EXPERT TUITION

Maths Questions By Topic:

Vectors
Mark Scheme

A-Level Edexcel

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Question	Scheme	Marks	AOs
1(a)	Attempts both $\lvert\overrightarrow{PQ}\rvert = \sqrt{2^2+3^2+(-4)^2}$ and $\lvert\overrightarrow{QR}\rvert = \sqrt{5^2+(-2)^2}$	M1	3.1a
	States that $\lvert\overrightarrow{PQ}\rvert = \lvert\overrightarrow{QR}\rvert = \sqrt{29}$ so $PQRS$ is a rhombus	A1	2.4
		(2)	
(b)	Attempts BOTH $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = 7\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$ AND $\overrightarrow{QS} = -\overrightarrow{PQ} + \overrightarrow{PS} = 3\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$	M1	3.1a
	Correct $\overrightarrow{PR} = 7\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$ and $\overrightarrow{QS} = 3\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$	A1	1.1b
	Correct method for area $PQRS$. E.g. $\frac{1}{2} \times \lvert\overrightarrow{PR}\rvert \times \lvert\overrightarrow{QS}\rvert$	dM1	2.1
	$= \sqrt{517}$	A1	1.1b
		(4)	
			(6 marks)

Alt (b) Example using the cosine rule	Attempts $\lvert\overrightarrow{QS}\rvert = \sqrt{3^2+(-3)^2+2^2} = \sqrt{22}$ and so $22 = 29 + 29 - 2\sqrt{29}\sqrt{29}\cos SPQ$	M1	3.1a
	$\cos PQR = -\frac{18}{29}$ or $\cos SPQ = \frac{18}{29}$ Condone angles in degrees 51.6, 128.4 (1dp) or radians 2.24, 0.901 (3sf) here	A1	1.1b
	Correct method for area $PQRS$. E.g. $PQ \times QR \sin PQR = \sqrt{2^2+3^2+(-4)^2} \times \sqrt{5^2+(-2)^2} \times \frac{\sqrt{517}}{29}$	dM1	2.1
	$= \sqrt{517}$	A1	1.1b
		(4)	

FYI

$\overrightarrow{QR} = 5\mathbf{i} + 0\mathbf{j} - 2\mathbf{k}$

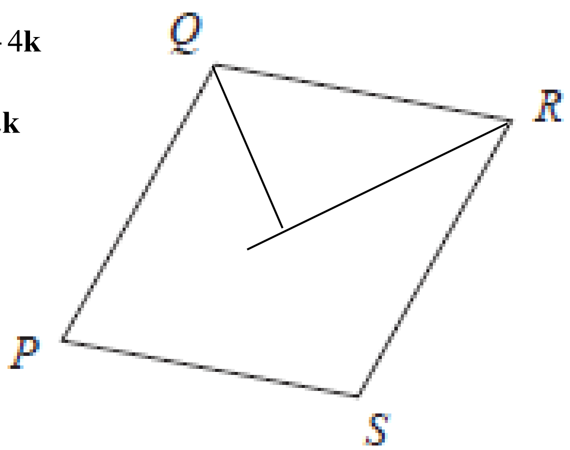
$\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$

$\overrightarrow{SQ} = -3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$

$\overrightarrow{MQ} = -1.5\mathbf{i} + 1.5\mathbf{j} - 1\mathbf{k}$

$\overrightarrow{PR} = 7\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$

$\overrightarrow{PM} = 3.5\mathbf{i} + 1.5\mathbf{j} - 3\mathbf{k}$



(a) Do not award marks in part (a) from work in part (b).

M1: Attempts both $\lvert\overrightarrow{PQ}\rvert = \sqrt{2^2+3^2+(\pm4)^2}$ and $\lvert\overrightarrow{QR}\rvert = \sqrt{5^2+(\pm2)^2}$ or PQ^2 and QR^2. For this mark only, condone just the correct answers $\lvert\overrightarrow{PQ}\rvert = \sqrt{29}$ and $\lvert\overrightarrow{QR}\rvert = \sqrt{29}$. Alternatively attempts $\overrightarrow{PR} \bullet \overrightarrow{QS}$ or PM^2, MQ^2 and PQ^2 where M is the mid point of PR

A1: Shows that $\lvert\overrightarrow{PQ}\rvert = \lvert\overrightarrow{QR}\rvert = \sqrt{29}$ (with calculations) and states $PQRS$ is a rhombus.

Condone poor notation such as $\overrightarrow{PQ}=\sqrt{29}$ here, So $\overrightarrow{PQ}=\overrightarrow{QR}=\sqrt{29}$ hence rhombus.
Requires both a reason and a conclusion. The reason may be given at the start of their solution.
In the alternatives $\overrightarrow{PR}\bullet\overrightarrow{QS}=(7\mathbf{i}+3\mathbf{j}-6\mathbf{k})\bullet(3\mathbf{i}-3\mathbf{j}+2\mathbf{k})=21-9-12=0$ so diagonals cross at $90°$ so $PQRS$ is a rhombus or $PM^2+MQ^2=PQ^2=23.5+5.5=29\Rightarrow\angle PMQ=90°\Rightarrow$ Rhombus

(b) **Candidates can transfer answers from (a) to use in part (b) to find the area**
Look through their complete solution first. The first two marks are for finding the elements that are required to calculate the area. The second set of two marks is for combining these elements correctly. If the method is NOT shown on how to find vector it can be implied by two correct components. Allow as column vectors.

M1: For a key step in solving the problem. It is scored for attempting to find both key vectors.
Attempts both $\overrightarrow{PR}=\overrightarrow{PQ}+\overrightarrow{QR}=(7\mathbf{i}+3\mathbf{j}-6\mathbf{k})$ **AND** $\overrightarrow{QS}=-\overrightarrow{PQ}+\overrightarrow{PS}=(3\mathbf{i}-3\mathbf{j}+2\mathbf{k})$
You may see $\overrightarrow{PM}=\frac{1}{2}\overrightarrow{PQ}+\frac{1}{2}\overrightarrow{QR}=\left(\frac{7}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}-3\mathbf{k}\right)$ **AND** $\overrightarrow{QM}=-\frac{1}{2}\overrightarrow{PQ}+\frac{1}{2}\overrightarrow{PS}=\left(\frac{3}{2}\mathbf{i}-\frac{3}{2}\mathbf{j}+1\mathbf{k}\right)$

A1: Accurately finds both key vectors whose lengths are required to solve the problem.
Score for both $\overrightarrow{PR}=7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$ and $\overrightarrow{QS}=3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$ (Allow either way around.)
or both $\overrightarrow{PM}=\frac{7}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}-3\mathbf{k}$ and $\overrightarrow{QM}=\frac{3}{2}\mathbf{i}-\frac{3}{2}\mathbf{j}+1\mathbf{k}$ (Allow either way around.)

dM1: Constructs a rigorous method leading to the area *PQRS.* Dependent upon previous M.
E.g. See scheme. Alt: the sum of the area of four right angled triangles e.g. $4\times\frac{1}{2}\times\left|\overrightarrow{PM}\right|\times\left|\overrightarrow{QM}\right|$,

A1: $\sqrt{517}$

Alternatives for (b). Two such ways are set out below

Alt 1-Examples via cosine rule but you may see use of scalar product via a Further Maths method.

M1: For a key step in solving the problem. In this case it for an attempt at cos *PQR* or cos *SPQ.*
Don't be too concerned with the labelling of the angle which may appear as θ.

$$\text{Attempts } \pm\begin{pmatrix}2\\3\\-4\end{pmatrix}\bullet\pm\begin{pmatrix}5\\0\\-2\end{pmatrix}=\sqrt{2^2+3^2+(-4)^2}\times\sqrt{5^2+(-2)^2}\cos PQR$$

A1: Finds the cosine of one of the angles in the Figure.
Look for $\cos\ldots=-\frac{18}{29}$ or $\cos\ldots=\frac{18}{29}$ which may have been achieved via the cosine rule.
Accept rounded answers and the angles in degrees 51.6, 128.4 (1dp) or radians 2.24, 0.901 (3sf) here.

dM1: Constructs a rigorous method leading to the area *PQRS.* Implied by awrt 22.7
E.g. $PQ\times QR\sin PQR=\sqrt{2^2+3^2+(-4)^2}\times\sqrt{5^2+(-2)^2}\times\frac{\sqrt{517}}{29}$

A1: $\sqrt{517}$

Alt 2-Example via vector product via a Further Maths method.

M1: For a key step in solving the problem. In this case it for an attempt at $\pm\overrightarrow{PQ}\times\overrightarrow{QR}$

$$\text{E.g.}\quad \overrightarrow{PQ}\times\overrightarrow{QR}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&3&-4\\5&0&-2\end{vmatrix}=(3\times-2-0\times-4)\mathbf{i}-(2\times-2-5\times-4)\mathbf{j}+(2\times0-3\times5)\mathbf{k}$$

A1: E.g. $\overrightarrow{PQ}\times\overrightarrow{QR}=-6\mathbf{i}-16\mathbf{j}-15\mathbf{k}$

dM1: Constructs a rigorous method leading to the area *PQRS.* In this case $\left|\overrightarrow{PQ}\times\overrightarrow{QR}\right|$

A1: $=\sqrt{(-6)^2+(-16)^2+(-15)^2}=\sqrt{517}$

Question	Scheme	Marks	AOs
2(a)	$\overrightarrow{QR}=\overrightarrow{PR}-\overrightarrow{PQ}=13\mathbf{i}-15\mathbf{j}-(3\mathbf{i}+5\mathbf{j})$	M1	1.1a
	$=10\mathbf{i}-20\mathbf{j}$	A1	1.1b
		(2)	
(b)	$\left\lvert\overrightarrow{QR}\right\rvert=\sqrt{"10"^2+"(-20)"^2}$	M1	2.5
	$=10\sqrt{5}$	A1ft	1.1b
		(2)	
(c)	$\overrightarrow{PS}=\overrightarrow{PQ}+\frac{3}{5}\overrightarrow{QR}=3\mathbf{i}+5\mathbf{j}+\frac{3}{5}("10\mathbf{i}-20\mathbf{j}")=...$ or $\overrightarrow{PS}=\overrightarrow{PR}+\frac{2}{5}\overrightarrow{RQ}=13\mathbf{i}-15\mathbf{j}+\frac{2}{5}("-10\mathbf{i}+20\mathbf{j}")=...$	M1	3.1a
	$=9\mathbf{i}-7\mathbf{j}$	A1	1.1b
		(2)	
		(6 marks)	

Notes

(a)

M1: Attempts subtraction either way round. This cannot be awarded for adding the two vectors. If no method shown it may be implied by one correct component. eg $10\mathbf{i}-10\mathbf{j}$ on its own can score M1.

A1: Correct answer. Allow $10\mathbf{i}-20\mathbf{j}$ and $\begin{pmatrix}10\\-20\end{pmatrix}$ but not $\begin{pmatrix}10\mathbf{i}\\-20\mathbf{j}\end{pmatrix}$

(b)

M1: Correct use of Pythagoras. Attempts to "square and add" before square rooting. The embedded values are sufficient. Follow through on their $\overrightarrow{QR}$

A1ft: $10\sqrt{5}$ following (a) of the form $\pm10\mathbf{i}\pm20\mathbf{j}$

(c)

M1: Full attempt at finding a $\overrightarrow{PS}$. They must be attempting $\overrightarrow{PQ}\pm\frac{3}{5}\overrightarrow{QR}$ or

$\overrightarrow{PS}=\overrightarrow{PR}\pm\frac{2}{5}\overrightarrow{RQ}$ but condone arithmetical slips after that.

Cannot be scored for just stating eg $\overrightarrow{PQ}\pm\frac{3}{5}\overrightarrow{QR}$

Follow through on their $\overrightarrow{QR}$. Terms do not need to be collected for this mark. If no method shown it may be implied by one correct component following through on their $\overrightarrow{QR}$

A1: Correct vector as shown. Allow $9\mathbf{i}-7\mathbf{j}$ and $\begin{pmatrix}9\\-7\end{pmatrix}$.

Only withhold the mark for $\begin{pmatrix}9\mathbf{i}\\-7\mathbf{j}\end{pmatrix}$ if the mark has not already been withheld in (a) for $\begin{pmatrix}10\mathbf{i}\\-20\mathbf{j}\end{pmatrix}$

Alt (c) (Expressing $\overrightarrow{PS}$ in terms of the given vectors) They must be attempting $\frac{2}{5}\overrightarrow{PQ}+\frac{3}{5}\overrightarrow{PR}$

M1: $(\overrightarrow{PS}=\overrightarrow{PQ}+\frac{3}{5}\overrightarrow{QR}=\overrightarrow{PQ}+\frac{3}{5}(\overrightarrow{PR}-\overrightarrow{PQ}))$

$\Rightarrow \frac{2}{5}\overrightarrow{PQ}+\frac{3}{5}\overrightarrow{PR}=\frac{2}{5}(3\mathbf{i}+5\mathbf{j})+\frac{3}{5}(13\mathbf{i}-15\mathbf{j})=\ldots$

A1: Correct vector as shown. Allow $9\mathbf{i}-7\mathbf{j}$ and $\begin{pmatrix}9\\-7\end{pmatrix}$.

Only withhold the mark for $\begin{pmatrix}9\mathbf{i}\\-7\mathbf{j}\end{pmatrix}$ if the mark has not already been withheld in (a) for $\begin{pmatrix}10\mathbf{i}\\-20\mathbf{j}\end{pmatrix}$

Question	Scheme	Marks	AOs
3(a)	$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=-3\mathbf{i}-4\mathbf{j}-5\mathbf{k}+\mathbf{i}+\mathbf{j}+4\mathbf{k}=...$	M1	1.1b
	$=-2\mathbf{i}-3\mathbf{j}-\mathbf{k}$	A1	1.1b
		(2)	
(b)	At least 2 of $\left(AC^2\right)="2^2+3^2+1^2",\left(AB^2\right)=3^2+4^2+5^2,\ \left(BC^2\right)=1^2+1^2+4^2$	M1	1.1b
	$2^2+3^2+1^2=3^2+4^2+5^2+1^2+1^2+4^2-2\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$14=50+18-2\sqrt{50}\sqrt{18}\cos ABC$ $\Rightarrow \cos ABC=\dfrac{50+18-14}{2\sqrt{50}\sqrt{18}}=\dfrac{9}{10}$*	A1*	2.1
		(3)	
	(b) Alternative		
	$AB^2=3^2+4^2+5^2,\ BC^2=1^2+1^2+4^2$	M1	1.1b
	$\overrightarrow{BA}.\overrightarrow{BC}=(3\mathbf{i}+4\mathbf{j}+5\mathbf{k})\bullet(\mathbf{i}+\mathbf{j}+4\mathbf{k})=27=\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$27=\sqrt{50}\sqrt{18}\cos ABC\Rightarrow\cos ABC=\dfrac{27}{\sqrt{50}\sqrt{18}}=\dfrac{9}{10}$*	A1*	2.1
			(5 marks)
	Notes		

(a)

M1: Attempts $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$

There must be attempt to add not subtract.

If no method shown it may be implied by **two** correct components

A1: Correct vector. Allow $-2\mathbf{i}-3\mathbf{j}-\mathbf{k}$ and $\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ but not $\begin{pmatrix}-2\mathbf{i}\\-3\mathbf{j}\\-1\mathbf{k}\end{pmatrix}$

(b)

M1: Attempts to "square and add" for at least 2 of the 3 sides. Follow through on their $\overrightarrow{AC}$

Look for an attempt at either $a^2+b^2+c^2$ or $\sqrt{a^2+b^2+c^2}$

M1: A correct attempt to apply a correct cosine rule to the given problem; Condone **slips** on the lengths of the sides but the sides must be in the correct position to find angle ABC

A1*: Correct completion with sufficient intermediate work to establish the printed result.

Condone different labelling, e.g. $ABC\leftrightarrow\theta$ as long as it is clear what is meant

It is OK to move from a correct cosine rule $14=50+18-2\sqrt{50}\sqrt{18}\cos ABC$

via $\cos ABC=\dfrac{54}{2\sqrt{50}\sqrt{18}}$ o.e. such as $\cos ABC=\dfrac{\left(5\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2-\left(\sqrt{14}\right)^2}{2\times5\sqrt{2}\times3\sqrt{2}}$ to $\cos ABC=\dfrac{9}{10}$

Alternative:

M1: Correct application of Pythagoras for sides AB and BC or their squares

M1: Recognises the requirement for and applies the scalar product

A1*: Correct completion with sufficient intermediate work to establish the printed result

Question	Scheme	Marks	AOs
4(a)	Attempts to compare the two position vectors. Allow an attempt using two of $\overrightarrow{AO}, \overrightarrow{OB}$ or $\overrightarrow{AB}$ E.g. $(-24\mathbf{i}-10\mathbf{j})=-2\times(12\mathbf{i}+5\mathbf{j})$	M1	1.1b
	Explains that as $\overrightarrow{AO}$ is parallel to $\overrightarrow{OB}$ (and the stone is travelling in a straight line) the stone passes through the point *O*.	A1	2.4
		(2)	
(b)	Attempts distance $AB=\sqrt{(12+24)^2+(10+5)^2}$	M1	1.1b
	Attempts speed = $\dfrac{\sqrt{(12+24)^2+(10+5)^2}}{4}$	dM1	3.1a
	Speed = $9.75\ \text{ms}^{-1}$	A1	3.2a
		(3)	
			(5 marks)
Alt(a)	Attempts to find the equation of the line which passes through *A* and *B* E.g. $y-5=\dfrac{5+10}{12+24}(x-12)\quad (y=\dfrac{5}{12}x)$	M1	1.1b
	Shows that when $x=0,\ y=0$ and concludes the stone passes through the point *O*.	A1	2.4

Notes

(a)

M1: Attempts to compare the two position vectors. Allow an attempt using two of $\overrightarrow{AO}, \overrightarrow{OB}$ or $\overrightarrow{AB}$ either way around.
E.g. States that $(-24\mathbf{i}-10\mathbf{j})=-2\times(12\mathbf{i}+5\mathbf{j})$
Alternatively, allow an attempt finding the gradient using any two of *AO*, *OB* or *AB*

Alternatively attempts to find the equation of the line through *A* and *B* proceeding as far as $y=...x$ Condone sign slips.

A1: States that as $\overrightarrow{AO}$ is parallel to $\overrightarrow{OB}$ or as *AO* is parallel to *OB* (and the stone is travelling in a straight line) the stone passes through the point *O*. Alternatively, shows that the point $(0,0)$ is on the line and concludes (the stone) passes through the point *O*.

(b)

M1: Attempts to find the distance *AB* using a correct method.
Condone slips but expect to see an attempt at $\sqrt{a^2+b^2}$ where *a* or *b* is correct

dM1: Dependent upon the previous mark. Look for an attempt at $\dfrac{\text{distance } AB}{4}$

A1: $9.75\ \text{ms}^{-1}$ Requires units

Question	Scheme	Marks	AOs
5 (a)	$\overrightarrow{AB} = (3\mathbf{i}-3\mathbf{j}-4\mathbf{k})-(2\mathbf{i}+5\mathbf{j}-6\mathbf{k})$	M1	1.1b
	$= \mathbf{i}-8\mathbf{j}+2\mathbf{k}$	A1	1.1b
		(2)	
(b)	States $\overrightarrow{OC} = 2\times\overrightarrow{AB}$	M1	1.1b
	Explains that as *OC* is parallel to *AB*, so *OABC* is a trapezium.	A1	2.4
		(2)	
			(4 marks)
Notes:			

(a)

M1: Attempts to subtract either way around. If no method is seen it is implied by two of $\pm 1\mathbf{i}\pm 8\mathbf{j}\pm 2\mathbf{k}$.

A1: $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ or $\begin{pmatrix}1\\-8\\2\end{pmatrix}$ but not $(1,-8,2)$

(b)

M1: Compares their $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ with $2\mathbf{i}-16\mathbf{j}+4\mathbf{k}$ by stating **any one of**

- $\overrightarrow{OC} = 2\times\overrightarrow{AB}$
- $\begin{pmatrix}2\\-16\\4\end{pmatrix} = 2\times\begin{pmatrix}1\\-8\\2\end{pmatrix}$
- $\overrightarrow{OC} = \lambda\times\overrightarrow{AB}$ or vice versa

This may be awarded if *AB* was subtracted "the wrong way around" or if there was one numerical slip

A1: A full explanation as to why *OABC* is a trapezium.

Requires fully correct calculations, so part (a) must be $\overrightarrow{AB} = (\mathbf{i}-8\mathbf{j}+2\mathbf{k})$

It requires a reason and minimal conclusion.

Example 1:

$\overrightarrow{OC} = 2\times\overrightarrow{AB}$, therefore *OC* is parallel to *AB* so *OABC* is a trapezium

Example 2:

A trapezium has one pair of parallel sides. As $\overrightarrow{OC} = 2\times\overrightarrow{AB}$, they are parallel, so ✓.

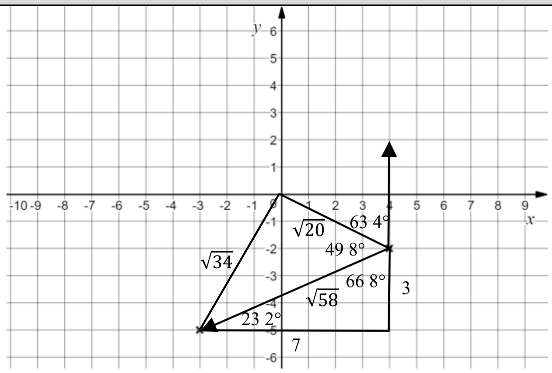
Example 3

As $\begin{pmatrix}2\\-16\\4\end{pmatrix} = 2\times\begin{pmatrix}1\\-8\\2\end{pmatrix}$, *OC* and *AB* are parallel, so proven

Example 4

Accept as $\overrightarrow{OC} = \lambda\times\overrightarrow{AB}$, they are parallel so true

Note: There are two definitions for a trapezium. One stating that it is a shape with one pair of parallel sides, the other with **only one** pair of parallel sides. Any calculations to do with sides *OA* and *CB* in this question may be ignored, even if incorrect.

Question	Scheme	Marks	AOs
6(a)			
	Attempts to find an "allowable" angle Eg $\tan\theta = \frac{7}{3}$	M1	1.1b
	A full attempt to find the bearing Eg $180° + "67°"$	dM1	3.1b
	Bearing = awrt $246.8°$	A1	1.1b
		(3)	
(b)	Attempts to find the distance travelled = $\sqrt{(4--3)^2 + (-2+5)^2} = (\sqrt{58})$	M1	1.1b
	Attempts to find the speed = $\frac{\sqrt{58}}{2.75}$	dM1	3.1b
	= awrt 2.77 km h^{-1}	A1	1.1b
		(3)	
			(6 marks)

Notes: Score these two parts together.

(a)

M1: Attempts an allowable angle. (Either the "66.8" , "23.2" or ("49.8" and "63.4"))

$\tan\theta = \pm\frac{7}{3}, \tan\theta = \pm\frac{3}{7}, \tan\theta = \pm\frac{-2--5}{4--3}$ etc

There must be an attempt to subtract the coordinates (seen or applied at least once)

If part (b) is attempted first, look for example for $\sin\theta = \pm\frac{7}{"\sqrt{58}"}, \cos\theta = \pm\frac{7}{"\sqrt{58}"}$, etc

They may use the cosine rule and trigonometry to find the two angles in the scheme. See above. Eg award for $\cos\theta = \frac{"58"+"20"-"34"}{2\times"\sqrt{58}"\times"\sqrt{20}"}$ **and** $\tan\theta = \pm\frac{4}{2}$ or equivalent.

dM1: A full attempt to find the bearing. $180° + \arctan\frac{7}{3}, 270° - \arctan\frac{3}{7}$, $360° - "49.8°" - "63.4°"$. It is dependent on the previous method mark.

A1: Bearing = awrt $246.8°$ oe. Allow S 66.8° W

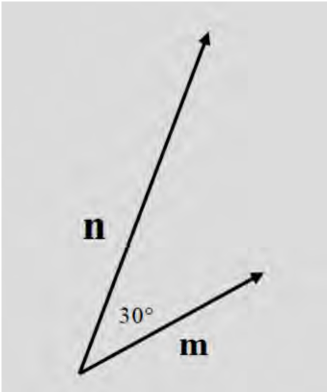
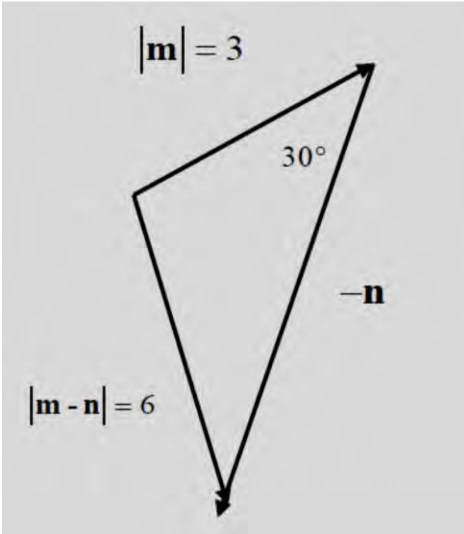
(b)

M1: Attempts to find the distance travelled. Allow for $d^2 = (4 - -3)^2 + (-2 + 5)^2$

You may see this on a diagram and allow if they attempt to find the magnitude from their "resultant vector" found in part (a).

dM1: Attempts to find the speed. There must have been an attempt to find the distance using the coordinates and then divide it by 2.75. Alternatively they could find the speed in km min^{-1} and then multiply by 60

A1: awrt 2.77 km h^{-1}

Question	Scheme	Marks	AOs
7(i)	Explains that $\mathbf{a}$ and $\mathbf{b}$ lie in the same direction oe	B1	2.4
		(1)	
(ii)	 	M1	1.1b
	Attempts $\frac{\sin 30°}{6} = \frac{\sin\theta}{3}$	M1	3.1a
	$\theta =$ awrt $14.5°$	A1	1.1b
	Angle between vector $\mathbf{m}$ and vector $\mathbf{m}-\mathbf{n}$ is awrt $135.5°$	A1	3.2a
		(4)	
			(5 marks)

Notes

(i)

B1: Accept any valid response E.g The lines are collinear. Condone "They are parallel"
Mark positively. ISW after a correct answer
Do not accept "the length of line a +b is the same as the length of line a + the length of line b
Do not accept $|\mathbf{a}|$ and $|\mathbf{b}|$ are parallel.

