EXPERT TUITION

Maths Questions By Topic:

Vectors

A-Level Edexcel

0207 060 4494
www.expert-tuition.co.uk
online.expert-tuition.co.uk
enquiries@expert-tuition.co.uk
The Foundry, 77 Fulham Palace Road, W6 8JA

Table Of Contents

1.

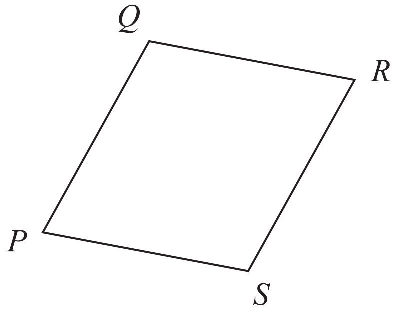

Figure 3

Figure 3 shows a sketch of a parallelogram *PQRS*.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$
- $\overrightarrow{QR} = 5\mathbf{i} - 2\mathbf{k}$

(a) show that parallelogram *PQRS* is a rhombus.

(2)

(b) Find the exact area of the rhombus *PQRS*.

(4)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 6 marks)

2. The triangle PQR is such that $\overrightarrow{PQ} = 3\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{PR} = 13\mathbf{i} - 15\mathbf{j}$

(a) Find $\overrightarrow{QR}$

(2)

(b) Hence find $|\overrightarrow{QR}|$ giving your answer as a simplified surd.

(2)

The point S lies on the line segment QR so that $QS : SR = 3 : 2$

(c) Find $\overrightarrow{PS}$

(2)

Question 2 continued

(Total for Question 2 is 6 marks)

3.

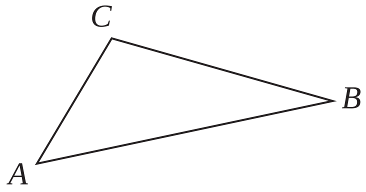

Figure 1

Figure 1 shows a sketch of triangle *ABC*.

Given that

- $\overrightarrow{AB} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$
- $\overrightarrow{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$

(a) find $\overrightarrow{AC}$

(2)

(b) show that $\cos ABC = \dfrac{9}{10}$

(3)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 5 marks)

4. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A stone slides horizontally across ice.

Initially the stone is at the point $A(-24\mathbf{i} - 10\mathbf{j})$ m relative to a fixed point O.

After 4 seconds the stone is at the point $B(12\mathbf{i} + 5\mathbf{j})$ m relative to the fixed point O.

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model,

(a) prove that the stone passes through O,

(2)

(b) calculate the speed of the stone.

(3)

Question 4 continued

(Total for Question 4 is 5 marks)

5. Relative to a fixed origin O

- point A has position vector $2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$
- point B has position vector $3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$
- point C has position vector $2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$

(a) Find $\overrightarrow{AB}$

(2)

(b) Show that quadrilateral $OABC$ is a trapezium, giving reasons for your answer.

(2)

Question 5 continued

(Total for Question 5 is 4 marks)

6. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A coastguard station O monitors the movements of a small boat.

At 10:00 the boat is at the point $(4\mathbf{i} - 2\mathbf{j})$ km relative to O.

At 12:45 the boat is at the point $(-3\mathbf{i} - 5\mathbf{j})$ km relative to O.

The motion of the boat is modelled as that of a particle moving in a straight line at constant speed.

(a) Calculate the bearing on which the boat is moving, giving your answer in degrees to one decimal place.

(3)

(b) Calculate the speed of the boat, giving your answer in km h^{-1}

(3)

Question 6 continued

(Total for Question 6 is 6 marks)

7. (i) Two non-zero vectors, **a** and **b**, are such that

$$|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$$

Explain, geometrically, the significance of this statement.

(1)

(ii) Two different vectors, **m** and **n**, are such that $|\mathbf{m}| = 3$ and $|\mathbf{m} - \mathbf{n}| = 6$
The angle between vector **m** and vector **n** is 30°

Find the angle between vector **m** and vector **m** – **n**, giving your answer, in degrees, to one decimal place.

