank

Question 43 continued

Leave blank

Question 43 continued

(Total 9 marks)

Leave blank

44. In the triangle ABC, $AB = 16$ cm, $AC = 13$ cm, angle $ABC = 50°$ and angle $BCA = x°$

Find the two possible values for x, giving your answers to one decimal place.

(4)

Leave blank

Question 44 continued

(Total 4 marks)

Leave blank

45.

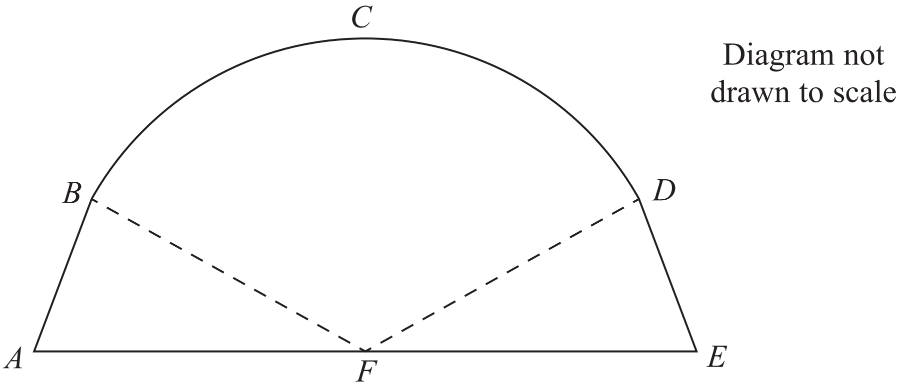


Figure 1

Figure 1 is a sketch representing the cross-section of a large tent *ABCDEF*.
AB and *DE* are line segments of equal length.
Angle *FAB* and angle *DEF* are equal.
F is the midpoint of the straight line *AE* and *FC* is perpendicular to *AE*.
BCD is an arc of a circle of radius 3.5 m with centre at *F*.
It is given that

$$AF = FE = 3.7\text{m}$$

$$BF = FD = 3.5\text{m}$$

$$\text{angle } BFD = 1.77 \text{ radians}$$

Find

(a) the length of the arc *BCD* in metres to 2 decimal places,

(2)

(b) the area of the sector *FBCD* in m^2 to 2 decimal places,

(2)

(c) the total area of the cross-section of the tent in m^2 to 2 decimal places.

(4)

Leave blank

Question 45 continued

Leave blank

Question 45 continued

Leave blank

Question 45 continued

(Total 8 marks)

Leave blank

46. (a) Show that the equation

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

can be written in the form

$$(3\sin x - 1)^2 = 2$$

(3)

(b) Hence solve, for $0 \leqslant x < 360°$,

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

giving your answers to 2 decimal places.

(5)

Leave blank

Question 46 continued

(Total 8 marks)

Leave blank

47. (i) Solve, for $-\pi < \theta \leqslant \pi$,

$$1 - 2\cos\left(\theta - \frac{\pi}{5}\right) = 0$$

giving your answers in terms of π.

(3)

(ii) Solve, for $0 \leqslant x < 360°$,

$$4\cos^2 x + 7\sin x - 2 = 0$$

giving your answers to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

Leave blank

Question 47 continued

(Total 9 marks)

48.

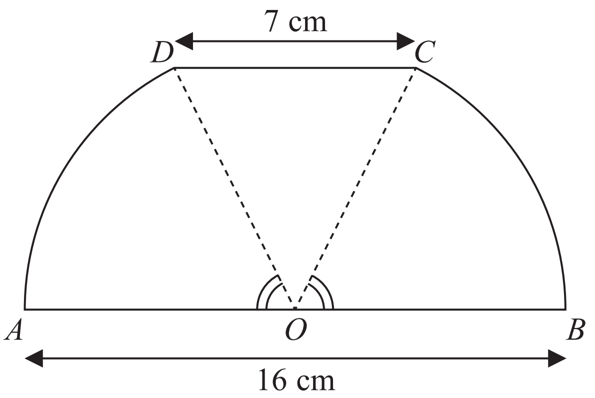


Figure 1

Figure 1 shows a sketch of a design for a scraper blade. The blade *AOBCDA* consists of an isosceles triangle *COD* joined along its equal sides to sectors *OBC* and *ODA* of a circle with centre *O* and radius 8 cm. Angles *AOD* and *BOC* are equal. *AOB* is a straight line and is parallel to the line *DC*. *DC* has length 7 cm.

(a) Show that the angle *COD* is 0.906 radians, correct to 3 significant figures.

(2)

(b) Find the perimeter of *AOBCDA*, giving your answer to 3 significant figures.

(3)

(c) Find the area of *AOBCDA*, giving your answer to 3 significant figures.

(3)

Leave blank

Leave blank

Question 48 continued

(Total 8 marks)

Leave blank

49. (i) Solve, for $0 \leqslant \theta < \pi$, the equation

$$\sin 3\theta - \sqrt{3}\cos 3\theta = 0$$

giving your answers in terms of π.

(3)

(ii) Given that

$$4\sin^2 x + \cos x = 4 - k, \qquad 0 \leqslant k \leqslant 3$$

(a) find $\cos x$ in terms of k.

(3)

(b) When $k = 3$, find the values of x in the range $0 \leqslant x < 360°$

(3)

Leave blank

Question 49 continued

(Total 9 marks)

Leave blank

50.

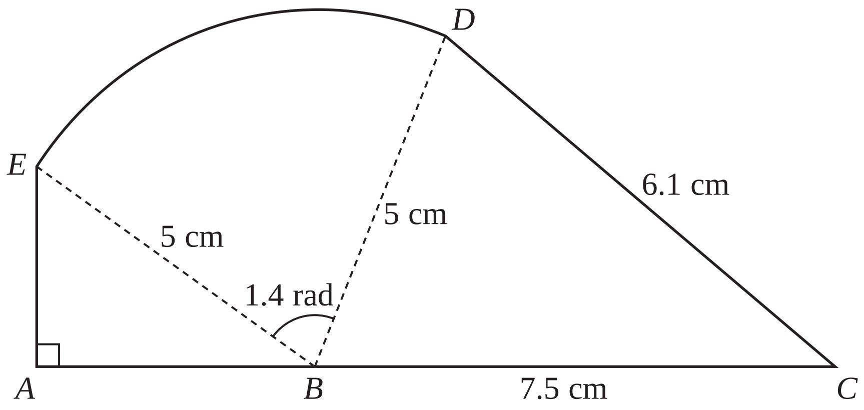


Figure 2

The shape *ABCDEA*, as shown in Figure 2, consists of a right-angled triangle *EAB* and a triangle *DBC* joined to a sector *BDE* of a circle with radius 5 cm and centre *B*.

