GCE A LEVEL MARKING SCHEME

SUMMER 2022

A LEVEL (NEW)
MATHEMATICS
UNIT 4 APPLIED MATHEMATICS B
1300U40-1

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE A LEVEL MATHEMATICS

UNIT 4 APPLIED MATHEMATICS B

SUMMER 2022 MARK SCHEME

SECTION A – Statistics

Qu. No.	Solution	Mark	Notes
1	Mark for selection to stage three $= 66 + k \times 14$	M1	k = 1.645 or better M1 implied by correct answer from calculator. Allow M1 for $\frac{x-66}{14} = 1.645$.
	$= 89.03$	A1	A1 for sight of either value Condone sight of 89.
	Mark for non-selection $= 66 - k \times 14$	(M1)	M1 may be awarded here if not previously awarded. Allow M1 for $\frac{x-66}{14} = -1.645$.
	$= 42.97$	(A1)	A1 for sight of either value Condone sight of 43.
	Candidates can obtain scores between 43 and 89 in order to be selected for stage two of the interview process.	A1	Must be a range. Accept 42.97 to 89.03 Allow calculation of range between highest and lowest scores. Correct answer only scores M1A1A1 SC1 for 44 to 88 from use of 1.64.
	Total for Question 1	**3**	

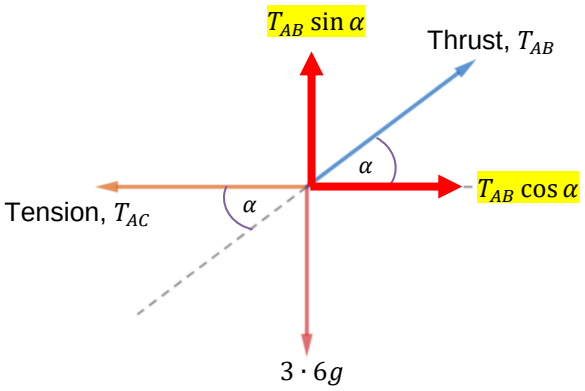
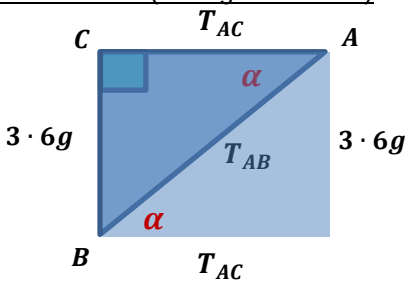
Qu. No.	Solution	Mark	Notes
2(a)	$P(F1) = 0.4 \times 0.70 + 0.35 \times 0.30$	M1	Allow one slip.
	$= 0.385$	A1	
	Part (a) Total	**[2]**	
(b)	$P(C \mid F1) = \frac{P(C \cap F1)}{P(F1)} = \frac{0.4 \times 0.7}{0.385}$	M1	FT their (a) provided it gives a valid probability as the final answer. If $P(F1) = 1$, must see division by 1.
	$= \frac{8}{11}$ or $0.\dot{7}\dot{2}$	A1	CAO (3sf required) Condone 0.73 from correct working
	Part (b) Total	**[2]**	
(c)	$P(F2') = 0.4 \times 0.8 + 0.35 \times 0.95 + 0.25 \times 0.15$	M1	si Allow one slip OR for $P(F2') = 1 - P(F2)$ with at most one slip in $P(F2)$ calculation
	$P(G \mid F2') = \frac{P(G \cap F2')}{P(F2')} = \frac{0.35 \times 0.95}{0.4 \times 0.8 + 0.35 \times 0.95 + 0.25 \times 0.15}$	M1	Correct numerator (calculation or sight of 0.3325). May be seen as $0.35 \times 0.3 + 0.35 \times 0.65$. (Must be part of fraction)
		m1	Dependent on first M1. Correct denominator (calculation or sight of 0.69) (Must be part of a fraction)
	$= \frac{133}{276}$ or 0.481884….	A1	CAO (3sf required) Condone 0.48 from correct working
	Part (c) Total	**[4]**	
	Total for Question 2	**8**	

