GCE A LEVEL MARKING SCHEME

SUMMER 2019

A LEVEL (NEW)
MATHEMATICS
UNIT 3 PURE MATHEMATICS B
1300U30-1

INTRODUCTION

This marking scheme was used by WJEC for the 2019 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE MATHEMATICS

A2 UNIT 3 PURE MATHEMATICS B

SUMMER 2019 MARK SCHEME

Q	Solution	Mark	Notes
1(a)	$\frac{9}{(x-1)(x+2)^2} \equiv \frac{A}{(x-1)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$	M1	
	$9 \equiv A(x+2)^2 + B(x-1)(x+2) + C(x-1)$	m1	correct method to remove denominator
	Put $x = 1$, $A = 1$	m1	correct method for finding A, B or C
	Coef. x^2, $0 = A + B$, $B = -1$		
	Put $x = -2$, $C = -3$	A1	all 3 values correct, cao
1(b)	$\int \frac{9}{(x-1)(x+2)^2}\mathrm{d}x$		
	$= \int \frac{1}{(x-1)}\mathrm{d}x - \int \frac{1}{(x+2)}\mathrm{d}x - \int \frac{3}{(x+2)^2}\mathrm{d}x$	M1	attempt to integrate PF
	$= \ln\lvert x-1\rvert - \ln\lvert x+2\rvert + \frac{3}{(x+2)} + Const$	A1	one correct term, ft (a)
		A1	all correct, -1 if no *Const.* ft(a), isw

<u>Note</u>: ft incorrect constants or $\frac{A}{(x-1)} + \frac{Bx+C}{(x+2)^2}$.

Q	Solution	Mark	Notes
2	$(4-x)(1+2x)^{-\frac{1}{2}}$		
	$= (4-x)\left[1+\left(-\frac{1}{2}\right)(2x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(2x)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3\times 2}(2x)^3\right]$		
		B2	-1 each error term $(4-x)$ not required.
	$=(4-x)[1-x+\frac{3}{2}x^2 - \frac{5}{2}x^3 + \ldots..]$		
	$= (4-4x+6x^2-10x^3)-(x - x^2+\frac{3}{2}x^3)+\ldots$	M1	correct method
	$= 4-5x+7x^2 - \frac{23}{2}x^3 + \ldots\ldots.$	A2	terms all correct -1 each incorrect term. Ignore further terms., isw
	Expansion is valid for $\lvert x\rvert < \frac{1}{2}$.	B1	oe, mark final answer
<u>Note</u>	$(1+2x)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)(2x)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(2x)^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3\times 2}(2x)^3+\ldots..$		
	$= 1+x - \frac{1}{2}x^2+\frac{1}{2}x^3+\ldots.$	(B0)	
	$(4-x)(1+2x)^{\frac{1}{2}} = 4+3x-3x^2+\frac{5}{2}x^3+\ldots.$	(M1)	correct method
		(A2)	terms all correct -1 each incorrect term. Ignore further terms.
	Expansion is valid for $\lvert x\rvert < \frac{1}{2}$.	(B1)	

Q	Solution	Mark	Notes
3(a)	$x_2 = 29$	B1	
	$x_1 = 8$	B1	ft 1 slip
3(b)	Not an AP because $113 - 29 \neq 29 - 8$		
	$x_3 - x_2 \neq x_2 - x_1$.	B1	either GP or AP statement, reason required, ft (a)
	Not a GP because $\frac{113}{29} \neq \frac{29}{8}$		
	$\frac{x_3}{x_2} \neq \frac{x_2}{x_1}$.	B1	for the other statement, Reason required, ft (a)

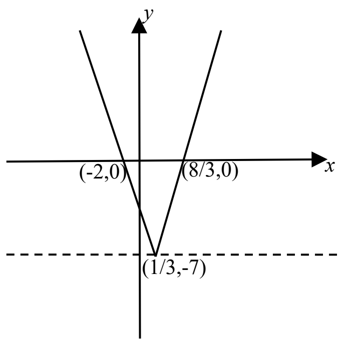
Note : Accept equivalent statement if clear.