(ii)

M1: A triangle showing 3, 6 and $30°$ in the correct positions.
Look for 6' opposite $30°$ with another side of 3.
Condone the triangle not being obtuse angled and not being to scale.
Do not condone negative lengths in the tringle. This would automatically be M0

M1: Correct sine rule statement with the sides and angles in the correct positions.
If a triangle is drawn then the angles and sides must be in the correct positions.
This is not dependent so allow recovery from negative lengths in the triangle.
If the candidate has not drawn a diagram then correct sine rule would be M1 M1
Do not accept calculations where the sides have a negative length. Eg $\frac{\sin 30°}{6} = \frac{\sin\theta}{-3}$ is M0

A1: $\theta =$ awrt $14.5°$

A1: CSO awrt $135.5°$

Question	Scheme	Marks	AOs
8(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = -9\mathbf{i} + 3\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB \rvert = \sqrt{(-9)^2 + (3)^2}$	M1	1.1b
	$\lvert AB \rvert = 3\sqrt{10}$	A1ft	1.1b
		(2)	
			(4 marks)

Notes

(a)

M1: Attempts subtraction either way around.

This may be implied by one correct component $\overrightarrow{AB} = \pm 9\mathbf{i} \pm 3\mathbf{j}$

There must be some attempt to write in vector form.

A1: cao (allow column vector notation but not the coordinate)

Correct notation should be used. Accept $-9\text{i} + 3\text{j}$ or $\begin{pmatrix} -9 \\ 3 \end{pmatrix}$ but not $\begin{pmatrix} -9\text{i} \\ 3\text{j} \end{pmatrix}$

(b)

M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a)

Note that $\lvert AB \rvert = \sqrt{(9)^2 + (3)^2}$ is also correct.

Condone missing brackets in the expression $\lvert AB \rvert = \sqrt{-9^2 + (3)^2}$

Also allow a restart usually accompanied by a diagram.

A1ft: $\lvert AB \rvert = 3\sqrt{10}$ ft from their answer to (a) as long as it has both an **i** and **j** component.

It must be simplified, if appropriate. Note that $\pm 3\sqrt{10}$ would be M1 A0

Note that, in cases where there is no working, the correct answer implies M1A1 in each part of this question

Question	Scheme	Marks	AOs
9 (a)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 6\mathbf{i} - 3\mathbf{j} - (4\mathbf{i} + 2\mathbf{j})$	M1	1.1b
	$= 2\mathbf{i} - 5\mathbf{j}$	A1	1.1b
		(2)	
9(b)	Explains that $\overrightarrow{OC}$ is parallel to $\overrightarrow{AB}$ as $8\mathbf{i} - 20\mathbf{j} = 4\times(2\mathbf{i} - 5\mathbf{j})$	M1	1.1b
	As $\overrightarrow{OC} = 4\times\overrightarrow{AB}$ it is parallel to it and not the same length Hence *OABC* is a trapezium.	A1	2.4
		(2)	
			(4 marks)

Notes:

(a)

M1: Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or equivalent. This may be implied by one correct component

A1: $2\mathbf{i} - 5\mathbf{j}$

(b)

M1: Attempts to compare vectors $\overrightarrow{OC}$ and $\overrightarrow{AB}$ by considering their directions

A1: Fully explains why *OABC* is a trapezium. (The candidate is required to state that *OC* is parallel to *AB* but not the same length as it.)

Question	Scheme	Marks	AOs
10 (a)	$\overrightarrow{OA} = \mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OC} = 2\mathbf{i} + 10\mathbf{j} + 9\mathbf{k}$		
	$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{BA} = (2\mathbf{i} + 10\mathbf{j} + 9\mathbf{k}) + (-3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k})$ or $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC} = (\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}) + (-2\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$	M1	3.1a
	So $\overrightarrow{OD} = -\mathbf{i} + 14\mathbf{j} + 4\mathbf{k}$	A1	1.1b
		(2)	
(b)	$\left\{\overrightarrow{AB} = 3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k} \Rightarrow \right\}$ $\lvert\overrightarrow{AB}\rvert = \sqrt{(3)^2 + (-4)^2 + (5)^2}$ $\left\{= \sqrt{50} = 5\sqrt{2}\right\}$	M1	1.1b
	As $\lvert\overrightarrow{AX}\rvert = 10\sqrt{2}$ then $\lvert\overrightarrow{AX}\rvert = 2\lvert\overrightarrow{AB}\rvert \Rightarrow \overrightarrow{AX} = 2\overrightarrow{AB}$		
	$\overrightarrow{OX} = \overrightarrow{OA} + 2\overrightarrow{AB} = (\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}) + 2(3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ or $\overrightarrow{OX} = \overrightarrow{OB} + \overrightarrow{AB} = (4 + 3\mathbf{j} + 3\mathbf{k}) + (3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$	M1	3.1a
	So $\overrightarrow{OX} = 7\mathbf{i} - \mathbf{j} + 8\mathbf{k}$ only	A1	1.1b
		(3)	
		(5 marks)	

Question **10** Notes:

(a)

M1: A complete method for finding the position vector of D

A1: $-\mathbf{i} + 14\mathbf{j} + 4\mathbf{k}$ or $\begin{pmatrix} -1 \\ 14 \\ 4 \end{pmatrix}$

(b)

M1: A complete attempt to find $\lvert\overrightarrow{AB}\rvert$ or $\lvert\overrightarrow{BA}\rvert$

M1: A complete process for finding the position vector of X

A1: $7\mathbf{i} - \mathbf{j} + 8\mathbf{k}$ or $\begin{pmatrix} 7 \\ -1 \\ 8 \end{pmatrix}$

Question	Scheme	Marks	AOs
11(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = 5\mathbf{i} + 10\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB \rvert = \sqrt{(5)^2 + (10)^2}$	M1	1.1b
	$\lvert AB \rvert = 5\sqrt{5}$	A1ft	1.1b
		(2)	
			(4 marks)

Notes:

(a)

M1: Attempts subtraction but may omit brackets

A1: cao (allow column vector notation)

(b)

M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a)

A1ft: $\lvert AB \rvert = 5\sqrt{5}$ ft from their answer to (a)

Note that the correct answer implies M1A1 in each part of this question

Question	Scheme	Marks	AOs
12	Attempts $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k} + \mathbf{i} - 9\mathbf{j} + 3\mathbf{k} = 3\mathbf{i} - 6\mathbf{j} + 4\mathbf{k}$	M1	3.1a
	Attempts to find any one length using 3-d Pythagoras	M1	2.1
	Finds all of $\lvert AB\rvert = \sqrt{14}$, $\lvert AC\rvert = \sqrt{61}$, $\lvert BC\rvert = \sqrt{91}$	A1ft	1.1b
	$\cos BAC = \dfrac{14+61-91}{2\sqrt{14}\sqrt{61}}$	M1	2.1
	angle $BAC = 105.9°$ *	A1*	1.1b
		(5)	
		(5 marks)	

Notes:

M1: Attempts to find $\overrightarrow{AC}$ by using $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

M1: Attempts to find any one length by use of Pythagoras' Theorem

A1ft: Finds all three lengths in the triangle. Follow through on their $\lvert AC\rvert$

M1: Attempts to find BAC using $\cos BAC = \dfrac{\lvert AB\rvert^2 + \lvert AC\rvert^2 - \lvert BC\rvert^2}{2\lvert AB\rvert\lvert AC\rvert}$

Allow this to be scored for other methods such as $\cos BAC = \dfrac{\overrightarrow{AB}.\overrightarrow{AC}}{\lvert AB\rvert\lvert AC\rvert}$

A1*: This is a show that and all aspects must be correct. Angle $BAC =$ 105.9 $^\circ$

Question	Scheme	Marks	AOs
13(a)	Attempts two of the relevant vectors $\pm\overrightarrow{AB} = \pm(-4\mathbf{i}+7\mathbf{j}+\mathbf{k})$ $\pm\overrightarrow{AC} = \pm(-20\mathbf{i}+(p+3)\mathbf{j}+5\mathbf{k})$ $\pm\overrightarrow{BC} = \pm(-16\mathbf{i}+(p-4)\mathbf{j}+4\mathbf{k})$	M1	3.1a
	Uses two of the three vectors in such a way as to find the value of p. E.g. $p+3=5\times 7$	dM1	2.1
	$p=32$	A1	1.1b
		(3)	
	(a) Alternative:		
	$r_{AB} = 4\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\lambda(-4\mathbf{i}+7\mathbf{j}+\mathbf{k})$	M1	3.1a
	$4\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\lambda(-4\mathbf{i}+7\mathbf{j}+\mathbf{k}) = -16\mathbf{i}+p\mathbf{j}+10\mathbf{k} \Rightarrow \lambda = 5$ $4\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\lambda(-4\mathbf{i}+7\mathbf{j}+\mathbf{k}) = -16\mathbf{i}+p\mathbf{j}+10\mathbf{k} \Rightarrow p = 35-3$	dM1	2.1
	$p=32$	A1	1.1b
(b)	Deduces that $\overrightarrow{OD} = \lambda\overrightarrow{OB} = 4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts $\overrightarrow{CD} = 16\mathbf{i}+(4\lambda-\text{"}32\text{"})\mathbf{j}+(6\lambda-10)\mathbf{k}$	M1	3.1a
	Correct attempt at λ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$ $\overrightarrow{CD} = 16\mathbf{i}+(4\lambda-\text{"}32\text{"})\mathbf{j}+(6\lambda-10)\mathbf{k}$ $\overrightarrow{OA} = 4\mathbf{i}-3\mathbf{j}+5\mathbf{k}$ $4\lambda-32=-12 \Rightarrow \lambda = ...$ OR $6\lambda-10=20 \Rightarrow \lambda = ...$	dM1	1.1b
	$\lvert\overrightarrow{OD}\rvert = 5\times\sqrt{4^2+6^2} = 10\sqrt{13}$	A1	1.1b
		(3)	
	(b) Alternative:		
	Deduces that $\overrightarrow{OD} = \lambda\overrightarrow{OB} = 4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts $\overrightarrow{OD} = \overrightarrow{OC}+\mu\overrightarrow{OA} = -16\mathbf{i}+32\mathbf{j}+10\mathbf{k}+\mu(4\mathbf{i}-3\mathbf{j}+5\mathbf{k})$	M1	3.1a
	Correct attempt at λ **or** μ using the fact that $\lambda\overrightarrow{OB} = \overrightarrow{OC}+\mu\overrightarrow{OA}$ E.g. $-16+4\mu=0 \Rightarrow \mu=4$	dM1	1.1b
	$\lvert\overrightarrow{OD}\rvert = 5\times\sqrt{4^2+6^2} = 10\sqrt{13}$	A1	1.1b
		(3)	
			(6 marks)
Notes:			

(a)

M1: Attempts two of the three relevant vectors by **subtracting** either way around. See scheme.

Allow equivalent work e.g. $\pm\overrightarrow{AB} = \pm(\overrightarrow{OB}+\overrightarrow{AO})$

If no working is shown, method can be implied by 2 correct components.

dM1: For the key step in using the fact that if the vectors are parallel, they will be multiples of each other (where the multiple is something other than 1) to find p.

E.g. $p+3=5\times 7$, $p-4=\frac{4}{5}(p+3)$, $p-4=4\times 7$

A1: $p=32$ (Condone 32**j**)

For reference, $\overrightarrow{BC}=4\overrightarrow{AB}$, $\overrightarrow{AC}=5\overrightarrow{AB}$, $\overrightarrow{BC}=\frac{4}{5}\overrightarrow{AC}$, $\overrightarrow{AC}=\frac{5}{4}\overrightarrow{BC}$

Note that candidates generally only need to use 2 components to find *p* and if the 3rd component has errors but is not used, full marks can be awarded.

Alternative:

M1: Forms the vector equation using A or B as position and $\pm\overrightarrow{AB}$ as the direction

dM1: For the key step in using the fact that C lies on the line to find p

A1: $p=32$ (Condone 32**j**)

For reference, $\overrightarrow{BC}=4\overrightarrow{AB}$, $\overrightarrow{AC}=5\overrightarrow{AB}$, $\overrightarrow{BC}=\frac{4}{5}\overrightarrow{AC}$, $\overrightarrow{AC}=\frac{5}{4}\overrightarrow{BC}$

Note that candidates generally only need to use 2 components to find *p* and if the 3rd component has errors but is not used, full marks can be awarded.

There will be other approaches e.g. using "gradients" and "ratios" and the method marks can be implied – if you are unsure if such attempts deserve credit use Review

(b) Vector approach

M1: Deduces that $\overrightarrow{OD}=\lambda\overrightarrow{OB}=4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts $\overrightarrow{CD}=16\mathbf{i}+(4\lambda-\text{"}32\text{"})\mathbf{j}+(6\lambda-10)\mathbf{k}$

dM1: Correct attempt at finding λ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

E.g. $16\mathbf{i}+(4\lambda-\text{"}32\text{"})\mathbf{j}+(6\lambda-10)\mathbf{k}=4\alpha\mathbf{i}-3\alpha\mathbf{j}+5\alpha\mathbf{k}\Rightarrow\alpha=4\Rightarrow 4\lambda-\text{"}32\text{"}=-3\times\text{"}4\text{"}\Rightarrow\lambda=...$

A1: $\left|\overrightarrow{OD}\right|=10\sqrt{13}$

Alternative:

M1: Deduces that $\overrightarrow{OD}=\lambda\overrightarrow{OB}=4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts

$\overrightarrow{OD}=\overrightarrow{OC}+\mu\overrightarrow{OA}=-16\mathbf{i}+32\mathbf{j}+10\mathbf{k}+\mu(4\mathbf{i}-3\mathbf{j}+5\mathbf{k})$

dM1: Correct attempt at finding λ or μ using the fact that $\lambda\overrightarrow{OB}=\overrightarrow{OC}+\mu\overrightarrow{OA}$

E.g. $(-16+4\mu)\mathbf{i}+(\text{"}32\text{"}-3\mu)\mathbf{j}+(10+5\mu)\mathbf{k}=4\lambda\mathbf{j}+6\lambda\mathbf{k}\Rightarrow -16+4\mu=0\Rightarrow\mu=...$

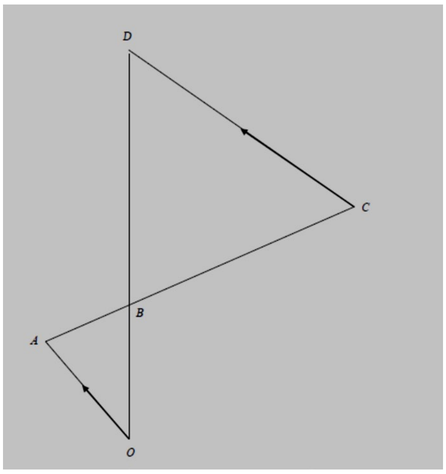
May also solve simultaneously using y and z components to find λ or μ

A1: $\left|\overrightarrow{OD}\right|=10\sqrt{13}$

Note that the correct vector is 20**j** + 30**k**

PTO for similar triangle approach

(b) Similar triangle approach



M1: For the key step in recognising that triangle *BCD* and triangle *BAO* are similar with a ratio of lengths of 4:1

dM1: States or uses the fact that $\left|\overrightarrow{OD}\right| = 5\times\left|\overrightarrow{OB}\right|$

Stating this will score M1 dM1 provided there is no evidence of incorrect work

Note that they may establish this result using the work from (a) but must be used here to score.

A1: $\left|\overrightarrow{OD}\right| = 10\sqrt{13}$

Question Number	Scheme	Marks	AO's
14	Attempts any one of $\left(\pm\overrightarrow{PQ}=\right)\pm(\mathbf{q}-\mathbf{p}),\ \left(\pm\overrightarrow{PR}=\right)\pm(\mathbf{r}-\mathbf{p}),\ \left(\pm\overrightarrow{QR}=\right)\pm(\mathbf{r}-\mathbf{q})$ Or e.g. $\left(\pm\overrightarrow{PQ}=\right)\pm\left(\overrightarrow{OQ}-\overrightarrow{OP}\right),\ \left(\pm\overrightarrow{PR}=\right)\pm\left(\overrightarrow{OR}-\overrightarrow{OP}\right),\ \left(\pm\overrightarrow{QR}=\right)\pm\left(\overrightarrow{OR}-\overrightarrow{OQ}\right)$	M1	1.1b
	Attempts e.g. $\mathbf{r}-\mathbf{q}=2(\mathbf{q}-\mathbf{p})$ $\mathbf{r}-\mathbf{p}=3(\mathbf{q}-\mathbf{p})$ $\frac{2}{3}(\mathbf{q}-\mathbf{p})=\frac{1}{3}(\mathbf{r}-\mathbf{q})$ $\mathbf{q}=\mathbf{p}+\frac{1}{3}(\mathbf{r}-\mathbf{p})$ $\mathbf{q}=\mathbf{r}+\frac{2}{3}(\mathbf{p}-\mathbf{r})$	dM1	3.1a
	E.g. $\Rightarrow \mathbf{r}-\mathbf{q}=2\mathbf{q}-2\mathbf{p}\Rightarrow 2\mathbf{p}+\mathbf{r}=3\mathbf{q}\Rightarrow \mathbf{q}=\frac{1}{3}(\mathbf{r}+2\mathbf{p})$ *	A1*	2.1
		(3)	
			(3 marks)

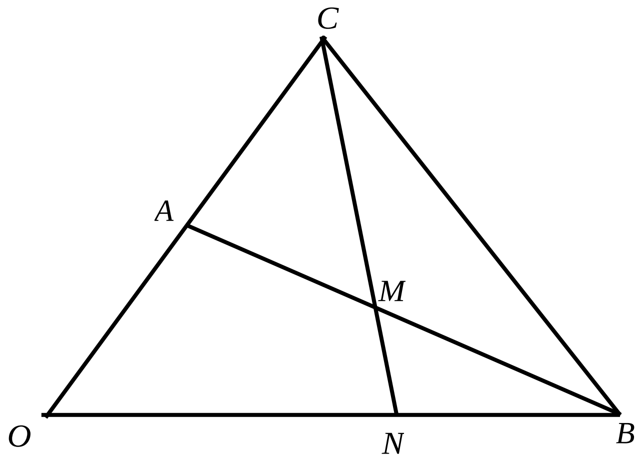
Notes:

M1: Attempts any of the relevant vectors by subtracting either way around. This may be implied by sight of any one of $\pm(\mathbf{q}-\mathbf{p}),\ \pm(\mathbf{r}-\mathbf{p}),\ \pm(\mathbf{r}-\mathbf{q})$ ignoring how they are labelled

dM1: Uses the given information and writes it correctly in vector form that if rearranged would give the printed answer

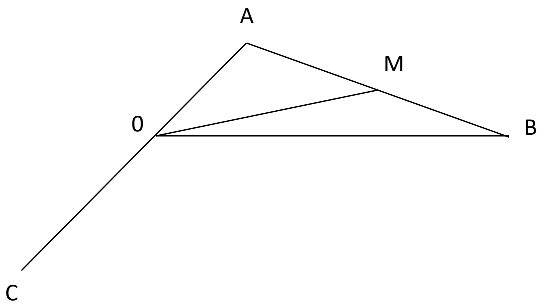
A1*: Fully correct work leading to the given answer. Allow OQ = … as long as OQ has been defined as **q** earlier.

In the working allow use of P instead of **p** and Q instead of **q** as long as the intention is clear.

Question	Scheme	Marks	AOs
15			
	$\overrightarrow{OA}=\mathbf{a},\ \overrightarrow{OB}=\mathbf{b}$		
(a)	$\left\{\overrightarrow{CM}=\overrightarrow{CA}+\overrightarrow{AM}=\overrightarrow{CA}+\frac{1}{2}\overrightarrow{AB}\Rightarrow\right\}\ \overrightarrow{CM}=-\mathbf{a}+\frac{1}{2}(\mathbf{b}-\mathbf{a})$	M1	3.1a
	$\left\{\overrightarrow{CM}=\overrightarrow{CB}+\overrightarrow{BM}=\overrightarrow{CB}+\frac{1}{2}\overrightarrow{BA}\Rightarrow\right\}\ \overrightarrow{CM}=(-2\mathbf{a}+\mathbf{b})+\frac{1}{2}(\mathbf{a}-\mathbf{b})$		
	$\Rightarrow\overrightarrow{CM}=-\frac{3}{2}\mathbf{a}+\frac{1}{2}\mathbf{b}$ (***needs to be simplified and seen in (a) only***)	A1	1.1b
		(2)	
(b)	$\overrightarrow{ON}=\overrightarrow{OC}+\overrightarrow{CN}\Rightarrow\ \overrightarrow{ON}=\overrightarrow{OC}+\lambda\overrightarrow{CM}$	M1	1.1b
	$\overrightarrow{ON}=2\mathbf{a}+\lambda\left(-\frac{3}{2}\mathbf{a}+\frac{1}{2}\mathbf{b}\right)\Rightarrow\overrightarrow{ON}=\left(2-\frac{3}{2}\lambda\right)\mathbf{a}+\frac{1}{2}\lambda\mathbf{b}$ *	A1*	2.1
		(2)	
(c) **Way 1**	$\left(2-\frac{3}{2}\lambda\right)=0\Rightarrow\lambda=...$	M1	2.2a
	$\lambda=\frac{4}{3}\Rightarrow\overrightarrow{ON}=\frac{2}{3}\mathbf{b}\ \left\{\Rightarrow\overrightarrow{NB}=\frac{1}{3}\mathbf{b}\right\}\Rightarrow ON:NB=2:1$ *	A1*	2.1
		(2)	
(c) **Way 2**	$\overrightarrow{ON}=\mu\mathbf{b}\Rightarrow\ \left(2-\frac{3}{2}\lambda\right)\mathbf{a}+\frac{1}{2}\lambda\mathbf{b}=\mu\mathbf{b}$		
	$\mathbf{a}:\ \left(2-\frac{3}{2}\lambda\right)=0\Rightarrow\lambda=...\ \left\{\mathbf{b}:\ \frac{1}{2}\lambda=\mu\ \ \&\ \lambda=\frac{4}{3}\Rightarrow\ \mu=\frac{2}{3}\right\}$	M1	2.2a
	$\lambda=\frac{4}{3}$ or $\mu=\frac{2}{3}\Rightarrow\overrightarrow{ON}=\frac{2}{3}\mathbf{b}\ \left\{\Rightarrow\overrightarrow{NB}=\frac{1}{3}\mathbf{b}\right\}\Rightarrow ON:NB=2:1$ *	A1*	2.1
		(2)	
			(6 marks)

Question	Scheme	Marks	AOs
15 (c) Way 3	$\overrightarrow{OB} = \overrightarrow{ON} + \overrightarrow{NB} \Rightarrow \mathbf{b} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} + K\mathbf{b}$		
	$\mathbf{a}:\ \left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = ...\ \left\{\mathbf{b}:\ 1 = \frac{1}{2}\lambda + K\ \ \&\ \ \lambda = \frac{4}{3}\ \Rightarrow K = \frac{1}{3}\right\}$	M1	2.2a
	$\lambda = \frac{4}{3}$ or $K = \frac{1}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}$ or $\overrightarrow{NB} = \frac{1}{3}\mathbf{b} \Rightarrow ON : NB = 2:1$ *	A1	2.1
		(2)	
15 (c) Way 4	$\overrightarrow{ON} = \mu\mathbf{b}\ \&\ \overrightarrow{CN} = k\overrightarrow{CM}\ \Rightarrow\ \overrightarrow{CO} + \overrightarrow{ON} = k\overrightarrow{CM}$		
	$-2\mathbf{a} + \mu\mathbf{b} = k\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$		
	$\mathbf{a}: -2 = -\frac{3}{2}k \Rightarrow k = \frac{4}{3},\quad \mathbf{b}:\ \mu = \frac{1}{2}k\ \Rightarrow \mu = \frac{1}{2}\left(\frac{4}{3}\right) = ...$	M1	2.2a
	$\mu = \frac{2}{3}\ \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}\ \left\{\Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b}\right\} \Rightarrow ON : NB = 2:1$ *	A1	2.1
		(2)	

Notes for Question **15**	
(a)	
M1:	Valid attempt to find $\overrightarrow{CM}$ using a combination of known vectors **a** and **b**
A1:	A simplified correct answer for $\overrightarrow{CM}$
Note:	Give M1 for $\overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$ or for $\left\{\overrightarrow{CM} = \overrightarrow{OM} - \overrightarrow{OC} \Rightarrow\right\}\ \overrightarrow{CM} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) - 2\mathbf{a}$ **only o.e.**
(b)	
M1:	Uses $\overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$
A1*:	Correct proof
Note:	**<u>Special Case</u>**
	Give SC M1 A0 for the solution $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \lambda\overrightarrow{CM}$
	$\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)\ \left\{= \left(\frac{1}{2} - \frac{3}{2}\lambda\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\lambda\right)\mathbf{b}\right\}$
Note:	**<u>Alternative 1:</u>** Give M1 A1 for the following alternative solution: $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \mu\overrightarrow{CM}$ $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \mu\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) = \left(\frac{1}{2} - \frac{3}{2}\mu\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\mu\right)\mathbf{b}$ $\mu = \lambda - 1 \Rightarrow\ \overrightarrow{ON} = \left(\frac{1}{2} - \frac{3}{2}(\lambda - 1)\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}(\lambda - 1)\right)\mathbf{b}\ \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$
(c)	**Way 1, Way 2 and Way 3**
M1:	Deduces that $\left(2 - \frac{3}{2}\lambda\right) = 0$ and attempts to find the value of λ
A1*:	Correct proof
(c)	**Way 4**
M1:	Complete attempt to find the value of μ
A1*:	Correct proof