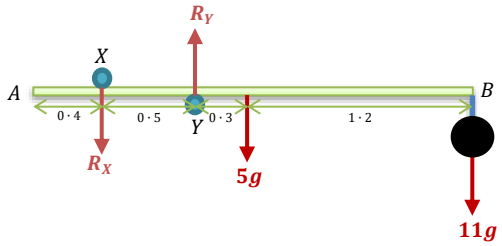
Qu. No.	Solution	Mark	Notes
3(a)	$X \sim \mathrm{U}(0,10)$	M1	Seen, or implied by correct values or calculation of $E(X)$ and $\mathrm{Var}(X)$
	$E(X) = 5$	A1	Must be from correct distribution
	$\mathrm{Var}(X) = \frac{25}{3}$	A1	Must be from correct distribution Condone 8.33 (condone 8.3)
			If no marks awarded, SC1 for $E(X)$ and $\mathrm{Var}(X)$ correct for their uniform distribution (stated or implied, e.g., implied by a diagram or for consistent use of a and b in the mean and variance formulae) e.g., SC1 for $E(X) = 10$ and $\mathrm{Var}(X) = \frac{100}{3}$.
	Part (a) Total	**[3]**	
(b)	$A = X(20 - X)$ $= 20X - X^2$	M1	Stating the area of the rectangle or consideration of relevant products.
	$P(20X - X^2 > 96) = P(X^2 - 20X + 96 < 0)$	M1	Forming a quadratic inequality or equation. Condone omission of $P()$ Stating $X > 8$ with no incorrect working scores M1M1.
	$= P(8 < X < 12)$	A1	Solving quadratic inequality or equation, may be implied by next A1 Condone $P(X > 8)$ if using correct distribution
	$= P(8 < X < 10)$	A1	si (may be implied by a diagram) FT if equivalent difficulty for restricting their range of values for X
	$= \frac{2}{10}$	A1	CSO (correct solution only)
			SC3 (M1M1A1A0A0) for 0.2 from $X \sim U(0,20)$ or with no working.
	Part (b) Total	**[5]**	
	Total for Question 3	**8**	

Qu. No.	Solution	Mark	Notes
4(a)	(Let the random variable W be the stopping distance in metres of a car travelling at 30mph.) $W \sim \mathrm{N}(23, 3.8^2)$ $\mathrm{P}(W < 30) = 0.96727$	M1A1	M1 implied by correct answer from calculator or for correctly standardising $Z = \frac{30-23}{3.8} = 1.84$. Gives 0.96712 from tables. 3sf required (0.97 earns M1A0).
	Part (a) Total	**[2]**	
(b)	(Let the random variable X be the stopping distance in metres of a car travelling at 20mph.) $X \sim \mathrm{N}(12, 3.5^2)$ $\mathrm{P}(X > 20) = 0.011135$	M1	Either method correct (see note above). M1 for 2.29 or -0.79 if standardised.
	$\mathrm{P}(W > 20) = 0.78508$	A1	A1 for both probabilities, with at least one probability to 3sf.
	$\frac{0.78508}{0.011135}$	M1	Alternatively, $\frac{0.78508}{50}$ or 0.011135×50. FT their probabilities. Condone division of $P(X < 20)$ by $P(W < 20)$ that leads to 4.6.
	Appropriate conclusion with a valid justification, e.g., You're about 70 times more likely to collide travelling at 30mph than 20mph, so Dafydd is incorrect.	A1	Allow e.g., $0.0157016 \neq 0.011135$ or $0.55675 \neq 0.78508$ so Dafydd is incorrect. FT their calculation for possible M1A1.
	Part (b) Total	**[4]**	
(c)	(Let μ be the population mean stopping distances for cars travelling at 30mph) $H_0: \mu = 23$ $\quad$ $H_1: \mu < 23$	B1	Allow other letters if defined. Allow worded hypotheses. B0 for $H_0: \mathrm{mean} = 23$, must imply or refer to population. B0 for omission of μ or use of $\bar{x}$ B0 for a non-strict inequality in H_1.
	$\bar{X} \sim \mathrm{N}\left(23, \frac{3.8^2}{40}\right)$ under H_0	B1	Distribution of $\bar{X}$ si (condone if used correctly). FT their hypotheses for 2nd B1 only
	$\mathrm{P}(\bar{X} < 21.5 \mid H_0)$	M1	M1 for $\mathrm{P}\left(Z < \frac{21.5-23}{\frac{3.8}{\sqrt{40}}}\right) = \mathrm{P}(Z < -2.50)$, -2.50 scores M1 only if using the p-value method
	$= 0.0062706$ ...	A1	0.00621 from tables, M0A0 for use of 21.5 and 23 the wrong way around.
	Since $0.00627 < 0.01$, there is sufficient evidence to reject H_0.	m1	Dependent on previous M1. FT their p-value. m0 for incorrect comparison such as p-value is in the critical region.
	Alternative 1: CV = 21.602	(M1A1)	M1 implied by correct answer from calculator or for correctly standardising $\frac{\mathrm{CV}-23}{\frac{3.8}{\sqrt{40}}} = -2.3263$
	Since $21.5 < 21.602$, there is sufficient evidence to reject H_0.	(m1)	Dependent on previous M1. FT their CV. m0 for incorrect comparison such as CV is less than significance level.
	Alternative 2: TS $= \frac{21.5-23}{\frac{3.8}{\sqrt{40}}}$	(M1)	
	$= -2.50$	(A1)	-2.50 scores M1A1 if used as a TS
	Since $-2.50 < -2.326$, there is sufficient evidence to reject H_0.	(m1)	Dependent on previous M1. FT their TS. m0 for incorrect comparison such as the TS is less than significance level. Condone accept H_1.
	There is sufficient evidence to suggest that stopping distances are less than previously thought.	A1	CSO (correct solution only). Do not allow categorical statements (condone categorical if "sufficient evidence" seen in m1 statement). Allow equivalent statements, e.g., there is sufficient evidence to support the claim.
	Part (c) Total	**[6]**	
(d)	Valid limitation, e.g. These are likely to be mostly young people which may mean they have a faster reaction time than average.	E1	Must address (young people having) faster reaction times. Condone reference to bias in the sample. Minimum response condoned "Only first year students used" or "Inexperienced drivers". Do not allow reference to sample size. Do not allow reference to driving slower.
	Part (d) Total	**[1]**	
	Total for Question 4	**13**	

