OCR
Oxford Cambridge and RSA

Wednesday 07 October 2020 – Afternoon

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

8166139396

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Answer **all** the questions.

1 Celia states that $n^2 + 2n + 10$ is always odd when n is a prime number.

Prove that Celia's statement is false. **[2]**

2 Fig. 2 shows a quadrilateral ABCD. The lengths AB and BC are 5 cm and 6 cm respectively. The angles ABC, ACD and DAC are 60°, 60° and 75° respectively.

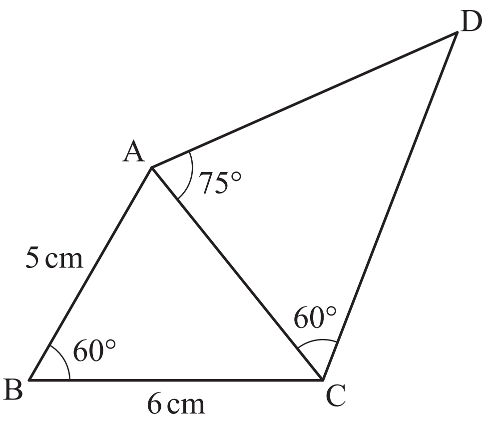

Fig. 2

Calculate the exact value of the length AD. **[4]**

3 Fig. 3 shows a triangle PQR. The vector $\overrightarrow{PQ}$ is $\mathbf{i} + 7\mathbf{j}$ and the vector $\overrightarrow{QR}$ is $4\mathbf{i} - 12\mathbf{j}$.

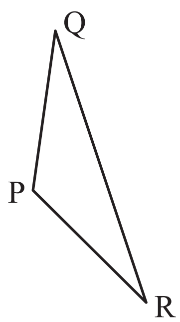

Fig. 3

(a) Show that the triangle PQR is isosceles. **[3]**

The point P has position vector $-3\mathbf{i} - \mathbf{j}$. The point S is added so that PQRS is a parallelogram.

(b) Find the position vector of S. **[2]**

4 In this question, the x and y directions are horizontal and vertically upwards respectively.

A particle of mass 1.5 kg is in equilibrium under the action of its weight and forces $\mathbf{F}_1 = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ N and $\mathbf{F}_2$.

(a) Find the force $\mathbf{F}_2$. **[3]**

The force $\mathbf{F}_2$ is changed to $\begin{pmatrix} 2 \\ 20 \end{pmatrix}$ N.

(b) Find the acceleration of the particle. **[2]**

5 Fig. 5.1 shows part of the curve $y = x^{\frac{1}{2}}$. P is the point (1, 1) and Q is the point on the curve with x-coordinate $1 + h$.

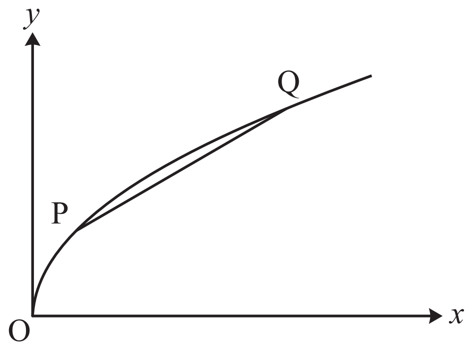

Fig. 5.1

Table 5.2 shows, for different values of h, the coordinates of P, the coordinates of Q, the change in y from P to Q and the gradient of the chord PQ.

x for P	y for P	h	x for Q	y for Q	change in y	gradient PQ
1	1	1				
1	1	0.1	1.1	1.048 809	0.048 809	0.488 088
1	1	0.01	1.01	1.004 988	0.004 988	0.498 756
1	1	0.001	1.001	1.000 500	0.000 500	0.499 875

Table 5.2

(a) Fill in the missing values for the case $h = 1$ in the copy of Table 5.2 in the Printed Answer Booklet. Give your answers correct to 6 decimal places where necessary. **[1]**

(b) Explain how the sequence of values in the last column of Table 5.2 relates to the gradient of the curve $y = x^{\frac{1}{2}}$ at the point P. **[1]**

(c) Use calculus to find the gradient of the curve at the point P. **[2]**

6 **In this question you must show detailed reasoning.**

A particle moves in a straight line. Its velocity $v\,\mathrm{m\,s^{-1}}$ after t s is given by $v = t^3 - 5t^2$.