Q	Solution	Mark	Notes
4(a)	$5\sin x - 12\cos x = R\sin x\cos\alpha - R\cos x\sin\alpha$		
	$R\cos\alpha = 5$		
	$R\sin\alpha = 12$	M1	
	$R = \sqrt{5^2 + 12^2} = 13$	B1	
	$\alpha = \tan^{-1}\left(\frac{12}{5}\right) = 67.380°$	A1	accept 1.176 rad not 1.176.
4(b)	$y = \frac{4}{13\sin(x - 67\cdot380) + 15}$		
	Min y when denominator is max,		
	ie when $\sin(x - 67.380) = 1$	M1	implied by correct min
	Min $y = \frac{4}{28}$ $(= \frac{1}{7} = 0.1429)$	A1	ft R
4(c)	$\sin(x - 67.380) = -\frac{3}{13}$	M1	
	$x - 67.380 = -13.342, 193.342$	A1	either value
	$x = 54.04°$	A1	0.943 rad
	$x = 260.72°$	A1	4.550 rad
			Ignore answers outside the range.
			-1 for any extra answers within range
			Accept answers rounding correctly to 54, 261.

Note: full follow through their R and α provided of equivalent difficulty, eg not $\alpha = 0$, $R=1$.

Q	Solution	Mark	Notes
5(a)	either $1 - 3x > 7$, or $1 - 3x < -7$	M1	one correct inequality
		A1	both correct
	either $x < -2$, or $x > \frac{8}{3}$	A1	cao oe, mark final answer
OR			
	$(1 - 3x)^2 > 7^2$	(M1)	
	$9x^2 - 6x - 48 > 0$	(A1)	
	$3x^2 - 2x - 16 > 0$		
	$(3x - 8)(x + 2) > 0$		
	either $x < -2$, or $x > \frac{8}{3}$	(A1)	cao oe

5(b)



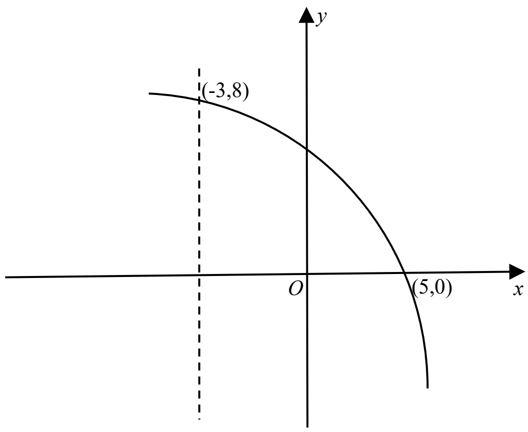
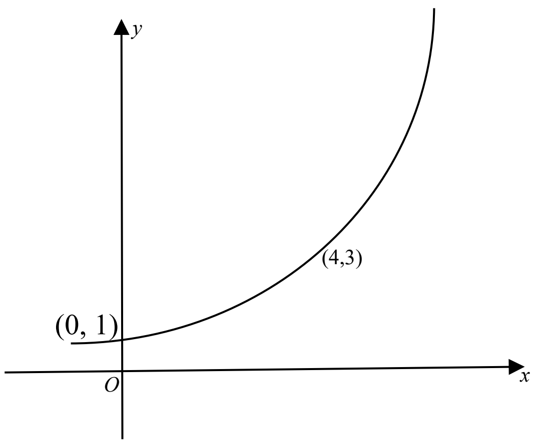
		G1	Shape, min below x-axis
		G1	(-2, 0), (8/3, 0), ft (a)
		B2	(1/3, -7) and drawn in correct quadrant. B1 for either coordinate.

Q	Solution	Mark	Notes
6(a)	$\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = \cos\theta$	B1	si
	$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = -2\sin 2\theta$	B1	si
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2\sin 2\theta}{\cos\theta}$	M1	
	When $\theta = \dfrac{\pi}{4}$, $x = \dfrac{1}{\sqrt{2}}$, $y = 0$	B1	si
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2\sin\frac{\pi}{2}}{\cos\frac{\pi}{4}} = -2\sqrt{2}$	A1	$m = -2\sqrt{2}$, oe
			ft $\mathrm{d}y/\mathrm{d}\theta = 2\sin 2\theta$ or $-\sin 2\theta$
	Eq^n of tgt is $y - 0 = -2\sqrt{2}\,(x - \dfrac{1}{\sqrt{2}})$		
	Eq^n of tgt is $y = -2\sqrt{2}\,x + 2$	A1	$c = 2$ cao
OR	$y = \cos 2\theta = 1 - 2\sin^2\theta$	(M1)	
	$y = 1 - 2x^2$	(A1)	
	$\dfrac{dy}{dx} = -4x$	(B1)	ft $2\sin^2\theta - 1$
	When $\theta = \dfrac{\pi}{4}$, $x = \dfrac{1}{\sqrt{2}}$, $y = 0$	(B1)	si
	$\dfrac{dy}{dx} = -4 \times \dfrac{1}{\sqrt{2}} = -2\sqrt{2}$	(B1)	ft $2\sin^2\theta - 1$
	Eq^n of tgt is $y - 0 = -2\sqrt{2}\,(x - \dfrac{1}{\sqrt{2}})$		
	Eq^n of tgt is $y = -2\sqrt{2}\,x + 2$	(A1)	$c = 2$, cao