(4)

Question 7 continued

Question 7 continued

(Total for Question 7 is 5 marks)

8. Given that the point A has position vector $4\mathbf{i} - 5\mathbf{j}$ and the point B has position vector $-5\mathbf{i} - 2\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$,

(2)

(b) find $|\overrightarrow{AB}|$.

Give your answer as a simplified surd.

(2)

Question 8 continued

(Total for Question 8 is 4 marks)

9. The quadrilateral $OABC$ has $\overrightarrow{OA} = 4\mathbf{i} + 2\mathbf{j}$, $\overrightarrow{OB} = 6\mathbf{i} - 3\mathbf{j}$ and $\overrightarrow{OC} = 8\mathbf{i} - 20\mathbf{j}$.

(a) Find $\overrightarrow{AB}$.

(2)

(b) Show that quadrilateral $OABC$ is a trapezium.

(2)

Question 9 continued

(Total for Question 9 is 4 marks)

10. Relative to a fixed origin O,

the point A has position vector $\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$,
the point B has position vector $4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$,
and the point C has position vector $2\mathbf{i} + 10\mathbf{j} + 9\mathbf{k}$.

Given that $ABCD$ is a parallelogram,

(a) find the position vector of point D.

(2)

The vector $\overrightarrow{AX}$ has the same direction as $\overrightarrow{AB}$.

Given that $|\overrightarrow{AX}| = 10\sqrt{2}$,

(b) find the position vector of X.

(3)

Question 10 continued

(Total for Question 10 is 5 marks)

11. Given that the point A has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point B has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$

(2)

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

(2)

(Total for Question 11 is 4 marks)

12.

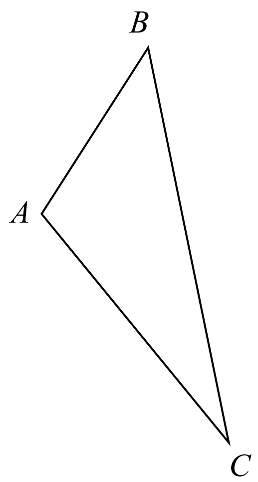

Figure 2

Figure 2 shows a sketch of a triangle *ABC*.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$,

show that $\angle BAC = 105.9°$ to one decimal place.

(5)

Question 12 continued

(Total for Question 12 is 5 marks)

13. Relative to a fixed origin O

- the point A has position vector $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $4\mathbf{j} + 6\mathbf{k}$
- the point C has position vector $-16\mathbf{i} + p\mathbf{j} + 10\mathbf{k}$

where p is a constant.

Given that A, B and C lie on a straight line,

(a) find the value of p.

(3)

The line segment OB is extended to a point D so that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

(b) Find $\left|\overrightarrow{OD}\right|$, writing your answer as a fully simplified surd.

(3)

Question 13 continued

Question 13 continued

Question 13 continued

(Total for Question 13 is 6 marks)

14. Relative to a fixed origin, points P, Q and R have position vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively.

Given that

- P, Q and R lie on a straight line
- Q lies one third of the way from P to R

show that

$$\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})$$

(3)

Question 14 continued

(Total for Question 14 is 3 marks)

15.

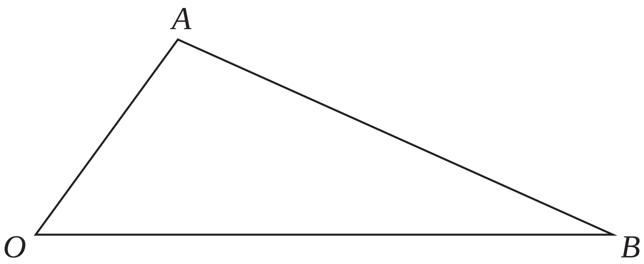

Figure 7

Figure 7 shows a sketch of triangle OAB.

The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point M is the midpoint of AB.

The straight line through C and M cuts OB at the point N.

Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{CM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

(2)

(b) Show that $\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant.

(2)

(c) Hence prove that $ON:NB = 2:1$

(2)

Question 15 continued

Question 15 continued

Question 15 continued

(Total for Question 15 is 6 marks)

16. Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(a) Find the position vector of D.

(2)

Given $|\overrightarrow{AC}| = 4$

(b) find the value of a.