The points *A*, *B* and *C* lie on a straight line with *BC* = 7.5 cm.

Angle *EAB* = $\frac{\pi}{2}$ radians, angle *EBD* = 1.4 radians and *CD* = 6.1 cm.

(a) Find, in cm^2, the area of the sector *BDE*.

(2)

(b) Find the size of the angle *DBC*, giving your answer in radians to 3 decimal places.

(2)

(c) Find, in cm^2, the area of the shape *ABCDEA*, giving your answer to 3 significant figures.

(5)

Leave blank

Question 50 continued

(Total 9 marks)

Leave blank

51. (i) Solve, for $0 \leqslant \theta < 360°$, the equation

$$9\sin(\theta + 60°) = 4$$

giving your answers to 1 decimal place.
You must show each step of your working.

(4)

(ii) Solve, for $-\pi \leqslant x < \pi$, the equation

$$2\tan x - 3\sin x = 0$$

giving your answers to 2 decimal places where appropriate.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(5)

Leave blank

Question 51 continued

(Total 9 marks)

Leave blank

52.

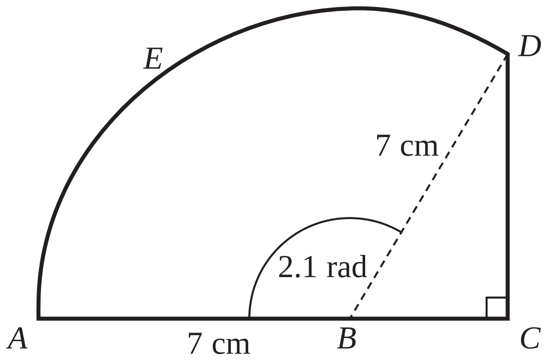


Figure 2

Figure 2 shows the shape *ABCDEA* which consists of a right-angled triangle *BCD* joined to a sector *ABDEA* of a circle with radius 7 cm and centre *B*.

A, *B* and *C* lie on a straight line with *AB* = 7 cm.

Given that the size of angle *ABD* is exactly 2.1 radians,

(a) find, in cm, the length of the arc *DEA*,

(2)

(b) find, in cm, the perimeter of the shape *ABCDEA*, giving your answer to 1 decimal place.

(4)

(Total 6 marks)

Leave blank

53. (i) Solve, for $0 \leqslant \theta < 180°$, the equation

$$\frac{\sin 2\theta}{(4\sin 2\theta - 1)} = 1$$

giving your answers to 1 decimal place.

(3)

(ii) Solve, for $0 \leqslant x < 2\pi$, the equation

$$5\sin^2 x - 2\cos x - 5 = 0$$

giving your answers to 2 decimal places.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Leave blank

Question 53 continued

(Total 8 marks)

EXPERT TUITION

54.

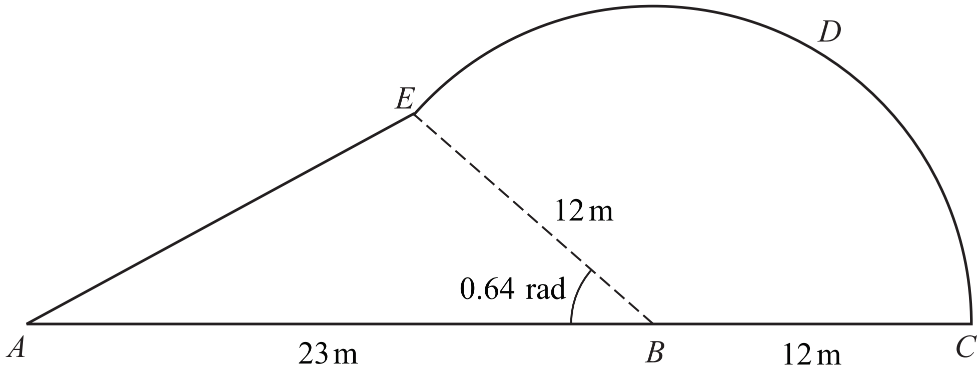


Figure 2

Figure 2 shows a plan view of a garden.
The plan of the garden *ABCDEA* consists of a triangle *ABE* joined to a sector *BCDE* of a circle with radius 12 m and centre *B*.
The points *A*, *B* and *C* lie on a straight line with *AB* = 23 m and *BC* = 12 m.

Given that the size of angle *ABE* is exactly 0.64 radians, find

(a) the area of the garden, giving your answer in m^2, to 1 decimal place,

(4)

(b) the perimeter of the garden, giving your answer in metres, to 1 decimal place.

(5)

Leave blank

Question 54 continued

(Total 9 marks)

Leave blank

55. (i) Solve, for $-180° \leqslant x < 180°$,

$$\tan(x - 40°) = 1.5$$

giving your answers to 1 decimal place.

(3)

(ii) (a) Show that the equation

$$\sin\theta \tan\theta = 3\cos\theta + 2$$

can be written in the form

$$4\cos^2\theta + 2\cos\theta - 1 = 0$$

(3)

(b) Hence solve, for $0 \leqslant \theta < 360°$,

$$\sin\theta \tan\theta = 3\cos\theta + 2$$

showing each stage of your working.

(5)

Leave blank

Question 55 continued

(Total 11 marks)

Leave blank

56.

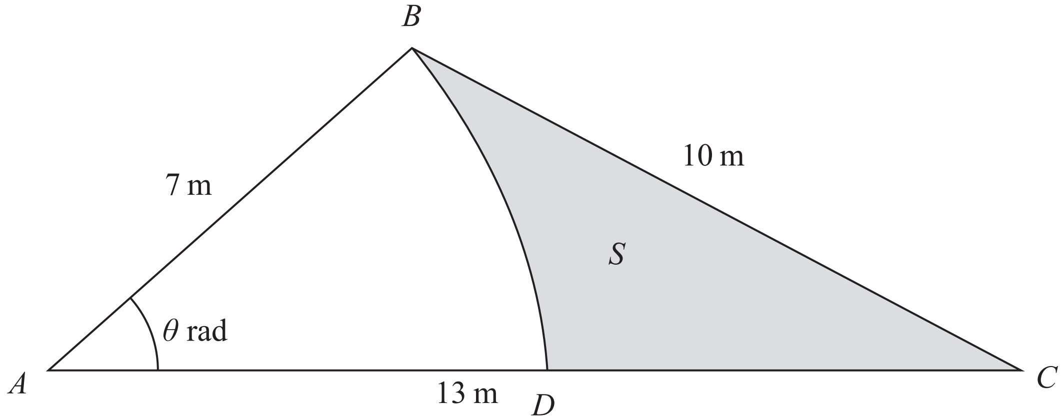


Figure 2

Figure 2 shows the design for a triangular garden *ABC* where $AB = 7$ m, $AC = 13$ m and $BC = 10$ m.