Notes for Question **15** Continued	
Note:	Part (b) and part (c) can be marked together.
(a) Special Case	**Special Case where the point *C* is believed to be below the origin *O*** 
	Give Special Case M1 A0 in part (a) for $\left\{\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} \Rightarrow\right\}$ $\overrightarrow{CM} = 3\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$
	$\left\{\text{which leads to } \overrightarrow{CM} = \frac{5}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right\}$

Question	Scheme	Marks	AOs
16	$\overrightarrow{OA} = 2\mathbf{i}+3\mathbf{j}-4\mathbf{k},\ \overrightarrow{OB} = 4\mathbf{i}-2\mathbf{j}+3\mathbf{k},\ \overrightarrow{OC} = a\mathbf{i}+5\mathbf{j}-2\mathbf{k},\ a<0$ $\overrightarrow{AB} = \overrightarrow{BD},\ \lvert\overrightarrow{AB}\rvert = 4$		
(a)	E.g. $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AB}$ **or** $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AB} = \overrightarrow{OB} + \overrightarrow{OB} - \overrightarrow{OA} = 2\overrightarrow{OB} - \overrightarrow{OA}$ **or** $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AB}$		
	$= \begin{pmatrix}4\\-2\\3\end{pmatrix} + \begin{pmatrix}4\\-2\\3\end{pmatrix} - \begin{pmatrix}2\\3\\-4\end{pmatrix} \left\{ = \begin{pmatrix}4\\-2\\3\end{pmatrix} + \begin{pmatrix}2\\-5\\7\end{pmatrix} \right\}$ **or** $= \begin{pmatrix}2\\3\\-4\end{pmatrix} + 2\left(\begin{pmatrix}4\\-2\\3\end{pmatrix} - \begin{pmatrix}2\\3\\-4\end{pmatrix}\right) \left\{ = \begin{pmatrix}2\\3\\-4\end{pmatrix} + 2\begin{pmatrix}2\\-5\\7\end{pmatrix} \right\}$	M1	3.1a
	$= \begin{pmatrix}6\\-7\\10\end{pmatrix}$ or $6\mathbf{i}-7\mathbf{j}+10\mathbf{k}$	A1	1.1b
		(2)	
(b)	$(a-2)^2 + (5-3)^2 + (-2--4)^2$	M1	1.1b
	$\left\{\lvert\overrightarrow{AC}\rvert = 4 \Rightarrow\right\} (a-2)^2 + (5-3)^2 + (-2--4)^2 = (4)^2$ $\Rightarrow (a-2)^2 = 8 \Rightarrow a = ...$ **or** $\Rightarrow a^2 - 4a - 4 = 0 \Rightarrow a = ...$	dM1	2.1
	(as $a<0 \Rightarrow$) $a = 2-2\sqrt{2}$ (or $a = 2-\sqrt{8}$)	A1	1.1b
		(3)	
		(5 marks)	

Notes for Question 16

(a)	
M1:	Complete ***applied*** strategy to find a vector expression for $\overrightarrow{OD}$
A1:	See scheme
Note:	Give M0 for subtracting the wrong way wrong to give e.g. $(4\mathbf{i}-2\mathbf{j}+3\mathbf{k}) + (2\mathbf{i}+3\mathbf{j}-4\mathbf{k}) - (4\mathbf{i}-2\mathbf{j}+3\mathbf{k}) = (4\mathbf{i}-2\mathbf{j}+3\mathbf{k}) + (-2\mathbf{i}+5\mathbf{j}-7\mathbf{k}) = (2\mathbf{i}+3\mathbf{j}-4\mathbf{k})$
Note:	Writing e.g. $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{AB}$ or $\overrightarrow{OD} = 2\overrightarrow{OB} - \overrightarrow{OA}$ with no other work is M0
Note:	Finding ***coordinates***, i.e. $(6, -7, 10)$ without reference to the correct position vectors is A0
Note:	Allow M1A1 for writing down $6\mathbf{i}-7\mathbf{j}+10\mathbf{k}$ with no working
Note:	M1 can be implied for at least two correct components in their position vector of D
(b)	
M1:	Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OC}$, then squares and adds each of the 3 components **Note:** Ignore labelling
dM1:	Complete method of ***correctly*** applying Pythagoras' Theorem on $\lvert\overrightarrow{AC}\rvert = 4$ and using a correct method of solving their resulting quadratic equation to find at least one of $a = ...$
Note:	Condone at least one of either awrt 4.8 or awrt -0.83 for the dM mark
A1:	Obtains **only one** exact value, $a = 2-2\sqrt{2}$
Note:	Writing $a = 2\pm2\sqrt{2}$, without evidence of rejecting $a = 2+2\sqrt{2}$ is A0
Note:	Allow exact alternatives such as $2-\sqrt{8}$ or $\frac{4-\sqrt{32}}{2}$ for A1, and isw can be applied
Note:	Writing $a = -0.828...$, without reference to a correct exact value is A0

Question Number	Scheme	Notes	Marks
17.	$\overrightarrow{OA}=\begin{pmatrix}-3\\7\\2\end{pmatrix}, \overrightarrow{AB}=\begin{pmatrix}4\\-6\\2\end{pmatrix}, \overrightarrow{OP}=\begin{pmatrix}9\\1\\8\end{pmatrix}; \overrightarrow{OQ}=\begin{pmatrix}9+4\mu\\1-6\mu\\8+2\mu\end{pmatrix}$ or $\overrightarrow{OQ}=\begin{pmatrix}9+2\mu\\1-3\mu\\8+\mu\end{pmatrix}$	Let $\theta=$ size of angle PAB. A, B lie on l_1 and P lies on l_2	
(a)	$\{\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}\Rightarrow\}$	Attempts to add $\overrightarrow{OA}$ to $\overrightarrow{AB}$	M1
	$\overrightarrow{OB}=\begin{pmatrix}-3\\7\\2\end{pmatrix}+\begin{pmatrix}4\\-6\\2\end{pmatrix}=\begin{pmatrix}1\\1\\4\end{pmatrix}\Rightarrow B(1,1,4)$	$(1,1,4)$ or $\begin{pmatrix}1\\1\\4\end{pmatrix}$ or $\mathbf{i}+\mathbf{j}+4\mathbf{k}$	A1
	Note: M1 can be implied by at least 2 correct components for B		**[2]**
(b)	$\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}9\\1\\8\end{pmatrix}-\begin{pmatrix}-3\\7\\2\end{pmatrix}=\begin{pmatrix}12\\-6\\6\end{pmatrix}$ or $\overrightarrow{PA}=\begin{pmatrix}-12\\6\\-6\end{pmatrix}$	An attempt to find $\overrightarrow{AP}$ or $\overrightarrow{PA}$	M1
	$\left\{\cos\theta=\dfrac{\overrightarrow{AP}\bullet\overrightarrow{AB}}{\lvert\overrightarrow{AP}\rvert\lvert\overrightarrow{AB}\rvert}\right\}=\dfrac{\begin{pmatrix}12\\-6\\6\end{pmatrix}\bullet\begin{pmatrix}4\\-6\\2\end{pmatrix}}{\sqrt{(12)^2+(-6)^2+(6)^2}.\sqrt{(4)^2+(-6)^2+(2)^2}}$	Applies dot product formula between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $(\overrightarrow{AB}$ or $\overrightarrow{BA})$ or a multiple of these vectors	dM1
	$\left\{\cos\theta=\dfrac{96}{\sqrt{216}.\sqrt{56}}\Rightarrow\cos\theta\right\}=\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$	$\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$	A1
			[3]
(c)	$\left\{\cos\theta=\dfrac{4}{\sqrt{21}}\right\}\Rightarrow\sin\theta=\dfrac{\sqrt{21-16}}{\sqrt{21}}=\dfrac{\sqrt{5}}{\sqrt{21}}=\dfrac{\sqrt{105}}{21}$	A correct method for converting an exact value for $\cos q$ to an exact value for $\sin q$	M1
	Area $PAB=\dfrac{1}{2}(\sqrt{216})(\sqrt{56})\left(\dfrac{\sqrt{5}}{\sqrt{21}}\right)\left\{=12\sqrt{21}\left(\dfrac{\sqrt{5}}{\sqrt{21}}\right)\right\}=12\sqrt{5}$	see notes	M1
		$12\sqrt{5}$	A1 **cao**
			[3]
(d)	$\{l_2:\}\ \mathbf{r}=\begin{pmatrix}9\\1\\8\end{pmatrix}+\mu\begin{pmatrix}4\\-6\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}9\\1\\8\end{pmatrix}+\mu\begin{pmatrix}2\\-3\\1\end{pmatrix}$	$\mathbf{p}+\lambda\mathbf{d}$ or $\mathbf{p}+\mu\mathbf{d}$, $\mathbf{p}\neq0$, $\mathbf{d}\neq0$ with either $\mathbf{p}=9\mathbf{i}+\mathbf{j}+8\mathbf{k}$ or $\mathbf{d}=4\mathbf{i}-6\mathbf{j}+2\mathbf{k}$ or $\mathbf{d}=$ multiple of $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$	M1
		Correct vector equation	A1
			[2]
(e)	$\overrightarrow{BQ}=\begin{pmatrix}9+4\mu\\1-6\mu\\8+2\mu\end{pmatrix}-\begin{pmatrix}1\\1\\4\end{pmatrix}\left\{=\begin{pmatrix}8+4\mu\\-6\mu\\4+2\mu\end{pmatrix}\right\}\ \left\{\overrightarrow{QB}=\begin{pmatrix}-8-4\mu\\6\mu\\-4-2\mu\end{pmatrix}\right\}$	Applies their $\overrightarrow{OQ}-$ their $\overrightarrow{OB}$ **or** their $\overrightarrow{OB}-$ their $\overrightarrow{OQ}$	M1
	$\overrightarrow{BQ}\bullet\overrightarrow{AP}=0\Rightarrow\begin{pmatrix}8+4\mu\\-6\mu\\4+2\mu\end{pmatrix}\bullet\begin{pmatrix}12\\-6\\6\end{pmatrix}=0\Rightarrow\mu=...$	Applies $\overrightarrow{BQ}\bullet\overrightarrow{AP}=0$, o.e. and ***solves*** the resulting equation to find a value for μ	dM1
	$\Rightarrow96+48\mu+36\mu+24+12\mu=0\Rightarrow96\mu+120=0\Rightarrow\mu=-\dfrac{5}{4}$	$\mu=-\dfrac{120}{96}$ or $\mu=-\dfrac{5}{4}$	A1 o.e.
	$\overrightarrow{OQ}=\begin{pmatrix}9+4(-1.25)\\1-6(-1.25)\\8+2(-1.25)\end{pmatrix}=\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}\Rightarrow Q(4,8.5,5.5)$	Substitutes their value of μ into $\overrightarrow{OQ}$	ddM1
		$(4,8.5,5.5)$ or $\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}$ or $4\mathbf{i}+8.5\mathbf{j}+5.5\mathbf{k}$	A1 o.e.
			[5]
			15

Question Number	Scheme	Notes	Marks
17.	$\overrightarrow{OA}=\begin{pmatrix}-3\\7\\2\end{pmatrix}$, $\overrightarrow{AB}=\begin{pmatrix}4\\-6\\2\end{pmatrix}$, $\overrightarrow{OP}=\begin{pmatrix}9\\1\\8\end{pmatrix}$; $\overrightarrow{OQ}=\begin{pmatrix}9+4\mu\\1-6\mu\\8+2\mu\end{pmatrix}$ or $\overrightarrow{OQ}=\begin{pmatrix}9+2\mu\\1-3\mu\\8+\mu\end{pmatrix}$	Let $\theta =$ size of angle PAB. A, B lie on l_1 and P lies on l_2	
(e) **Alt 1**	$\overrightarrow{BQ}=\begin{pmatrix}9+2\mu\\1-3\mu\\8+\mu\end{pmatrix}-\begin{pmatrix}1\\1\\4\end{pmatrix}\left\{=\begin{pmatrix}8+2\mu\\-3\mu\\4+\mu\end{pmatrix}\right\}$ $\left\{\overrightarrow{QB}=\begin{pmatrix}-8-2\mu\\3\mu\\-4-\mu\end{pmatrix}\right\}$	Applies their $\overrightarrow{OQ}$ − their $\overrightarrow{OB}$ **or** their $\overrightarrow{OB}$ − their $\overrightarrow{OQ}$	M1
	$\overrightarrow{BQ}\bullet\overrightarrow{AP}=0\Rightarrow\begin{pmatrix}8+2\mu\\-3\mu\\4+\mu\end{pmatrix}\bullet\begin{pmatrix}12\\-6\\6\end{pmatrix}=0\Rightarrow\mu=...$	Applies $\overrightarrow{BQ}\bullet\overrightarrow{AP}=0$, o.e. and ***solves*** the resulting equation to find a value for μ	dM1
	$\Rightarrow 96+24\mu+18\mu+24+6\mu=0\Rightarrow 48\mu+120=0\Rightarrow\mu=-\frac{5}{2}$	$\mu=-\frac{5}{2}$	A1 o.e.
	$\overrightarrow{OQ}=\begin{pmatrix}9+2(-2.5)\\1-3(-2.5)\\8+1(-2.5)\end{pmatrix}=\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}\Rightarrow Q(4, 8.5, 5.5)$	Substitutes their value of μ into $\overrightarrow{OQ}$	ddM1
		$(4, 8.5, 5.5)$ or $\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}$ or $4\mathbf{i}+8.5\mathbf{j}+5.5\mathbf{k}$	A1 o.e.
			[5]
(b) **Alt 1**	**<u>Vector Cross Product:</u>** Use this scheme if a vector cross product method is being applied		
	$\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}9\\1\\8\end{pmatrix}-\begin{pmatrix}-3\\7\\2\end{pmatrix}=\begin{pmatrix}12\\-6\\6\end{pmatrix}$ or $\overrightarrow{PA}=\begin{pmatrix}-12\\6\\-6\end{pmatrix}$	An attempt to find $\overrightarrow{AP}$ or $\overrightarrow{PA}$	M1
	$\mathbf{d}_1\times\mathbf{d}_2=\begin{pmatrix}12\\-6\\6\end{pmatrix}\times\begin{pmatrix}4\\-6\\2\end{pmatrix}=\left\{\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\12&-6&6\\4&-6&2\end{vmatrix}=24\mathbf{i}+0\mathbf{j}-48\mathbf{k}\right\}$		
	$\sin\theta=\dfrac{\sqrt{(24)^2+(0)^2+(-48)^2}}{\sqrt{(12)^2+(-6)^2+(6)^2}.\sqrt{(4)^2+(-6)^2+(2)^2}}$	Applies vector cross product formula between their $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\left(\overrightarrow{AB}\text{ or }\overrightarrow{BA}\right)$ or a multiple of these vectors	dM1
	$\left\{\sin\theta=\dfrac{\sqrt{2880}}{\sqrt{216}.\sqrt{56}}=\sqrt{\dfrac{5}{21}}\right\}\left\{\Rightarrow\cos\theta\right\}=\sqrt{\dfrac{16}{21}}=\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$	$\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$	A1
			[3]
(b) **Alt 2**	**<u>Cosine Rule</u>**		
	$\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}9\\1\\8\end{pmatrix}-\begin{pmatrix}-3\\7\\2\end{pmatrix}=\begin{pmatrix}12\\-6\\6\end{pmatrix}$ or $\overrightarrow{PA}=\begin{pmatrix}-12\\6\\-6\end{pmatrix}$	An attempt to find $\overrightarrow{AP}$ or $\overrightarrow{PA}$	M1
	Note: $\left\lvert\overrightarrow{PA}\right\rvert=\sqrt{216}$, $\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{56}$ and $\left\lvert\overrightarrow{PB}\right\rvert=\sqrt{80}$		
	$\left(\sqrt{80}\right)^2=\left(\sqrt{216}\right)^2+\left(\sqrt{56}\right)^2-2\left(\sqrt{216}\right)\left(\sqrt{56}\right)\cos\theta$	Applies the cosine rule the correct way round	dM1
	$\cos\theta=\dfrac{216+56-80}{2\sqrt{216}\sqrt{56}}=\dfrac{192}{2\sqrt{216}\sqrt{56}}$		
	$\left\{\Rightarrow\cos\theta\right\}=\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$	$\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$	A1
			[3]

		Question 17 Notes
17. (b)	**Note**	If no "subtraction" seen, you can award 1st M1 for 2 out of 3 correct components of the difference
	Note	For dM1 the dot product formula can be applied as $\sqrt{(12)^2+(-6)^2+(6)^2}.\sqrt{(4)^2+(-6)^2+(2)^2}\cos\theta = \begin{pmatrix}12\\-6\\6\end{pmatrix}\bullet\begin{pmatrix}4\\-6\\2\end{pmatrix}$
	Note	***Evaluation*** of the dot product for $12\mathbf{i}-6\mathbf{j}+6\mathbf{k}$ & $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ is not required for the dM1 mark
	A1	For either $\frac{4}{\sqrt{21}}$ or $\frac{4}{21}\sqrt{21}$ or $\cos\theta=\frac{4}{\sqrt{21}}$ or $\frac{4}{21}\sqrt{21}$
	Note	Using $12\mathbf{i}-6\mathbf{j}+6\mathbf{k}$ & $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ gives $\cos\theta=\frac{24+18+6}{\sqrt{216}.\sqrt{14}}=\frac{48}{12\sqrt{21}}=\underline{\underline{\frac{4}{\sqrt{21}}}}$ or $\underline{\underline{\frac{4}{21}\sqrt{21}}}$
	Note	Using $2\mathbf{i}-\mathbf{j}+\mathbf{k}$ & $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ gives $\cos\theta=\frac{4+3+1}{\sqrt{6}.\sqrt{14}}=\frac{8}{2\sqrt{21}}=\underline{\underline{\frac{4}{\sqrt{21}}}}$ or $\underline{\underline{\frac{4}{21}\sqrt{21}}}$
	Note	Give M1M1A0 for finding $\theta=$ awrt 29.2 without reference to $\cos\theta=\underline{\underline{\frac{4}{\sqrt{21}}}}$ or $\underline{\underline{\frac{4}{21}\sqrt{21}}}$
	Note	Condone taking the dot product between vectors the wrong way round for the M1 dM1 marks
	Note	**<u>Vectors the wrong way round</u>**
		• E.g. taking the dot product between $\overrightarrow{PA}$ and $\overrightarrow{AB}$ to give $\cos\theta=-\frac{4}{\sqrt{21}}$ or $-\frac{4}{21}\sqrt{21}$ ***with no other working*** is final A0 • E.g. taking the dot product between $\overrightarrow{PA}$ and $\overrightarrow{AB}$ to give $\cos\theta=-\frac{4}{\sqrt{21}}$ or $-\frac{4}{21}\sqrt{21}$ **followed by** $\cos\theta=\frac{4}{\sqrt{21}}$ or $\frac{4}{21}\sqrt{21}$ or just simply writing $\frac{4}{\sqrt{21}}$ or $\frac{4}{21}\sqrt{21}$ is final A1
	Note	In part (b), give M0dM0 for finding and using $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{AB}=(5\mathbf{i}+7\mathbf{j}+6\mathbf{k})$
(c)	**Note**	Give 1st M0 for $\sin\theta=\sin\left(\cos^{-1}\left(\frac{4\sqrt{21}}{21}\right)\right)$ or $\sin\theta=1-\left(\frac{4}{21}\sqrt{21}\right)^2$ ***unless recovered***
	M1	Give 2nd M1 for either • $\frac{1}{2}$(their length AP)(their length AB)(their attempt at $\sin\theta$) • $\frac{1}{2}$(their length AP)(their length AB)sin(their 29.2° from part (b)) • $\frac{1}{2}$(their length AP)(their length AB)$\sin\theta$; where $\cos\theta=...$ in part (b)
	Note	$\frac{1}{2}\left(\sqrt{216}\right)\left(\sqrt{56}\right)$sin(awrt 29.2° or awrt 150.8°) {= awrt 26.8} without reference to finding $\sin\theta$ as an exact value if M0 M1 A0
	Note	Anything that rounds to 26.8 without reference to finding $\sin\theta$ as an exact value is M0 M1 A0
	Note	Anything that rounds to 26.8 without reference to $12\sqrt{5}$ is A0
	Note	If they use $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{AB}=(5\mathbf{i}+7\mathbf{j}+6\mathbf{k})$ in part (b), then this can be followed through in part (c) for the 2nd M mark as e.g. $\frac{1}{2}\left(\sqrt{110}\right)\left(\sqrt{56}\right)\sin\theta$
	Note	Finding $12\sqrt{5}$ in part (c) is M1 dM1 A1, even if there is little or no evidence of finding an exact value for $\sin\theta$. So $\frac{1}{2}\left(\sqrt{216}\right)\left(\sqrt{56}\right)\sin(29.2°)=12\sqrt{5}$ is M1 dM1 A1

	Question 17 Notes Continued	
17. (d)	**Note**	Writing $\mathbf{r} = ...$ or $l_2 = ...$ or $l = ...$ or Line 2 = … is not required for the M mark
	A1	Writing $\mathbf{r} = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} + \mu\mathbf{d}$, where $\mathbf{d} =$ a multiple of $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$
	Note	Writing $\mathbf{r} = ...$ or $l_2 = ...$ or $l = ...$ or Line 2 = … is required for the A mark
	Note	Other valid $\mathbf{p} = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix}$ are e.g. $\mathbf{p} = \begin{pmatrix} 13 \\ -5 \\ 10 \end{pmatrix}$ or $\mathbf{p} = \begin{pmatrix} 5 \\ 7 \\ 6 \end{pmatrix}$. So $\mathbf{r} = \begin{pmatrix} 13 \\ -5 \\ 10 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$ is M1 A1
	Note	Give A0 for writing $l_2 : \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$ or ans $= \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$ unless recovered
	Note	Using scalar parameter λ or other scalar parameters (e.g. μ or *s* or *t*) is fine for M1 and/or A1
(e)	**ddM1**	Substitutes their value of μ into $\overrightarrow{OQ}$, where $\overrightarrow{OQ}$ = their equation for l_2
	Note	If they use $\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{AB} = (5\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$ in part (b), then this can be followed through in part (e) for the 2nd M mark and the 3rd M mark
	Note	You imply the final M mark in part (e) for at least 2 correctly followed through components for *Q* from their μ

Question Number	Scheme	Notes	Marks
17. (c) **Alt 1**	**Vector Cross Product:** Use this scheme if a vector cross product method is being applied		
	$\overrightarrow{AP} \times \overrightarrow{AB} = \begin{pmatrix} 12 \\ -6 \\ 6 \end{pmatrix} \times \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & -6 & 6 \\ 4 & -6 & 2 \end{vmatrix} = 24\mathbf{i} + 0\mathbf{j} - 48\mathbf{k} \right\}$		
	Area $PAB = \frac{1}{2}\sqrt{(24)^2 + (-48)^2}$	Uses a vector product and $\sqrt{("24")^2 + ("0")^2 + ("-48")^2}$	M1
		Uses a vector product and $\frac{1}{2}\sqrt{("24")^2 + ("0")^2 + ("-48")^2}$	M1
	$= 12\sqrt{5}$	$12\sqrt{5}$	A1 **cao**
			[3]
17. (c) **Alt 2**	**Note:** $\cos APB = \frac{5}{\sqrt{30}}$ or $\frac{1}{6}\sqrt{30}$ **Note:** $\lvert\overrightarrow{PA}\rvert = \sqrt{216}$ and $\lvert\overrightarrow{PB}\rvert = \sqrt{80}$		
	$\sin\theta = \frac{\sqrt{30 - 25}}{\sqrt{30}} = \frac{\sqrt{5}}{\sqrt{30}} = \frac{\sqrt{6}}{6}$	A correct method for converting an exact value for $\cos q$ to an exact value for $\sin q$	M1
	Area $PAB = \frac{1}{2}(\sqrt{216})(\sqrt{80})\left(\frac{\sqrt{5}}{\sqrt{30}}\right)\left\{= 12\sqrt{30}\left(\frac{\sqrt{5}}{\sqrt{30}}\right)\right\} = 12\sqrt{5}$	$\frac{1}{2}$(their *PA*)(their *PB*)$\sin\theta$	M1
		$12\sqrt{5}$	A1 **cao**
			[3]