(3)

Question 16 continued

(Total for Question 16 is 5 marks)

Leave blank

17. The point A with coordinates $(-3, 7, 2)$ lies on a line l_1
The point B also lies on the line l_1

Given that $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$,

(a) find the coordinates of point B.

(2)

The point P has coordinates $(9, 1, 8)$

(b) Find the cosine of the angle PAB, giving your answer as a simplified surd.

(3)

(c) Find the exact area of triangle PAB, giving your answer in its simplest form.

(3)

The line l_2 passes through the point P and is parallel to the line l_1

(d) Find a vector equation for the line l_2

(2)

The point Q lies on the line l_2

Given that the line segment AP is perpendicular to the line segment BQ,

(e) find the coordinates of the point Q.

(5)

Leave blank

Question 17 continued

Leave blank

Question 17 continued

Leave blank

Question 17 continued

(Total 15 marks)

Leave blank

18. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}, \qquad l_2 : \mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$$

where λ and μ are scalar parameters.

The lines l_1 and l_2 intersect at the point X.

(a) Find the coordinates of the point X.

(3)

(b) Find the size of the acute angle between l_1 and l_2, giving your answer in degrees to 2 decimal places.

(3)

The point A lies on l_1 and has position vector $\begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix}$

(c) Find the distance AX, giving your answer as a surd in its simplest form.

(2)

The point Y lies on l_2. Given that the vector $\overrightarrow{YA}$ is perpendicular to the line l_1

(d) find the distance YA, giving your answer to one decimal place.

(2)

The point B lies on l_1 where $|\overrightarrow{AX}| = 2|\overrightarrow{AB}|$.

(e) Find the two possible position vectors of B.

(3)

Leave blank

Question 18 continued

Leave blank

Question 18 continued

Leave blank

Question 18 continued

(Total 13 marks)

EXPERT TUITION

Leave blank

19. With respect to a fixed origin O, the line l_1 is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

where μ is a scalar parameter.

The point A lies on l_1 where $\mu = 1$

(a) Find the coordinates of A.

(1)

The point P has position vector $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$.

The line l_2 passes through the point P and is parallel to the line l_1

(b) Write down a vector equation for the line l_2

(2)

(c) Find the exact value of the distance AP.
Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined.

(2)

The acute angle between AP and l_2 is θ.

(d) Find the value of $\cos\theta$

(3)

A point E lies on the line l_2
Given that $AP = PE$,

(e) find the area of triangle APE,

(2)

(f) find the coordinates of the two possible positions of E.

(5)

Leave blank

Question 19 continued

(Total 15 marks)

Leave blank

20. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$$

where λ and μ are scalar parameters and p is a constant.

The lines l_1 and l_2 intersect at the point A.

(a) Find the coordinates of A.

(2)

(b) Find the value of the constant p.

(3)

(c) Find the acute angle between l_1 and l_2, giving your answer in degrees to 2 decimal places.

(3)

The point B lies on l_2 where $\mu = 1$

(d) Find the shortest distance from the point B to the line l_1, giving your answer to 3 significant figures.

(3)

Leave blank

Question 20 continued

(Total 11 marks)

Leave blank

21. Relative to a fixed origin O, the point A has position vector $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$

and the point B has position vector $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$

The line l_1 passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Hence find a vector equation for the line l_1

(1)

The point P has position vector $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$

Given that angle PBA is θ,

(c) show that $\cos\theta = \frac{1}{3}$

(3)

The line l_2 passes through the point P and is parallel to the line l_1

(d) Find a vector equation for the line l_2

(2)

The points C and D both lie on the line l_2

Given that $AB = PC = DP$ and the x coordinate of C is positive,

(e) find the coordinates of C and the coordinates of D.

(3)

(f) find the exact area of the trapezium $ABCD$, giving your answer as a simplified surd.

(4)

Leave blank

Question 21 continued

(Total 15 marks)

Leave blank

22. With respect to a fixed origin, the point A with position vector $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ lies on the line l_1 with equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \quad \text{where } \lambda \text{ is a scalar parameter,}$$

and the point B with position vector $4\mathbf{i} + p\mathbf{j} + 3\mathbf{k}$, where p is a constant, lies on the line l_2 with equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}, \quad \text{where } \mu \text{ is a scalar parameter.}$$

(a) Find the value of the constant p.