Qu. No.	Solution	Mark	Notes
5(a)	(Let ρ denote the population correlation coefficient between average house price and average score in the national reading test). $H_0: \rho = 0$ $\quad$ $H_1: \rho > 0$	B1	Allow other letters if defined. Allow worded hypotheses. B0 for H_0: correlation = 0. Population must be stated or implied. B0 for omission of ρ or use of r B0 for a non-strict inequality in H_1
	TS = 0.86371…	B1	Labelled as TS or used in comparison B0 for $TS = \pm 0.86371$ unless the positive value correctly used later.
	CV = 0.3687	B1	FT their hypotheses (e.g., 0.4329 for two-tailed)
	Since TS > 0.3687, there is sufficient evidence to reject H_0.	B1	FT for using 0.746 FT their CV
	Sufficient evidence to suggest there is **positive** correlation between the average house price and average national reading test score.	E1	CSO (correct solution only). E0 for categorical statements or omission of the word positive (unless positive implied by contextualised comment). E0 for conclusion not in context
	Part (a) Total	**[5]**	
(b)	Valid comment saying the two variables are linked, i.e., giving a reason for the headline. e.g., The data support the idea that the more expensive houses are correlated with better reading scores.	E1	
	Valid comment saying why the headline is unreasonable. e.g., It's unreasonable to suggest that a more expensive house will improve a child's reading ability.	E1	E0 for correlation does not imply causation unless explained in context. Condone responses that give a valid alternative explanation for the correlation.
	Part (b) Total	**[2]**	
(c)	Possible explanation. e.g., parents who can afford better houses may have a better education so are more likely to help their children to read.	E1	E0 for comments such as "those with higher reading scores can afford better houses". Do not accept "small sample size" or "it is a coincidence". Condone a repetition of a valid alternative explanation that was given in (b). Do not condone a valid alternative explanation given in (b) only.
	Part (c) Total	**[1]**	
	Total for Question 5	**8**	