Question Number	Scheme	Notes	Marks
18.	$l_1 : \mathbf{r} = \begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}, \quad l_2 : \mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$; $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix}$ lies on l_1	Let θ_{Acute} be the acute angle between l_1 and l_2	
(a)	$\{l_1 = l_2 \Rightarrow\}\ 28 - 5\lambda = 3\ \{\Rightarrow \lambda = 5\}$ or $4 - \lambda = 5 + 3\mu$ and $4 + \lambda = 1 - 4\mu\ \{\Rightarrow \mu = -2\}$	$28 - 5\lambda = 3$ or $4 - \lambda = 5 + 3\mu$ and $4 + \lambda = 1 - 4\mu$ or $\lambda = 5$ or $\mu = -2$ (Can be implied).	B1
	$\{\overrightarrow{OX} =\}\ \begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + 5\begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} - 2\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$	Puts $l_1 = l_2$ and solves to find λ and/or μ **and** substitutes their value for λ into l_1 or their value for μ into l_2	M1
	So, $X(-1, 3, 9)$	$(-1, 3, 9)$ or $\begin{pmatrix} -1 \\ 3 \\ 9 \end{pmatrix}$ or $-\mathbf{i} + 3\mathbf{j} + 9\mathbf{k}$ or condone $\begin{matrix} -1 \\ 3 \\ 9 \end{matrix}$	A1 cao
			[3]
(b) **Way 1**	$\mathbf{d}_1 = \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix},\ \mathbf{d}_2 = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$	Realisation that the dot product is required between $\mathbf{d_1}$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d_1}$ and $\mathbf{d}_2$	M1
	$\cos\theta = \dfrac{\pm \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}}{\sqrt{(-1)^2 + (-5)^2 + (1)^2}\,.\sqrt{(3)^2 + (0)^2 + (-4)^2}}\ \left\{= \dfrac{-7}{\sqrt{27}.\sqrt{25}}\right\}$	**dependent on the 1st M mark.** Applies dot product formula between $\mathbf{d_1}$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d_1}$ and $\mathbf{d}_2$	dM1
	$\{\theta = 105.6303588... \Rightarrow\}\ \theta_{\text{Acute}} = 74.36964117... = 74.37$ (2 dp)	awrt 74.37 seen in (b) only	A1
			[3]
(c)	$\overrightarrow{AX} = "\overrightarrow{OX}" - \overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \\ 9 \end{pmatrix} - \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ -15 \\ 3 \end{pmatrix}$ or $A_{\lambda=2}, X_{\lambda=5} \Rightarrow AX = 3\lvert\mathbf{d}_1\rvert, \{\lvert\mathbf{d}_1\rvert = \sqrt{27}\}$		
	$AX = \sqrt{(-3)^2 + (-15)^2 + (3)^2}$ or $3\sqrt{27}\ \{= \sqrt{243}\} = 9\sqrt{3}$	Full method for finding AX or XA	M1
		$9\sqrt{3}$ seen in (c) only	A1 cao
	Note: You cannot recover work for part (c) in either part (d) or part (e).		**[2]**
(d) **Way 1**	$\dfrac{YA}{"9\sqrt{3}"} = \tan("74.36964...")$	$\dfrac{YA}{\text{their } \lvert\overrightarrow{AX}\rvert} = \tan\theta$ or $YA = \left(\text{their } \lvert\overrightarrow{AX}\rvert\right)\tan\theta$, where θ is their acute or obtuse angle between l_1 and l_2	M1
	$YA = 55.71758... = 55.7$ (1 dp)	anything that rounds to 55.7	A1
			[2]
(e) **Way 1**	$\{A_{\lambda=2}, X_{\lambda=5} \Rightarrow \text{So } AX = 2AB \Rightarrow \text{So at } B,\ \lambda = 3.5 \text{ or } \lambda = 0.5)\}$		
	$\overrightarrow{OB} = \begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + 3.5\begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}; = \begin{pmatrix} 0.5 \\ 10.5 \\ 7.5 \end{pmatrix}$	Substitutes **either** $\lambda = \dfrac{(\text{their } \lambda_X \text{ found in } (a)) + 2}{2}$ **or** $\lambda_b = 3 - \dfrac{(\text{their } \lambda_X \text{ found in } (a))}{2}$ into l_1	M1;
	$\overrightarrow{OB} = \begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + 0.5\begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}; = \begin{pmatrix} 3.5 \\ 25.5 \\ 4.5 \end{pmatrix}$	At least one position vector is correct. (Also allow coordinates).	A1
		Both position vectors are correct. (Also allow coordinates).	A1
			[3]
			13

Question Number	Scheme	Notes	Marks
18. (e) **Way 2**	$\left\{AX = 2AB \Rightarrow AB = \tfrac{1}{2}AX.\ \text{So, } \overrightarrow{OB} = \overrightarrow{OA} \pm \overrightarrow{AB} \Rightarrow \overrightarrow{OB} = \overrightarrow{OA} \pm \tfrac{1}{2}\overrightarrow{AX}\right\}$		
	$\overrightarrow{OB} = \begin{pmatrix}2\\18\\6\end{pmatrix} + 0.5\begin{pmatrix}-3\\-15\\3\end{pmatrix}; = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$	Applies either $\overrightarrow{OA} + 0.5\overrightarrow{AX}$ **or** $\overrightarrow{OA} - 0.5\overrightarrow{AX}$ where $(\text{their } \overrightarrow{AX}) = \pm\left[(\text{their } \overrightarrow{OX}) - \overrightarrow{OA}\right]$	M1;
	$\overrightarrow{OB} = \begin{pmatrix}2\\18\\6\end{pmatrix} - 0.5\begin{pmatrix}-3\\-15\\3\end{pmatrix}; = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$	At least one position vector is correct (Also allow coordinates)	A1
		Both position vectors are correct (Also allow coordinates)	A1
			[3]
18. (e) **Way 3**	$\overrightarrow{AB} = \begin{pmatrix}4-\lambda\\28-5\lambda\\4+\lambda\end{pmatrix} - \begin{pmatrix}2\\18\\6\end{pmatrix} = \begin{pmatrix}2-\lambda\\10-5\lambda\\-2+\lambda\end{pmatrix} = \begin{pmatrix}1(2-\lambda)\\5(2-\lambda)\\-1(2-\lambda)\end{pmatrix}$; $\overrightarrow{AX} = \begin{pmatrix}-3\\-15\\3\end{pmatrix}$ $\quad AX^2 = 243 \Rightarrow$ $AB^2 = 27(2-\lambda)^2$ $AX = 2AB \Rightarrow AX^2 = 4AB^2 \Rightarrow 243 = 4(27)(2-\lambda)^2 \Rightarrow (2-\lambda)^2 = \frac{9}{4}$ **or** $27\lambda^2 - 108\lambda + \frac{189}{4} = 0$ **or** $108\lambda^2 - 432\lambda + 189 = 0$ **or** $4\lambda^2 - 16\lambda + 7 = 0 \Rightarrow \lambda = 3.5$ or $\lambda = 0.5$		
	$\overrightarrow{OB} = \begin{pmatrix}4\\28\\4\end{pmatrix} + 3.5\begin{pmatrix}-1\\-5\\1\end{pmatrix}; = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$	Full method of solving for λ the equation $AX^2 = 4AB^2$ using $(\text{their } \overrightarrow{AX})$ and $\overrightarrow{AB}$ **and** substitutes at least one of their values for λ into l_1	M1;
	$\overrightarrow{OB} = \begin{pmatrix}4\\28\\4\end{pmatrix} + 0.5\begin{pmatrix}-1\\-5\\1\end{pmatrix}; = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$	At least one position vector is correct (Also allow coordinates)	A1
		Both position vectors are correct (Also allow coordinates)	A1
	Note: $AX = 2AB \Rightarrow \overrightarrow{AX} = \pm 2\overrightarrow{AB}$. Hence, $\lambda = 3.5$ or $\lambda = 0.5$ can be found from solving either $x: -3 = \pm 2(2-\lambda)$ or $y: -15 = \pm 2(10-5\lambda)$ or $z: -3 = \pm 2(-2+\lambda)$		**[3]**
18. (e) **Way 4**	$\overrightarrow{OB} = \begin{pmatrix}-1\\3\\9\end{pmatrix} + 0.5\begin{pmatrix}3\\15\\-3\end{pmatrix}; = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$	Applies **either** $(\text{their } \overrightarrow{OX}) + 0.5\overrightarrow{XA}$ **or** $(\text{their } \overrightarrow{OX}) + 1.5\overrightarrow{XA}$ where $(\text{their } \overrightarrow{XA}) = \overrightarrow{OA} - (\text{their } \overrightarrow{OX})$	M1;
	$\overrightarrow{OB} = \begin{pmatrix}-1\\3\\9\end{pmatrix} + 1.5\begin{pmatrix}3\\15\\-3\end{pmatrix}; = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$	At least one position vector is correct (Also allow coordinates)	A1
		Both position vectors are correct (Also allow coordinates)	A1
			[3]
18. (e) **Way 5**	$\overrightarrow{OB} = 0.5\left(\begin{pmatrix}-1\\3\\9\end{pmatrix} + \begin{pmatrix}2\\18\\6\end{pmatrix}\right); = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$	Applies $\frac{1}{2}\left[(\text{their } \overrightarrow{OX}) + \overrightarrow{OA}\right]$	M1;
	$\overrightarrow{OB} = \begin{pmatrix}2\\18\\6\end{pmatrix} - 0.5\begin{pmatrix}-3\\-15\\3\end{pmatrix}; = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$	At least one position vector is correct (Also allow coordinates)	A1
		Both position vectors are correct (Also allow coordinates)	A1
			[3]

Question Number	Scheme	Notes	Marks
18. (e) **Way 6**	$\left\{\left\|\overrightarrow{AX}\right\| = 9\sqrt{3}, \left\|d_1\right\| = 3\sqrt{3} \Rightarrow K = \frac{9\sqrt{3}}{3\sqrt{3}} = 3 \Rightarrow \overrightarrow{AX} = 3\mathbf{d}_1; \text{So, } \overrightarrow{OB} = \overrightarrow{OA} \pm \frac{1}{2}\overrightarrow{AX} = \overrightarrow{OA} \pm \frac{1}{2}(3\mathbf{d}_1)\right\}$		
	$\overrightarrow{OB} = \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix} + 0.5\left(3\begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}\right); = \begin{pmatrix} 0.5 \\ 10.5 \\ 7.5 \end{pmatrix}$	Applies either $\overrightarrow{OA} + 0.5(K\mathbf{d}_1)$ **or** $\overrightarrow{OA} - 0.5(K\mathbf{d}_1)$, where $K = \frac{\text{their } \left\|\overrightarrow{AX}\right\|}{3\sqrt{3}}$	M1;
	$\overrightarrow{OB} = \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix} - 0.5\left(3\begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}\right); = \begin{pmatrix} 3.5 \\ 25.5 \\ 4.5 \end{pmatrix}$	At least one position vector is correct (Also allow coordinates)	A1
		Both position vectors are correct (Also allow coordinates)	A1
			[3]

Question 18 Notes

18. (a)	**Note**	M1 can be implied by at least two correct follow through coordinates from their l or from their m.
(b)	**Note**	***Evaluating*** the dot product (i.e. $(-1)(3) + (-5)(0) + (1)(-4)$) is not required for the M1, dM1 marks.
	Note	**For M1 dM1:** Allow one slip in writing down their direction vectors, $\mathbf{d}_1$ and $\mathbf{d}_2$
	Note	Allow M1 dM1 for $\left(\sqrt{(-1)^2 + (-5)^2 + (1)^2}.\sqrt{(3)^2 + (0)^2 + (-4)^2}\right)\cos\theta = \pm\begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}\cdot\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$
	Note	$\theta = 1.297995...^c$, (without evidence of awrt 74.37) is A0

18. (b) **Way 2**	**<u>Alternative Method: Vector Cross Product</u>**		
	Only apply this scheme if it is clear that a vector cross product method is being applied.		
	$\mathbf{d}_1 \times \mathbf{d}_2 = \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} = \left\{\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -5 & 1 \\ 3 & 0 & -4 \end{vmatrix} = 20\mathbf{i} - \mathbf{j} + 15\mathbf{k}\right\}$	Realisation that the vector cross product is required between $\mathbf{d}_1$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d}_1$ and $\mathbf{d}_2$	M1
	$\sin\theta = \frac{\sqrt{(20)^2 + (-1)^2 + (15)^2}}{\sqrt{(-1)^2 + (-5)^2 + (1)^2}.\sqrt{(3)^2 + (0)^2 + (-4)^2}}$	Applies the vector product formula between $\mathbf{d}_1$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d}_1$ and $\mathbf{d}_2$	dM1
	$\sin\theta = \frac{\sqrt{626}}{\sqrt{27}.\sqrt{25}} \Rightarrow \theta = 74.36964117... = 74.37$ (2 dp)	awrt 74.37 seen in (b) only	A1
			[3]

18. (c)	**M1**	Finds the difference between their $\overrightarrow{OX}$ and $\overrightarrow{OA}$ and applies Pythagoras to the result to find AX or XA **OR** applies $\left\|(\text{their } l_X \text{ found in } (a)) - 2\right\|.\sqrt{(-1)^2 + (-5)^2 + (1)^2}$
	Note	For M1: Allow one slip in writing down their $\overrightarrow{OX}$ and $\overrightarrow{OA}$
	Note	Allow M1A1 for $\begin{pmatrix} 3 \\ 15 \\ 3 \end{pmatrix}$ leading to $AX = \sqrt{(3)^2 + (15)^2 + (3)^2} = \sqrt{243} = 9\sqrt{3}$
(e)	**Note**	Imply M1 for no working leading to any two components of one of the $\overrightarrow{OB}$ which are correct.

Question Number	Scheme	Notes	Marks
18. (d) **Way 2**	$\dfrac{"9\sqrt{3}"}{YA} = \tan(90 - "74.36964...")$	$\dfrac{\text{their } \lvert\overrightarrow{AX}\rvert}{YA} = \tan(90-\theta)$ or $AY = \dfrac{\text{their } \lvert\overrightarrow{AX}\rvert}{\tan(90-\theta)}$, where θ is the acute or obtuse angle between l_1 and l_2	M1
	$YA = 55.71758... = 55.7$ (1 dp)	anything that rounds to 55.7	A1
			[2]
18. (d) **Way 3**	$\dfrac{YA}{\sin("74.36964...")} = \dfrac{"9\sqrt{3}"}{\sin(90 - "74.36964...")}$	$\dfrac{YA}{\sin\theta} = \dfrac{\text{their } \lvert\overrightarrow{AX}\rvert}{\sin(90-\theta)}$ o.e., where θ is the acute or obtuse angle between l_1 and l_2	M1
	$YA = \dfrac{9\sqrt{3}\sin(74.36964...)}{\sin(15.63036...)} = 55.71758... = 55.7$ (1 dp)	anything that rounds to 55.7	A1
			[2]
18. (d) **Way 4**	$\mathbf{d}_1 = \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}$, $\overrightarrow{OY} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + \mu\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 5+3\mu \\ 3 \\ 1-4\mu \end{pmatrix}$		
	$\overrightarrow{YA} = \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix} - \begin{pmatrix} 5+3\mu \\ 3 \\ 1-4\mu \end{pmatrix} = \begin{pmatrix} -3-3\mu \\ 15 \\ 5+4\mu \end{pmatrix}$ $\overrightarrow{YA}\bullet\mathbf{d}_1 = 0 \Rightarrow \begin{pmatrix} -3-3\mu \\ 15 \\ 5+4\mu \end{pmatrix}\bullet\begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix} = 0$ $\Rightarrow 3+3\mu-75+5+4\mu=0 \Rightarrow \mu=\dfrac{67}{7}$ $YA^2 = \left(-3-3\left(\dfrac{67}{7}\right)\right)^2 + (15)^2 + \left(5+4\left(\dfrac{67}{7}\right)\right)^2$	(Allow a sign slip in copying $\mathbf{d}_1$) Applies $\overrightarrow{YA}\bullet\mathbf{d}_1 = 0$ or $\overrightarrow{AY}\bullet\mathbf{d}_1 = 0$ or $\overrightarrow{YA}\bullet(K\mathbf{d}_1) = 0$ or $\overrightarrow{AY}\bullet(K\mathbf{d}_1) = 0$ to find μ and applies Pythagoras to find a numerical expression for AY^2 or for the distance AY	M1
	So, $YA = \sqrt{\left(-\dfrac{222}{7}\right)^2 + (15)^2 + \left(\dfrac{303}{7}\right)^2}$		
	$= 55.71758... = 55.7$ (1 dp)	anything that rounds to 55.7	A1
	Note: $\overrightarrow{OY} = \dfrac{236}{7}\mathbf{i} + 3\mathbf{j} - \dfrac{261}{7}\mathbf{k}$, $\overrightarrow{AY} = -\dfrac{222}{7}\mathbf{i} + 15\mathbf{j} + \dfrac{303}{7}\mathbf{k}$		**[2]**

Question Number	Scheme	Notes	Marks
19.	$l_1 : \mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ So $\mathbf{d}_1 = \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$. $\overrightarrow{OA}$ occurs when $\mu = 1$. $\overrightarrow{OP} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$		
(a)	$A(3, 5, 0)$	$(3, 5, 0)$	B1
			[1]
(b)	$\{l_2 :\}\ \mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$	$\mathbf{a} + \lambda\mathbf{d}$ or $\mathbf{a} + \mu\mathbf{d}$, $\mathbf{a} + t\mathbf{d}$, $\mathbf{a} \neq 0$, $\mathbf{d} \neq 0$ with either $\mathbf{a} = \mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ or $\mathbf{d} = -5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, or a multiple of $-5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$	M1
		Correct vector equation using $\mathbf{r} =$ **or** $l =$ **or** $l_2 =$	A1
	$\mathbf{d}_2$ is the direction vector of l_2	Do not allow $l_2 :$ **or** $l_2 \rightarrow$ **or** $l_1 =$ for the A1 mark.	**[2]**
(c)	$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$		
	$AP = \sqrt{(-2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$	Full method for finding AP	M1
		$2\sqrt{2}$	A1
			[2]
(d)	So $\overrightarrow{AP} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$ and $\mathbf{d}_2 = \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$	Realisation that the dot product is required between $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	M1
	$\{\cos\theta =\}\ \dfrac{\overrightarrow{AP} \bullet \mathbf{d}_2}{\lvert\overrightarrow{AP}\rvert . \lvert\mathbf{d}_2\rvert} = \dfrac{\pm\left(\begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}\right)}{\sqrt{(-2)^2 + (0)^2 + (2)^2} . \sqrt{(-5)^2 + (4)^2 + (3)^2}}$	**dependent on the previous M mark.** Applies dot product formula between their $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	dM1
	$\{\cos\theta\} = \dfrac{\pm(10 + 0 + 6)}{\sqrt{8}.\sqrt{50}} = \dfrac{4}{5}$	$\{\cos\theta\} = \dfrac{4}{5}$ or 0.8 or $\dfrac{8}{10}$ or $\dfrac{16}{20}$	A1 cso
			[3]
(e)	$\{\text{Area } APE =\}\ \dfrac{1}{2}(\text{their } 2\sqrt{2})^2 \sin\theta$	$\dfrac{1}{2}(\text{their } 2\sqrt{2})^2 \sin\theta$ or $\dfrac{1}{2}(\text{their } 2\sqrt{2})^2 \sin(\text{their } \theta)$	M1
	$= 2.4$	2.4 or $\dfrac{12}{5}$ or $\dfrac{24}{10}$ or awrt 2.40	A1
			[2]
(f)	$\overrightarrow{PE} = (-5\lambda)\mathbf{i} + (4\lambda)\mathbf{j} + (3\lambda)\mathbf{k}$ and $PE =$ their $2\sqrt{2}$ from part (c)		
	$\{PE^2 =\}\ (-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})^2$	This mark can be implied.	M1
	$\left\{\Rightarrow 50\lambda^2 = 8 \Rightarrow \lambda^2 = \dfrac{4}{25} \Rightarrow\right\}\ \lambda = \pm\dfrac{2}{5}$	Either $\lambda = \dfrac{2}{5}$ or $\lambda = -\dfrac{2}{5}$	A1
	$l_2 : \mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} \pm \dfrac{2}{5}\begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$	**dependent on the previous M mark** Substitutes at least one of their values of λ into l_2.	dM1
	$\{\overrightarrow{OE}\} = \begin{pmatrix} 3 \\ \frac{17}{5} \\ \frac{4}{5} \end{pmatrix}$ or $\begin{pmatrix} 3 \\ 3.4 \\ 0.8 \end{pmatrix}$, $\{\overrightarrow{OE}\} = \begin{pmatrix} -1 \\ \frac{33}{5} \\ \frac{16}{5} \end{pmatrix}$ or $\begin{pmatrix} -1 \\ 6.6 \\ 3.2 \end{pmatrix}$	At least one set of coordinates are correct.	A1
		Both sets of coordinates are correct.	A1
			[5]
			15

		Question 19 Notes
19. (a)	**B1**	Allow $A(3, 5, 0)$ or $3\mathbf{i}+5\mathbf{j}$ or $3\mathbf{i}+5\mathbf{j}+0\mathbf{k}$ or $\begin{pmatrix}3\\5\\0\end{pmatrix}$ or benefit of the doubt $\begin{matrix}3\\5\\0\end{matrix}$
(b)	**A1**	Correct vector equation using $\mathbf{r} =$ **or** $l =$ **or** $l_2 =$ **or** Line 2 = i.e. Writing $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\mathbf{d}$, where $\mathbf{d}$ is a multiple of $\begin{pmatrix}-5\\4\\3\end{pmatrix}$.
	Note	Allow the use of parameters μ or t instead of λ.
(c)	**M1**	Finds the difference between $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ and applies Pythagoras to the result to find AP
	Note	Allow M1A1 for $\begin{pmatrix}2\\0\\2\end{pmatrix}$ leading to $AP = \sqrt{(2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$.
(d)	**Note**	For both the M1 and dM1 marks $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) must be the vector used in part (c) or the difference $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ from part (a).
	Note	Applying the dot product formula correctly without $\cos\theta$ as the subject is fine for M1dM1
	Note	***Evaluating*** the dot product (i.e. $(-2)(-5) + (0)(4) + (2)(3)$) is not required for M1 and dM1 marks.
	Note	In part (d) allow one slip in writing $\overrightarrow{AP}$ and $\mathbf{d}_2$
	Note	$\cos\theta = \dfrac{-10+0-6}{\sqrt{8}.\sqrt{50}} = -\dfrac{4}{5}$ followed by $\cos\theta = \dfrac{4}{5}$ is fine for A1 cso
	Note	Give M1dM1A1 for $\{\cos\theta =\} = \dfrac{\begin{pmatrix}-2\\0\\2\end{pmatrix}\bullet\begin{pmatrix}-10\\8\\6\end{pmatrix}}{\sqrt{8}.10\sqrt{2}} = \dfrac{20+12}{40} = \dfrac{4}{5}$
	Note	Allow final A1 (ignore subsequent working) for $\cos\theta = 0.8$ followed by $36.869...^{\circ}$
		<u>Alternative Method: Vector Cross Product</u>
		Only apply this scheme if it is clear that a candidate is applying a vector cross product method.
	$\overrightarrow{AP}\times\mathbf{d}_2 = \begin{pmatrix}-2\\0\\2\end{pmatrix}\times\begin{pmatrix}-5\\4\\3\end{pmatrix} = \left\{\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-2 & 0 & 2\\-5 & 4 & 3\end{vmatrix} = -8\mathbf{i} - 4\mathbf{j} - 8\mathbf{k}\right\}$	Realisation that the vector cross product is required between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ — M1
	$\sin\theta = \dfrac{\sqrt{(-8)^2+(-4)^2+(-8)^2}}{\sqrt{(-2)^2+(0)^2+(2)^2}.\sqrt{(-5)^2+(4)^2+(3)^2}}$	Applies the vector product formula between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ — dM1
	$\sin\theta = \dfrac{12}{\sqrt{8}.\sqrt{50}} = \dfrac{3}{5} \Rightarrow \cos\theta = \dfrac{4}{5}$	$\cos\theta = \dfrac{4}{5}$ or 0.8 or $\dfrac{8}{10}$ or $\dfrac{16}{20}$ — A1
(e)	**Note**	Allow M1;A1 for $\frac{1}{2}(2\sqrt{2})^2\sin(36.869...^{\circ})$ or $\frac{1}{2}(2\sqrt{2})^2\sin(180^{\circ} - 36.869...^{\circ})$; = awrt 2.40
	Note	Candidates must use their θ from part (d) or apply a correct method of finding their $\sin\theta = \dfrac{3}{5}$ from their $\cos\theta = \dfrac{4}{5}$

		Question 19 Notes continued	
19. (f)	**Note**	Allow the first M1A1 for deducing $\lambda = \frac{2}{5}$ or $\lambda = -\frac{2}{5}$ from no incorrect working	
	SC	Allow special case 1st M1 for $\lambda = 2.5$ from comparing lengths or from no working	
	Note	Give 1st M1 for $\sqrt{(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2} = (\text{their } 2\sqrt{2})$	
	Note	Give 1st M0 for $(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})$ or equivalent	
	Note	Give 1st M1 for $\lambda = \dfrac{\text{their } AP = \text{"}2\sqrt{2}\text{"}}{\sqrt{(-5)^2 + (4)^2 + (3)^2}}$ and 1st A1 for $\lambda = \dfrac{2\sqrt{2}}{5\sqrt{2}}$	
	Note	So $\left\{ \hat{\mathbf{d}}_1 = \frac{1}{5\sqrt{2}}\begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} \Rightarrow \right\}$ "vector" $= \frac{2\sqrt{2}}{5\sqrt{2}}\begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ is M1A1	
	Note	The 2nd dM1 in part (f) can be implied for at least 2 (out of 6) correct *x*, *y*, *z* ordinates from their values of λ.	
	Note	Giving their "coordinates" as a column vector or position vector is fine for the final A1A1.	
	CAREFUL	Putting l_2 equal to *A* gives $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \lambda = \frac{2}{5} \\ \lambda = 0 \\ \lambda = -\frac{2}{3} \end{pmatrix}$	Give M0 dM0 for finding and using $\lambda = \frac{2}{5}$ from this incorrect method.
	CAREFUL	Putting $\lambda\mathbf{d}_2 = \overrightarrow{AP}$ gives $\lambda\begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} \rightarrow \begin{pmatrix} \lambda = -\frac{2}{5} \\ \lambda = 0 \\ \lambda = -\frac{2}{3} \end{pmatrix}$	Give M0 dM0 for finding and using $\lambda = -\frac{2}{5}$ from this incorrect method.
	General	You can follow through the part (c) answer of their $AP = 2\sqrt{2}$ for (d) M1dM1, (e) M1, (f) M1dM1	
	General	You can follow through their $\mathbf{d}_2$ in part (b) for (d) M1dM1, (f) M1dM1.	