(1)

(b) Show that l_1 and l_2 intersect and find the position vector of their point of intersection, C.

(4)

(c) Find the size of the angle ACB, giving your answer in degrees to 3 significant figures.

(3)

(d) Find the area of the triangle ABC, giving your answer to 3 significant figures.

(2)

Leave blank

Question 22 continued

(Total 10 marks)

Leave blank

23. With respect to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \text{ where } \lambda \text{ is a scalar parameter.}$$

The point A lies on l and has coordinates $(3, -2, 6)$.

The point P has position vector $(-p\mathbf{i} + 2p\mathbf{k})$ relative to O, where p is a constant.

Given that vector $\overrightarrow{PA}$ is perpendicular to l,

(a) find the value of p.

(4)

Given also that B is a point on l such that $\angle BPA = 45°$,

(b) find the coordinates of the two possible positions of B.

(5)

Leave blank

Question 23 continued

(Total 9 marks)

Leave blank

24. Relative to a fixed origin O, the point A has position vector $21\mathbf{i} - 17\mathbf{j} + 6\mathbf{k}$ and the point B has position vector $25\mathbf{i} - 14\mathbf{j} + 18\mathbf{k}$.

The line l has vector equation

$$\mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$$

where a, b and c are constants and λ is a parameter.

Given that the point A lies on the line l,

(a) find the value of a.

(3)

Given also that the vector $\overrightarrow{AB}$ is perpendicular to l,

(b) find the values of b and c,

(5)

(c) find the distance AB.

(2)

The image of the point B after reflection in the line l is the point B'.

(d) Find the position vector of the point B'.

(2)

Leave blank

Question 24 continued

(Total 12 marks)

Leave blank

25. With respect to a fixed origin O, the line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} -9 \\ 8 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix}$$

where μ is a scalar parameter.

The point A is on l_1 where $\mu = 2$.

(a) Write down the coordinates of A.

(1)

The acute angle between OA and l_1 is θ, where O is the origin.

(b) Find the value of $\cos\theta$.

(3)

The point B is such that $\overrightarrow{OB} = 3\overrightarrow{OA}$.

The line l_2 passes through the point B and is parallel to the line l_1.

(c) Find a vector equation of l_2.

(2)

(d) Find the length of OB, giving your answer as a simplified surd.

(1)

The point X lies on l_2. Given that the vector $\overrightarrow{OX}$ is perpendicular to l_2,

(e) find the length of OX, giving your answer to 3 significant figures.

(3)

Leave blank

Question 25 continued

(Total 10 marks)

Leave blank

26. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (9\mathbf{i} + 13\mathbf{j} - 3\mathbf{k}) + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$$

$$l_2 : \mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

where λ and μ are scalar parameters.

(a) Given that l_1 and l_2 meet, find the position vector of their point of intersection.

(5)

(b) Find the acute angle between l_1 and l_2, giving your answer in degrees to 1 decimal place.

(3)

Given that the point A has position vector $4\mathbf{i} + 16\mathbf{j} - 3\mathbf{k}$ and that the point P lies on l_1 such that AP is perpendicular to l_1,

(c) find the exact coordinates of P.

(6)

Leave blank

Question 26 continued

(Total 14 marks)

Leave blank

27. Relative to a fixed origin O, the point A has position vector $(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$, and the point B has position vector $(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l.

(2)

The point C has position vector $(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$.

The point P lies on l. Given that the vector $\overrightarrow{CP}$ is perpendicular to l,

(c) find the position vector of the point P.

(6)

Leave blank

Question 27 continued

(Total 10 marks)

Leave blank

28. Relative to a fixed origin O, the point A has position vector $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$, the point B has position vector $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$, and the point D has position vector $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l.

(2)

(c) Show that the size of the angle BAD is 109°, to the nearest degree.

(4)

The points A, B and D, together with a point C, are the vertices of the parallelogram $ABCD$, where $\overrightarrow{AB} = \overrightarrow{DC}$.

(d) Find the position vector of C.

(2)

(e) Find the area of the parallelogram $ABCD$, giving your answer to 3 significant figures.