SECTION B – Mechanics

Q6	Solution	Mark	Notes
			$\tan\alpha=\frac{3}{4}$ $\sin\alpha=\frac{3}{5}=0\cdot6$ $\cos\alpha=\frac{4}{5}=0\cdot8$ $3\cdot6g=\frac{882}{25}=35\cdot28$
	Resolving horizontally **OR** vertically	M1	Attempt at resolution to get at least one dim. correct equation with no missing or extra forces
	$T_{AB}\sin\alpha=3\cdot6g$ $\quad(T_{AB}\times0\cdot6=3\cdot6g)$ $T_{AB}\cos\alpha=T_{AC}$ $\quad(T_{AB}\times0\cdot8=T_{AC})$	A1 A1	First correct equation Second correct equation
	$T_{AB}=58\cdot8$ (N) $\quad\left(T_{AB}=\frac{294}{5}=6g\right)$	A1	cso, allow answer rounding to $58\cdot8$ (1dp)
	$T_{AC}=47\cdot04$ (N) $\quad\left(T_{AC}=\frac{1176}{25}=4\cdot8g\right)$	A1 **[5]**	FT their T_{AB} if substituted into a correct equation (if M awarded)
	Alternative Solution (Triangle of forces) 		$\tan\alpha=\frac{3}{4}$ $\sin\alpha=\frac{3}{5}=0\cdot6$ $\cos\alpha=\frac{4}{5}=0\cdot8$ $3\cdot6g=\frac{882}{25}=35\cdot28$
	Evidence of one of the trig. ratios below (Resolving horizontally **OR** vertically)	M1	Attempt at resolution to get at least one dim. correct equation with no missing or extra forces
	$\sin\alpha=\frac{3\cdot6g}{T_{AB}}$ $\quad\cos\alpha=\frac{T_{AC}}{T_{AB}}$ $\quad\tan\alpha=\frac{3\cdot6g}{T_{AC}}$	A1 A1	First correct equation Second correct equation
	$T_{AB}=58\cdot8$ (N) **or** $T_{AC}=47\cdot04$ (N) $\left(T_{AB}=\frac{294}{5}=6g\right)$ $\quad\left(T_{AC}=\frac{1176}{25}=4\cdot8g\right)$	A1	cso, allow answer rounding to $58\cdot8$, allow answer rounding to $47\cdot0$
	$T_{AC}=47\cdot04$ (N) **or** $T_{AB}=58\cdot8$ (N)	A1 **[5]**	FT their T_{AB}/T_{AC} if substituted into a correct equation even if $\alpha=37°$
	Total for Question 6	**5**	

Q7	Solution	Mark	Notes
(a)			
		B1	Any correct moment with pivot clearly indicated
	Moments about X	M1	Dim. correct equation, oe, no extra/missing forces
	$0 \cdot 5R_Y = 0 \cdot 8 \times 5g + 2 \times 11g$	A1	Correct equation
	$0 \cdot 5R_Y = 4g + 22g$		$0 \cdot 5R_Y = 26g$
	$R_Y = 52g$	A1	cao
	Moments about Y	(M1)	Dim. correct equation, oe, no extra/no missing forces
	$0 \cdot 5R_X = 0 \cdot 3 \times 5g + 1 \cdot 5 \times 11g$	(A1)	Correct equation
	$0 \cdot 5R_X = 1 \cdot 5g + 16 \cdot 5g$		$0 \cdot 5R_X = 18g$
	$R_X = 36g$	(A1)	cao
	Resolve vertically	M1	Equation attempted, no extra/missing forces (or 2nd moment equation)
	$R_Y = R_X + 5g + 11g$ $(R_Y = R_X + 16g)$	A1	oe
	$R_X = 36g$ OR $R_Y = 52g$	A1	FT R_X or R_Y
		[7]	
(b)	On the point of turning about Y, $R_X = 0$.	M1	si
	Moments about Y $Mg \times 0 \cdot 9 = 5g \times 0 \cdot 3 + 11g \times 1 \cdot 5$	m1	Equation, no additional forces
	$0 \cdot 9Mg = 1 \cdot 5g + 16 \cdot 5g$ $(0 \cdot 9Mg = 18g)$		
	$M = 20$	A1	cao
		[3]	
	Total for Question 7	**10**	