Question Number	Scheme		Marks
20.	$l_1: \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda\begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}$, $l_2: \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$. Let θ = acute angle between l_1 and l_2. **Note: You can mark parts (a) and (b) together.**		
(a)	$\{l_1 = l_2 \Rightarrow \mathbf{i}:\}\ 5 = 8 + 3\mu \Rightarrow \mu = -1$	Finds μ and substitutes their μ into l_2	M1
	So, $\{\overrightarrow{OA}\} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} - 1\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$	$5\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or $\begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$ or $(5, 1, 3)$	A1
			[2]
(b)	$\{\mathbf{j}:\ -3 + \lambda = 5 + 4\mu \Rightarrow\}\ -3 + \lambda = 5 + 4(-1) \Rightarrow \lambda = 4$	Equates **j** components, substitutes their μ and solves to give $\lambda = ...$	M1
	$\mathbf{k}:\ p - 3\lambda = -2 - 5\mu \Rightarrow$ $p - 3(4) = -2 - 5(-1) \Rightarrow \underline{p = 15}$ or $\mathbf{k}:\ p - 3\lambda = 3 \Rightarrow$ $p - 3(4) = 3 \Rightarrow \underline{p = 15}$	Equates **k** components, substitutes their λ and their μ and solves to give $p = ...$ **or** equates **k** components to give their "$p - 3\lambda$ = the **k** value of A found in part (a)", substitutes their λ and solves to give $p = ...$	M1
		$p = 15$	A1
			[3]
(c)	$\mathbf{d}_1 = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}$, $\mathbf{d}_2 = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$	Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.	M1
	$\cos\theta = \pm K\left(\dfrac{0(3) + (1)(4) + (-3)(-5)}{\sqrt{(0)^2 + (1)^2 + (-3)^2}\,.\sqrt{(3)^2 + (4)^2 + (-5)^2}}\right)$	An attempt to apply the dot product formula between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.	dM1 **(A1 on ePEN)**
	$\cos\theta = \dfrac{19}{\sqrt{10}.\sqrt{50}} \Rightarrow \theta = 31.8203116... = 31.82$ (2 dp)	anything that rounds to 31.82	A1
			[3]
(d)	$\overrightarrow{OB} = \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix}$; $\overrightarrow{AB} = \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -10 \end{pmatrix}$ or $\overrightarrow{AB} = 2\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -10 \end{pmatrix}$ $\lvert\overrightarrow{AB}\rvert = \sqrt{6^2 + 8^2 + (-10)^2}\ \{= 10\sqrt{2}\}$	**See notes**	M1
	$\dfrac{d}{10\sqrt{2}} = \sin\theta$	Writes down a correct trigonometric equation involving the shortest distance, d. Eg: $\dfrac{d}{\text{their } AB} = \sin\theta$, oe.	dM1
	$\{d = 10\sqrt{2}\sin 31.82... \Rightarrow\}\ d = 7.456540753... = 7.46$ (3sf)	anything that rounds to 7.46	A1
			[3] **11**

20. (b)	**<u>Alternative method for part (b)</u>**		
	$\left\{\begin{array}{l} 3\times\mathbf{j}: \ -9+3\lambda=15+12\mu \\ \mathbf{k}: \ p-3\lambda=-2+5\mu \end{array}\right\}$ $p-9=13+7\mu$	Eliminates λ to write down an equation in p and μ	M1
	$p-9=13+7(-1) \Rightarrow \underline{p=15}$	Substitutes their μ and solves to give $p=...$	M1
		$p=15$	A1
20. (d)	**<u>Alternative Methods for part (d)</u>** Let X be the foot of the perpendicular from B onto l_1		
	$\mathbf{d}_1=\begin{pmatrix}0\\1\\-3\end{pmatrix}$, $\overrightarrow{OX}=\begin{pmatrix}5\\-3\\15\end{pmatrix}+\lambda\begin{pmatrix}0\\1\\-3\end{pmatrix}=\begin{pmatrix}5\\-3+\lambda\\15-3\lambda\end{pmatrix}$		
	$\overrightarrow{BX}=\begin{pmatrix}5\\-3+\lambda\\15-3\lambda\end{pmatrix}-\begin{pmatrix}11\\9\\-7\end{pmatrix}=\begin{pmatrix}-6\\-12+\lambda\\22-3\lambda\end{pmatrix}$		
	<u>Method 1</u>		
	$\overrightarrow{BX}\bullet\mathbf{d}_1=0 \Rightarrow \begin{pmatrix}-6\\-12+\lambda\\22-3\lambda\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-3\end{pmatrix}=-12+\lambda-66+9\lambda=0$ leading to $10\lambda-78=0 \Rightarrow \lambda=\frac{39}{5}$	(Allow a sign slip in copying $\mathbf{d}_1$) Applies $\overrightarrow{BX}\bullet\mathbf{d}_1=0$ and solves the resulting equation to find a value for λ.	M1
	$\overrightarrow{BX}=\begin{pmatrix}-6\\-12+\frac{39}{5}\\22-3\left(\frac{39}{5}\right)\end{pmatrix}=\begin{pmatrix}-6\\-\frac{21}{5}\\-\frac{7}{5}\end{pmatrix}$	Substitutes their value of λ into their $\overrightarrow{BX}$. **Note:** This mark is dependent upon the previous M1 mark .	dM1
	$d=BX=\sqrt{(-6)^2+\left(-\frac{21}{5}\right)^2+\left(-\frac{7}{5}\right)^2}=7.456540753...$	awrt 7.46	A1
	<u>Method 2</u>		
	Let $\beta=\left\lvert\overrightarrow{BX}\right\rvert^2=36+144-24\lambda+\lambda^2+484-132\lambda+9\lambda^2$ $=10\lambda^2-156\lambda+664$ So $\frac{\mathrm{d}\beta}{\mathrm{d}\lambda}=20\lambda-156=0 \Rightarrow \lambda=\frac{39}{5}$	Finds $\beta=\left\lvert\overrightarrow{BX}\right\rvert^2$ in terms of λ, finds $\frac{\mathrm{d}\beta}{\mathrm{d}\lambda}$ and sets this result equal to 0 and finds a value for λ.	M1
	$\left\lvert\overrightarrow{BX}\right\rvert^2=10\left(\frac{39}{5}\right)^2-156\left(\frac{39}{5}\right)+664=\frac{278}{5}$	Substitutes their value of λ into their $\left\lvert\overrightarrow{BX}\right\rvert^2$. **Note:** This mark is dependent upon the previous M1 mark .	dM1
	$d=BX=\sqrt{\frac{278}{5}}=7.456540753...$	awrt 7.46	A1

	Question 20 Notes	
20. (a)	**M1**	Finds μ and substitutes their μ into l_2
	A1	Point of intersection of $5\mathbf{i}+\mathbf{j}+3\mathbf{k}$. Allow $\begin{pmatrix}5\\1\\3\end{pmatrix}$ or $(5, 1, 3)$.
	Note	You cannot recover the answer for part (a) in part (c) or part (d).
(b)	**M1**	Equates **j** components, substitutes their μ and solves to give $\lambda = ...$
	M1	Equates **k** components, substitutes their λ and their μ and solves to give $p = ...$ **or** equates **k** components to give their "$p-3\lambda =$ the **k** value of A" found in part (b).
	A1	$p = 15$
(c)	**NOTE**	**Part (c) appears as M1A1A1 on ePEN, but now is marked as M1M1A1.**
	M1	Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.
	Note	Allow one slip in candidates copying down their direction vectors, $\mathbf{d}_1$ and $\mathbf{d}_2$.
	dM1	**dependent on the FIRST method mark being awarded.** An attempt to apply the dot product formula between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.
	A1	anything that rounds to 31.82. This can also be achieved by $180 - 148.1796... =$ awrt 31.82
	Note	$\theta = 0.5553...^c$ is A0.
	Note	**M1A1 for** $\cos\theta = \left(\dfrac{0-16-60}{\sqrt{(0)^2+(4)^2+(-12)^2}.\sqrt{(-3)^2+(-4)^2+(5)^2}}\right) = \dfrac{-76}{\sqrt{160}.\sqrt{50}}$

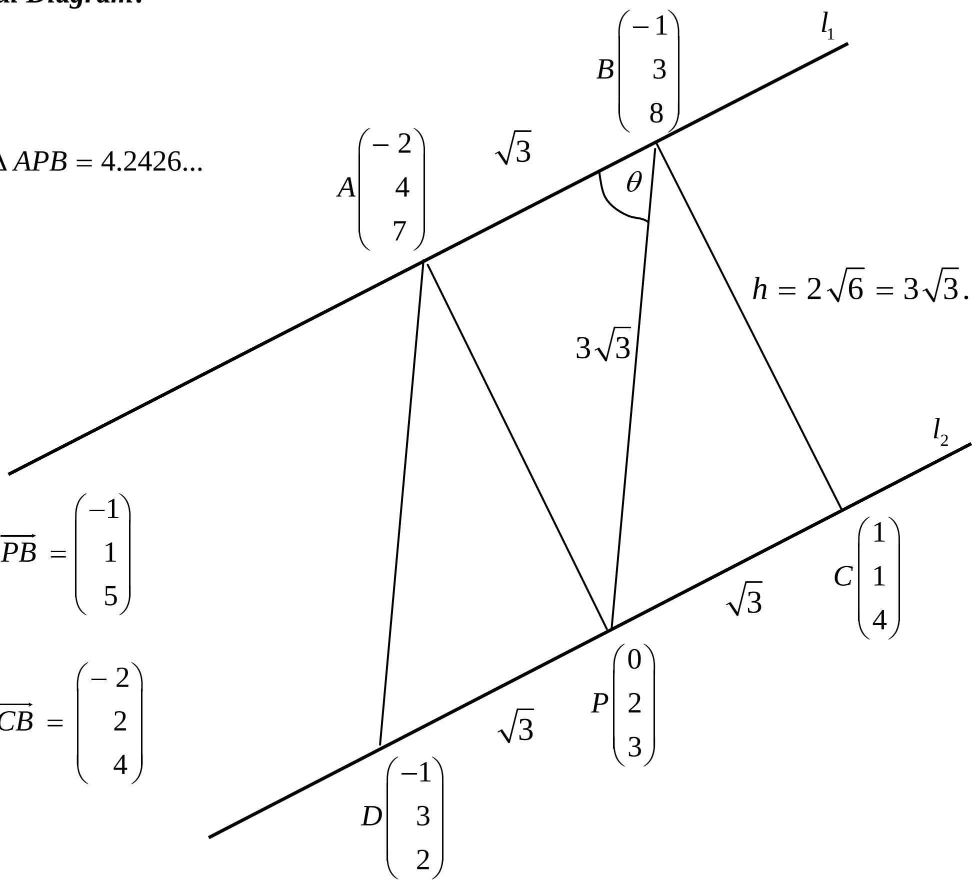
Alternative Method: Vector Cross Product

Only apply this scheme if it is clear that a candidate is applying a vector cross product method.

$\mathbf{d}_1\times\mathbf{d}_2 = \begin{pmatrix}0\\1\\-3\end{pmatrix}\times\begin{pmatrix}3\\4\\-5\end{pmatrix} = \left\{\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\0&1&-3\\3&4&-5\end{vmatrix} = 7\mathbf{i}-9\mathbf{j}-3\mathbf{k}\right\}$	Realisation that the vector cross product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.	M1
$\sin\theta = \dfrac{\sqrt{(7)^2+(-9)^2+(3)^2}}{\sqrt{(0)^2+(1)^2+(-3)^2}.\sqrt{(3)^2+(4)^2+(-5)^2}}$	An attempt to apply the vector cross product formula	dM1 **(A1 on ePEN)**
$\sin\theta = \dfrac{\sqrt{139}}{\sqrt{10}.\sqrt{50}} \Rightarrow \theta = 31.8203116... = 31.82$ (2 dp)	anything that rounds to 31.82	A1

(d)	**M1**	Full method for finding B **and** for finding the magnitude of $\overrightarrow{AB}$ or the magnitude of $\overrightarrow{BA}$.
	dM1	**dependent on the first method mark being awarded.** Writes down correct trigonometric equation involving the shortest distance, d. Eg: $\dfrac{d}{\text{their } AB} = \sin\theta$ **or** $\dfrac{d}{\text{their } AB} = \cos(90-\theta)$, o.e., where " their AB" is a value. and $\theta =$ "their θ" or stated as θ
	A1	anything that rounds to 7.46

Question Number	Scheme		Marks
21.	$\overrightarrow{OA} = -2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$, $\overrightarrow{OB} = -\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ & $\overrightarrow{OP} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$		
(a)	$\overrightarrow{AB} = = \pm((-\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}) - (-2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}));= \mathbf{i} - \mathbf{j} + \mathbf{k}$		M1; A1
			[2]
(b)	$\{l_1 : \mathbf{r}\} = \begin{pmatrix}-2\\4\\7\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$ or $\{\mathbf{r}\} = \begin{pmatrix}-1\\3\\8\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$		B1ft
			[1]
(c)	$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix}-1\\3\\8\end{pmatrix} - \begin{pmatrix}0\\2\\3\end{pmatrix} = \begin{pmatrix}-1\\1\\5\end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix}1\\-1\\-5\end{pmatrix}$		M1
	$\{\cos\theta =\}\ \dfrac{\overrightarrow{AB} \bullet \overrightarrow{PB}}{\lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{PB}\rvert} = \dfrac{\begin{pmatrix}1\\-1\\1\end{pmatrix} \bullet \begin{pmatrix}-1\\1\\5\end{pmatrix}}{\sqrt{(1)^2 + (-1)^2 + (1)^2} . \sqrt{(-1)^2 + (1)^2 + (5)^2}}$	Applies dot product formula between their $(\overrightarrow{AB}$ or $\overrightarrow{BA})$ and their $(\overrightarrow{PB}$ or $\overrightarrow{BP})$.	M1
	$\{\cos\theta\} = \dfrac{-1-1+5}{\sqrt{3}.\sqrt{27}} = \dfrac{3}{9} = \underline{\dfrac{1}{3}}$	Correct proof	A1 cso
			[3]
(d)	$\{l_2 : \mathbf{r} =\} \begin{pmatrix}0\\2\\3\end{pmatrix} + \mu\begin{pmatrix}1\\-1\\1\end{pmatrix}$	$\mathbf{p} + \lambda\mathbf{d}$ or $\mathbf{p} + \mu\mathbf{d}$, $\mathbf{p} \neq 0$, $\mathbf{d} \neq 0$ with either $\mathbf{p} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ or $\mathbf{d}$ = their $\overrightarrow{AB}$, or a multiple of their $\overrightarrow{AB}$.	M1
		Correct vector equation.	A1 ft
			[2]
(e)	$\overrightarrow{OC} = \begin{pmatrix}0\\2\\3\end{pmatrix} + \begin{pmatrix}1\\-1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\4\end{pmatrix}$ or $\overrightarrow{OD} = \begin{pmatrix}0\\2\\3\end{pmatrix} - \begin{pmatrix}1\\-1\\1\end{pmatrix}$	Either $\overrightarrow{OP}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OP}$ − their $\overrightarrow{AB}$	M1
		At least one set of coordinates are correct.	A1 ft
	$\{C(1, 1, 4),\ D(-1, 3, 2)\}$	Both sets of coordinates are correct.	A1 ft
			[3]
(f) **Way 1**	$\dfrac{h}{\sqrt{(-1)^2 + (1)^2 + (5)^2}} = \sin\theta$	$\dfrac{h}{\text{their } \lvert\overrightarrow{PB}\rvert} = \sin\theta$	M1
	$h = \sqrt{27}\sin(70.5...) \left\{= \sqrt{27}\dfrac{\sqrt{8}}{3} = 2\sqrt{6} = \text{awrt } 4.9\right\}$	$\sqrt{27}\sin(70.5...)$ or $\sqrt{27}.\dfrac{\sqrt{8}}{3}$ or $2\sqrt{6}$ or awrt 4.9 or equivalent	A1 oe
	Area $ABCD = \dfrac{1}{2} 2\sqrt{6}\left(\sqrt{3} + 2\sqrt{3}\right)$	$\dfrac{1}{2}$(their h)(their AB + their CD)	dM1
	$\left\{= \dfrac{1}{2} 2\sqrt{6}\left(3\sqrt{3}\right) = 3\sqrt{18}\right\} = \underline{9\sqrt{2}}$	$9\sqrt{2}$	A1 **cao**
			[4] **15**

Question	Scheme	Notes	Marks
21. (f)	***Helpful Diagram!*** Area $\Delta APB = 4.2426...$  $h = 2\sqrt{6} = 3\sqrt{3}.\left(\frac{\sqrt{8}}{3}\right) = 4.8989...$ $\overrightarrow{DA} = \overrightarrow{PB} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$ $\overrightarrow{PA} = \overrightarrow{CB} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}$		
	$\overrightarrow{PA} = \overrightarrow{CB} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, so $BC \perp AB$	Candidates do not need to prove this result for part (f)	
21. (f) **Way 2**	$h = \lvert\overrightarrow{CB}\rvert = \sqrt{(-2)^2 + (2)^2 + (4)^2} = \sqrt{24} = 2\sqrt{6} = 4.8989...$	Attempts $\lvert\overrightarrow{PA}\rvert$ or $\lvert\overrightarrow{CB}\rvert$	M1
		$\lvert\overrightarrow{PA}\rvert = \lvert\overrightarrow{CB}\rvert = \sqrt{24}$	A1 oe
	Area $ABCD = \frac{1}{2}\sqrt{24}\left(\sqrt{3} + 2\sqrt{3}\right)$ or $\frac{1}{2}\sqrt{24}\sqrt{3} + \sqrt{24}\sqrt{3}$	$\frac{1}{2}h(\text{their } AB + \text{their } CD)$	dM1 oe
	$= \underline{9\sqrt{2}}$	$9\sqrt{2}$	A1 **cso**
			[4]
Way3 **21.** (f)	**<u>Finds the area of either triangle *APB* or *APD* or *BCP* and triples the result.</u>**		
	Area $\Delta APB = \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin\theta$	Attempts $\frac{1}{2}$(their AB)(their PB)$\sin\theta$	M1
	$= \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin(70.5...)$	$\frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin(70.5...)$ or $3\sqrt{2}$ or awrt 4.24 or equivalent	A1
	Area $ABCD = 3\left(3\sqrt{2}\right)$	$3 \times$ Area of ΔAPB	dM1
	$= \underline{9\sqrt{2}}$	$9\sqrt{2}$	A1 **cso**
			[4]

	Question 21 Notes	
21. (a)	**M1**	Finding the difference (either way) between OB and OA. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference.
	A1	$\mathbf{i}-\mathbf{j}+\mathbf{k}$ or $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ or $(1, -1, 1)$ or benefit of the doubt $\begin{matrix}1\\-1\\1\end{matrix}$
(b)	**B1ft**	$\{\mathbf{r}\} = \begin{pmatrix}-2\\4\\7\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$ **or** $\{\mathbf{r}\} = \begin{pmatrix}-1\\3\\8\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$, with $\overrightarrow{AB}$ or $\overrightarrow{BA}$ correctly followed through from (a).
	Note	$\mathbf{r} =$ is not needed.
(c)	**M1**	An attempt to find either the vector $\overrightarrow{PB}$ or $\overrightarrow{BP}$. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference.
	M1	Applies dot product formula between their $\left(\overrightarrow{AB}\text{ or }\overrightarrow{BA}\right)$ and their $\left(\overrightarrow{PB}\text{ or }\overrightarrow{BP}\right)$.
	A1	Obtains $\{\cos\theta\} = \frac{1}{3}$ ***by correct solution only.***
	Note	If candidate starts by applying $\frac{\overrightarrow{AB}\bullet\overrightarrow{PB}}{\lvert\overrightarrow{AB}\rvert.\lvert\overrightarrow{PB}\rvert}$ correctly (without reference to $\cos\theta = ...$) they can gain both 2nd M1 and A1 mark.
	Note	Award the final A1 mark if candidate achieves $\{\cos\theta\} = \frac{1}{3}$ by either taking the dot product between (i) $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\\5\end{pmatrix}$ **or** (ii) $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\-5\end{pmatrix}$. Ignore if any of these vectors are labelled incorrectly.
	Note	Award final A0, cso for those candidates who take the dot product between (iii) $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\-5\end{pmatrix}$ or (iv) $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\\5\end{pmatrix}$ They will usually find $\{\cos\theta\} = -\frac{1}{3}$ or may fudge $\{\cos\theta\} = \frac{1}{3}$. If these candidates give a convincing detailed explanation which must include reference to the direction of their vectors then this can be given A1 cso

(c)	**Alternative Method 1: The Cosine Rule**		
	$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix}-1\\3\\8\end{pmatrix} - \begin{pmatrix}0\\2\\3\end{pmatrix} = \begin{pmatrix}-1\\1\\5\end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix}1\\-1\\-5\end{pmatrix}$	Mark in the same way as the main scheme.	M1
	Note $\lvert\overrightarrow{PB}\rvert = \sqrt{27}$, $\lvert\overrightarrow{AB}\rvert = \sqrt{3}$ and $\lvert\overrightarrow{PA}\rvert = \sqrt{24}$		
	$\left(\sqrt{24}\right)^2 = \left(\sqrt{27}\right)^2 + \left(\sqrt{3}\right)^2 - 2\left(\sqrt{27}\right)\left(\sqrt{3}\right)\cos\theta$	Applies the cosine rule the correct way round	M1 oe
	$\cos\theta = \frac{27 + 3 - 24}{18} = \frac{1}{3}$	Correct proof	A1 cso
			[3]

21. (c)	**Alternative Method 2: Right-Angled Trigonometry**		
	$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$	Mark in the same way as the main scheme.	M1
	Either $\left(\sqrt{24}\right)^2 + \left(\sqrt{3}\right)^2 = \left(\sqrt{27}\right)^2$ or $\overrightarrow{AB} \bullet \overrightarrow{PA} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix} = -2 - 2 + 4 = 0$	Confirms ΔPAB is right-angled	M1
	So, $\left\{\cos\theta = \dfrac{AB}{PB} \Rightarrow \right\} \cos\theta = \dfrac{\sqrt{3}}{\sqrt{27}} = \dfrac{1}{3}$	Correct proof	A1 cso
			[3]

(d)	**M1**	Writing down a line in the form $\mathbf{p} + \lambda\mathbf{d}$ or $\mathbf{p} + \mu\mathbf{d}$ with either $\mathbf{a} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$ or $\mathbf{d}$ = their $\overrightarrow{AB}$ $\mathbf{d}$ = their $\overrightarrow{AB}$, or a multiple of their $\overrightarrow{AB}$ found in part (a).
	A1ft	Writing $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \mu\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \mu\mathbf{d}$, where $\mathbf{d}$ = their $\overrightarrow{AB}$ or a multiple of their $\overrightarrow{AB}$ found in part (a).
	Note	$\mathbf{r} =$ is not needed.
	Note	Using the same scalar parameter as in part (b) is fine for A1.
(e)	**M1**	Either $\overrightarrow{OP}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OP}$ − their $\overrightarrow{AB}$.
	Note	This can be implied at least two out of three correct components for either their *C* or their *D*.
	A1ft	At least one set of coordinates are correct. Ignore labelling of C, D
	A1ft	Both sets of coordinates are correct. Ignore labelling of C, D
	Note	You can follow through either or both accuracy marks in this part using their $\overrightarrow{AB}$ from part (a).
(f)	**M1**	Way 1: $\dfrac{h}{\text{their } \left\lvert\overrightarrow{PB}\right\rvert} = \sin\theta$ Way 2: Attempts $\left\lvert\overrightarrow{PA}\right\rvert$ or $\left\lvert\overrightarrow{CB}\right\rvert$ Way 3: Attempts $\dfrac{1}{2}$(their PB)(their AB)$\sin\theta$
	Note	Finding *AD* by itself is M0.
	A1	Either • $h = \sqrt{27}\sin(70.5...)$ or $\left\lvert\overrightarrow{PA}\right\rvert = \left\lvert\overrightarrow{CB}\right\rvert = \sqrt{24}$ or equivalent. (See Way 1 and Way 2) or • the area of either triangle *APB* or *APD* or *BDP* $= \dfrac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin\left(70.5...\right)$ o.e. (See Way 3).
	dM1	**which is dependent on the 1st M1 mark.** A full method to find the area of trapezium *ABCD*. (See Way 1, Way 2 and Way 3).
	A1	$9\sqrt{2}$ from a correct solution only.
	Note	A decimal answer of 12.7279... (without a correct **exact** answer) is A0.