(3)

(f) Find the shortest distance from the point D to the line l, giving your answer to 3 significant figures.

(2)

Leave blank

Question 28 continued

(Total 15 marks)

Leave blank

29. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection A.

(6)

(b) Find, to the nearest $0.1°$, the acute angle between l_1 and l_2.

(3)

The point B has position vector $\begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$.

(c) Show that B lies on l_1.

(1)

(d) Find the shortest distance from B to the line l_2, giving your answer to 3 significant figures.

(4)

Leave blank

Question 29 continued

(Total 14 marks)

Leave blank

30. Relative to a fixed origin O, the point A has position vector $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and the point B has position vector $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The points A and B lie on a straight line l.

(a) Find $\overrightarrow{AB}$.

(2)

(b) Find a vector equation of l.

(2)

The point C has position vector $2\mathbf{i} + p\mathbf{j} - 4\mathbf{k}$ with respect to O, where p is a constant. Given that AC is perpendicular to l, find

(c) the value of p,

(4)

(d) the distance AC.

(2)

Leave blank

Question 30 continued

(Total 10 marks)

EXPERT TUITION

Leave blank

31. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C, find

(a) the coordinates of C.

(3)

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$.

(b) Find the size of the angle ACB. Give your answer in degrees to 2 decimal places.

(4)

(c) Hence, or otherwise, find the area of the triangle ABC.

(5)

Leave blank

Question 31 continued

(Total 12 marks)

Leave blank

32. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point A and the acute angle between l_1 and l_2 is θ.

(a) Write down the coordinates of A.

(1)

(b) Find the value of $\cos\theta$.

(3)

The point X lies on l_1 where $\lambda = 4$.

(c) Find the coordinates of X.

(1)

(d) Find the vector $\overrightarrow{AX}$.

(2)

(e) Hence, or otherwise, show that $|\overrightarrow{AX}| = 4\sqrt{26}$.

(2)

The point Y lies on l_2. Given that the vector $\overrightarrow{YX}$ is perpendicular to l_1,

(f) find the length of AY, giving your answer to 3 significant figures.

(3)

Leave blank

Question 32 continued

(Total 12 marks)

Leave blank

33. Relative to a fixed origin O, the point A has position vector $(8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k})$, the point B has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point C has position vector $(9\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$.

The line l passes through the points A and B.

(a) Find a vector equation for the line l.

(3)

(b) Find $|\overrightarrow{CB}|$.

(2)

(c) Find the size of the acute angle between the line segment CB and the line l, giving your answer in degrees to 1 decimal place.

(3)

(d) Find the shortest distance from the point C to the line l.

(3)

The point X lies on l. Given that the vector $\overrightarrow{CX}$ is perpendicular to l,

(e) find the area of the triangle CXB, giving your answer to 3 significant figures.

(3)

Leave blank

Question 33 continued

(Total 14 marks)

Leave blank

34. With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where λ and μ are parameters and p and q are constants. Given that l_1 and l_2 are perpendicular,

(a) show that $q = -3$.

(2)

Given further that l_1 and l_2 intersect, find

(b) the value of p,

(6)

(c) the coordinates of the point of intersection.

(2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point C lies on l_2.

Given that a circle, with centre C, cuts the line l_1 at the points A and B,

(d) find the position vector of B.

(3)

Leave blank

Question 34 continued

(Total 13 marks)

Leave blank

35. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2: \quad \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection.
(6)

(b) Show that l_1 and l_2 are perpendicular to each other.
(2)

The point A has position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

(c) Show that A lies on l_1.
(1)

The point B is the image of A after reflection in the line l_2.

(d) Find the position vector of B.
(3)

Leave blank

Question 35 continued

(Total 12 marks)

Leave blank

36. The points A and B have position vectors $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ respectively.

The line l_1 passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l_1.

(2)

A second line l_2 passes through the origin and is parallel to the vector $\mathbf{i} + \mathbf{k}$. The line l_1 meets the line l_2 at the point C.

(c) Find the acute angle between l_1 and l_2.

(3)

(d) Find the position vector of the point C.

(4)

Leave blank

Question 36 continued

(Total 11 marks)