Q	Solution	Mark	Notes	
6(b)	$x + y = 1$			
	$\cos 2\theta + \sin\theta - 1 = 0$	M1		
	$1 - 2\sin^2\theta + \sin\theta - 1 = 0$	M1	$\cos 2\theta = 1 - 2\sin^2\theta$	
	$2\sin^2\theta - \sin\theta = 0$			
	$\sin\theta(2\sin\theta - 1) = 0$	m1	si, ft for correct factorisation	
	$x = \sin\theta = 0,\ \frac{1}{2}$	A1	one correct pair	cao
	$y = 1 - x = 1,\ \frac{1}{2}$	A1	all correct	cao
	required coordinates are (0, 1), $\left(\frac{1}{2}, \frac{1}{2}\right)$			

OR

	$y = \cos 2\theta = 1 - 2\sin^2\theta$	(M1)	$\cos 2\theta = 1 - 2\sin^2\theta$
	$y = 1 - 2x^2$	(m1)	$x = \sin\theta$
	$y = 1 - x$		
	Solving simultaneously	(m1)	
	$x(2x - 1) = 0$		
	$x = 0, x = \frac{1}{2}$	(A1)	one correct pair
	$y = 0, y = \frac{1}{2}$	(A1)	all correct
	required coordinates are (0, 1), $\left(\frac{1}{2}, \frac{1}{2}\right)$		

Q	Solution	Mark	Notes
7(a)		G1	shape of graph
		B1	(5,0)
		B1	(-3, 8)
7(b)		G1	shape, intersecting y-axis at a positive value of y.
		B1	(4, 3)
			(0, 1) not required.

Q	Solution	Mark	Notes
8(a)	$T_3 = 3 + 2d$	B1	T_3, T_{19} or T_{67} correct
	$T_{19} = 3 + 18d$		
	$T_{67} = 3 + 66d$	B1	all correct
	$\frac{3+66d}{3+18d} = \frac{3+18d}{3+2d}$ $(= r)$	M1	method for d or r
	$(3+66d)(3+2d) = (3+18d)(3+18d)$	m1	method for d
	$9 + 204d + 132d^2 = 9 + 108d + 324d^2$		
	$192d^2 = 96d$		
	$d = \frac{1}{2}$	A1	cao, condone presence of $d = 0$
8(b)(i)	AP $a = 100$, $d = 12$	M1	si
	8 weeks = 40 working days.		
	Total no. employees = $100 + 39 \times 12$	m1	
	Total no. employees = 568	A1	
8(b)(ii)	Wage bill =		
	$55[100 + 112 + 124 + \ldots(40 \text{ terms})]$	M1	55 not required, implied by 13360
	Wage bill = $55\left[\frac{40}{2}(2\times100+39\times12)\right]$	m1	
	Wage bill = $55\left[\frac{40}{2}(100+568)\right]$	(m1)	ft (b)(i)
	Wage bill = (£)734 800	A1	cao

Q	Solution	Mark	Notes
9(a)	$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$		
	$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - 2\cot\beta\tan\beta}$	M1	$\tan\alpha\tan\beta=2$
	$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - 2}$		
	$\tan(\alpha+\beta) = -(\tan\alpha + \tan\beta)$	A1	convincing
9(b)	$4\tan\theta = 3(1 + \tan^2\theta) - 7$	M1	$\sec^2\theta = 1 + \tan^2\theta$
	$3\tan^2\theta - 4\tan\theta - 4 = 0$	A1	
	$(3\tan\theta + 2)(\tan\theta - 2) = 0$	m1	allow $(3\tan\theta - 2)(\tan\theta + 2)$
	$\tan\theta = -\dfrac{2}{3}, 2$	A1	cao
	<u>Note :</u> No working shown m0 A0		
	$\theta = 63.4°, 243.4°$	B1	ft tan value, -1 each extra value in range
	$\theta = 146.3°, 326.3°$	B1	ft tan value if different sign. -1 each extra value in range

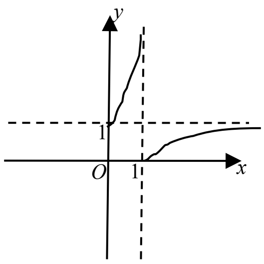
<u>Note</u> : Do not ft for other trig functions.