Question Number	Scheme		Marks
22.	$l_1: \mathbf{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}0\\2\\-1\end{pmatrix}$, $l_2: \mathbf{r} = \begin{pmatrix}7\\0\\7\end{pmatrix} + \mu\begin{pmatrix}3\\-5\\4\end{pmatrix}$ $\overrightarrow{OA} = \begin{pmatrix}1\\2\\3\end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix}4\\p\\3\end{pmatrix}$	A lies on l_1 and B lies on l_2	
(a)	$\{B \text{ lies on } l_2 \Rightarrow \mu = -1 \Rightarrow\}$ $p = 5$	$p = 5$	B1 **[1]**
(b)	$\{l_1 = l_2 \Rightarrow\}$ $\begin{Bmatrix}\mathbf{i}: & 1 = 7 + 3\mu\\ \mathbf{j}: & 2 + 2\lambda = -5\mu\\ \mathbf{k}: & 3 - \lambda = 7 + 4\mu\end{Bmatrix}$		
	e.g. $\mathbf{i}$: $7 + 3\mu = 1$	Writes down an equation involving only one parameter.	M1
	So, $\mu = -2$	$\mu = -2$	A1
	Point of intersection is $\overrightarrow{OC} = \mathbf{i} + 10\mathbf{j} - \mathbf{k}$	$\mathbf{i} + 10\mathbf{j} - \mathbf{k}$	B1
	Finds $\lambda = 4$ and either • checks $\lambda = 4$ and $\mu = -2$ is true for the third component. • substitutes $\mu = -2$ into l_1 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$ **and** substitutes $\lambda = 4$ into l_2 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$		B1 **[4]**
(b)	**<u>Alternative Method:</u>** Solving $\mathbf{j}$ and $\mathbf{k}$ simultaneously gives		
	$8 = 14 + 3\mu$ or $23 + 3\lambda = 35$	Writes down an equation involving only one parameter.	M1
	So, $\mu = -2$ or $\lambda = 4$	Either $\mu = -2$ or $\lambda = 4$	A1
	Point of intersection is $\overrightarrow{OC} = \mathbf{i} + 10\mathbf{j} - \mathbf{k}$	$\mathbf{i} + 10\mathbf{j} - \mathbf{k}$	B1
	Finds $\lambda = 4$ and either • checks $\mu = -2$ is true for the $\mathbf{i}$ component. • substitutes $\mu = -2$ into l_1 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$ **and** substitutes $\lambda = 4$ into l_2 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$		B1 **[4]**
(c)	$\overrightarrow{AC} = \begin{pmatrix}1\\10\\-1\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}0\\8\\-4\end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix}1\\10\\-1\end{pmatrix} - \begin{pmatrix}4\\5\\3\end{pmatrix} = \begin{pmatrix}-3\\5\\-4\end{pmatrix}$	An attempt to find both the vectors $(\overrightarrow{AC}$ or $\overrightarrow{CA})$ **and** $(\overrightarrow{BC}$ or $\overrightarrow{CB})$.	M1
	$\cos ACB = \dfrac{\overrightarrow{AC} \bullet \overrightarrow{BC}}{\lvert\overrightarrow{AC}\rvert . \lvert\overrightarrow{BC}\rvert} = \dfrac{\pm\left(\begin{pmatrix}0\\8\\-4\end{pmatrix} \bullet \begin{pmatrix}-3\\5\\-4\end{pmatrix}\right)}{\sqrt{(0)^2 + (8)^2 + (-4)^2} . \sqrt{(-3)^2 + (5)^2 + (-4)^2}}$	Applies dot product formula between their $(\overrightarrow{AC}$ or $\overrightarrow{CA})$ **and** their $(\overrightarrow{BC}$ or $\overrightarrow{CB})$.	M1
	$\left\{\cos ACB = \dfrac{0 + 40 + 16}{\sqrt{80}.\sqrt{50}} = \dfrac{56}{\sqrt{4000}} \Rightarrow\right\}$ $ACB = 27.69446... = 27.7$ (3 sf)	Anything that rounds to 27.7	A1 **[3]**
(d)	Area $ACB = \dfrac{1}{2}(\sqrt{80})(\sqrt{50})\sin 27.69446...^\circ = 14.696888...$	See notes Anything that rounds to 14.7	M1 A1 **[2]** **10**

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	Question 22: Alternative Methods for Part (c)		
22. (c)	**<u>Alternative Method 1: Using the direction vectors of Line 1 and Line 2</u>** $\mathbf{d}_1 = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \quad \mathbf{d}_2 = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}$		
	$\cos\theta = \dfrac{\mathbf{d}_1 \bullet \mathbf{d}_1}{\lvert\mathbf{d}_1\rvert . \lvert\mathbf{d}_2\rvert} = \dfrac{\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}}{\sqrt{(0)^2+(2)^2+(-1)^2} . \sqrt{(3)^2+(-5)^2+(4)^2}}$	Applies dot product formula between their $\mathbf{d}_1$ and $\mathbf{d}_2$	M2
	$\left\{\cos\theta = \dfrac{0-10-4}{\sqrt{5}.\sqrt{50}} = \dfrac{-7\sqrt{10}}{25} \Rightarrow\right\}\ \theta = 152.3054385...$		
	Angle $ACB = 180 - 152.3054385... = 27.69446145... = 27.7$ (3 sf)	Anything that rounds to 27.7	A1
			[3]
	<u>Alternative Method 2: The Cosine Rule</u> $\overrightarrow{AC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix}$	An attempt to find both the vectors $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$.	M1
	Also $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$		
	Note $\lvert\overrightarrow{AC}\rvert = \sqrt{80}$, $\lvert\overrightarrow{BC}\rvert = \sqrt{50}$ and $\lvert\overrightarrow{AB}\rvert = \sqrt{18}$		
	$\left(\sqrt{18}\right)^2 = \left(\sqrt{80}\right)^2 + \left(\sqrt{50}\right)^2 - 2\left(\sqrt{80}\right)\left(\sqrt{50}\right)\cos\theta$	Applies the cosine rule the correct way round.	M1 oe
	$\left\{\cos\theta = \dfrac{7\sqrt{10}}{25}\right\} \Rightarrow \theta = 27.69446145... = 27.7$ (3 sf)	Anything that rounds to 27.7	A1
			[3]
	<u>Alternative Method 3: Vector Cross Product</u> **Only apply this scheme if it is clear that a candidate is applying a vector cross product method.**		
	$\overrightarrow{AC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix}$	An attempt to find both the vectors $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$.	M1
	$\overrightarrow{AC} \times \overrightarrow{BC} = \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix} \times \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & -4 \\ -3 & 5 & -4 \end{vmatrix} = 24\mathbf{i} + 12\mathbf{j} + 24\mathbf{k} \right\}$ $\sin ACB = \dfrac{\sqrt{(24)^2+(12)^2+(12)^2}}{\sqrt{(0)^2+(8)^2+(-4)^2} . \sqrt{(-3)^2+(5)^2+(-4)^2}}$	Full method for applying the vector cross product formula between their $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** their $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$.	M1
	$\left\{\sin ACB = \dfrac{\sqrt{864}}{\sqrt{80}.\sqrt{50}} = \dfrac{3\sqrt{15}}{25} \Rightarrow\right\} \theta = 27.69446145... = 27.7$ (3 sf)	Anything that rounds to 27.7	A1
			[3]

		Question 22 Notes
22. (a)	**B1**	$p = 5$ (Ignore working.)
(b)		**Method 1**
	M1	Writes down an equation involving only one parameter. This equation will usually be $7 + 3\mu = 1$ which is found from equating the **i** components of l_1 and l_2.
	A1	Finds $\mu = -2$
	B1	Point of intersection of $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$. Allow $(1, 10, -1)$ or $\begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix}$.
	B1	Finds $\lambda = 4$ and either • checks $\lambda = 4$ and $\mu = -2$ is true for the third component. • substitutes $\mu = -2$ into l_1 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$ **and** substitutes $\lambda = 4$ into l_2 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$
(b)		**Alternative Method**
	M1	Writes down an equation involving only one parameter. Solving the **j** and **k** components simultaneously will usually give either $8 = 14 + 3\mu$ or $23 + 3\lambda = 35$
	A1	Finds either $\mu = -2$ or $\lambda = 4$
	B1	Point of intersection of $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$. Allow $(1, 10, -1)$ or $\begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix}$.
	B1	Finds $\lambda = 4$ and either • checks $\mu = -2$ is true for the **i** component. • substitutes $\mu = -2$ into l_1 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$ **and** substitutes $\lambda = 4$ into l_2 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$
(c)	**M1**	An attempt to find both the vectors $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$ by subtracting.
	M1	Applies dot product **formula** between their $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** their $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$.
	A1	anything that rounds to 27.7
	Note	An answer of 0.48336... in radians without the correct answer in degrees is A0.
	Note	Some candidates will apply the dot product formula between vectors which are the wrong way round and achieve $152.3054385...^{\circ}$. If they give the acute equivalent of awrt 27.7 then award A1.
(d)	**M1**	$\frac{1}{2}(\text{their length } AC)(\text{their length } BC)\sin\left(\text{their } 27.7^{\circ} \text{ from part (c)}\right)$
	A1	anything that rounds to 14.7. Also allow $6\sqrt{6}$.
	Note	Area $ACB = \frac{1}{2}\left(\sqrt{80}\right)\left(\sqrt{50}\right)\sin(152.3054385...^{\circ}) =$ awrt 14.7 is M1A1.

Question Number	Scheme		Marks
23.	$l: \mathbf{r} = \begin{pmatrix}13\\8\\1\end{pmatrix} + \lambda\begin{pmatrix}2\\2\\-1\end{pmatrix}$, $A(3, -2, 6)$, $\overrightarrow{OP} = \begin{pmatrix}-p\\0\\2p\end{pmatrix}$		
(a)	$\{\overrightarrow{PA}\} = \begin{pmatrix}3\\-2\\6\end{pmatrix} - \begin{pmatrix}-p\\0\\2p\end{pmatrix}$ | $\{\overrightarrow{AP}\} = \begin{pmatrix}-p\\0\\2p\end{pmatrix} - \begin{pmatrix}3\\-2\\6\end{pmatrix}$	Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OP}$. Ignore labelling.	M1
	$= \begin{pmatrix}3+p\\-2\\6-2p\end{pmatrix}$ | $= \begin{pmatrix}-3-p\\2\\2p-6\end{pmatrix}$	Correct difference.	A1
	$\begin{pmatrix}3+p\\-2\\6-2p\end{pmatrix} \bullet \begin{pmatrix}2\\2\\-1\end{pmatrix} = 6 + 2p - 4 - 6 + 2p = 0$	See notes.	M1
	$p = 1$		A1 **cso**
			[4]
(b)	$\lvert AP\rvert = \sqrt{4^2 + (-2)^2 + 4^2}$ or $\lvert AP\rvert = \sqrt{(-4)^2 + 2^2 + (-4)^2}$	See notes.	M1
	So, PA or $AP = \sqrt{36}$ or 6 **cao**		A1 **cao**
	It follows that, $AB = "6"$ $\{= PA\}$ or $PB = "6\sqrt{2}"$ $\{= \sqrt{2}\,PA\}$	See notes.	B1 ft
	{Note that $AB = "6" = 2$(the modulus of the direction vector of l)}		
	$\overrightarrow{OB} = \begin{pmatrix}3\\-2\\6\end{pmatrix} \pm 2\begin{pmatrix}2\\2\\-1\end{pmatrix}$ or $\overrightarrow{OB} = \begin{pmatrix}13\\8\\1\end{pmatrix} - 3\begin{pmatrix}2\\2\\-1\end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix}13\\8\\1\end{pmatrix} - 7\begin{pmatrix}2\\2\\-1\end{pmatrix}$	Uses a correct method in order to find both possible sets of coordinates of B.	M1
	$= \begin{pmatrix}7\\2\\4\end{pmatrix}$ and $\begin{pmatrix}-1\\-6\\8\end{pmatrix}$	Both coordinates are correct.	A1 **cao**
			[5] **9**

Notes for Question 23

23. (a)	**M1:** Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OP}$. Ignore labelling. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference. **A1:** Accept any of $\begin{pmatrix}3+p\\-2\\6-2p\end{pmatrix}$ or $(3+p)\mathbf{i} - 2\mathbf{j} + (6-2p)\mathbf{k}$ or $\begin{pmatrix}-3-p\\2\\2p-6\end{pmatrix}$ or $(-3-p)\mathbf{i} + 2\mathbf{j} + (2p-6)\mathbf{k}$

Notes for Question 23 Continued

23. (a)

M1: ***Applies*** the formula $\overrightarrow{PA}\bullet\begin{pmatrix}2\\2\\-1\end{pmatrix}$ or $\overrightarrow{AP}\bullet\begin{pmatrix}2\\2\\-1\end{pmatrix}$ correctly to give a linear equation in p which is set equal to zero. **Note:** The dot product can also be with $\pm k\begin{pmatrix}2\\2\\-1\end{pmatrix}$. Eg: Some candidates may find $\begin{pmatrix}13\\8\\1\end{pmatrix}-\begin{pmatrix}3\\-2\\6\end{pmatrix}=\begin{pmatrix}10\\10\\-5\end{pmatrix}$, for instance, and use this in their dot product which is fine for M1.

A1: Finds $p = 1$ from ***a correct solution only.***

Note: The direction of subtraction is not important in part (a).

(b)

M1: Uses their value of p and Pythagoras to obtain a numerical expression for either AP or PA or AP^2 or PA^2. Eg: PA or AP $= \sqrt{4^2+(-2)^2+4^2}$ or $\sqrt{(-4)^2+2^2+(-4)^2}$ or $\sqrt{4^2+2^2+4^2}$

or PA^2 or $AP^2 = 4^2+(-2)^2+4^2$ or $(-4)^2+2^2+(-4)^2$ or $4^2+2^2+4^2$

A1: AP or $PA = \sqrt{36}$ or 6 **cao** or $AP^2 = 36$ **cao**

B1ft: States or it is clear from their working that $AB = \text{"6"}$ $\{=$ their evaluated $PA\}$ or $PB = \text{"6"}\sqrt{2}$ $\{=\sqrt{2}$ (their evaluated PA)$\}$.

Note: So a correct follow length is required here for either AB or PB using their evaluated PA.

Note: This mark may be found on a diagram.

Note: If a candidate states that $|\overrightarrow{AP}| = |\overrightarrow{AB}|$ and then goes on to find $|\overrightarrow{AP}| = 6$ then the B1 mark can be implied.

IMPORTANT: This mark may be implied as part of expressions such as:

$\{AB =\}\ \sqrt{(10+2\lambda)^2+(10+2\lambda)^2+(-5-\lambda)^2} = \mathbf{6}$ or $\{AB^2 =\}\ (10+2\lambda)^2+(10+2\lambda)^2+(-5-\lambda)^2 = \mathbf{36}$

or $\{PB =\}\ \sqrt{(14+2\lambda)^2+(8+2\lambda)^2+(-1-\lambda)^2} = \mathbf{6\sqrt{2}}$ or $\{PB^2 =\}\ (14+2\lambda)^2+(8+2\lambda)^2+(-1-\lambda)^2 = \mathbf{72}$

M1: Uses a full method in order to find **both** possible sets of coordinates of B:

Eg 1: $\overrightarrow{OB} = \begin{pmatrix}3\\-2\\6\end{pmatrix} \pm 2\begin{pmatrix}2\\2\\-1\end{pmatrix}$ Eg 2: $\overrightarrow{OB} = \begin{pmatrix}13\\8\\1\end{pmatrix} - 3\begin{pmatrix}2\\2\\-1\end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix}13\\8\\1\end{pmatrix} - 7\begin{pmatrix}2\\2\\-1\end{pmatrix}$

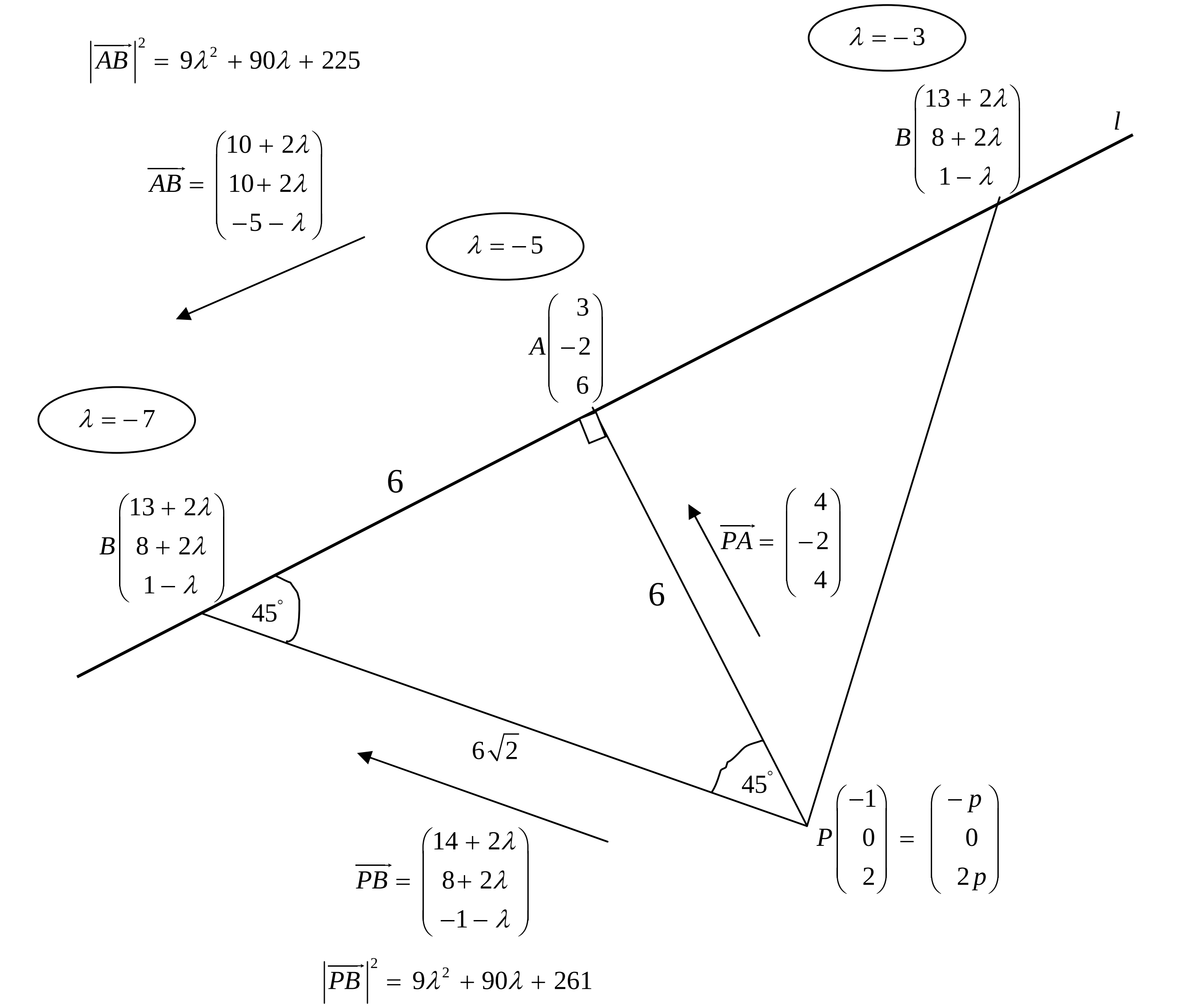
Note: If a candidate achieves at least one of the correct $(7, 2, 4)$ or $(-1, -6, 8)$ then award SC M1 here.

Note: $\overrightarrow{OB} = \begin{pmatrix}3\\-2\\6\end{pmatrix} - 3\begin{pmatrix}2\\2\\-1\end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix}3\\-2\\6\end{pmatrix} - 7\begin{pmatrix}2\\2\\-1\end{pmatrix}$ is M0.

A1: For both $(7, 2, 4)$ and $(-1, -6, 8)$. Accept vector notation or $\mathbf{i}, \mathbf{j}, \mathbf{k}$ notation.

Note: All the marks are accessible in part (b) if $p = 1$ is found from incorrect working in part (a).

Note: **Imply M1A1B1 and award M1** for candidates who write: $\overrightarrow{OB} = \begin{pmatrix}3\\-2\\6\end{pmatrix} \pm 2\begin{pmatrix}2\\2\\-1\end{pmatrix}$, with little or no earlier working.

<table>
<tr><th colspan="2">Notes for Question 23 Continued</th></tr>
<tr><td>23.</td><td>Helpful Diagram!

</td></tr>
<tr><td>23. (b)</td><td>Way 2: Setting $AB = "6"$ or $AB^2 = "36"$ Note: It is possible for you to apply the main scheme for Way 2.
$\left\{AB = "6" \Rightarrow AB^2 = "36" \Rightarrow\right\}$ $(10+2\lambda)^2 + (10+2\lambda)^2 + (-5-\lambda)^2 = "36"$ B1ft could be implied here.
$9\lambda^2 + 90\lambda + 225 = 36 \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0$
$\lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda+3)(\lambda+7) = 0$
$\lambda = -3, -7$
Then apply final M1 A1 as in the original scheme. | ... M1 A1</td></tr>
<tr><td>23. (b)</td><td>Way 3: Setting $PB = "6\sqrt{2}"$ or $PB^2 = "72"$ Note: It is possible for you to apply the main scheme for Way 3.
$\left\{PB = "6"\sqrt{2} \Rightarrow PB^2 = "72" \Rightarrow\right\}$ $(14+2\lambda)^2 + (8+2\lambda)^2 + (-1-\lambda)^2 = "72"$ B1ft could be implied here.
$9\lambda^2 + 90\lambda + 261 = 72 \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0$
$\lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda+3)(\lambda+7) = 0$
$\lambda = -3, -7$
Then apply final M1 A1 as in the original scheme. | ... M1 A1</td></tr>
</table>

Notes for Question 23 Continued	
23. (b)	**(You need to be convinced that a candidate is applying this method before you apply the Mark Scheme for Way 4).** **Way 4:** Using the dot product formula between $\overrightarrow{PA}$ and $\overrightarrow{PB}$, ie: $\cos 45^\circ = \dfrac{\overrightarrow{PA} \bullet \overrightarrow{PB}}{\lvert\overrightarrow{PA}\rvert . \lvert\overrightarrow{PB}\rvert}$. $\overrightarrow{PA} \bullet \overrightarrow{PB} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 14+2\lambda \\ 8+2\lambda \\ -1-\lambda \end{pmatrix} = 56 + 8\lambda - 16 - 4\lambda - 4 - 4\lambda = 36$ $\left\{\cos 45^\circ = \right\} \dfrac{1}{\sqrt{2}} = \dfrac{36}{6\sqrt{9\lambda^2 + 90\lambda + 261}}$ For finding $\lvert\overrightarrow{PA}\rvert$ as before. — M1 $\sqrt{36}$ or 6 — A1 **cao** $\lvert\overrightarrow{PB}\rvert = \sqrt{9\lambda^2 + 90\lambda + 261}$ — B1 oe $\dfrac{1}{2} = \dfrac{36}{9\lambda^2 + 90\lambda + 261}$ $9\lambda^2 + 90\lambda + 261 = 72 \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0$ $\lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda+3)(\lambda+7) = 0$ $\lambda = -3, -7$ Then apply final M1 A1 as in the original scheme. — ... M1 A1
23. (b)	**(You need to be convinced that a candidate is applying this method before you apply the Mark Scheme for Way 5).** **Way 5:** Using the dot product formula between $\overrightarrow{AB}$ and $\overrightarrow{PB}$, ie: $\cos 45^\circ = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{PB}}{\lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{PB}\rvert}$ $\cos 45^\circ = \dfrac{1}{\sqrt{2}} = \dfrac{\begin{pmatrix} 10+2\lambda \\ 10+2\lambda \\ -5-\lambda \end{pmatrix} \bullet \begin{pmatrix} 14+2\lambda \\ 8+2\lambda \\ -1-\lambda \end{pmatrix}}{\sqrt{9\lambda^2 + 90\lambda + 225}\sqrt{9\lambda^2 + 90\lambda + 261}}$ Attempts the dot product formula between $\overrightarrow{AB}$ and $\overrightarrow{PB}$. — M1 Correct statement with $\lvert\overrightarrow{AB}\rvert$ and $\lvert\overrightarrow{PB}\rvert$ simplified as shown. — A1 Either $\lvert\overrightarrow{AB}\rvert = \sqrt{9\lambda^2 + 90\lambda + 225}$ or $\lvert\overrightarrow{PB}\rvert = \sqrt{9\lambda^2 + 90\lambda + 261}$ — B1 $\left\{\cos 45^\circ = \right\} \dfrac{1}{\sqrt{2}} = \dfrac{140 + 20\lambda + 28\lambda + 4\lambda^2 + 80 + 20\lambda + 16\lambda + 4\lambda^2 + 5 + 5\lambda + \lambda + \lambda^2}{\sqrt{9\lambda^2 + 90\lambda + 225}\sqrt{9\lambda^2 + 90\lambda + 261}}$ $\left\{\cos 45^\circ = \right\} \dfrac{1}{\sqrt{2}} = \dfrac{9\lambda^2 + 90\lambda + 225}{\sqrt{9\lambda^2 + 90\lambda + 225}\sqrt{9\lambda^2 + 90\lambda + 261}}$ $\dfrac{1}{2} = \dfrac{(9\lambda^2 + 90\lambda + 225)^2}{(9\lambda^2 + 90\lambda + 225)(9\lambda^2 + 90\lambda + 261)}$ $\dfrac{1}{2} = \dfrac{(9\lambda^2 + 90\lambda + 225)}{(9\lambda^2 + 90\lambda + 261)}$ $9\lambda^2 + 90\lambda + 261 = 2(9\lambda^2 + 90\lambda + 225) \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0$ $\lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda+3)(\lambda+7) = 0$ $\lambda = -3, -7$ Then apply final M1 A1 as in the original scheme. — ... M1 A1

Notes for Question 23 Continued	
23. (b)	**Way 6:** $\overrightarrow{PA} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} = 2\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ and direction vector of l is $\mathbf{d} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ So, $\lvert\overrightarrow{PA}\rvert = 2\,\lvert\mathbf{d}\rvert$ or $PA = 2\,\lvert\mathbf{d}\rvert$ — A correct statement relating these distances (and not vectors) — M1 A1 B1 Apply final M1 A1 as in the original scheme. — ... M1 A1 **Note:** $\overrightarrow{PA} = 2\mathbf{d}$ with no other creditable working is M0A0B0... **Note:** $\overrightarrow{PA} = 2\mathbf{d}$, followed by $\overrightarrow{OB} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} \pm 2\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ is M1A1B1M1 and the final A1 mark is for both sets of correct coordinates.

Question Number	Scheme		Marks
24.	$l: \mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$, $\overrightarrow{OA} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix}$		
(a)	A is on l, so $\begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$		
	$\{\mathbf{k}: \ 10 - \lambda = 6 \Rightarrow\} \ \lambda = 4$	$\lambda = 4$	B1
	$\{\mathbf{i}: \ a + 6\lambda = 21 \Rightarrow\} \ a + 6(4) = 21$	Substitutes their value of λ into $a + 6\lambda = 21$	M1
	$a = -3$	$a = -3$	A1 **cao**
			[3]
(b)	$\{\overrightarrow{AB}\} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix} - \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix}$ | $\{\overrightarrow{BA}\} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} - \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix}$	Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Ignore labelling.	M1
	$\{\overrightarrow{AB}\} = \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix}$ | $\{\overrightarrow{BA}\} = \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix}$		
	$\{\overrightarrow{AB} \perp l \Rightarrow \overrightarrow{AB} \bullet \mathbf{d} = 0\} \Rightarrow \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix} \bullet \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix} = 24 + 3c - 12 = 0; \Rightarrow c = -4$	See notes.	M1; A1 ft
	$\{\mathbf{j}: \ b + c\lambda = -17 \Rightarrow\} \ b + (-4)(4) = -17; \Rightarrow b = -1$	See notes.	ddM1; A1 **cso cao**
			[5]
(c)	$\lvert AB \rvert = \sqrt{4^2 + 3^2 + 12^2}$ or $\lvert AB \rvert = \sqrt{(-4)^2 + (-3)^2 + (-12)^2}$	See notes.	M1
	So, $\lvert AB \rvert = 13$		A1 **cao**
			[2]
(d)	$\overrightarrow{OB'} \{= \overrightarrow{OA} + \overrightarrow{BA}\} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix}; = \begin{pmatrix} 17 \\ -20 \\ -6 \end{pmatrix}$	See notes for alternative methods.	M1;A1 **cao**
			[2] **12**

Notes for Question 24

(a)

B1: $\lambda = 4$ seen or implied.