Given that angle $BAC = \theta$ radians,

(a) show that, to 3 decimal places, $\theta = 0.865$

(3)

The point *D* lies on *AC* such that *BD* is an arc of the circle centre *A*, radius 7 m.

The shaded region *S* is bounded by the arc *BD* and the lines *BC* and *DC*. The shaded region *S* will be sown with grass seed, to make a lawned area.

Given that 50 g of grass seed are needed for each square metre of lawn,

(b) find the amount of grass seed needed, giving your answer to the nearest 10 g.

(7)

Leave blank

Question 56 continued

(Total 10 marks)

EXPERT TUITION

Leave blank

57. (i) Solve, for $0 \leqslant \theta < 180°$

$$\sin(2\theta - 30°) + 1 = 0.4$$

giving your answers to 1 decimal place.

(5)

(ii) Find all the values of x, in the interval $0 \leqslant x < 360°$, for which

$$9\cos^2 x - 11\cos x + 3\sin^2 x = 0$$

giving your answers to 1 decimal place.

(7)

You must show clearly how you obtained your answers.

Leave blank

Question 57 continued

(Total 12 marks)

Leave blank

58. Solve, for $0 \leqslant x < 180°$,

$$\cos(3x - 10°) = -0.4$$

giving your answers to 1 decimal place. You should show each step in your working.

(7)

(Total 7 marks)

59.

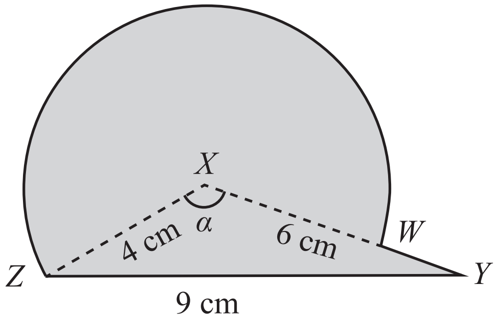


Figure 1

The triangle XYZ in Figure 1 has $XY = 6$ cm, $YZ = 9$ cm, $ZX = 4$ cm and angle $ZXY = \alpha$. The point W lies on the line XY.

The circular arc ZW, in Figure 1 is a major arc of the circle with centre X and radius 4 cm.

(a) Show that, to 3 significant figures, $\alpha = 2.22$ radians.

(2)

(b) Find the area, in cm^2, of the major sector $XZWX$.

(3)

The region enclosed by the major arc ZW of the circle and the lines WY and YZ is shown shaded in Figure 1.

Calculate

(c) the area of this shaded region,

(3)

(d) the perimeter $ZWYZ$ of this shaded region.

(4)

Leave blank

Question 59 continued

(Total 12 marks)

Leave blank

60. (a) Show that the equation

$$\tan 2x = 5 \sin 2x$$

can be written in the form

$$(1 - 5 \cos 2x) \sin 2x = 0$$

(2)

(b) Hence solve, for $0 \leqslant x \leqslant 180°$,

$$\tan 2x = 5 \sin 2x$$

giving your answers to 1 decimal place where appropriate.
You must show clearly how you obtained your answers.

(5)

(Total 7 marks)

Leave blank

61.

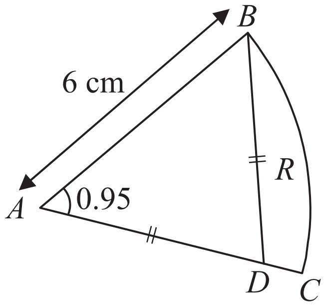


Figure 2

Figure 2 shows ABC, a sector of a circle of radius 6 cm with centre A. Given that the size of angle BAC is 0.95 radians, find

(a) the length of the arc BC,

(2)

(b) the area of the sector ABC.

(2)

The point D lies on the line AC and is such that $AD = BD$. The region R, shown shaded in Figure 2, is bounded by the lines CD, DB and the arc BC.

(c) Show that the length of AD is 5.16 cm to 3 significant figures.

(2)

Find

(d) the perimeter of R,

(2)

(e) the area of R, giving your answer to 2 significant figures.

(4)

Leave blank

Question 61 continued

(Total 12 marks)

Leave blank

62. (i) Find the solutions of the equation $\sin(3x - 15^\circ) = \frac{1}{2}$, for which $0 \leqslant x \leqslant 180^\circ$

(6)

(ii)

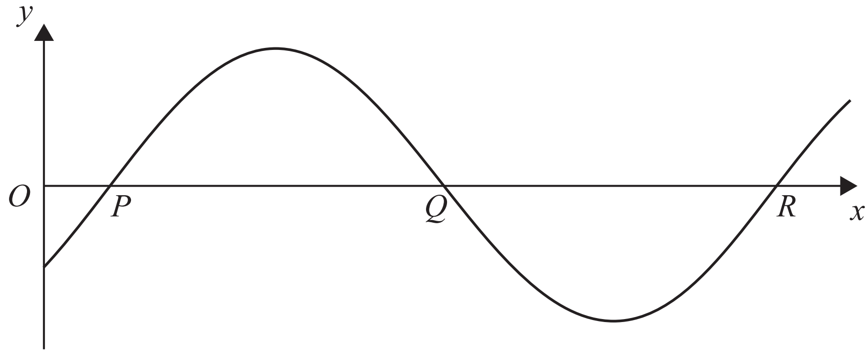


Figure 4

Figure 4 shows part of the curve with equation

$$y = \sin(ax - b), \text{ where } a > 0, \ 0 < b < \pi$$

The curve cuts the x-axis at the points P, Q and R as shown.

Given that the coordinates of P, Q and R are $\left(\frac{\pi}{10}, 0\right)$, $\left(\frac{3\pi}{5}, 0\right)$ and $\left(\frac{11\pi}{10}, 0\right)$ respectively, find the values of a and b.

(4)

Leave blank

Question 62 continued

(Total 10 marks)

Leave blank

63.

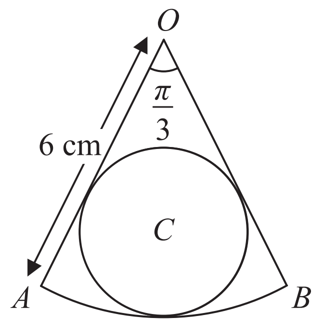


Figure 1

The shape shown in Figure 1 is a pattern for a pendant. It consists of a sector OAB of a circle centre O, of radius 6 cm, and angle $AOB = \frac{\pi}{3}$. The circle C, inside the sector, touches the two straight edges, OA and OB, and the arc AB as shown.