(a) Find the times at which the particle is stationary. **[2]**

(b) Find the total distance travelled by the particle in the first 6 seconds. **[3]**

7 **In this question you must show detailed reasoning.**

A curve has equation $y = 4x^3 - 6x^2 - 9x + 4$.

(a) Sketch the gradient function for this curve, clearly indicating the points where the gradient is zero. **[4]**

(b) Find the set of values of x for which the gradient function is decreasing. Give your answer using set notation. **[2]**

8 The point A has coordinates $(-1, -2)$ and the point B has coordinates $(7, 4)$. The perpendicular bisector of AB intersects the line $y + 2x = k$ at P.

Determine the coordinates of P in terms of k. **[7]**

9 A car travelling in a straight line accelerates uniformly from rest to $V\,\mathrm{m\,s^{-1}}$ in T s. It then slows down uniformly, coming to rest after a further $2T$ s.

(a) Sketch a velocity-time graph for the car. **[2]**

The acceleration in the first stage of the motion is $2.5\,\mathrm{m\,s^{-2}}$ and the total distance travelled is $240\,\mathrm{m}$.

(b) Calculate the values of V and T. **[4]**

10 An astronaut on the surface of the moon drops a ball from a point 2 m above the surface.

(a) Without any calculations, explain why a standard model using $g = 9.8\,\mathrm{m\,s^{-2}}$ will not be appropriate to model the fall of the ball. **[1]**

The ball takes 1.6 s to hit the surface.

(b) Find the acceleration of the ball which best models its motion. Give your answer correct to 2 significant figures. **[2]**

(c) Use this value to predict the maximum height of the ball above the point of projection when thrown vertically upwards with an initial velocity of $15\,\mathrm{m\,s^{-1}}$. **[2]**

11 **In this question you must show detailed reasoning.**

(a) A student is asked to solve the inequality $x^{\frac{1}{2}} < 4$.

The student argues that $x^{\frac{1}{2}} < 4 \Leftrightarrow x < 16$, so that the solution is $\{x : x < 16\}$.

Comment on the validity of the student's argument. **[1]**

(b) Solve the inequality $\left(\frac{1}{2}\right)^x < 4$. **[3]**

(c) Show that the equation $2\log_2(x+8) - \log_2(x+6) = 3$ has only one root. **[5]**

12 In this question you must show detailed reasoning.

Fig. 12 shows part of the graph of $y = x^2 + \frac{1}{x^2}$.

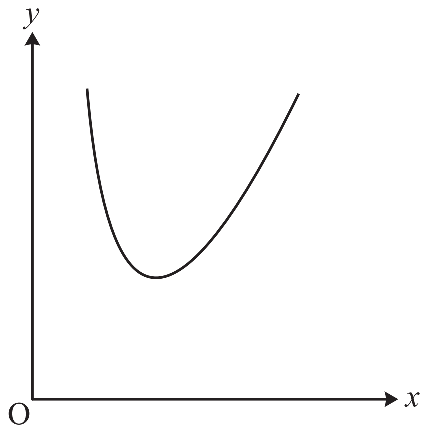

Fig. 12

The tangent to the curve $y = x^2 + \frac{1}{x^2}$ at the point $(2, \frac{17}{4})$ meets the x-axis at A and meets the y-axis at B. O is the origin.

(a) Find the exact area of the triangle OAB. **[6]**

(b) Use calculus to prove that the complete curve has two minimum points and no maximum point. **[6]**

END OF QUESTION PAPER