Q	Solution	Mark	Notes
10a(i)	Use of product rule	M1	$x^5\mathrm{f}(x)+\mathrm{g}(x)\ln x$
	$x^5\times\dfrac{1}{x}+5x^4\ln x$	A1	$\mathrm{f}(x)=\dfrac{1}{x}$
		A1	$\mathrm{g}(x)=5x^4$ isw
10a(ii)	Use of quotient rule	M1	$\dfrac{(x^3-1)\mathrm{f}(x) - e^{3x}\mathrm{g}(x)}{(x^3-1)^2}$
	$\dfrac{(x^3-1)3e^{3x} - e^{3x}(3x^2)}{(x^3-1)^2}$	A1	$\mathrm{f}(x)=3e^{3x}$
		A1	$\mathrm{g}(x)=3x^2$ isw
10a(iii)	Use of chain rule	M1	$\dfrac{1}{2}(\tan x + 7x)^{-1/2}\mathrm{f}(x)$
	$\dfrac{1}{2}(\tan x + 7x)^{-1/2}(\sec^2 x + 7)$	A1	$\mathrm{f}(x)= (\sec^2 x + 7)$ isw

<u>Note</u> : $\mathrm{f}(x)$, $\mathrm{g}(x) \neq 0$ or 1.

Q	Solution	Mark	Notes
10(b)	$3\dfrac{\mathrm{d}y}{\mathrm{d}x} + 4y^2 + 8xy\dfrac{\mathrm{d}y}{\mathrm{d}x} - 15x^2 = 0$	B1	$3\dfrac{\mathrm{d}y}{\mathrm{d}x} - 15x^2$, 0
		B1	$4y^2 + 8xy\dfrac{\mathrm{d}y}{\mathrm{d}x}$
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{15x^2-4y^2}{3+8xy}$	B1	correct $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ cao

Q	Solution	Mark	Notes
11(a)	$y = \dfrac{\sqrt{x^2 - 1}}{x}$	M1	
	$x^2y^2 = x^2 - 1$		
	$x^2(1 - y^2) = 1$		
	$x = \pm\dfrac{1}{\sqrt{1-y^2}}$,	A1	
	$f^{-1}(x) = \dfrac{1}{\sqrt{1-x^2}}$, +ve since $x \geq 1$	A1	ft above for similar expression
	Domain [0, 1)	B1	



		G1	for $f(x)$ starting at (1,0) with horizontal asymptote y=1 Or for $f^{-1}(x)$ starting at (0,1) with vertical asymptote x=1 (does not need to be shown)
		G1	reflection in $y = x$, provided curve passes through (1,0) or (0,1)
11(b)	$ff(x)$ cannot be formed because the range of $f(x)$ is not in the domain of $f(x)$.	E1	oe eg consideration of a single point.

Q	Solution	Mark	Notes
12(a)	Area of sector $OAB = \frac{1}{2}r^2\theta$		
	Area triangle $OAB = \frac{1}{2}r^2\sin\theta$	B1	either si
	Area of segment = $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$	B1	si
	$3(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta) = \pi r^2$	M1	oe
	$\sin\theta = \theta - \frac{2\pi}{3}$	A1	convincing
12(b)(i)	$f(\theta) = \theta - \sin\theta - \frac{2\pi}{3}$		
	$f(2.6) = -0.00989647\ldots < 0$	M1	
	$f(2.7) = 0.178225\ldots > 0$		
	Change of sign, therefore $2.6 < \theta < 2.7$	A1	
12(b)(ii)	$f'(\theta) = 1 - \cos\theta$	B1	
	$\theta_{n+1} = \theta_n - \dfrac{\theta_n - \sin\theta_n - \frac{2\pi}{3}}{1 - \cos\theta_n}$	M1	si
	$\theta_0 = 2.6$		
	$\theta_1 = 2.6053296$	A1	1st iteration correct si ft $f'(\theta) = 1 + \cos\theta$ only (2.6691…)
	$\theta_2 = 2.605325675$		
	$\theta = 2.605$ (correct to 3 d.p.)	A1	cao

Note: No marks for unsupported answer of 2.605.