Find

(a) the area of the sector OAB,

(2)

(b) the radius of the circle C.

(3)

The region outside the circle C and inside the sector OAB is shown shaded in Figure 1.

(c) Find the area of the shaded region.

(2)

Leave blank

Question 63 continued

(Total 7 marks)

Leave blank

64. (a) Solve for $0 \leqslant x < 360°$, giving your answers in degrees to 1 decimal place,

$$3\sin(x + 45°) = 2$$

(4)

(b) Find, for $0 \leqslant x < 2\pi$, all the solutions of

$$2\sin^2 x + 2 = 7\cos x$$

giving your answers in radians.

You must show clearly how you obtained your answers.

(6)

Leave blank

Question 64 continued

(Total 10 marks)

Leave blank

65. In the triangle ABC, $AB = 11$ cm, $BC = 7$ cm and $CA = 8$ cm.

(a) Find the size of angle C, giving your answer in radians to 3 significant figures.
(3)

(b) Find the area of triangle ABC, giving your answer in cm^2 to 3 significant figures.
(3)

(Total 6 marks)

Leave blank

66. (a) Show that the equation

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

can be written in the form

$$4\sin^2 x + 7\sin x + 3 = 0$$

(2)

(b) Hence solve, for $0 \leqslant x < 360°$,

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

giving your answers to 1 decimal place where appropriate.

(5)

Leave blank

Question 66 continued

(Total 7 marks)

Leave blank

67. (a) Given that $5\sin\theta = 2\cos\theta$, find the value of $\tan\theta$.

(1)

(b) Solve, for $0 \leqslant x < 360°$,

$$5\sin 2x = 2\cos 2x,$$

giving your answers to 1 decimal place.

(5)

(Total 6 marks)

68.

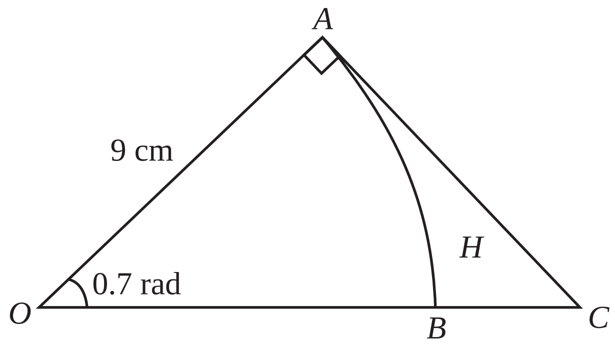


Figure 1

Figure 1 shows the sector *OAB* of a circle with centre *O*, radius 9 cm and angle 0.7 radians.

(a) Find the length of the arc *AB*.

(2)

(b) Find the area of the sector *OAB*.

(2)

The line *AC* shown in Figure 1 is perpendicular to *OA*, and *OBC* is a straight line.

(c) Find the length of *AC*, giving your answer to 2 decimal places.

(2)

The region *H* is bounded by the arc *AB* and the lines *AC* and *CB*.

(d) Find the area of *H*, giving your answer to 2 decimal places.

(3)

Leave blank

Question 68 continued

(Total 9 marks)

Leave blank

69. (a) Show that the equation

$$5\sin x = 1 + 2\cos^2 x$$

can be written in the form

$$2\sin^2 x + 5\sin x - 3 = 0$$

(2)

(b) Solve, for $0 \leqslant x < 360°$,

$$2\sin^2 x + 5\sin x - 3 = 0$$

(4)

(Total 6 marks)

Leave blank

70.

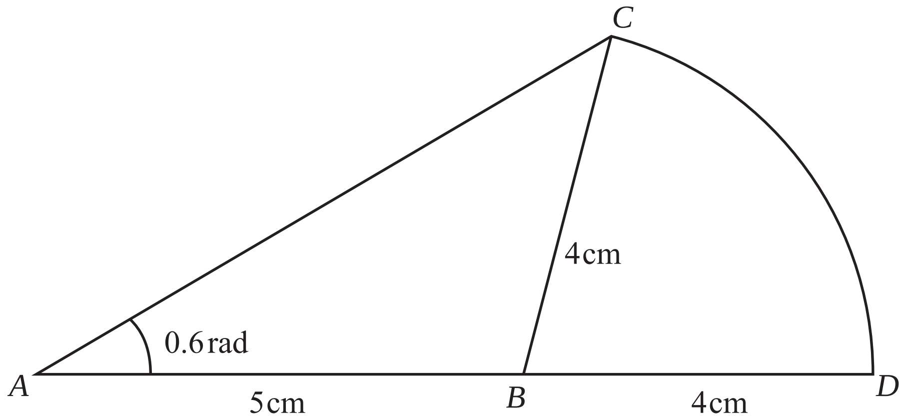


Figure 1

An emblem, as shown in Figure 1, consists of a triangle *ABC* joined to a sector *CBD* of a circle with radius 4 cm and centre *B*. The points *A*, *B* and *D* lie on a straight line with $AB = 5$ cm and $BD = 4$ cm. Angle $BAC = 0.6$ radians and *AC* is the longest side of the triangle *ABC*.

(a) Show that angle $ABC = 1.76$ radians, correct to 3 significant figures.

(4)

(b) Find the area of the emblem.

(3)

(Total 7 marks)

Leave blank

71. (i) Solve, for $-180° \leqslant \theta < 180°$,

$$(1+\tan\theta)(5\sin\theta-2)=0.$$

(4)

Leave blank

(ii) Solve, for $0 \leqslant x < 360°$,

$$4\sin x = 3\tan x.$$

(6)

(Total 10 marks)

Leave blank

72.