M1: Substitutes their value of λ into $a + 6\lambda = 21$

A1: $a = -3$.

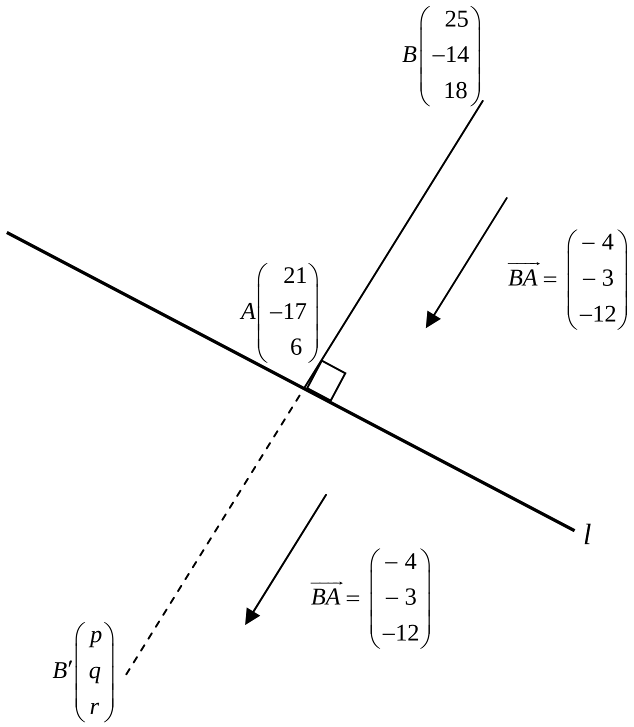
Note: Award B1M1A1 if the candidate states $a = -3$ from no working.

Alternative Method Using Simultaneous equations for part (a).

B1: For $60 - 6\lambda = 36$

M1: $60 - 6\lambda = 36$ and $a + 6\lambda = 21$ solved simultaneously to give $a = ...$

A1: $a = -3$, **cao**.

Notes for Question 24 Continued	
24. (b) ctd	**M1:** Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Ignore labelling. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference. **M1:** ***Applies*** the formula $\overrightarrow{AB}\bullet\begin{pmatrix}6\\c\\-1\end{pmatrix}$ or $\overrightarrow{BA}\bullet\begin{pmatrix}6\\c\\-1\end{pmatrix}$ correctly to give a linear equation in c which is set equal to zero. **Note:** The dot product can also be with $\pm k\begin{pmatrix}6\\c\\-1\end{pmatrix}$. **A1ft:** $c = -4$ **or** for finding a correct follow through c. **ddM1:** Substitutes their value of λ and their value of c into $b + c\lambda = -17$ **Note** that this mark is dependent on the two previous method marks being awarded. **A1:** $b = -1$
(c)	**M1:** An attempt to apply a three term Pythagoras in order to find $\lvert AB \rvert$, so taking the square root is required here. **A1:** 13 **cao** **Note:** Don't recover work for part (b) in part (c).
(d)	**M1:** For a full ***applied*** method of finding the coordinates of B'. **Note:** You can give M1 for 2 out of 3 correct components of B'. **A1:** For either $\begin{pmatrix}17\\-20\\-6\end{pmatrix}$ or $17\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}$ or $(17, -20, -6)$ **cao**. ***Helpful diagram!*** 

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Notes for Question 24 Continued	
	Acceptable Methods for the Method mark in part (d)
Way 1	$\overrightarrow{OB'}\left\{= \overrightarrow{OA} + \overrightarrow{BA}\right\} = \begin{pmatrix}21\\-17\\6\end{pmatrix} + \begin{pmatrix}-4\\-3\\-12\end{pmatrix}$ (using their $\overrightarrow{BA}$)
Way 2	$\overrightarrow{OB'}\left\{= \overrightarrow{OA} - \overrightarrow{AB}\right\} = \begin{pmatrix}21\\-17\\6\end{pmatrix} - \begin{pmatrix}4\\3\\12\end{pmatrix}$ (using their $\overrightarrow{AB}$)
Way 3	$\overrightarrow{OB'}\left\{= \overrightarrow{OB} + 2\overrightarrow{BA}\right\} = \begin{pmatrix}25\\-14\\18\end{pmatrix} + 2\begin{pmatrix}-4\\-3\\-12\end{pmatrix}$ (using their $\overrightarrow{BA}$)
Way 4	$\overrightarrow{OB'}\left\{= \overrightarrow{OB} - 2\overrightarrow{AB}\right\} = \begin{pmatrix}25\\-14\\18\end{pmatrix} - 2\begin{pmatrix}4\\3\\12\end{pmatrix}$ (using their $\overrightarrow{AB}$)
Way 5	$\begin{pmatrix}25\\-14\\18\end{pmatrix} \rightarrow \begin{pmatrix}\text{Minus 4}\\\text{Minus 3}\\\text{Minus 12}\end{pmatrix} \rightarrow \begin{pmatrix}21\\-17\\6\end{pmatrix} \rightarrow \begin{pmatrix}\text{Minus 4}\\\text{Minus 3}\\\text{Minus 12}\end{pmatrix} \left\{\rightarrow \begin{pmatrix}17\\-20\\-6\end{pmatrix}\right\}$, so $\overrightarrow{OA}$ + their $\overrightarrow{BA}$
Way 6	$\overrightarrow{OB'}\left\{= 2\overrightarrow{OA} - \overrightarrow{OB}\right\} = 2\begin{pmatrix}21\\-17\\6\end{pmatrix} - \begin{pmatrix}25\\-14\\18\end{pmatrix}$
Way 7	$\overrightarrow{OB} = 25\mathbf{i} - 14\mathbf{j} + 18\mathbf{k}$, $\overrightarrow{OA} = 21\mathbf{i} - 17\mathbf{j} + 6\mathbf{k}$ and $\overrightarrow{OB'} = p\mathbf{i} + q\mathbf{j} + r\mathbf{k}$, $(21, -17, 6) = \left(\frac{25+p}{2}, \frac{-14+q}{2}, \frac{18+r}{2}\right)$ $p = 21(2) - 25 = 17$ $q = -17(2) + 14 = -20$ $r = 6(2) - 18 = -6$ **M1:** Writing down any two equations correctly and an attempt to find at least two of *p*, *q* or *r*.

Question Number	Scheme		Marks
25.	$l_1 : \mathbf{r} = \begin{pmatrix} -9 \\ 8 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix}$		
(a)	$A(1, 0, -1)$	correct coordinates	B1 **[1]**
(b)	$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $\mathbf{d}_1 = \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix}$ and θ is angle		
	$\cos\theta = \dfrac{\overrightarrow{OA} \bullet \mathbf{d}_1}{\lvert\overrightarrow{OA}\rvert . \lvert\mathbf{d}_1\rvert} = \dfrac{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix}}{\sqrt{(1)^2 + (0)^2 + (-1)^2} . \sqrt{(5)^2 + (-4)^2 + (-3)^2}}$	Applies dot product formula between $\overrightarrow{OA}$ and $\mathbf{d}_1$.	M1
	$\cos\theta = \dfrac{5 + 0 + 3}{\sqrt{(1)^2 + (0)^2 + (-1)^2} . \sqrt{(5)^2 + (-4)^2 + (-3)^2}} \left\{ = \dfrac{8}{(\sqrt{2})(5\sqrt{2})} \right.$	Correct ft expression or equation.	A1 ft
	$\cos\theta = \underline{\dfrac{8}{10}}$ or $\underline{\dfrac{4}{5}}$ or $\underline{0.8}$	$\underline{\dfrac{8}{10}}$ or $\underline{\dfrac{4}{5}}$ or $\underline{0.8}$ isw	A1 **cao** **[3]**
(c)	$\overrightarrow{OB} = 3\overrightarrow{OA} = 3\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$		
	$l_2 : \mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix}$	In the form of their $\overrightarrow{OB} + \lambda\mathbf{d}$ with any one of either $\mathbf{d}_1$ or their ft $\overrightarrow{OB}$ correct.	M1
		Correct equation and $\mathbf{r} =$	A1ft oe **[2]**
(d)	$OB = \sqrt{(3)^2 + (0)^2 + (-3)^2}$ $= \sqrt{18} = 3\sqrt{2}$	$3\sqrt{2}$	B1 ft **[1]**
(e)	So, $\dfrac{OX}{3\sqrt{2}} = \sin\theta$	$\dfrac{OX}{\text{their } OB} = \sin\theta$	M1
	$\left\{\cos\theta = \dfrac{4}{5} \Rightarrow \right\} \sin\theta = \dfrac{3}{5}$	Converts $\cos\theta$ into an expression for $\sin\theta$	M1 oe
	$OX \left\{ = 3\sqrt{2}\left(\dfrac{3}{5}\right) = \dfrac{9}{5}\sqrt{2} \right\} = 2.5455844...$	$OX =$ awrt 2.55	A1 **[3]** **10**

	Notes on Question 25
(b)	**Note:** Obtaining $\cos\theta = -\frac{4}{5}$ is M1A1A0.
(e)	**Note:** 2nd M1 mark can be awarded instead for candidate using $\sin(\text{awrt } 37)$

(e) Alternative Method 1 for part (e)

$\mathbf{d}_2 = \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix}, \quad \overrightarrow{OX} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix} = \begin{pmatrix} 3+5\lambda \\ -4\lambda \\ -3-3\lambda \end{pmatrix}$

$\overrightarrow{OX} \bullet \mathbf{d}_2 = 0 \Rightarrow \begin{pmatrix} 3+5\lambda \\ -4\lambda \\ -3-3\lambda \end{pmatrix} \bullet \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix} = 15 + 25\lambda + 16\lambda + 9 + 9\lambda = 0$

M1: Applies $\overrightarrow{OX} \bullet \mathbf{d}_2 = 0$ and solves the resulting equation to find a value for λ.

leading to $50\lambda + 24 = 0 \Rightarrow \lambda = -\frac{12}{25}$

Position vector $\overrightarrow{OX} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} - \frac{12}{25}\begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} \\ \frac{48}{25} \\ -\frac{39}{25} \end{pmatrix}$

dM1: Substitutes their value of λ into $\begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix}$.

Note: This mark is dependent upon the previous M1 mark if a candidate uses this alternative method.

$OX = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{48}{25}\right)^2 + \left(-\frac{39}{25}\right)^2} = 2.5455844...$

A1: For $OX =$ awrt 2.55

(e) Alternative Method 2 for part (e)

$\frac{BX}{3\sqrt{2}} = \cos\theta \left\{ \Rightarrow BX = 3\sqrt{2}\left(\frac{4}{5}\right) = \frac{12\sqrt{2}}{5} \right\}$

M1: $\frac{BX}{\text{their } OB} = \cos\theta$

So, $OX = \sqrt{\left(3\sqrt{2}\right)^2 - \left(2.4\sqrt{2}\right)^2}$

$OX = 2.5455844...$

M1: Subtracts using Pythagoras to find OX.

A1: For $OX =$ awrt 2.55

Question Number	Scheme		Marks
26. (a)	**i:** $9+\lambda = 2+2\mu$ **(1)** **j:** $13+4\lambda = -1+\mu$ **(2)** **k:** $-3-2\lambda = 1+\mu$ **(3)**	Any two equations. (Allow one slip).	M1
	Eg: **(2) − (3)**: $16+6\lambda = -2$ or **(2) − 4(1)**: $-23 = -9-7\mu$	An attempt to eliminate one of the parameters.	dM1
	Leading to $\lambda = -3$ or $\mu = 2$	Either $\lambda = -3$ or $\mu = 2$	A1
	$l_1 : \mathbf{r} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} - 3\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix}$ or $l_2 : \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix}$	See notes	ddM1 A1
			[5]
(b)	$\mathbf{d_1} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$, $\mathbf{d}_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$	Realisation that the dot product is required between $\pm A\mathbf{d_1}$ and $\pm B\mathbf{d}_2$.	M1
	$\cos\theta = \pm\left(\dfrac{2+4-2}{\sqrt{(1)^2+(4)^2+(-2)^2}\,.\sqrt{(2)^2+(1)^2+(1)^2}}\right)$	Correct **equation**.	A1
	$\cos\theta = \dfrac{4}{\sqrt{21}.\sqrt{6}} \Rightarrow \theta = 69.1238974... = 69.1$ (1 dp)	awrt 69.1	A1
			[3]
(c)	$\overrightarrow{OA} = \begin{pmatrix} 4 \\ 16 \\ -3 \end{pmatrix}$, $\overrightarrow{OP} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 9+\lambda \\ 13+4\lambda \\ -3-2\lambda \end{pmatrix}$		
	$\overrightarrow{AP} = \begin{pmatrix} 9+\lambda \\ 13+4\lambda \\ -3-2\lambda \end{pmatrix} - \begin{pmatrix} 4 \\ 16 \\ -3 \end{pmatrix} = \begin{pmatrix} \lambda+5 \\ 4\lambda-3 \\ -2\lambda \end{pmatrix}$		M1 A1
	$\overrightarrow{AP} \bullet \mathbf{d_1} = 0 \Rightarrow \begin{pmatrix} \lambda+5 \\ 4\lambda-3 \\ -2\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \lambda+5+16\lambda-12+4\lambda = 0$		dM1
	leading to $\{21\lambda - 7 = 0 \Rightarrow\}\ \lambda = \dfrac{1}{3}$	$\lambda = \dfrac{1}{3}$	A1
	Position vector $\overrightarrow{OP} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \dfrac{1}{3}\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 9\frac{1}{3} \\ 14\frac{1}{3} \\ -3\frac{2}{3} \end{pmatrix}$ or $\begin{pmatrix} \frac{28}{3} \\ \frac{43}{3} \\ -\frac{11}{3} \end{pmatrix}$		ddM1 A1
			[6] **14**

26. (a) **M1:** Writes down any two equations. Allow one slip.

dM1: Attempts to eliminate either λ or μ to form an equation in one parameter only.

A1: For either $\lambda = -3$ or $\mu = 2$. **Note**: candidates only need to find one of the parameters.

ddM1: For either substituting their value of λ into l_1 or their μ into l_2.

2nd A1: For either $\begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix}$ or $6\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or $(6 \quad 1 \quad 3)$.

Note: Each of the method marks in this part are dependent upon the previous method marks.

(b) **M1:** Realisation that the dot product is required between $\pm A\mathbf{d_1}$ and $\pm B\mathbf{d}_2$. Allow one slip in $\mathbf{d}_1 = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$.

A1: Correct application of the dot product formula $\mathbf{d_1} \bullet \mathbf{d_2} = \pm |\mathbf{d_1}||\mathbf{d_2}|\cos\theta$ or $\cos\theta = \pm\left(\frac{\mathbf{d_1} \bullet \mathbf{d_2}}{|\mathbf{d_1}||\mathbf{d_2}|}\right)$

The dot product must be correctly applied and the square roots although they can be un-simplified must be correctly applied.

A1: awrt 69.1 . This can be also be achieved by $180 - 110.876 = \text{awrt } 69.1$. $\theta = 1.2064...^c$ is A0.

Common response: $\cos\theta = \left(\frac{-12 - 24 + 12}{\sqrt{(-3)^2 + (-12)^2 + (6)^2} \cdot \sqrt{(4)^2 + (2)^2 + (2)^2}}\right) = \frac{-24}{\sqrt{189} \cdot \sqrt{24}}$ is M1A1...

Alternative Method: Vector Cross Product

Only apply this scheme if it is clear that a candidate is applying a vector cross product method.

$$\mathbf{d_1} \times \mathbf{d_2} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4 & -2 \\ 2 & 1 & 1 \end{vmatrix} = 6\mathbf{i} - 5\mathbf{j} - 7\mathbf{k} \right\}$$

M1: Realisation that the vector cross product is required between $\pm A\mathbf{d_1}$ and $\pm B\mathbf{d}_2$. Allow one slip in $\mathbf{d}_1 = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$.

$$\sin\theta = \frac{\sqrt{(6)^2 + (5)^2 + (-7)^2}}{\sqrt{(1)^2 + (4)^2 + (-2)^2} \cdot \sqrt{(2)^2 + (1)^2 + (1)^2}}$$

A1: Correct applied **equation**.

$$\sin\theta = \frac{\sqrt{110}}{\sqrt{21} \cdot \sqrt{6}} \Rightarrow \theta = 69.1238974... = 69.1 \text{ (1 dp)}$$

A1: awrt 69.1

(c)

M1: Attempts to find $\overrightarrow{AP}$ in terms of the parameter by subtracting the components of $\overrightarrow{OP}$ from l_1 and $\overrightarrow{OA}$. Ignore the direction of subtraction and ignore any confusion between $\overrightarrow{OP}$ and $\overrightarrow{PO}$ or between $\overrightarrow{OA}$ and $\overrightarrow{AO}$. The correct subtraction of two components is enough to establish that subtraction is intended. The coordinates or position vector of *P* must be given in terms of a parameter. Taking $P:(x, y, z)$ gains no marks although this can be recovered later. See **Additional Solutions**.

A1: (M1 on epen) A correct expression for $\overrightarrow{AP}$. Again accept the reverse direction.

dM1: Depends on the previous M. Taking the scalar product of their expression for $\overrightarrow{AP}$ with $\mathbf{d_1}$ or a multiple of $\mathbf{d_1}$ and equating to 0 and obtaining an equation for λ. The equation must derive from an expression of the form $x_1x_2 + y_1y_2 + z_1z_2 = 0$. Differentiation can be used. See **Additional Solutions**.

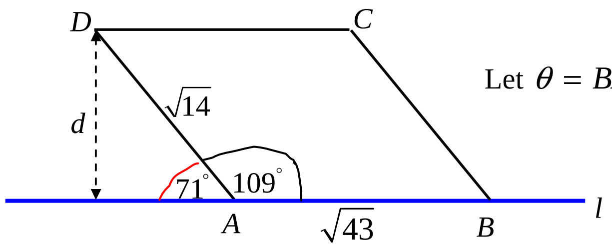
A1: Solving to find $\lambda = \frac{1}{3}$.

ddM1: Depends on both previous Ms. Substitutes their value of the parameter into their expression for $\overrightarrow{OP}$. Substituting into $\overrightarrow{AP}$ is a common error which loses the mark.

Note: Needs 2 correct co-ordinates if $\lambda = \frac{1}{3}$ found and then *P* stated without method to gain ddM1.

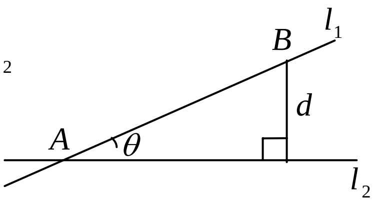
	A1: $9\frac{1}{3}\mathbf{i}+14\frac{1}{3}\mathbf{j}-3\frac{2}{3}\mathbf{k}$. Accept vector notation or coordinates. ***Must be exact.***

Question Number	Scheme	Marks
27.	(a) $\overrightarrow{AB} = \begin{pmatrix} 8 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 10 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$	M1 A1 **(2)**
	(b) $\mathbf{r} = \begin{pmatrix} 10 \\ 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ $\qquad$ $\mathbf{r} = \begin{pmatrix} 8 \\ 3 \\ 4 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$	M1 A1ft **(2)**
	(c) $\overrightarrow{CP} = \begin{pmatrix} 10-2t \\ 2+t \\ 3+t \end{pmatrix} - \begin{pmatrix} 3 \\ 12 \\ 3 \end{pmatrix} = \begin{pmatrix} 7-2t \\ t-10 \\ t \end{pmatrix}$	M1 A1
	$\begin{pmatrix} 7-2t \\ t-10 \\ t \end{pmatrix} . \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = -14+4t+t-10+t=0$	M1
	Leading to $\quad t = 4$	A1
	Position vector of P is $\begin{pmatrix} 10-8 \\ 2+4 \\ 3+4 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 7 \end{pmatrix}$	M1 A1 **(6)**
		[10]
	Alternative working for (c)	
	$\overrightarrow{CP} = \begin{pmatrix} 8-2t \\ 3+t \\ 4+t \end{pmatrix} - \begin{pmatrix} 3 \\ 12 \\ 3 \end{pmatrix} = \begin{pmatrix} 5-2t \\ t-9 \\ t+1 \end{pmatrix}$	M1 A1
	$\begin{pmatrix} 5-2t \\ t-9 \\ t+1 \end{pmatrix} . \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = -10+4t+t-9+t+1=0$	M1
	Leading to $\quad t = 3$	A1
	Position vector of P is $\begin{pmatrix} 8-6 \\ 3+3 \\ 4+3 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 7 \end{pmatrix}$	M1 A1 **(6)**

Question Number	Scheme	Marks
28.	$\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$, $\overrightarrow{OB} = 5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}$, $\{\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}\}$ & $\overrightarrow{OD} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$	
(a)	$\overrightarrow{AB} = = \pm\left((5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 5\mathbf{k})\right); = 3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$	M1; A1
		[2]
(b)	$l: \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$ See notes	M1 A1ft
		[2]
	 Let $\theta = B\hat{A}D$ Let d be the shortest distance from C to l.	
(c)	$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$	M1
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{AD}}{\lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{AD}\rvert} = \dfrac{\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}$ Applies dot product formula between their $(\overrightarrow{AB}$ or $\overrightarrow{BA})$ and their $(\overrightarrow{AD}$ or $\overrightarrow{DA})$.	M1
	$\cos\theta = \pm\left(\dfrac{-9 + 6 - 5}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}\right)$ Correct followed through expression or **equation**.	A1 $\sqrt{}$
	$\cos\theta = \dfrac{-8}{\sqrt{43}.\sqrt{14}} \Rightarrow \theta = 109.029544... = 109$ (nearest °) awrt 109	A1 **cso AG**
		[4]
(d)	$\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{DC} = \overrightarrow{OD} + \overrightarrow{AB} = (-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + (3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$ $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OB} + \overrightarrow{AD} = (5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) + (-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$	M1
	So, $\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$	A1
		[2]
(e)	Area $ABCD = \left(\frac{1}{2}(\sqrt{43})(\sqrt{14})\sin 109^\circ\right); \times 2 = 23.19894905$ awrt 23.2	M1; dM1 A1
		[3]
(f)	$\dfrac{d}{\sqrt{14}} = \sin 71$ or $\sqrt{43}\,d = 23.19894905...$	M1
	$\therefore\ d = \sqrt{14}\sin 71^\circ = 3.537806563...$ awrt 3.54	A1
		[2] **15**

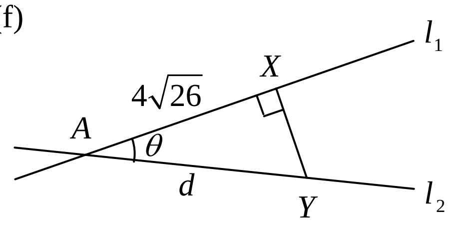
28. (a)	**M1:** Finding the difference between $\overrightarrow{OB}$ and $\overrightarrow{OA}$. Can be implied by two out of three components correct in $3\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ or $-3\mathbf{i}-3\mathbf{j}-5\mathbf{k}$ **A1:** $3\mathbf{i}+3\mathbf{j}+5\mathbf{k}$
(b)	**M1:** An expression of the form $(3\text{ component vector}) \pm \lambda(3\text{ component vector})$ **A1ft:** $\mathbf{r}=\overrightarrow{OA}+\lambda(\text{their } \pm\overrightarrow{AB})$ or $\mathbf{r}=\overrightarrow{OB}+\lambda(\text{their } \pm\overrightarrow{AB})$. **Note:** Candidate must begin writing their line as $\mathbf{r}=$ or $l=\dots$ or $\begin{pmatrix}x\\y\\z\end{pmatrix}=\dots$ So, Line = ... would be A0.
(c)	**M1:** An attempt to find either the vector $\overrightarrow{AD}$ or $\overrightarrow{DA}$. Can be implied by two out of three components correct in $-3\mathbf{i}+2\mathbf{j}-\mathbf{k}$ or $3\mathbf{i}-2\mathbf{j}+\mathbf{k}$, respectively. **M1:** Applies dot product formula between their $(\overrightarrow{AB}$ or $\overrightarrow{BA})$ and their $(\overrightarrow{AD}$ or $\overrightarrow{DA})$. **A1ft:** Correct followed through expression or **equation**. The dot product must be correctly followed through correctly and the square roots although they can be un-simplified must be followed through correctly. **A1:** Obtains an angle of awrt 109 ***by correct solution only***. Award the final A1 mark if candidate achieves awrt 109 by either taking the dot product between: (i) $\begin{pmatrix}3\\3\\5\end{pmatrix}$ and $\begin{pmatrix}-3\\2\\-1\end{pmatrix}$ or (ii) $\begin{pmatrix}-3\\-3\\-5\end{pmatrix}$ and $\begin{pmatrix}3\\-2\\1\end{pmatrix}$. Ignore if any of these vectors are labelled incorrectly. Award A0, cso for those candidates who take the dot product between: (iii) $\begin{pmatrix}-3\\-3\\-5\end{pmatrix}$ and $\begin{pmatrix}-3\\2\\-1\end{pmatrix}$ or (iv) $\begin{pmatrix}3\\3\\5\end{pmatrix}$ and $\begin{pmatrix}3\\-2\\1\end{pmatrix}$. They will usually find awrt 71 and apply 180 – awrt 71 to give awrt 109. If these candidates give a convincing detailed explanation which must include reference to the direction of their vectors then this can be given A1 cso. If still in doubt, here, send to review.
(d)	**M1:** Applies either $\overrightarrow{OD}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OB}$ + their $\overrightarrow{AD}$. This mark can be implied by two out of three correctly followed through components in their $\overrightarrow{OD}$. **A1:** For $2\mathbf{i}+4\mathbf{j}+9\mathbf{k}$.
(e)	**M1:** $\frac{1}{2}(\text{their } AB)(\text{their } CB)\sin(\text{their } 109^\circ \text{ or } 71^\circ \text{ from (b)})$. Awrt 11.6 will usually imply this mark. **dM1:** Multiplies this by 2 for the parallelogram. Can be implied. **Note:** $\frac{1}{2}((\text{their } AB + \text{their } AB))(\text{their } CB)\sin(\text{their } 109^\circ \text{ or } 71^\circ \text{ from (b)})$ **A1:** awrt 23.2
(f)	**M1:** $\dfrac{d}{\text{their } AD}=\sin(\text{their } 109^\circ \text{ or } 71^\circ \text{ from (b)})$ or $(\text{their } AB)\, d=(\text{their Area } ABCD)$ Award M0 for $(\text{their } AB)$ in part (f), if the area of their parallelogram in part (e) is $(\text{their } AB)(\text{their } CB)$. Award M0 for $\dfrac{d}{\text{their } \sqrt{43}}=\sin 71$ or $(\text{their } \sqrt{14})\,d=23.19894905\dots$ **A1:** awrt 3.54 **Note: Some candidates will use their answer to part (f) in order to answer part (e).**