Q8	Solution	Mark	Notes
(a)	$R = 90g\cos\alpha \quad (= 90g\cos 10° = 868\cdot 600\ldots)$	B1	si
	$F = \frac{2}{9}\times R \quad \left(F = \frac{2}{9}\times 90g\cos\alpha = 20g\cos\alpha\right)$	B1	si
	Apply N2L up slope	M1	Dim. correct, no missing/extra forces,
	$380 - 90g\sin 10° - F = 90a$	A1	
	$380 - 153\cdot 15769\ldots - 193\cdot 02231\ldots = 90a$		$(F = 20g\cos 10° = 193\cdot 022\ldots)$
	$a = 0\cdot 375(7776\ldots) \quad (\text{ms}^{-2})$	A1	cso, allow answers rounding to $0\cdot 38$
		[5]	
(b)	If object remains stationary, component of weight down slope $\leq$ Limiting Friction $90g\sin\alpha \leq F$	M1	si
	$90g\sin\alpha \leq 20g\cos\alpha$	A1	$882\sin\alpha \leq 196\cos\alpha$
	$\alpha_{max} = \tan^{-1}\left(\frac{2}{9}\right)$		$\frac{2}{9} = \frac{196}{882}$
	$= 12\cdot 5(288\ldots)^0$	A1	cao
		[3]	
	Total for Question 8	**8**	

Q9	Solution	Mark	Notes
(a)	$\frac{d\theta}{dt} = -k(\theta + 18)$	B1 **[1]**	oe
(b)	$\int \frac{1}{\theta+18} d\theta = -k \int dt$ $\ln(\theta + 18) = -kt \ (+C)$ When $t = 0$, $\theta = 10$ $C = \ln(28)$ $\quad (k = 3 \cdot 3322 \ldots)$ $kt = \ln(28) - \ln(\theta + 18)$ $kt = \ln\left(\frac{28}{\theta + 18}\right)$	M1 A1 m1 A1 **[4]**	Separating variables Correct integration $\ln\lvert\theta + 18\rvert$ not needed as $\theta > -18$. FT Used Convincing
(c)	Using $t = 1$, $\theta = 6$, in given result $k = \ln\left(\frac{28}{6+18}\right)$ $\quad \left(k = \ln\left(\frac{28}{24}\right) = \ln\left(\frac{7}{6}\right) = 0 \cdot 15415 \ldots\right)$ At $\theta = -5$, $kt = \ln\left(\frac{28}{-5 + 18}\right)$ $t = 4 \cdot 9773 \ldots$ $t = 5$ hours	 M1 m1 A1 **[3]**	 Conditions used Their k sustituted $t = \frac{1}{\ln\left(\frac{7}{6}\right)} \ln\left(\frac{28}{13}\right)$ cao
	Total for Question 9	**8**	

Q10	Solution	Mark	Notes
(a)	Horizontally $t = \dfrac{x}{35\cos\theta}$ Vertically $y = (35\sin\theta)t \pm \frac{1}{2}gt^2$ $y = (35\sin\theta)\left(\dfrac{x}{35\cos\theta}\right) + \dfrac{1}{2}(-9\cdot 8)\left(\dfrac{x}{35\cos\theta}\right)^2$ $y = x\tan\theta - \dfrac{x^2}{250}\sec^2\theta$ $y = x\tan\theta - \dfrac{x^2}{250}(1+\tan^2\theta)$	B1 M1 A1 A1 **[4]**	oe, $x = (35\cos\theta)t$ $s = ut + \frac{1}{2}at^2$, $a = \pm g$, $u = 35\sin\theta$ /$35\cos\theta$ Correct equation Convincing with evidence, e.g $\dfrac{1}{\cos^2\theta} = 1 + \tan^2\theta$
(b)	(i) $20 = 100\tan\theta - \frac{100^2}{250}(1+\tan^2\theta)$ $2\tan^2\theta - 5\tan\theta + 3 = 0$ $(2\tan\theta - 3)(\tan\theta - 1) = 0$ $\tan\theta = \frac{3}{2}, 1$	M1 m1 A1	Correct use of $(100\mathbf{i} + 20\mathbf{j})$ i.e. $x = 100$, $y = 20$ An attempt to collect terms, form and solve a quadratic equation in $\tan\theta$. Both values, isw
	(ii) $0 = x(a) - \frac{x^2}{250}(1+a^2)$ $x = 125$ (or $x = \frac{1500}{13} = 115\cdot 38\ldots$) Shortest distance from F is $130 - 125 = 5$ (m)	M1 A1 **[5]**	Using $a = \tan\theta = 1$ **or** $\frac{3}{2}$ **and** $y = 0$ FT their $\tan\theta$ from (i) cao
	Total for Question 10	**9**	