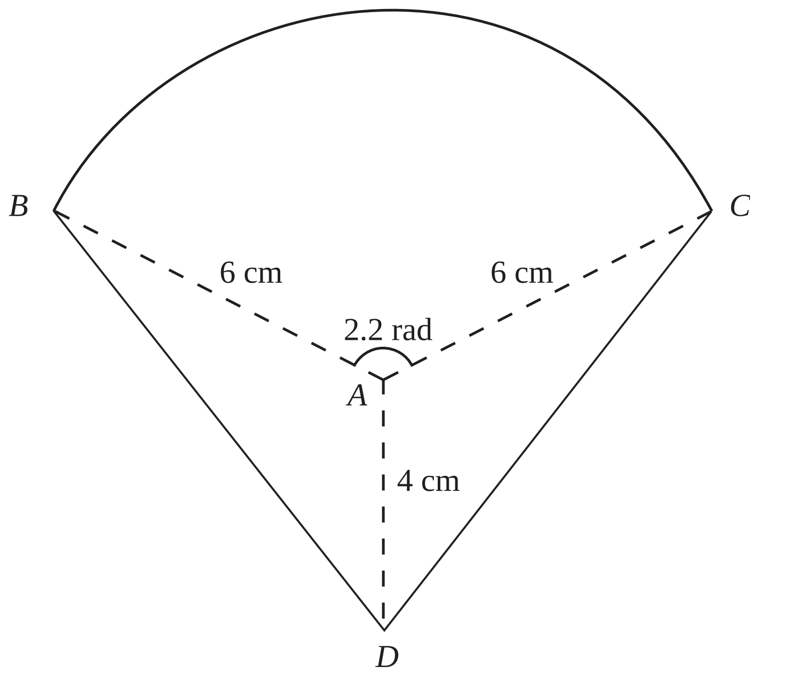


Figure 3

The shape *BCD* shown in Figure 3 is a design for a logo.

The straight lines *DB* and *DC* are equal in length. The curve *BC* is an arc of a circle with centre *A* and radius 6 cm. The size of $\angle BAC$ is 2.2 radians and $AD = 4$ cm.

Find

(a) the area of the sector *BAC*, in cm^2,

(2)

(b) the size of $\angle DAC$, in radians to 3 significant figures,

(2)

(c) the complete area of the logo design, to the nearest cm^2.

(4)

Leave blank

Question 72 continued

(Total 8 marks)

EXPERT TUITION

Leave blank

73. (a) Show that the equation

$$4\sin^2 x + 9\cos x - 6 = 0$$

can be written as

$$4\cos^2 x - 9\cos x + 2 = 0.$$

(2)

(b) Hence solve, for $0 \leqslant x < 720°$,

$$4\sin^2 x + 9\cos x - 6 = 0,$$

giving your answers to 1 decimal place.

(6)

Leave blank

Question 73 continued

(Total 8 marks)

Leave blank

74.

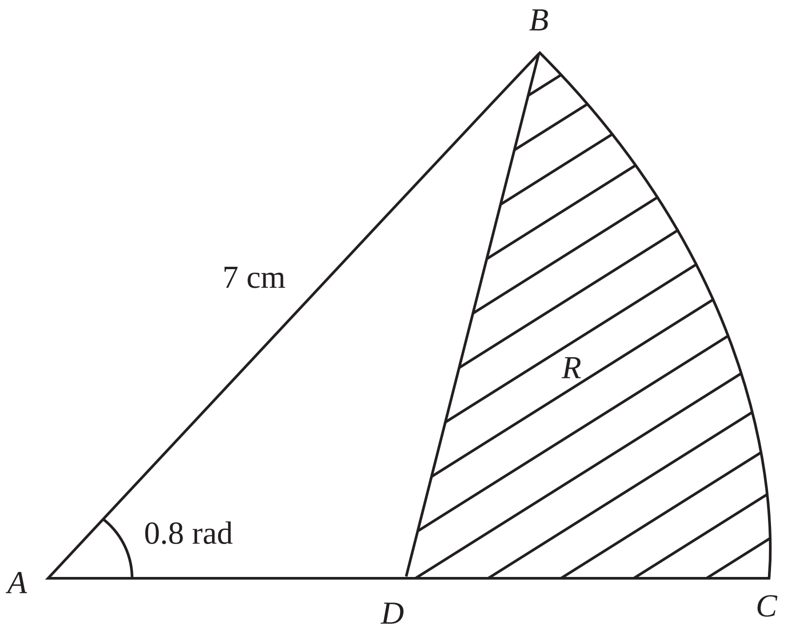


Figure 1

Figure 1 shows *ABC*, a sector of a circle with centre *A* and radius 7 cm.

Given that the size of $\angle BAC$ is exactly 0.8 radians, find

(a) the length of the arc *BC*,

(2)

(b) the area of the sector *ABC*.

(2)

The point *D* is the mid-point of *AC*. The region *R*, shown shaded in Figure 1, is bounded by *CD*, *DB* and the arc *BC*.

Find

(c) the perimeter of *R*, giving your answer to 3 significant figures,

(4)

(d) the area of *R*, giving your answer to 3 significant figures.

(4)

Leave blank

Question 74 continued

(Total 12 marks)

EXPERT TUITION

Leave blank

75. Solve, for $0 \leqslant x < 360°$,

(a) $\sin(x - 20°) = \frac{1}{\sqrt{2}}$ **(4)**

(b) $\cos 3x = -\frac{1}{2}$ **(6)**

Leave blank

Question 75 continued

(Total 10 marks)

Leave blank

76. (a) Show that the equation

$$3\sin^2\theta - 2\cos^2\theta = 1$$

can be written as

$$5\sin^2\theta = 3.$$

(2)

(b) Hence solve, for $0° \leqslant \theta < 360°$, the equation

$$3\sin^2\theta - 2\cos^2\theta = 1,$$

giving your answers to 1 decimal place.

(7)

Leave blank

Question 76 continued

(Total 9 marks)

Leave blank

77.

Figure 1

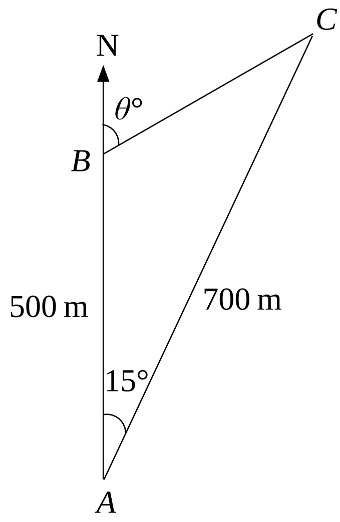


Figure 1 shows 3 yachts A, B and C which are assumed to be in the same horizontal plane. Yacht B is 500 m due north of yacht A and yacht C is 700 m from A. The bearing of C from A is 015°.

(a) Calculate the distance between yacht B and yacht C, in metres to 3 significant figures.

(3)

The bearing of yacht C from yacht B is θ°, as shown in Figure 1.

(b) Calculate the value of θ.

(4)

Leave blank

Question 77 continued

(Total 7 marks)

Leave blank

78. (i) Using the identity for tan $(A \pm B)$, solve, for $-90° < x < 90°$,

$$\frac{\tan 2x + \tan 32°}{1 - \tan 2x \tan 32°} = 5$$

Give your answers, in degrees, to 2 decimal places.