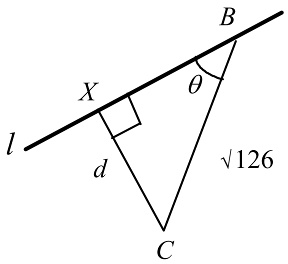
28.	*Alternative method for part (c): Applying the cosine rule:*	
	$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix}-1\\1\\4\end{pmatrix} - \begin{pmatrix}2\\-1\\5\end{pmatrix} = \begin{pmatrix}-3\\2\\-1\end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix}3\\-2\\1\end{pmatrix}$	**M1:** as above.
	$\overrightarrow{DB} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix}5\\2\\10\end{pmatrix} - \begin{pmatrix}-1\\1\\4\end{pmatrix} = \begin{pmatrix}6\\1\\6\end{pmatrix}$ or $\overrightarrow{BD} = \begin{pmatrix}-6\\-1\\-6\end{pmatrix}$	
	So $\lvert\overrightarrow{AB}\rvert = \sqrt{43}$, $\lvert\overrightarrow{AD}\rvert = \sqrt{14}$ and $\lvert\overrightarrow{DB}\rvert = \sqrt{73}$	
	$\cos\theta = \dfrac{(\sqrt{43})^2 + (\sqrt{14})^2 - (\sqrt{73})^2}{2\sqrt{43}.\sqrt{14}}$	**M1:** Cosine rule structure of $\cos\theta = \dfrac{a^2+b^2-c^2}{2ab}$ assigned each of $\lvert\overrightarrow{AB}\rvert$, $\lvert\overrightarrow{AD}\rvert$ and $\lvert\overrightarrow{DB}\rvert$ in any order as their *a*, *b* and *c*. **A1:** Correct application of cosine rule.
	$\left\{\cos\theta = \dfrac{-16}{2\sqrt{43}.\sqrt{14}} \Rightarrow \theta = 109.029544...\right\} = 109 \text{ (nearest}^\circ)$	A1: awrt 109 (no errors seen). **AG**
	Alternative method for part (d):	
	$\overrightarrow{OE} = \begin{pmatrix}2\\-1\\5\end{pmatrix} + \lambda\begin{pmatrix}3\\3\\5\end{pmatrix}$	
	$\overrightarrow{DE} = \begin{pmatrix}2+3\lambda\\-1+3\lambda\\5+5\lambda\end{pmatrix} - \begin{pmatrix}-1\\1\\4\end{pmatrix} = \begin{pmatrix}3+3\lambda\\-2+3\lambda\\1+5\lambda\end{pmatrix}$	
	$\overrightarrow{DE} \bullet \overrightarrow{AB} = 0 \Rightarrow \begin{pmatrix}3+3\lambda\\-2+3\lambda\\1+5\lambda\end{pmatrix} \bullet \begin{pmatrix}3\\3\\5\end{pmatrix} = 0$	**M1:** Takes the dot product between $\overrightarrow{DE}$ and $\overrightarrow{AB}$ and progresses to find a value of λ
	$9 + 9\lambda - 6 + 9\lambda + 5 + 3\lambda = 0 \Rightarrow \lambda = -\dfrac{8}{43}$	
	$\overrightarrow{DE} = \begin{pmatrix}2+3\lambda\\-1+3\lambda\\5+5\lambda\end{pmatrix} - \begin{pmatrix}-1\\1\\4\end{pmatrix} = \begin{pmatrix}\frac{103}{43}\\-\frac{110}{43}\\\frac{3}{43}\end{pmatrix}$	**dM1:** Uses their value of λ to find $\overrightarrow{DE}$
	Length DE = 3.537806563...	**A1:** awrt 3.54

Question Number	Scheme	Marks
29.	(a) **i**: $6-\lambda=-5+2\mu$	
	j: $-3+2\lambda=15-3\mu$ Any two equations	M1
	leading to $\lambda=3,\ \mu=4$	M1 A1
	$\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+3\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$ *or* $\mathbf{r}=\begin{pmatrix}-5\\15\\3\end{pmatrix}+4\begin{pmatrix}2\\-3\\1\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$	M1 A1
	k: LHS $=-2+3(3)=7$, RHS $=3+4(1)=7$	B1 **(6)**
	(As LHS = RHS, lines intersect)	
	Alternatively for B1, showing that $\lambda=3$ and $\mu=4$ both give $\begin{pmatrix}3\\3\\7\end{pmatrix}$	
	(b) $\begin{pmatrix}-1\\2\\3\end{pmatrix}.\begin{pmatrix}2\\-3\\1\end{pmatrix}=-2-6+3=\surd 14\surd 14\cos\theta$ $(\theta\approx 110.92°)$	M1 A1
	Acute angle is $69.1°$ awrt 69.1	A1 **(3)**
	(c) $\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+1\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}5\\-1\\1\end{pmatrix}$ $(\Rightarrow B$ lies on $l_1)$	B1 **(1)**
	(d) Let d be shortest distance from B to l_2 	
	$\overrightarrow{AB}=\begin{pmatrix}5\\-1\\1\end{pmatrix}-\begin{pmatrix}3\\3\\7\end{pmatrix}=\begin{pmatrix}2\\-4\\-6\end{pmatrix}$	M1
	$\lvert\overrightarrow{AB}\rvert=\surd\left(2^2+(-4)^2+(-6)^2\right)=\surd 56$ awrt 7.5	A1
	$\dfrac{d}{\surd 56}=\sin\theta$	M1
	$d=\surd 56\sin 69.1°\approx 6.99$ awrt 6.99	A1 **(4)**
		[14]

Question Number	Scheme	Marks
30. (a)	$\overrightarrow{AB} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} - (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = -3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$	M1 A1 (2)
(b)	$\mathbf{r} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$ or $\mathbf{r} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$	M1 A1ft (2)
(c)	$\overrightarrow{AC} = 2\mathbf{i} + p\mathbf{j} - 4\mathbf{k} - (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$ $= \mathbf{i} + (p+3)\mathbf{j} - 6\mathbf{k}$ or $\overrightarrow{CA}$ $\overrightarrow{AC}.\overrightarrow{AB} = \begin{pmatrix} 1 \\ p+3 \\ -6 \end{pmatrix} . \begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix} = 0$ $-3 + 5p + 15 + 18 = 0$ Leading to $p = -6$	 B1 M1 M1 A1 (4)
(d)	$AC^2 = (2-1)^2 + (-6+3)^2 + (-4-2)^2 \quad (= 46)$ $AC = \sqrt{46}$ accept awrt 6.8	M1 A1 (2) [10]

Question Number	Scheme	Marks
31.	(a) **j** components $3+2\lambda=9 \;\Rightarrow \lambda=3$ $(\mu=1)$	M1 A1
	Leading to $C:(5,9,-1)$ accept vector forms	A1 **(3)**
	(b) Choosing correct directions or finding $\overrightarrow{AC}$ and $\overrightarrow{BC}$	M1
	$\begin{pmatrix}1\\2\\1\end{pmatrix}.\begin{pmatrix}5\\0\\2\end{pmatrix}=5+2=\sqrt{6}\sqrt{29}\cos\angle ACB$ use of scalar product	M1 A1
	$\angle ACB=57.95°$ awrt $57.95°$	A1 **(4)**
	(c) $A:(2,3,-4)$ $B:(-5,9,-5)$	
	$\overrightarrow{AC}=\begin{pmatrix}3\\6\\3\end{pmatrix}$, $\overrightarrow{BC}=\begin{pmatrix}10\\0\\4\end{pmatrix}$	
	$AC^2=3^2+6^2+3^2 \;\Rightarrow\; AC=3\sqrt{6}$	M1 A1
	$BC^2=10^2+4^2 \;\Rightarrow\; BC=2\sqrt{29}$	A1
	$\triangle ABC=\frac{1}{2}AC\times BC\sin\angle ACB$	
	$=\frac{1}{2}3\sqrt{6}\times 2\sqrt{29}\sin\angle ACB\approx 33.5$ $15\sqrt{5}$, awrt 34	M1 A1 **(5)**
		[12]
	Alternative method for (b) *and* (c)	
	(b) $A:(2,3,-4)$ $B:(-5,9,-5)$ $C:(5,9,-1)$	
	$AB^2=7^2+6^2+1^2=86$	
	$AC^2=3^2+6^2+3^2=54$	
	$BC^2=10^2+0^2+4^2=116$ Finding all three sides	M1
	$\cos\angle ACB=\frac{116+54-86}{2\sqrt{116}\sqrt{54}}$ $(=0.53066\ldots)$	M1 A1
	$\angle ACB=57.95°$ awrt $57.95°$	A1 **(4)**
	If this method is used some of the working may gain credit in part (c) and appropriate marks may be awarded if there is an attempt at part (c).	

Question Number	Scheme	Marks
32	(a) $A: (-6, 4, -1)$ Accept vector forms	B1 (1)
	(b) $\begin{pmatrix}4\\-1\\3\end{pmatrix}.\begin{pmatrix}3\\-4\\1\end{pmatrix} = 12+4+3 = \sqrt{4^2+(-1)^2+3^2}\sqrt{3^2+(-4)^2+1^2}\cos\theta$	M1 A1
	$\cos\theta = \frac{19}{26}$ awrt 0.73	A1 (3)
	(c) $X: (10, 0, 11)$ Accept vector forms	B1 (1)
	(d) $\overrightarrow{AX} = \begin{pmatrix}10\\0\\11\end{pmatrix} - \begin{pmatrix}-6\\4\\-1\end{pmatrix}$ Either order	M1
	$= \begin{pmatrix}16\\-4\\12\end{pmatrix}$ cao	A1 (2)
	(e) $\lvert\overrightarrow{AX}\rvert = \sqrt{16^2+(-4)^2+12^2}$	M1
	$= \sqrt{416} = \sqrt{16\times 26} = 4\sqrt{26}$ * Do not penalise if consistent incorrect signs in (d)	A1 (2)
	(f) 	
	Use of correct right angled triangle	M1
	$\frac{\lvert\overrightarrow{AX}\rvert}{d} = \cos\theta$	M1
	$d = \frac{4\sqrt{26}}{\frac{19}{26}} \approx 27.9$ awrt 27.9	A1 (3)
		[12]

Question Number	Scheme	Marks
33 (a)	$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}10\\14\\-4\end{pmatrix}-\begin{pmatrix}8\\13\\-2\end{pmatrix}=\begin{pmatrix}2\\1\\-2\end{pmatrix}$ or $\overrightarrow{BA}=\begin{pmatrix}-2\\-1\\2\end{pmatrix}$	M1
	$\mathbf{r}=\begin{pmatrix}8\\13\\-2\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}$ *or* $\mathbf{r}=\begin{pmatrix}10\\14\\-4\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}$ accept equivalents	M1 A1ft (3)
(b)	$\overrightarrow{CB}=\overrightarrow{OB}-\overrightarrow{OC}=\begin{pmatrix}10\\14\\-4\end{pmatrix}-\begin{pmatrix}9\\9\\6\end{pmatrix}=\begin{pmatrix}1\\5\\-10\end{pmatrix}$ or $\overrightarrow{BC}=\begin{pmatrix}-1\\-5\\10\end{pmatrix}$	
	$CB=\sqrt{\left(1^2+5^2+(-10)^2\right)}=\sqrt{(126)}\quad(=3\sqrt{14}\approx 11.2)$ awrt 11.2	M1 A1 (2)
(c)	$\overrightarrow{CB}.\overrightarrow{AB}=\lvert\overrightarrow{CB}\rvert\lvert\overrightarrow{AB}\rvert\cos\theta$	
	$(\pm)(2+5+20)=\sqrt{126}\sqrt{9}\cos\theta$	M1 A1
	$\cos\theta=\dfrac{3}{\sqrt{14}}\Rightarrow\theta\approx 36.7°$ awrt 36.7°	A1 (3)
(d)		
	$\dfrac{d}{\sqrt{126}}=\sin\theta$	M1 A1ft
	$d=3\sqrt{5}(\approx 6.7)$ awrt 6.7	A1 (3)
(e)	$BX^2=BC^2-d^2=126-45=81$	M1
	$!\ CBX=\dfrac{1}{2}\times BX\times d=\dfrac{1}{2}\times 9\times 3\sqrt{5}=\dfrac{27\sqrt{5}}{2}(\approx 30.2)$ awrt 30.1 or 30.2	M1 A1 (3)
		[14]
	Alternative for (e)	
	$!\ CBX=\dfrac{1}{2}\times d\times BC\sin\angle XCB$	M1
	$=\dfrac{1}{2}\times 3\sqrt{5}\times\sqrt{126}\sin(90-36.7)°$ sine of correct angle	M1
	≈ 30.2 $\quad\dfrac{27\sqrt{5}}{2}$, awrt 30.1 or 30.2	A1 (3)

Question Number	Scheme		Marks
34. (a)	$\mathbf{d}_1 = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $\mathbf{d}_2 = q\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$		
	As $\left\{\mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix}-2\\1\\-4\end{pmatrix} \bullet \begin{pmatrix}q\\2\\2\end{pmatrix}\right\} = \underline{(-2\times q) + (1\times 2) + (-4\times 2)}$	Apply dot product calculation between two direction vectors, ie. $\underline{(-2\times q) + (1\times 2) + (-4\times 2)}$	M1
	$\mathbf{d}_1 \bullet \mathbf{d}_2 = 0 \Rightarrow -2q + 2 - 8 = 0$ $-2q = 6 \Rightarrow \underline{q = -3}$ AG	Sets $\mathbf{d}_1 \bullet \mathbf{d}_2 = 0$ and solves to find $\underline{q = -3}$	A1 **cso**
			[2]
(b)	Lines meet where:		
	$\begin{pmatrix}11\\2\\17\end{pmatrix} + \lambda\begin{pmatrix}-2\\1\\-4\end{pmatrix} = \begin{pmatrix}-5\\11\\p\end{pmatrix} + \mu\begin{pmatrix}q\\2\\2\end{pmatrix}$		
	First two of $\mathbf{i}: \; 11 - 2\lambda = -5 + q\mu$ (1) $\mathbf{j}: \; 2 + \lambda = 11 + 2\mu$ (2) $\mathbf{k}: \; 17 - 4\lambda = p + 2\mu$ (3)	Need to see equations (1) and (2). Condone one slip. (Note that $q = -3$.)	M1
	(1) + 2**(2)** gives: $15 = 17 + \mu \Rightarrow \mu = -2$	Attempts to solve (1) and (2) to find one of either λ or μ	dM1
		Any one of $\underline{\lambda = 5}$ or $\underline{\mu = -2}$	A1
	(2) gives: $2 + \lambda = 11 - 4 \Rightarrow \lambda = 5$	Both $\underline{\lambda = 5}$ and $\underline{\mu = -2}$	A1
	(3) $\Rightarrow 17 - 4(5) = p + 2(-2)$	Attempt to substitute their λ and μ into their $\mathbf{k}$ component to give an equation in p alone.	ddM1
	$\Rightarrow p = 17 - 20 + 4 \Rightarrow \underline{p = 1}$	$\underline{p = 1}$	A1 **cso**
			[6]
(c)	$\mathbf{r} = \begin{pmatrix}11\\2\\17\end{pmatrix} + 5\begin{pmatrix}-2\\1\\-4\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}-5\\11\\1\end{pmatrix} - 2\begin{pmatrix}-3\\2\\2\end{pmatrix}$	Substitutes their value of λ or μ into the correct line l_1 or l_2 .	M1
	Intersect at $\mathbf{r} = \underline{\begin{pmatrix}1\\7\\-3\end{pmatrix}}$ or $\underline{(1, 7, -3)}$	$\underline{\begin{pmatrix}1\\7\\-3\end{pmatrix}}$ or $\underline{(1, 7, -3)}$	A1
			[2]

Question Number	Scheme	Marks
(d)	Let $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection	
	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$ Finding vector $\overrightarrow{AX}$ by finding the difference between $\overrightarrow{OX}$ and $\overrightarrow{OA}$. Can be ft using candidate's $\overrightarrow{OX}$.	M1 $\sqrt{\ }$ ±
	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AX}$	
	$\overrightarrow{OB} = \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$ $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\begin{pmatrix} \text{their } \overrightarrow{AX} \end{pmatrix}$	dM1 $\sqrt{\ }$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ $\underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1
		[3]
		13 marks

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Question Number	Scheme		Marks
35. (a)	Lines meet where:		
	$\begin{pmatrix}-9\\0\\10\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}3\\1\\17\end{pmatrix}+\mu\begin{pmatrix}3\\-1\\5\end{pmatrix}$		
	Any two of $\mathbf{i}: -9+2\lambda=3+3\mu$ (1) $\mathbf{j}: \lambda=1-\mu$ (2) $\mathbf{k}: 10-\lambda=17+5\mu$ (3)	Need any two of these correct equations seen anywhere in part (a).	M1
	(1) – 2(2) gives: $-9=1+5\mu \Rightarrow \mu=-2$	Attempts to solve simultaneous equations to find one of either λ or μ	dM1
	(2) gives: $\lambda=1--2=3$	Both $\underline{\lambda=3}$ & $\underline{\mu=-2}$	A1
	$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+3\begin{pmatrix}2\\1\\-1\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\1\\17\end{pmatrix}-2\begin{pmatrix}3\\-1\\5\end{pmatrix}$	Substitutes their value of either λ or μ into the line l_1 or l_2 respectively. This mark can be implied by any two correct components of $(-3, 3, 7)$.	ddM1
	Intersect at $\mathbf{r}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\mathbf{r}=\underline{-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}}$	$\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\underline{-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}}$ or $(-3, 3, 7)$	A1
	Either check k: $\lambda=3$: $\text{LHS}=10-\lambda=10-3=7$ $\mu=-2$: $\text{RHS}=17+5\mu=17-10=7$	Either check that $\lambda=3$, $\mu=-2$ in a third equation or check that $\lambda=3$, $\mu=-2$ give the same coordinates on the other line. Conclusion not needed.	B1
	(As LHS = RHS then the lines intersect.)		[6]
(b)	$\mathbf{d}_1=2\mathbf{i}+\mathbf{j}-\mathbf{k}$, $\mathbf{d}_2=3\mathbf{i}-\mathbf{j}+5\mathbf{k}$		
	As $\mathbf{d}_1\bullet\mathbf{d}_2=\begin{pmatrix}2\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}3\\-1\\5\end{pmatrix}=\underline{(2\times3)+(1\times-1)+(-1\times5)}=0$	Dot product calculation between the ***two direction vectors***: $\underline{(2\times3)+(1\times-1)+(-1\times5)}$ or $\underline{6-1-5}$	M1
	Then l_1 is perpendicular to l_2.	Result '=0' and appropriate conclusion	A1
			[2]

Question Number	Scheme		Marks
35. (c)	Equating i ; $-9+2\lambda=5 \Rightarrow \lambda=7$		
	$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+7\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}5\\7\\3\end{pmatrix}$ $(=\overrightarrow{OA}.$ Hence the point A lies on l_1.)	Substitutes candidate's $\lambda=7$ into the line l_1 and finds $5\mathbf{i}+7\mathbf{j}+3\mathbf{k}$. The conclusion on this occasion is not needed.	B1 [1]
(d)	Let $\overrightarrow{OX}=-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX}=\overrightarrow{OX}-\overrightarrow{OA}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}-\begin{pmatrix}5\\7\\3\end{pmatrix}}=\begin{pmatrix}-8\\-4\\4\end{pmatrix}$	Finding the difference between their $\overrightarrow{OX}$ (can be implied) and $\overrightarrow{OA}$. $\overrightarrow{AX}=\pm\left(\begin{pmatrix}-3\\3\\7\end{pmatrix}-\begin{pmatrix}5\\7\\3\end{pmatrix}\right)$	M1$\sqrt{}$ $\pm$
	$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OA}+2\overrightarrow{AX}$		
	$\overrightarrow{OB}=\begin{pmatrix}5\\7\\3\end{pmatrix}+2\begin{pmatrix}-8\\-4\\4\end{pmatrix}$	$\begin{pmatrix}5\\7\\3\end{pmatrix}+2\left(\text{their }\overrightarrow{AX}\right)$	dM1$\sqrt{}$
	Hence, $\overrightarrow{OB}=\underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\overrightarrow{OB}=\underline{-11\mathbf{i}-\mathbf{j}+11\mathbf{k}}$	$\underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\underline{-11\mathbf{i}-\mathbf{j}+11\mathbf{k}}$ or $\underline{(-11,-1,11)}$	A1 [3]
			12 marks

Question Number	Scheme		Marks
36. (a)	$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix}$ & $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$		
	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} = \underline{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}}$	Finding the difference between $\overrightarrow{OB}$ and $\overrightarrow{OA}$.	M1 $\pm$
		Correct answer.	A1
			[2]
(b)	$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$	An expression of the form $(\text{vector}) \pm \lambda(\text{vector})$	M1
	$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$	$\mathbf{r} = \overrightarrow{OA} \pm \lambda(\text{their } \overrightarrow{AB})$ or $\mathbf{r} = \overrightarrow{OB} \pm \lambda(\text{their } \overrightarrow{AB})$ or $\mathbf{r} = \overrightarrow{OA} \pm \lambda(\text{their } \overrightarrow{BA})$ or $\mathbf{r} = \overrightarrow{OB} \pm \lambda(\text{their } \overrightarrow{BA})$ ($\mathbf{r}$ is needed.)	A1 $\sqrt{}$ aef
			[2]
(c)	$l_2 : \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \mathbf{r} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$		
	$\overrightarrow{AB} = \mathbf{d}_1 = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{d}_2 = \mathbf{i} + 0\mathbf{j} + \mathbf{k}$ & θ is angle		
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \mathbf{d}_2}{\left(\lvert\overrightarrow{AB}\rvert . \lvert\mathbf{d}_2\rvert\right)} = \dfrac{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{\left(\sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2}\right)}$	← Considers dot product between $\mathbf{d}_2$ and their $\overrightarrow{AB}$.	M1 $\sqrt{}$
	$\cos\theta = \dfrac{1 + 0 + 2}{\sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2}}$	Correct followed through expression or **equation**.	A1 $\sqrt{}$
	$\cos\theta = \dfrac{3}{3.\sqrt{2}} \Rightarrow \underline{\theta = 45^\circ \text{ or } \frac{\pi}{4} \text{ or awrt } 0.79}.$	$\underline{\theta = 45^\circ \text{ or } \frac{\pi}{4} \text{ or awrt } 0.79}$	A1 **cao**
			[3]

This means that $\cos\theta$ does not necessarily have to be the subject of the equation. It could be of the form $3\sqrt{2}\cos\theta = 3$.

Question Number	Scheme		Marks
36. (d)	If l_1 and l_2 intersect then: $\begin{pmatrix}2\\6\\-1\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\2\end{pmatrix} = \mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}: \quad 2+\lambda = \mu \qquad (1)$ $\mathbf{j}: \quad 6-2\lambda = 0 \qquad (2)$ $\mathbf{k}: -1+2\lambda = \mu \qquad (3)$	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{}$
	(2) yields $\lambda = 3$ Any two yields $\lambda = 3, \quad \mu = 5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	$l_1: \mathbf{r} = \begin{pmatrix}2\\6\\-1\end{pmatrix} + 3\begin{pmatrix}1\\-2\\2\end{pmatrix} = \underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r} = 5\begin{pmatrix}1\\0\\1\end{pmatrix} = \underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
Aliter **36.** (d) **Way 2**	If l_1 and l_2 intersect then: $\begin{pmatrix}3\\4\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\2\end{pmatrix} = \mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}: \quad 3+\lambda = \mu \qquad (1)$ $\mathbf{j}: \quad 4-2\lambda = 0 \qquad (2)$ $\mathbf{k}: \quad 1+2\lambda = \mu \qquad (3)$	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{}$
	(2) yields $\lambda = 2$ Any two yields $\lambda = 2, \quad \mu = 5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	$l_1: \mathbf{r} = \begin{pmatrix}3\\4\\1\end{pmatrix} + 2\begin{pmatrix}1\\-2\\2\end{pmatrix} = \underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r} = 5\begin{pmatrix}1\\0\\1\end{pmatrix} = \underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
			11 marks

Note: Be careful! λ and μ are not defined in the question, so a candidate could interchange these or use different scalar parameters.

Question Number	Scheme		Marks
Aliter **36.** (d) **Way 3**	If l_1 and l_2 intersect then: $\begin{pmatrix}2\\6\\-1\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}: \ 2-\lambda=\mu \quad (1)$ $\mathbf{j}: \ 6+2\lambda=0 \quad (2)$ $\mathbf{k}: -1-2\lambda=\mu \quad (3)$	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{\ }$
	(2) yields $\lambda=-3$ Any two yields $\lambda=-3,\ \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	$l_1: \mathbf{r}=\begin{pmatrix}2\\6\\-1\end{pmatrix}-3\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
Aliter **36.** (d) **Way 4**	If l_1 and l_2 intersect then: $\begin{pmatrix}3\\4\\1\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}: \ 3-\lambda=\mu \quad (1)$ $\mathbf{j}: \ 4+2\lambda=0 \quad (2)$ $\mathbf{k}: \ 1-2\lambda=\mu \quad (3)$	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{\ }$
	(2) yields $\lambda=-2$ Any two yields $\lambda=-2,\ \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	$l_1: \mathbf{r}=\begin{pmatrix}3\\4\\1\end{pmatrix}-2\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
			11 marks