$1 + \cos\theta$ in denominator, series is divergent.

Q	Solution	Mark	Notes
13(a)	$\frac{\mathrm{d}A}{\mathrm{d}t} = kA$	B1	
13(b)	$\int \frac{dA}{A} = \int k dt$	M1	separate variables
	$\ln A = kt + (C)$	A1	
	$t = 0$, $A = 0.2$	m1	use of initial conditions
	$C = \ln 0.2$		
	$\ln \frac{A}{0 \cdot 2} = kt$		
	$t = 1$, $A = 1.48$	m1	used
	$k = \ln(7.4) = 2.00148$		
	$e^k = 7.4$	A1	either k or e^k
	$(A =)\ 0.2e^{kt}$	A1	$k = 2$, 2.00148, $\ln(7.4)$
	$(A =)\ 0.2(7.4)^t$	A1	cao

Q	Solution	Mark	Notes
14(a)	$\frac{1}{2}e^{2x} - 2\cos 3x + \mathrm{C}$	B1	one correct term
		B1	second correct term
			-1 if no $+C$.
14(b)	$(x^2 + \sin x)^7 + \mathrm{C}$	B1	-1 if no $+C$(only once) .
14(c)	$\mathrm{I}=\int x^{-2}\ln x\,\mathrm{d}x = \left[\frac{x^{-1}}{-1}\ln x\right] - \int -x^{-1}\times\frac{1}{x}\mathrm{d}x$	M1	$\mathrm{f}(x)\ln x - \int \mathrm{f}(x)\frac{1}{x}\mathrm{d}x$
		A1	1st term
		A1	2nd term
	$\mathrm{I} = -\frac{1}{x}\ln x + \int x^{-2}\,\mathrm{d}x$		
	$\mathrm{I} = -\frac{1}{x}\ln x - \frac{1}{x} + \mathrm{C}$	A1	-1 if no $+C$ (only once)
14(d)	$u = 2\cos x + 1$; $\quad \mathrm{d}u = -2\sin x\,\mathrm{d}x$		
	$x = 0, u = 3$; $\quad x = \frac{\pi}{3}, u = 2$		
	$\mathrm{I} = \int_3^2 -\frac{1}{2u^2}\mathrm{d}u = \frac{1}{2}\int_2^3 u^{-2}\mathrm{d}u$	M1	integrand au^{-2}
	$\mathrm{I} = \frac{1}{2}\left[-\frac{1}{u}\right]_2^3$	A1	correct integration of u^{-2}
	$\mathrm{I} = \frac{1}{2}\left[\frac{1}{2}-\frac{1}{3}\right]$	m1	correct use of correct limits
	$\mathrm{I} = \frac{1}{12}$	A1	cao

Note: No marks for unsupported answer of 1/12.

Q	Solution	Mark	Notes
15	Assume that $\sqrt{6}$ is rational.	M1	
	Then there are (integers) a and b,		
	with no common factor (except 1)		
	such that $\sqrt{6} = \dfrac{a}{b}$	m1	
OR			
	Assume $\sqrt{6} = \dfrac{a}{b}$, where a and b,		
	are integers.	(M1)	
	a and b have no common factor (except 1).	(m1)	
THEN			
	Square both sides, $6 = \dfrac{a^2}{b^2}$		
	$6b^2 = a^2$		
	So (a^2 and thus)a is an even number,	A1	Dep on M1
	$(a = 2k,)$		
	$6b^2 = a^2 = (2k)^2 = 4k^2$		
	$3b^2 = 2k^2$		
	So (b^2 and thus)b is an even number.	A1	Dep on M1
	$(b = 2h)$		
	So, a and b have a common factor 2.		
	This is a contradiction.		
	Hence $\sqrt{6}$ is irrational.	A1	cso
OR			
	$6b^2 = a^2$		
	So (a^2 and thus)a has a factor of 6, $a = 6k$	(A1)	Dep on M1
	$6b^2 = a^2 = (6k)^2 = 36k^2$		
	$b^2 = 6k^2$		
	So (b^2 and thus)b has a factor of 6, $b = 6h$	(A1)	Dep on M1
	So, a and b have a common factor 6.		
	This is a contradiction.		
	Hence $\sqrt{6}$ is irrational.	(A1)	cso

Note: Also accept factor of 3.