(4)

(ii) (a) Using the identity for $\tan(A \pm B)$, show that

$$\tan(3\theta - 45°) \equiv \frac{\tan 3\theta - 1}{1 + \tan 3\theta}, \qquad \theta \neq (60n + 45)°,\ n \in \mathbb{Z}$$

(2)

(b) Hence solve, for $0 < \theta < 180°$,

$$(1 + \tan 3\theta)\tan(\theta + 28°) = \tan 3\theta - 1$$

(5)

Leave blank

Question 78 continued

Leave blank

Question 78 continued

Leave blank

Question 78 continued

(Total 11 marks)

Leave blank

79. (a) Express $\sin\theta - 2\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α, in radians, to 3 decimal places.

(3)

$$M(\theta) = 40 + (3\sin\theta - 6\cos\theta)^2$$

(b) Find

(i) the maximum value of $M(\theta)$,

(ii) the smallest value of θ, in the range $0 < \theta \leqslant 2\pi$, at which the maximum value of $M(\theta)$ occurs.

(3)

$$N(\theta) = \frac{30}{5 + 2(\sin 2\theta - 2\cos 2\theta)^2}$$

(c) Find

(i) the maximum value of $N(\theta)$,

(ii) the largest value of θ, in the range $0 < \theta \leqslant 2\pi$, at which the maximum value of $N(\theta)$ occurs.

(3)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

Leave blank

Question 79 continued

Leave blank

Question 79 continued

Leave blank

Question 79 continued

Leave blank

Question 79 continued

(Total 9 marks)

Leave blank

80. (a) Write $5\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 \leqslant \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

(b) Show that the equation

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

can be rewritten in the form

$$5\cos 2x - 2\sin 2x = c$$

where c is a positive constant to be determined.

(2)

(c) Hence or otherwise, solve, for $0 \leqslant x < \pi$,

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

giving your answers to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

Leave blank

Question 80 continued

Leave blank

Question 80 continued

Leave blank

Question 80 continued

(Total 9 marks)

Leave blank

81. (a) Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \qquad x \neq (2n+1)90°, \qquad n \in \mathbb{Z}$$

(4)

(b) Given that $x \neq 90°$ and $x \neq 270°$, solve, for $0 \leqslant x < 360°$,

$$\sin 2x - \tan x = 3\tan x \sin x$$

Give your answers in degrees to one decimal place where appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Leave blank

Question 81 continued

Leave blank

Question 81 continued

(Total 9 marks)

Leave blank

82. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

(b) Hence solve, for $0 \leqslant \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15$$

Give your answers to one decimal place.

(5)

(c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15$$

Give your answer to one decimal place.

(2)

EXPERT TUITION

Leave blank

Question 82 continued

(Total 10 marks)

EXPERT TUITION

Leave blank

83. (a) Prove that

$$2\cot 2x + \tan x \equiv \cot x \qquad x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise, solve, for $-\pi \leqslant x < \pi$,

$$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2$$

Give your answers to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

Leave blank

Question 83 continued

(Total 10 marks)

Leave blank

84. Given that

$$\tan\theta° = p, \text{ where } p \text{ is a constant, } p \neq \pm 1$$

use standard trigonometric identities, to find in terms of p,

(a) $\tan 2\theta°$

(2)

(b) $\cos\theta°$

(2)

(c) $\cot(\theta - 45)°$

(2)

Write each answer in its simplest form.

(Total 6 marks)

Leave blank

85.

$$g(\theta) = 4\cos 2\theta + 2\sin 2\theta$$

Given that $g(\theta) = R\cos(2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(a) find the exact value of R and the value of α to 2 decimal places.

(3)

(b) Hence solve, for $-90° < \theta < 90°$,

$$4\cos 2\theta + 2\sin 2\theta = 1$$

giving your answers to one decimal place.

(5)

Given that k is a constant and the equation $g(\theta) = k$ has no solutions,

(c) state the range of possible values of k.

(2)

Leave blank

Question 85 continued

(Total 10 marks)

EXPERT TUITION

Leave blank

86. (a) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \qquad A \neq \frac{(2n+1)\pi}{4}, \ n \in \mathbb{Z}$$

(5)

(b) Hence solve, for $0 \leqslant \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}$$

Give your answers to 3 decimal places.

(4)

Leave blank

Question 86 continued

(Total 9 marks)

Leave blank

87. (a) Show that

$$\text{cosec } 2x + \cot 2x = \cot x, \qquad x \neq 90n°, \qquad n \in \mathbb{Z}$$

(5)

(b) Hence, or otherwise, solve, for $0 \leqslant \theta < 180°$,

$$\text{cosec } (4\theta + 10°) + \cot (4\theta + 10°) = \sqrt{3}$$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Leave blank

Question 87 continued

(Total 10 marks)

EXPERT TUITION

Leave blank

88. (a) Express $2\sin\theta - 4\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of α to 3 decimal places.

(3)

$$H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find

(b) (i) the maximum value of $H(\theta)$,

(ii) the smallest value of θ, for $0 \leqslant \theta < \pi$, at which this maximum value occurs.

(3)

Find

(c) (i) the minimum value of $H(\theta)$,

(ii) the largest value of θ, for $0 \leqslant \theta < \pi$, at which this minimum value occurs.

(3)

Leave blank

Question 88 continued

(Total 9 marks)

Leave blank

89. (i) (a) Show that $2\tan x - \cot x = 5\operatorname{cosec} x$ may be written in the form

$$a\cos^2 x + b\cos x + c = 0$$

stating the values of the constants a, b and c.

(4)

(b) Hence solve, for $0 \leqslant x < 2\pi$, the equation

$$2\tan x - \cot x = 5\operatorname{cosec} x$$

giving your answers to 3 significant figures.

(4)

(ii) Show that

$$\tan\theta + \cot\theta \equiv \lambda\operatorname{cosec} 2\theta, \qquad \theta \neq \frac{n\pi}{2}, \quad n \in \mathbb{Z}$$

stating the value of the constant λ.

(4)

Leave blank

Question 89 continued

(Total 12 marks)

90.

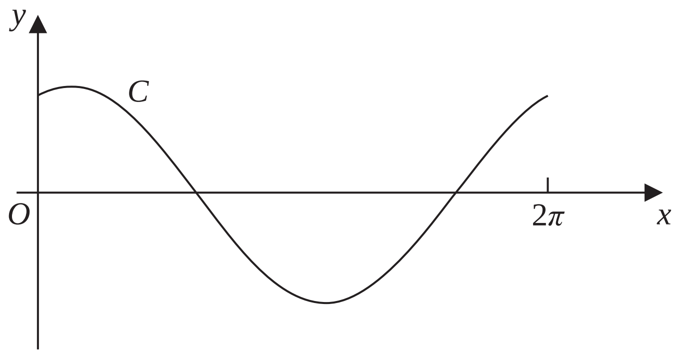


Figure 1

Figure 1 shows the curve C, with equation $y = 6\cos x + 2.5\sin x$ for $0 \leqslant x \leqslant 2\pi$

(a) Express $6\cos x + 2.5\sin x$ in the form $R\cos(x - \alpha)$, where R and α are constants with $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your value of α to 3 decimal places.

(3)

(b) Find the coordinates of the points on the graph where the curve C crosses the coordinate axes.

(3)

A student records the number of hours of daylight each Sunday throughout the year. She starts on the last Sunday in May with a recording of 18 hours, and continues until her final recording 52 weeks later.

She models her results with the continuous function given by

$$H = 12 + 6\cos\left(\frac{2\pi t}{52}\right) + 2.5\sin\left(\frac{2\pi t}{52}\right), \quad 0 \leqslant t \leqslant 52$$

where H is the number of hours of daylight and t is the number of weeks since her first recording.

Use this function to find

(c) the maximum and minimum values of H predicted by the model,

(3)

(d) the values for t when $H = 16$, giving your answers to the nearest whole number.

[*You must show your working. Answers based entirely on graphical or numerical methods are not acceptable.*]

(6)

Leave blank

Question 90 continued

(Total 15 marks)

Leave blank

91. Given that

$$2\cos(x + 50)° = \sin(x + 40)°$$

(a) Show, without using a calculator, that

$$\tan x° = \frac{1}{3}\tan 40°$$

(4)

(b) Hence solve, for $0 \leqslant \theta < 360$,

$$2\cos(2\theta + 50)° = \sin(2\theta + 40)°$$

giving your answers to 1 decimal place.

(4)

(Total 8 marks)

Leave blank

92.

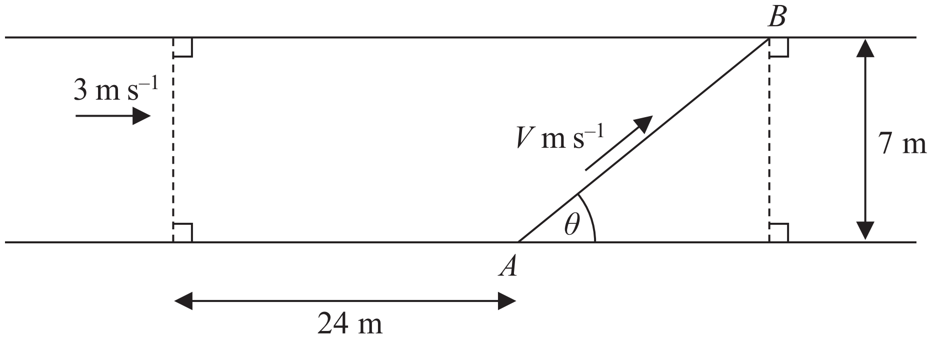


Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1}.
Kate is 24 m ahead of John when she starts to cross the road from the fixed point *A*.
John passes her as she reaches the other side of the road at a variable point *B*, as shown in Figure 2.
Kate's speed is $V \text{ m s}^{-1}$ and she moves in a straight line, which makes an angle θ, $0 < \theta < 150°$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24\sin\theta + 7\cos\theta}, \qquad 0 < \theta < 150°$$

(a) Express $24\sin\theta + 7\cos\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.
(3)

Given that θ varies,

(b) find the minimum value of V.
(2)

Given that Kate's speed has the value found in part (b),

(c) find the distance *AB*.
(3)

Given instead that Kate's speed is 1.68 m s^{-1},

(d) find the two possible values of the angle θ, given that $0 < \theta < 150°$.
(6)

Leave blank

Question 92 continued

(Total 14 marks)

Leave blank

93. Given that $\tan 40° = p$, find in terms of p

(a) $\cot 40°$

(1)

(b) $\sec 40°$

(2)

(c) $\tan 85°$

(2)

(Total 5 marks)

Leave blank

94. (a) Prove that

$$\frac{\cos x}{1-\sin x}+\frac{1-\sin x}{\cos x}=2\sec x, \qquad x \neq (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z}$$

(4)

(b) Hence find, for $0 < x < \frac{\pi}{4}$, the exact solution of

$$\frac{\cos x}{1-\sin x}+\frac{1-\sin x}{\cos x}=8\sin x$$

(4)

Leave blank

Question 94 continued

(Total 8 marks)

Leave blank

95. (a) Express $9\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the exact value of R and give the value of α to 4 decimal places.

(3)

(b) (i) State the maximum value of $9\cos\theta - 2\sin\theta$

(ii) Find the value of θ, for $0 < \theta < 2\pi$, at which this maximum occurs.

(3)

Ruth models the height H above the ground of a passenger on a Ferris wheel by the equation

$$H = 10 - 9\cos\left(\frac{\pi t}{5}\right) + 2\sin\left(\frac{\pi t}{5}\right)$$

where H is measured in metres and t is the time in minutes after the wheel starts turning.

(c) Calculate the maximum value of H predicted by this model, and the value of t, when this maximum first occurs. Give your answers to 2 decimal places.

(4)

(d) Determine the time for the Ferris wheel to complete two revolutions.

(2)

Leave blank

Question 95 continued

(Total 12 marks)

Leave blank

96. $$f(x) = 7\cos x + \sin x$$

Given that $f(x) = R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(a) find the exact value of R and the value of α to one decimal place.

(3)

(b) Hence solve the equation

$$7\cos x + \sin x = 5$$

for $0 \leqslant x < 360°$, giving your answers to one decimal place.

(5)

(c) State the values of k for which the equation

$$7\cos x + \sin x = k$$

has only one solution in the interval $0 \leqslant x < 360°$

(2)

Leave blank

Question 96 continued

(Total 10 marks)

Leave blank

97. (a) Express $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

(b)
$$p(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, \qquad 0 \leqslant \theta \leqslant 2\pi$$

Calculate

(i) the maximum value of $p(\theta)$,

(ii) the value of θ at which the maximum occurs.

(4)

(Total 8 marks)

Leave blank

98. (i) Without using a calculator, find the exact value of

$$(\sin 22.5° + \cos 22.5°)^2$$

You must show each stage of your working.

(5)

(ii) (a) Show that $\cos 2\theta + \sin\theta = 1$ may be written in the form

$$k\sin^2\theta - \sin\theta = 0,$$ stating the value of k.

(2)

(b) Hence solve, for $0 \leqslant \theta < 360°$, the equation

$$\cos 2\theta + \sin\theta = 1$$

(4)

Leave blank

Question 98 continued

(Total 11 marks)

Leave blank

99. (a) Express $4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta$ in terms of $\sin\theta$ and $\cos\theta$.

(2)

(b) Hence show that

$$4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = \sec^2\theta$$

(4)

(c) Hence or otherwise solve, for $0 < \theta < \pi$,

$$4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = 4$$

giving your answers in terms of π.

(3)

(Total 9 marks)

Leave blank

100. $$f(x) = 7\cos 2x - 24\sin 2x$$

Given that $f(x) = R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$,

(a) find the value of R and the value of α.

(3)

(b) Hence solve the equation

$$7\cos 2x - 24\sin 2x = 12.5$$

for $0 \leqslant x < 180^\circ$, giving your answers to 1 decimal place.

(5)

(c) Express $14\cos^2 x - 48\sin x\cos x$ in the form $a\cos 2x + b\sin 2x + c$, where a, b, and c are constants to be found.

(2)

(d) Hence, using your answers to parts (a) and (c), deduce the maximum value of

$$14\cos^2 x - 48\sin x\cos x$$

(2)

Leave blank

Question 100 continued

(Total 12 marks)

Leave blank

101. Solve, for $0 \leqslant \theta \leqslant 180°$,

$$2\cot^2 3\theta = 7\operatorname{cosec} 3\theta - 5$$

Give your answers in degrees to 1 decimal place.

(10)

(Total 10 marks)

Leave blank

102. (a) Starting from the formulae for $\sin(A+B)$ and $\cos(A+B)$, prove that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

(4)

(b) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3}\tan\theta}{\sqrt{3} - \tan\theta}$$

(3)

(c) Hence, or otherwise, solve, for $0 \leqslant \theta \leqslant \pi$,

$$1 + \sqrt{3}\tan\theta = (\sqrt{3} - \tan\theta)\tan(\pi - \theta)$$

Give your answers as multiples of π.

(6)

Leave blank

Question 102 continued

(Total 13 marks)

Leave blank

103. (a) Express $7\cos x - 24\sin x$ in the form $R\cos(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
Give the value of α to 3 decimal places.
(3)

(b) Hence write down the minimum value of $7\cos x - 24\sin x$.
(1)

(c) Solve, for $0 \leqslant x < 2\pi$, the equation

$$7\cos x - 24\sin x = 10$$

giving your answers to 2 decimal places.
(5)

(Total 9 marks)

Leave blank

104. Find all the solutions of

$$2\cos 2\theta = 1 - 2\sin\theta$$

in the interval $0 \leqslant \theta < 360°$.

(6)

(Total 6 marks)

EXPERT TUITION

Leave blank

105. (a) Show that

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$$

(2)

(b) Hence find, for $-180° \leqslant \theta < 180°$, all the solutions of

$$\frac{2\sin 2\theta}{1+\cos 2\theta} = 1$$

Give your answers to 1 decimal place.

(3)

(Total 5 marks)

106. (a) Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

(b) (i) Find the maximum value of $2\sin\theta - 1.5\cos\theta$.

(ii) Find the value of θ, for $0 \leqslant \theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), \quad 0 \leqslant t < 12,$$

where t hours is the number of hours after midday.

(c) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs.

(3)

(d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)

Leave blank

Question 106 continued

(Total 15 marks)

Leave blank

107. (a) Express $5\cos x - 3\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

(4)

(b) Hence, or otherwise, solve the equation

$$5\cos x - 3\sin x = 4$$

for $0 \leqslant x < 2\pi$, giving your answers to 2 decimal places.

(5)

(Total 9 marks)

Leave blank

108. Solve

$$\text{cosec}^2 2x - \cot 2x = 1$$

for $0 \leqslant x \leqslant 180°$.

(7)

(Total 7 marks)

Leave blank

109. (a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$.

(2)

(b) Solve, for $0 \leqslant \theta < 360°$, the equation

$$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2$$

(6)

(Total 8 marks)

Leave blank

110. (a) Use the identity $\cos(A+B) = \cos A\cos B - \sin A\sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A$$

(2)

The curves C_1 and C_2 have equations

$$C_1: \quad y = 3\sin 2x$$

$$C_2: \quad y = 4\sin^2 x - 2\cos 2x$$

(b) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$

(3)

(c) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

(d) Hence find, for $0 \leqslant x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2$$

giving your answers to 1 decimal place.

(4)

Leave blank

Question 110 continued

(Total 12 marks)

EXPERT TUITION

Leave blank

111. (a) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$.

(1)

(b) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8\cos x = 0$$

giving your answers to 2 decimal places.

(5)

(Total 6 marks)

Leave blank

112. (a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8\sin^3\theta - 6\sin\theta + 1 = 0.$$

Give your answers in terms of π.

(5)

(b) Using $\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)

Leave blank

Question 112 continued

(Total 13 marks)

Leave blank

113. (a) Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Hence find the maximum value of $3\cos\theta + 4\sin\theta$ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3\cos(15t)° + 4\sin(15t)°,$$

where t is the time in hours from midday and $0 \leqslant t < 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model.

(2)

(d) Find the value of t when this minimum temperature occurs.

(3)

Leave blank

Question 113 continued

(Total 12 marks)

Leave blank

114.

$$f(x) = 5\cos x + 12\sin x$$

Given that $f(x) = R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

(a) find the value of R and the value of α to 3 decimal places.

(4)

(b) Hence solve the equation

$$5\cos x + 12\sin x = 6$$

for $0 \leqslant x < 2\pi$.

(5)

(c) (i) Write down the maximum value of $5\cos x + 12\sin x$.

(1)

(ii) Find the smallest positive value of x for which this maximum value occurs.

(2)

Leave blank

Question 114 continued

(Total 12 marks)

Leave blank

115. (a) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \text{cosec}^2\theta$.

(2)

(b) Solve, for $0 \leqslant \theta < 180°$, the equation

$$2\cot^2\theta - 9\,\text{cosec}\,\theta = 3,$$

giving your answers to 1 decimal place.

(6)

(Total 8 marks)

Leave blank

116. (a) Use the double angle formulae and the identity

$$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(4)

(b) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2\sec x, \qquad x \neq (2n+1)\frac{\pi}{2}.$$

(4)

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4.$$

(3)

Leave blank

Question 116 continued

(Total 11 marks)

Leave blank

117. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x, \ -\pi \leqslant x \leqslant \pi .$$

The point $A(0, 4)$ lies on C.

(a) Find an equation of the normal to the curve C at A.

(5)

(b) Express y in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

Give the value of α to 3 significant figures.

(4)

(c) Find the coordinates of the points of intersection of the curve C with the x-axis. Give your answers to 2 decimal places.

(4)

Leave blank

Question 117 continued

(Total 13 marks)